Granular dispersion rheology as constraint counting

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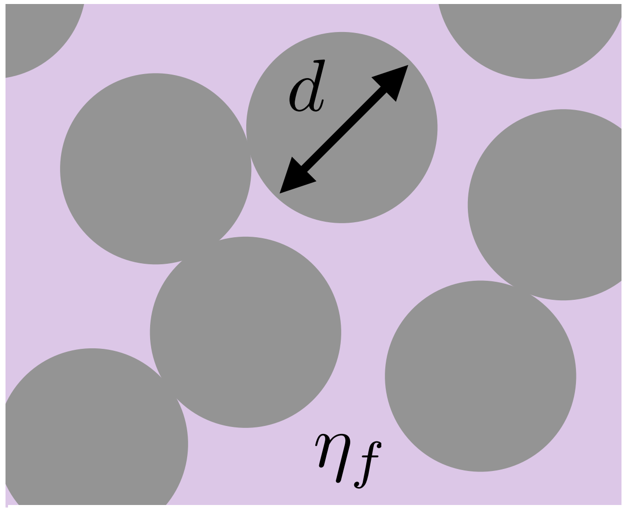
and W. C. K. Poon

Granular dispersions: simple, right?

- Hard particles with $d \gtrsim 2\ \mu\mathrm{m}$
- Non-Brownian ($\mathrm{Pe} \to \infty$)
- Viscous flow (negligible particle and fluid inertia)

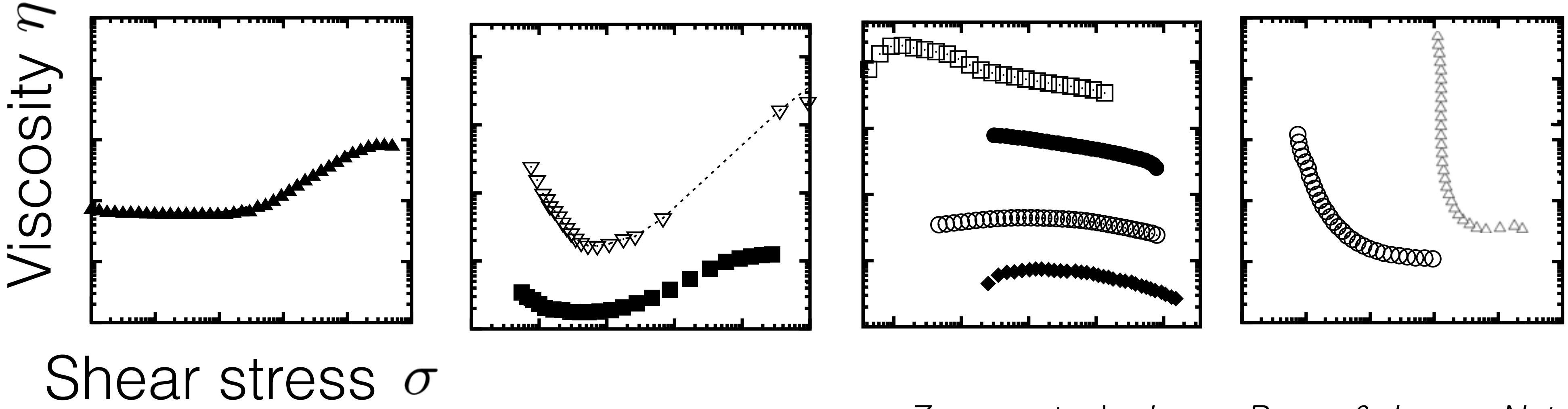


<u>Expectation</u>: universal, *Newtonian* rheology:

Dimensional analysis $\implies$ $\eta = \eta_f f(\phi) \propto \dot{\gamma}^0$

Reality:

Experimental phenomenology is *capricious*



Guy et. al., *PRL (2016)*

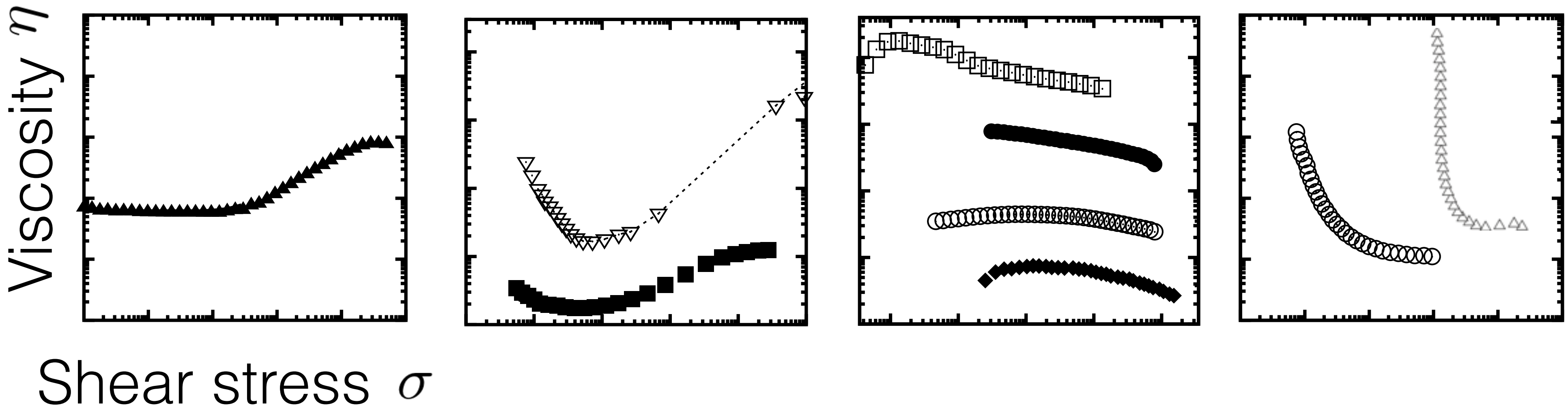
Zarraga et. al., *J. Rheol. (2000)*

Gamonpilas et. al., *J. Rheol. (2016)*

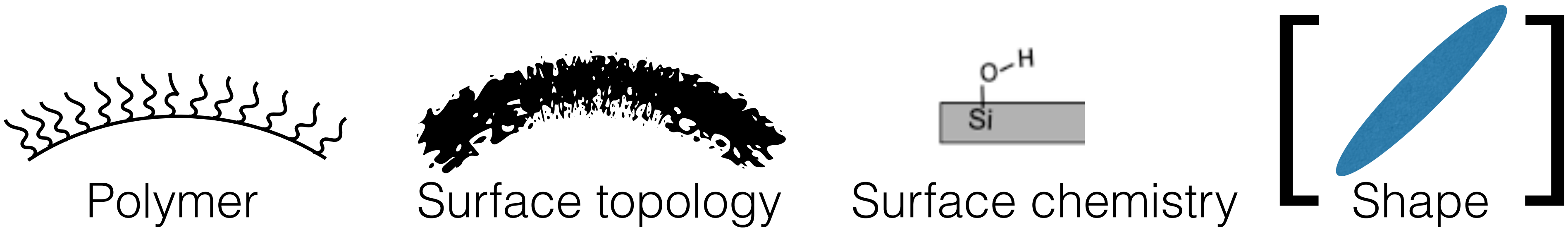
Brown & Jaeger, Nat. Mater. (2010)

Reality:

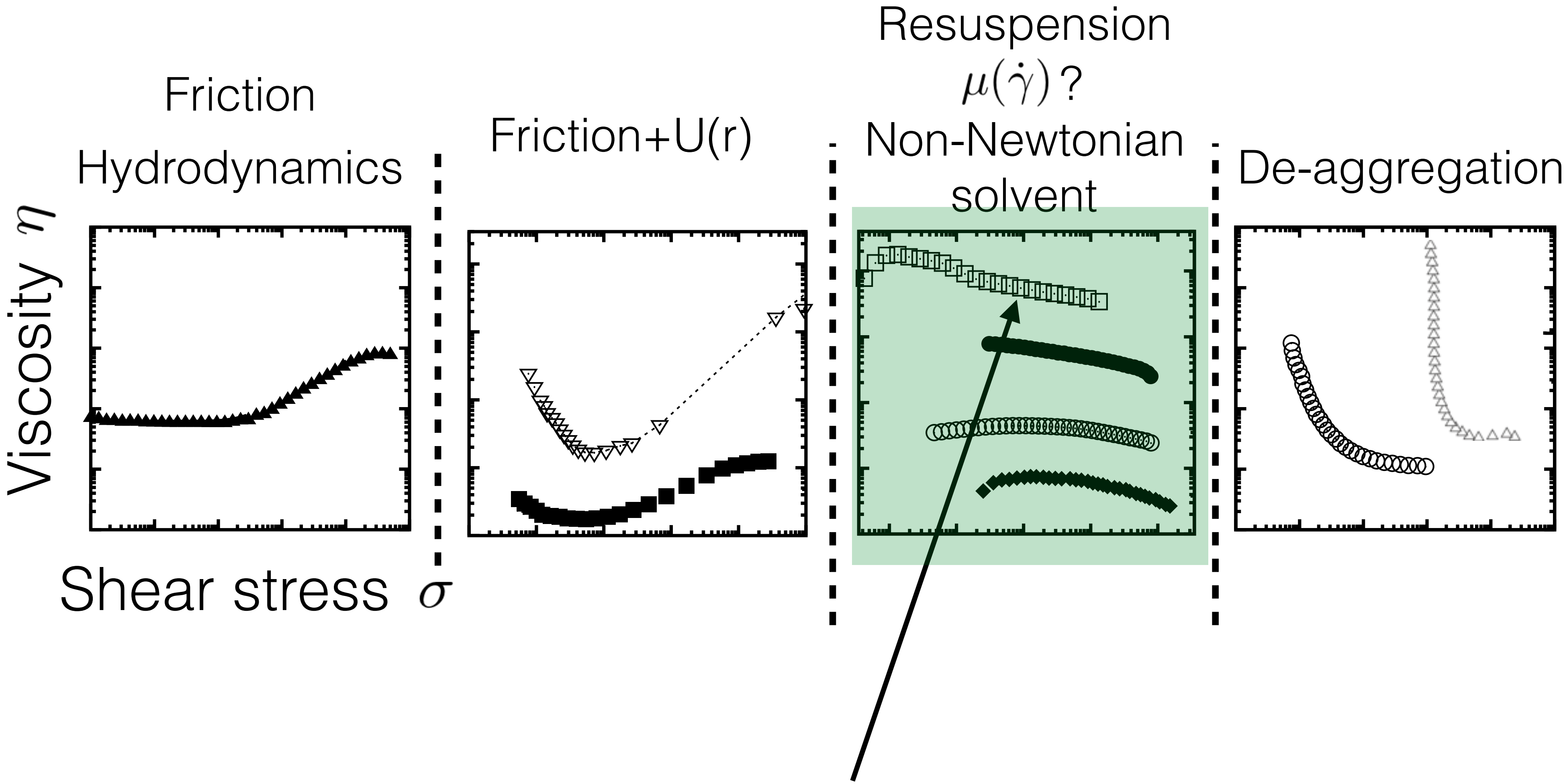
Experimental phenomenology is *capricious*



Particle-level details are important

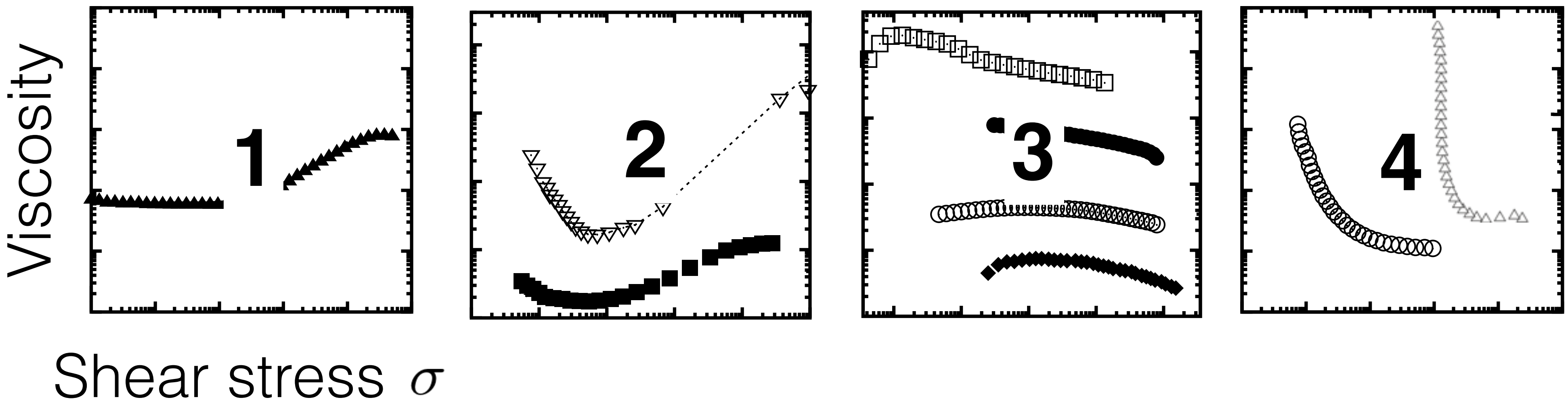


In most cases, explanations are bespoke



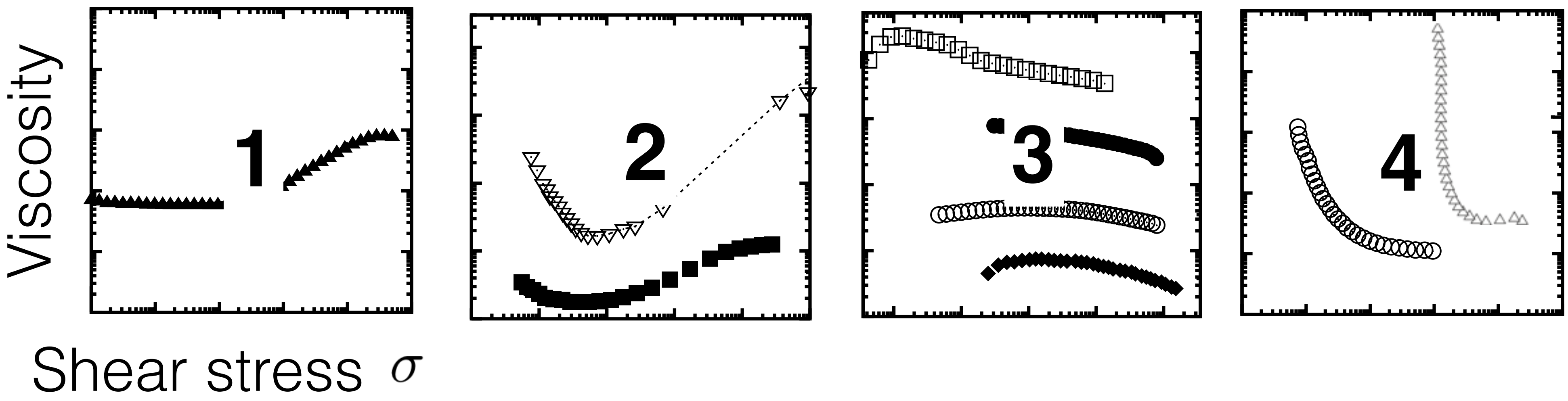
… and highly debated

Is there a generic, underlying physics?



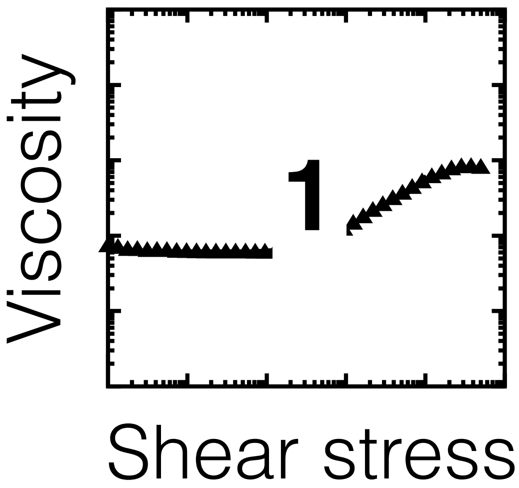
Connection?

Granular dispersion rheology is about making and breaking of ***constraints*** with stress



Details —> $\mathcal{Z}(\sigma)$

Wyart and Cates theory: constraint-driven version



Borrow ideas from dry granular packings

Reformulated phenomenological WC theory (3-d)

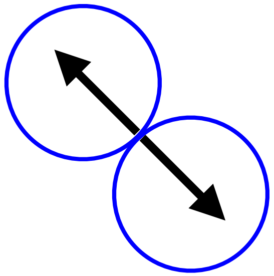
Isostaticity:

Minimum number of contacts per sphere $\mathcal{Z}$ for mechanical stability

force/torque balance **equations** per particle = # force/torque **degrees of freedom** per particle

Frictionless

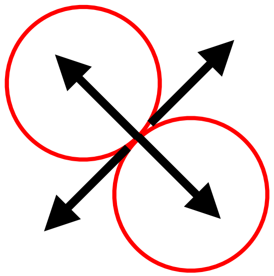
$\mu = 0$



$\mathcal{Z} = 6$

Frictional

$\mu > 0$



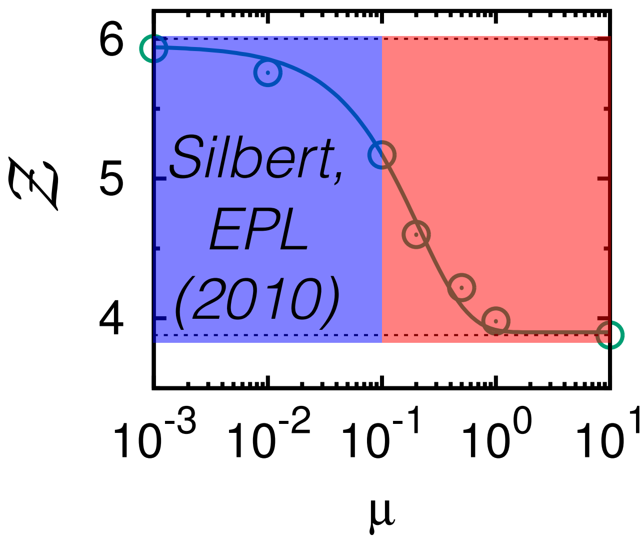
$\mathcal{Z} = 4$

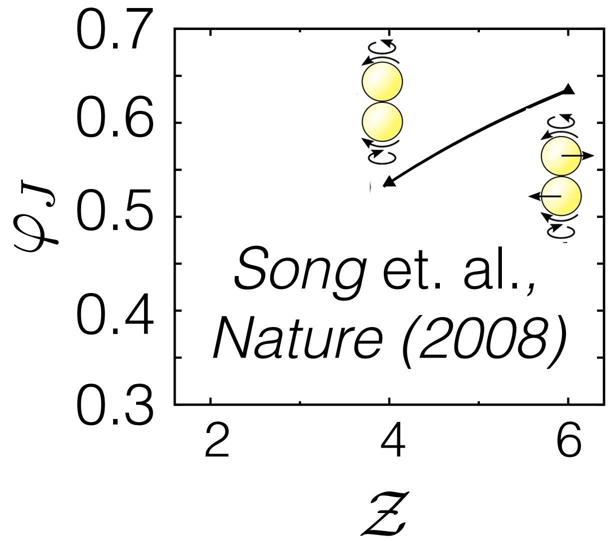
Constrains sliding

Reformulated phenomenological WC theory (3-d)

Frictionless Frictional

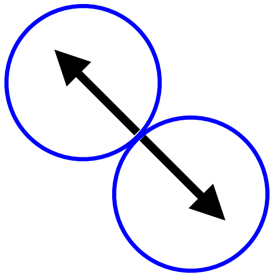
In practice:





Frictionless

$\mu \ll 0.1$

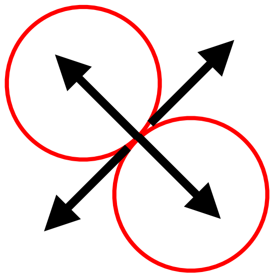


$Z \approx 6$ $\rightarrow \varphi_J^{(6)} \approx 0.64$

(RCP)

Frictional

$\mu \gg 0.1$



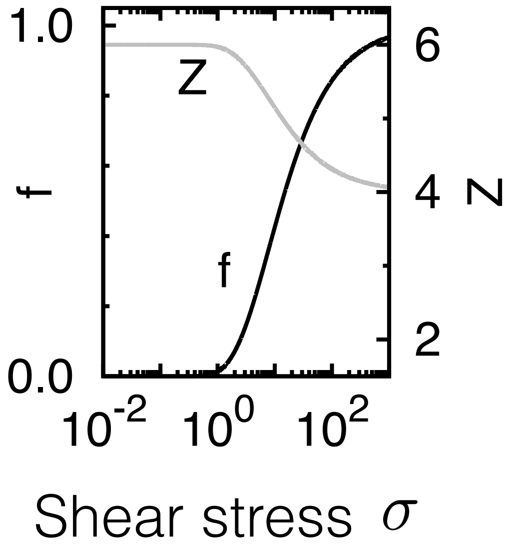
$Z \approx 4$ $\rightarrow \varphi_J^{(4)} \approx 0.54$

(RLP)

Dry packing → Sheared suspension

Stress-dependent isostatic coordination number

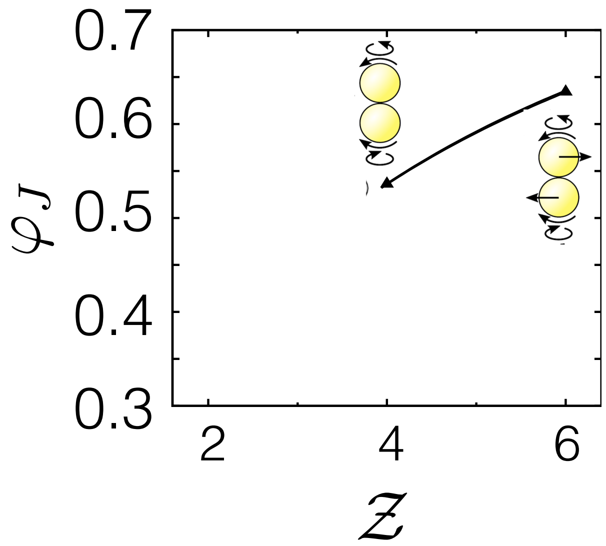
$\mathcal{Z}(\sigma)$



$$\mathcal{Z} = 6 - 2f$$

$$f(\sigma) = e^{-(\sigma^*/\sigma)^{\beta}}$$

$\phi_{\mathrm{J}}(\mathcal{Z})$



$$\phi_{\mathrm{J}} = \frac{\mathcal{Z}}{\mathcal{Z} + C}$$

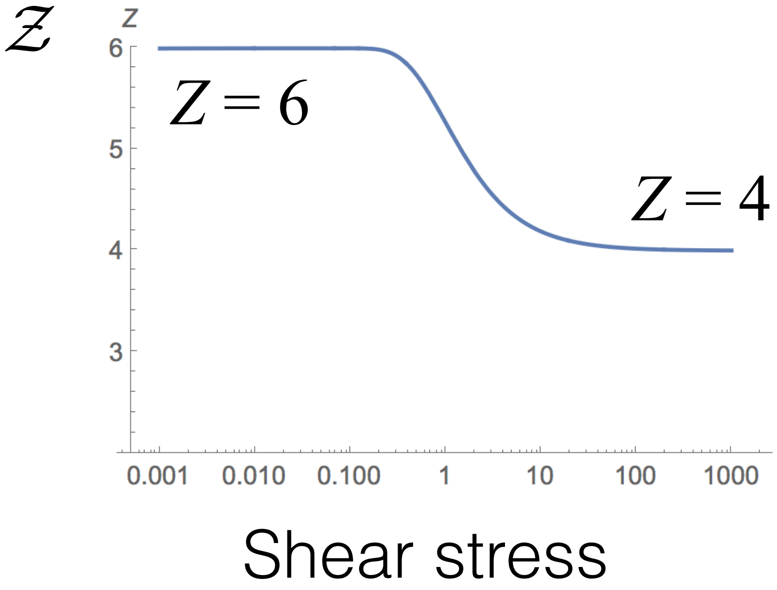
$= 2\sqrt{3}$ for monodisperse spheres

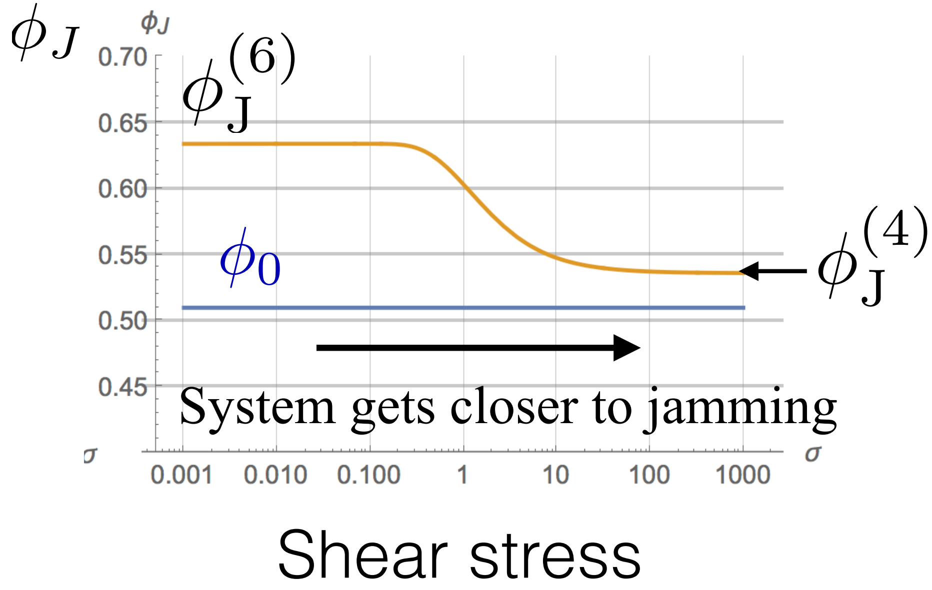
$\eta(\phi_{\mathrm{J}}(\mathcal{Z}))$

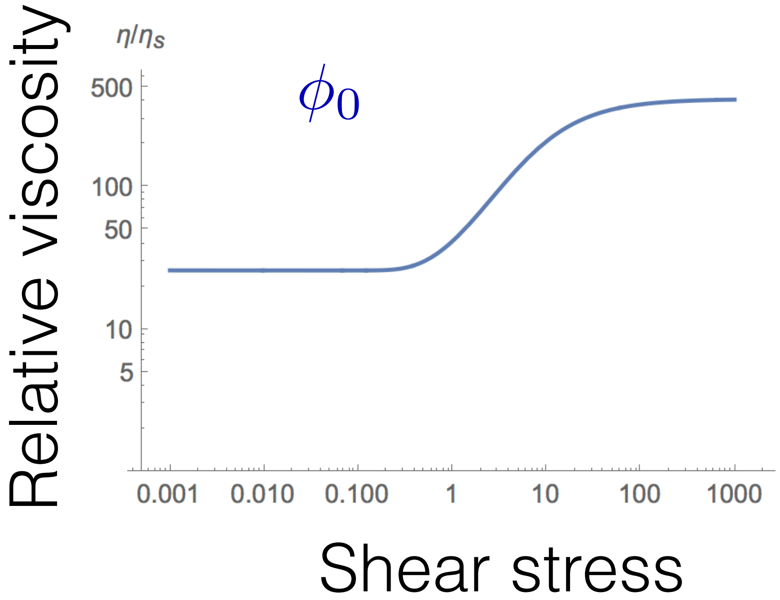
$$\eta = \left(1 - \frac{\phi}{\phi_J}\right)^{-2}$$

$$\eta(\sigma, \phi; \beta, \sigma^*, C)$$

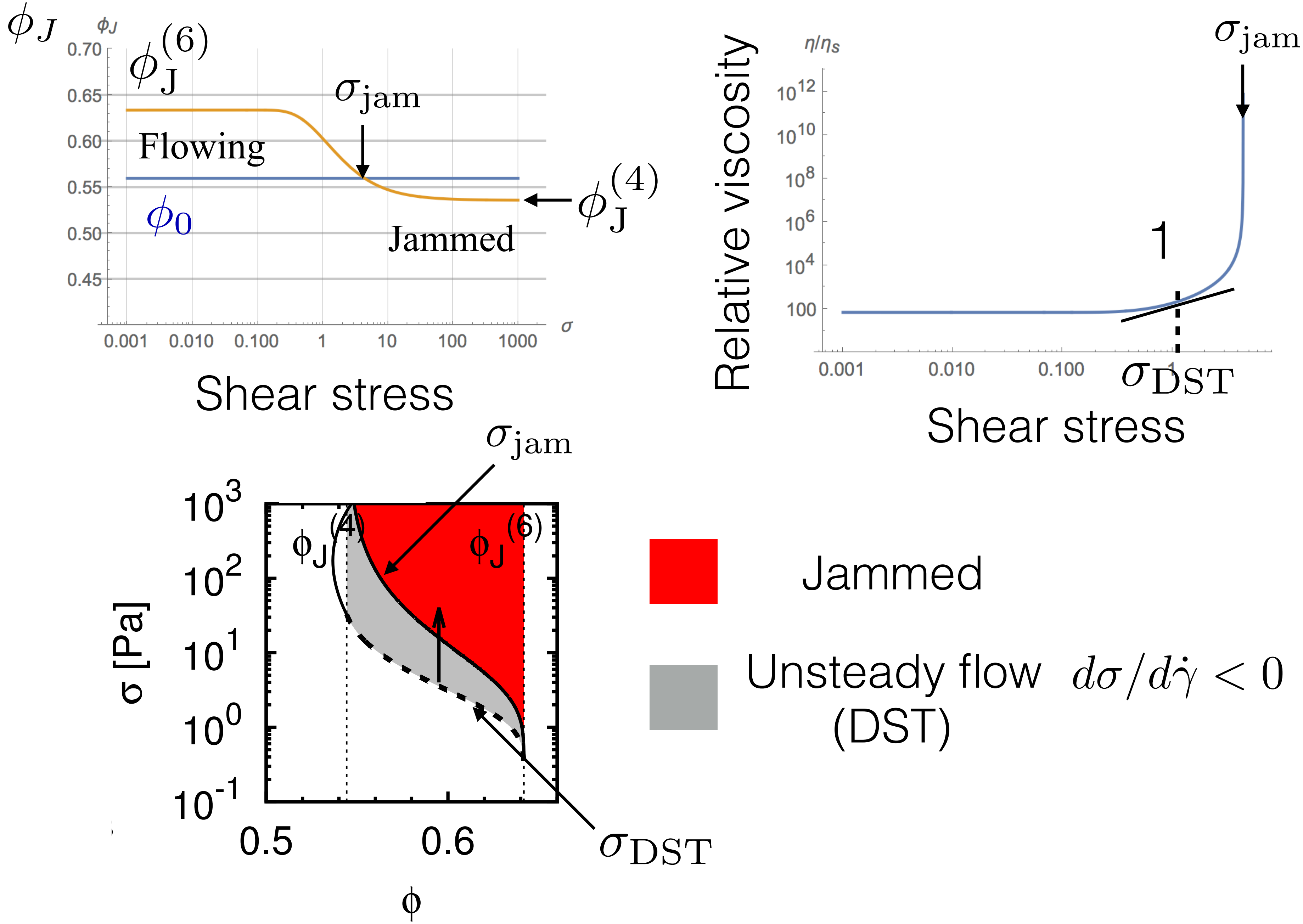
Walkthrough: $\phi_0 < \varphi_J^{(4)}$



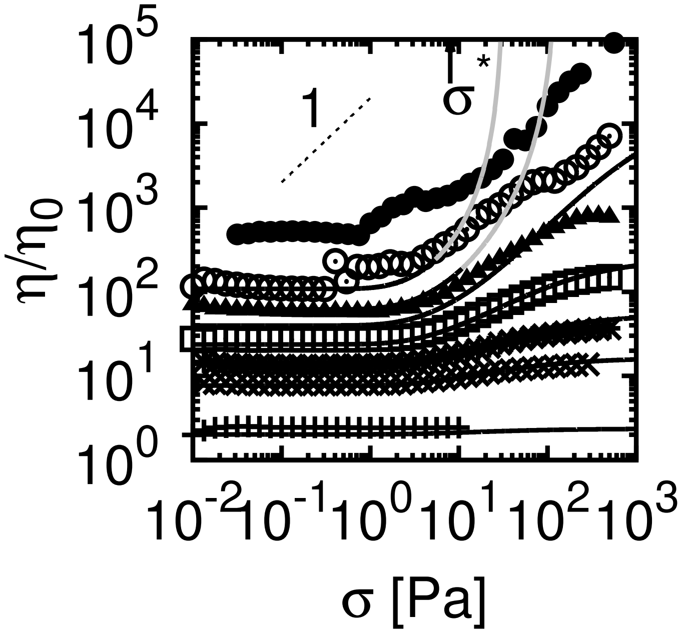




Walkthrough: $\phi_0 \geq \varphi_J^{(4)}$



Example: 4 micron PMMA in CXB+decalin



Guy, Hermes and Poon, PRL (2015)

✓ Quantitatively captures experimental phenomenology

Real particles are usually sticky!

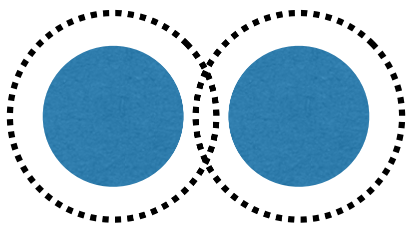
(e.g., due to van der Waals interactions)

BUT

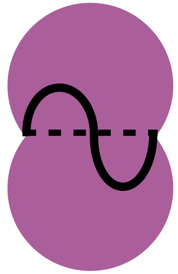
Attraction resulting from a central potential

$U_{\mathrm{vdW}}(r)$ *does not* constrain rotations

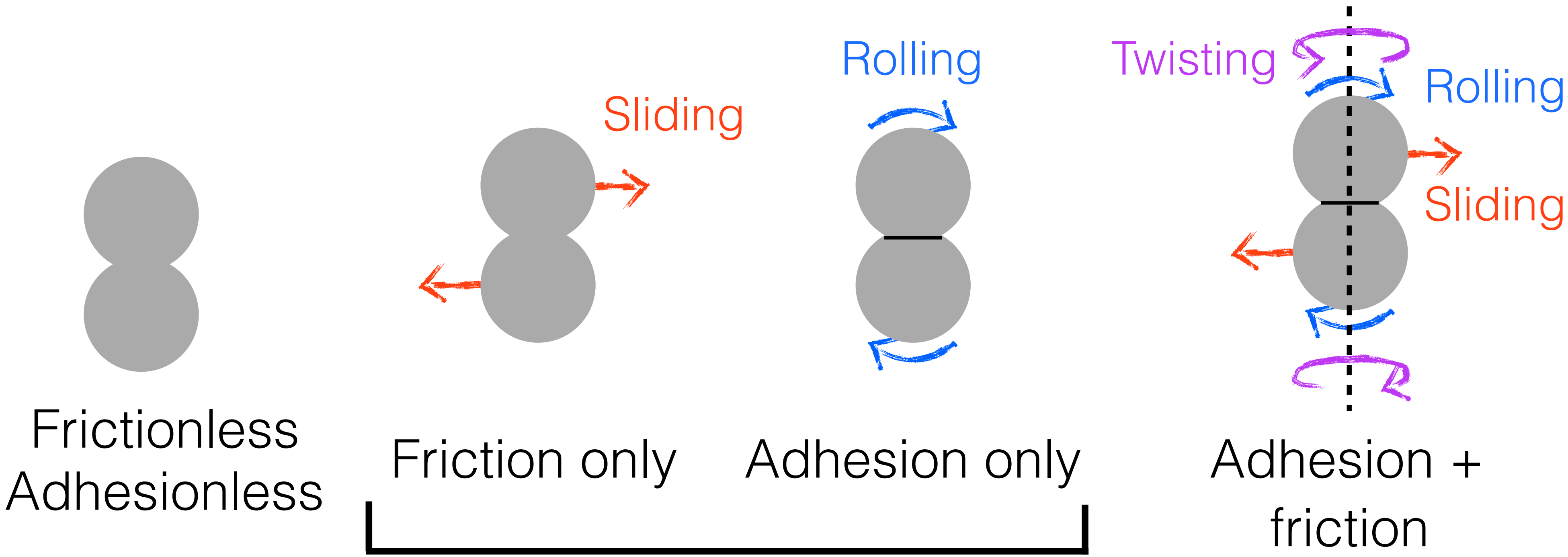
$\Longrightarrow \mathcal{Z}$ unaffected



Adhesion: rotations cost energy



Literature unclear for friction + adhesion. We propose:



$$\mathcal{Z} = 6 \qquad \mathcal{Z} = 4 \qquad \mathcal{Z} = 2$$

$$\varphi_{\mathrm{J}}^{(6)} \qquad \varphi_{\mathrm{J}}^{(4)} \qquad \varphi_{\mathrm{J}}^{(2)}$$

WC-like theory

Fraction of adhesive contacts $a(\sigma) = 1 - e^{-(\sigma_A/\sigma)^\kappa}$

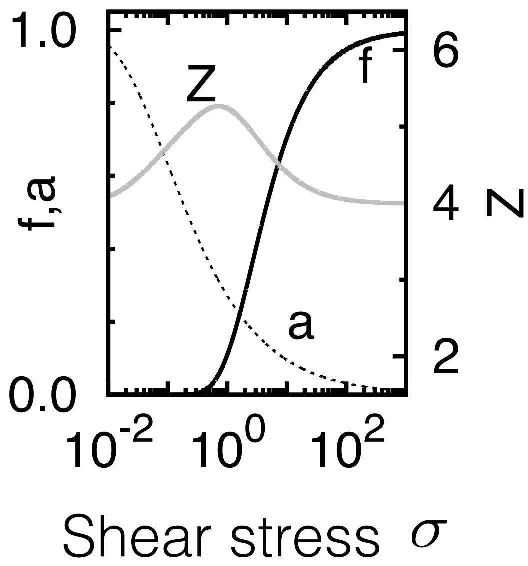
$\sigma_A = \text{Characteristic adhesive stress}$

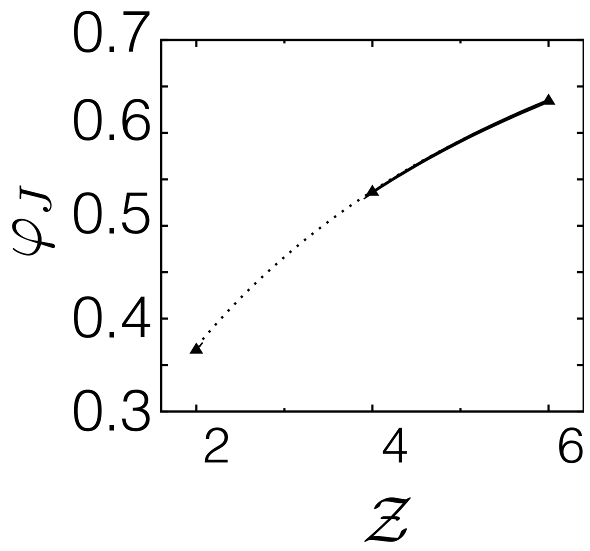
Fraction of frictional contacts $f(\sigma) = e^{-(\sigma^*/\sigma)^\beta}$

$$\mathcal{Z} = 6 - 2f - 2a$$

$$\phi_\text{J} = \frac{\mathcal{Z}}{\mathcal{Z} + C}$$

$$\eta = \left(1 - \frac{\phi}{\phi_J}\right)^{-2}$$





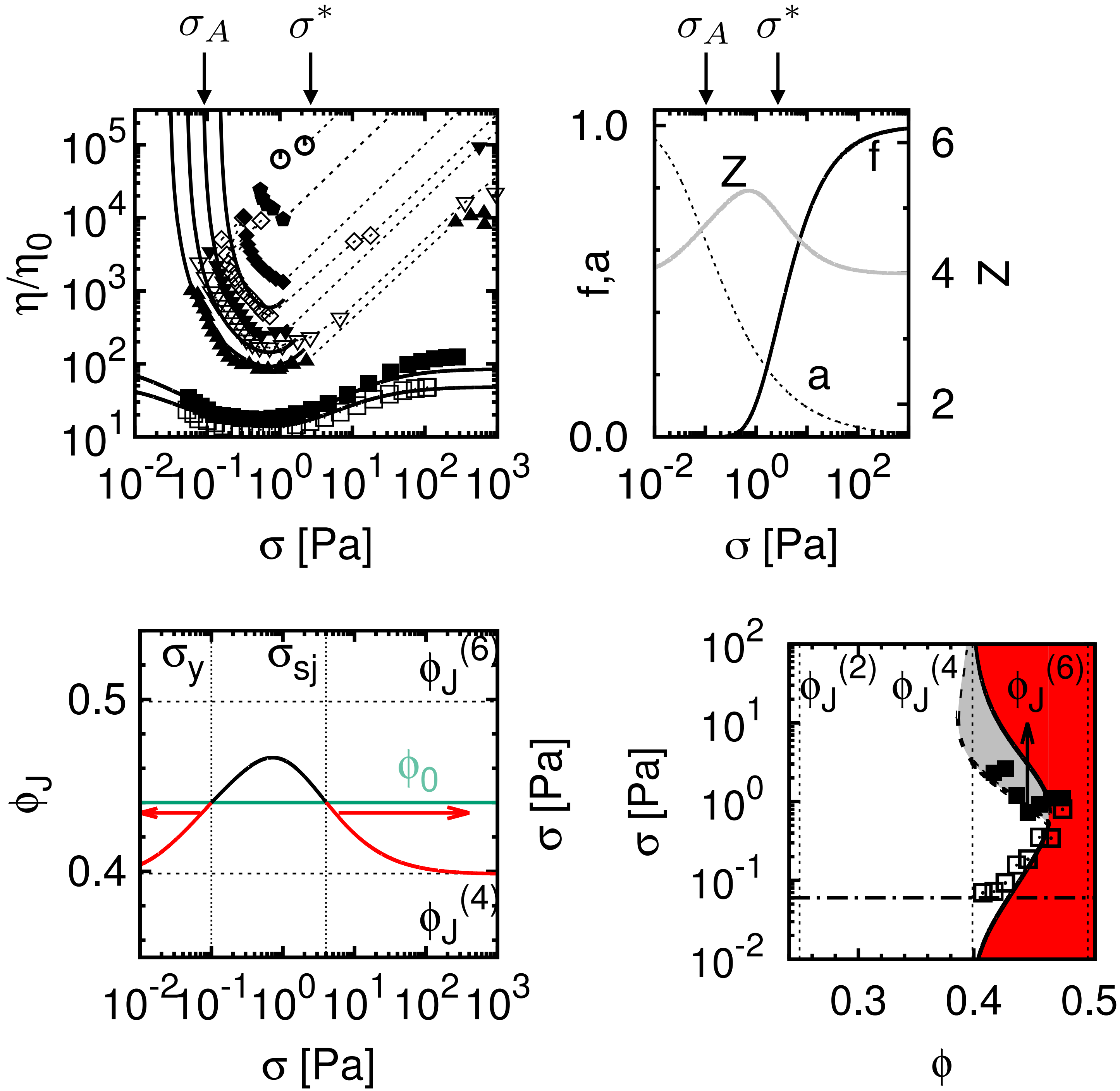
Song et. al., *Nature (2008)*

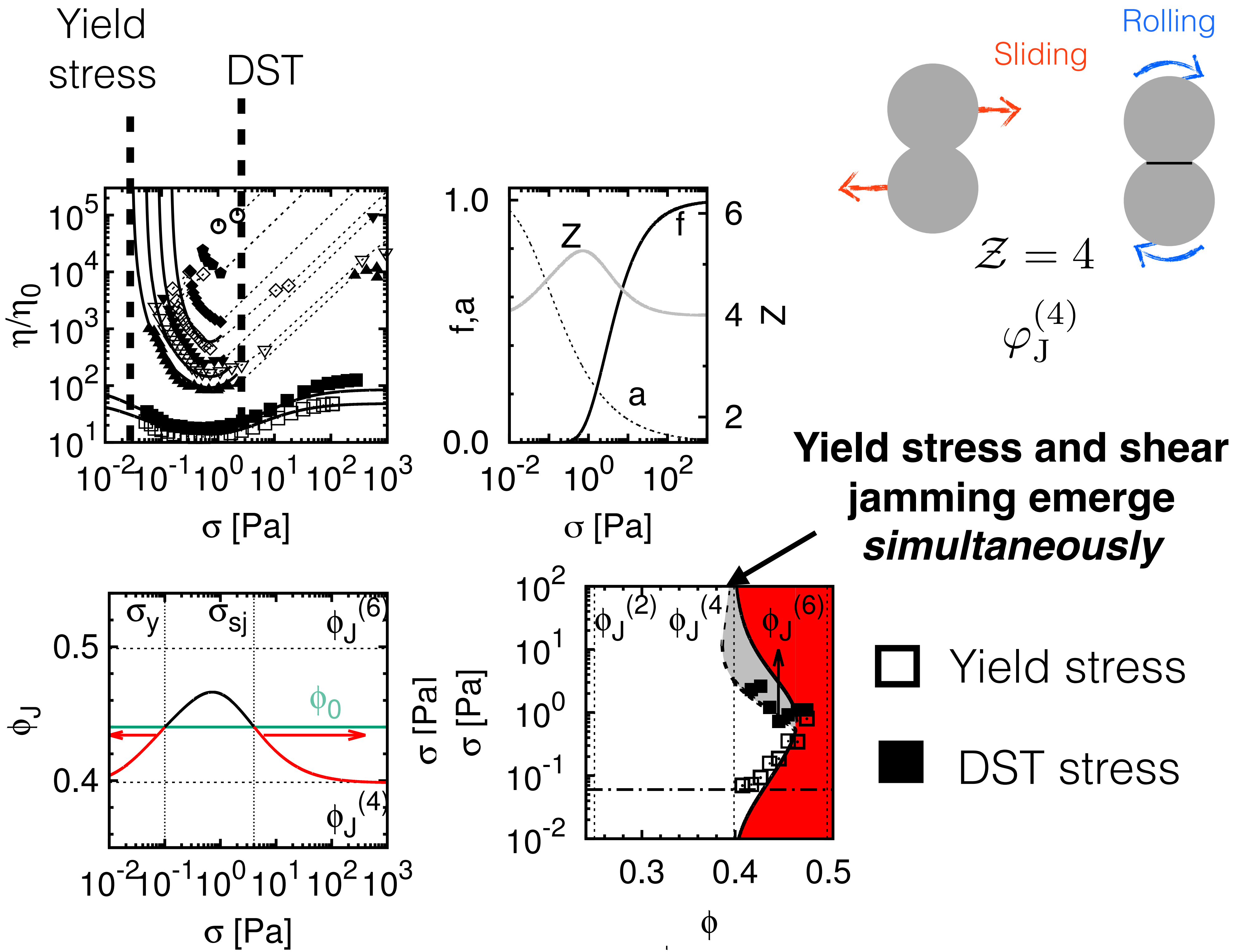
Details depend on

σ_A/σ^*

κ/β

Case 1: $\sigma_A/\sigma^* \ll 1$ (Cornstarch in glycerol/water)

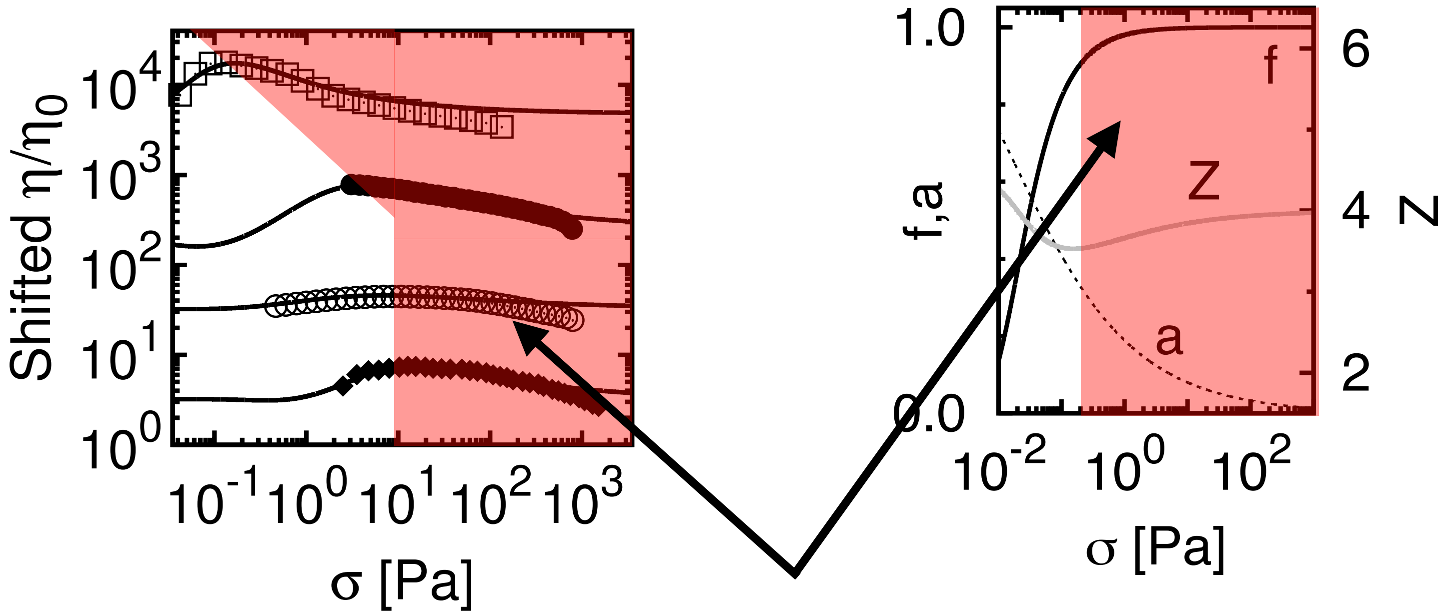




Yield stress
DST
Sliding
Rolling
Z = 4
φJ(4)
η/η0
σ [Pa]
f,a
Z
f
a
Z
Yield stress and shear jamming emerge *simultaneously*
φJ
σy
σsj
φJ(6)
φ0
φJ(4)
σ [Pa]
σ [Pa]
φJ(2)
φJ(4)
φJ(6)
φ
Yield stress
DST stress

Case 2: $\sigma_A \approx \sigma^*$ (various systems)

Predicts *peaked* flow curves!



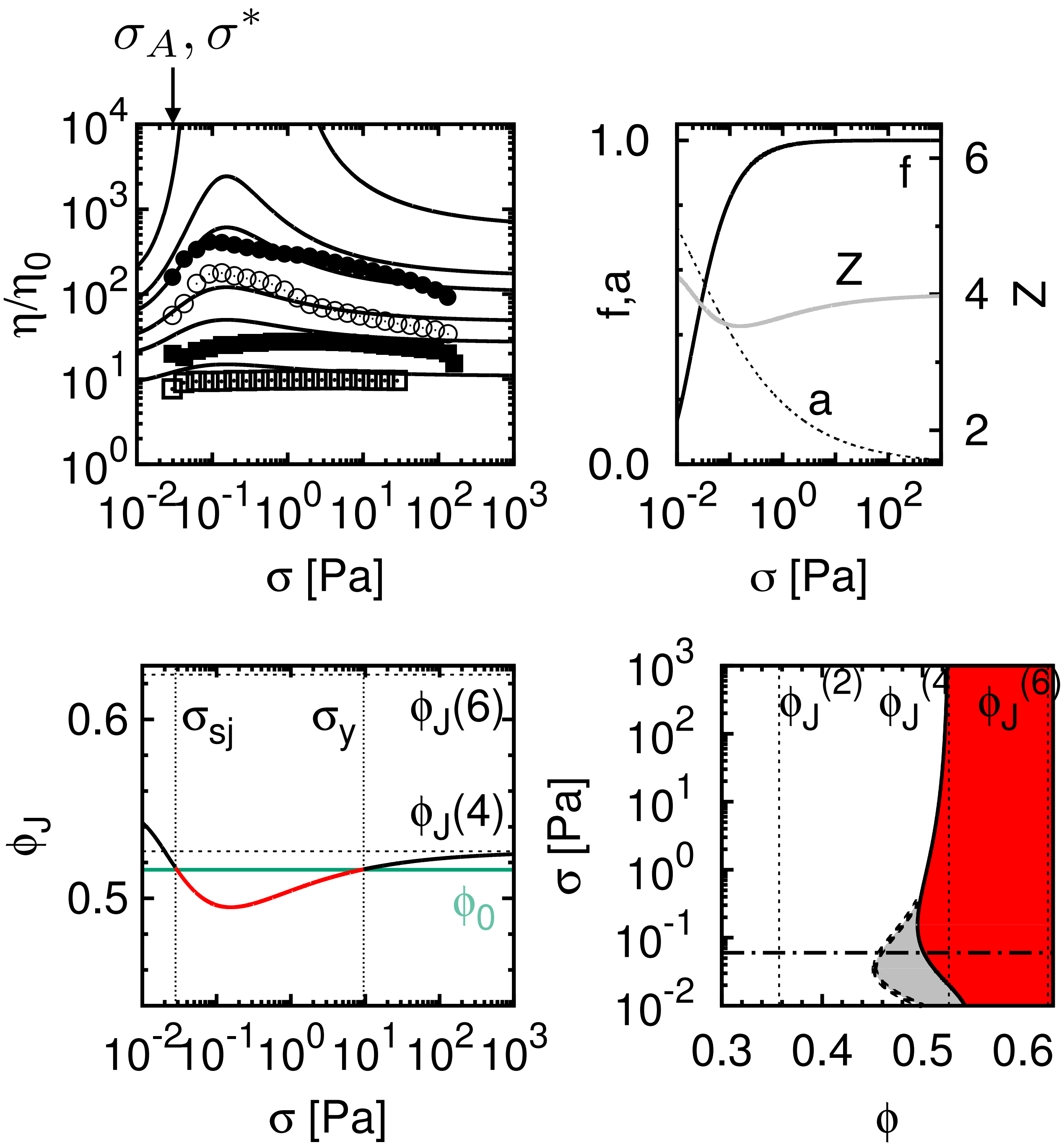
Gradual breakage of adhesive constraints

Exact form sensitive to σ_A/σ^* and κ/β

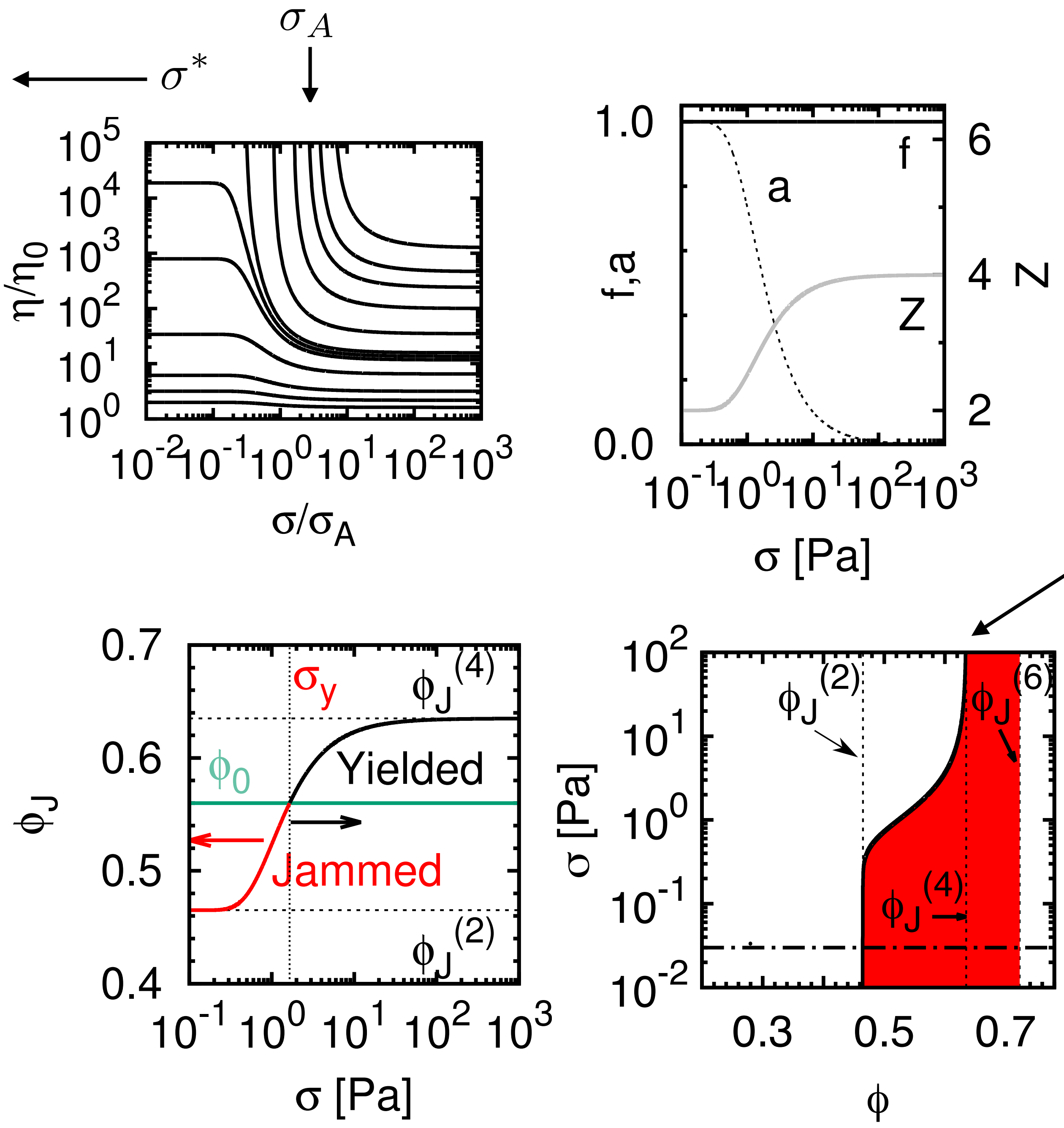
(and shear history)

Case 2: $\sigma_A \approx \sigma^*$ (sterically-stabilised 45 μm PMMA in hydrocarbons)

$\kappa \ll \beta = 1$

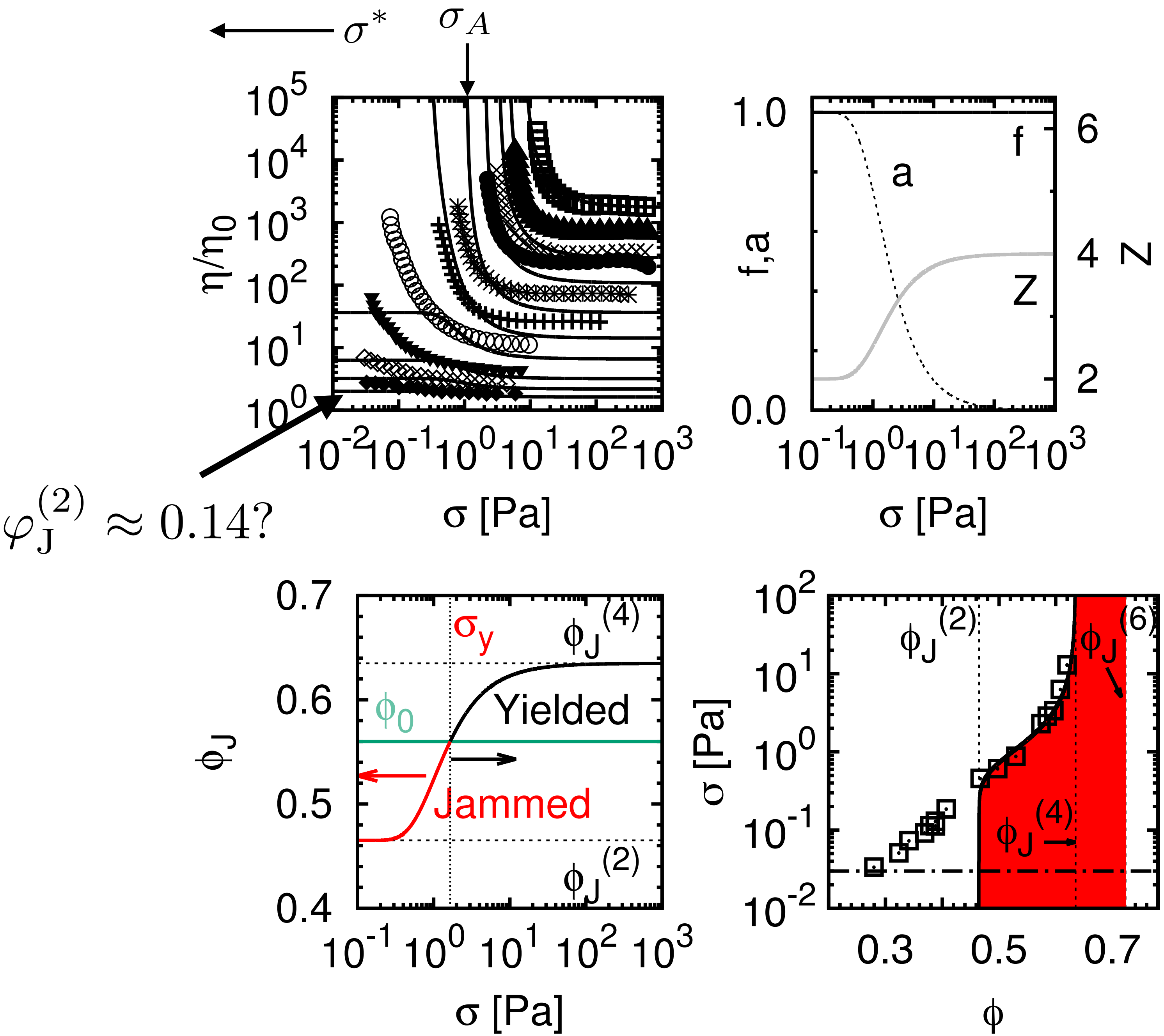


Case 3: $\sigma_A/\sigma^* \gg 1$



Yield stress diverges at $\varphi_J^{(4)}$

Case 3: $\sigma_A/\sigma^* \gg 1$ (Cornstarch in sunflower oil + lecithin)

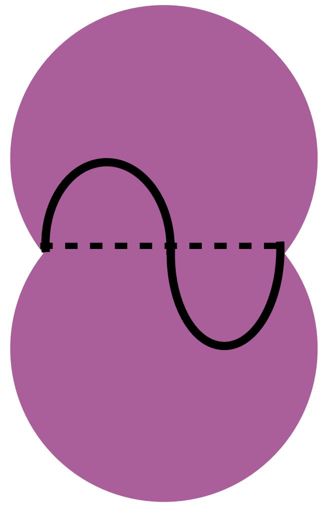


Derived for $4 \leq Z \leq 6$

$$\phi_J = \frac{\mathcal{Z}}{\mathcal{Z}+C}$$

Dubious validity for $Z < 4$!

Open questions



- σ_A = stress to break adhesive bonds

  e.g., JKR+``pinning" ⟶ Bonds break by ``peeling"

  Dominik and Tielens, Phil. Mag. A (1995)

- Z provides a "common language" for tribologists and rheologists — how we think about details.

- Hydrodynamics and timescales: implications for $\mathcal{Z}$

  $\dot{\gamma} > 0 \implies$ hydrodynamic forces and torques

  Standard Reynolds lubrication $\rightarrow \mathcal{Z} = 6$

Thank you for your attention!

Edinburgh team

- J. A. Richards **(poster)**
- D. Hodgson
- E. Blanco
- W. C. K. Poon

Funding

AkzoNobel

EPSRC