Instability of a falling jet of concentrated suspensions

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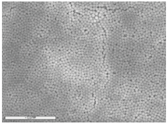
Outline

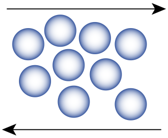
1. Introduction : shear thickening
    - Shear thickening in extension
    - Arrested state life time
2. Falling jet of a suspension : solid-like regime
    - Jet shape
    - Weight loss
    - Transverse waves propagation
3. Conclusion

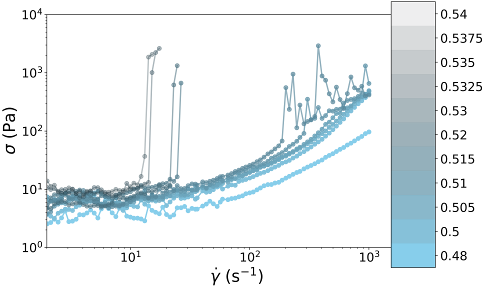
Shear thickening of concentrated suspensions

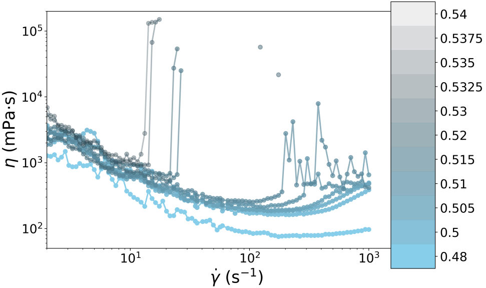
- d=2 μm silica particles,
- water pH=7



- Cone-plate
- Controlled shear rate







Shear thickening of concentrated suspensions

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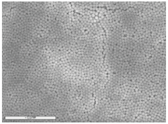

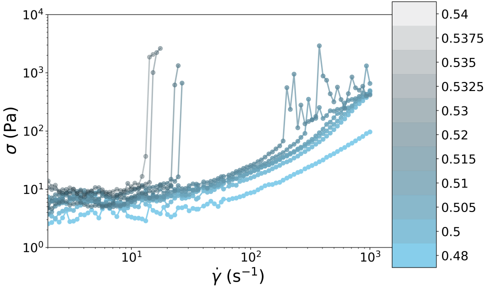

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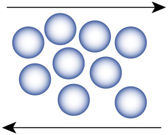
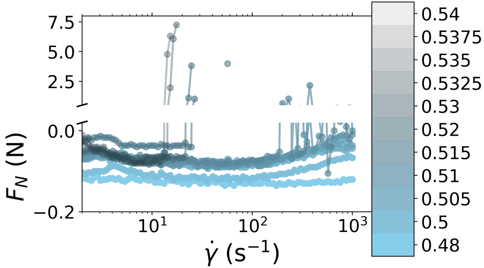
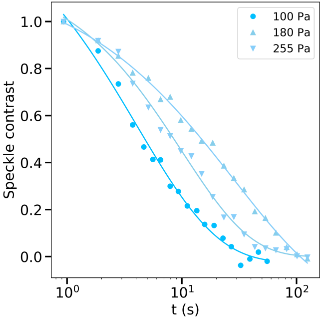
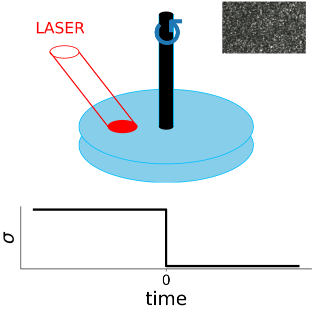




Life time of the arrested state





- rough silica particles :
  - diameter : 650 nm
  - roughness : $\sqrt{\langle \delta r^2 \rangle} = 6.2$ nm

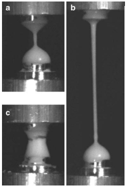
F.Ianni *et al P.R.E.*, 2006

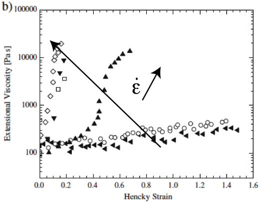
- Mustispeckle Diffusing Wave Spectroscopy
- Relaxation of the contrast of the speckles after a stress step

Shear thickening under extension

Suspensions of corn starch particles

- $\dot{\epsilon}$ imposed
- Force and filament widths measured

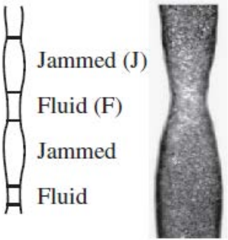




$\phi = 55\ \%$

E. E. Bischoff White, M. Chellamuthu, J.P. Rothstein, *Rheol. Acta*, 2010

Observation of the extended filament



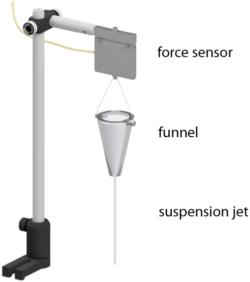
M. Roché, H. Kellay, H.A. Stone, *P.R.L.*, 2011

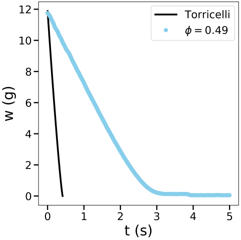
- pulling a column of sample
- heterogeneities : jammed regions linked with liquid bridges

Falling jet of a suspension

How does the suspension flow under its own weight ?
If g is sufficient :

- Flow under $g \longrightarrow$ shear-thicken
- Flow stops $\longrightarrow$ Relaxes back to the liquid state





Acquisition rate 500 im/s

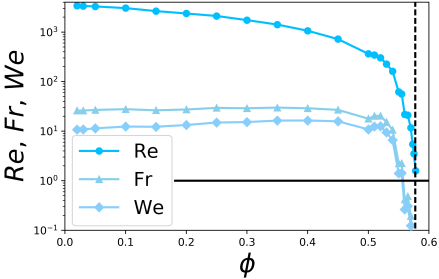
Non dimensional numbers

Flow regimes

$$Re = \frac{\text{Inertia}}{\text{Viscosity}} = \frac{\rho U a}{\eta}$$

$$Fr = \frac{\text{Inertia}}{\text{Gravity}} = \frac{U^2}{ga}$$

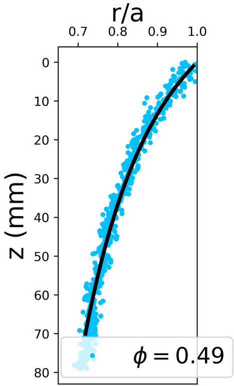
$$We = \frac{\text{Inertia}}{\text{Surface}} = \frac{\rho U^2 a}{\gamma}$$



- small ϕ : $Re, Fr, We \gg 1$: inertial regime,
- $\phi \longrightarrow \phi_m$: surface effects

Inertial regime

Shape of the jet



Assuming a perfect fluid :

- Bernoulli's theorem
- Laplace pressure

$$\frac{r(z)}{r_0} = \left[1 + \frac{2}{Fr}\frac{z}{a} + \frac{2}{We}\left(1 - \frac{a}{r(z)}\right)\right]^{-1/4}$$

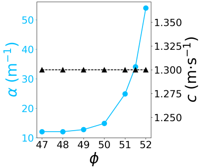
$$= \left[1 + \frac{2}{Fr}\frac{z}{a}\right]^{-1/4}$$

Capillary regime

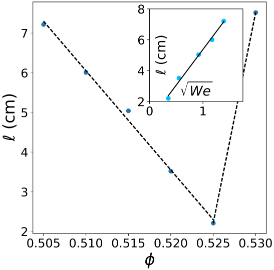
Growth of the capillary wave amplitudes :

$$r(z) = r_0 + \epsilon e^{\alpha z}$$

- Constant celerity $c = v_{jet}$
- Decreasing α^{-1}



$$L \propto \frac{1}{\alpha} = \frac{U}{\sqrt{\Omega_c}} \propto \sqrt{We}$$



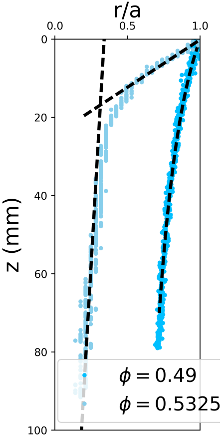
At high ϕ, the jet ruptures at longer length.

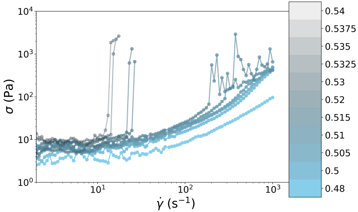
Jet shape in the "solid-like" regime

Jet instability

speed$\approx 1/10$
$\phi = 0.535$

Average jet shape

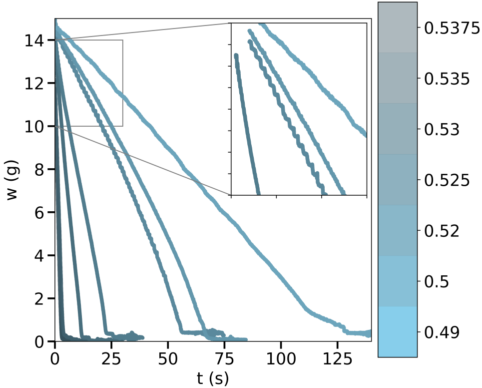


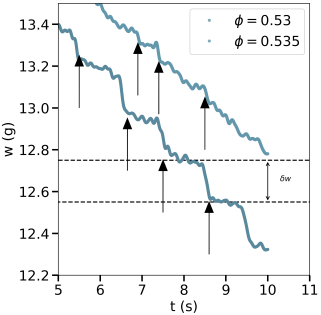


At the output of the funnel :
$\dot{\epsilon} \approx 10^2 \ s^{-1}$
$1 \gg We, Fr$: inertia irrelevant

- Rapid decrease of the jet radius at the output of the funnel,
- Thinner jet, smaller radius decrease

Weight loss in the "solid like" regime

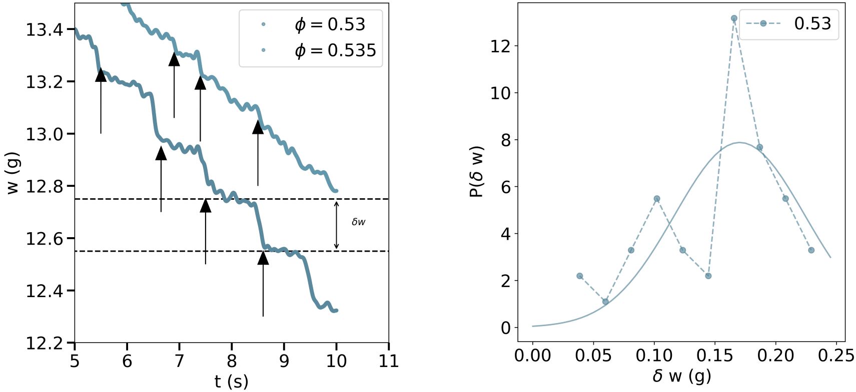




At $\phi > 53\ \%$:

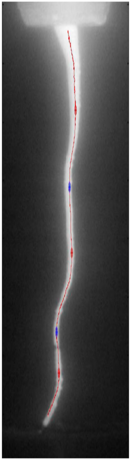
- the weight of the entire column is measured : tensile stress is transmitted
- well defined weigth scales

Weight loss in the "solid like" regime



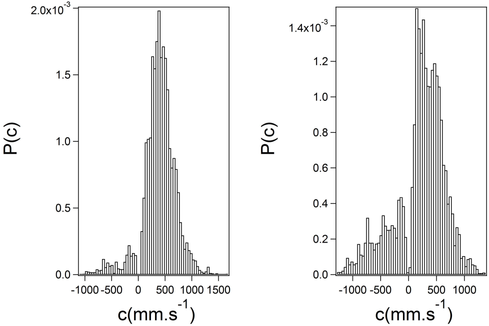
$\langle \delta w \rangle = 0.17\ g$ lineic mass : $4.7 \cdot$ g$\cdot$m^{-1} $\longrightarrow \delta\ell = 3.4$ cm

Wave propagation : solid rope model



Identify and follow maxima

Distribution of velocities



$\phi = 53.25$ %, $\phi = 53.5$ %

Superposition of up and down travelling waves

$c_{max} \approx 0.5\mathrm{m} \cdot \mathrm{s}^{-1}$ whereas $v_{jet} \approx 0.1 m \cdot s^{-1}$

Wave propagation along the rope

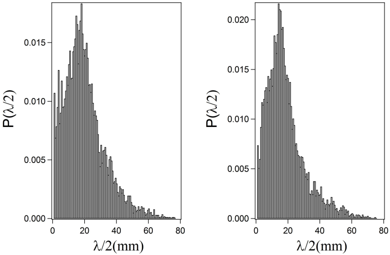
Wave propagation along a solid rope

$$\frac{\partial^2 x}{\partial t^2} = \frac{T(x)}{\mu}\frac{\partial^2 x}{\partial z^2}$$

$$c = \sqrt{\frac{T}{\mu}} = \sqrt{Lg} \approx 1\mathrm{m}\cdot\mathrm{s}^{-1}$$

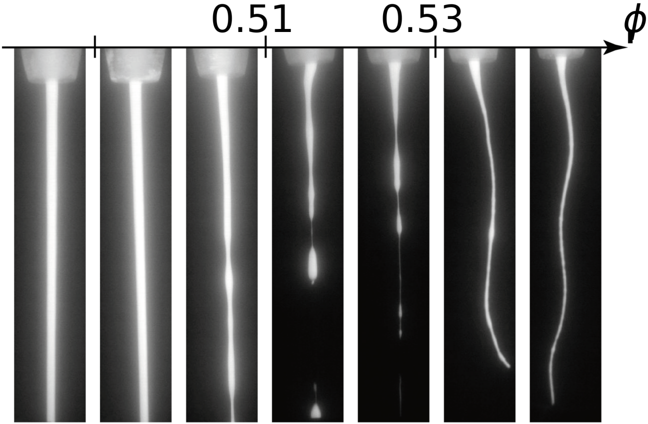
We observe :
$A(x,t) = \sum A_i \cos(\omega t \pm kx)$
Assuming one wave travelling up and one down :
Reflexion coeff : $R \approx 0.5$

Distribution of wavelengths



$\phi = 53.25$ %, $\phi = 53.5$ %
$\lambda_{max} = 20$ mm

Conclusion



Conclusion

At the shear thickening transition **under gravity** :

- the jet is heterogeneous
- it sustains tensile stress
- it lets solid chunks fall
- becomes radially unstable