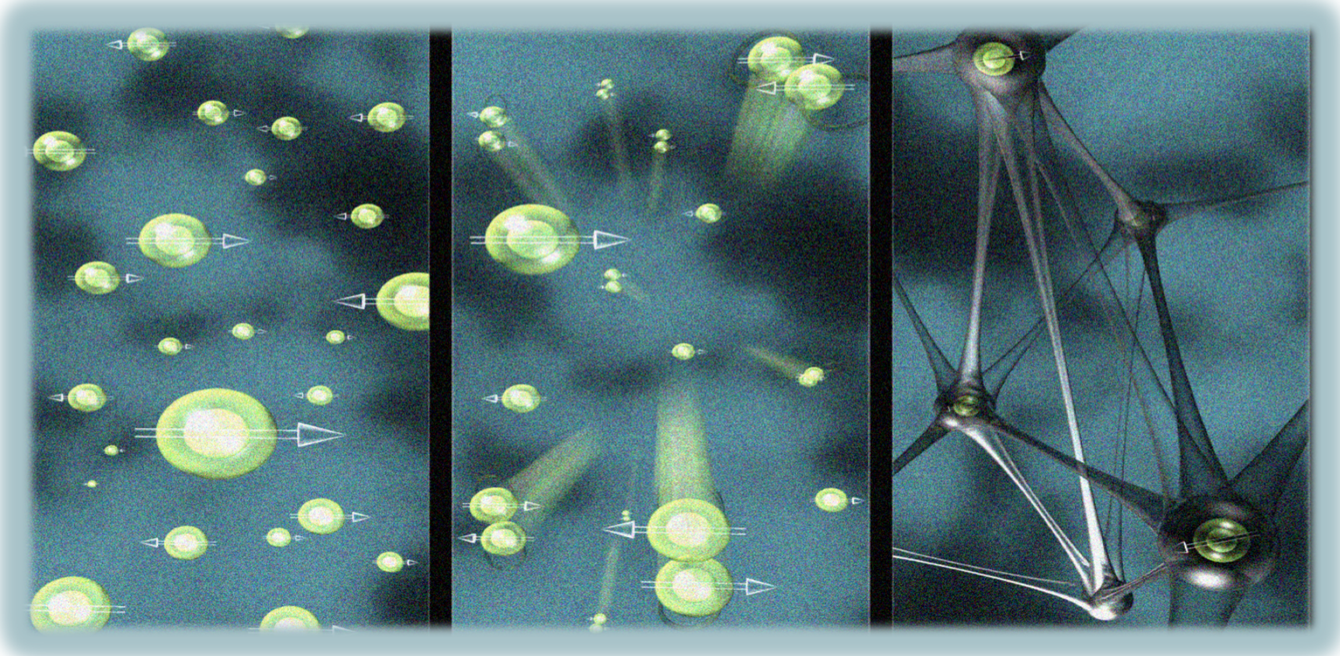
Quantum spin dynamics, coherences and entanglement in trapped ion arrays

Ana Maria Rey



Designer Quantum Systems Out of Equilibrium

KITP, Nov 13, 2016

Theory


M. Wall


M. Gärttner


M. Foss-Feig


A. Safavi-Naini


R. Lewis-Swam

Experiment


J. Bollinger


J. Bohnet


J. Britton

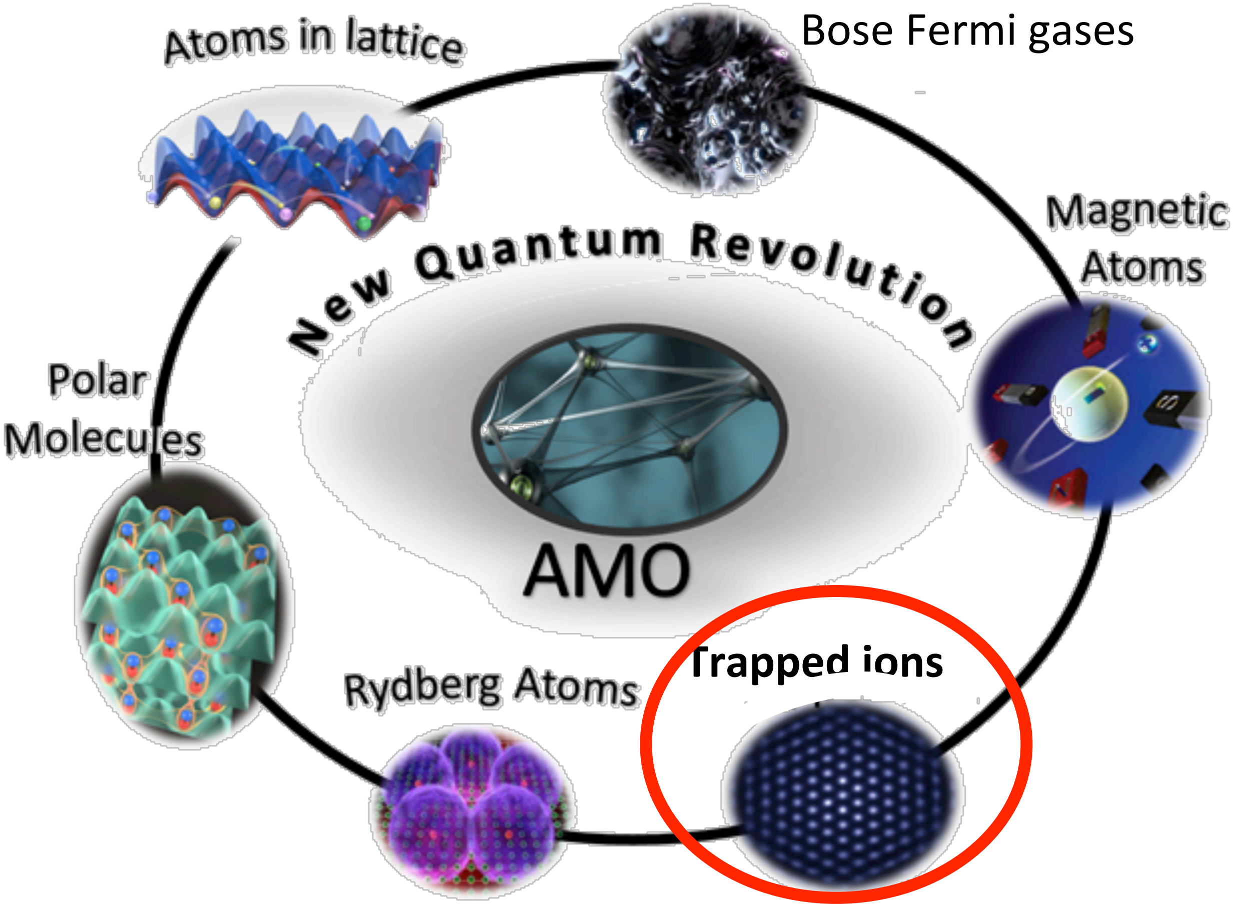

K. Gilmore


B. Sawyer

Highly non-equilibrium quantum matter

Far from-equilibrium strongly interacting quantum systems are complex and at the heart of modern quantum science

- Can be strongly correlated and entangled
- Can not be described by standard theoretical tools developed for equilibrium physics
- Dynamics is now accessible in current experiments

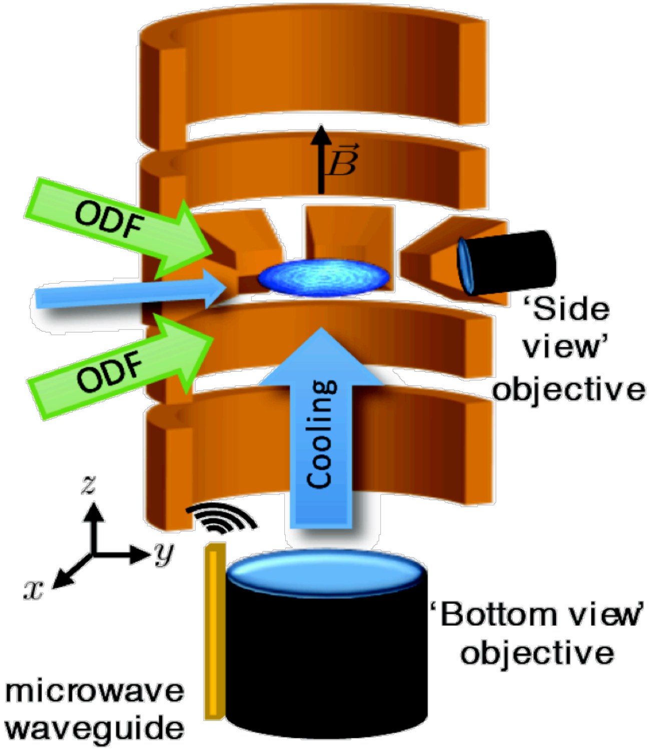


Long-range interacting systems: Now available

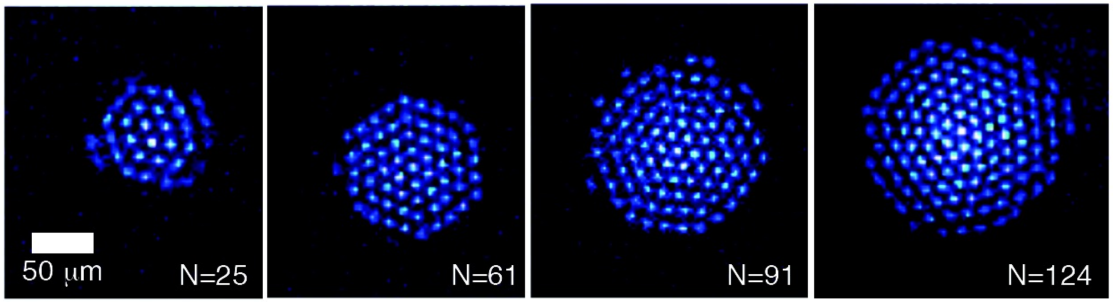
How fast entanglement and correlations propagate in the presence of long-range interactions as contrasted with short-range-interacting systems?

- ✓ Speed limits on quantum state transfer and thermalization.
- ✓ Complexity that one faces when simulating quantum dynamics with classical computers.
- ✓ How can we best characterize and measure multi-particle entanglement?
- ✓ What happens in the presence of decoherence?

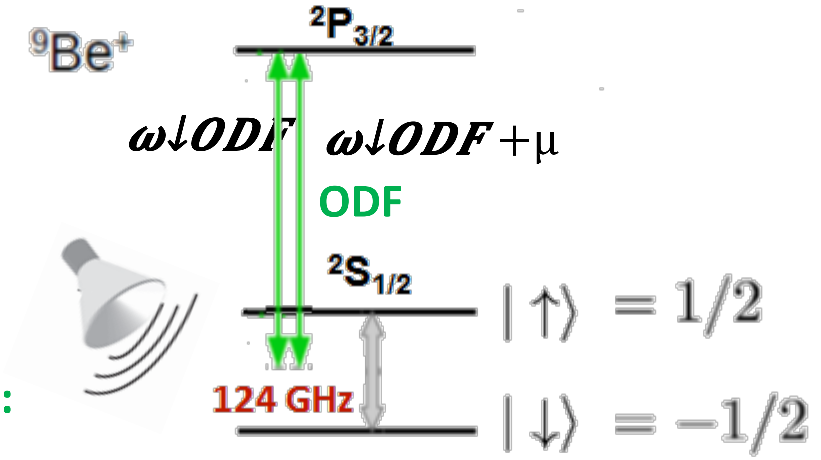
Penning Trap Experiments: $^9Be^+$



- Penning trap: 2D triangular crystals of 20-300 ions



- Two hyperfine states used as spin ½ system



- **Spin dependent force applied by controlling frequency and polarization of Raman laser beams:**
- **Excites phonon modes**
- **Phonons mediate long-range spin-spin interactions between ions**

Phonons & Spin-spin interactions

$$\hat{H}_{ODF}(t) = -F_0\cos(\mu t)\sum_{j=1}^{N}\underbrace{\hat{z}_j}_{\sum_{m=1}^{N} b_{jm}\sqrt{\frac{\hbar}{2M\omega_m}}\left(\hat{a}_m^{\dagger}e^{i\omega_m t}+\hat{a}_m e^{-i\omega_m t}\right)}\cdot\hat{\sigma}_j^z$$

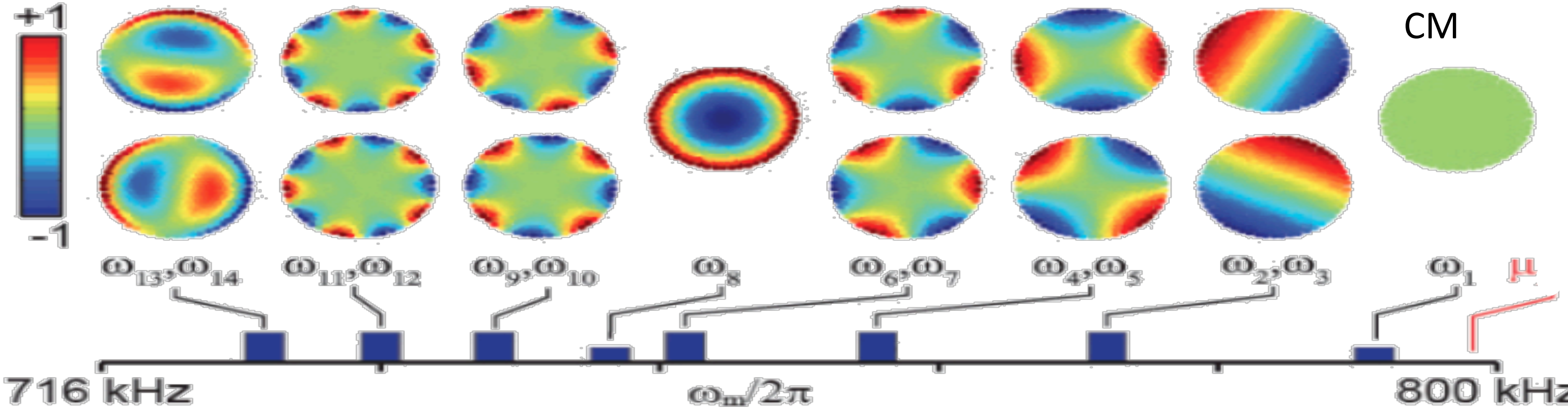
N drumhead eigenvalues ω_m and eigenvector b_m

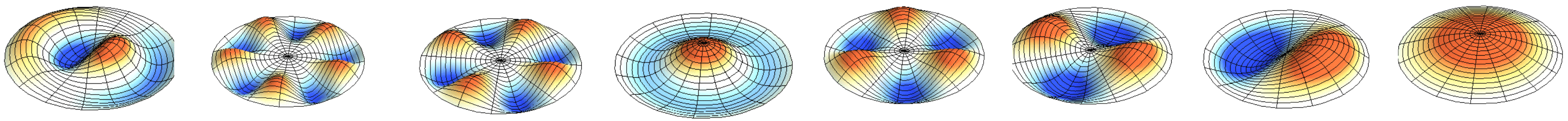
$$\hat{U}_{ODF} = \hat{U}_{SP}(t)\cdot\hat{U}_{SS}(t)$$

spin-phonon coupling

spin-spin interactions

Dominant for large
$\delta = |\mu - \omega_1|$





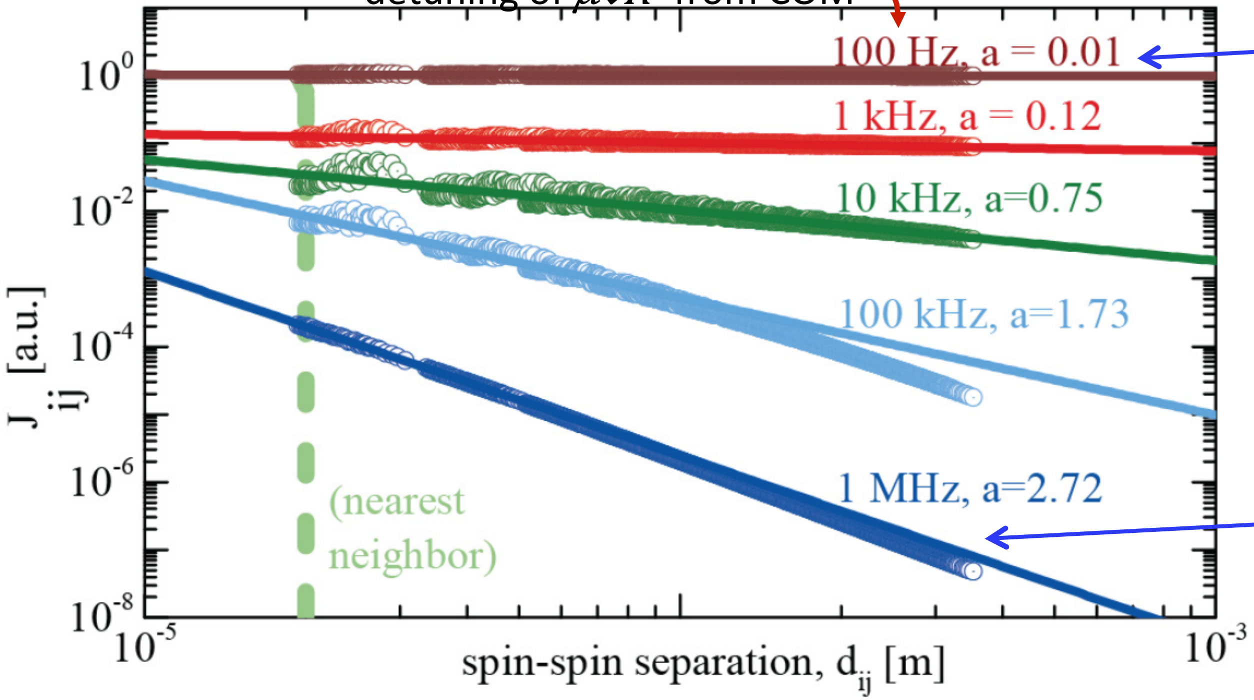
<a href="http://www.acs.psu.edu/drussell/Demos/waves/wavemotion.html">

Ising coupling coefficients determined by transverse modes

$$H_{SS} = \sum_{i<j} J_{ij}(t)\,\sigma_i^z\sigma_j^z \qquad J_{ij} \sim \frac{\Omega^2}{\hbar}\sum_{\mu} \frac{\hbar}{4M}\, b_{\mu i}\, b_{\mu j}\, \frac{1}{\mu^2-\omega_\mu^2}$$

$J_{i,j}$ depends on eigenmodes and ODF beating note (μ)

detuning of μ_R from COM



infinite range $J_{i,j} \approx J/N$

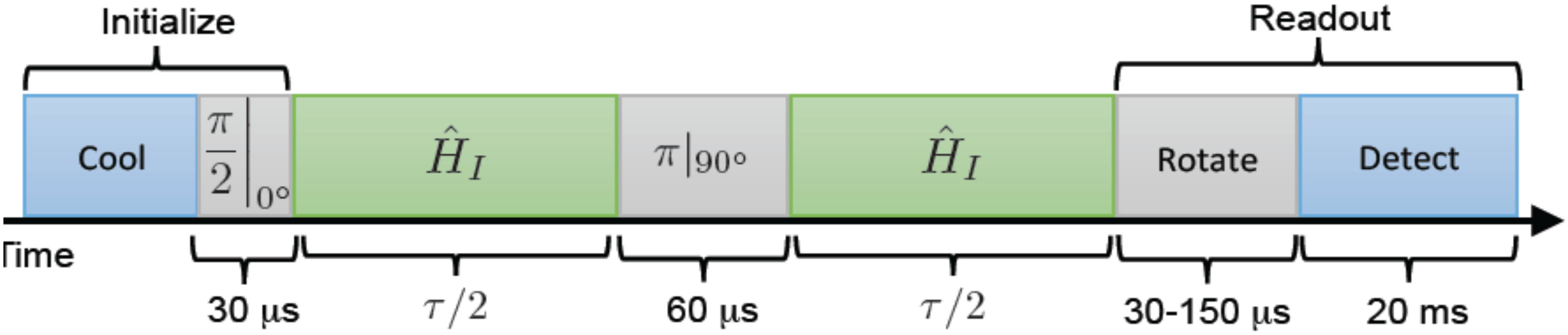
power-law interaction range

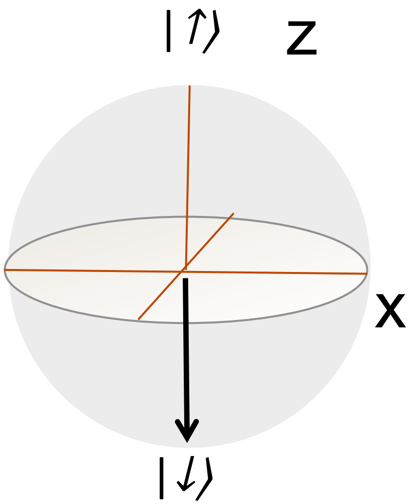
$$J_{i,j} \propto d_{i,j}^{-a}$$

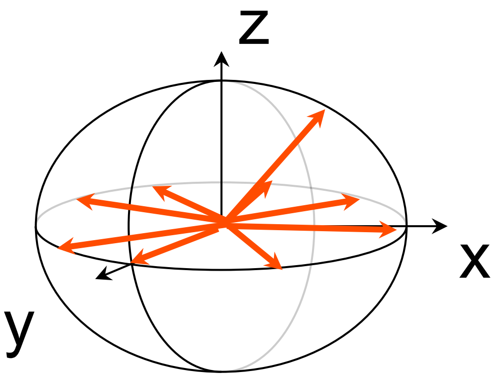
dipole-dipole coupling

J. W. Britton *et al* Nature 484, 489 (2012)

Probing Spin model with dynamics







Initial $|\downarrow\downarrow\downarrow\downarrow\rangle$

Rotate:θ

Wait τ/2

echo

Wait τ/2

Read

Goals:

Verify spin model

Create strong correlations

Explore regime intractable to theory

All-to-All Case: One Axis Twisting

$$H_{SS}=\frac{1}{N}\sum_{i<j} J\,\sigma_i^z\sigma_j^z=\frac{2J}{N}(S^z)^2 \qquad S^{x,y,z}=\frac{1}{2}\sum_i \sigma_i^{x,y,z}$$

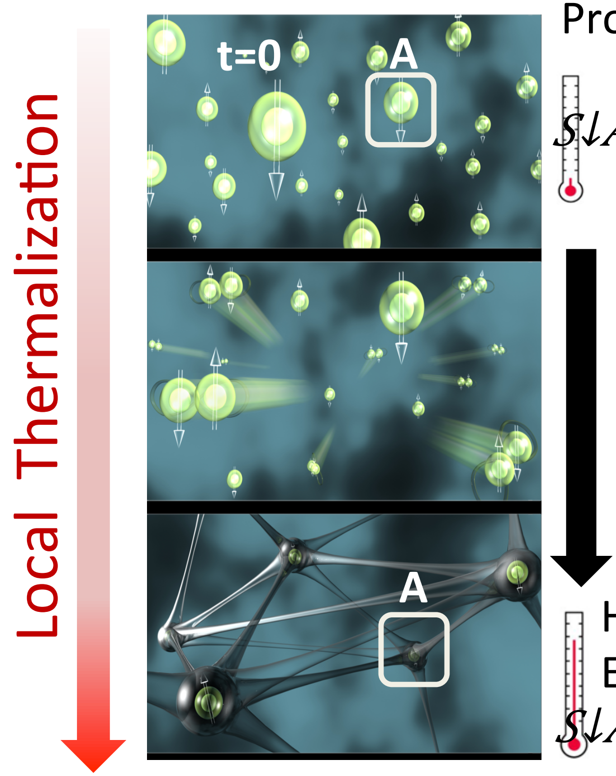
mean field limit

$$H_{MF}=B_{eff}S^z$$

$$B_{eff}=2J\cos\theta$$

No mean field dynamics at θ=π/2

- Quantum correlations?



Product state

$S_A=0$

Global Unitary dynamics

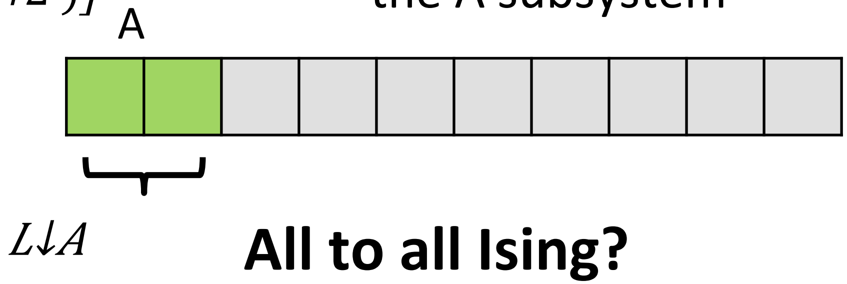
Highly Entangled

$S_A\to L_A$

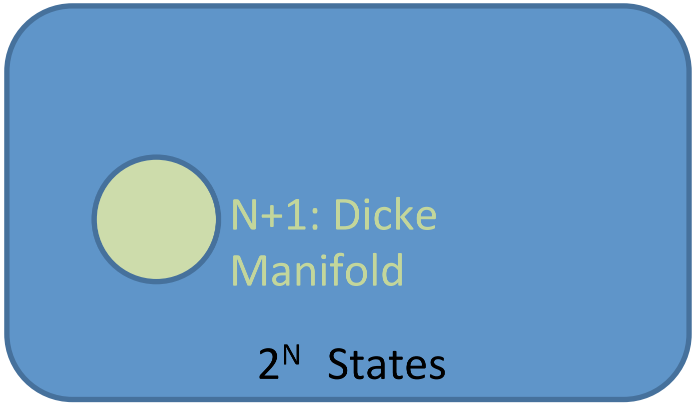
Measured by entanglement entropy

$$S_A=-\ln[\mathrm{Tr}(\rho_A^2)]$$

Renyi entropy: Purity of the A subsystem

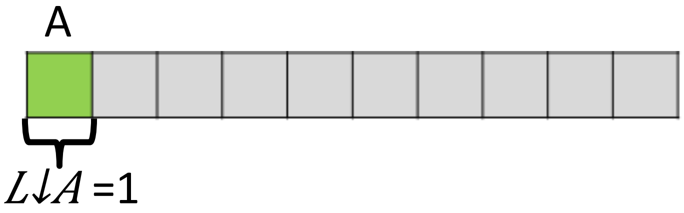


All to all Ising?

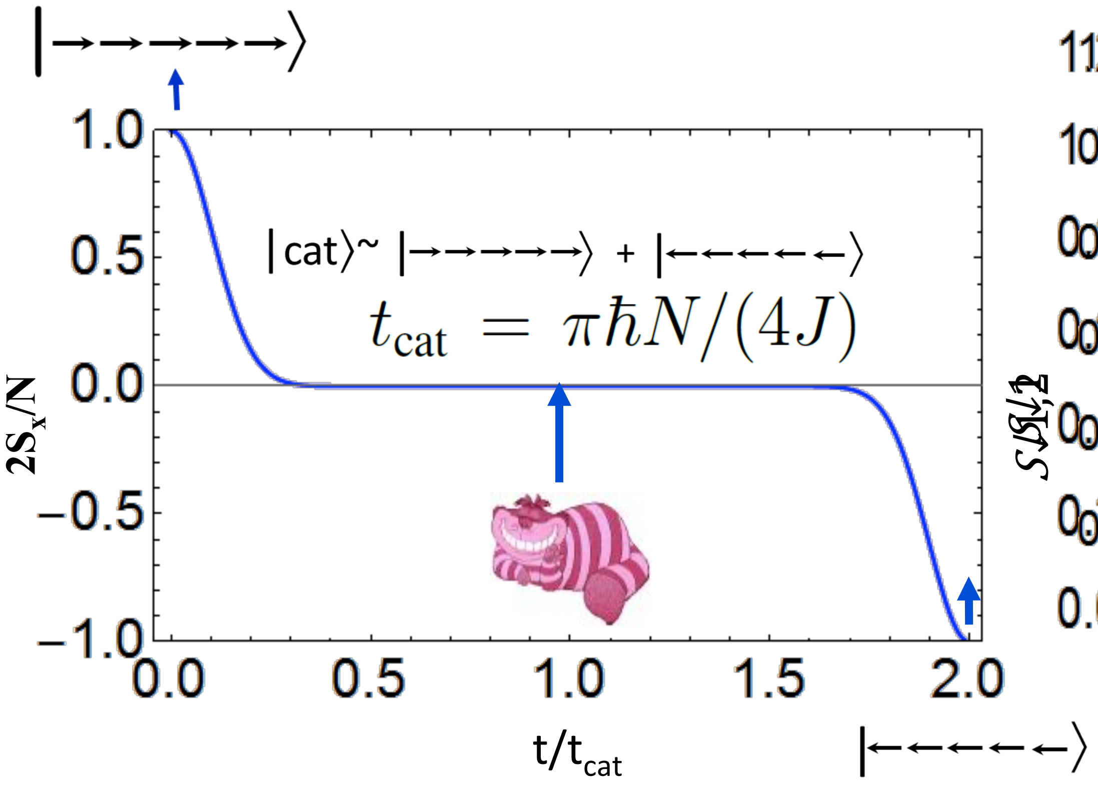


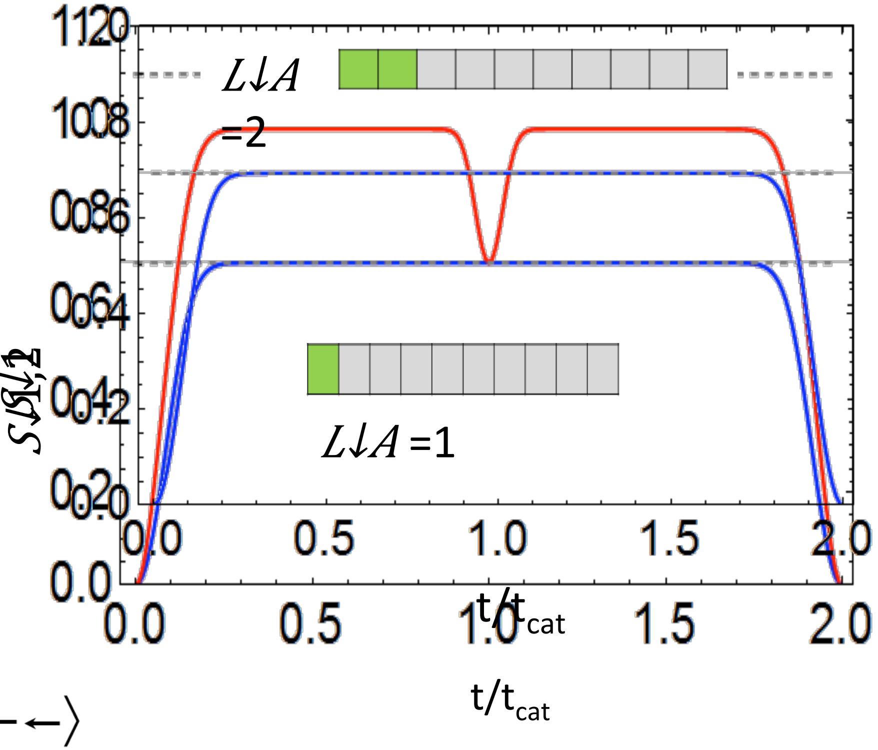
A. Kaufman *et al* Science 353, 794 (2016)

ALL-to-All Ising local thermalization?



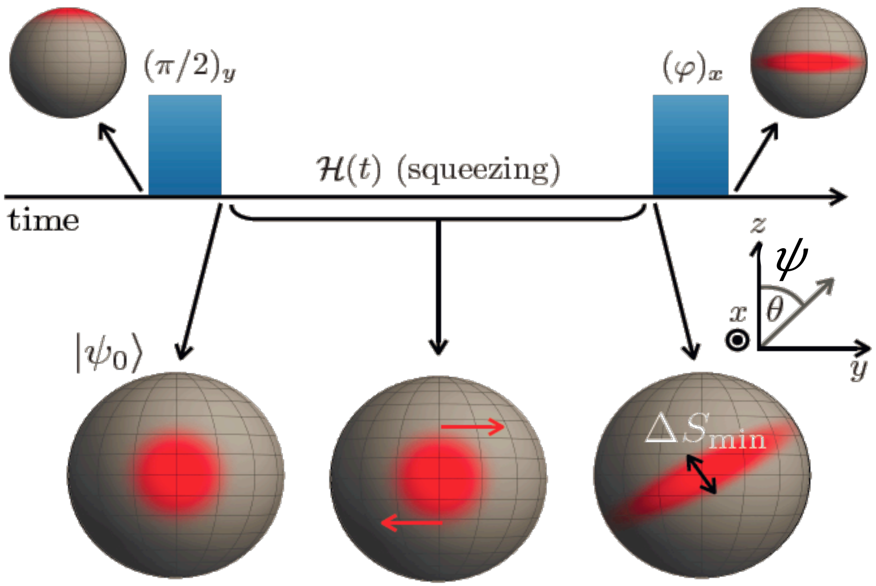
$S_1 = -\log[\frac{1}{2}(1+(\frac{2}{N}\langle S_x\rangle)^2)]$ One-tangle





K. Mølmer and A. Sørensen, PRL, 82,1835(1999)

Spin Squeezing



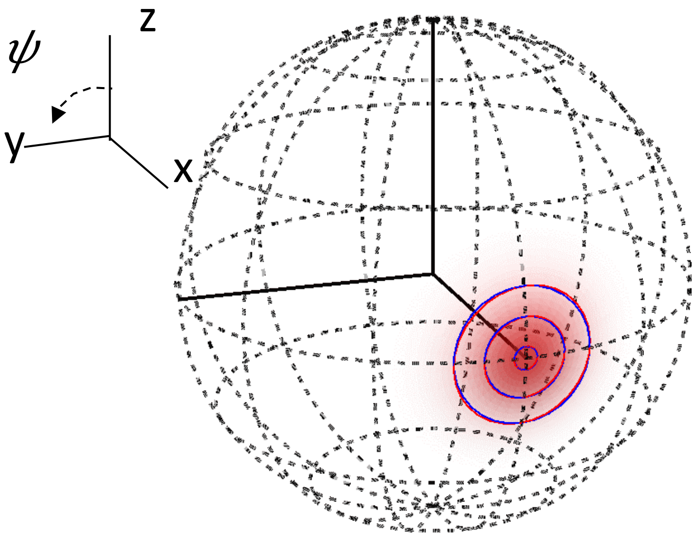
Squeezing parameter

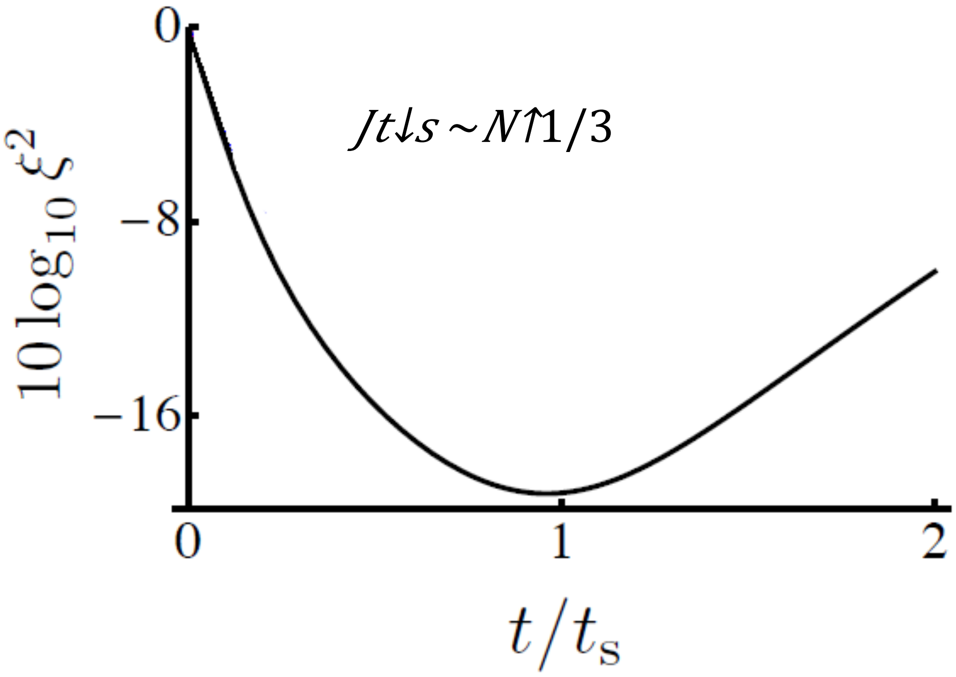
$$\xi(\psi)=\sqrt{N}\,\Delta S^{\psi}/\langle S^{x}\rangle$$

$$\xi^{2}<1$$

A. Sørensen *et al* Nature 409, 63 (2001)

- Entanglement witness
- Enhanced phase sensitivity
- Useful only for Gaussian states





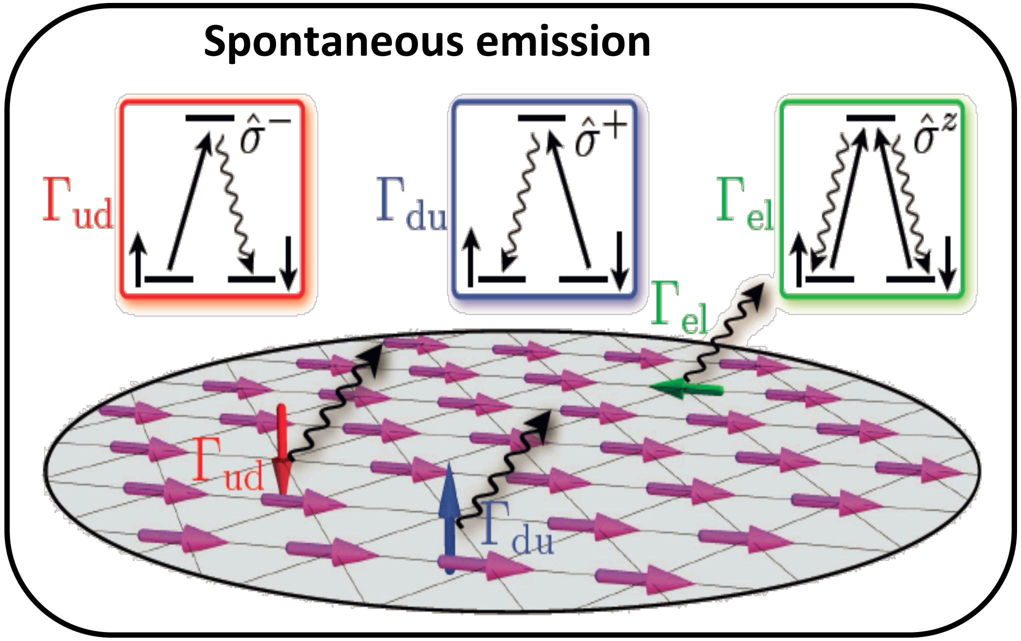
$Jt_{s}\sim N^{1/3}$

Exact Ising solution with decoherence

Relevant in trapped ion experiments

$$\dot{\rho} = -\mathrm{i}[H,\rho] + \boxed{\sum_{\nu=-1}^{1} \mathcal{L}_{\Gamma^{\nu}}[\rho]}$$

↓ =∑i,j↑▒ J↓ij ↓ σ↓i↑z σ↓j↑z



Spontaneous emission

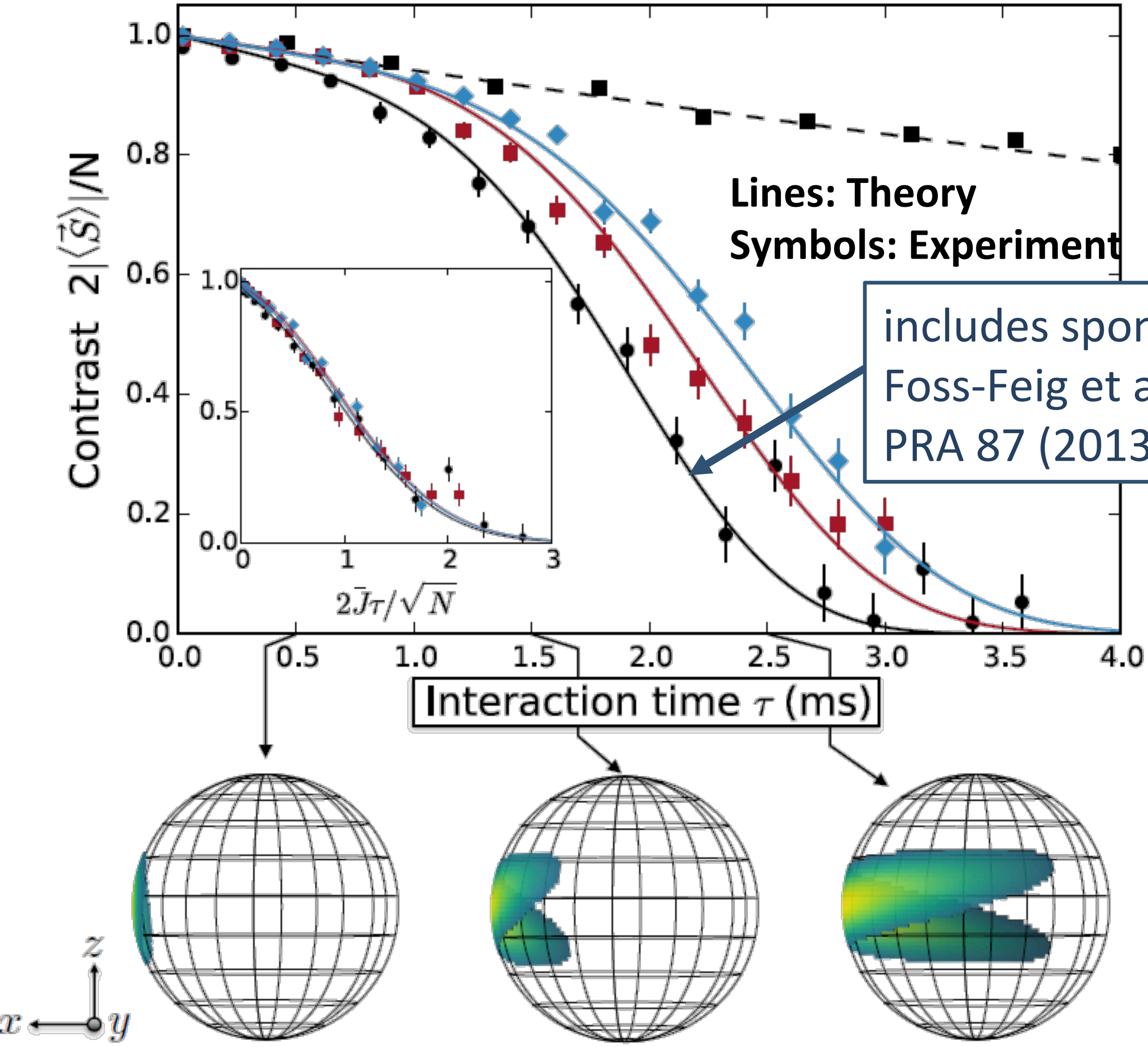
$$\mathcal{L}_{\Gamma^{\nu}}[\rho] = -\sum_j \left(A_j^{\nu\dagger} A_j^{\nu} \rho + \rho A_j^{\nu\dagger} A_j^{\nu} - 2 A_j^{\nu} \rho A_j^{\nu\dagger} \right)$$

We have been able to derive exact solutions for all correlation functions

M. Foss-Feig, *et al* PRA 87, 042101 (2013).

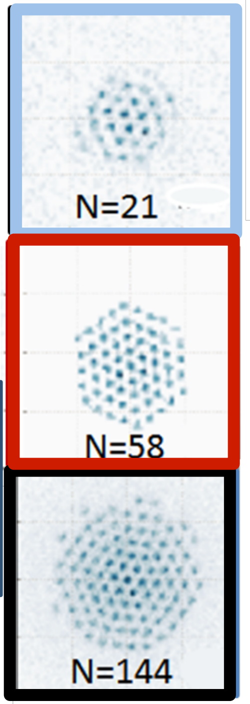
Experiment: Depolarization

- Coherent spin depolarization: Bloch vector length $|\langle S \rangle|$ vs time



Lines: Theory
Symbols: Experiment

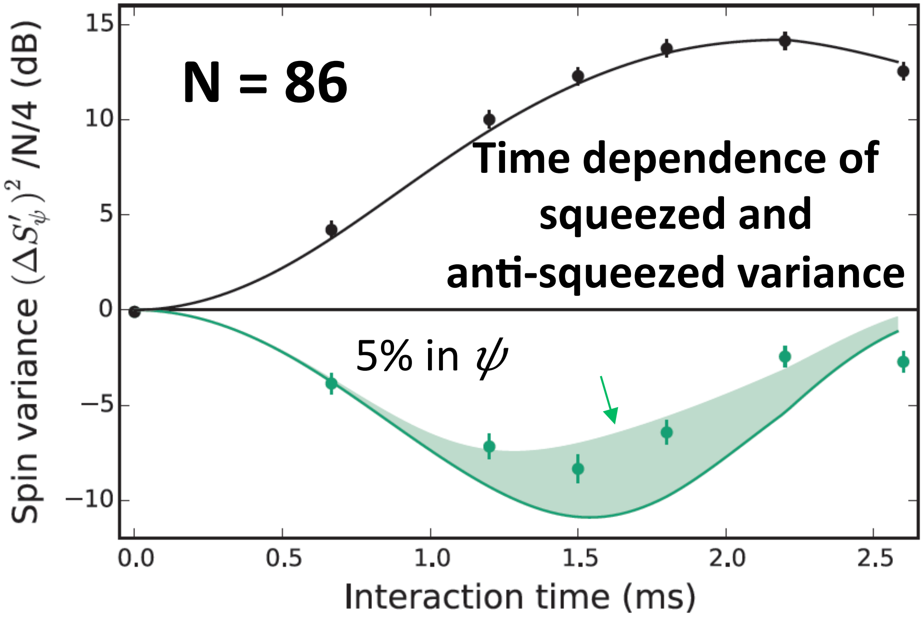
includes spontaneous emission
Foss-Feig et al., NJP 15 (2013)
PRA 87 (2013)



Bohnet *et al.*, Science352,1297(2016).

Beyond mean field effects at $\theta=\pi/2$!!

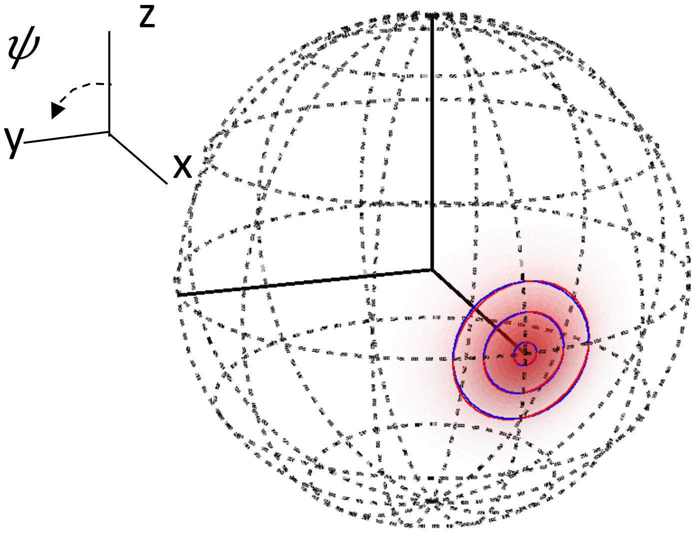
Experiment: Squeezing



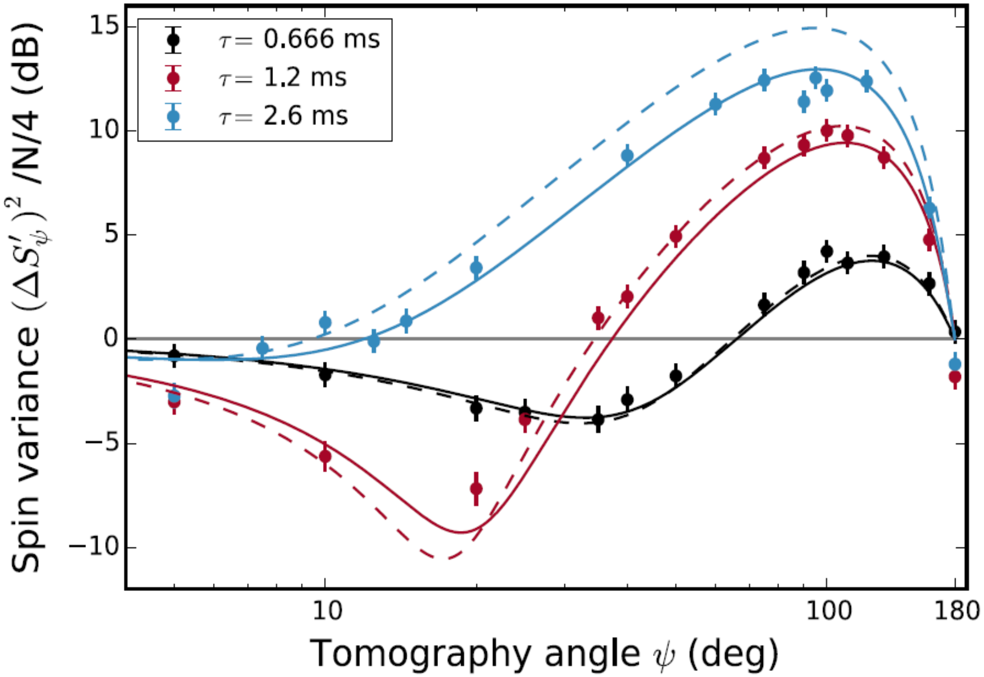
N = 86

Time dependence of squeezed and anti-squeezed variance

- largest inferred squeezing: $10\log_{10}\xi^2 =$ -6.0 dB

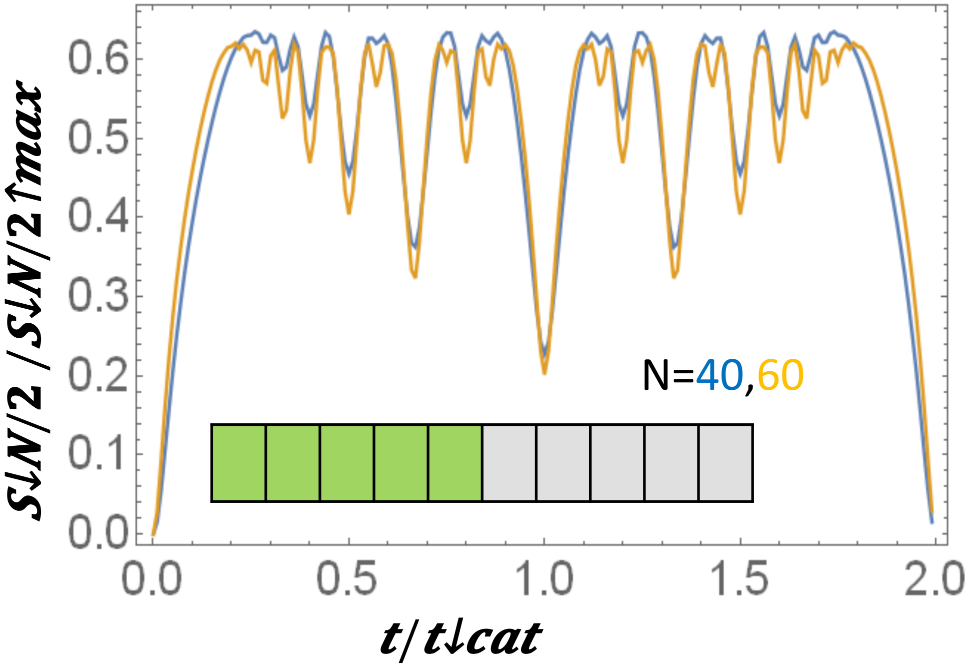


solid: Full
Dashed: No decoherence

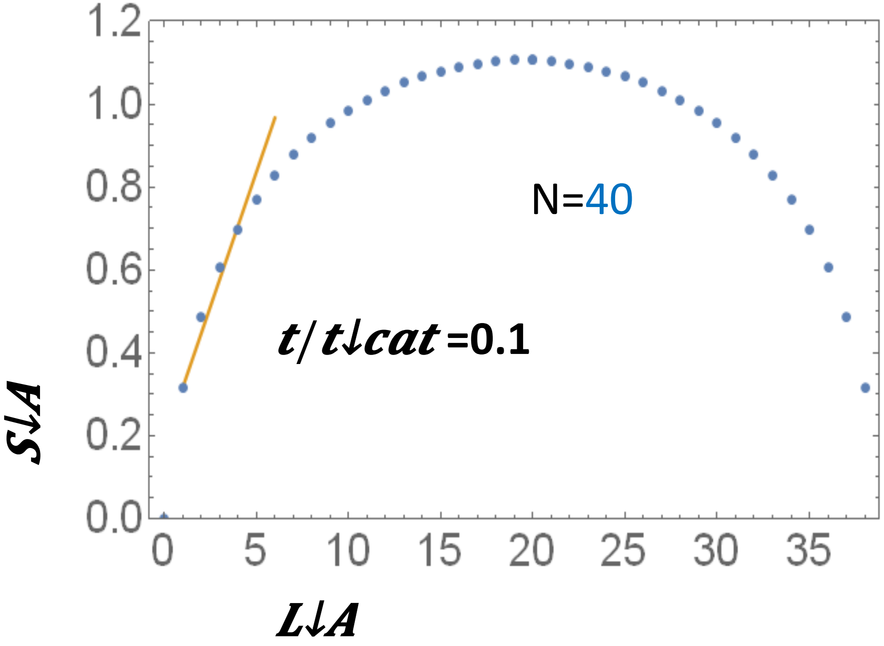


ALL-to-All Ising entanglement entropy?

- Growth of S_A for $L_A=N/2$



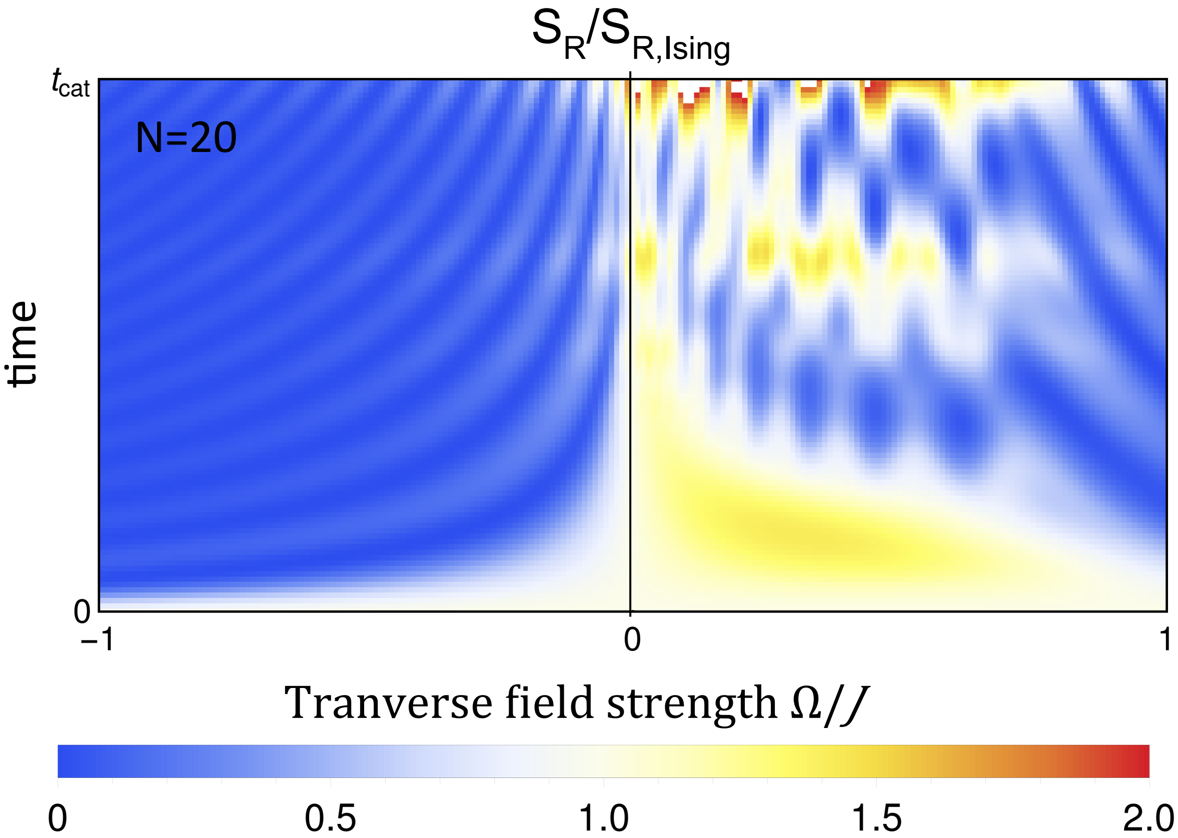
- Linear growth with L_A



How fast entanglement develops in the Ising case compare with other collective models?

Renyi entropy: Ising vs Transverse field Ising

$$H = -\frac{J}{N} S_z^2 \qquad\qquad H = -\frac{J}{N} S_z^2 - \Omega S_x$$

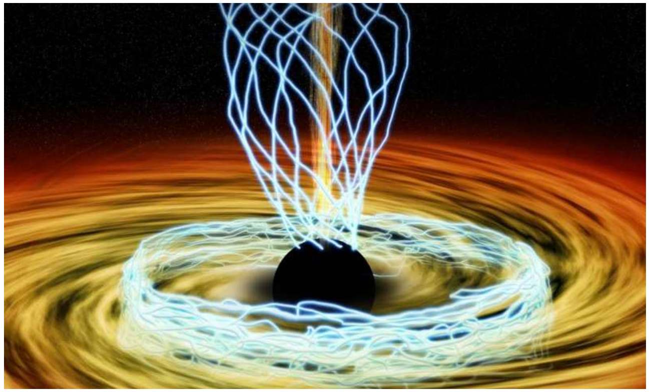


- ✓ Ising buildups up in general faster entanglement than the transverse field Ising
- ✓ Can we measure in the experiment entanglement buildup?

Quantum Scrambling

- Scrambling occurs when local quantum information, e.g. a local perturbation, is spread over all the degrees of freedom of a system, becoming inaccessible to local measurements
- Link to entanglement entropy: thermalization
- Cousin of classical chaos [Maldacena-Shenker-Stanford][Martinis'16]
- Connections to quantum gravity: Black holes scramble quantum information as fast as possible

  [Hayden-Preskill, Sekino-Susskind, Shenker-Stanford '13, Kitaev '14]



- Scrambling: Measured by out-of-time order correlations (OTOCs)
- Bounds on scrambling: Pure states? Quantum quenches?

Out-of-time-order correlators (OTOCs)

Given two commuting operators V and W, define the OTO correlator:

$$F(t) = \langle W_t^\dagger V^\dagger W_t V \rangle$$

$$W_t = e^{iHt} W e^{-iHt}$$ Heisenberg operator

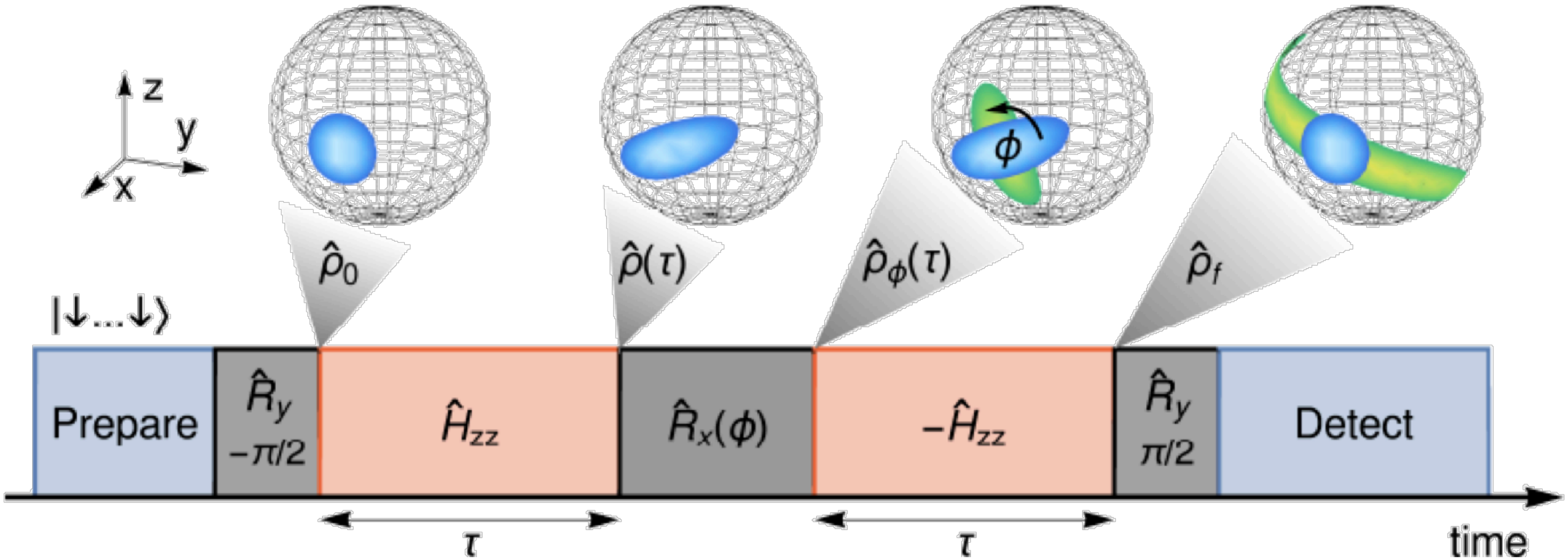
F measures the degree of non-commutativity of V and the time evolved version of W:

$$C(t) = \langle [W_t, V]^\dagger [W_t, V] \rangle = 2 - 2\,\mathrm{Re}[F]$$

- Can we asses them in an experiment?

[Swingle-Bentsen-Schleier-Smith-Hayden ‘16]
[Zhu-Hafezi-Grover ‘16]
[Yao-Grusdt-BGS-Lukin-StamperKurn-Moore-Demler ‘16]

Measuring OTOCS in trapped ions



J↓ ~J↓0 /δ↓

J↓ ~−J↓0 /δ↓

|+⟩=|↑⟩+|↓⟩/√2

Measure initial state overlap: ρ ↓0↑ =|++...+⟩⟨++...+|

↓ϕ (τ)=⟨ρ↓0 ⟩=⟨Ψ↓0 | e↑−iτH↓zz R↓x↑† ↓↑ (ϕ) e↑−iτH↓zz ρ ↓0↑ e↑i

H↓zz R↓x↑ (ϕ)e↑−iτH↓zz |Ψ↓0 ⟩

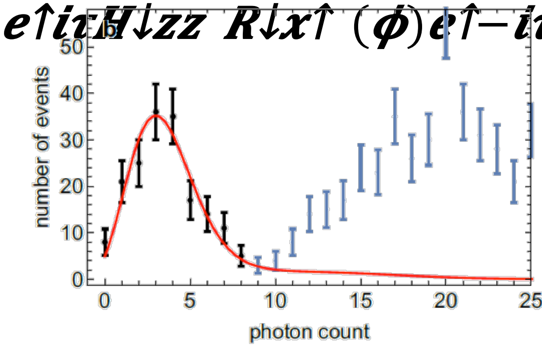
=⟨Ψ↓0 | e↑iτH↓zz R↓x↑† ↓↑ (ϕ) e↑−iτH↓zz ρ ↓0↑ ↓↑ e↑iτH↓zz R↓x↑ (ϕ)e↑−iτH

V

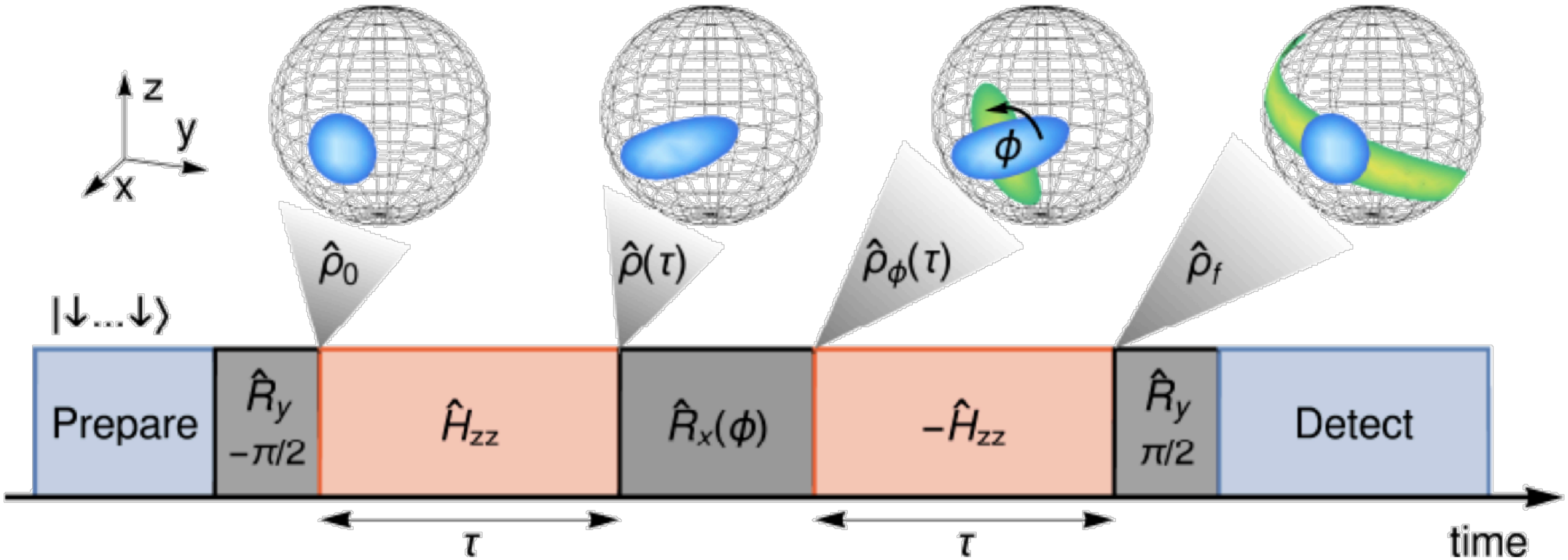
W↓t

V

W↓t
↑†



Measuring OTOCS in trapped ions



$J\downarrow \sim J\downarrow 0 /\delta\downarrow$ $J\downarrow \sim -J\downarrow 0 /\delta\downarrow$

Measure Magnetization: $S\downarrow x\uparrow$

$$F\downarrow\phi\ (\tau)=2/N\ \langle S\downarrow x\ \rangle=\langle\Psi\downarrow 0\ |\ e\uparrow -i\tau H\downarrow zz\ \ R\downarrow x\uparrow\dagger\downarrow\uparrow\ (\phi)\ e\uparrow -i\tau H\downarrow zz\ \ \sigma\downarrow i\uparrow x$$
$$e\uparrow i\tau H\downarrow zz\ \ R\downarrow x\uparrow\ (\phi) e\uparrow -i\tau H\downarrow zz\ |\Psi\downarrow 0\ \rangle$$

$$=2/N\ \langle\Psi\downarrow 0\ |\ e\uparrow i\tau H\downarrow zz\ \ R\downarrow x\uparrow\dagger\downarrow\uparrow\ (\phi)\ e\uparrow -i\tau H\downarrow zz\ \sigma\downarrow i\uparrow x\downarrow\uparrow\ \ e\uparrow i\tau H\downarrow zz\ R\downarrow x\uparrow\ (\phi$$

V $W\downarrow t$ V

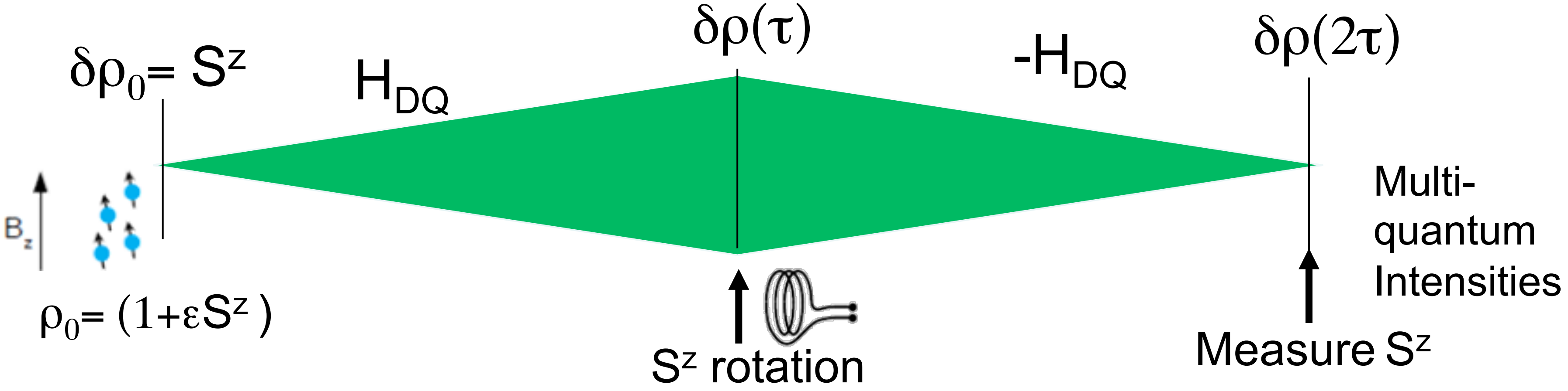
$W\downarrow t$
$\uparrow\dagger$

Same measurements done in NMR: Multiple Quantum Coherence

Multi-quantum Coherences

Multi-Quantum coherence spectrum (NMR)

$$H_{DQ} \propto \sum J_{ij} (\sigma_i^+ \sigma_j^+ + \sigma_i^- \sigma_j^-)$$



$\delta\rho_0 = S^z$

H_{DQ}

$\delta\rho(\tau)$

$-H_{DQ}$

$\delta\rho(2\tau)$

B_z

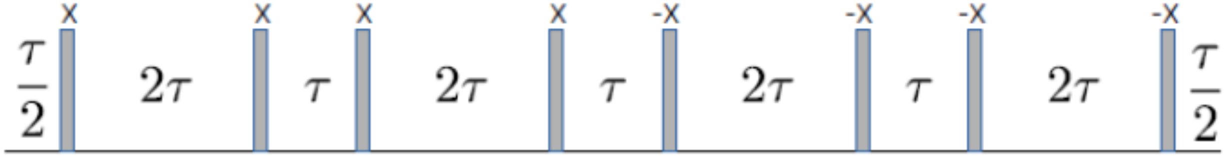
$\rho_0 = (1+\varepsilon S^z)$

S^z rotation

Measure S^z

Multi-quantum Intensities

H_{DQ}: Is obtained from the dipole-dipole H_{ZZ} by pules:



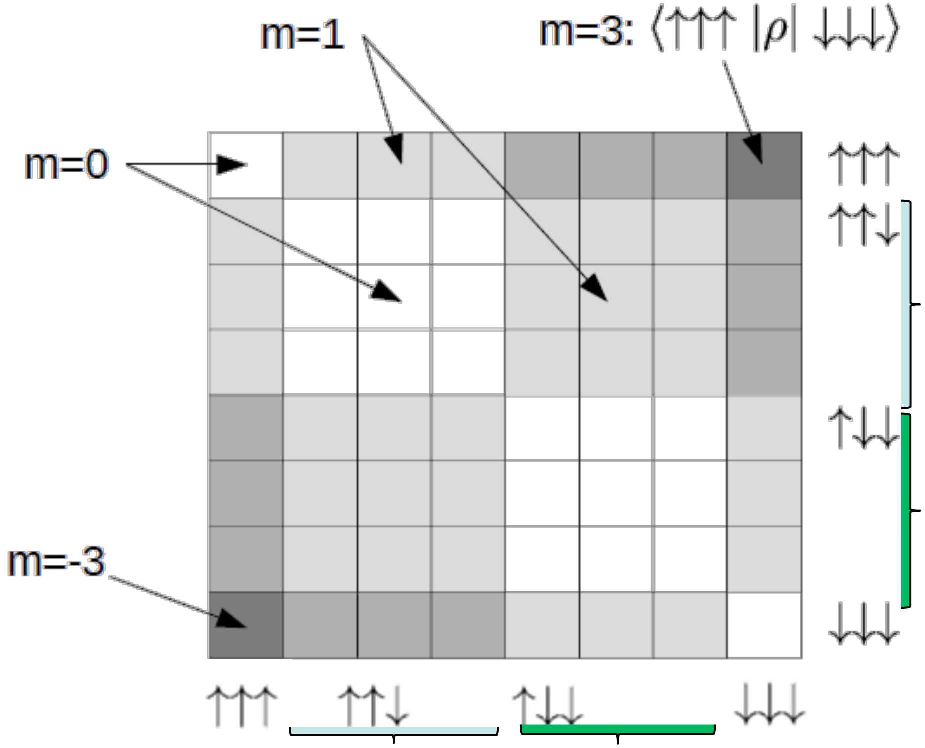
The same sequence with y- instead of x-rotations gives $H_{zz} \to -H_{DQ}$

M. Munowitz and M. Mehring , Sol. St. Com., 64, 605 (1987)

Multi-quantum Coherences

- Divide the density matrix into blocks wrt. to the multi-quantum order m:

$\rho=\sum_{m}\rho_{m}$ ρ_m contains all matrix elements with coherences between states differing in S_z by m



- Double Quantum Hamiltonian changes m by ±2 in each time step.

→ “Multi-Quantum spectrum” builds up and broadens with time.

$$\langle S_z\rangle=\langle\delta\rho_0\rangle=\sum_{m=-N}^{N}\underbrace{\mathrm{Tr}[\rho_{-m}(t)\rho_m(t)]}\,e^{-im\phi}$$

I_m=Multi-quantum intensities

Requirements:
1) Invert many-body time evolution.
2) Measure initial state.

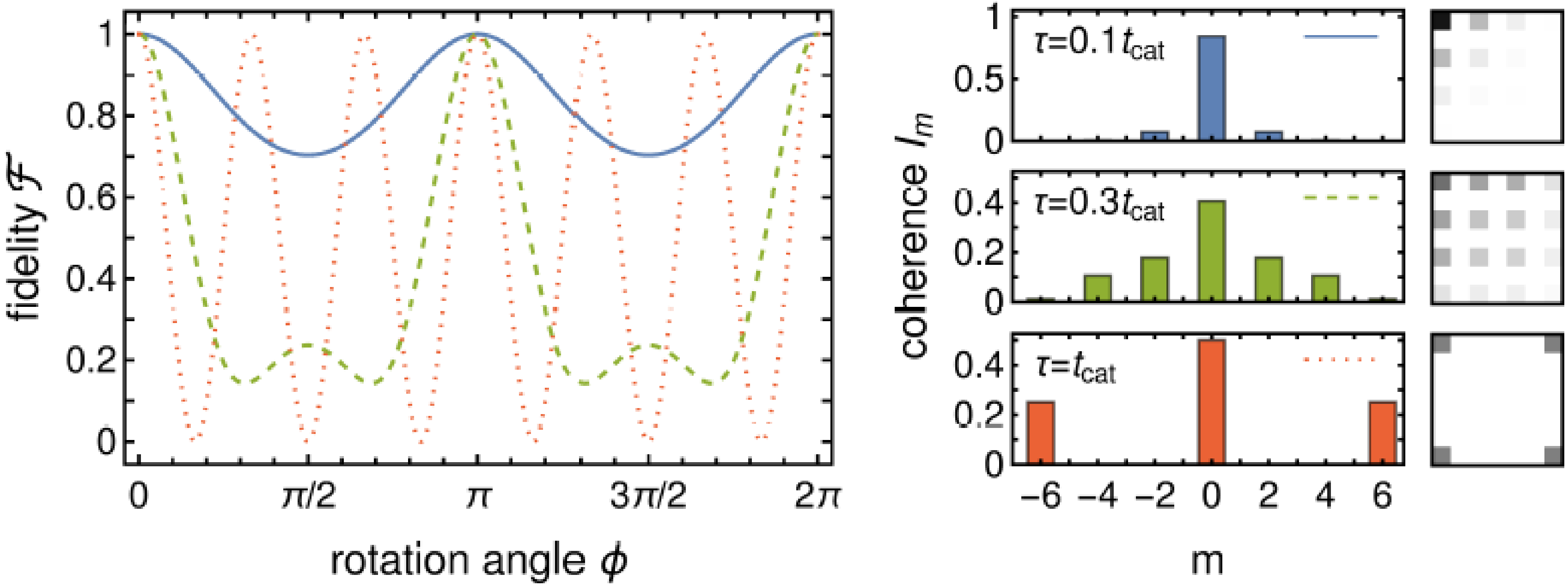
Fourier transform: ϕ gives the Multi-Quantum spectrum.

M. Munowitz and M. Mehring , Sol. St. Com., 64, 605 (1987)

Inspired by NMR we measure family of OTOCs

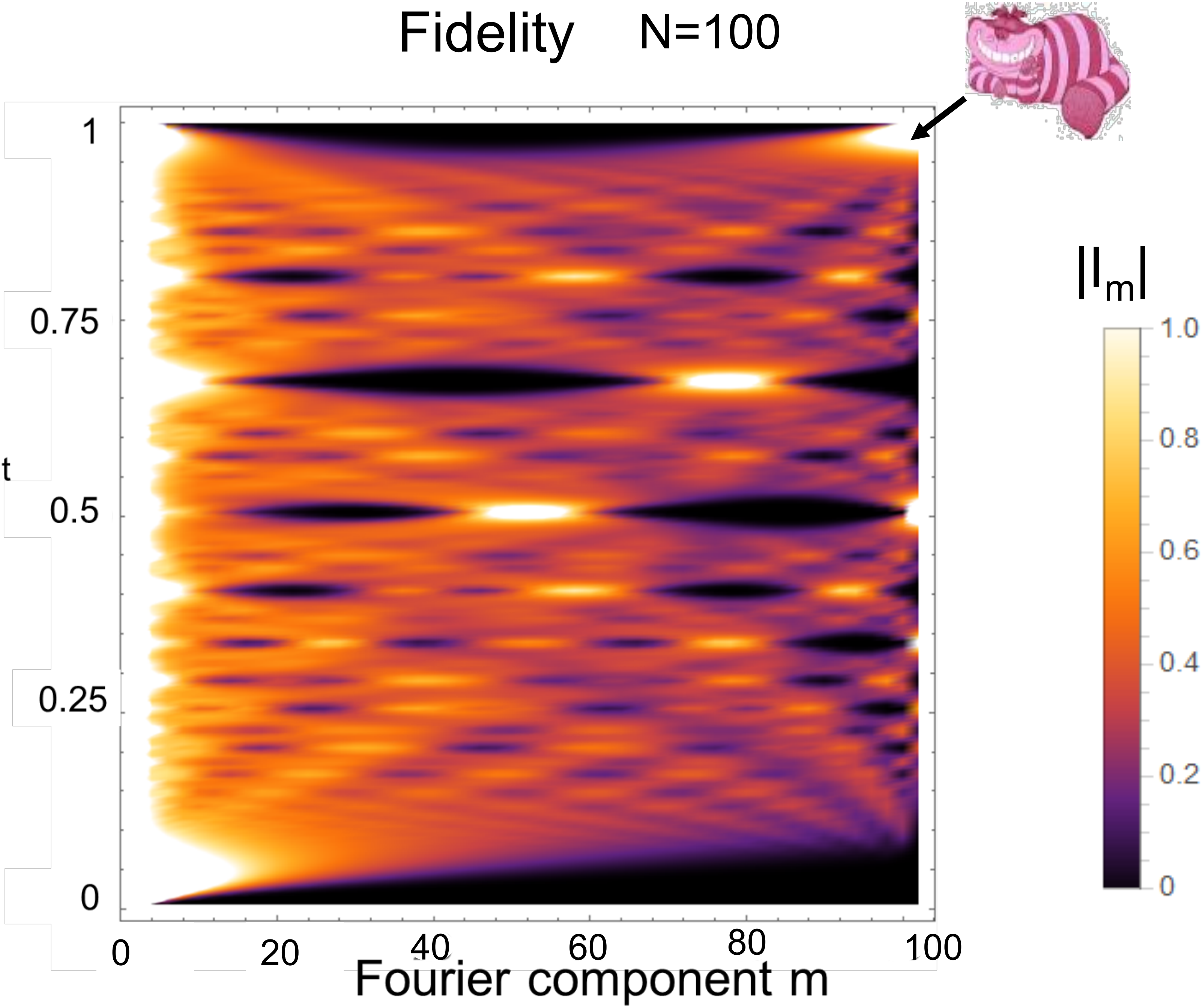
$$\Im_{\boldsymbol{\phi}}(\boldsymbol{\tau})=\langle\boldsymbol{\rho}_{\mathbf{0}}\rangle=\sum_{\boldsymbol{m}=-\boldsymbol{N}}^{\boldsymbol{N}} \boldsymbol{I}_{\boldsymbol{m}}(\boldsymbol{\tau})\boldsymbol{e}^{-\boldsymbol{im\phi}}$$

Fourier component: $\boldsymbol{I}_{\boldsymbol{m}}$ →m-body coherences



$$N = 6$$

Fidelity N=100

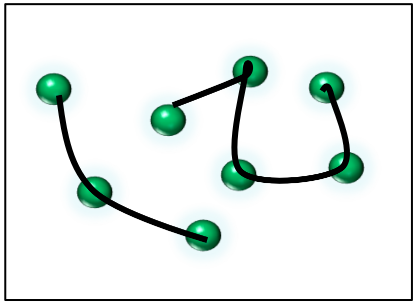


Inspired by NMR we measure family of OTOCs

$$F_{\phi}(\tau)=2/N\,\langle S_{x}\rangle=\sum_{m=-N}^{N} A_{m}(\tau)e^{-im\phi}$$

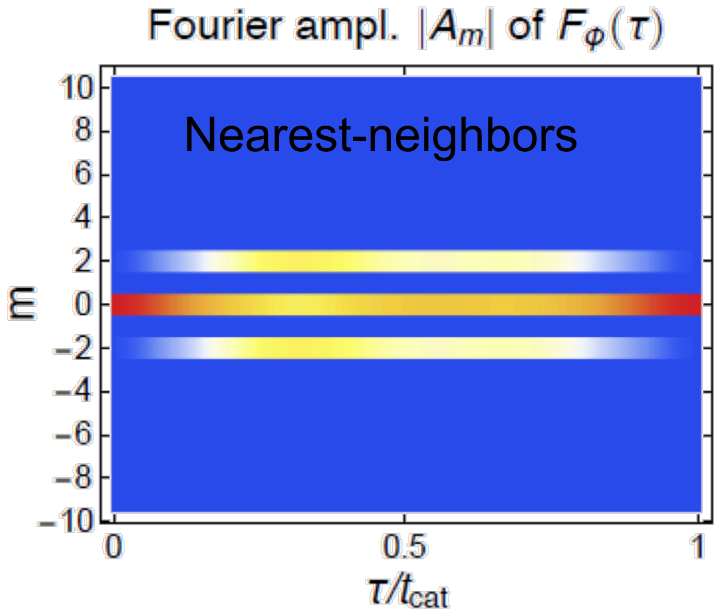
Fourier component: A_m →m-body correlation

- In the case of the Ising model:
- A non-zero A_m signals the existence of m spins directly coupled by the Hamiltonian.

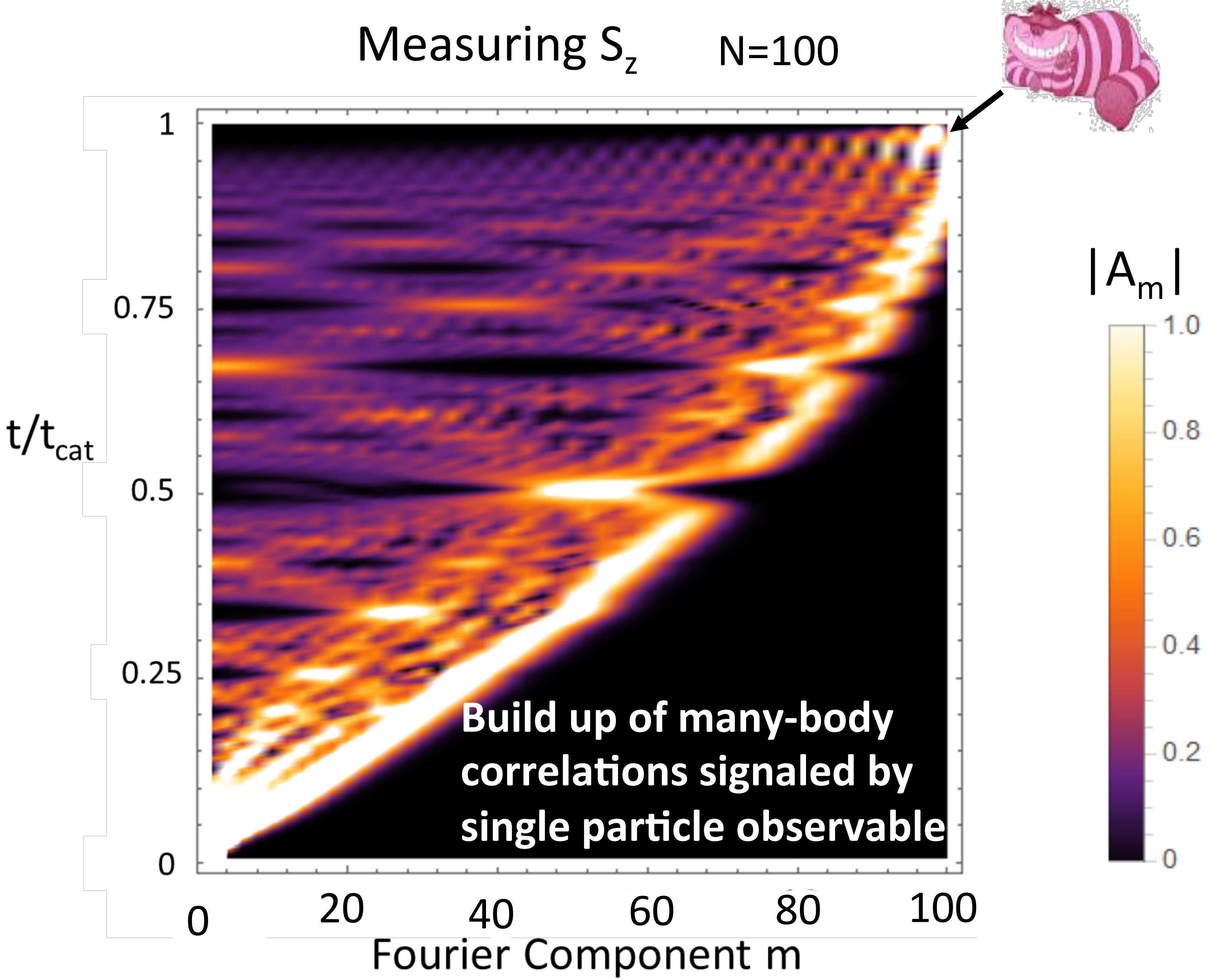


$A_{m>5}=0$

- A_{m+1} grows as t^p with p>m



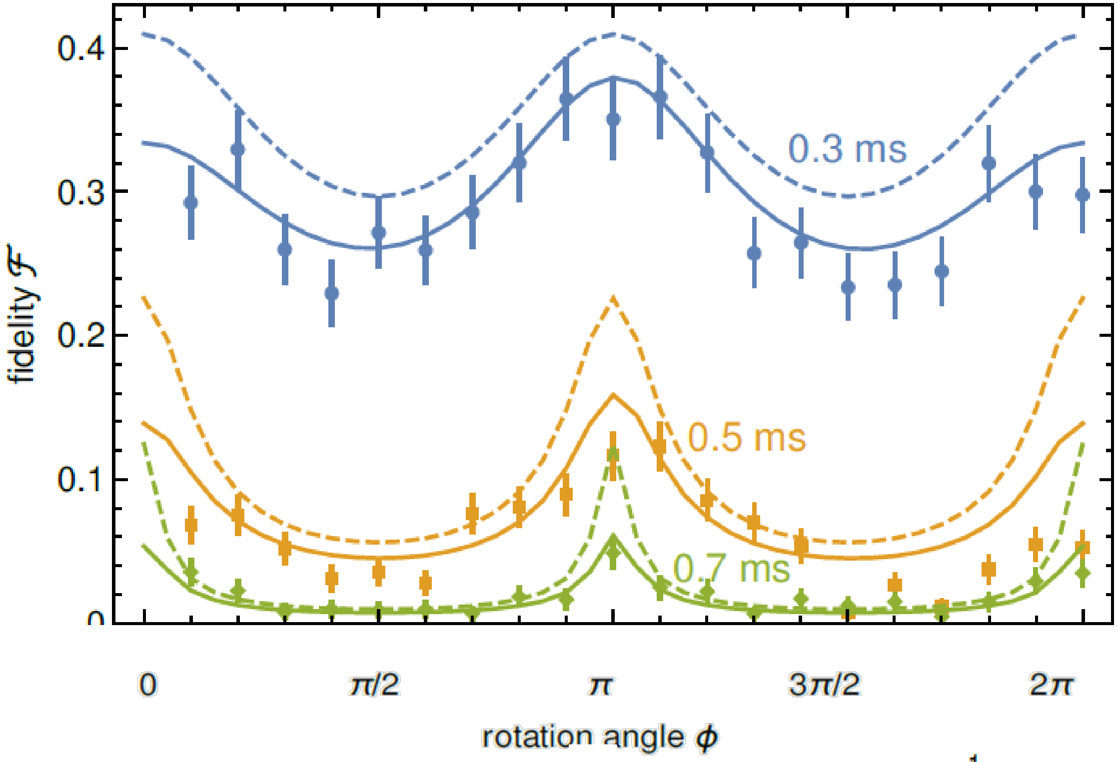
Measuring S_z N=100



Build up of many-body correlations signaled by single particle observable

Fidelity Measurements

N=48



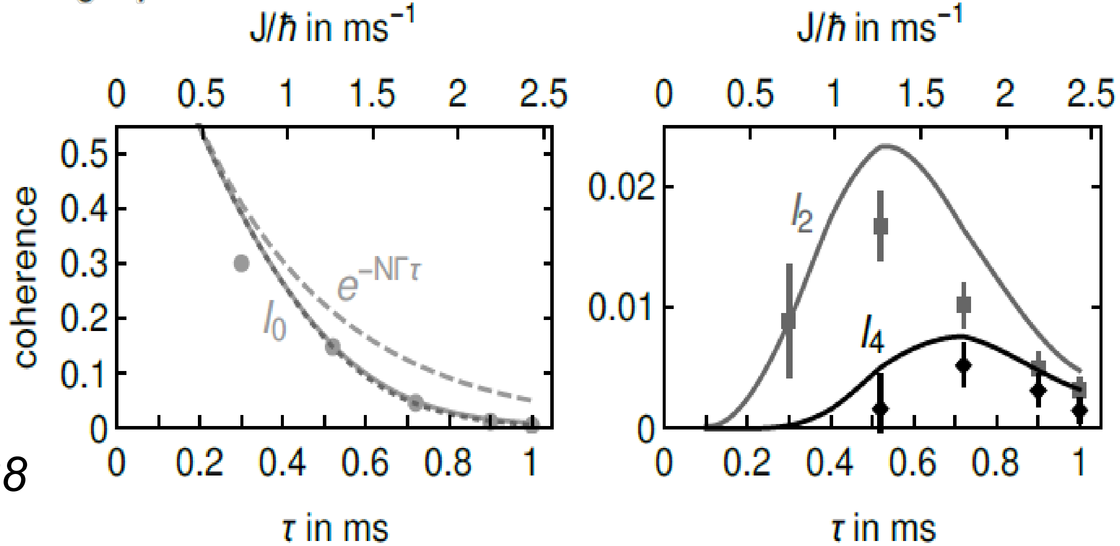
Solid lines: decoherence + phonons

Dashed Lines: decoherence

τ_{cat}~15 ms

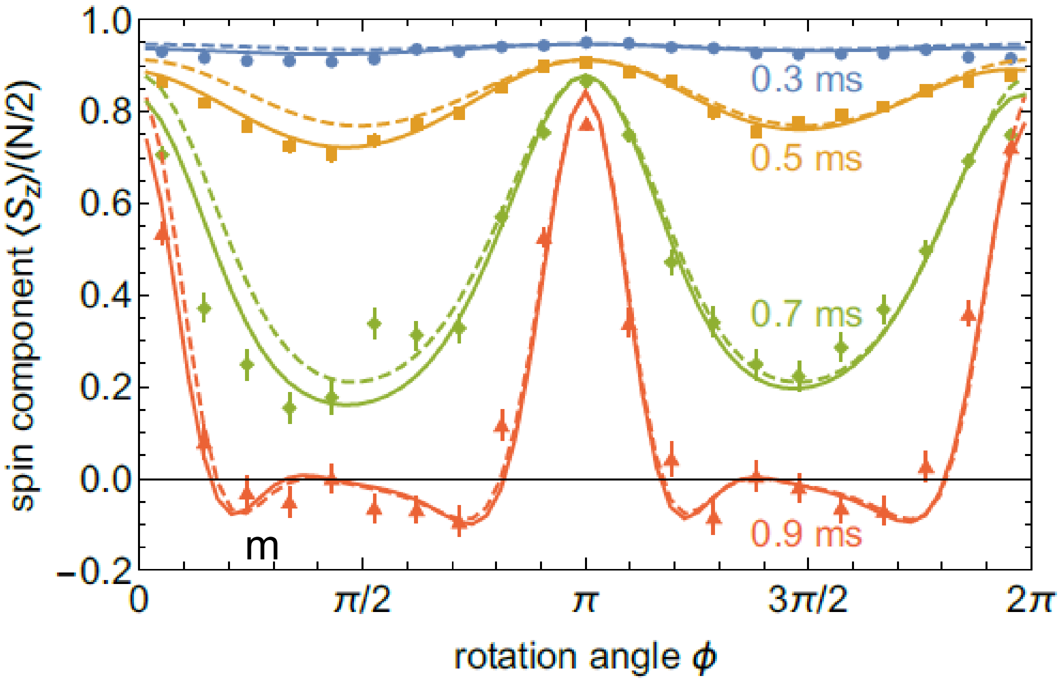
/↓0↑ (τ)=e↑−ΓNτ
/↓0↑pure

I↓0↑pure (τ)=(1+J↑2
τ↑2)↑−1



Arxiv: Garttner et *al1608.08938*

Polarization Measurements

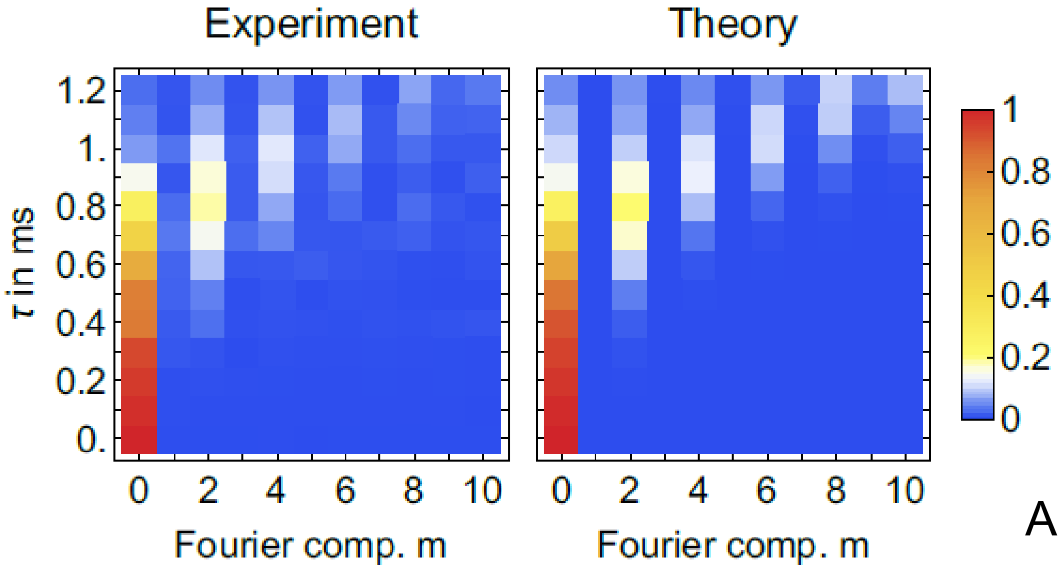


N=111

Solid lines: decoherence + phonons

Dasehd lines: decoherence

τ_{cat}=17 ms



Up to m=8 significant correlations!!

Arxiv: Garttner *et al*,1608.08938

Entanglement Witness

Quantum Fisher Information, F_Q: Sensitivity of a quantum state with respect to an unitary transformation parametrized by a classical parameter:

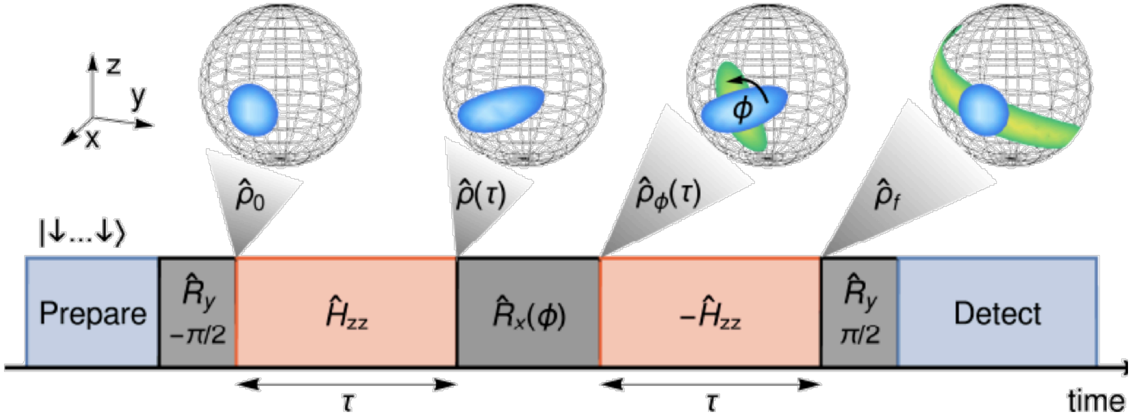
- It has been shown to be a multipartite entanglement criteria
- It determines the phase sensitivity of state with respect to SU(2) rotations[classical parameter: phase]

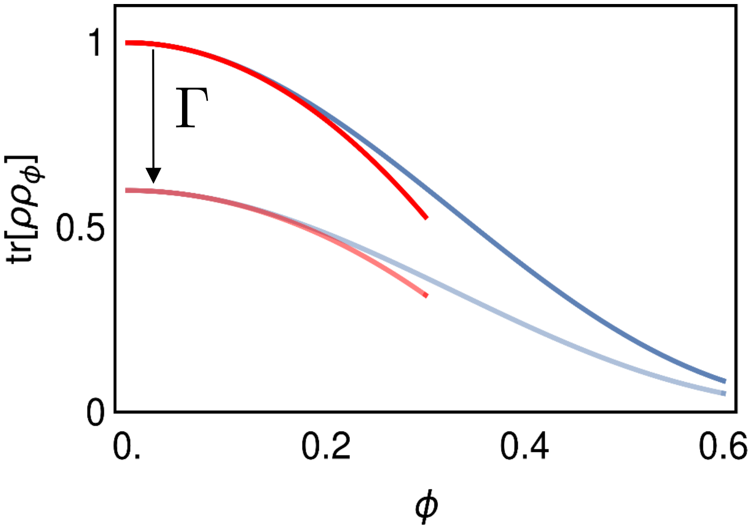
$\leq F_Q$

$$F_Q \geq 2\sum_{m=-N}^{m=N} m^2 I_m$$

Lower bound of Quantum Fisher Information

Pure states saturate equality





$$\Im_\phi(\tau) = Tr[\rho\rho_\phi$$

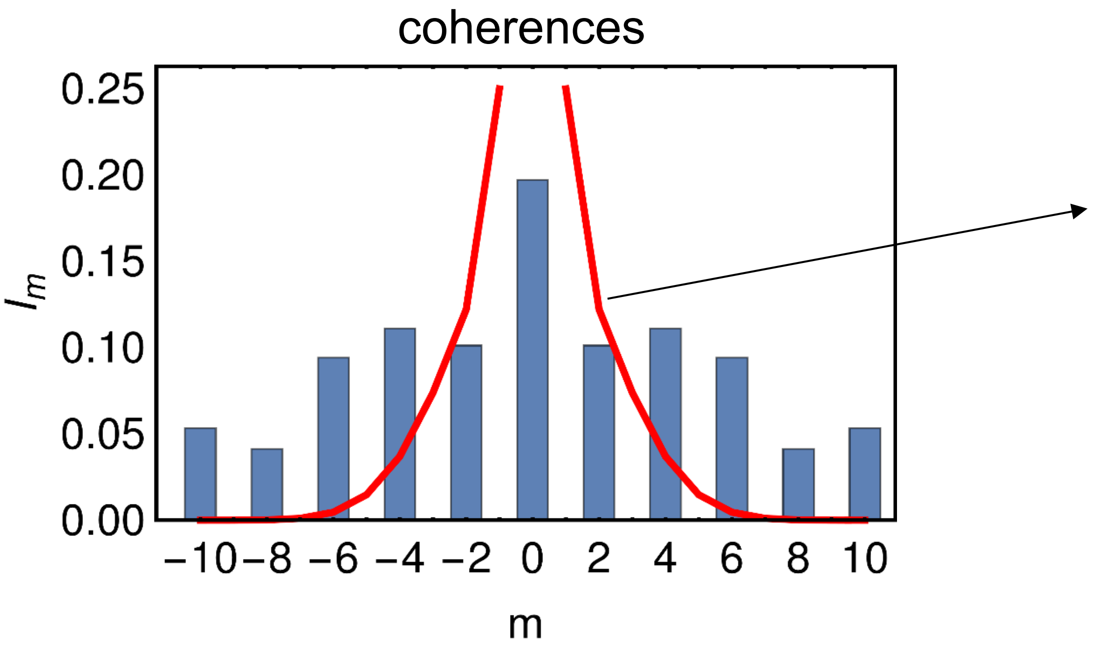
$$\mathrm{tr}[\rho\rho_\phi] = 1 - \frac{A}{4}\phi^2$$

$$A \leq F_Q$$

Entanglement Witness

Individual *Im can detect entanglement*

- For separable states coherences are bounded
- Robust :degree of violation of the bound increases exponentially with m.
- $I{\downarrow}m=N\downarrow \propto|GHZ\rangle\langle GHZ|$ *witness of N−partite entanglement*



Separable states

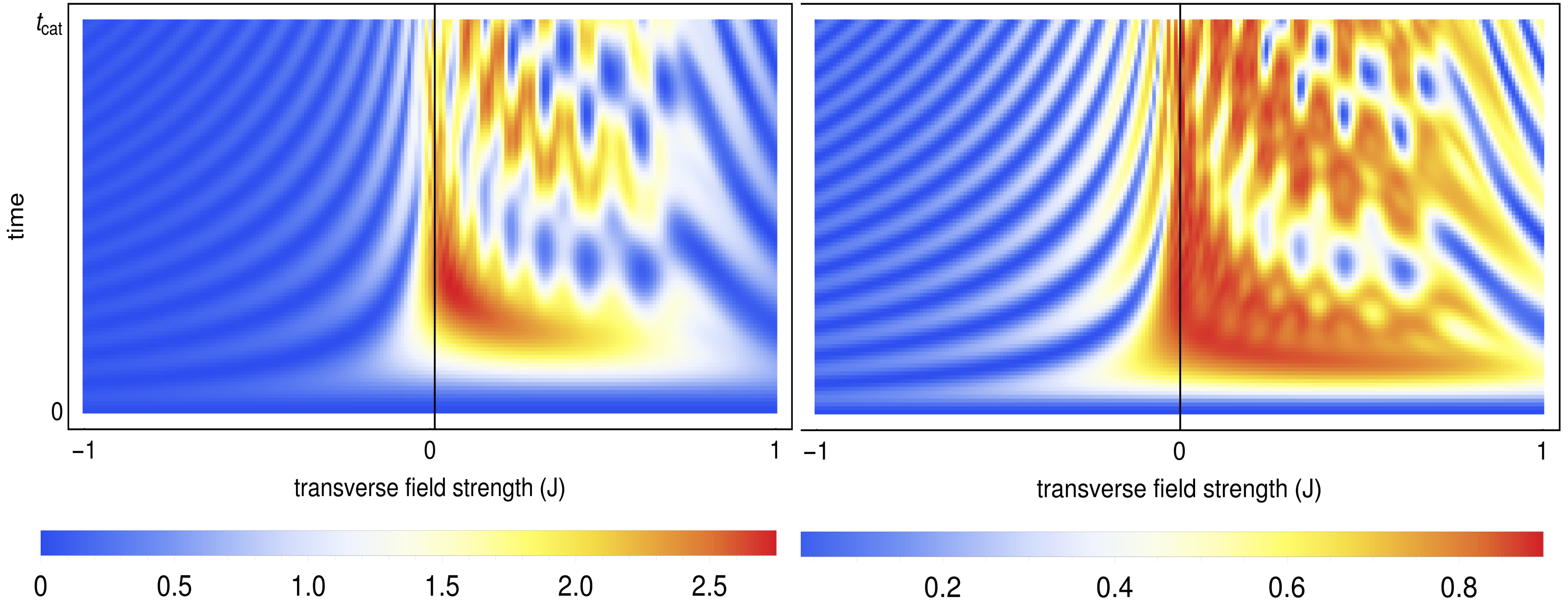
Connection to Entanglement Entropy

$$H = -\frac{J}{N} S_z^2 - \Omega S_x$$

Renyi Entropy: S_R

1-I_0

$N = 20$



Why?

I↓0 (τ)↑ =1/2π ∫0↑2π▒ dϕℑ↓ϕ (τ)

e↑−S↓A =∑W∈A↑▒Tr [W↓t↑† ↓↑ O↓↑ ↑ e↑−βH↓ O↓↑† W↓t↑ O↓↑ e↑−βH↓ O↓ ↑†]

We sum over an incomplete set of operators

V=O↓↑ e↑−βH↓ O↓↑† =ρ↓0

Penning trap simulator: A great vista ahead !

Future Directions

- **Transverse field**, and variable range
- **Mitigate decoherence : sub-Doppler**
- **Spatial correlations –single ion readout**

Thank You!

- **Measure OTOCS**

- Generate and observe **spin squeezed states**

- **Implement time-reversible Ising interactions in 2D arrays of 100's of ions**