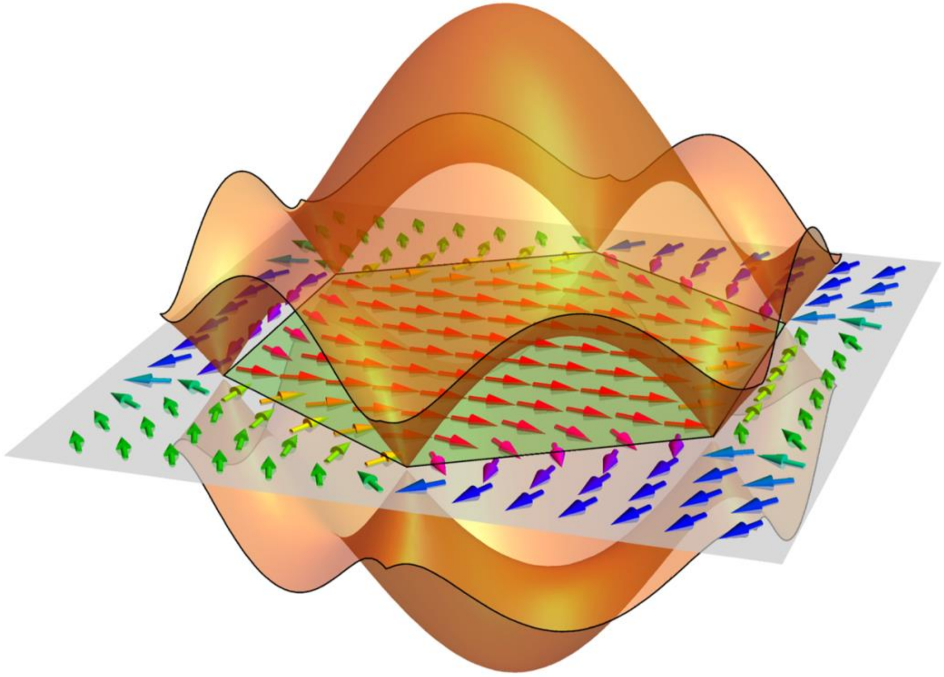
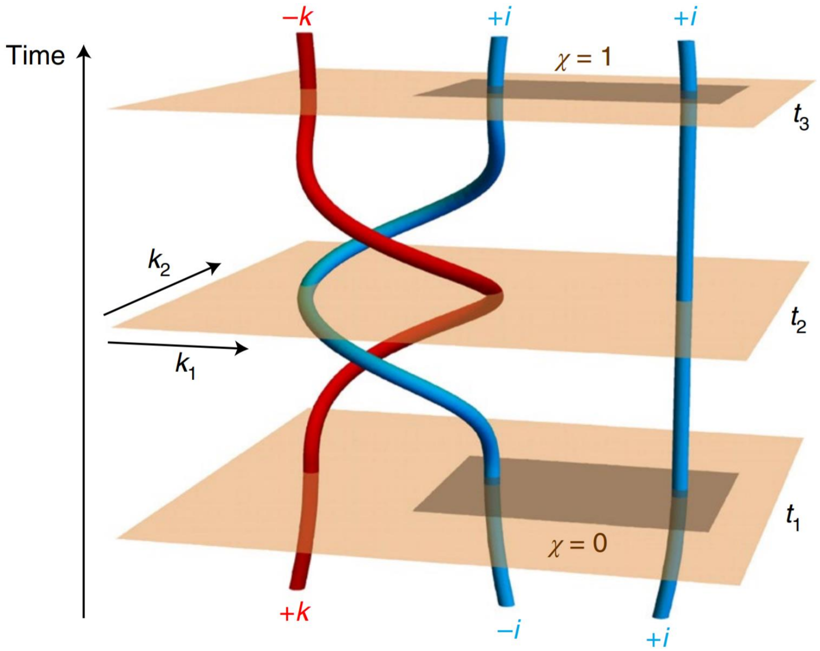
Homotopic insights into topological band invariants

Tomáš Bzdušek; 22. July 2021

KITP Program: Interacting Topological Matter





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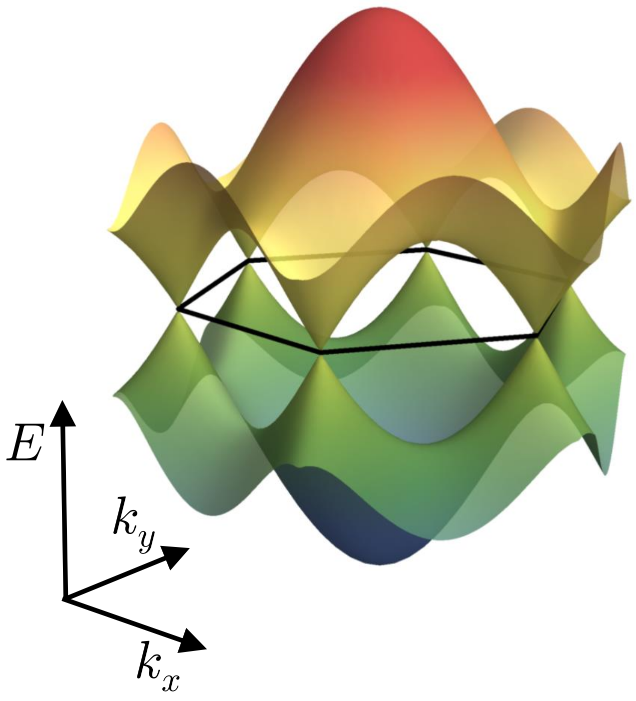
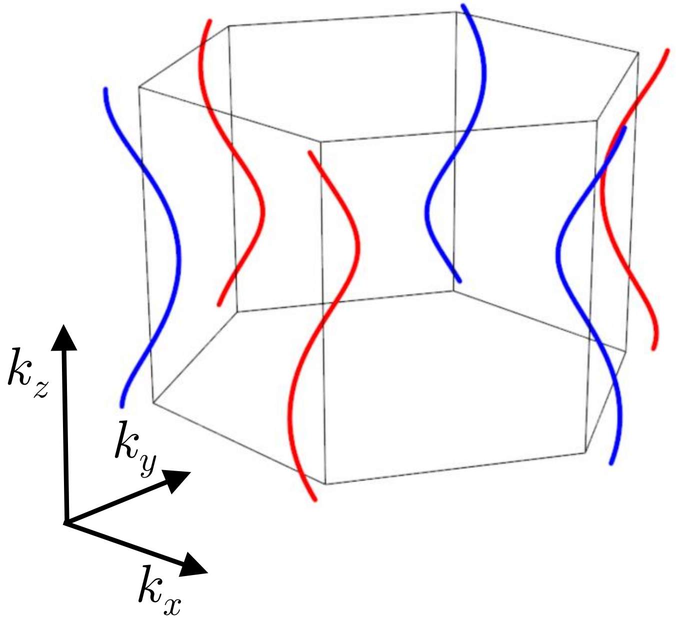
Band degeneracies (“nodes”) in energy spectra

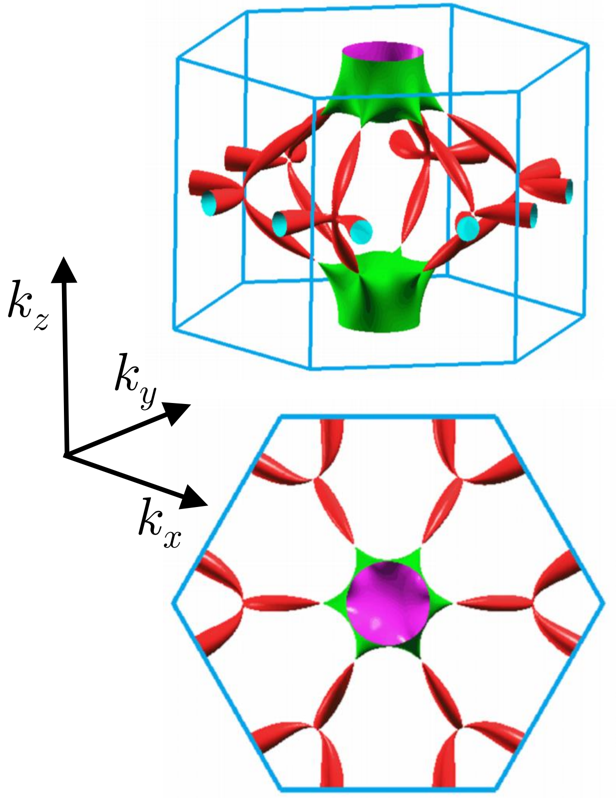
[X. Feng et al., *Phys. Rev. Mat.* **2**, 014202 (2018)]



Nodal points
in graphene



Nodal lines
in ABC-graphite



Nodal chain
in ZrB_2

Techniques to characterize band topology

K-theory

→ assumes ∞ of bands

→ uses <u>complete</u> information

→ handles crystalline symmetry

Homotopy theory

→ for arbitrarily many bands

→ uses <u>complete</u> information

→ hard to handle symmetries

> Few-band (fragile/delicate) topologies,
> $\boldsymbol{k}$-local obstructions (nodes),
> symmetry non-indicated phases, …

Symmetry indicators / TQC

→ for arbitrarily many bands

→ uses <u>reduced</u> information

→ built around symmetries

FEW — # BANDS — INFINITE

LOW — SYMMETRY — HIGH

Structure of the presentation

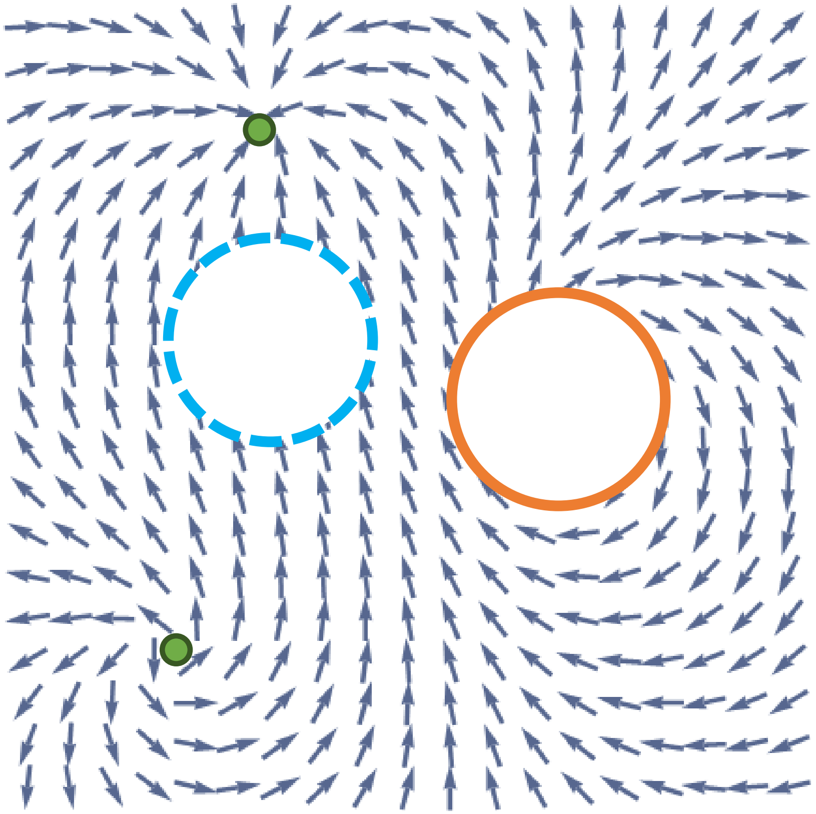
I. Intro to **topological defects in ordered media**
(vortices, hedgehogs, disclinations, etc.)

II. Formalization using **homotopy groups**
(mathematical theory of defects developed in late 1970s)

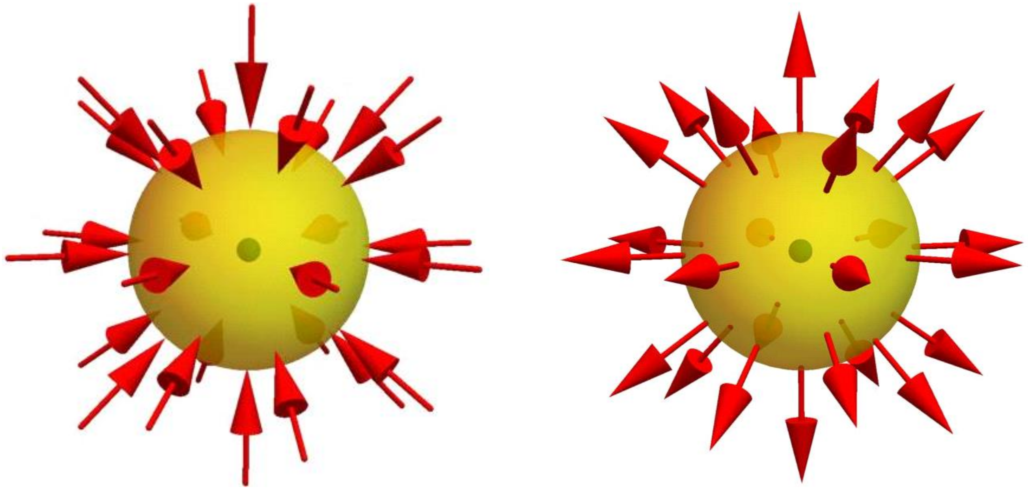
III. Use mathematical analogy to **characterize band nodes in k-space**
(classification of band nodes + their non-Abelian properties)

IV. Introduce **"delicate" topological insulators**
(Few-band topologies invisible to both K-theory and symmetry indicators)

Topological defects in ordered media

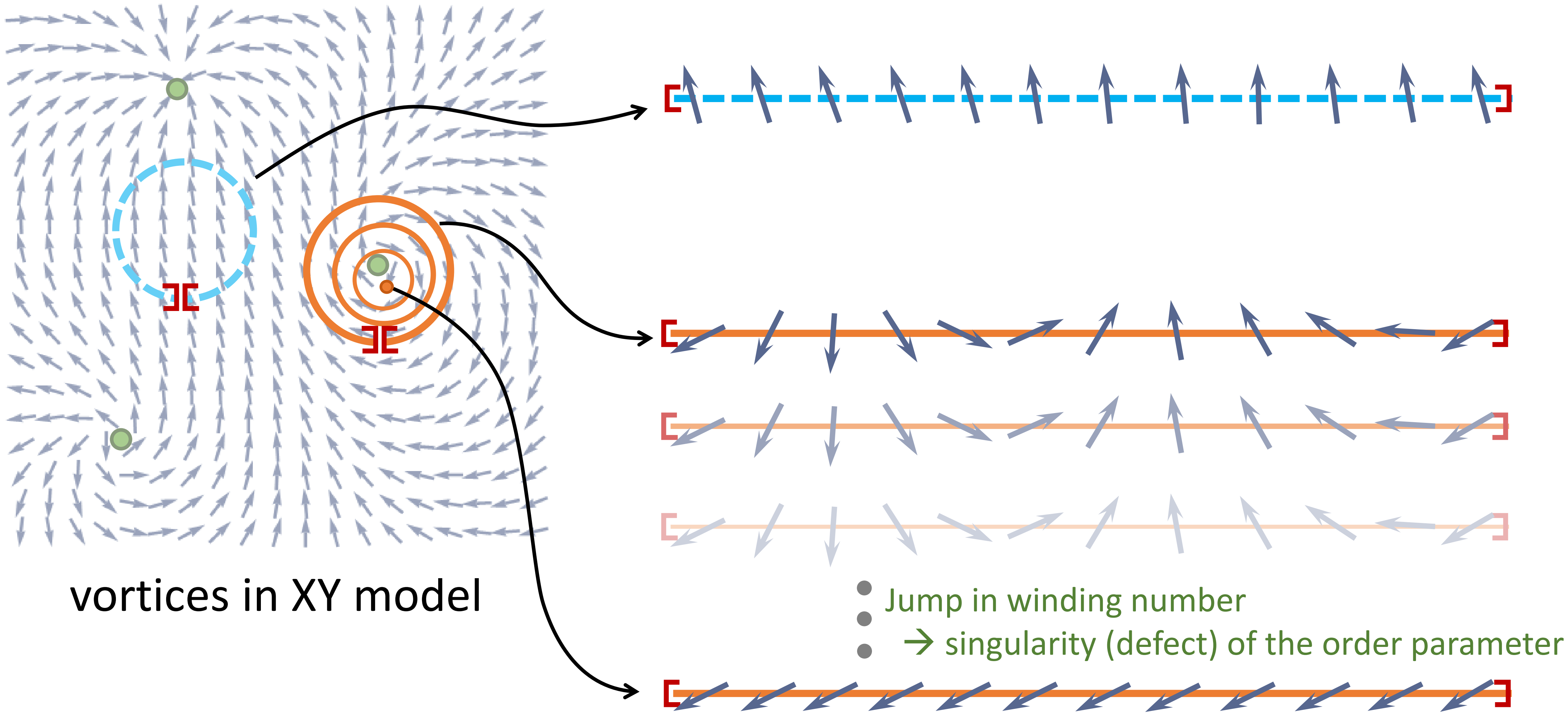


vortices in XY model
(detected on a closed loop)



hedgehogs in Heisenberg model
(detected on a spheres)

Relation of topological *defects* to topological *textures*



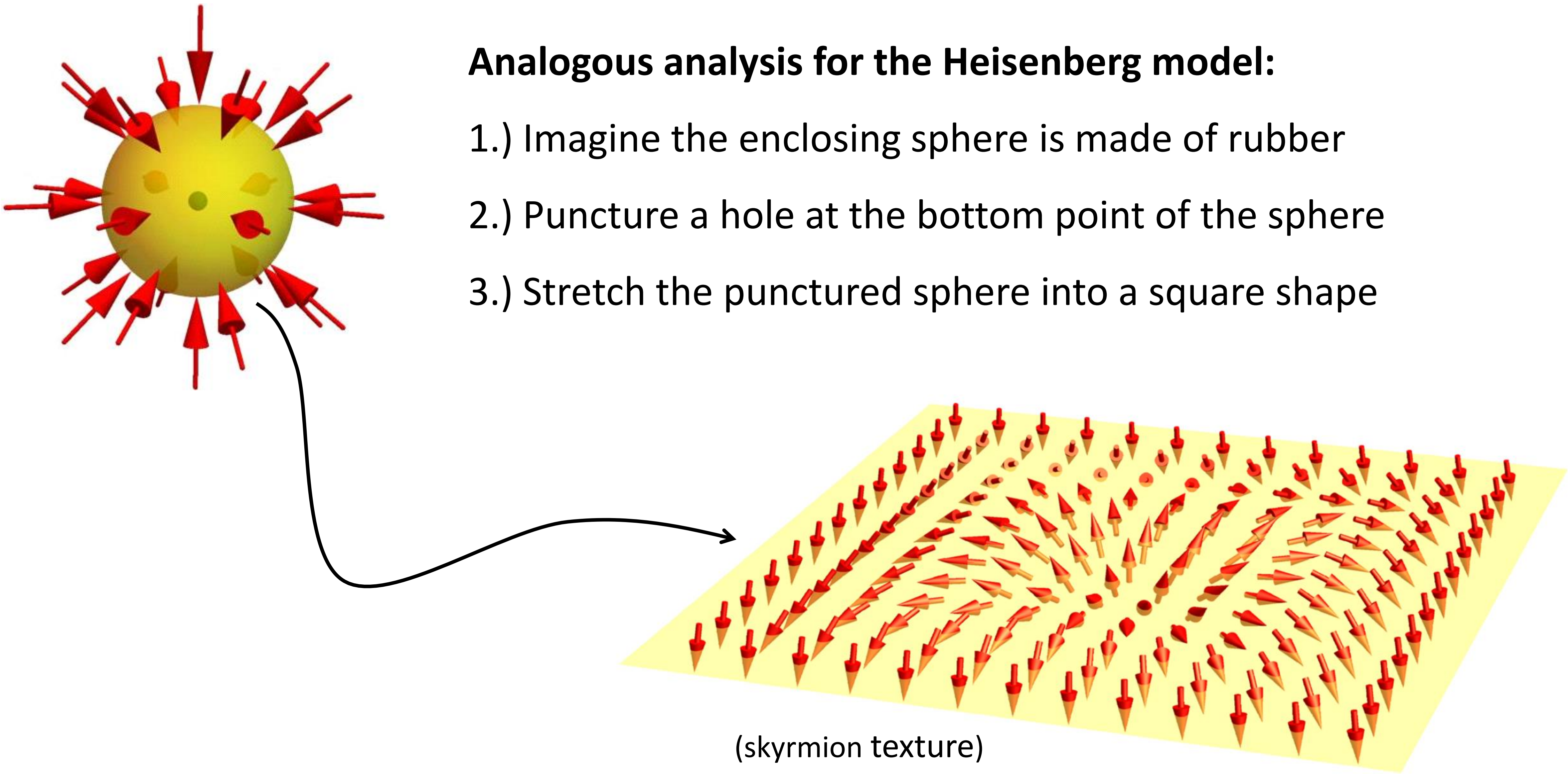
Relation of topological *defects* to topological *textures*

Analogous analysis for the Heisenberg model:

1.) Imagine the enclosing sphere is made of rubber

2.) Puncture a hole at the bottom point of the sphere

3.) Stretch the punctured sphere into a square shape

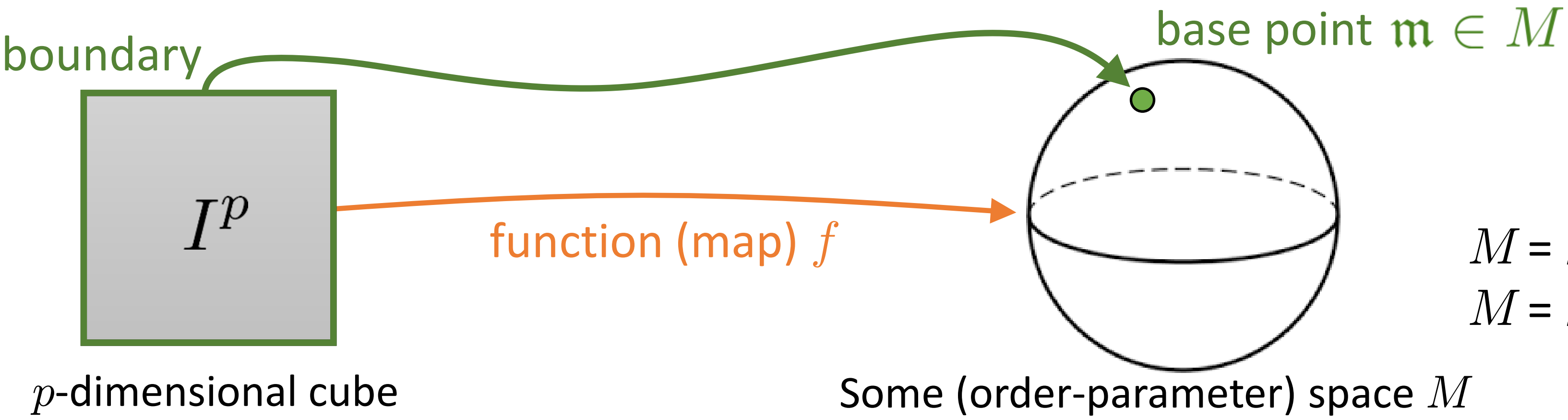


(skyrmion texture)

Basics of homotopy theory

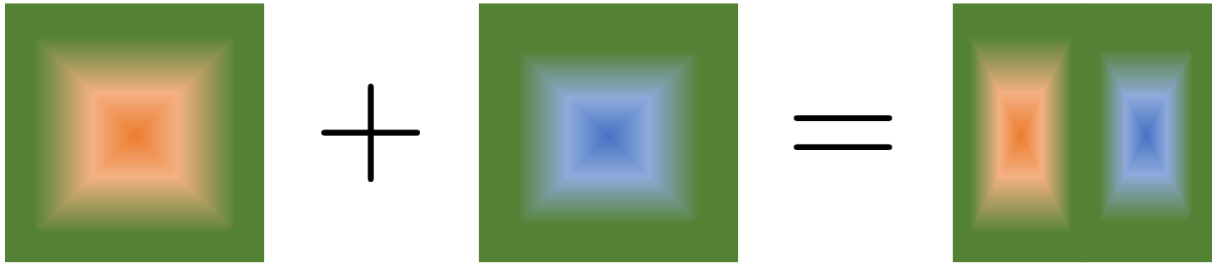
[Review: N. D. Mermin, *The topological theory of defects in ordered media*, Rev. Mod. Phys. **51**, 591–648 (1979)]

- Formal definition of *p^th^* *based homotopy group* $\pi_p(M, \mathfrak{m})$:



M = S^1 for XY moel
M = S^2 for Heisenberg

- **Group structure:** Composition rule is *stacking*:

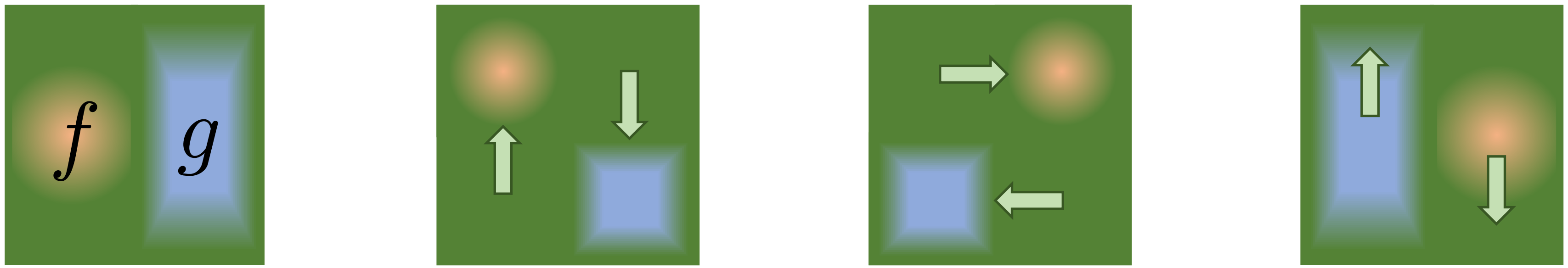


(equivalence class $[f]$ = continuous deformations of f; identity & inverse element left for homework!)

Basics of homotopy theory

[Review: N. D. Mermin, *The topological theory of defects in ordered media*, Rev. Mod. Phys. **51**, 591–648 (1979)]

- Second (and higher) based homotopy groups are <u>Abelian</u>:



$$f \circ g \sim g \circ f$$

→ The two stacking of functions f and g are equivalent under continuous deformations.

$$[f] \circ [g] = [g] \circ [f]$$

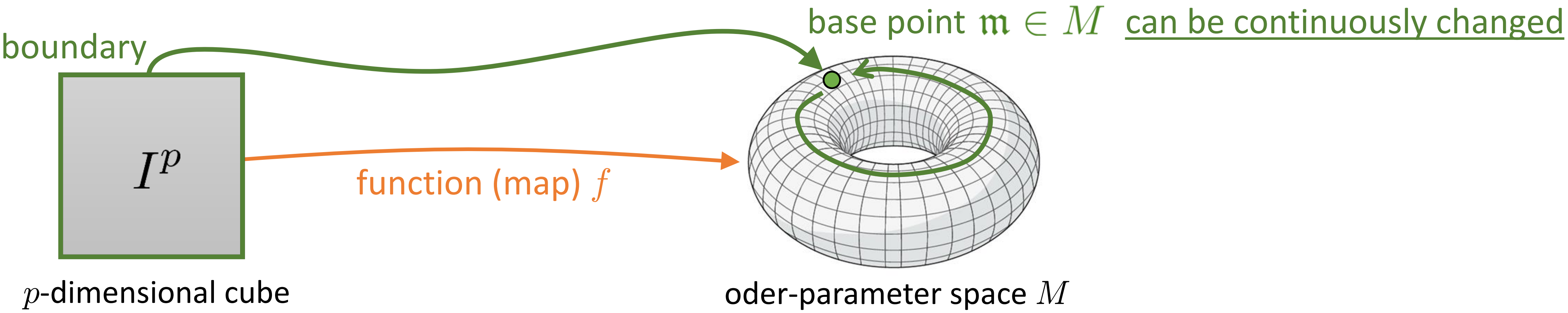
→ Equivalence classes $[f]$ of f and $[g]$ of g (elements of the homotopy group) commute.

Basics of homotopy theory

[Review: N. D. Mermin, *The topological theory of defects in ordered media*, Rev. Mod. Phys. **51**, 591–648 (1979)]

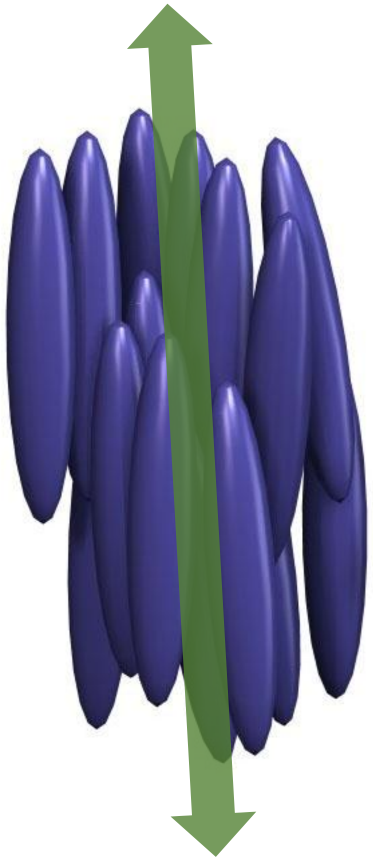
- First homotopy group may be non-Abelian

- In practice: *free homotopy classes* $\pi_p(M, \mathfrak{m})$; may lose group structure



Key notion in constructing the free homotopy group: *action of* $\pi_1(M, \mathfrak{m})$ *on* $\pi_p(M, \mathfrak{m})$.

Non-trivial example #1: Uniaxial nematic liquid



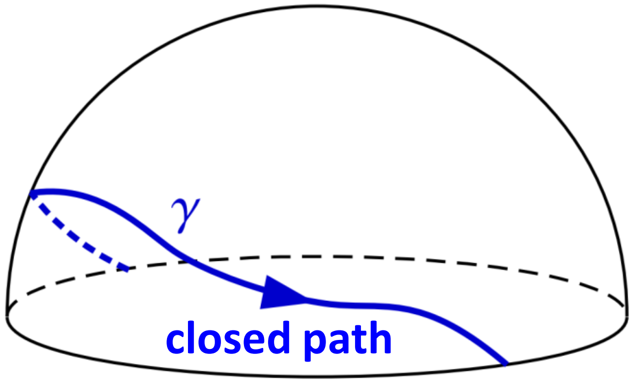
“director” d
(headless arrow)

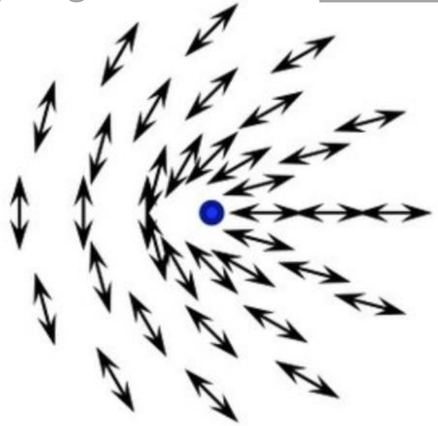
Orientational order; order parameter space: $M = S^2/\mathbb{Z}_2 \simeq \mathbb{R}P^2$

[image source DOI:10.1117/12.2007269]

$$\pi_1(\mathbb{R}P^2, \mathfrak{m}) = \mathbb{Z}_2$$

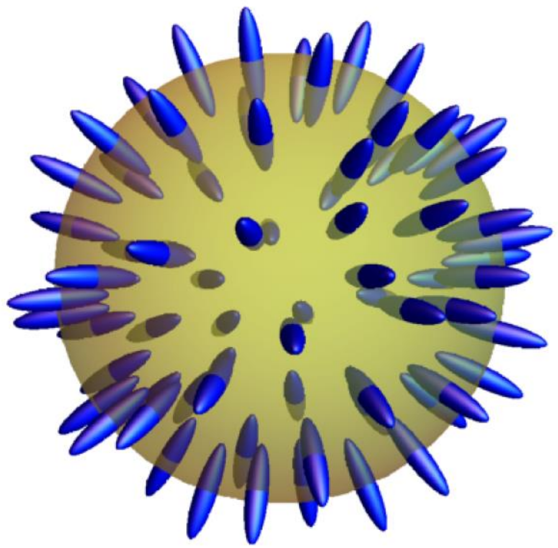
results in disclination defects (lines in 3D)





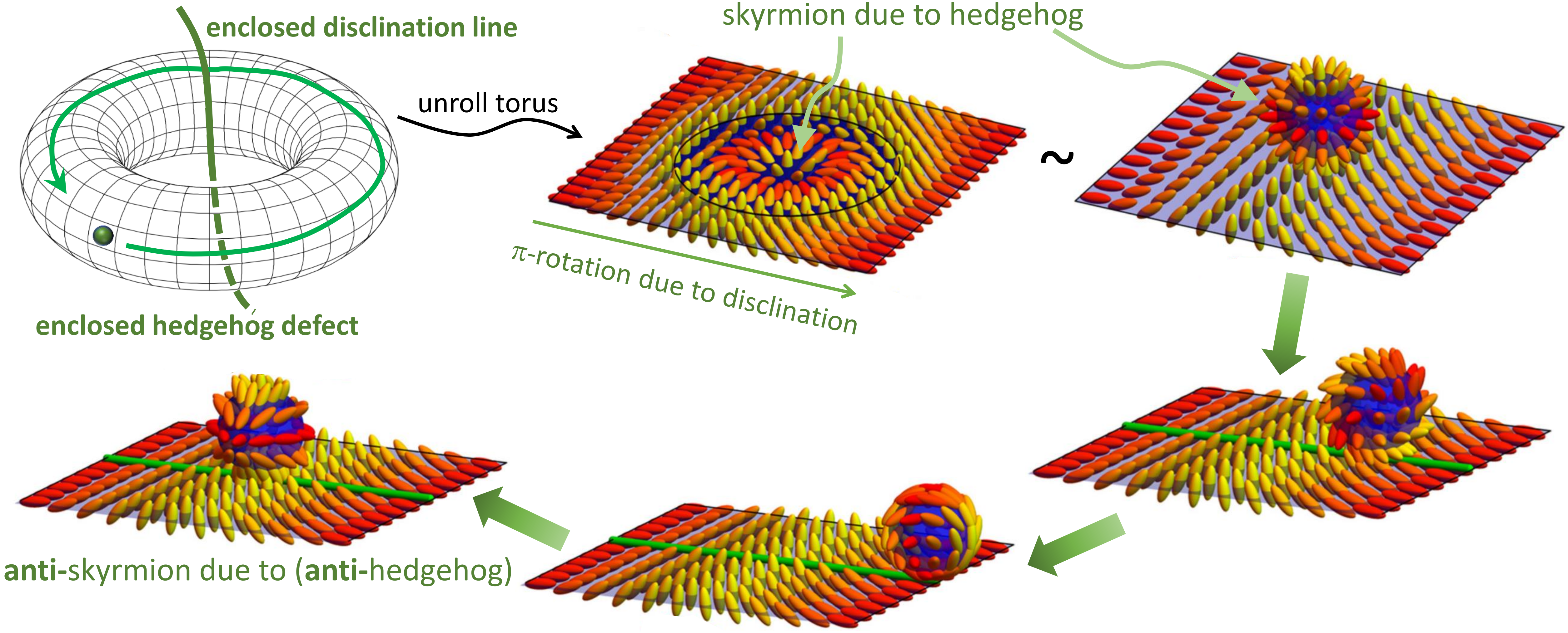
$$\pi_2(\mathbb{R}P^2, \mathfrak{m}) = \mathbb{Z}$$

results in hedgehog defects



Non-trivial example #1: Uniaxial nematic liquid

Action of π_1 on π_2 results in *non-trivial braiding effects*!



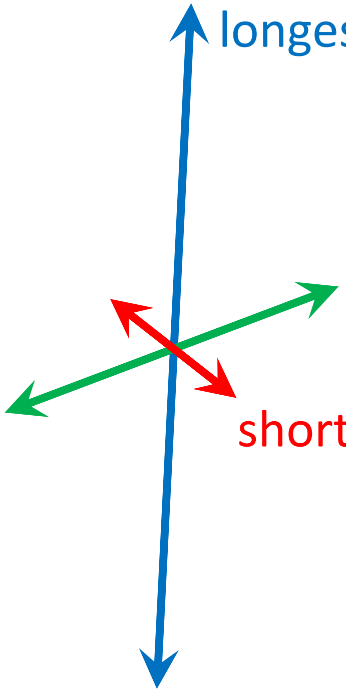
[G. E. Volovik *et al.*, Zh. Exp. Teor. Fiz. **72**, 2256 (1977) ; illustrations: C. C. Wojcik, T.B., *et al.*, Phys. Rev. B **101**, 205419 (2020)]

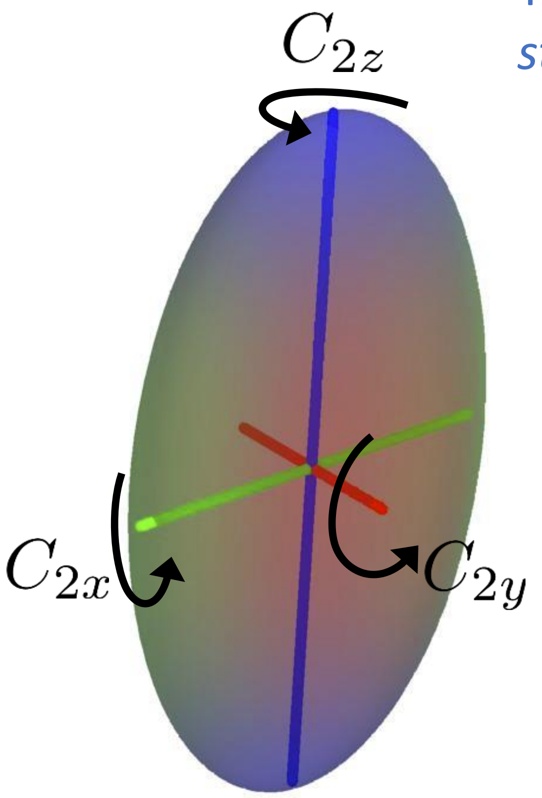
Non-trivial example #2: Biaxial nematic liquid

Orientational order; order parameter space: $M = \mathrm{SO}(3)/\mathrm{D}_2$

The quotient is the *stabilizer subgroup*.

spin-½ representations of the pi-rotation.





$$= \{E, C_{2x}, C_{2y}, C_{2z}\}$$

"dihedral group"

$$= \{\pm\mathbf{1}, \mp \mathrm{i}\sigma_x, \mp \mathrm{i}\sigma_y, \mp \mathrm{i}\sigma_z\}$$

$$= \{\pm 1, \pm \mathrm{i}, \pm \mathrm{j}, \pm \mathrm{k}\}$$

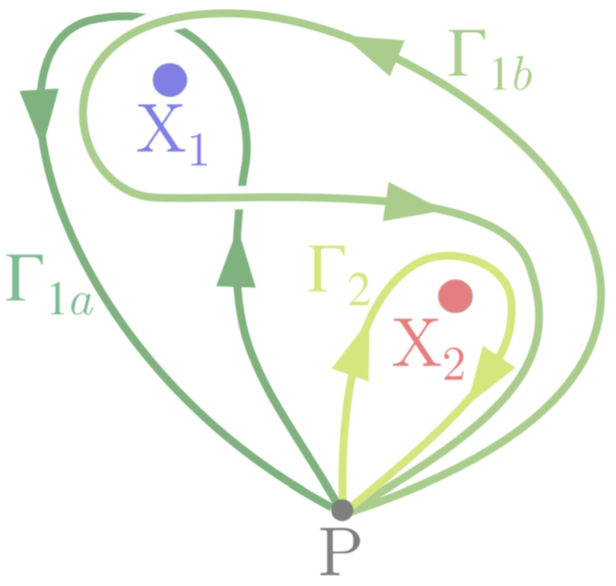
"quaternion group"

It can be shown that $\pi_1(\ldots) = \overline{\mathrm{D}}_2$.

[M. Kléman *et al.*, J. Phys. Lett. **38**, 195—197 (1977)]

Non-trivial example #2: Biaxial nematic liquid

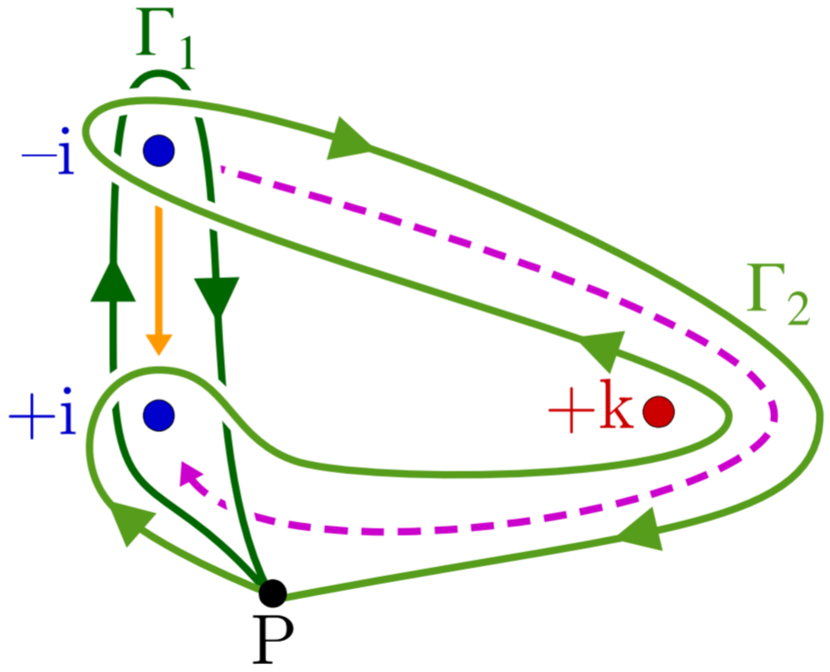
To study *braiding of defects*, let us fix a base point P in the coordinate space.



$$\Gamma_{1b} \sim \Gamma_2 \circ \Gamma_{1a} \circ \Gamma_2^{-1}$$

$$c_{1b} = c_2 \cdot c_{1a} \cdot c_2^{-1}$$

$$-\mathrm{i} = \mathrm{k} \cdot \mathrm{i} \cdot (-\mathrm{k})$$



It can be shown that $\pi_1(\ldots) = \overline{\mathrm{D}}_2 = \{\pm 1, \pm \mathrm{i}, \pm \mathrm{j}, \pm \mathrm{k}\}$.

This group is *non-commutative*, e.g. $\mathrm{i} \cdot \mathrm{j} = \mathrm{k} = -\mathrm{j} \cdot \mathrm{i}$.

[M. Kléman *et al.*, J. Phys. Lett. **38**, 195—197 (1977); Q.S. Wu, T.B., *et al.* Science **365**, 1273—1277 (2019)]

From topological defects to band-structure nodes

Isomorphism:

Playground:	coordinate space	→	momentum space
Texture:	order parameter	→	gapped Hamiltonian
Target space:	order-parameter space	→	classifying space

Mathematical intermezzo: Grassmannian spaces

Consider Hamiltonian with N filled and M empty bands with no symmetry:

eigensystem decomposition

$$\mathcal{H}(\boldsymbol{k}) = \sum_{a=1}^{N+M} |u_a(\boldsymbol{k})\rangle\, \varepsilon_a(\boldsymbol{k})\, \langle u_a(\boldsymbol{k})|$$

spectral normalization

$$\mathcal{H}(\boldsymbol{k}) = \sum_{a=1}^{N+M} |u_a(\boldsymbol{k})\rangle\, \mathrm{sign}[\varepsilon_a(\boldsymbol{k})]\, \langle u_a(\boldsymbol{k})|$$

space of spectrally normalized Hamiltonians

$$M = \mathsf{U}(N+M)/\mathsf{U}(N)\times\mathsf{U}(M)$$

Some symmetries (C_2T, spinless PT) impose *reality condition*, in which case:

$$M = \mathsf{O}(N+M)/\mathsf{O}(N)\times\mathsf{O}(M)$$

Simple instances:

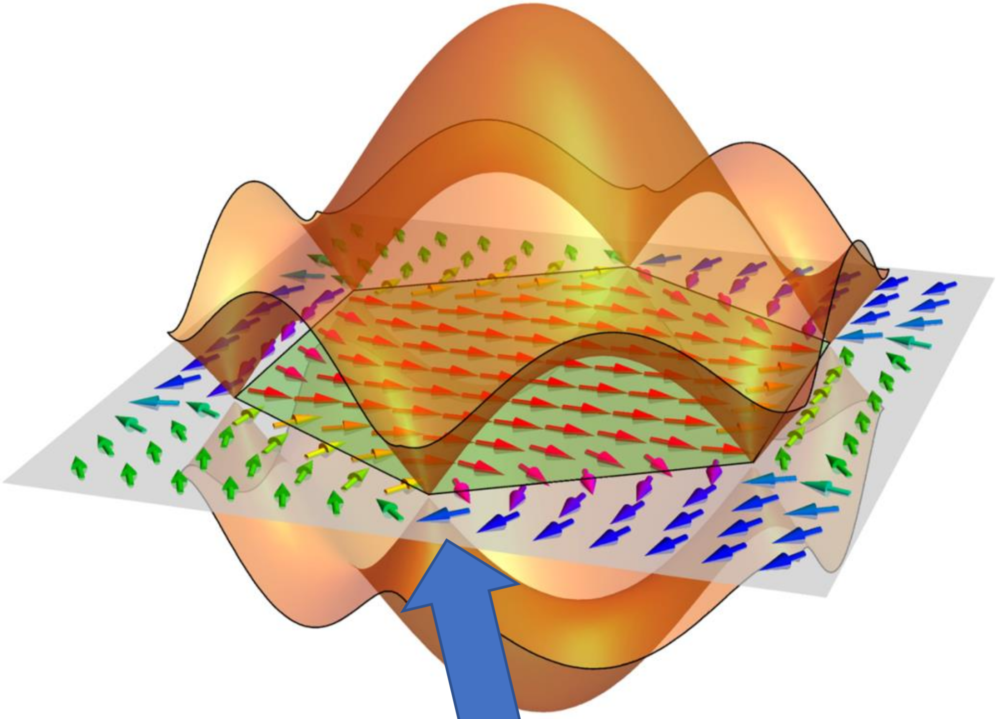
$\mathsf{Gr}_{\mathbb{C}}(1,2) \simeq S^2$
like Heisenberg model

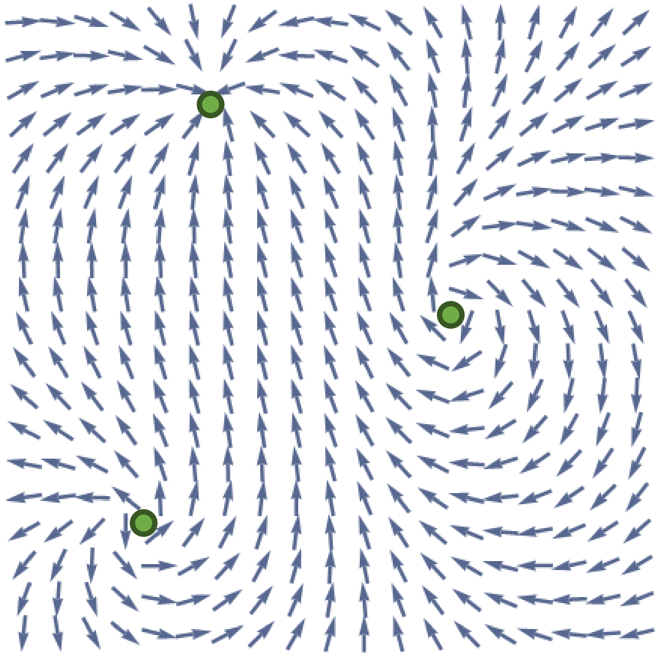
$\mathsf{Gr}_{\mathbb{R}}(1,2) \simeq S^1$
like XY model

$\mathsf{Gr}_{\mathbb{R}}(1,3) = \mathsf{Gr}_{\mathbb{R}}(2,3) \simeq S^2/\mathbb{Z}_2 \simeq \mathbb{R}P^2$
like uniaxial nematic liquid

$\mathsf{Gr}_{\mathbb{R}}(2,4) \simeq (S^2\times S^2)/\mathbb{Z}_2$

Graphene vs. the XY model





Some symmetries (C_2T, spinless PT) impose *reality condition*, in which case:

$$M = \mathsf{O}(N+M)/\mathsf{O}(N) \times \mathsf{O}(M)$$

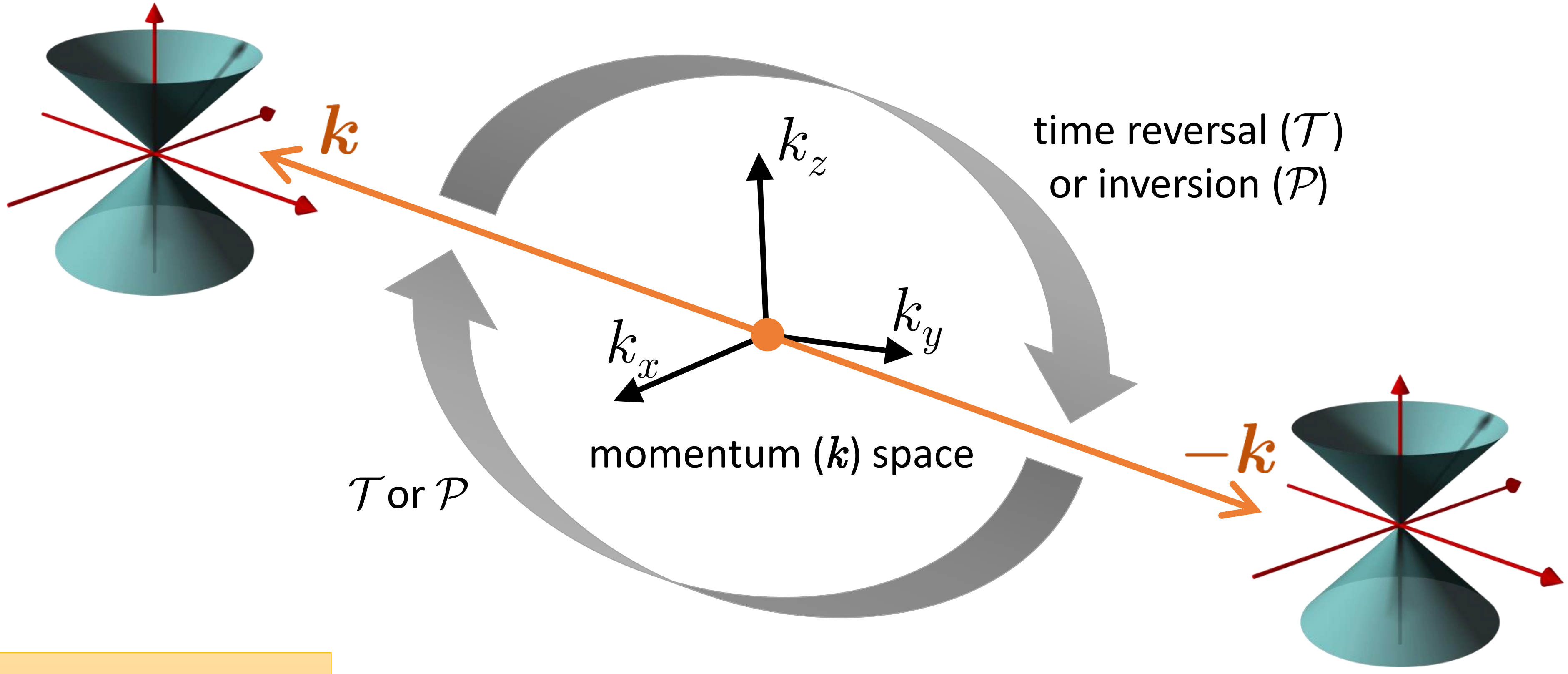
$$\mathsf{Gr}_{\mathbb{R}}(1,3) = \mathsf{Gr}_{\mathbb{R}}(2,3) \simeq S^2/\mathbb{Z}_2 \simeq \mathbb{R}P^2$$

like uniaxial nematic liquid

$$\mathsf{Gr}_{\mathbb{R}}(1,2) \simeq S^1$$

like XY model

Band nodes protected by local-in-$\boldsymbol{k}$ symmetries



<u>Three such symmetries</u>

$\mathcal{PT}$, $\mathcal{PC}$, and $\mathcal{CT}$

$$(\mathcal{PT})\mathcal{H}(\boldsymbol{k})(\mathcal{PT})^{-1} = \mathcal{H}(\boldsymbol{k})$$

local-in-$\boldsymbol{k}$ symmetry

[<u>T.B.</u> & M. Sigrist, Phys. Rev. B **96**, 155105 (2017)]

Band nodes protected by local-in-$\boldsymbol{k}$ symmetries

label	$\mathcal{PT}$	$\mathcal{PC}$	$\mathcal{CT}$	Space M of Hamiltonians	(stable) homotopy group of M π_0	π_1	π_2	π_3	π_4
A	$\times$	$\times$	$\times$	$\mathsf{U}(n+\ell)/\mathsf{U}(n)\times\mathsf{U}(\ell)$			$\mathbb{Z}$		$\mathbb{Z}$
AIII	$\times$	$\times$	1	$\mathsf{U}(n)$		$\mathbb{Z}$		$\mathbb{Z}$	
AI	$+1$	$\times$	$\times$	$\mathsf{O}(n+\ell)/\mathsf{O}(n)\times\mathsf{O}(\ell)$		$\mathbb{Z}_2$	$\mathbb{Z}_2$		$2\mathbb{Z}$
BDI	$+1$	$+1$	1	$\mathsf{O}(n)$	$\mathbb{Z}_2$	$\mathbb{Z}_2$		$2\mathbb{Z}$	
D	$\times$	$+1$	$\times$	$\mathsf{O}(2n)/\mathsf{U}(n)$	$\mathbb{Z}_2$		$2\mathbb{Z}$		
DIII	-1	$+1$	1	$\mathsf{U}(2n)/\mathsf{Sp}(n)$		$2\mathbb{Z}$			
AII	-1	$\times$	$\times$	$\mathsf{Sp}(n+\ell)/\mathsf{Sp}(n)\times\mathsf{Sp}(\ell)$					$\mathbb{Z}$
CII	-1	-1	1	$\mathsf{Sp}(n)$				$\mathbb{Z}$	$\mathbb{Z}_2$
C	$\times$	-1	$\times$	$\mathsf{Sp}(n)/\mathsf{U}(n)$			$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$
CI	$+1$	-1	1	$\mathsf{U}(n)/\mathsf{O}(n)$		$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	

band nodes
nodal points
Interpret as *topological charges*
PAIR charges?
nodal lines
carried by the band nodes
nodal surfaces

[T.B. & M. Sigrist, Phys. Rev. B **96**, 155105 (2017)]

Band nodes protected by local-in-$\boldsymbol{k}$ symmetries

label	$\mathcal{PT}$	$\mathcal{PC}$	$\mathcal{CT}$	Space M of Hamiltonians	(stable) homotopy group of M				
					π_0	π_1	π_2	π_3	π_4
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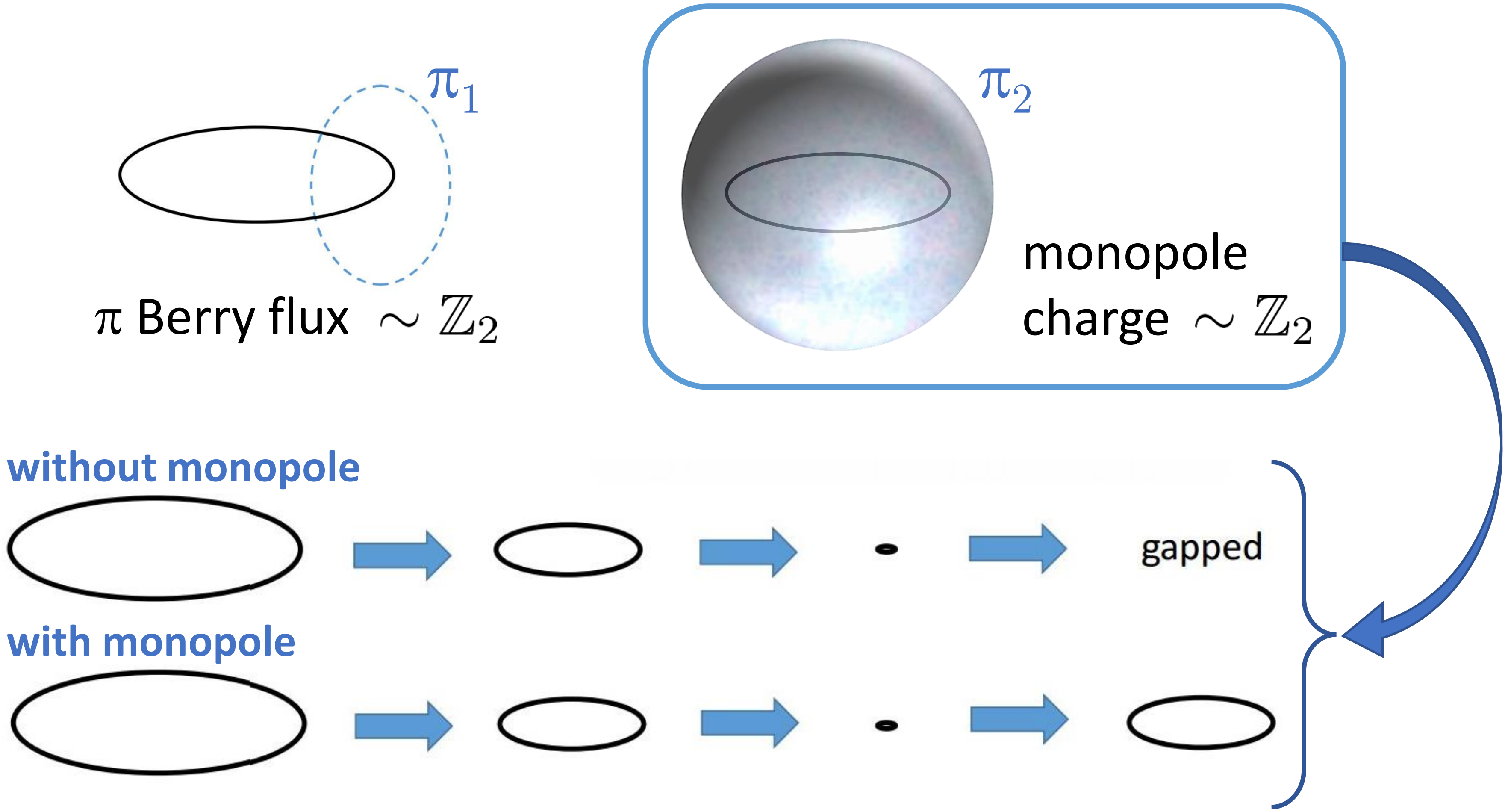
AIII	$\times$	$\times$	1	$\mathsf{U}(n)$		$\mathbb{Z}$		$\mathbb{Z}$	
AI	$+1$	$\times$	$\times$	$\mathsf{O}(n+\ell)/\mathsf{O}(n)\times\mathsf{O}(\ell)$		$\mathbb{Z}_2$	$\mathbb{Z}_2$		$2\mathbb{Z}$
BDI	$+1$	$+1$	1	$\mathsf{O}(n)$	$\mathbb{Z}_2$	$\mathbb{Z}_2$		$2\mathbb{Z}$	
D	$\times$	$+1$	$\times$	$\mathsf{O}(2n)/\mathsf{U}(n)$	$\mathbb{Z}_2$		$2\mathbb{Z}$		
DIII	-1	$+1$	1	$\mathsf{U}(2n)/\mathsf{Sp}(n)$		$2\mathbb{Z}$			
AII	-1	$\times$	$\times$	$\mathsf{Sp}(n+\ell)/\mathsf{Sp}(n)\times\mathsf{Sp}(\ell)$					$\mathbb{Z}$
CII	-1	-1	1	$\mathsf{Sp}(n)$				$\mathbb{Z}$	$\mathbb{Z}_2$
C	$\times$	-1	$\times$	$\mathsf{Sp}(n)/\mathsf{U}(n)$			$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$
CI	$+1$	-1	1	$\mathsf{U}(n)/\mathsf{O}(n)$		$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	

[T.B. & M. Sigrist, Phys. Rev. B **96**, 155105 (2017)]

Nodal-line rings with **two** topological charges

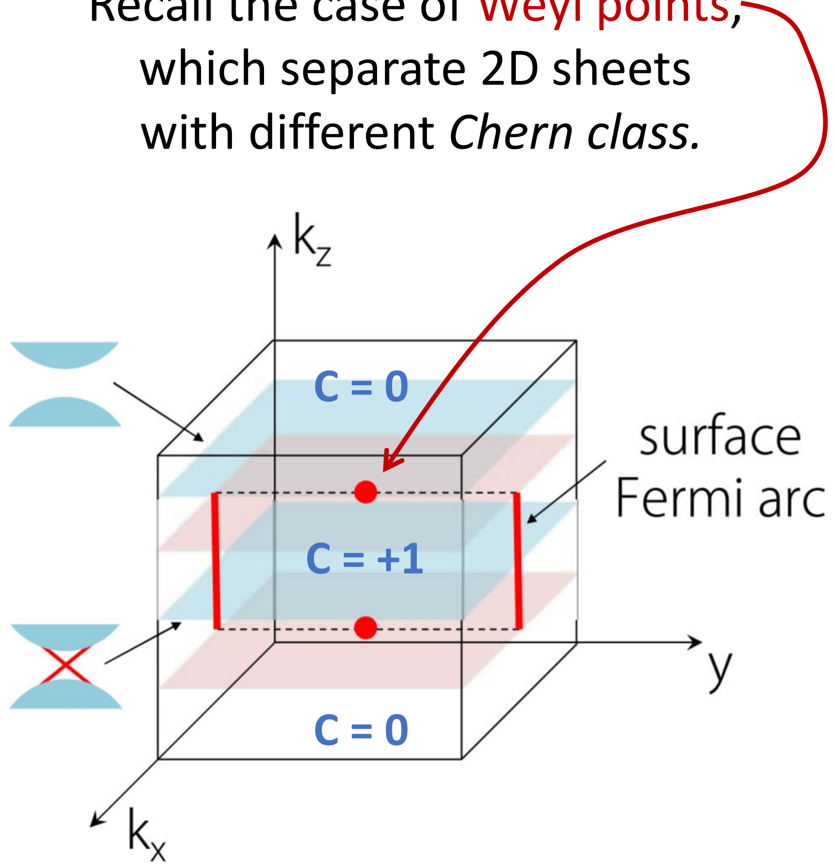


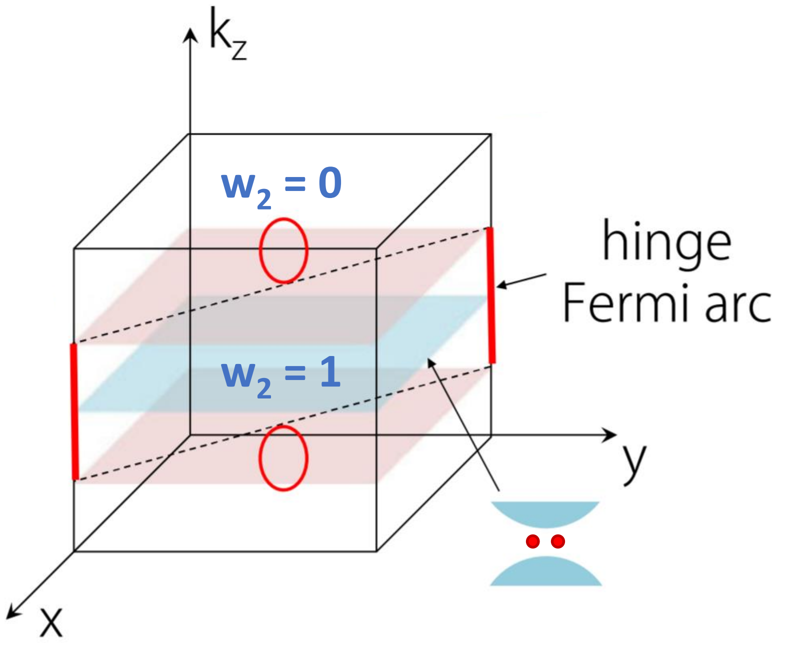
C. Fang et al., *Phys. Rev. B* **92**, 081201(R) (2015)

Manifestation of the monopole charge: higher-order topology

Recall the case of Weyl points, which separate 2D sheets with different *Chern class.*

Similarly, nodal loops with monopole charge separate 2D sheets with different values of *second Stiefel-Whitney class.*





Manifested by surface Fermi arcs.

Manifested by hinge Fermi arcs or by filling anomaly.

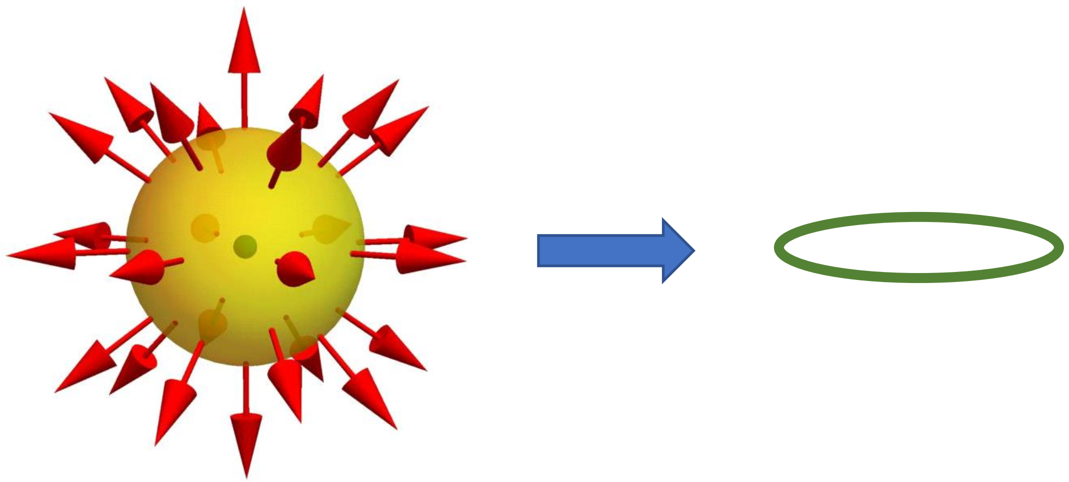
[J. Ahn *et al.*, Phys. Rev. Lett. **121**, 106403 (2018); K. Wang *et al.*, Phys. Rev. Lett. **125**, 126403 (2020); E. Lee *et al.*, npj Quantum Mater. **5**, 1 (2020) P. Lenggenhager, T.B., *et al.*, arXiv:2104.11254 (2021)]

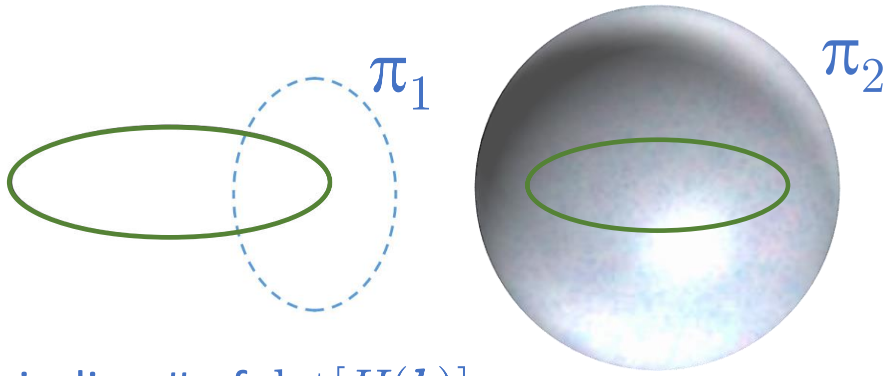
Non-trivial band node braiding example #1:
Chern class vs. exceptional lines of non-Hermitian models

(my only slide about non-Herm. models!)

Consider Hermitian model with no local-in-$\boldsymbol{k}$ symmetries → *Weyl points*.

Non-Hermitian perturbation expands the WP into an *exceptional ring*.





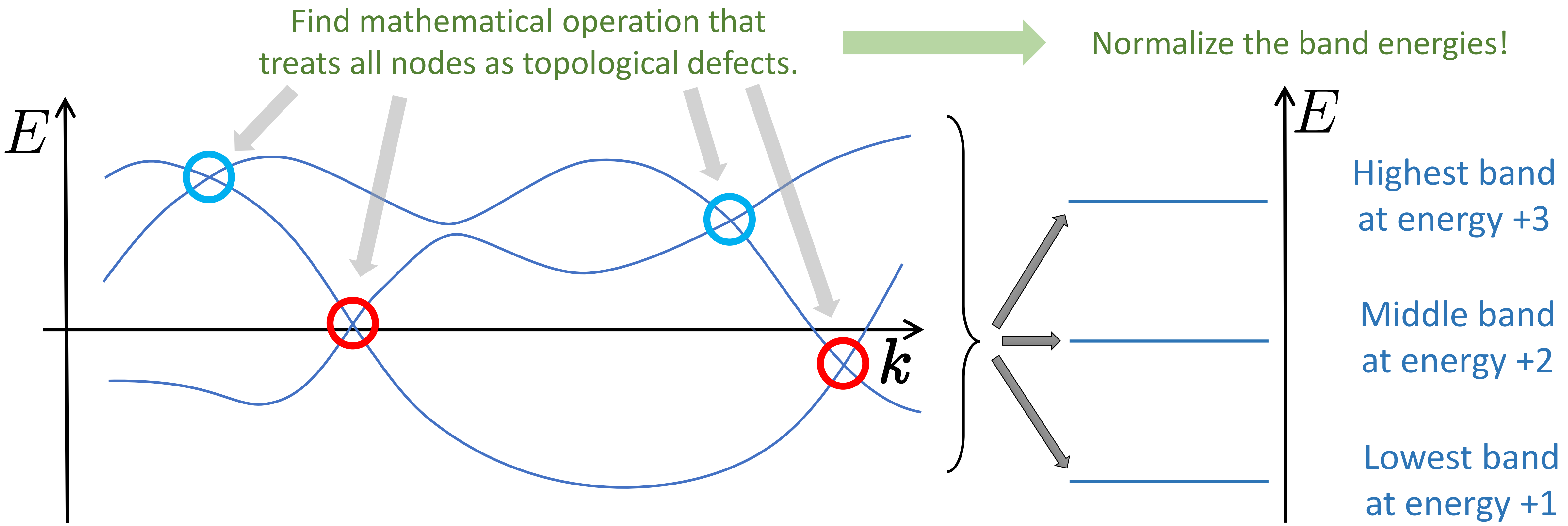
Space of spectrally normalized (non-degenerate, two-band) non-Hermitian Hamiltonians

$$M = (S^2 \times S^1)/\mathbb{Z}_2 \qquad \pi_1(\ldots) = \mathbb{Z} \qquad \pi_2(\ldots) = \mathbb{Z}$$

… and there is a non-trivial action of π_1 (its parity) on π_2 (sign reversal)

[X.-Q. Sun, T.B., *et al.*, Phys. Rev. Research **2**, 023226 (2020); C. C. Wojcik, T.B., *et al.*, Phys. Rev. B **101**, 205417 (2020)]

Non-trivial band node braiding example #2: Nodal points of C_2T-symmetric 3-band model.

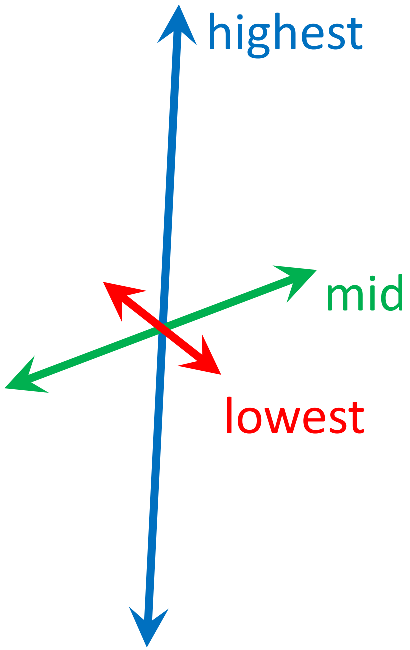


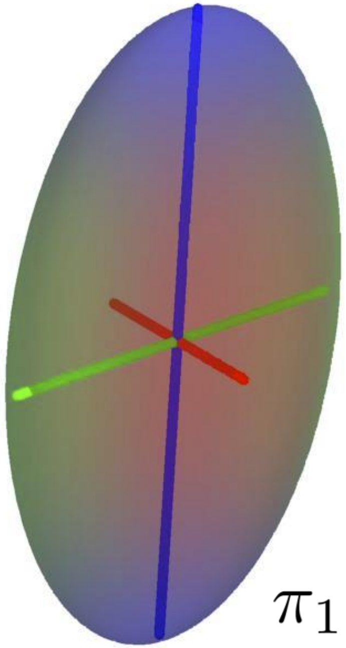
<u>Mathematically:</u> Normalize the spectral decomposition of the Hamiltonian

$$\mathcal{H}(\boldsymbol{k}) = \sum_{n=1}^{3} |u^n_{\boldsymbol{k}}\rangle E^n_{\boldsymbol{k}} \langle u^n_{\boldsymbol{k}}| \quad \Longrightarrow \quad \mathcal{H}(\boldsymbol{k}) = \sum_{n=1}^{3} |u^n_{\boldsymbol{k}}\rangle \, n \, \langle u^n_{\boldsymbol{k}}|$$

[Q.S. Wu, A. A. Soluyanov, and <u>T.B.</u>, *et al.*, Science **365** (2019); related work: J. Ahn *et al.*, Phys. Rev. X **9**, 021013 (2019)]

Non-trivial band node braiding example #2:
Nodal points of C_2T-symmetric 3-band model.



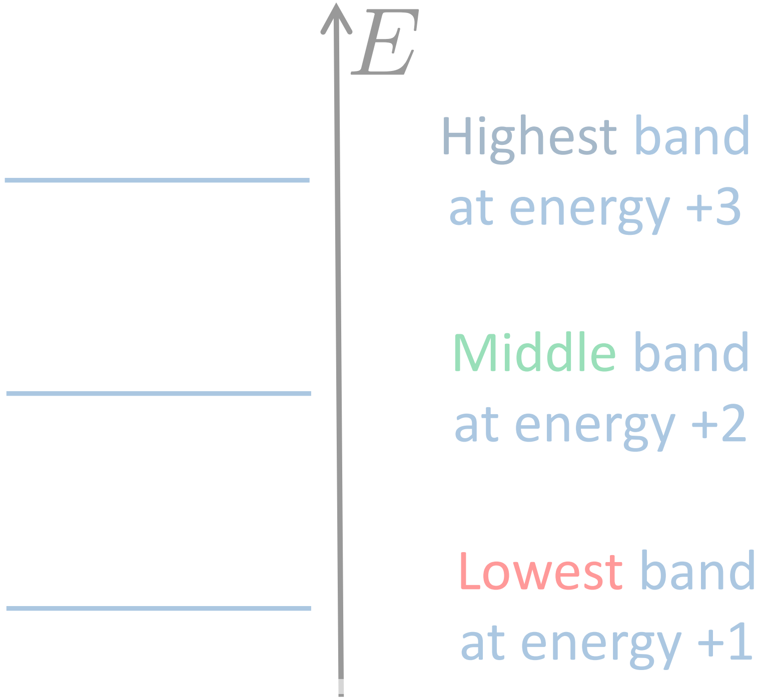


(reality condition!)

$$M = \mathrm{SO}(3)/\mathrm{D}_2$$

$$\pi_1(M) = \{\pm 1, \pm \mathrm{i}, \pm \mathrm{j}, \pm \mathrm{k}\}$$

Three real orthogonal states – each defined up to overall sign!

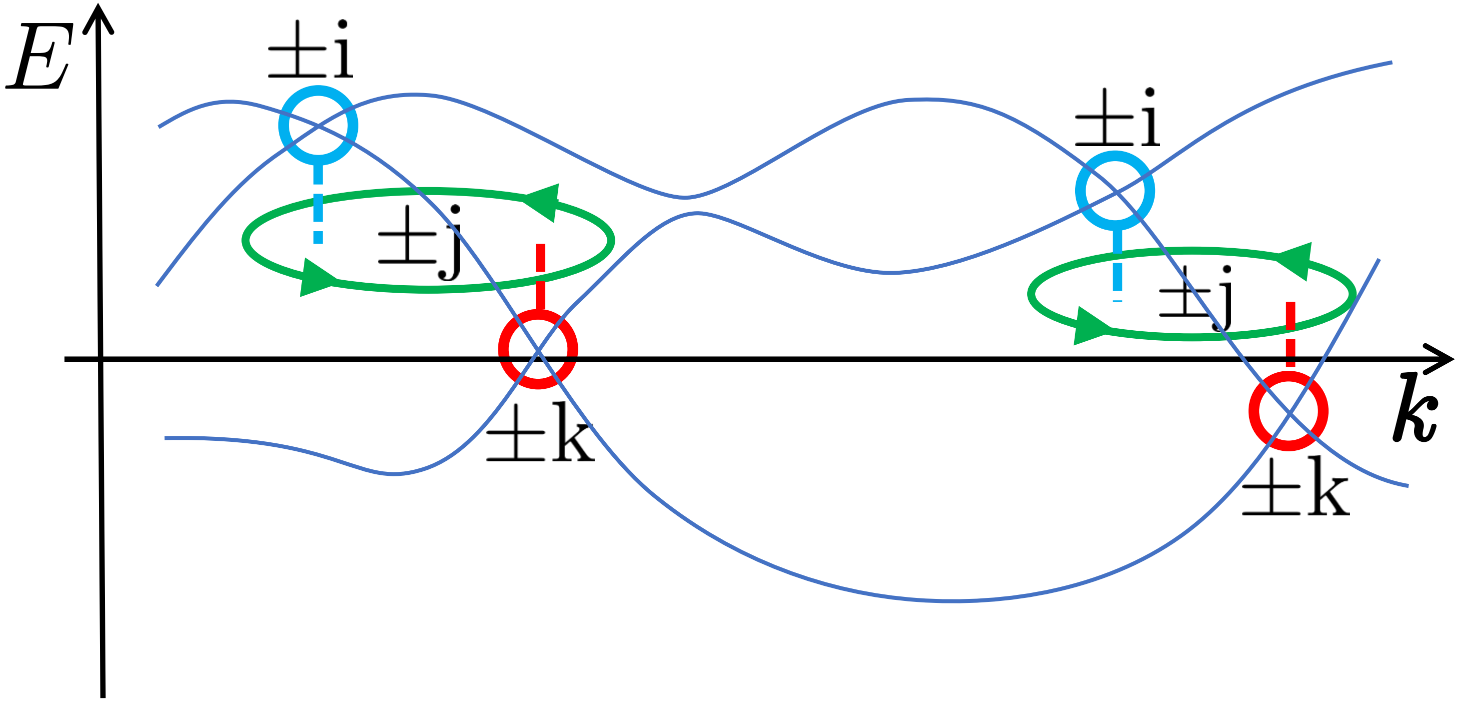


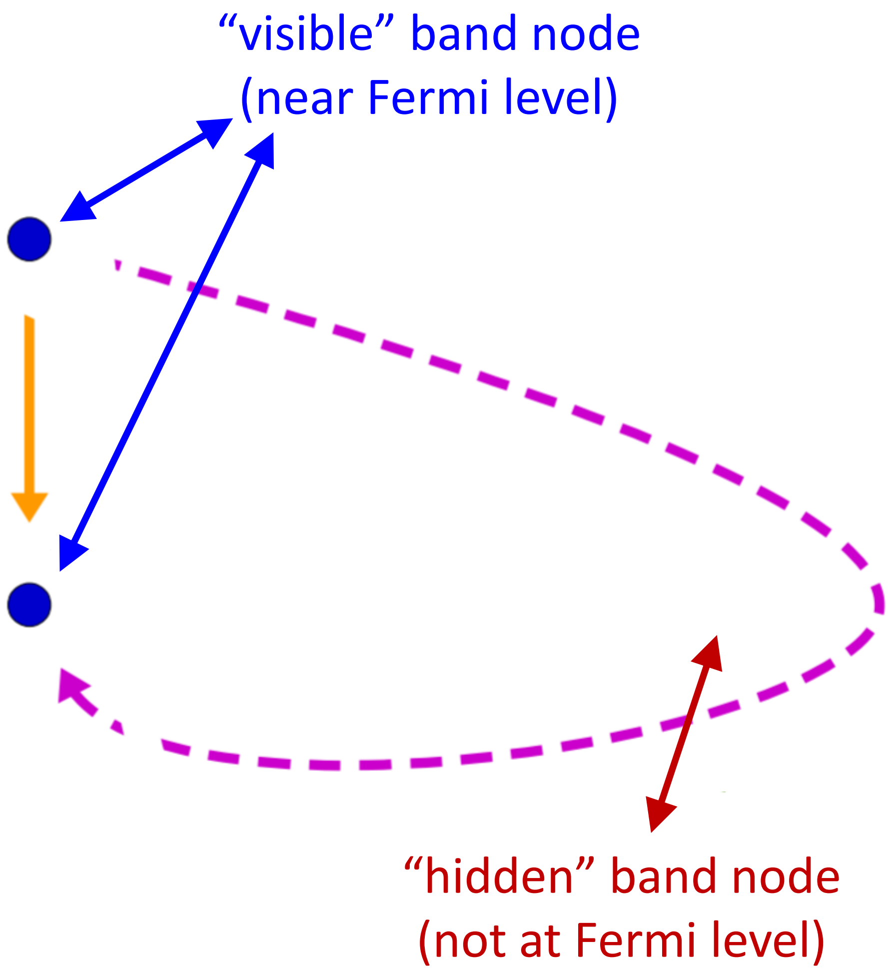
Mathematically: Normalize the spectral decomposition of the Hamiltonian

$$\mathcal{H}(\boldsymbol{k}) = \sum_{n=1}^{3} |u_{\boldsymbol{k}}^n\rangle\, E_{\boldsymbol{k}}^n\, \langle u_{\boldsymbol{k}}^n| \quad \longrightarrow \quad \mathcal{H}(\boldsymbol{k}) = \sum_{n=1}^{3} |u_{\boldsymbol{k}}^n\rangle\, n\, \langle u_{\boldsymbol{k}}^n|$$

[Q.S. Wu, A. A. Soluyanov, and T.B., *et al.*, Science **365** (2019)]

Non-Abelian charges of band nodes





$$M = \mathsf{SO}(3)/\mathsf{D}_2$$

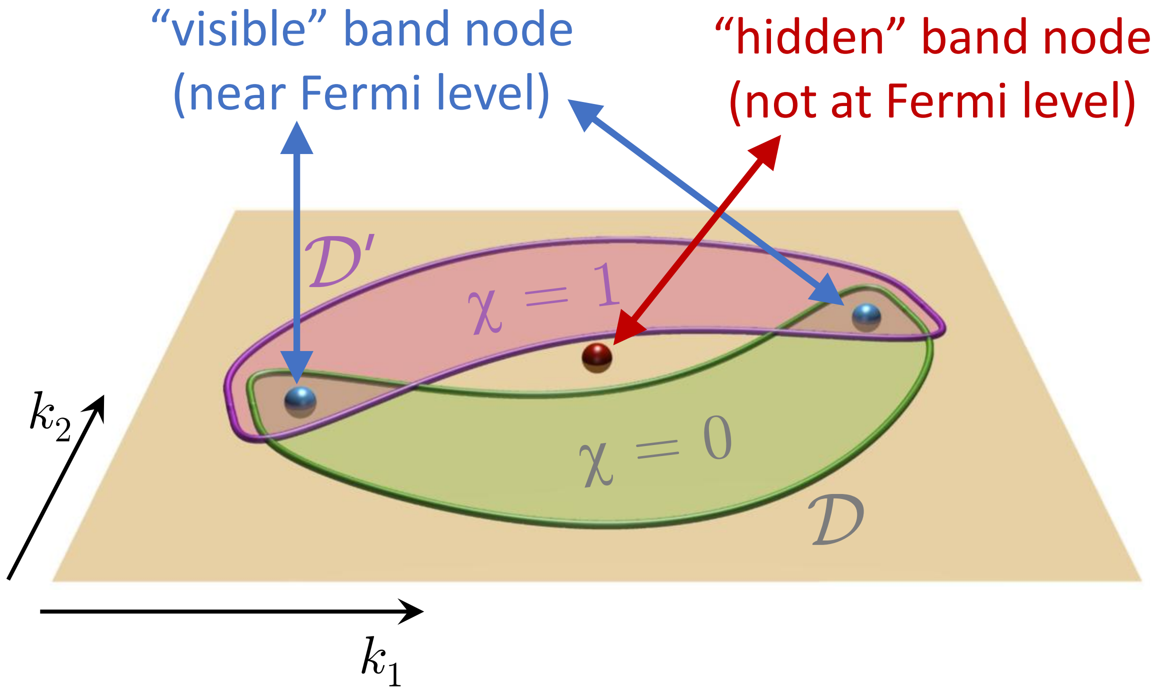
$$\pi_1(M, \mathfrak{m}) = \{\pm 1, \pm \mathrm{i}, \pm \mathrm{j}, \pm \mathrm{k}\}$$

Nodes characterized by the conjugacy classes

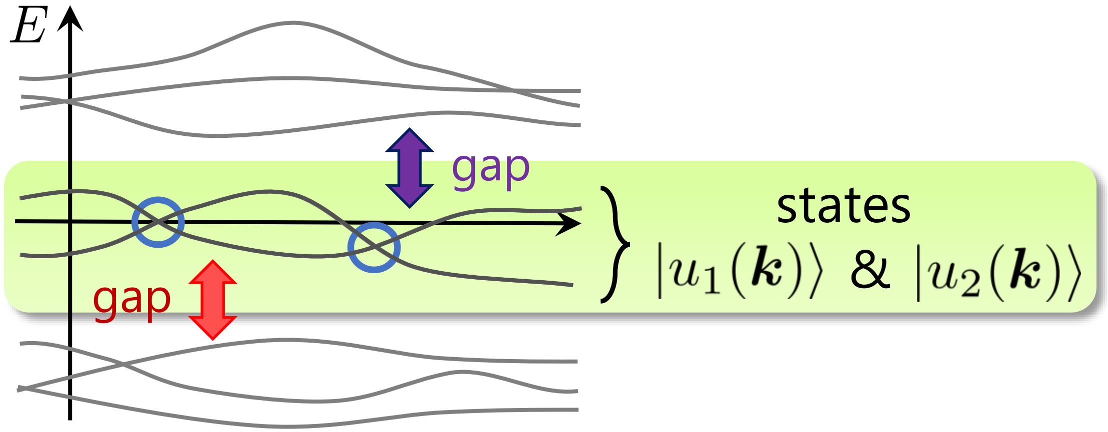
$$\pi_1(M) = \Big\{\{+1\}, \{\pm \mathrm{i}\}, \{\pm \mathrm{j}\}, \{\pm \mathrm{k}\}, \{-1\}\Big\}$$

Note: energy (E) and time progression (t) in principle correspond to additional dimensions (not plotted).

Generalization to many bands: *patch Euler class*



Required:



$$\chi(\mathcal{D}) = \frac{1}{2\pi}\left[\int_{\mathcal{D}} \mathrm{Eu}(\boldsymbol{k})\,\mathrm{d}k_1\mathrm{d}k_2 - \oint_{\partial\mathcal{D}} \mathbf{a}(\boldsymbol{k})\cdot\mathrm{d}\boldsymbol{k}\right]$$

Euler connection

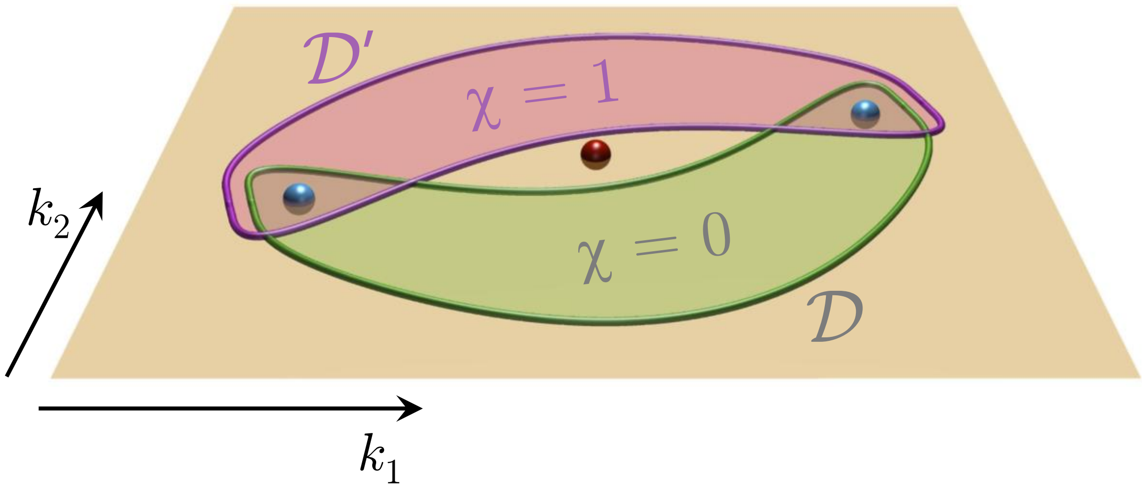
$$\mathbf{a}(\boldsymbol{k}) = \langle u_1(\boldsymbol{k})|\boldsymbol{\nabla} u_2(\boldsymbol{k})\rangle$$

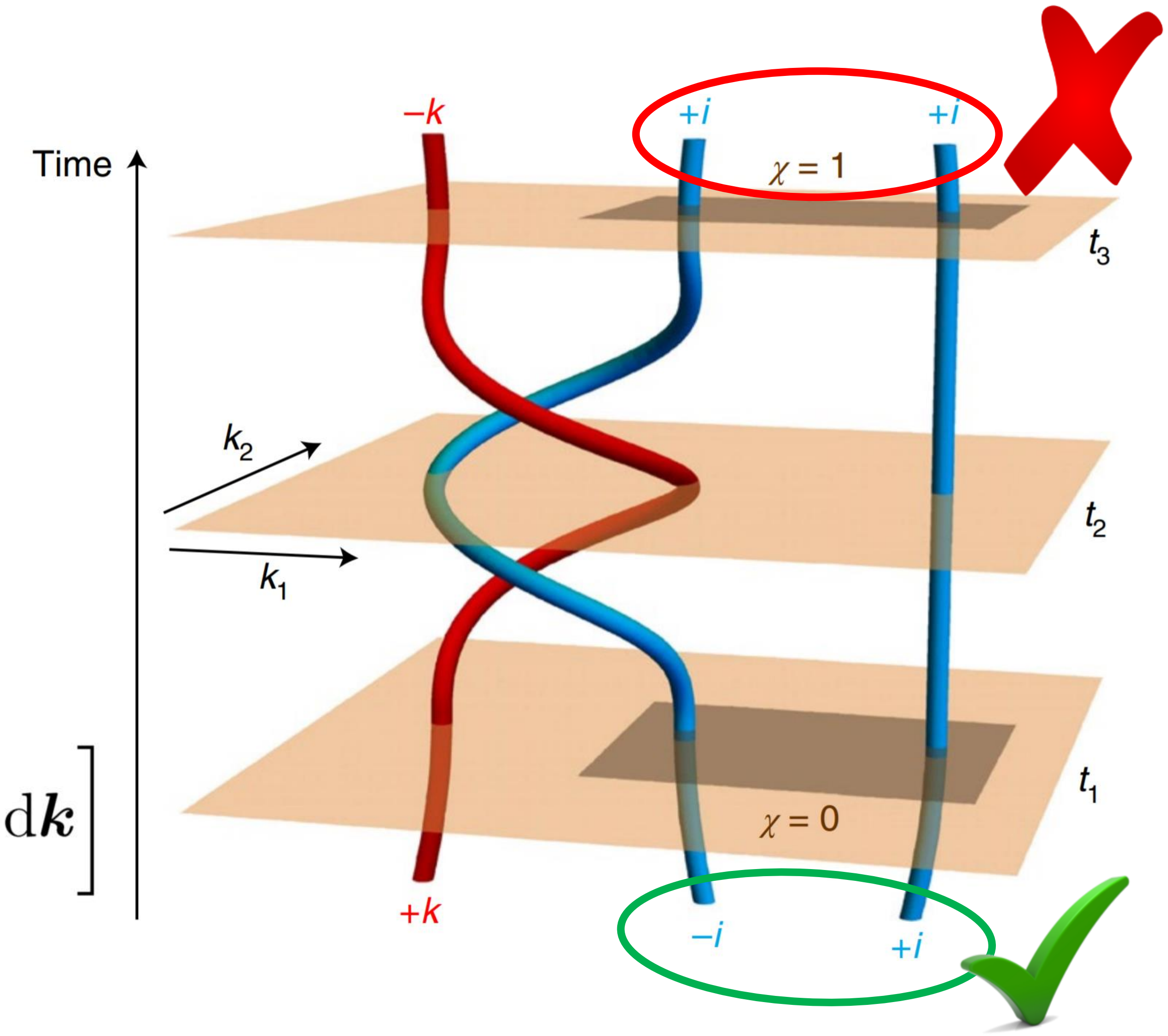
Euler curvature

$$\mathrm{Eu}(\boldsymbol{k}) = \boldsymbol{\nabla}\times\mathbf{a}(\boldsymbol{k})$$

A. Bouhon, Q.S. Wu, R.-J. Slager, H. Weng. O. V. Yazyev, and T.B., *et al.*, Nat. Phys. **16**, 1137—1143 (2020)
[See also: Y. X. Zhao and Y. Lu, Phys. Rev. Lett. **118**, 056401 (2017) & J. Ahn et al., Phys. Rev. X **9**, 021013 (2019)]

Generalization to many bands: *patch Euler class*





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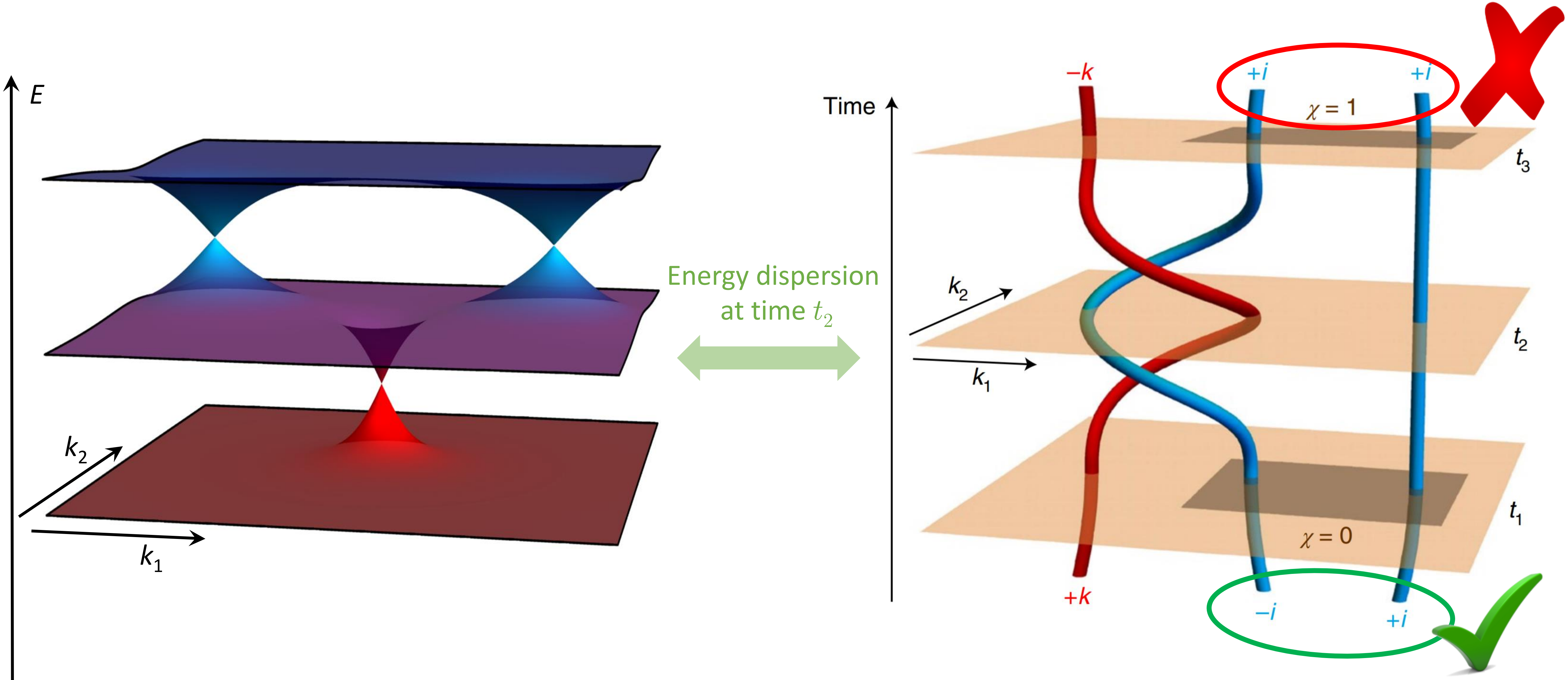
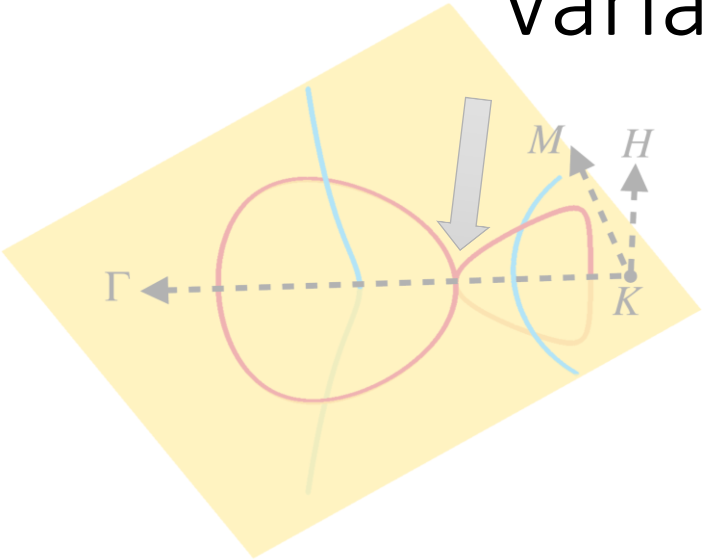
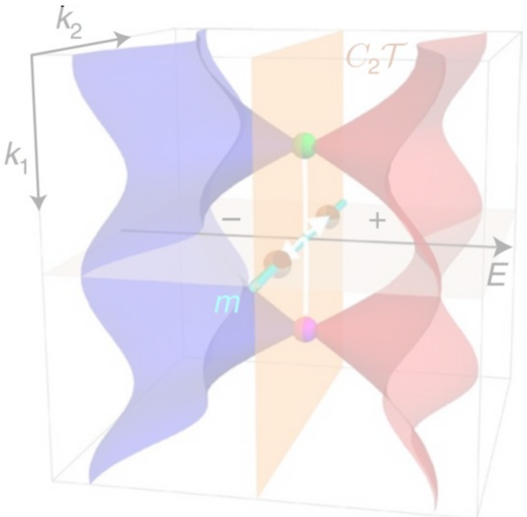

E
k2
k1
Energy dispersion
at time t2
Time
k2
k1
−k
+k
+i
+i
−i
+i
χ = 1
χ = 0
t3
t2
t1

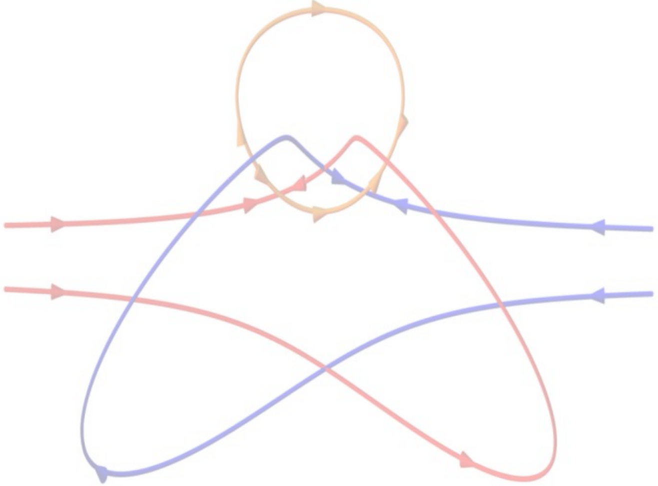
Variations on the non-Abelian story



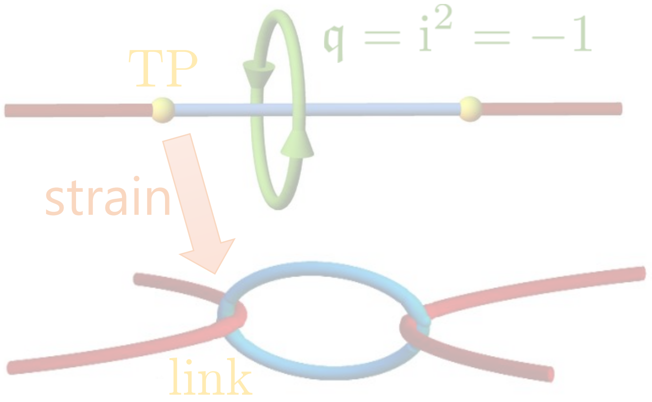
Topological stability of nodal chains; application to elemental Sc.
[Science **365**, 1273-1277 (2019)]



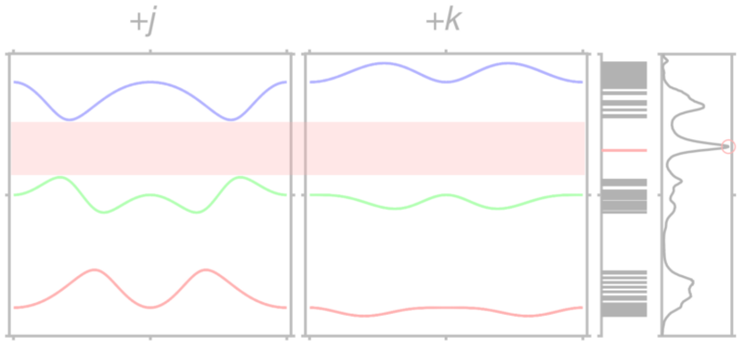
Colliding Weyl points of opposite chirality in ZrTe *fail to annihilate*.
[Nat. Phys. **16**, 1137—1143 (2020)]



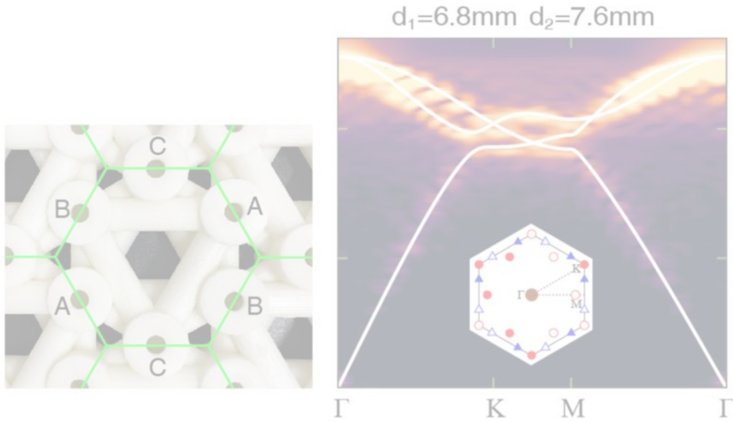
Non-trivial braiding of Dirac points in twisted bilayer graphene.
[Phys. Rev B **102**, 035161 (2020)]



Conversion of triple nodal points into linked nodal rings (Li_2NaN).
[Phys. Rev. B **103**, L121101 (2021)]



Photonics realization of “non-Abelian 1D topogical Insulators”.
[Nature **594**, 195—200 (2021)]



Observed non-trivial braiding of Dirac points in metamaterial phonons.
[arXiv:2104.13397 (2021)]

Mathematical analogies:

Defects of order parameter in coordinates space. ⟷ Degeneracies of spectral gap in momentum space.

XY-model vortices <---------> Dirac points in graphene

[non-trivial π_1]

Heisenberg hedgehogs <---------> Weyl points

[trivial π_1 and non-trivial π_2]

hedgehogs & disclinations of uniaxial nematics <---------> Chern # & exceptional lines of non-Herm. Hamiltonian

[non-trivial action of π_1 on π_2]

disclination defects of biaxial nematics <---------> band nodes of three-band real-sym. Hamiltonian

[non-Abelian π_1]

Domain walls <---------> (Bogoliubov-Fermi) nodal surfaces

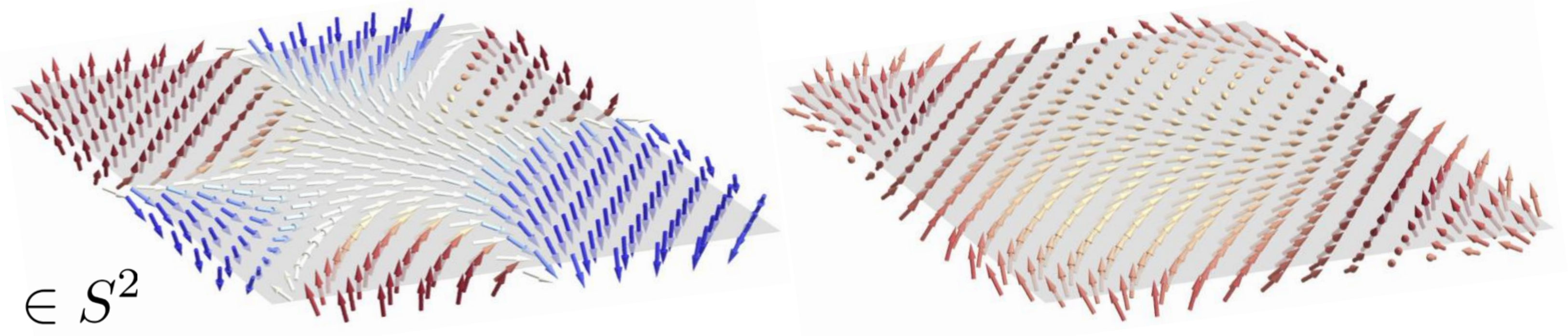
[non-trivial π_0]

Mathematical analogies:

[see also: R. Kennedy, *et al.*, Phys. Rev. B **91**, 245148 (2015)]

Topological textures of order parameter in coordinates space. ⟷ *Topological insulators* in momentum space.

topological phase | trivial phase



Homotopy group $\pi_2(\mathrm{Gr}_{\mathbb{C}}) = \mathbb{Z}$ detects the Chern number.

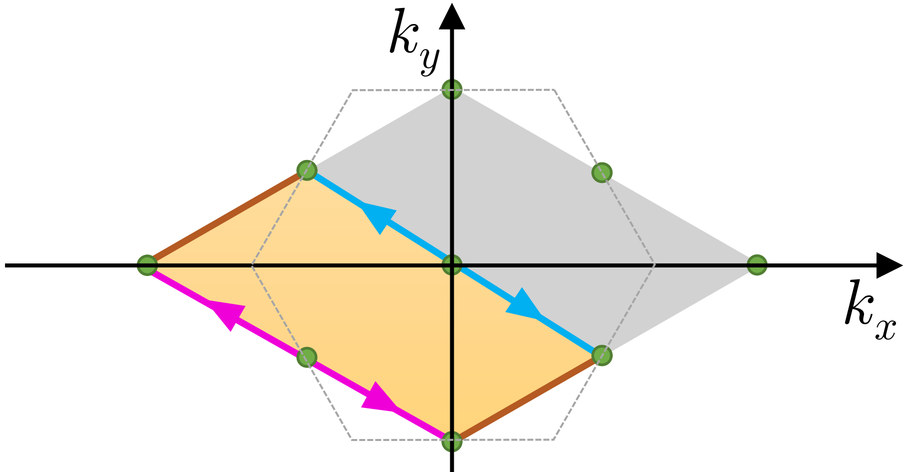
Example #1: Haldane model

$$H(\boldsymbol{k}) = h_0(\boldsymbol{k})\mathbf{1} + \boldsymbol{h}(\boldsymbol{k}) \cdot \boldsymbol{\sigma}$$

$$(n_x, n_y, n_z) := \frac{(h_x, h_y, h_z)}{\sqrt{h_x^2 + h_y^2 + h_z^2}} \in S^2$$

Example #2: Kane-Mele model

$$\mathcal{T}^{-1} H(\boldsymbol{k}) \mathcal{T}^{-1} = H(-\boldsymbol{k})$$!



Techniques to characterize band topology

K-theory

→ assumes ∞ of bands

→ uses <u>complete</u> information

→ handles crystalline symmetry

Homotopy theory

→ for arbitrarily many bands

→ uses <u>complete</u> information

→ hard to handle symmetries

Symmetry indicators / TQC

→ for arbitrarily many bands

→ uses <u>reduced</u> information

→ built around symmetries

FEW **# BANDS** INFINITE

LOW **SYMMETRY** HIGH

Canonical model of "unstable" topology

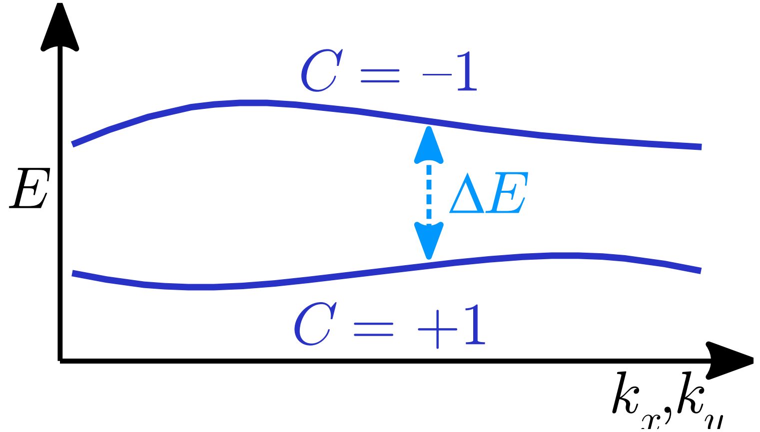
<u>Hopf insulator:</u> Two-band class-A model in 3D, based on $\pi_3(S^2) = \mathbb{Z}$.

[J. E. Moore, Y. Ran, and X.-G. Wen, Phys. Rev. Lett. **101**, 186805 (2008)]

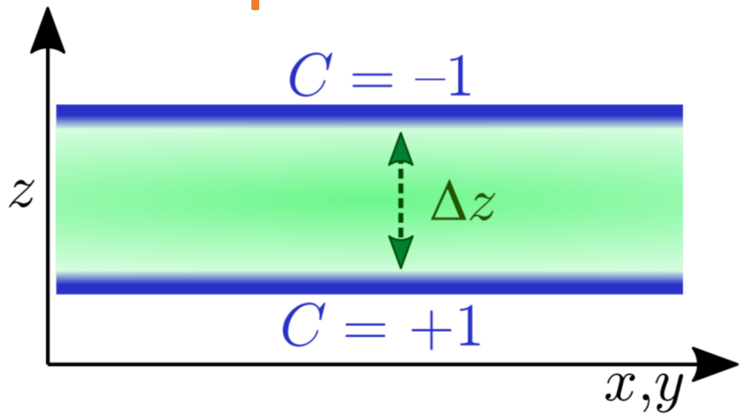
$$z(\boldsymbol{k}) = \begin{pmatrix} \sin k_x + \mathrm{i} \sin k_y \\ \sin k_z + \mathrm{i}[\cos k_x + \cos k_y + \cos k_z - m] \end{pmatrix} \qquad H(\boldsymbol{k}) = [z^\dagger \boldsymbol{\sigma} z] \cdot \boldsymbol{\sigma}$$

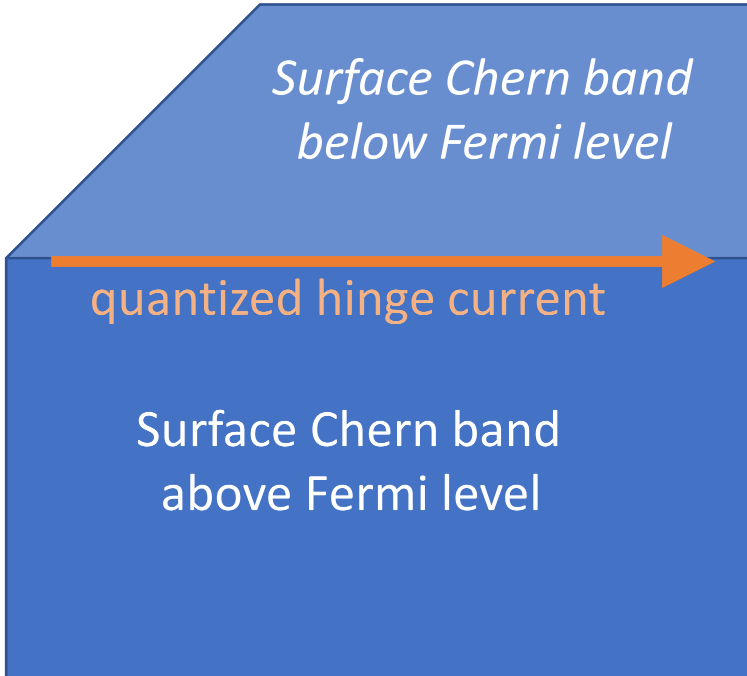
New paradigm for topological boundary anomaly:

Chern insulator



Hopf insulator

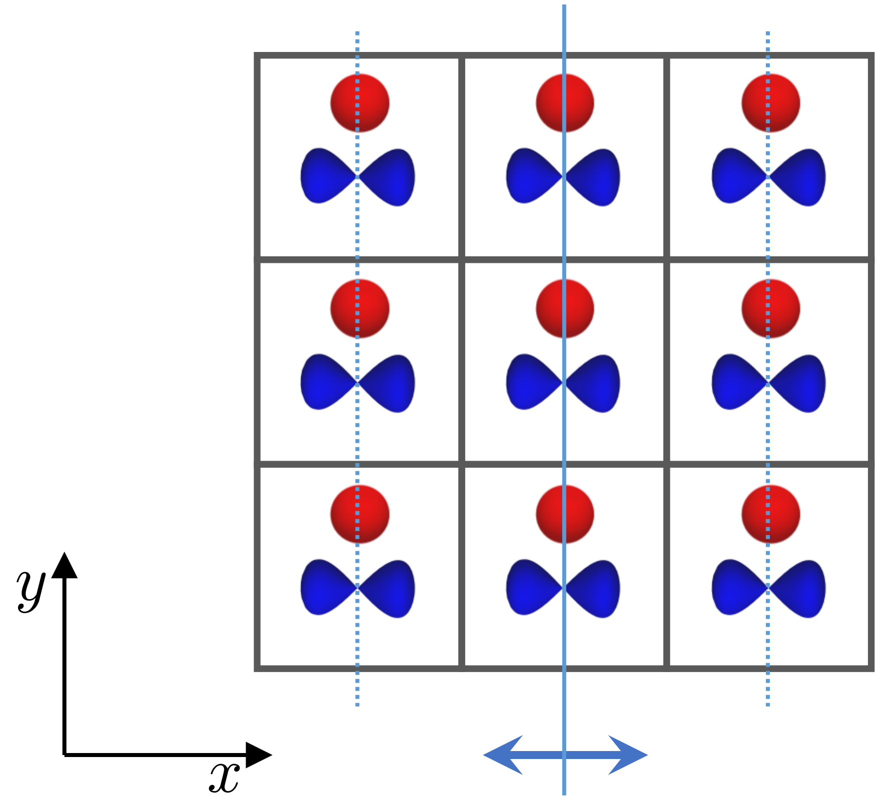




A. Alexandradinata *et al.*, Phys. Rev. B **103**, 045107 (2021)
B. Lapierre *et al.*, Phys. Rev. Research **3**, 033045 (2021)
P. Zhu *et al.*, Phys. Rev. B **103**, 014417 (2021)

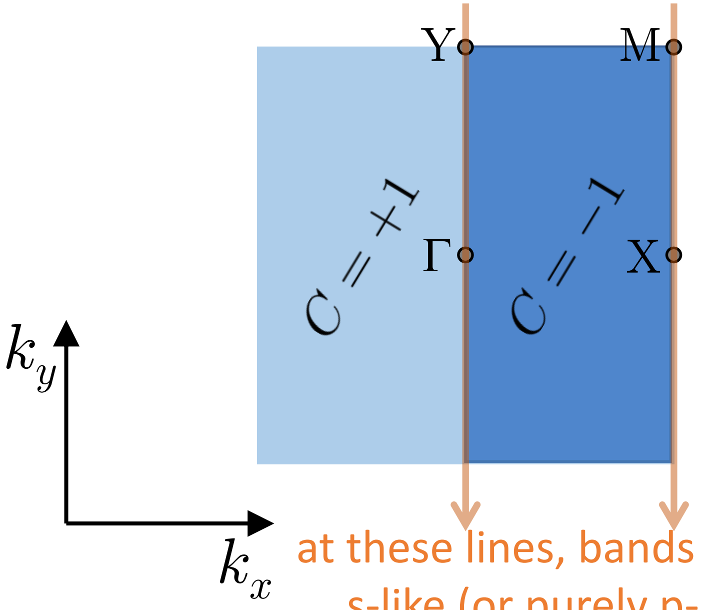
Returning Thouless pump in 2D

real space:

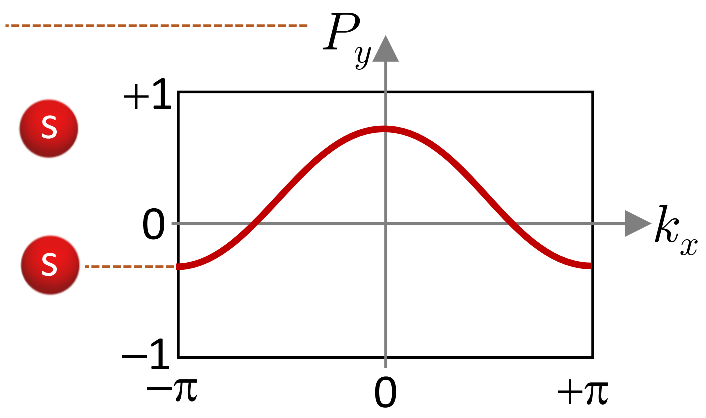


Left-right (m_x) mirror symmetry

momentum space:



polarization:



at these lines, bands purely s-like (or purely p-like)

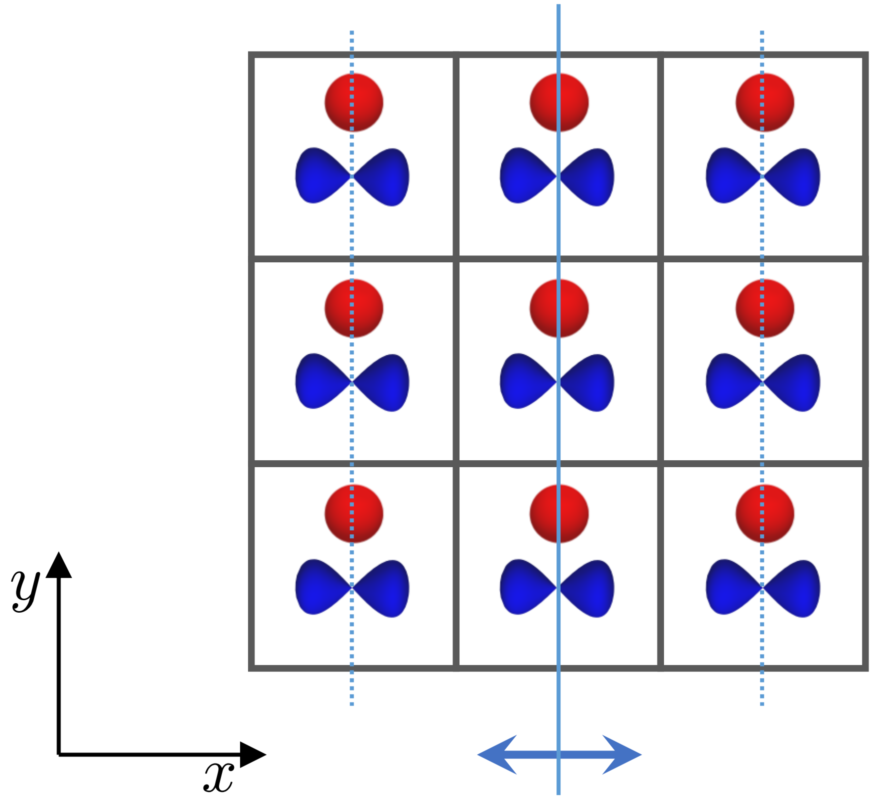
<u>Assume:</u> valence s-like & conduction p-like at both high-sym. lines.

The same (projected) Hamiltonian

A. Nelson, T. Neupert, <u>T.B.</u>, and A. Alexandradinata, Phys. Rev. Lett. **126**, 216404 (2021)

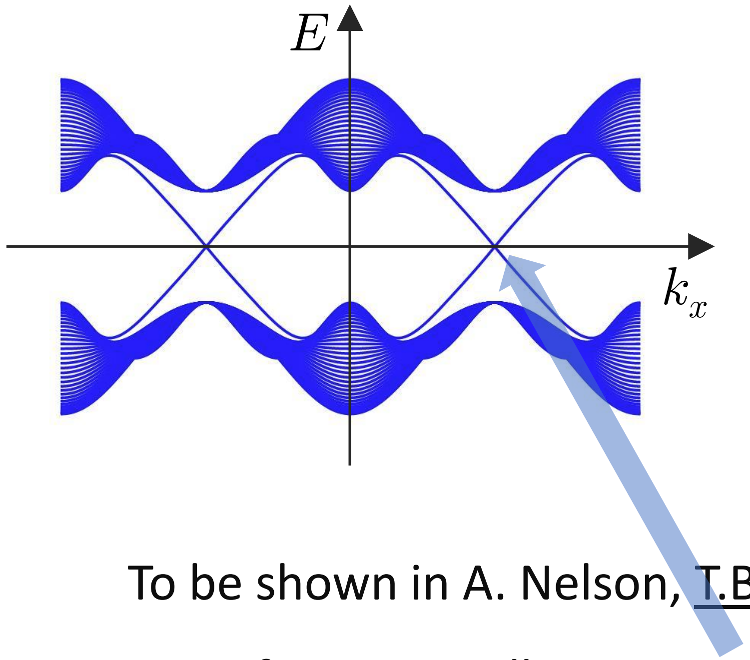
Returning Thouless pump in 2D

real space:

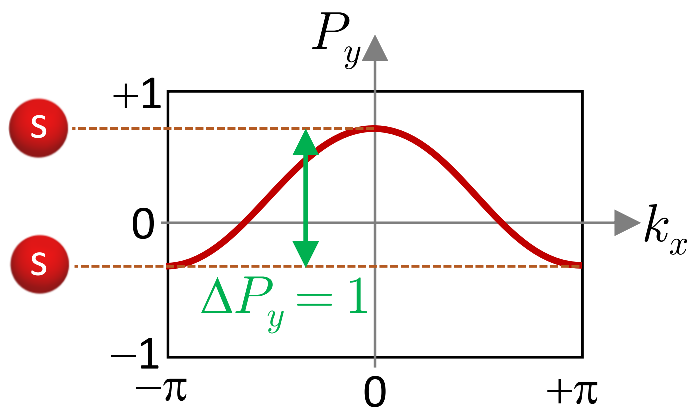


Left-right (m_x) mirror symmetry

surface states:



polarization:



To be shown in A. Nelson, T.B., *et. al.* (in preparation):

- Enforces metallic in-gap states *at sharp boundaries*.
- In 2D leads to edge-localized Zak-phase.
- In 3D leads to surface-localized Chern number (like Hopf).

The topological obstruction *remains stable* if added conduction/valence bands respect the mirror-even/odd dichotomy → neither stable, nor fragile, but *delicate topology*.

Open questions…

Observable fingerprints of the non-commutative exchange of band nodes?

Cataloguing homotopy-revealed delicate topologies?

Stability against disorder & interactions?

Material realizations?

(…)

Works in collaborations with:

Theory:

A. Alexandradinata (UIUC, USA)

A. Bouhon (Nordita, Sweden)

S. Fan (Stanford, USA)

P. M. Lenggenhager (PSI, Switzerland)

A. Nelson (Uni Zürich, Switzerland)

T. Neupert (Uni Zürich, Switzerland)

M. Sigrist (ETH Zürich, Switzerland)

R.-J. Slager (Harvard → Cambridge)

A. A. Soluyanov† (Uni Zürich, Switzerland)

X.-Q. Sun (Stanford → UIUC)

A. Tiwari (PSI, Switzerland)

C. C. Wojcik (Stanford, USA)

First-principles calculations:

X. Liu (Uni Zürich, Switzerland)

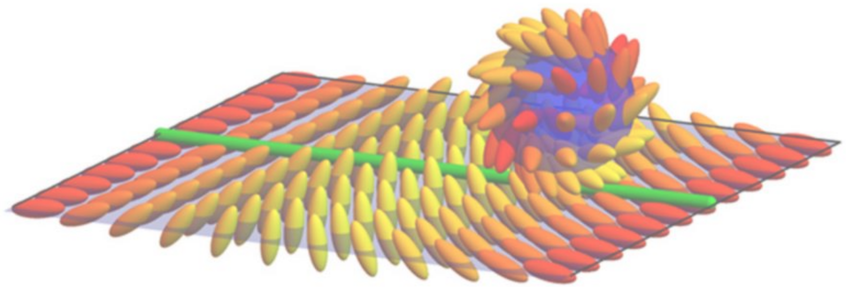
S. S. Tsirkin (Uni Zürich, Switzerland)

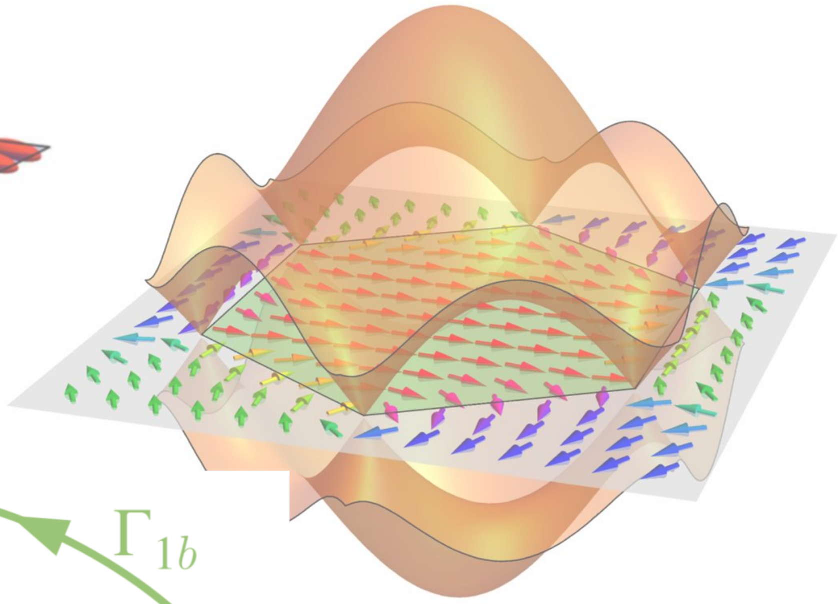
H. Weng (IoP Beijing, China)

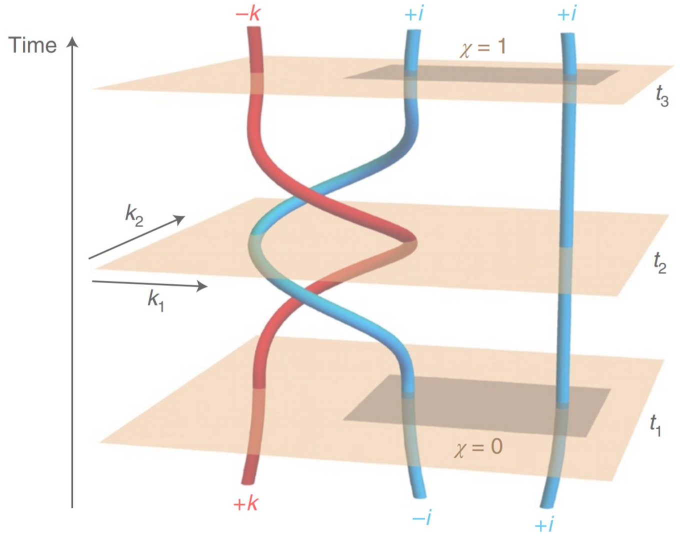
Q.S. Wu (EPFL, Switzerland)

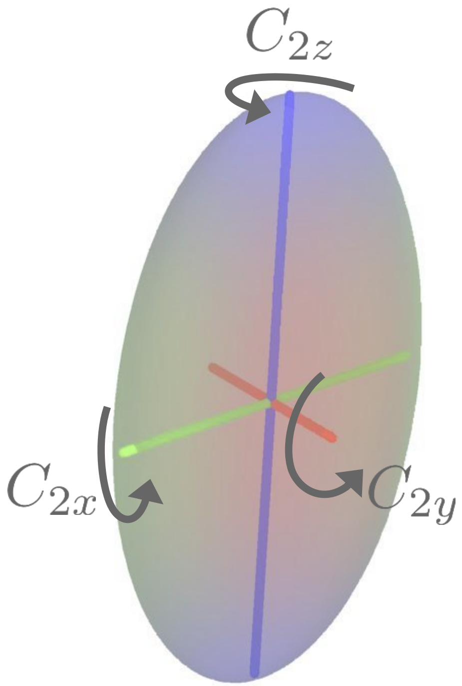
O. V. Yazyev (EPFL, Switzerland)

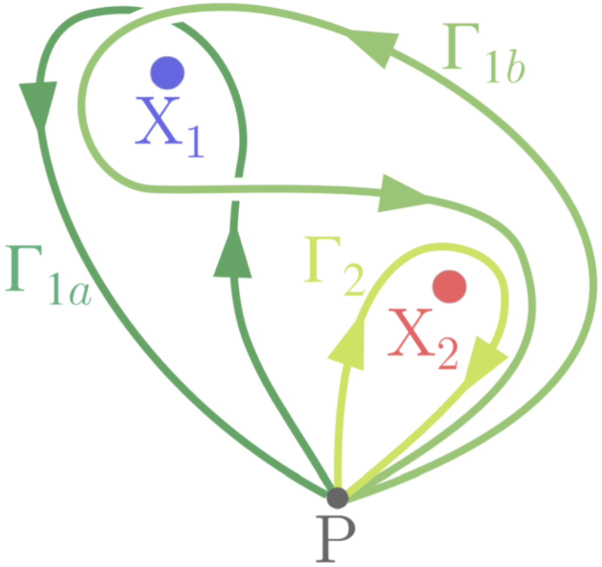
Summary & thank you for attention!

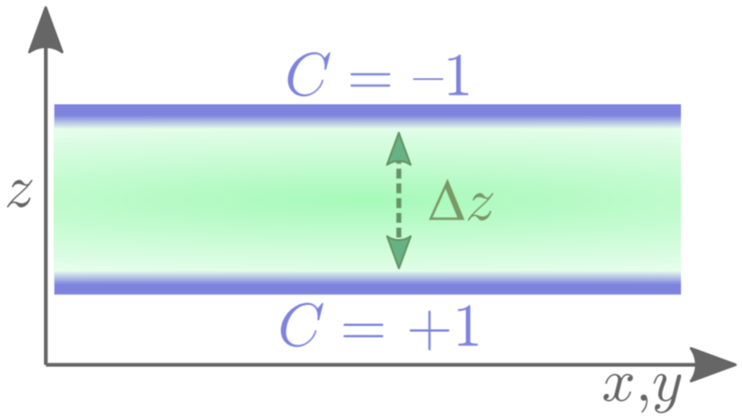


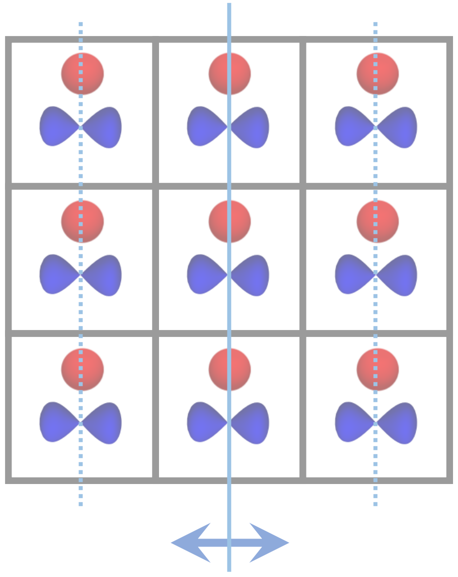










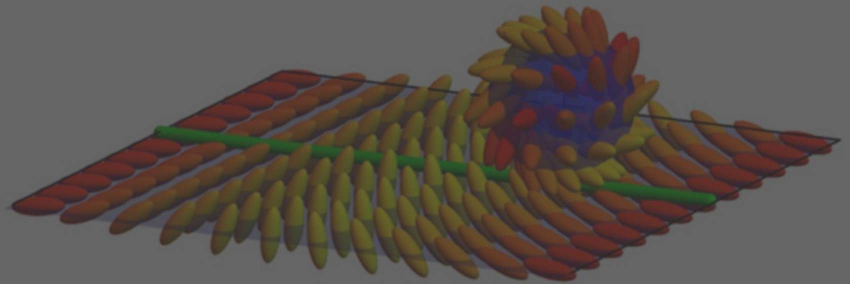
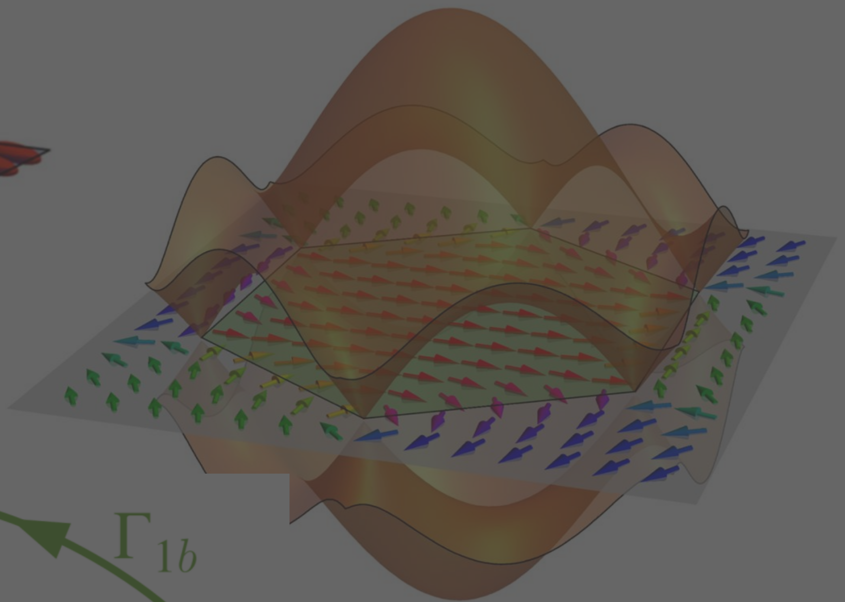
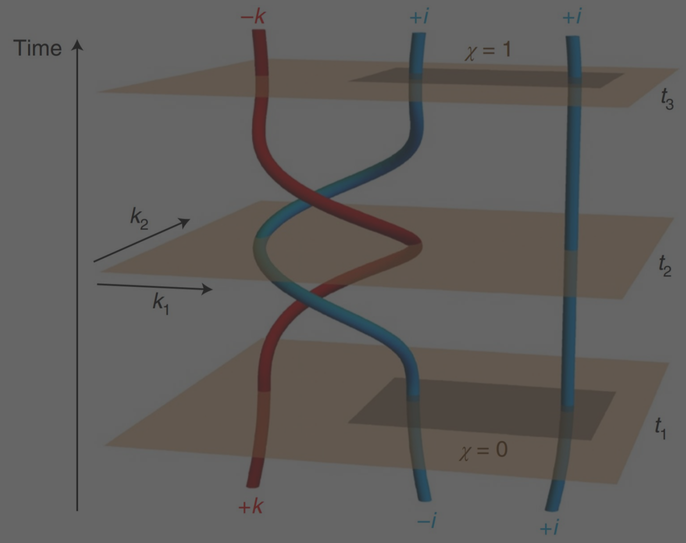


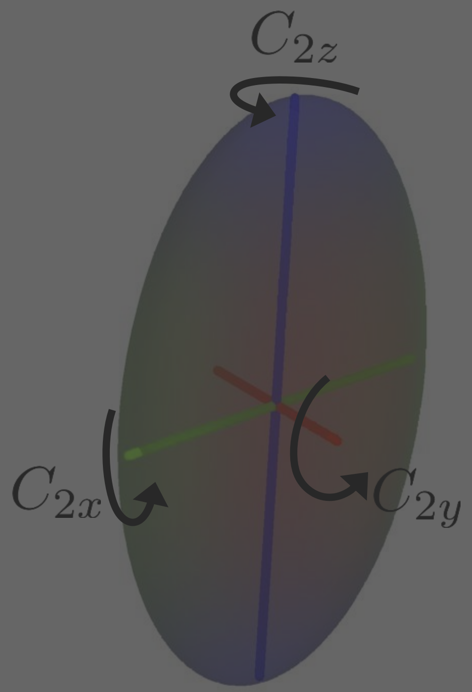
Tomáš Bzdušek

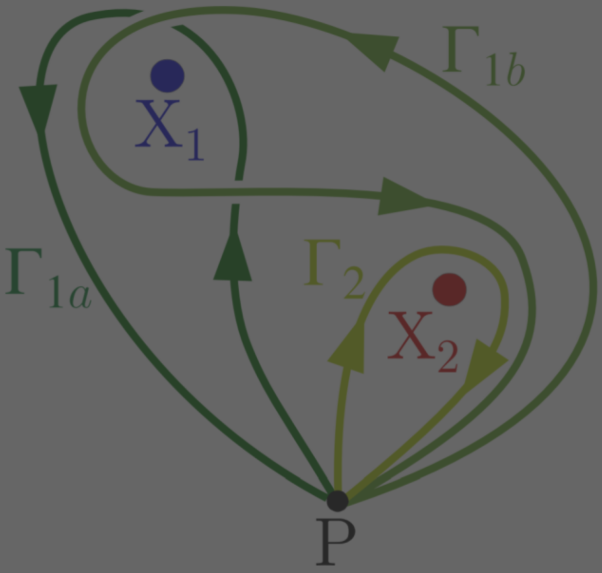
22. July 2021

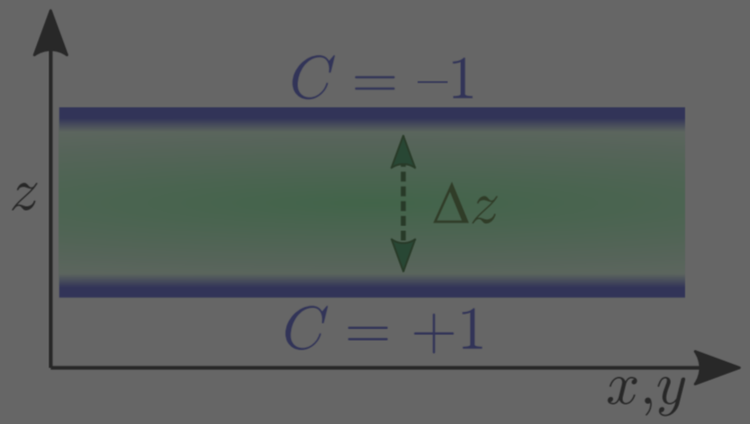
PAUL SCHERRER INSTITUT
PSI
University of Zurich[UZH]
FNSNF
SWISS NATIONAL SCIENCE FOUNDATION

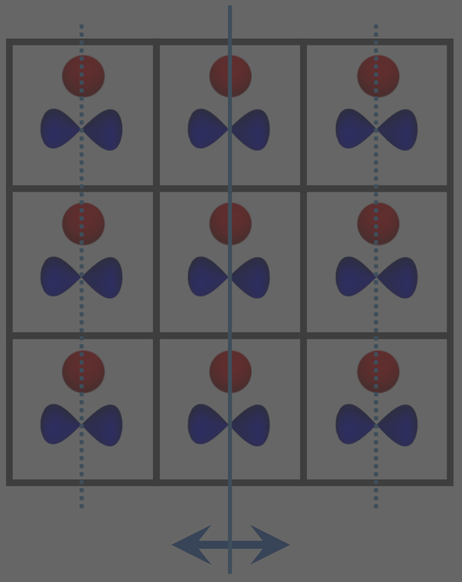
Summary & thank you for attention!









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22. July 2021

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