Warped Models in String Theory

Roberto Valandro

SISSA/ISAS
Trieste (Italy)

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(Work in progress with B.S.Acharya and F.Benini)

- 5D warped models (in a slice of AdS_5) with bulk and boundary mass terms:
  - they address the hierarchy problem;
  - fermion zero modes are localised in the bulk;
  - Yukawa couplings.

$\Downarrow$

- String realisation of models presenting analogous characteristics.

Introduction

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- String realisation of models presenting analogous characteristics.

The Model

- $D3$-branes background $\Rightarrow$ warping and splitting of 10dim space;
- $D7$-branes with SYM theory living on the 8-dim worldvolume:
  - induced metric $\rightarrow$ warped product of (3,1) space and (4) euclidean space:

$$ds^2 \quad = \quad h(r)^{-1}\eta_{\mu\nu}dx^\mu dx^\nu + h(r)\,\delta_{\alpha\beta}\,dy^\alpha dy^\beta$$

  - turn on background gauge field, living only in (4)-space

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    EOM's of the 8dim warped theory gives 4dim gauge field:

$$*_4 F = -F$$

    $\Rightarrow$ Instanton anti-selfduality condition.
- Consider reduction of this theory to $(3,1)$ dimensions.

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Fermion Zero Modes

In the $D7$-brane spectrum $\rightarrow$ 8dim massless fermions:

$$\not{D}_8 \Psi = 0$$

Under the splitting it becomes:

$$(\not{D}_{3,1} + \not{D}_4) \sum_k \chi_k(x) \otimes \psi_k(y)$$

Written in terms of (flat) $\tilde{\not{D}}_{3,1}$ and $\tilde{\not{D}}_4$, and of the warp factor:

$$\not{D}_8 = h^{1/2} \tilde{\not{D}}_{3,1} + \frac{1}{h^{1/2}} \tilde{\not{D}}_4 - \frac{1}{4h^{1/2}} \frac{h'}{h} \gamma_r$$

Massless fermions in $(3,1)$dim $\leftrightarrow$ zero modes of $(\tilde{\not{D}}_4 - \frac{h'}{4h}\gamma_r)$:

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Fermion Zero Modes

The ψ_0's are fermion zero modes in an instanton background. In the 't Hooft solution, they are given by:

$$(v^\dagger bf)_h = \left[1 + \sum_{\ell=1}^{k} \frac{\rho_\ell^2}{(x - X_\ell)^2}\right]^{-3/2} \frac{\rho_h}{(x - X_h)^2} \times$$
$$\times \left\{ \left[1 + \sum_{\ell=1}^{k} \frac{\rho_\ell^2}{(x - X_\ell)^2}\right] \frac{\mathbf{x} - \mathbf{X}_h}{(x - X_h)^2} - \sum_{j=1}^{k} \frac{\rho_j^2}{(x - X_j)^4} (\mathbf{x} - \mathbf{X}_j) \right\}$$

In the limit of well separated k instantons, *i.e.* $(X_i - X_j)^2 \gg \rho_i \rho_j$:

$$(v^\dagger bf)_h \sim \frac{\rho_h}{(\rho_h^2 + (x - X_h)^2)^{3/2}} \frac{\mathbf{x} - \mathbf{X}_h}{|x - X_h|} .$$

$\rightarrow$ Localisation.

Normalisation → consider the 8dim kinetic term:

$$-\int d^8X\sqrt{-G}G^{\mu\nu}\bar{\Psi}\Gamma_\mu\partial_\nu\Psi + \ldots \tag{1}$$
$$= -\int d^4y\, h(y)^{1/2}\psi(y)^\dagger\psi(y)\int d^4x\,\eta^{\mu\nu}\bar{\chi}(x)\gamma_\mu\partial_\nu\chi(x) + \ldots$$

Canonical 4dim kinetic term implies:

$$d_\psi^2\int d^4y\, h(y)\psi_0(y)^\dagger\psi_0(y) = 1 \tag{2}$$

$$\Downarrow$$

- If X_h in region of negligible warping, then $d_\psi \sim 1$.
- If X_h in region of large warping, then $d_\psi \sim h(\rho_h)^{-1/2}$.

⇒ Possible large suppression.

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$\Rightarrow$ Possible large suppression.

Yukawa couplings

Generally they are given by product of zero modes and warp factor (localised functions):

$$\lambda_{ij} \sim \int d^4y\, h(y)^\beta\, \psi_i^\dagger(y)\Phi(y)\psi_j(y)\,.$$

$\Rightarrow$ Varying instanton parameters (relative positions and sizes) can give suppression and so Yukawa hierarchy.

Proposal: 8dim Kinetic term gives a scalar-(fermion)^2 $(3,1)$-dim term:

$$g\int d^8X\sqrt{-G}\,\bar\Psi\,\not{\delta A}\,\Psi \quad\supset\quad \int d^4x\,\bar\chi_i(x)\chi_j(x)H_k(x)\times g\int d^4y\,\psi_i^\dagger(y)\,\not{\delta a}_k(y)\psi_j(y)$$

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Yukawa couplings

One can do approximate integrals in different asymptotic regions of the parameters space:

LIMIT	$g\int d^4y\, \psi_i^\dagger(y)\, \not{a}_k(y)\psi_j(y)$	d_ψ^2
$\rho_H \sim \rho_1 \ll X$	$\frac{g}{\rho_H^2}\left(\frac{\rho_H}{X}\right)^3$	$\left(\frac{\rho_H}{L}\right)^2$
$\rho_H \ll \rho_1 \sim X$	$\frac{g}{\rho_H^2}\left(\frac{\rho_H}{X}\right)^2$	$\left(\frac{\rho_1}{L}\right)^2$
$\rho_H \ll \rho_1 \ll X$	$\frac{g}{\rho_H^2}\left(\frac{\rho_H}{X}\right)^2\left(\frac{\rho_1}{X}\right)^2\left[1+\frac{X}{\rho_1}\left(\frac{\rho_H}{\rho_1}\right)^2\right]$	$\left(\frac{\rho_1}{L}\right)^2$
$\rho_H \ll X \ll \rho_1$	$\frac{g}{\rho_H^2}\left(\frac{\rho_H}{\rho_1}\right)^4\left[1+\left(\frac{X}{\rho_H}\right)^2\left(\frac{X}{\rho_1}\right)^2\right]$	$\left(\frac{\rho_1}{L}\right)^2$
$\rho_H \sim X \ll \rho_1$	$\frac{g}{\rho_H^2}\left(\frac{\rho_H}{\rho_1}\right)^4$	$\left(\frac{\rho_1}{L}\right)^2$
$X \ll \rho_H \sim \rho_1$	$\frac{g}{\rho_H^2}$	$\left(\frac{\rho_H}{L}\right)^2$

Conclusions

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- Realisation of the fermion masses hierarchy $\longrightarrow$ Instanton parameters & warp factor can give suppression or enhancement of Yukawa coupling.
- The most urgent *open problem*:
  - Higgs nature and top Yukawa coupling.

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