Topological Phenomena in Periodically Driven Systems: Disorder, Interactions, and Quasi-Steady States

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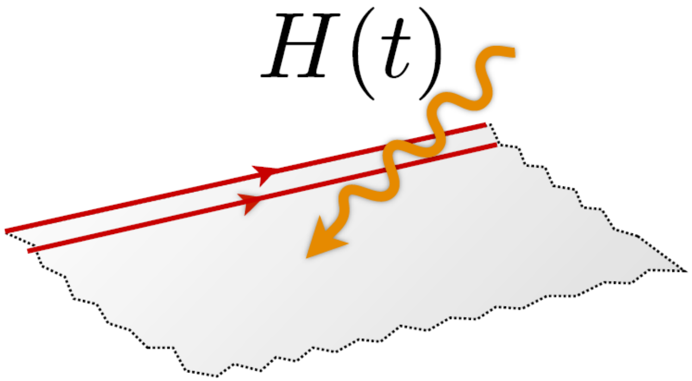
מכון ויצמן למדע
Weizmann Institute of Science

New types of non-equilibrium topological "phases" unique to periodically-driven systems?



Outline

- **Review**: Floquet, Bloch, Floquet-Bloch
- **Disorder**, the Anomalous Floquet-Anderson insulator (AFAI), and non-adiabatic quantized pumping
- **Interactions**, thermalization in Floquet bands, and universal current carrying quasi-steady states

Floquet states and the quasi-energy

No ground state, energy conservation for driven system

$$i\frac{d}{dt}|\psi\rangle = H(t)|\psi\rangle; \quad H(t+T) = H(t)$$

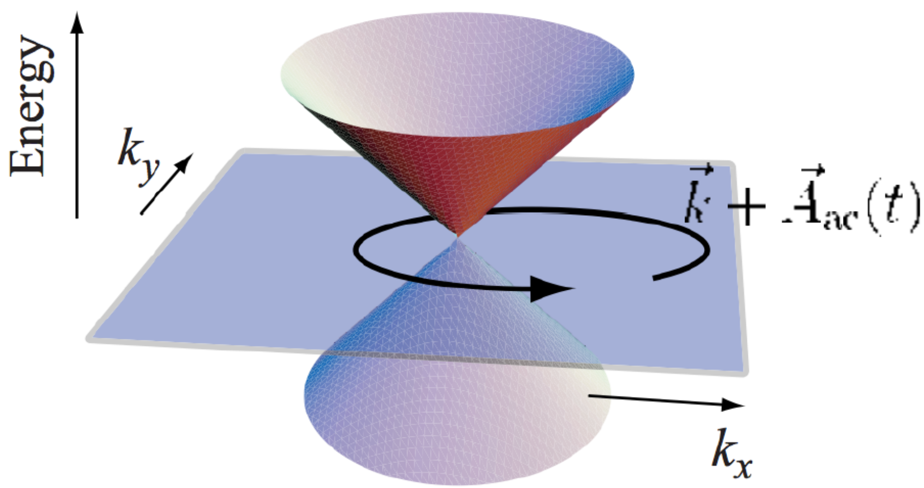
Floquet formalism:

$$\boxed{U(T)|\psi_n\rangle = e^{-i\varepsilon_n T}|\psi_n\rangle}$$

Eigenvalue invariant under $\varepsilon_n \to \varepsilon_n + 2\pi N/T$: quasi-energy lives on a <u>circle</u>

Floquet band topology induced by periodic driving

Circularly-polarized light opens Haldane gap in graphene



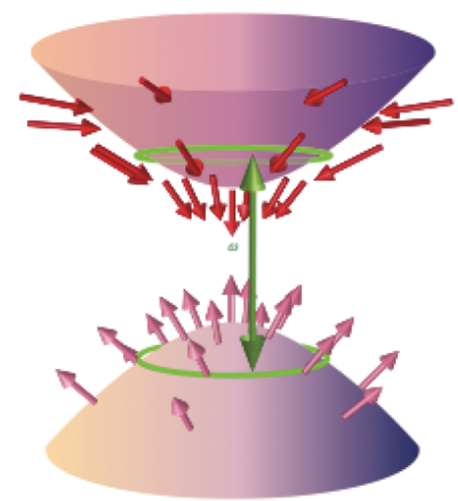
T. Oka and H. Aoki, Phys. Rev. B **79**, 081406 (2009).
T. Kitagawa, et al., Phys. Rev. B **84**, 235108 (2011).
Z. Gu et al., Phys. Rev. Lett. 107, 216601 (2011).

Resonant driving used to create band inversion

ARTICLES
PUBLISHED ONLINE: 13 MARCH 2011 | DOI: 10.1038/NPHYS1926
nature physics

Floquet topological insulator in semiconductor quantum wells

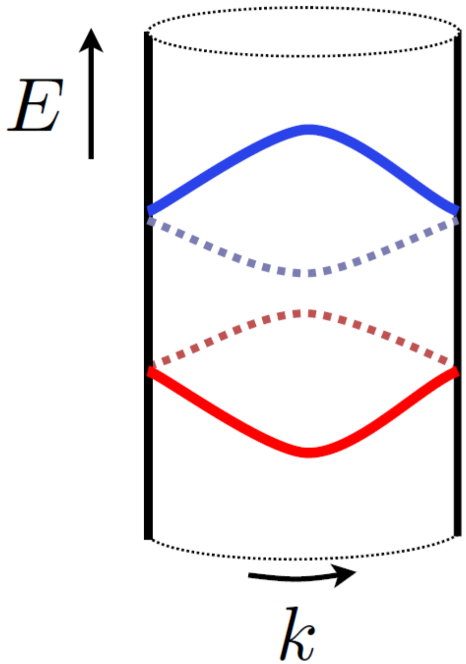
Netanel H. Lindner[1,2]★, Gil Refael[1,2] and Victor Galitski[3,4]



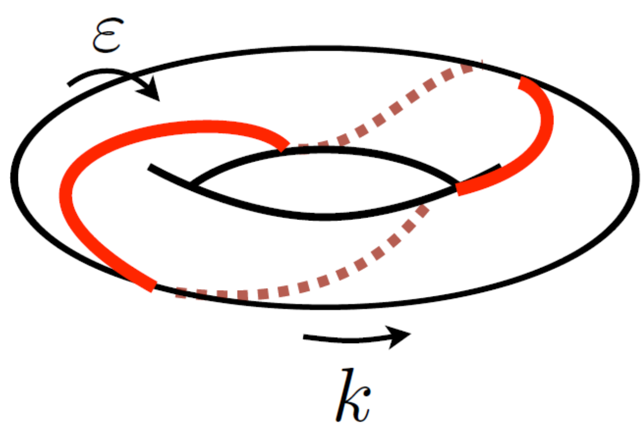
N. Lindner, G. Refael, and V. Galitski, Nature Physics **7**, 490 (2011).

New topological configurations possible in driven systems

Normal band structure: <u>cylinder</u>



Quasi-band structure: <u>torus</u>



Quasi-energy winding and adiabatic quantized transport

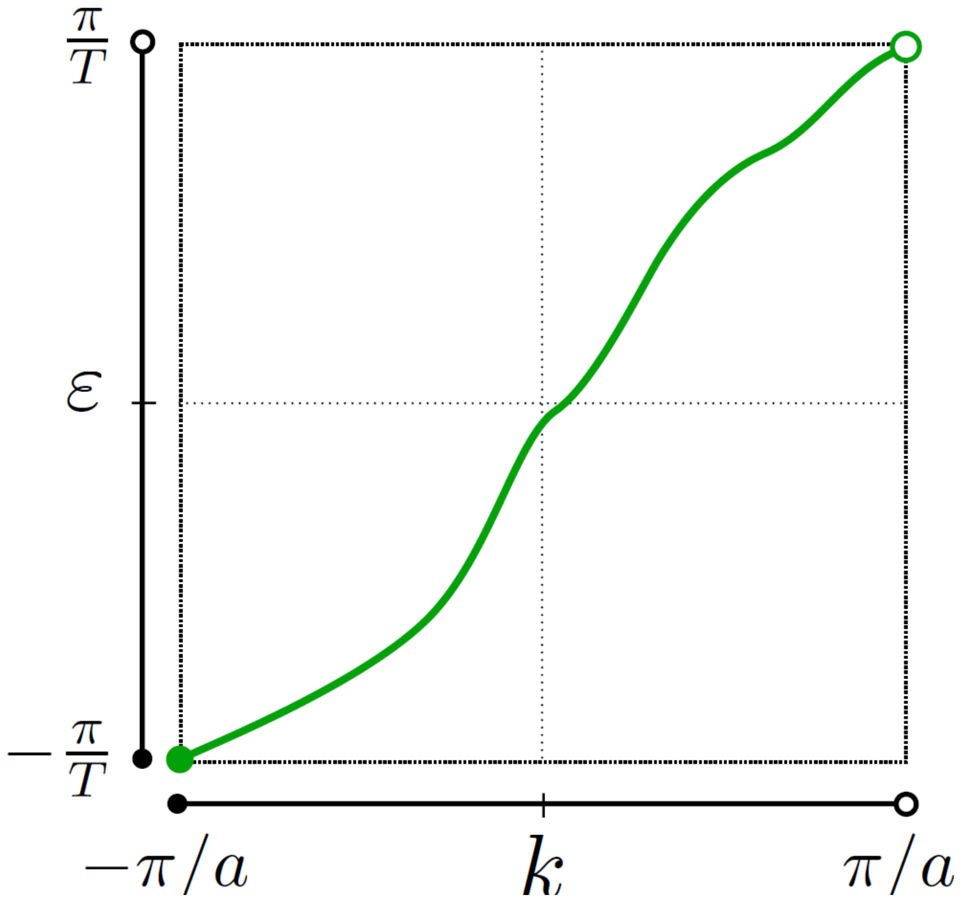
Average group velocity <u>quantized</u>

$$\bar{v}_g = \overline{\frac{d\varepsilon_k}{dk}} = a/T$$

Average displacement:

$$\overline{\Delta x} = \bar{v}_g T = a$$

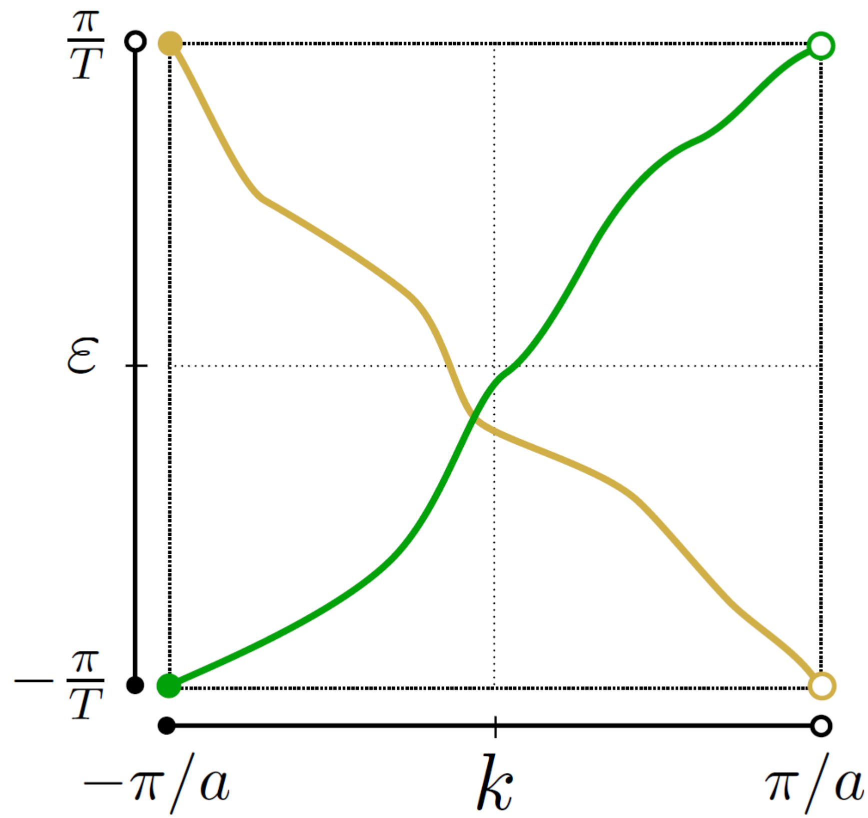
shift by one unit cell



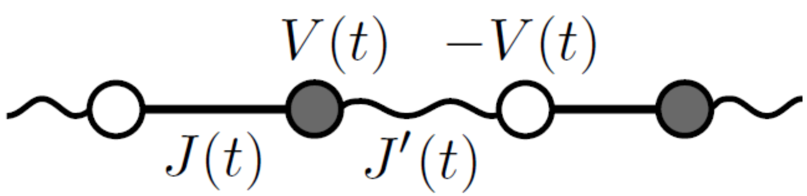
Quasi-energy winding and adiabatic quantized transport

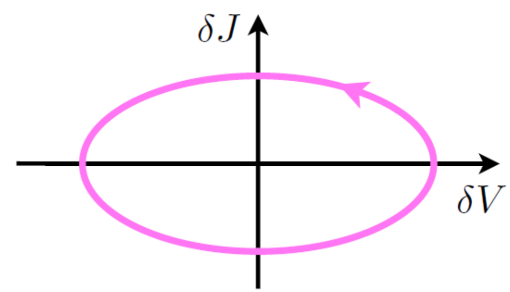
Winding number

$$W = \oint \frac{dk}{2\pi} \mathrm{Tr}\left[U^{-1}\partial_k U\right]$$



Adiabatic
Thouless pump (83')



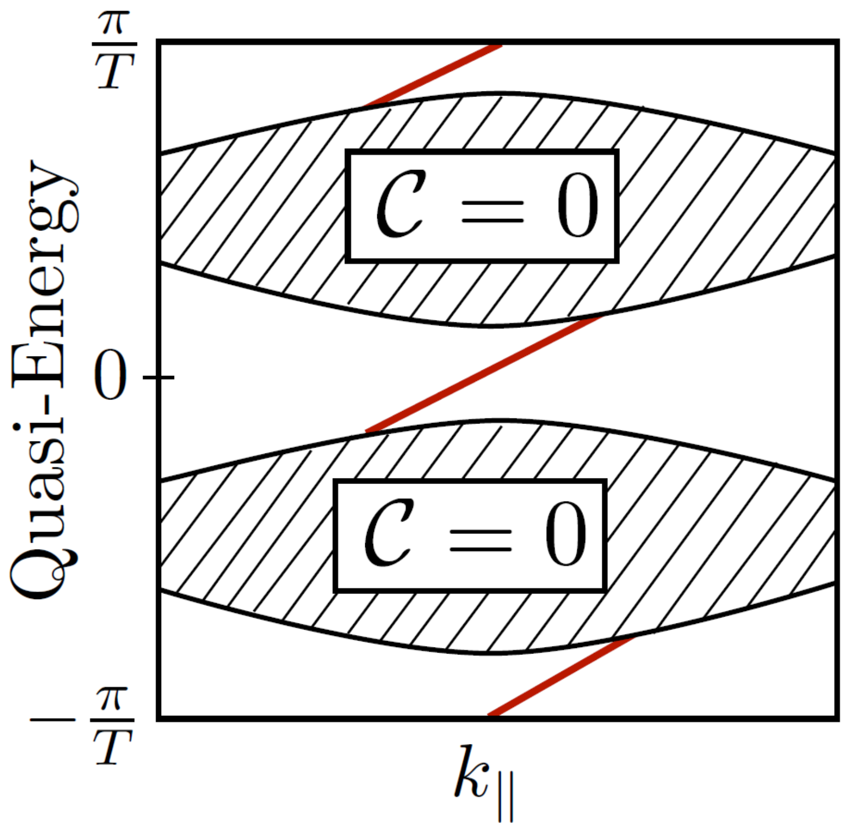


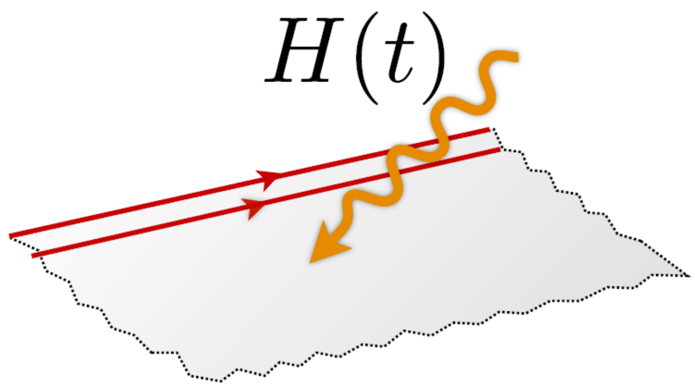
Recent experiments of Thouless pump in an optical lattice

S. Nakajima et. al., Nature Physics (2016)

M. Lohse, C. Schweizer, O. Zilberberg, M. Aidelsburger, I. Bloch, Nature Physics (2016)

Driven 2D systems may support chiral edge modes even when all Chern numbers are <u>zero</u>!





Chiral edge modes for $\mathcal{C} = 0$ bands

T. Kitagawa, EB, M. Rudner, and E. Demler PRB (2010)
M. Rudner, N. Lindner, EB, and M. Levin, PRX (2013)

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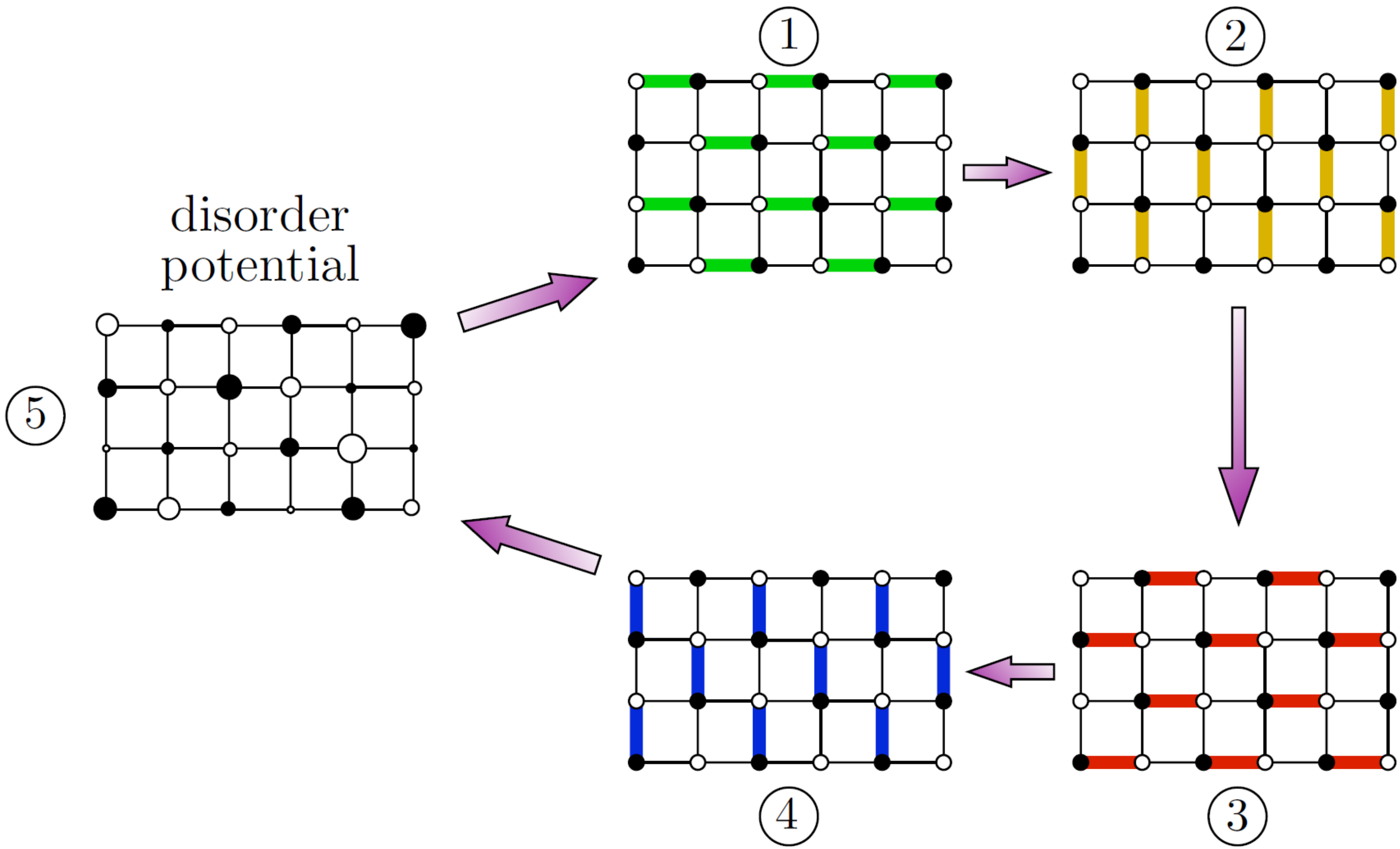
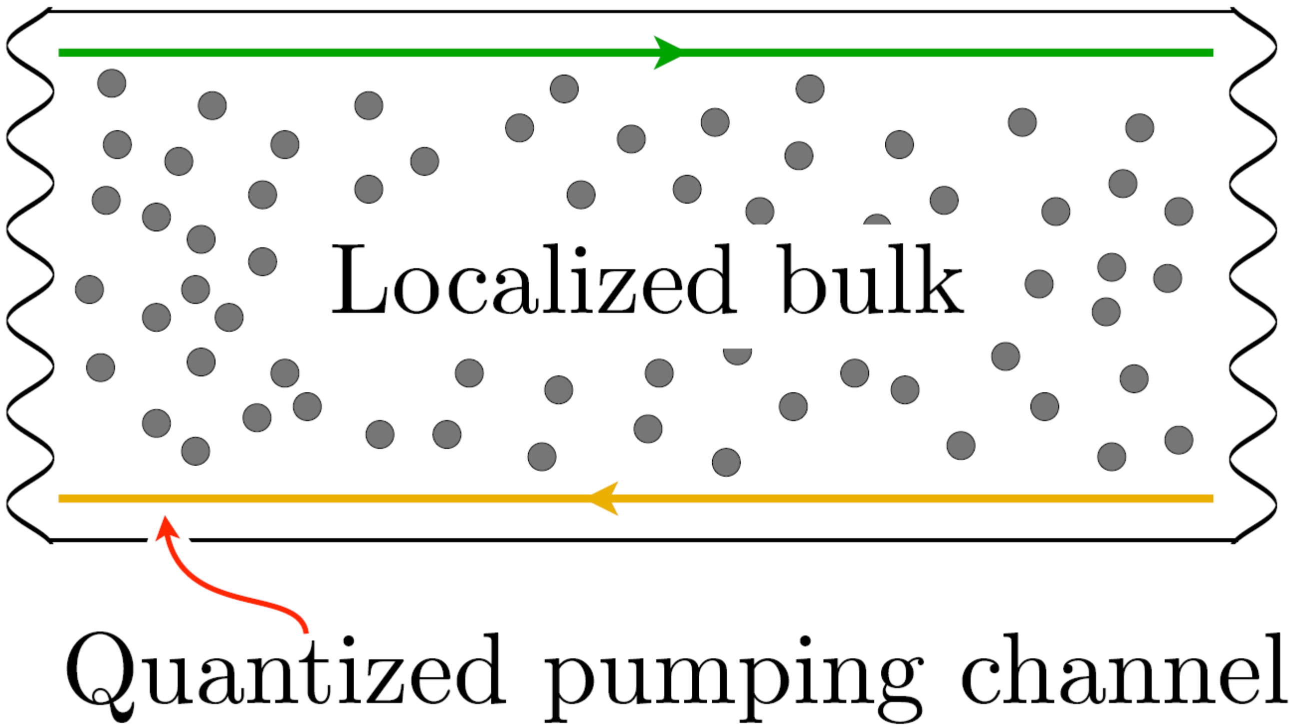
Disorder localizes all bulk states

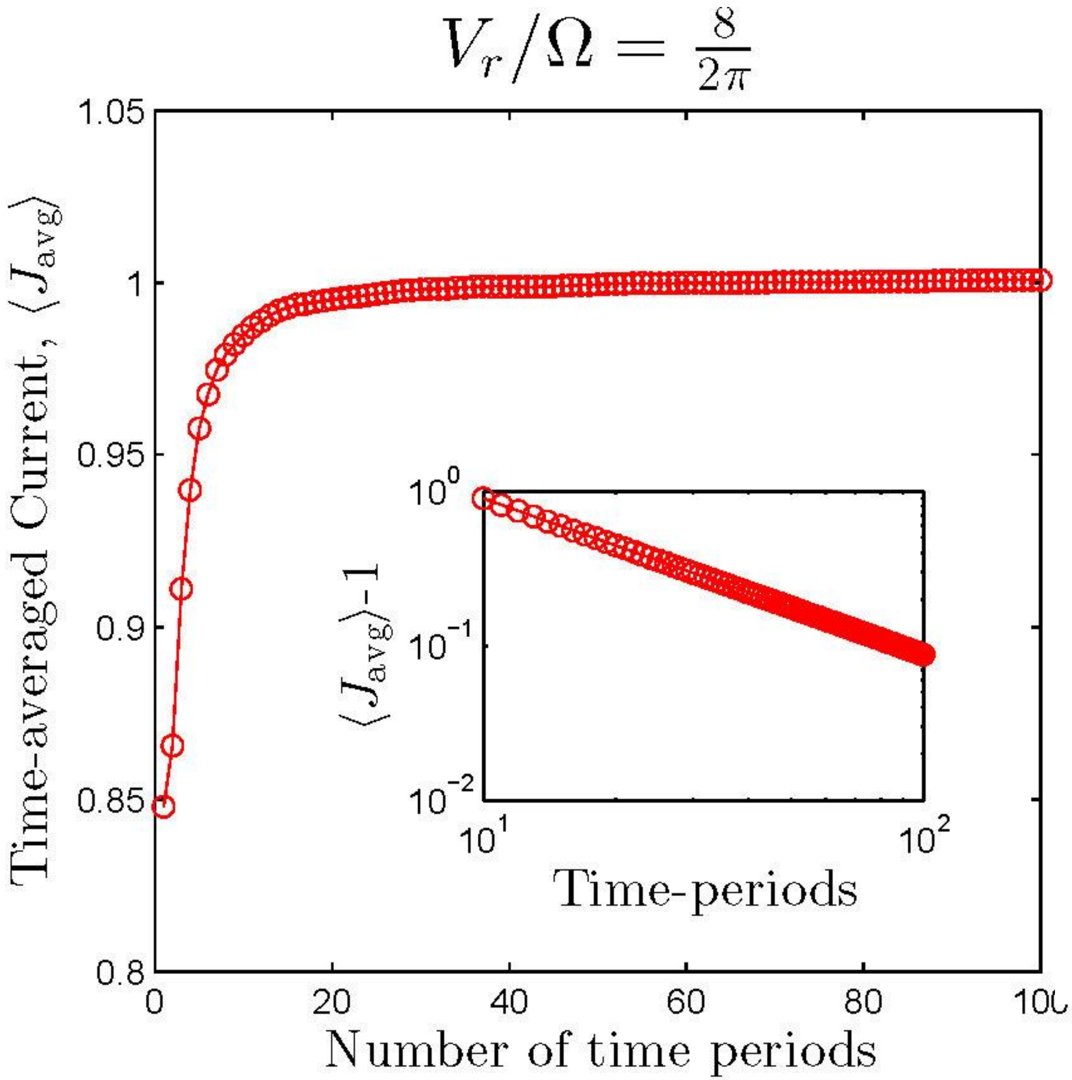


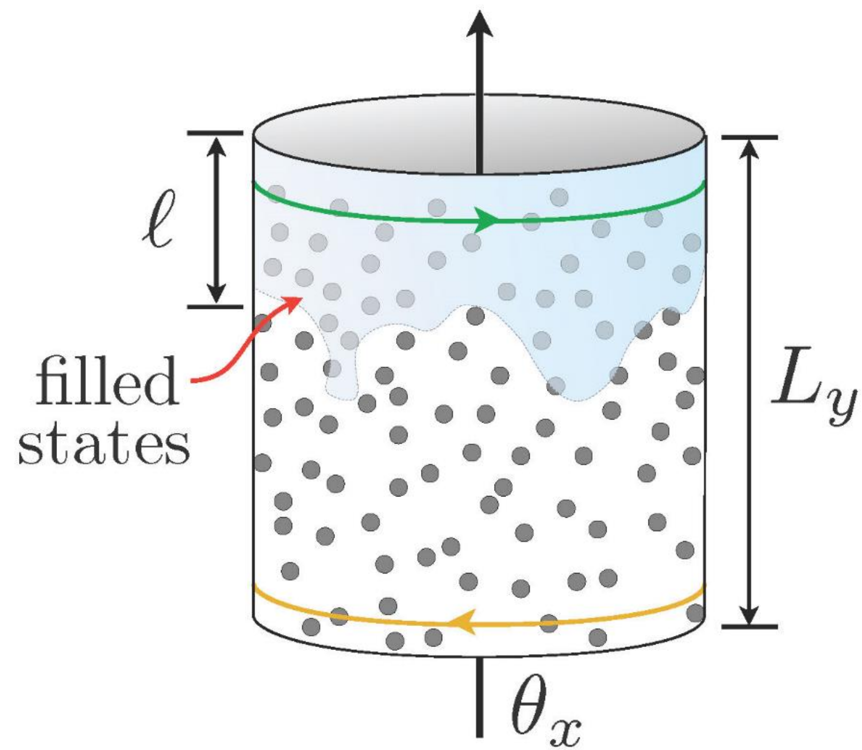
Anomalous Floquet-Anderson Insulator: fully localized bulk with propagating chiral edge states



P. Titum, EB, M. Rudner, G. Refael, and N. Lindner, PRX (2015)

Non-adiabatic Quantized pumping current in the AFAI





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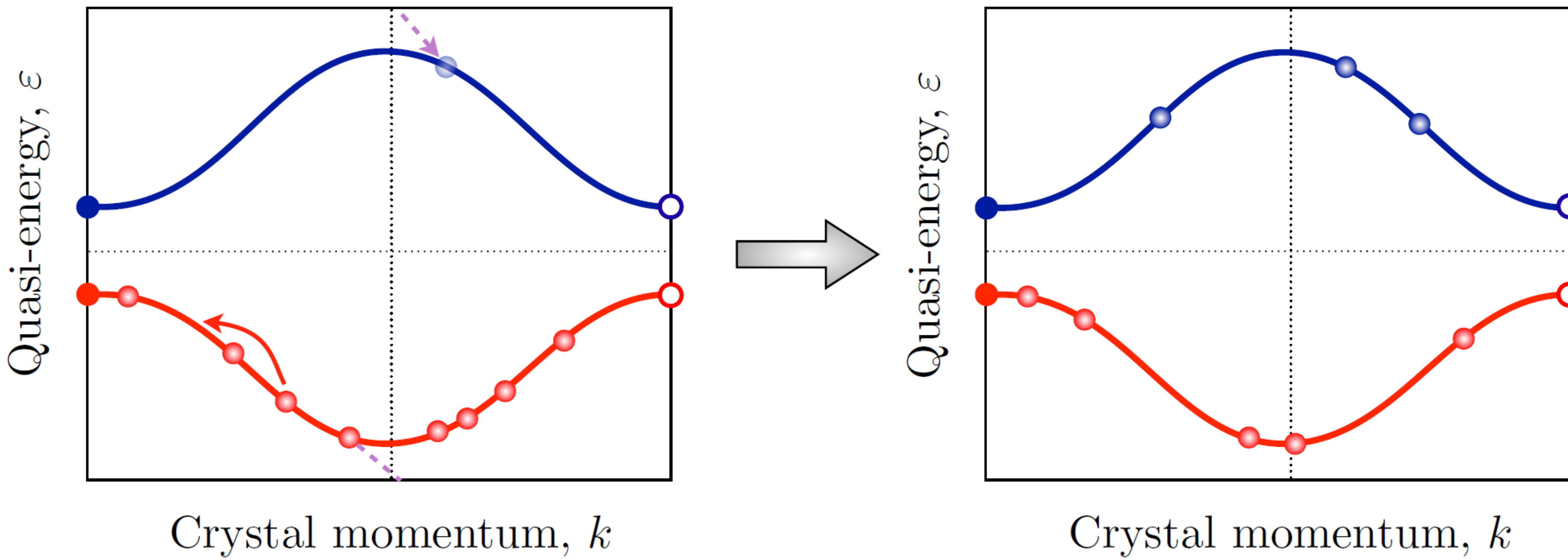
At long times, driven interacting (closed) system generically heats to infinite temperature



See examples (plus others)

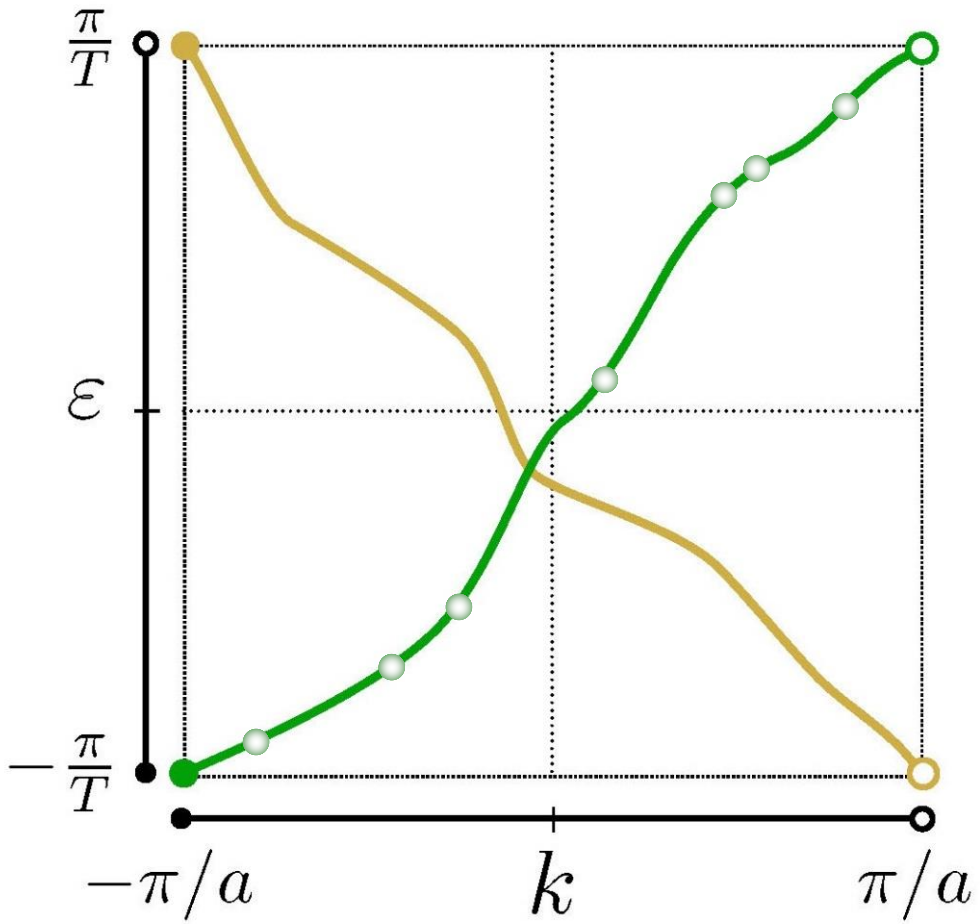
A. Lazarides, A. Das, R. Moessner (PRL, 2014)

L. D`Alessio, M. Rigol (PRX, 2014)

P. Ponte, A. Chandran, Z. Papic, D. Abanin (Ann. Phys., 2014)

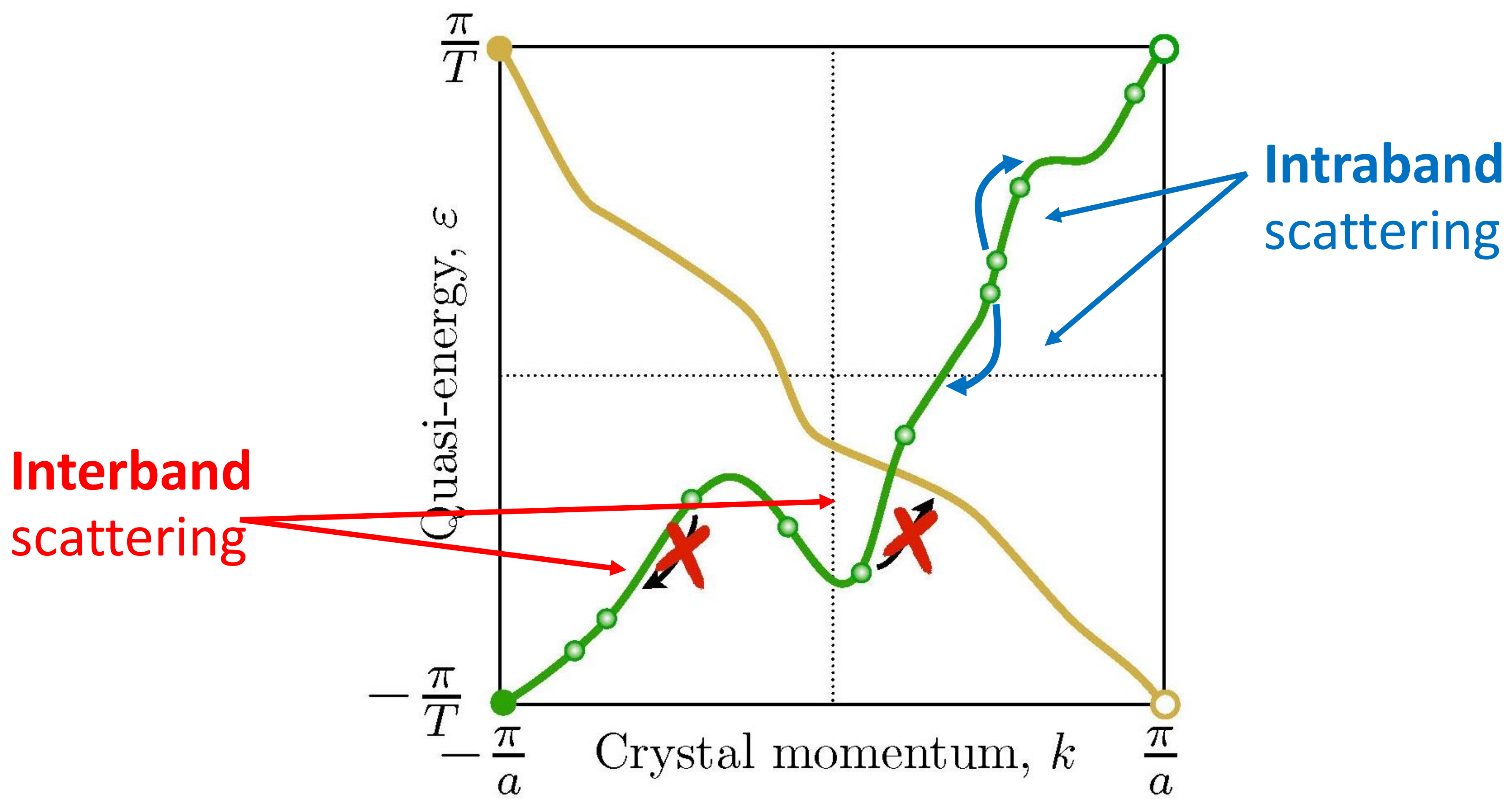
Topological features in a closed, interacting Floquet system?

- 1D system with non-zero winding numbers (Thouless pump)
- Initial state:
  partially filled right moving band
  (or filled with bosons)
- Gapless system:
  adiabatic theorem not applicable!



Chiral “thermalization”?

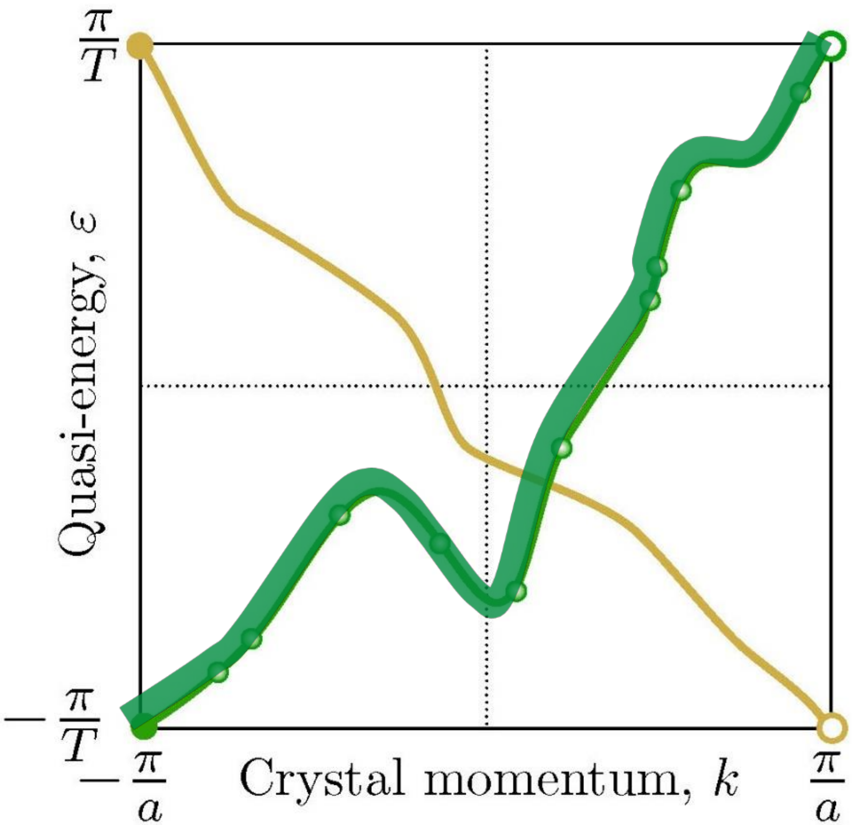
- If **interband** scattering is suppressed, particles can thermalize only within one of the chiral bands



Chiral quasi-steady state

Intermediate time “quasi-steady state”:

Thermalization *in a single Floquet band*



$$t_{\text{intra}} < t < t_{\text{inter}}$$

$$\text{current} = \frac{1}{T} \times \text{density}$$

M. Rudner, N. Lindner and EB, arXiv:1603.03053

Outline

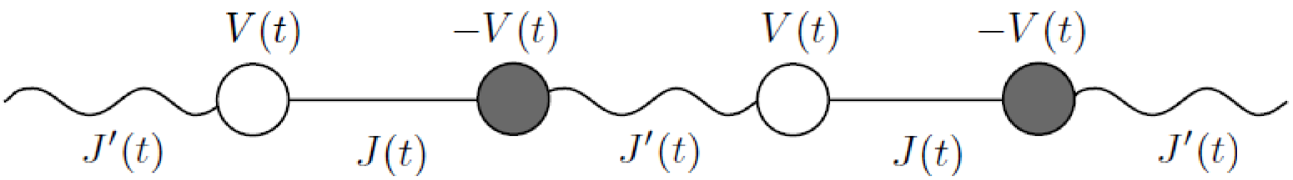
- **Review**: quantum driven systems: Floquet, Bloch, Floquet-Bloch

- **Interactions** in closed systems and thermalization

- **Current-carrying quasi steady states**

Simulation

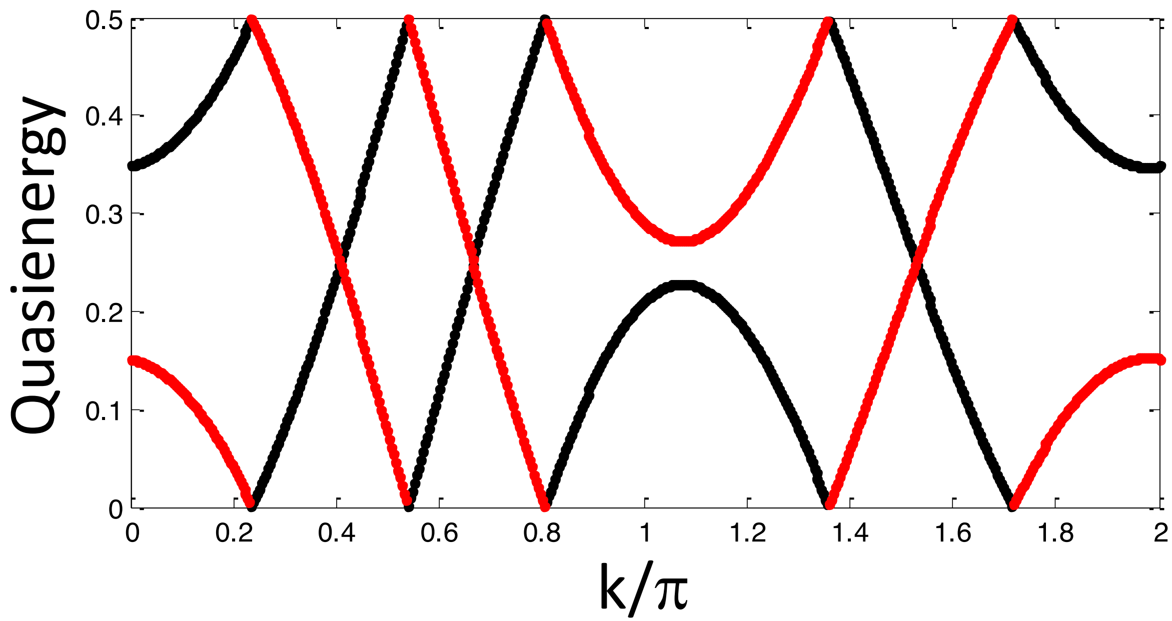
Numerics: 8 fermionic particles, 32 sites (16 unit cells)



$$J_1 = \frac{2}{3} J_0$$
$$V_1 = 2 J_0$$
$$\omega = \frac{1}{3} J_0$$

$$J(t) = J_0 + J_1 \cos \omega t \qquad J'(t) = J_0 - J_1 \cos \omega t \qquad V(t) = V_1 \sin \omega t$$

Single particle quasienergy spectrum:



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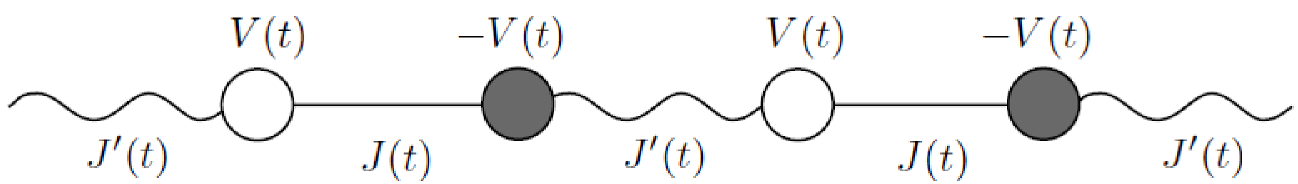

$J_1 = \frac{2}{3} J_0$

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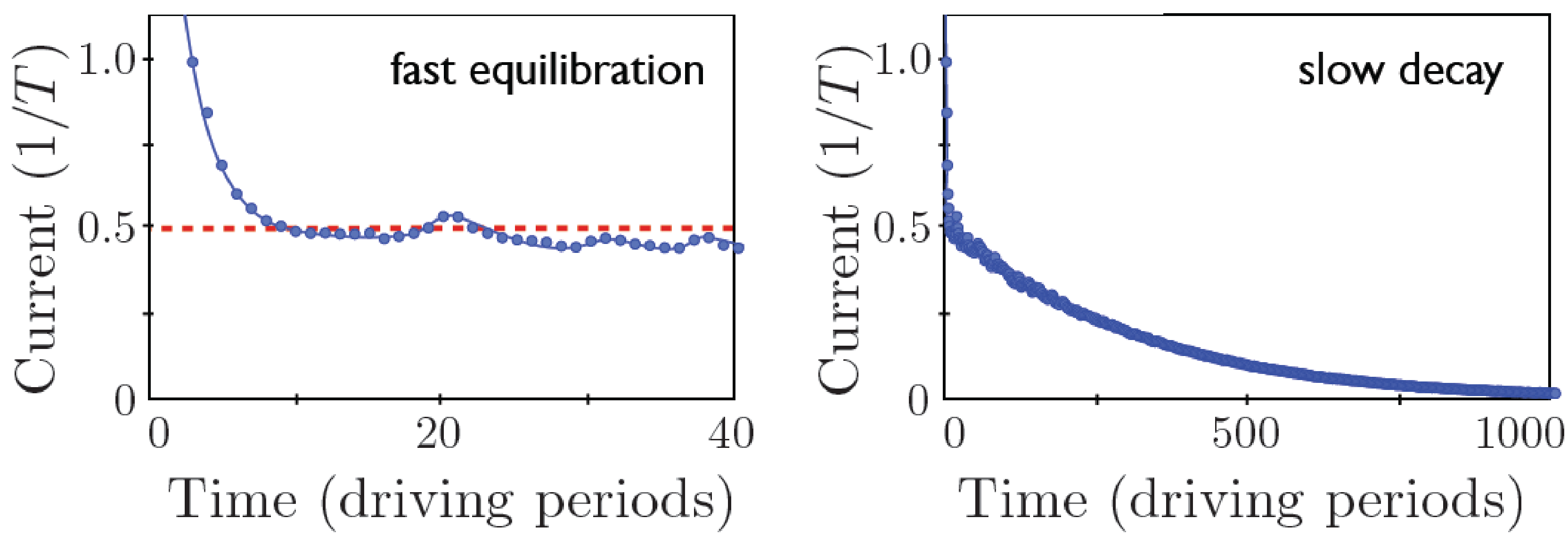
$J(t) = J_0 + J_1 \cos\omega t \qquad J'(t) = J_0 - J_1 \cos\omega t \qquad V(t) = V_1 \sin\omega t$

Intra-unit cell interaction, $U = 3J_0$



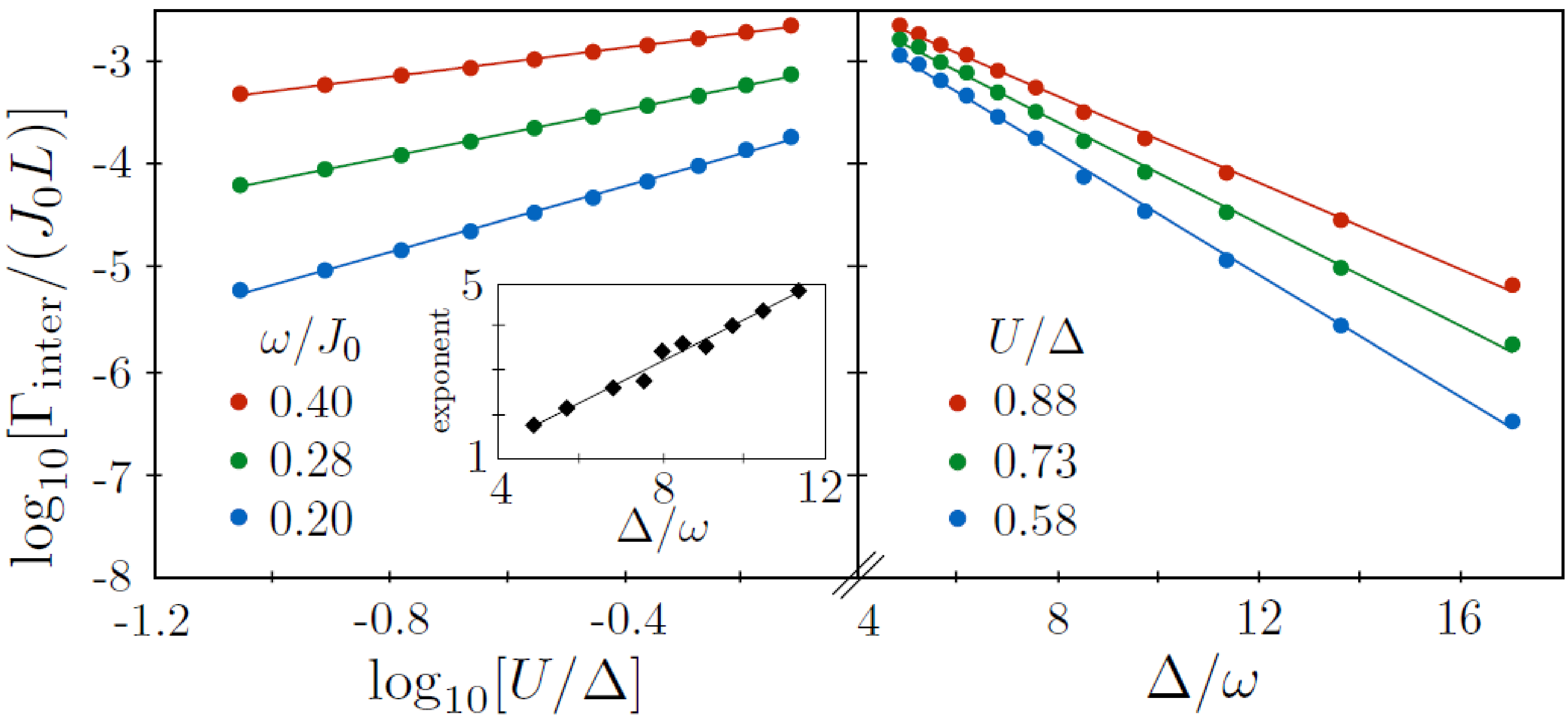
Inter-band relaxation rate

$$\Gamma_{\rm inter} \sim \left(\frac{\alpha U}{\Delta}\right)^{\frac{\Delta}{\delta m \omega}}$$

interaction

minimal order of photon absorption

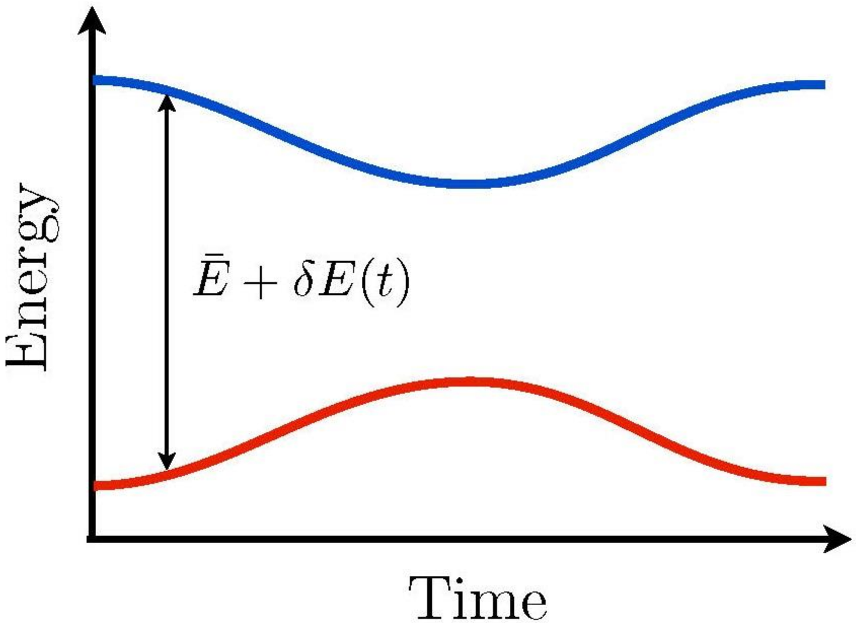
minimal instantaneous band gap

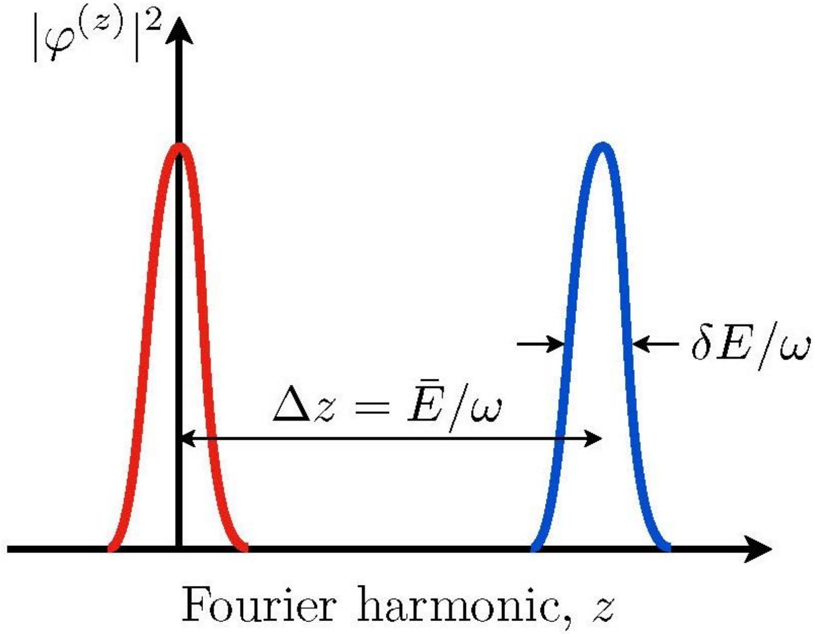


Frequency space analysis

- Matrix element for scattering controlled by the overlap of Fourier components of the scattering Floquet states

$$|\psi(t)\rangle = e^{i\varepsilon t} \sum_{z=-\infty}^{\infty} e^{iz\omega t} |\varphi_z\rangle$$





Frequency space analysis

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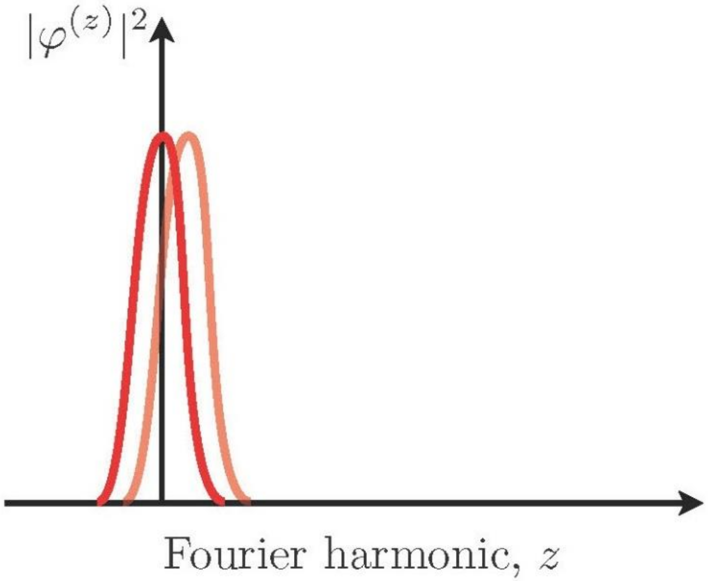
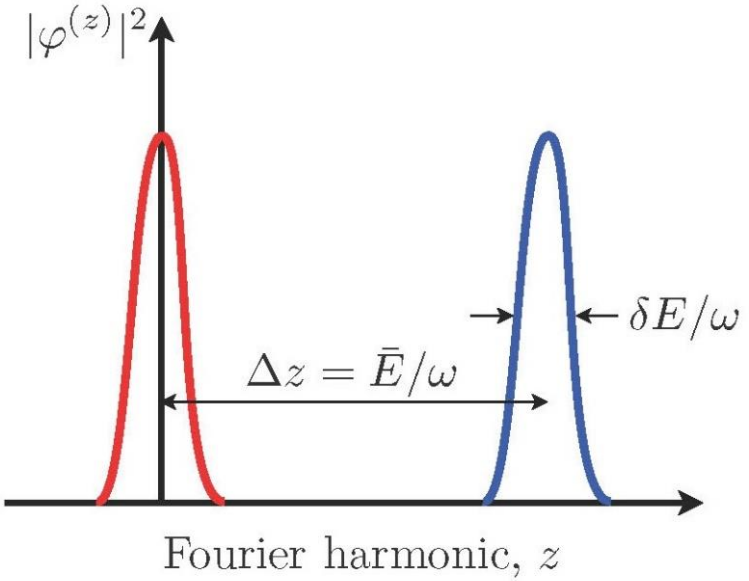
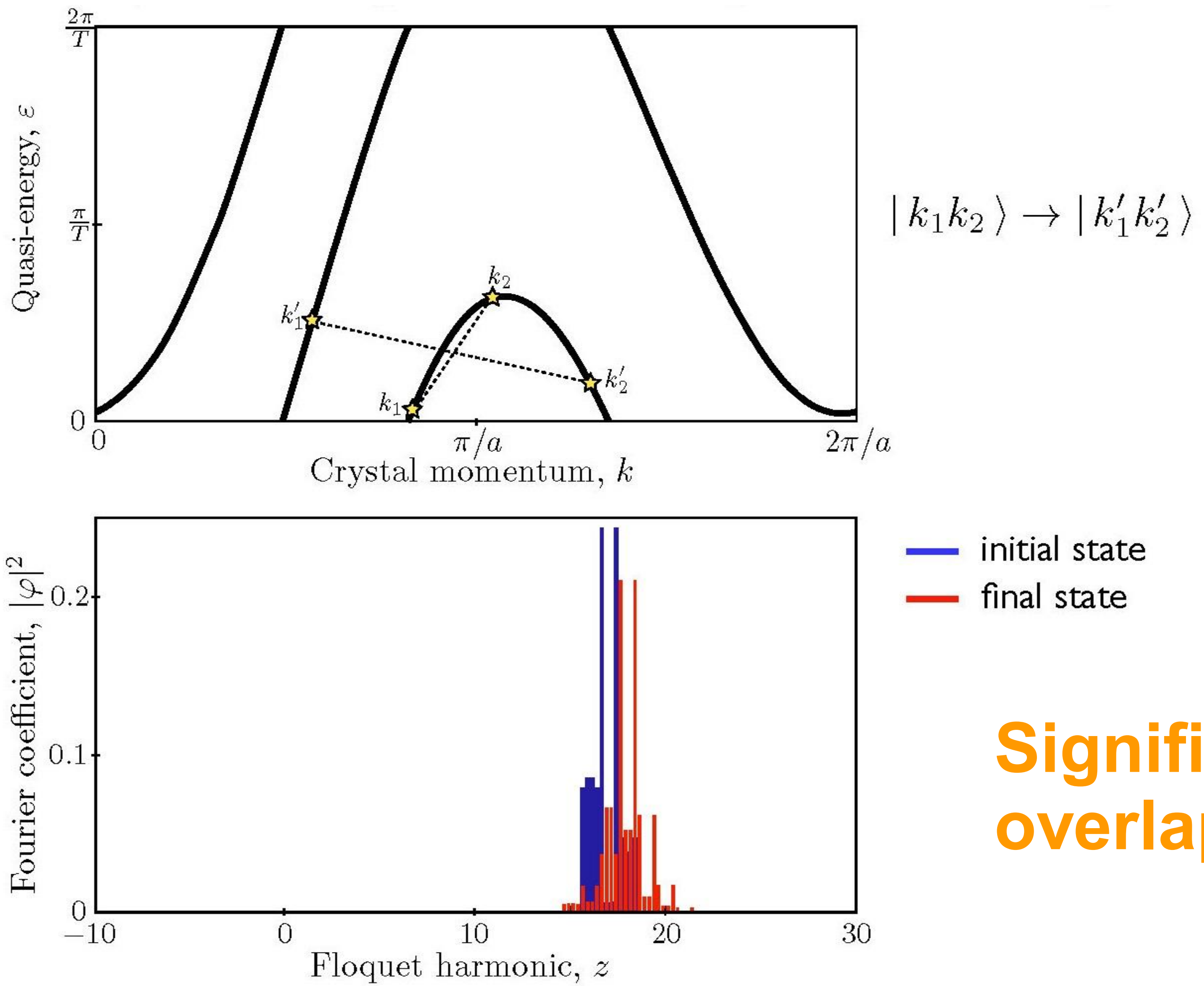
Intraband scattering



Interband scattering

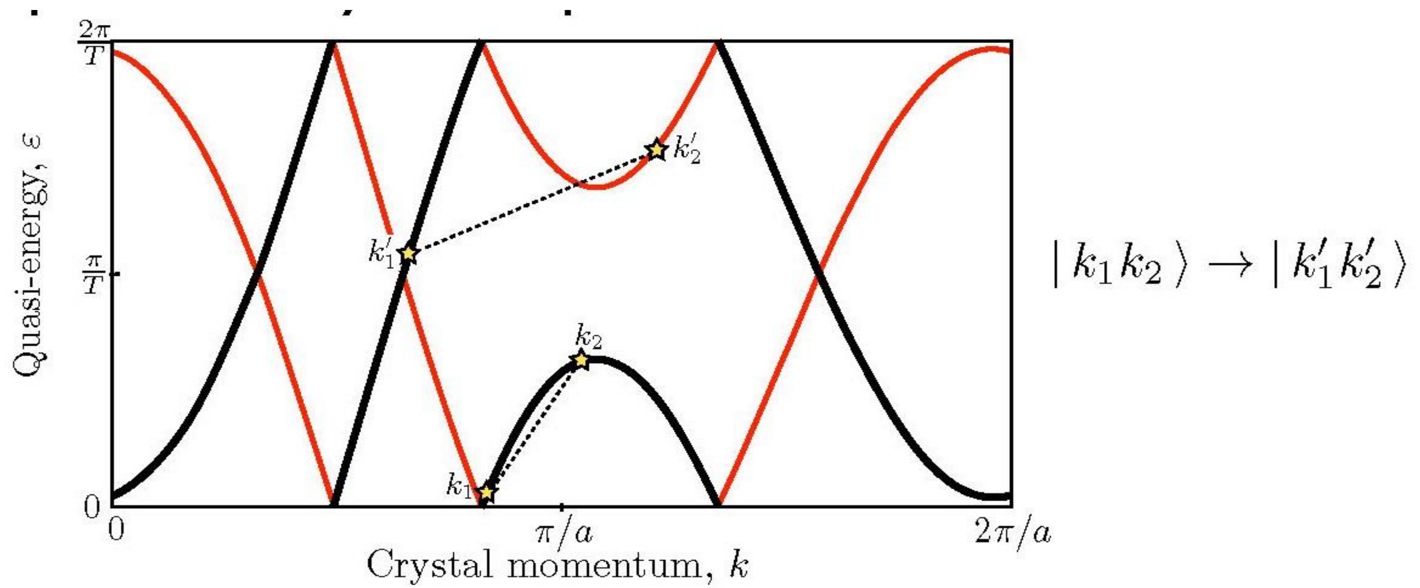


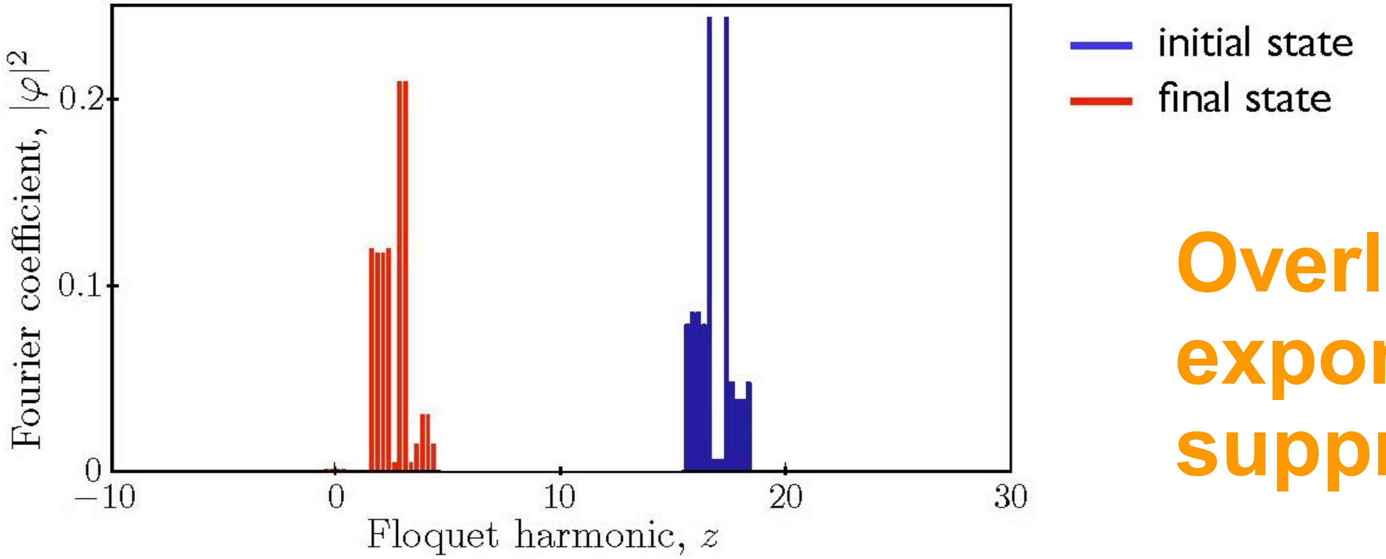
Intraband scattering



Significant overlap

Interband scattering



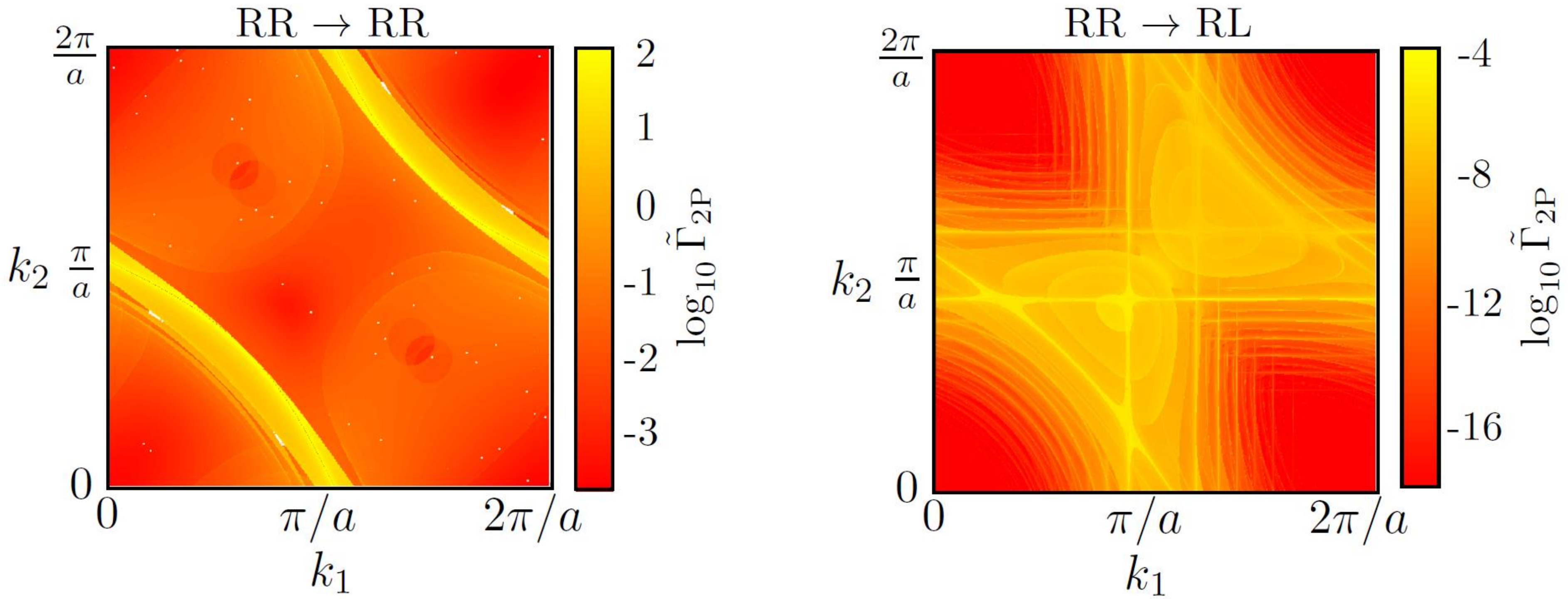


Overlap exponentially suppressed

Interband scattering: Born approximation

$$|\, k_1 k_2 \,\rangle \rightarrow |\, k_1' k_2' \,\rangle$$

$$\frac{\Gamma_{2\mathrm{P}}}{L} = \sum_f \frac{1}{|\Delta v_f|} |\varphi^\dagger_{2\mathrm{P},f} \mathcal{U} \varphi_{2\mathrm{P},0}|^2$$



$\Gamma_{2P} \propto U^2$: does not account for observed $\Gamma_{inter} \propto \left(\frac{U}{\Delta}\right)^{\frac{\Delta}{\omega}} \ldots$

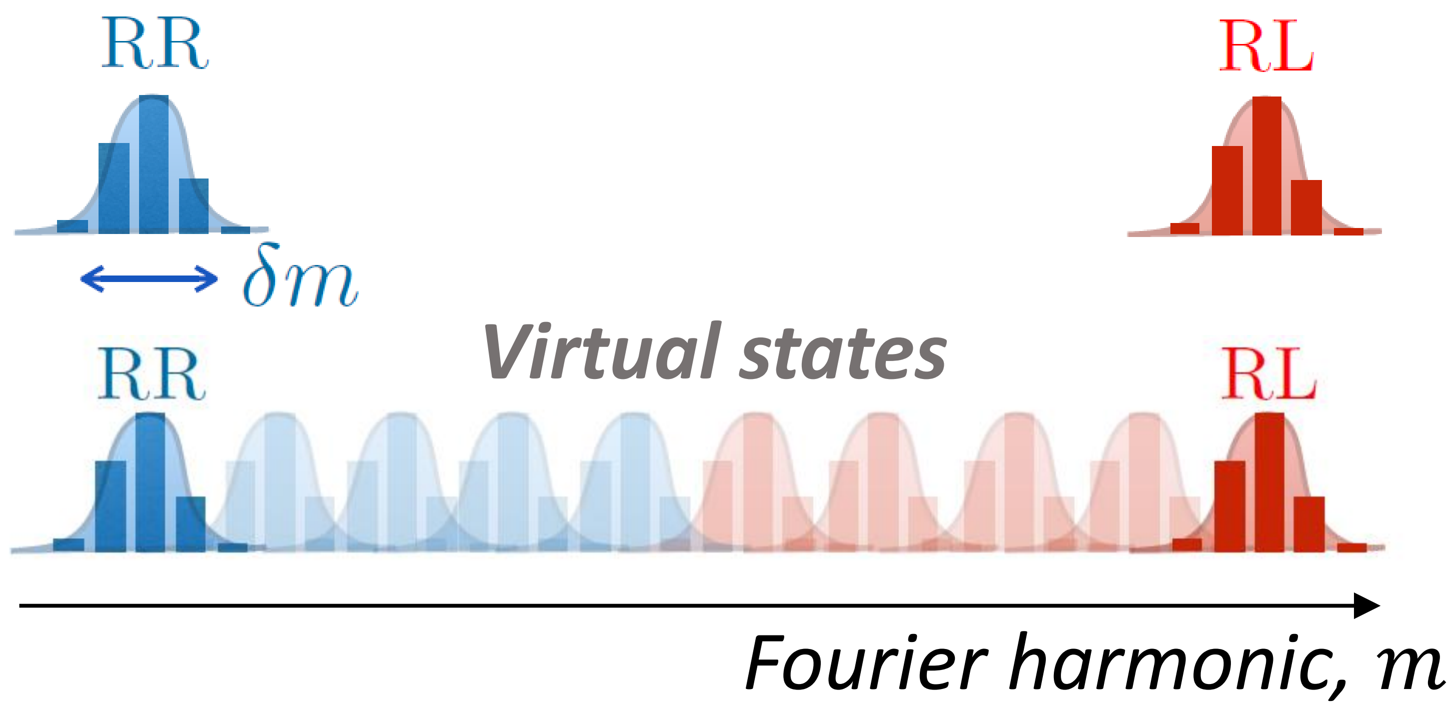
High order scattering

T matrix formulation in extended (Fourier harmonic) Hilbert space:

$$T(\Omega) = U + UG_0U + UG_0UG_0U + \cdots$$

Born Approx.:

Nth order:



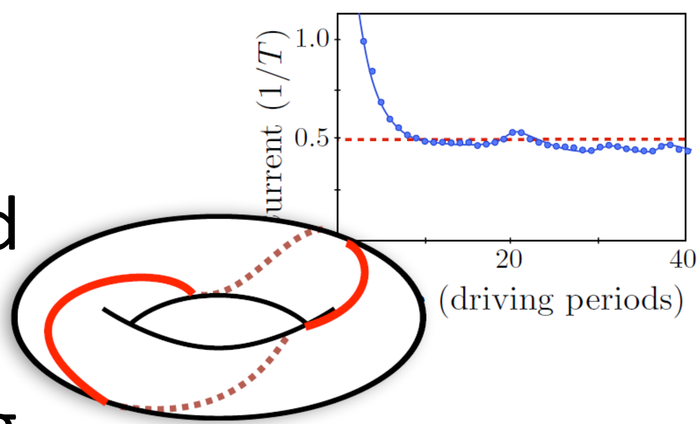
Optimal order: $N \sim \dfrac{\Delta}{\delta m \omega}$

$$\Longrightarrow \Gamma_{\text{inter}} \sim \left(\frac{U}{\Delta}\right)^{\Delta/(\delta m \omega)}$$

Summary

Periodically driven systems host a variety of topological phenomena, with no analogues in static systems.

- Chiral edge states with no Chern numbers
- **Disorder**: chiral edge states with fully localized bulk, non-adiabatic quantized charge pumping
- **Interactions:** in closed systems, driving generically leads to indefinite heating… …But unusual long-lived quasi-steady states are possible



Thank you.