Gauging the Kitaev Chain

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Together with Ruben Verresen, Jeet Shah, Sergej Moroz

Introduction - Gauge Symmetry Revisited

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- Condensed matter: real crystal or optical lattices.

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- High energy physics: discretization for numerics and regularization.
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(Not so usual) questions

- Behavior of gauge theories in the presence of boundaries.
- Is there a way to interpolate between the gauged and ungauged theory? How do they differ?

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1D spinless fermions with NN hopping and pairing

$$H = it\sum_j \tilde{\gamma}_j \gamma_{j+1} + \frac{i\ \mu}{2}\sum_j \tilde{\gamma}_j \gamma_j .$$

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- Trivial, $|t/\mu| < 1/2.$ One ground state, no edge modes.
- Topological, $|t/\mu| > 1/2.$ Two ground states, edge modes.
- Falls into the SPT paradigm, but is protected by fermion parity which can never be broken!

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Do the edge-modes survive gauging?

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Hamiltonian

$$H = i\,t \sum_j \tilde{\gamma}_j \, \sigma^z_{j+1/2} \gamma_{j+1} + \frac{i\,\mu}{2} \sum_j \tilde{\gamma}_j \, \gamma_j - h \sum_j \sigma^x_{j+1/2},$$

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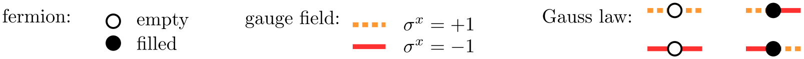
- Invariant under $G_i = \sigma^x_{i-1/2} (-1)^{n_i} \, \sigma^x_{j+1/2}$
- We restrict to the even sector $G_i = +1 \rightarrow$ Gauss' Law.



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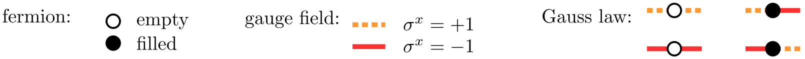

Symmetries

- Magnetic symmetry: $W := \prod_j \sigma^z_{j+1/2}$
- Fermion parity acts non-trivially only at the boundary.

$$P = \prod_i (-1)^{n^i} = \sigma^x_{1/2} \prod_i (\sigma^x_{i+1/2})^2 \sigma^x_{L+1/2} = \sigma^x_{1/2} \sigma^x_{L+1/2}$$

Spins in the bulk, Fermions at the edge

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- Local mapping: introduce gauge-invariant spin 1/2 variables:

$$X_{i+1/2} = \sigma^x_{i+1/2}, \qquad Y_{i+1/2} = -i\tilde{\gamma}_i \sigma^y_{i+1/2} \gamma_{i+1} \qquad Z_{i+1/2} = -i\tilde{\gamma}_i \sigma^z_{i+1/2} \gamma_{i+1}.$$

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- The Hamiltonian becomes:

$$H = \frac{\mu}{2} \sum_j X_{j-1/2} X_{j+1/2} - t \sum_j Z_{j+1/2} - h \sum_j X_{j+1/2}.$$

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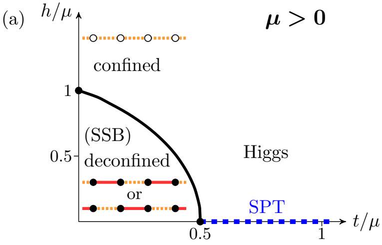
- Phase diagram is known, $\mathbb{Z}_2$ gauge theory interpretation is not!



Higgs = SPT

$$H = \frac{\mu}{2}\sum_{j=1}^{L} X_{j-1/2}X_{j+1/2} - t\sum_{j=1}^{L-1} Z_{j+1/2},$$

Edge operators #1

- Does not depend on $Z_{1/2}$ and $Z_{L+1/2}$ which are fermionic:

$$Z_{1/2} = \sigma^z_{1/2}\gamma_1, \qquad Z_{L+1/2} = \tilde{\gamma}_L \sigma^z_{L+1/2}$$

- Two additional symmetries localized at the edge: $X_{1/2}$ and $X_{L+1/2}$
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Edge operators #2

- One can construct

$$\gamma_{\mathsf{left}} = Y_{1/2} - \frac{\mu}{2t} Z_{1/2}Y_{1+1/2} + \left(-\frac{\mu}{2t}\right)^2 Z_{1/2}Z_{1+1/2}Y_{2+1/2} + \cdots$$

- $[\gamma_{\mathsf{left}}, H] = O\left(\left(-\frac{\mu}{2t}\right)^L\right) \longrightarrow 0$ in the Higgs phase.
- $X_{1/2}$ and γ_{left} anticommute: 2-fold GS degeneracy at left edge!
- Same for right edge. 4-fold in total!

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Motivation

- Gauging is a drastic operation, radically changes the physics at play.
- Can the gauged and ungauged models emerge from a unified framework?

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Hamiltonian

- We consider the Hamiltonian

$$H = \sum_j \left(it\tilde{\gamma}_j \sigma^z_{j+1/2}\gamma_{j+1} + i\frac{\mu}{2}\tilde{\gamma}_j\gamma_j - h\sigma^x_{j+1/2} - iK\sigma^x_{j-1/2}\tilde{\gamma}_j\gamma_j\sigma^x_{j+1/2} - \frac{t^2}{K}\sigma^z_{i+1/2} \right)$$

- As $K \to 0$, we recover the Kitaev chain. Gauge d.o.f. are still present, but frozen to $\sigma^z = 1$ due to the $1/K$ term.
- For $K \to \infty$ we have the gauged Kitaev chain. The Gauss law is enforced energetically by the K term.

Phase diagram at $h = 0$

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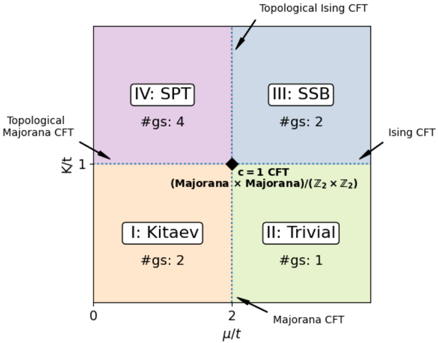
Global $\mathbb{Z}_2^f \times \mathbb{Z}_2$ symmetry

- Parity $P = \prod_j \left(i\tilde{\gamma}_j\gamma_j\right)$
- Wilson Loop $W = \prod_j \sigma^z_{j+1/2}$



- Phases are characterized in terms of these two symmetries.

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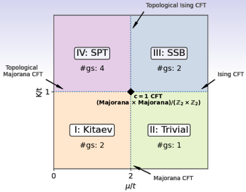

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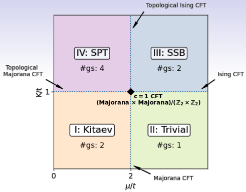

Phases I and II

These are the topological and trivial Kitaev phases respectively. Link spins form a paramagnet.

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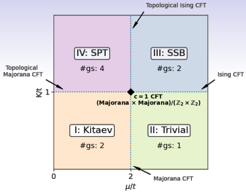

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Phase III

As $\mu \to \infty$ all sites are occupied: $i\tilde{\gamma}_i \gamma_i = -1$. Hamiltonian for the links:

$$H = \sum_j K \sigma^x_{j-1/2} \sigma^x_{j+1/2} - \frac{t^2}{K} \sigma^z_{j+1/2}$$

Ising transition in the link variables. Phase III ($|K/t| > 1$) is ordered (SSB). Connects to the deconfined phase of the gauged Kitaev chain.

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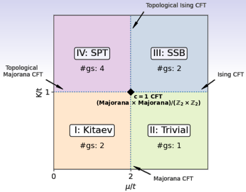

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Phase IV

This is the SPT (Higgs) phase of the gauged Kitaev chain. It can be mapped to a stack of two Kitaev chains through a *local* mapping.

Phase diagram at finite h

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- $K \to 0$ physics is unchanged. The h-term here is negligible.
- As $K \to \infty$ the symmetry broken phase survives only for $\mu > 0$. The SPT character of the Higgs phase is lost.
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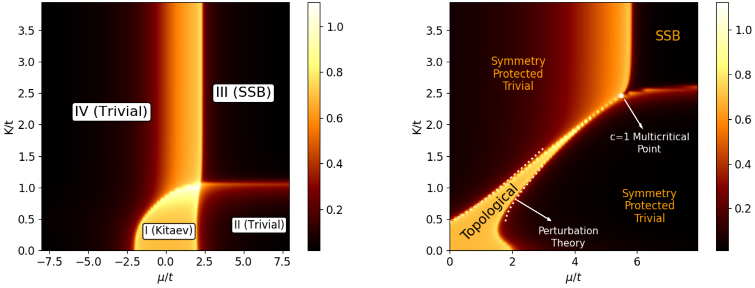

- Why are the trivial phases not connected? Distinct, protected by fermion parity and translational symmetry! *(Fuji et al. 2015)*

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Future directions

- Claim (Higgs=SPT): *any magnetic-symmetry-preserving phase in the gauge theory must be a nontrivial SPT!* (Work in progress)
- Investigate similar phenomenology in related models. Example: $\mathbb{Z}_3$ parafermions.