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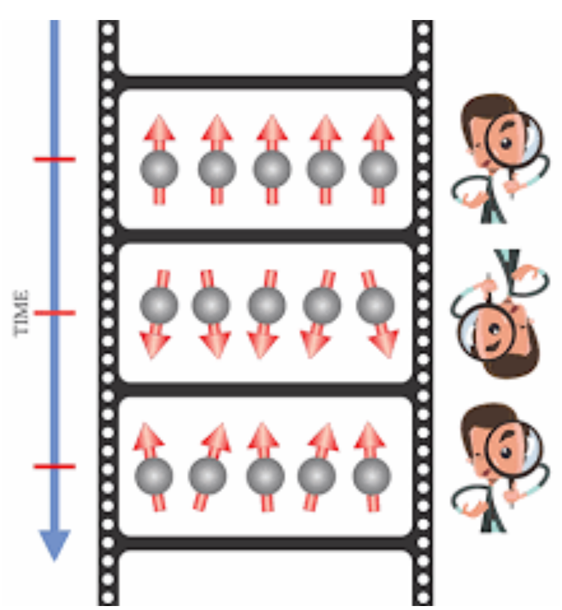
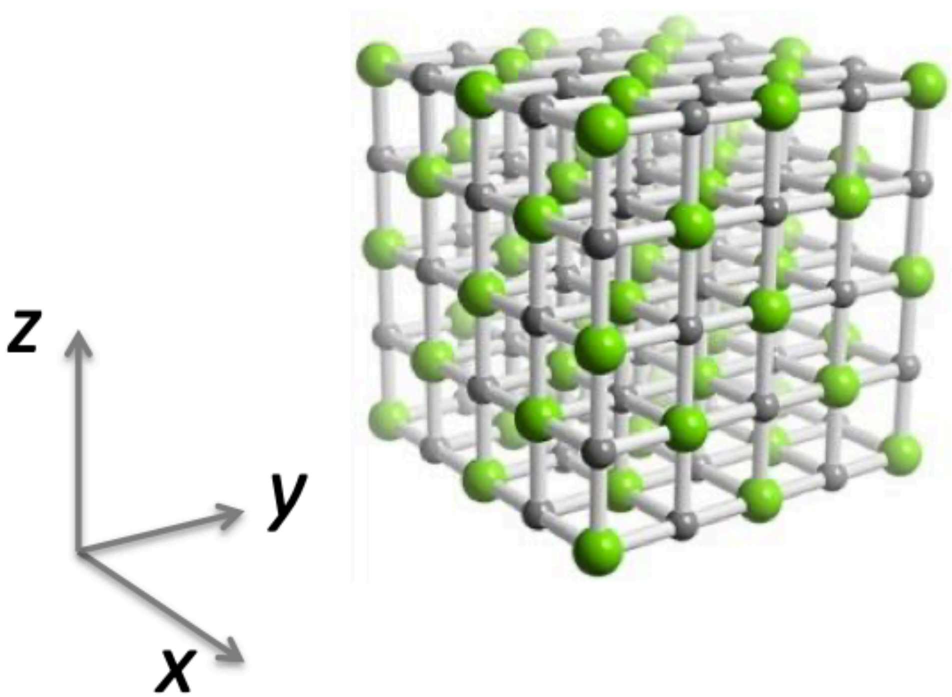
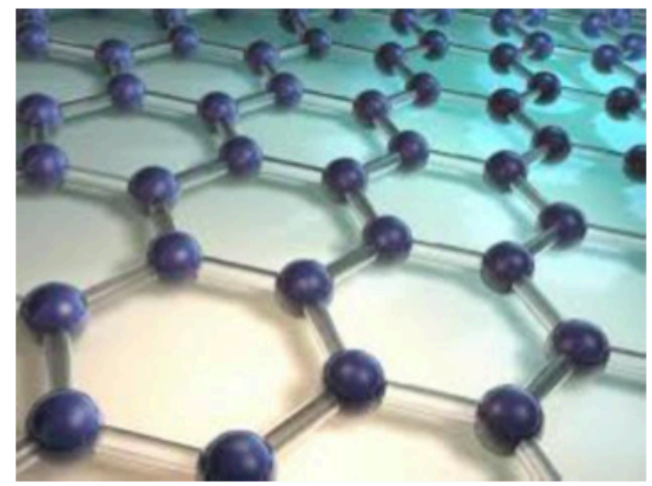
An Eternal Discrete Time Crystal

Sayan Choudhury
University of Pittsburgh



A Primer on Crystals





$$\sum_i \frac{\mathbf{p}_i^2}{2m_i} + \sum_{i \neq j} V(\mathbf{r}_i - \mathbf{r}_j)$$

Crystals are ubiquitous in nature and they represent a striking example of Landau's spontaneous symmetry breaking paradigm.

Space-Translation Symmetry is spontaneously broken in crystals!

Consequence: Phonon Modes!

Time Crystals

Selected for a Viewpoint in *Physics*
PRL **109,** 160401 (2012) PHYSICAL REVIEW LETTERS week ending 19 OCTOBER 2012

Quantum Time Crystals

Frank Wilczek

Selected for a Viewpoint in *Physics*
PRL **109,** 163001 (2012) PHYSICAL REVIEW LETTERS week ending 19 OCTOBER 2012

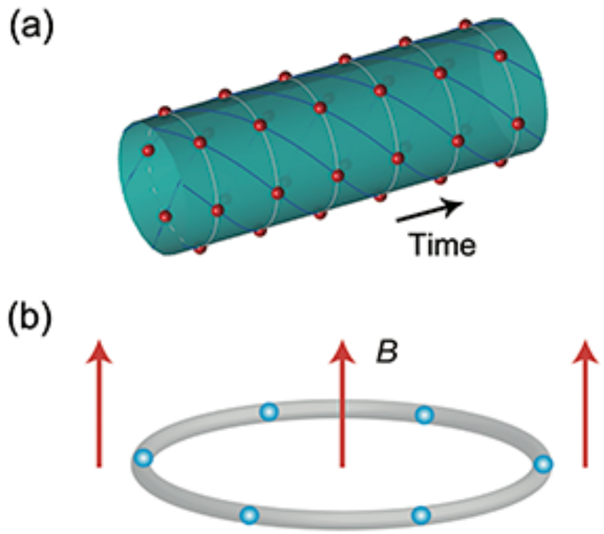
Space-Time Crystals of Trapped Ions

Tongcang Li,[1] Zhe-Xuan Gong,[2,3] Zhang-Qi Yin,[3,4] H. T. Quan,[5] Xiaobo Yin,[1] Peng Zhang,[1] L.-M. Duan,[2,3] and Xiang Zhang[1,6,*]

In 2012, Frank Wilczek introduced the notion of Time Translation Symmetry Breaking in Quantum Many-body Systems.

He dubbed this new phase of matter a "Quantum Time Crystal".

Wilczek's Model

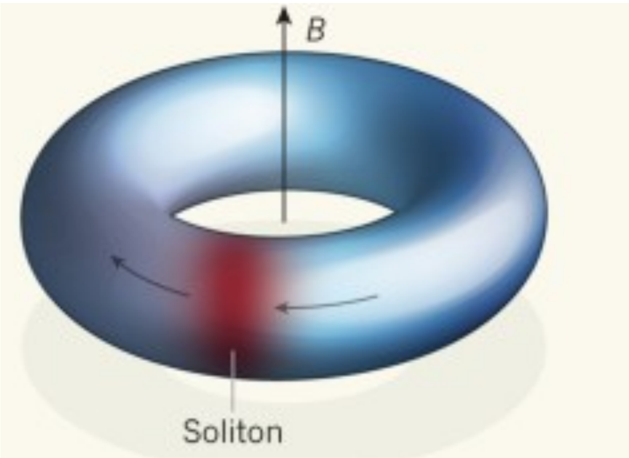


$$H = \sum_{i=1}^{N} \frac{(p_i - \alpha)^2}{2} + \frac{g_0}{2} \sum_{i \neq j} \delta(x_i - x_j)$$

Ground State Ansatz: $\psi(x) = \prod_{i=1}^{N} \phi_0(x_i)$

T. Li *et al.* Phys. Rev. Lett. **109**, 163001 (2012)

Wilczek considered N attractively interacting bosons in an Aharonov-Bohm (AB) ring of unit length.



When $g_0(N-1) < -\pi^2$ and $\alpha = 0$, ϕ_0 becomes non-uniform and the ground state comprises bright solitons.

Central idea: A finite AB flux would cause the lump of charge to feel a simple torque. This will cause the lump to move and lead to spontaneous breaking of Time-Translation Symmetry

Problems with Wilczek's Model

PRL **111,** 070402 (2013) PHYSICAL REVIEW LETTERS week ending 16 AUGUST 2013

Impossibility of Spontaneously Rotating Time Crystals: A No-Go Theorem

Patrick Bruno*

Time crystals: Can diamagnetic currents drive a charge density wave into rotation?

EPL **103** 57008 (2013).

Philippe Nozières

PRL **114,** 251603 (2015) PHYSICAL REVIEW LETTERS week ending 26 JUNE 2015

Absence of Quantum Time Crystals

Haruki Watanabe[1,*] and Masaki Oshikawa[2,†]

The state posited by Wilczek was not the ground state of the model. The ground state of this system is one where the soliton does not move.

More generally, time translation symmetry breaking is not possible in the ground state or in equilibrium.

Circumventing the Issue: Resort to Non-Equilibrium

doi:10.1038/nature21413

Observation of a discrete time crystal

J. Zhang[1], P. W. Hess[1], A. Kyprianidis[1], P. Becker[1], A. Lee[1], J. Smith[1], G. Pagano[1], I.-D. Potirniche[2], A. C. Potter[3], A. Vishwanath[2,4], N. Y. Yao[2] & C. Monroe[1,5]

PRL **119,** 250602 (2017) PHYSICAL REVIEW LETTERS week ending 22 DECEMBER 2017

Time Crystal Behavior of Excited Eigenstates

Andrzej Syrwid,[1] Jakub Zakrzewski,[1,2] and Krzysztof Sacha[1,2]

PHYSICAL REVIEW LETTERS **121,** 035301 (2018)

Boundary Time Crystals

F. Iemini,[1] A. Russomanno,[2,1] J. Keeling,[3] M. Schirò,[4] M. Dalmonte,[1] and R. Fazio[1,2]

Non-stationarity and dissipative time crystals: spectral properties and finite-size effects

Cameron Booker[1], Berislav Buča[1] and Dieter Jaksch[1]

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New Journal of Physics, Volume 22, August 2020

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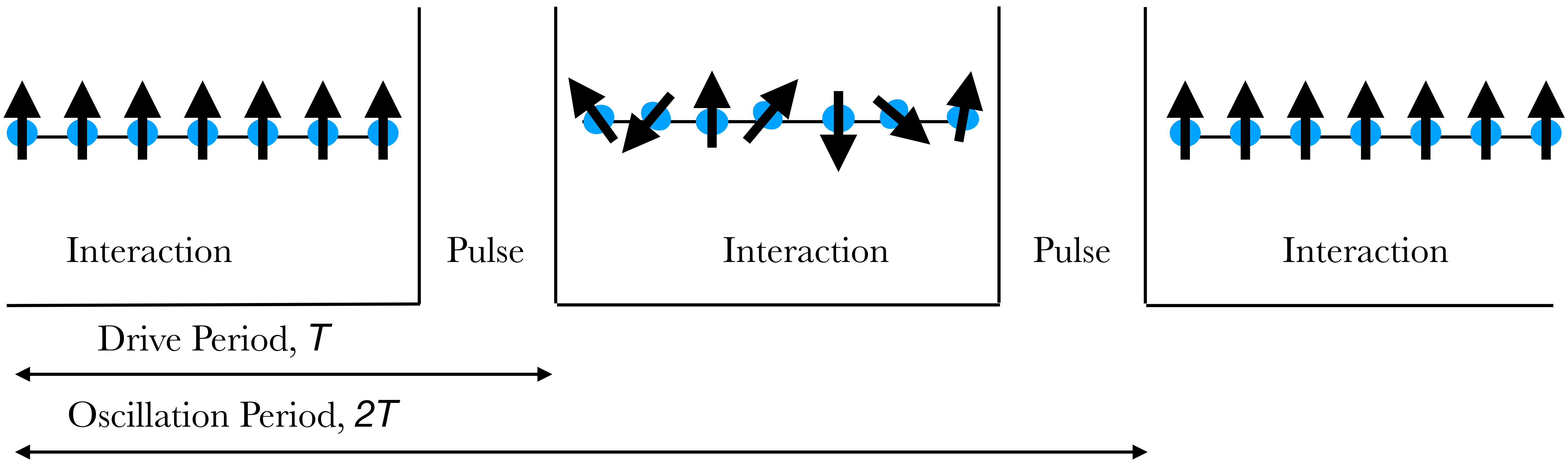

A Discrete Time Crystal (DTC) is a periodically driven system, where physical observables oscillate with a subharmonic frequency.

Discrete Time Crystals

In order to qualify as a Discrete Time Crystal, there should be a class of observables, O and initial states $|\psi\rangle$ such that $f(t) = \langle\psi|O(t)|\psi\rangle$ satisfy the following 3 conditions:

A. Time Translation Symmetry Breaking: $f(t+T) \neq f(t)$, while $H(t+T) = H(t)$

B. Rigidity: $f(t)$ shows a fixed oscillation frequency without fine-tuned Hamiltonian parameters.

C. Persistence: The nontrivial oscillation with fixed frequency must persist indefinitely when $L \to \infty$.

Ref: Biao Huang, Ying-Hai Wu, and W. Vincent Liu, Phys. Rev. Lett. **120**, 110603 (2018)

Recipe for Realizing a DTC

Periodically driven systems typically heat up to a featureless infinite temperature state. This heating can be evaded in a many-body localized system.

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Periodically driven systems typically heat up to a featureless infinite temperature state. This heating can be evaded in a many-body localized system.

Model:

$$H = H_1 + V\delta(t - nT)$$

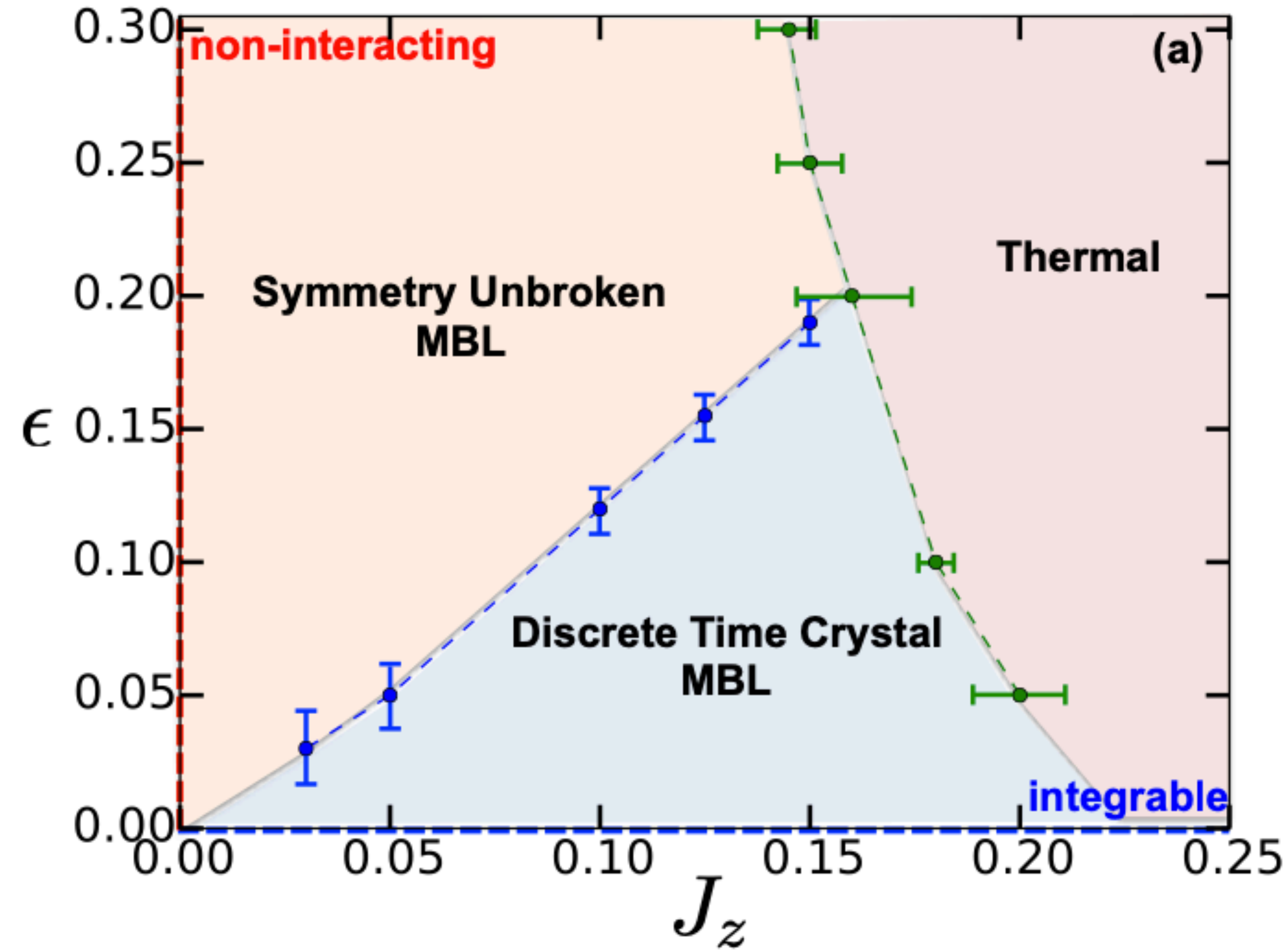
$$H_1 = \sum_i J_i^z \sigma_i^z \sigma_{i+1}^z + B_i^z \sigma_i^z$$

Disordered Interactions

Onsite Disorder

Imperfect π-pulse

$$V = (\frac{\pi}{2} - \epsilon)\sum_i \sigma_i^x$$



Yao *et al.* PRL **118**, 030401 (2017)

Time Crystal Dynamics

Perfect π -Pulse, $\epsilon = 0$

$$|\uparrow\uparrow\ldots\uparrow\uparrow\rangle \longrightarrow |\downarrow\downarrow\ldots\downarrow\downarrow\rangle \longrightarrow |\uparrow\uparrow\ldots\uparrow\uparrow\rangle$$

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Period doubling oscillations

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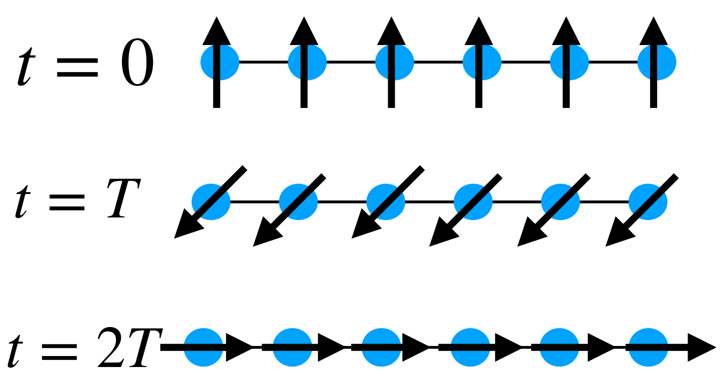
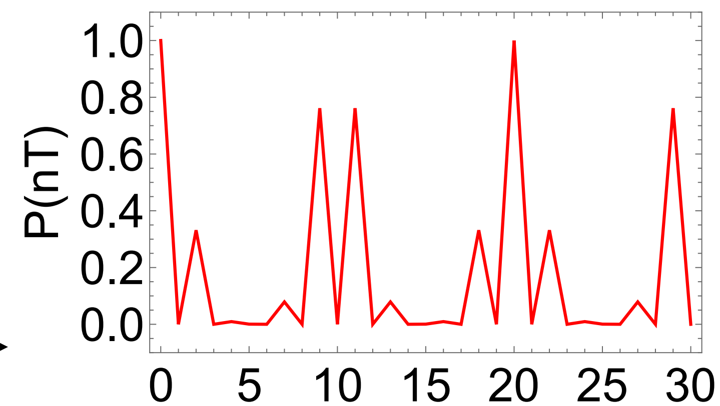
No MBL:





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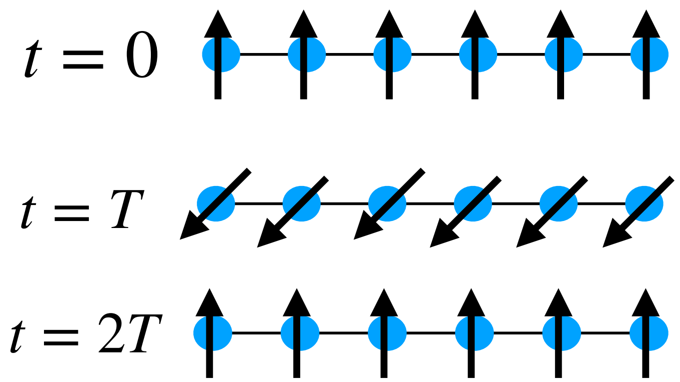

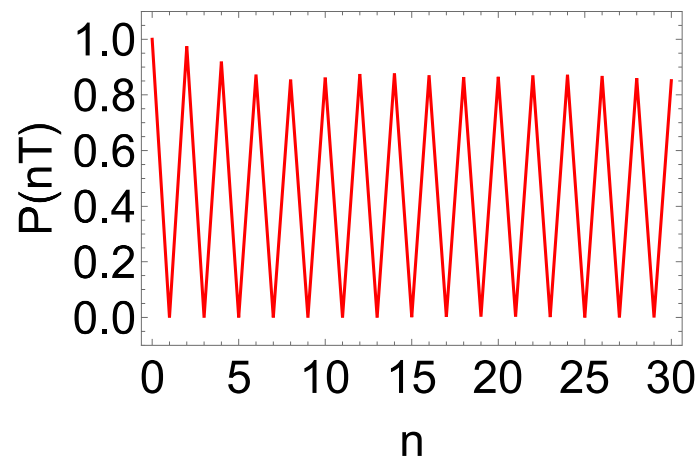

Period doubling oscillations

Issues With Many-Body Localization

- Many-Body localization places very stringent bounds on the range of interactions, symmetries and dimensionality of the system (see: Gopalakrishnan and Parameswaran, Phys. Rep. **862**, 1 (2020)).

- Furthermore, MBL may not even be possible in the Thermodynamic Limit (see: Jan Šuntajs, Janez Bonča, Tomaž Prosen, and Lev Vidmar, Phys. Rev. E **102**, 062144 (2020))

- Practically, MBL can lead to long lived transient dynamics, making it very difficult to access long-time dynamics.

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Is MBL necessary to realize a DTC?

DTC without MBL

Observation of discrete time-crystalline order in a disordered dipolar many-body system

Soonwon Choi, Joonhee Choi, Renate Landig, Georg Kucsko, Hengyun Zhou, Junichi Isoya, Fedor Jelezko, Shinobu Onoda, Hitoshi Sumiya, Vedika Khemani, Curt von Keyserlingk, Norman Y. Yao, Eugene Demler & Mikhail D. Lukin

Nature **543**, 221–225(2017) | Cite this article

PHYSICAL REVIEW LETTERS **120,** 110603 (2018)

Clean Floquet Time Crystals: Models and Realizations in Cold Atoms

Biao Huang,[1,*] Ying-Hai Wu,[2] and W. Vincent Liu[1,3,4,†]

PHYSICAL REVIEW LETTERS **120,** 180602 (2018)

Editors' Suggestion | Featured in Physics

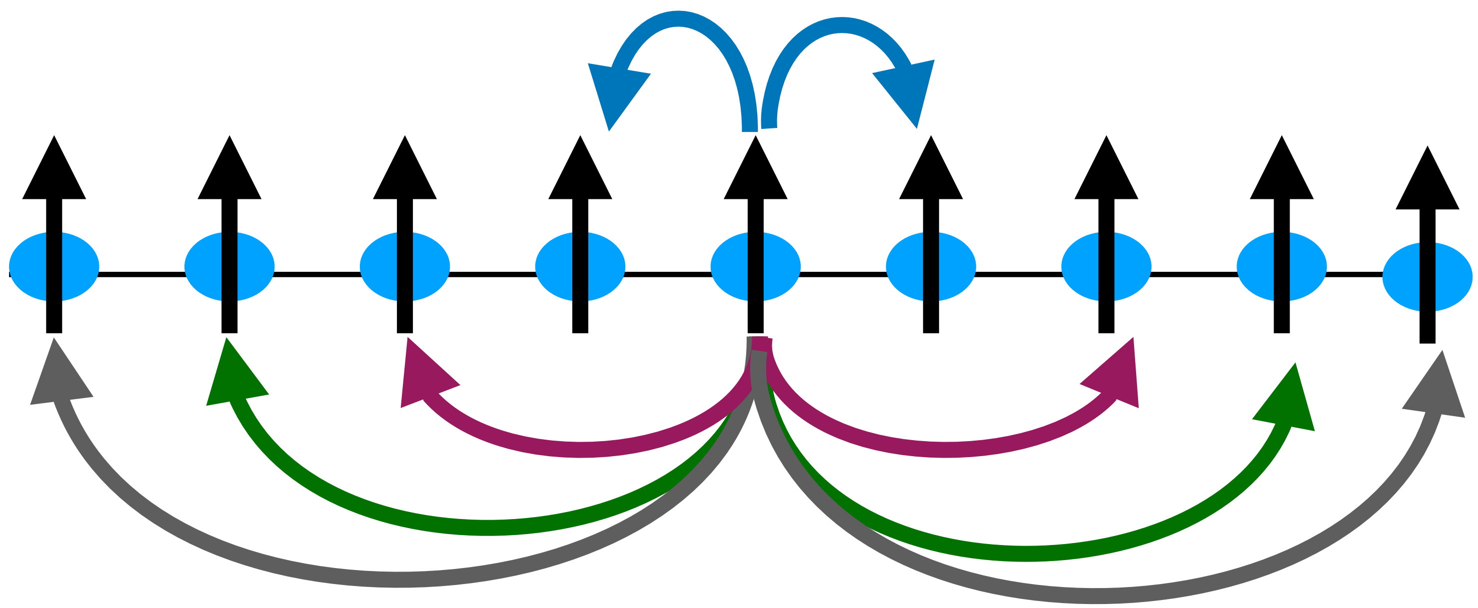
Temporal Order in Periodically Driven Spins in Star-Shaped Clusters

Soham Pal, Naveen Nishad, T. S. Mahesh, and G. J. Sreejith

Question: What is the simplest model where a DTC can arise in the absence of MBL?

Our Model

We consider a periodically driven one-dimensional spin-chain



$$H(t) = \sum_i \sum_{j \neq i} J_{ij} S_i^z S_j^z + \sum_i (\pi - \epsilon) S_i^x \delta(t - nT)$$

We consider 2 different types of interaction:

(a) Nearest Neighbor Ising interaction
(b) Infinite Range Ising interaction

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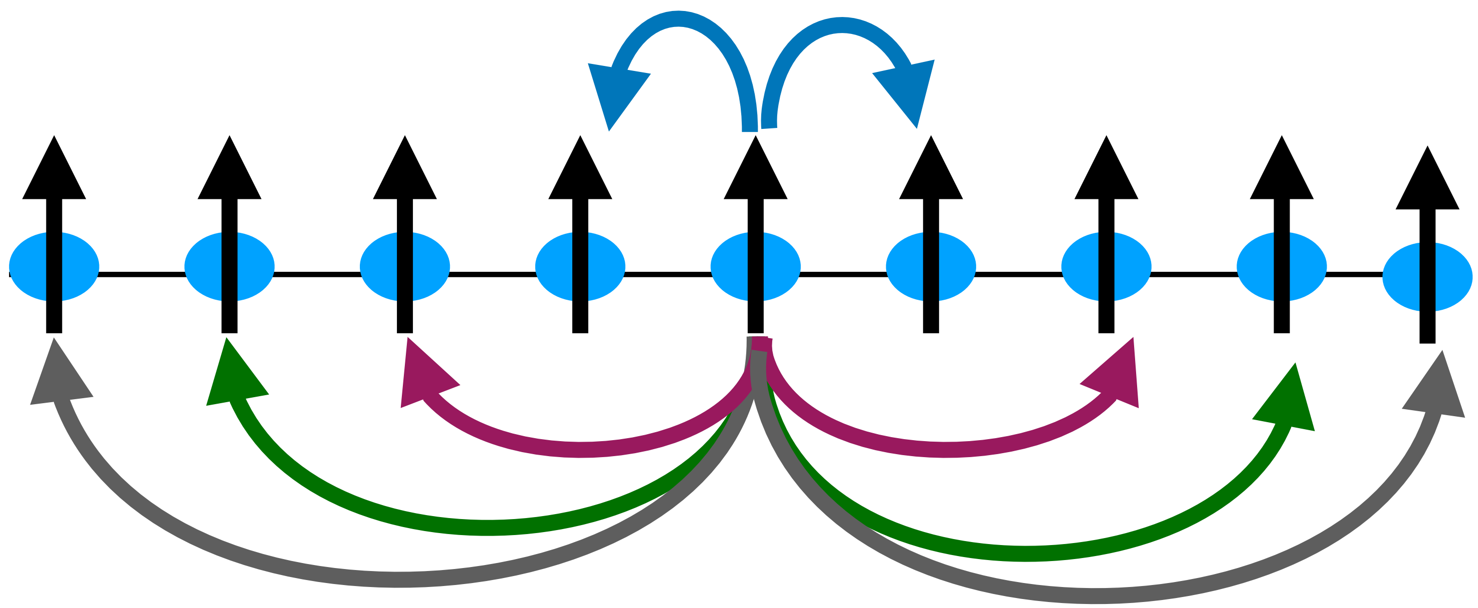

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We consider 2 different types of interaction:

(a) Nearest Neighbor Ising interaction
(b) Infinite Range Ising interaction

When $\epsilon = 0$, the chain exhibits a sub-harmonic response with frequency $\omega/2$.

2-Pulse Analysis

Perfect π Pulse

$$|\uparrow\uparrow\ldots\uparrow\uparrow\rangle \longrightarrow |\downarrow\downarrow\ldots\downarrow\downarrow\rangle \longrightarrow |\uparrow\uparrow\ldots\uparrow\uparrow\rangle$$

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Imperfect π Pulse, $\epsilon \neq 0$

$$|\uparrow\uparrow\dots\uparrow\uparrow\rangle \xrightarrow{\cos(\epsilon)^L} |\downarrow\downarrow\dots\downarrow\downarrow\rangle$$

$$\xrightarrow{-\cos(\epsilon)^{L-1}\sin(\epsilon)} |\downarrow\downarrow\dots\uparrow\dots\downarrow\downarrow\rangle$$

$$\xrightarrow{(-1)^m\cos(\epsilon)^{L-m}\sin(\epsilon)^m} \bullet\bullet\bullet$$

$$\xrightarrow{\sin(\epsilon)^L} |\downarrow\downarrow\dots\downarrow\downarrow\rangle$$

In general, no perfect revival

Question: How can we recover the initial state after 2 pulses?

Creating a time-crystal

For a small deviation from the perfect π pulse (when $L\epsilon^2 \ll 1$), we can just consider 2 pathways to examine the dynamics.

We consider the nearest neighbor Ising model where the Ising coupling is J

$$
\begin{tikzcd}
\lvert\uparrow\uparrow\dots\uparrow\uparrow\rangle \arrow[r, "\cos(\epsilon)^L"] \arrow[dr, "-\cos(\epsilon)^{L-1}\sin(\epsilon)"'] & \lvert\downarrow\downarrow\dots\downarrow\downarrow\rangle \arrow[r, "\cos(\epsilon)^{2L}"] & \lvert\uparrow\uparrow\dots\uparrow\uparrow\rangle \\
& \lvert\downarrow\downarrow\dots\uparrow\dots\downarrow\downarrow\rangle \arrow[ur, "-\cos(\epsilon)^{2L-2}\sin(\epsilon)^2\exp(-iJT)"'] &
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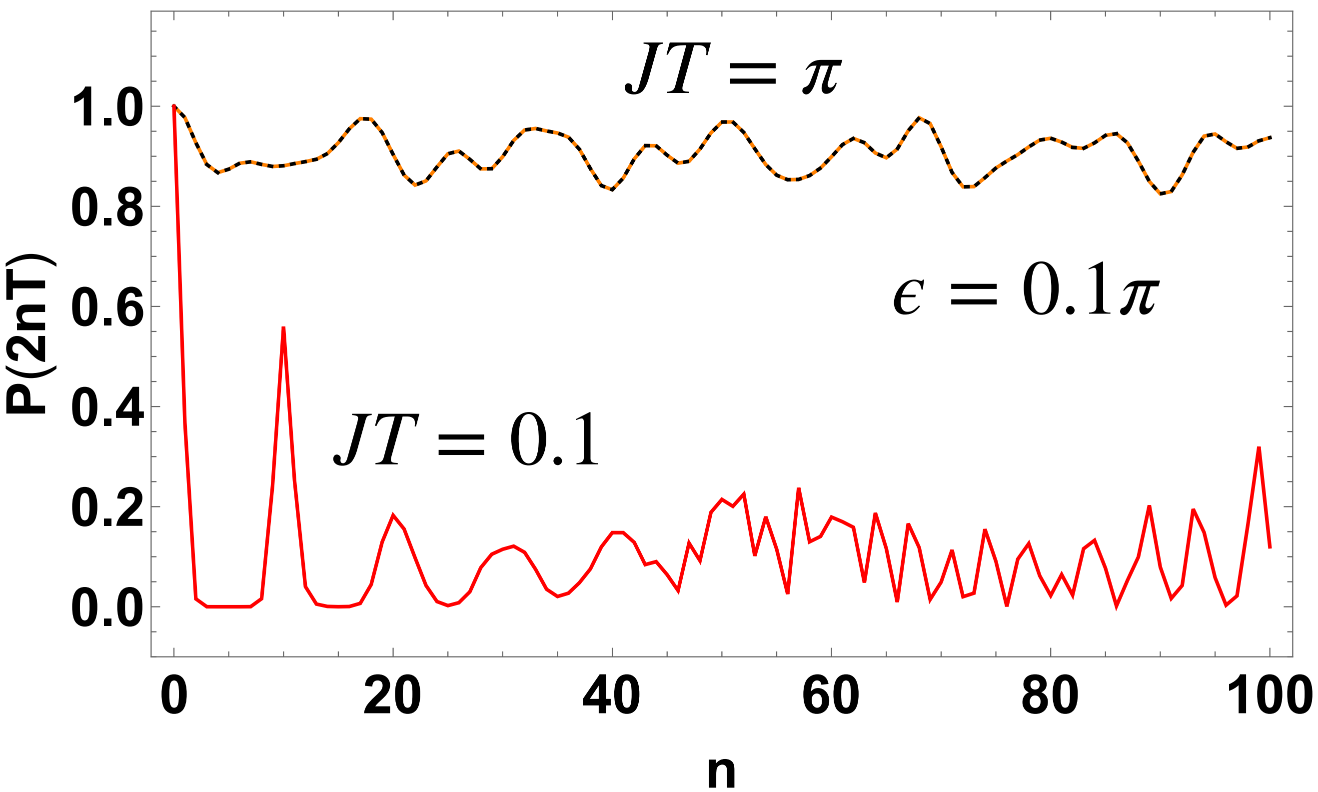
\end{tikzcd}$$

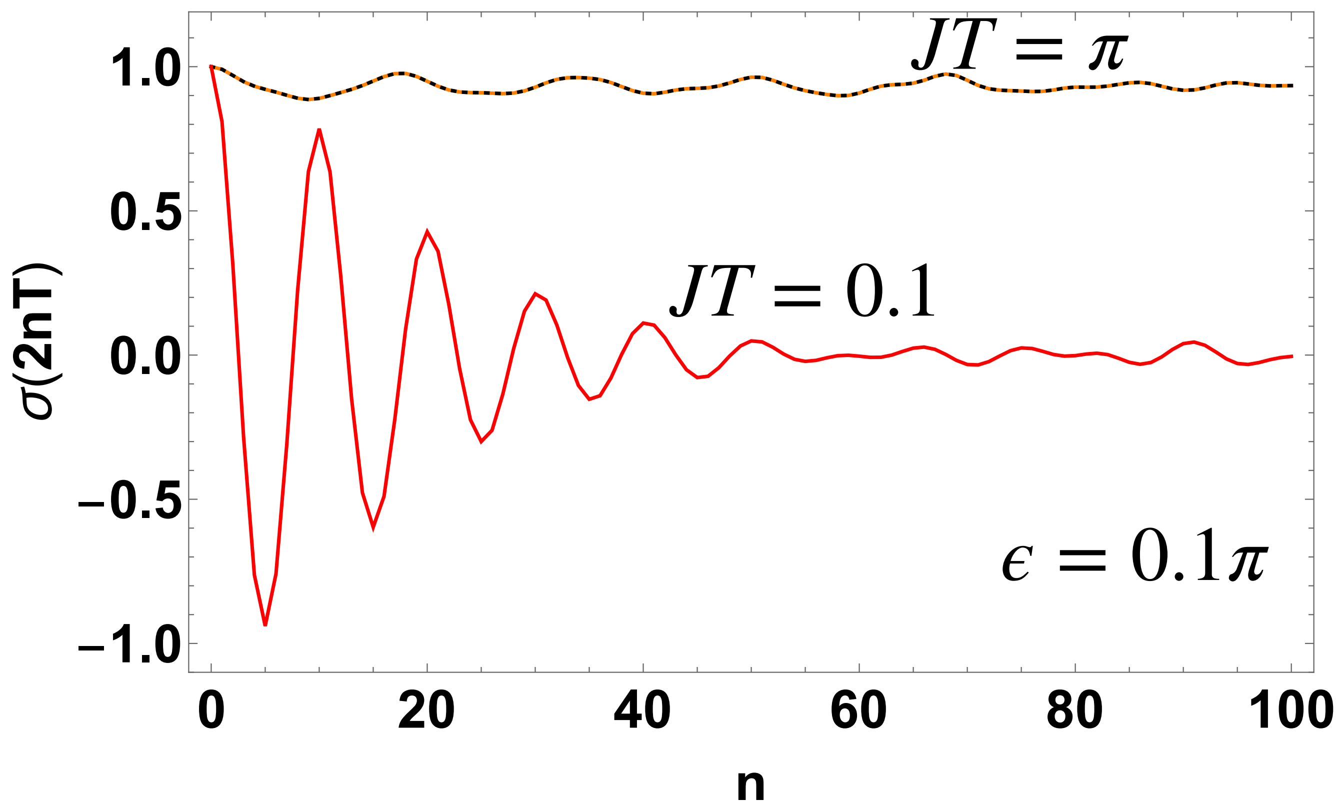
When $JT = \pi$, these two pathways interfere constructively.

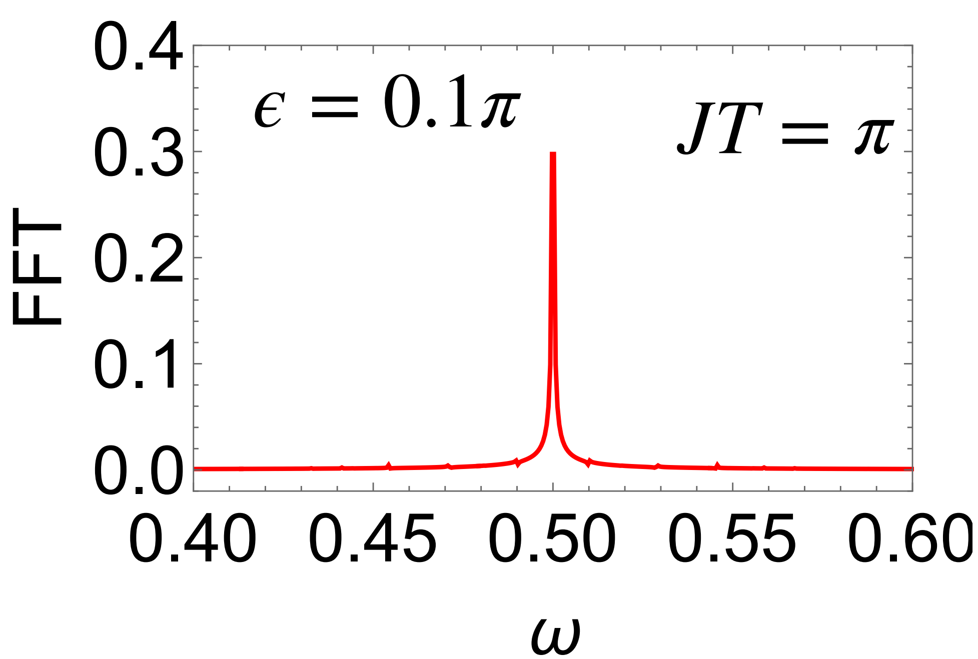
Results: DTC Dynamics

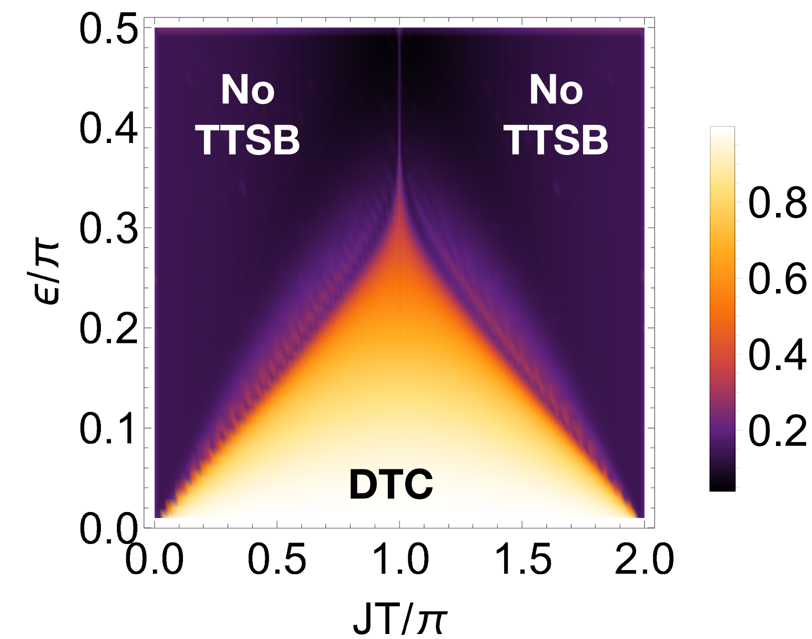
$$P(2nT) = |\langle \psi(2nT) | \psi(0) \rangle|^2$$



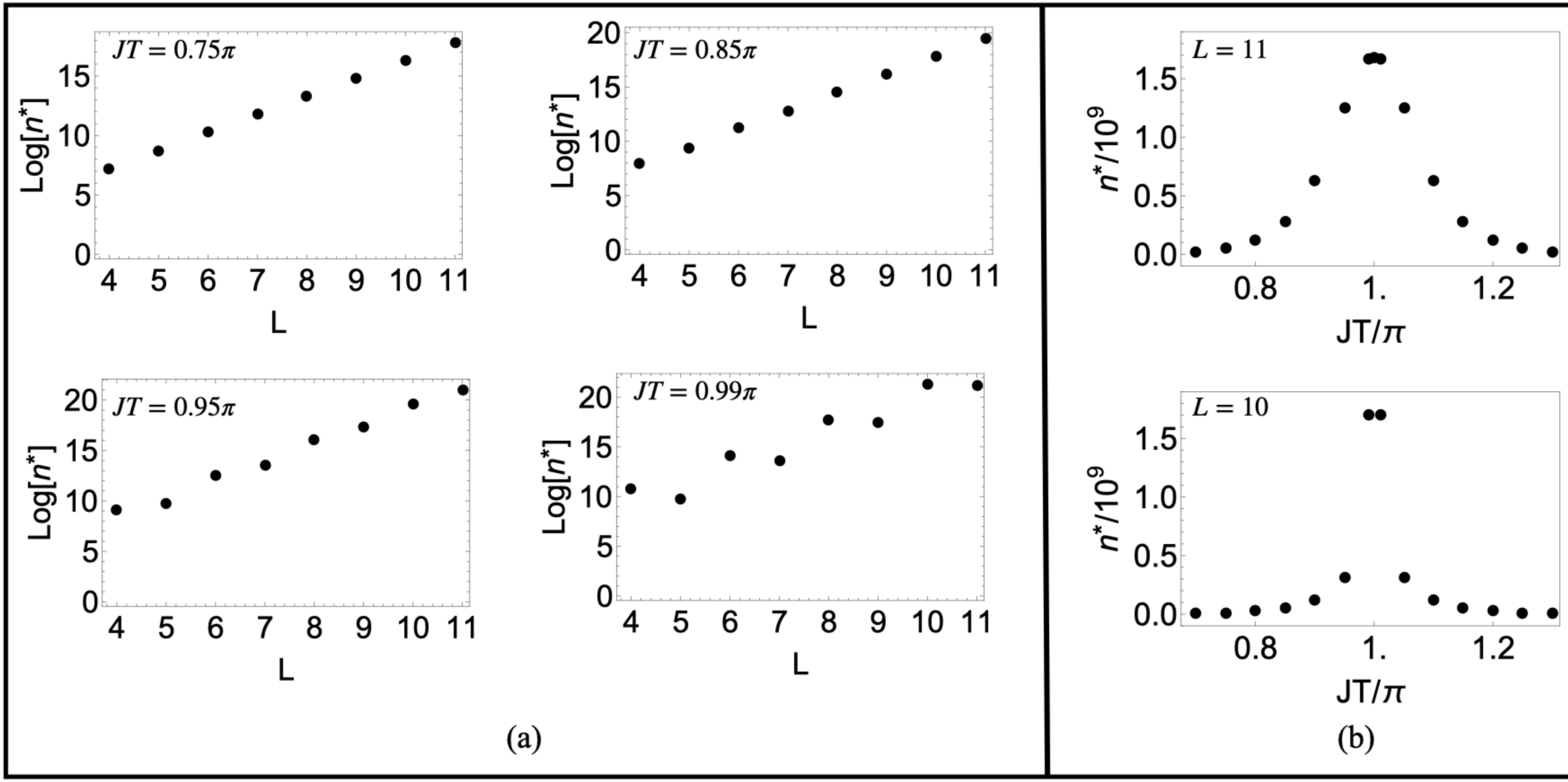
$$\sigma(2nT) = \frac{1}{L} \langle \psi(2nT) | \sigma_i^z | \psi(2nT) \rangle$$







Results: Lifetime



Ref: **Sayan Choudhury** (arXiv:2104.05201)

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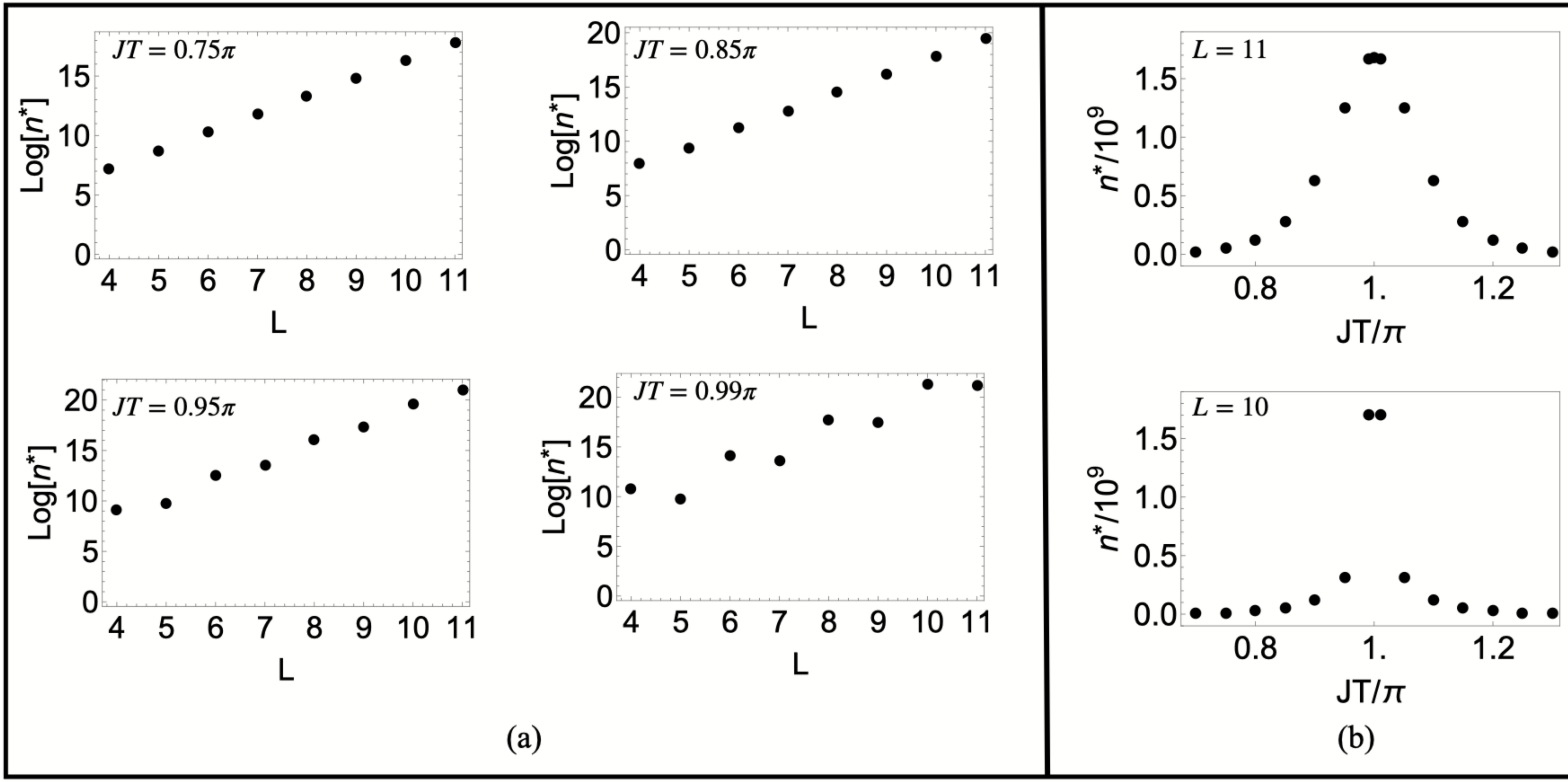

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The DTC lifetime is maximum when $JT = \pi$

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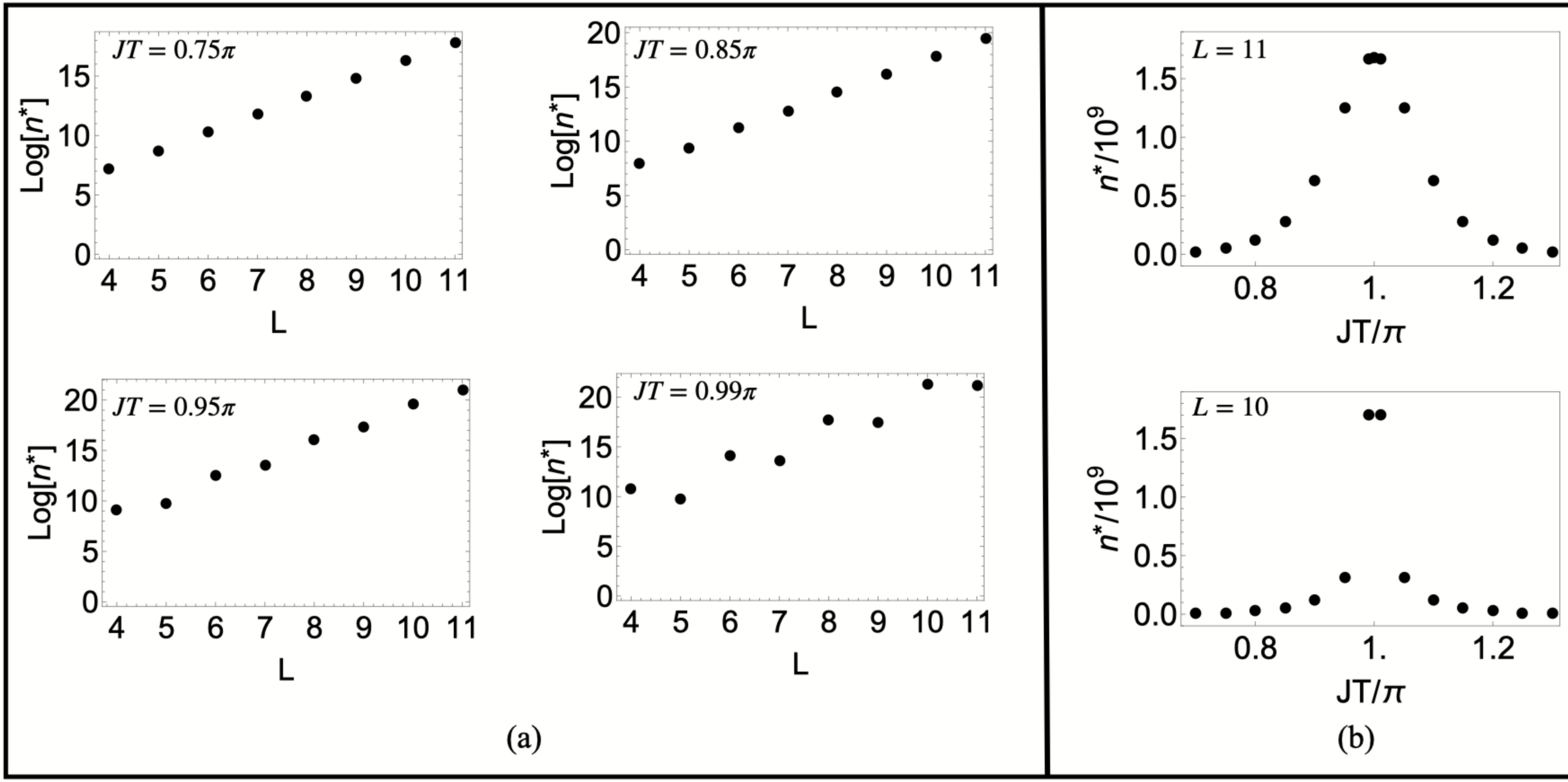

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The DTC lifetime shows a significant enhancement for even size chains when $JT \sim \pi$.

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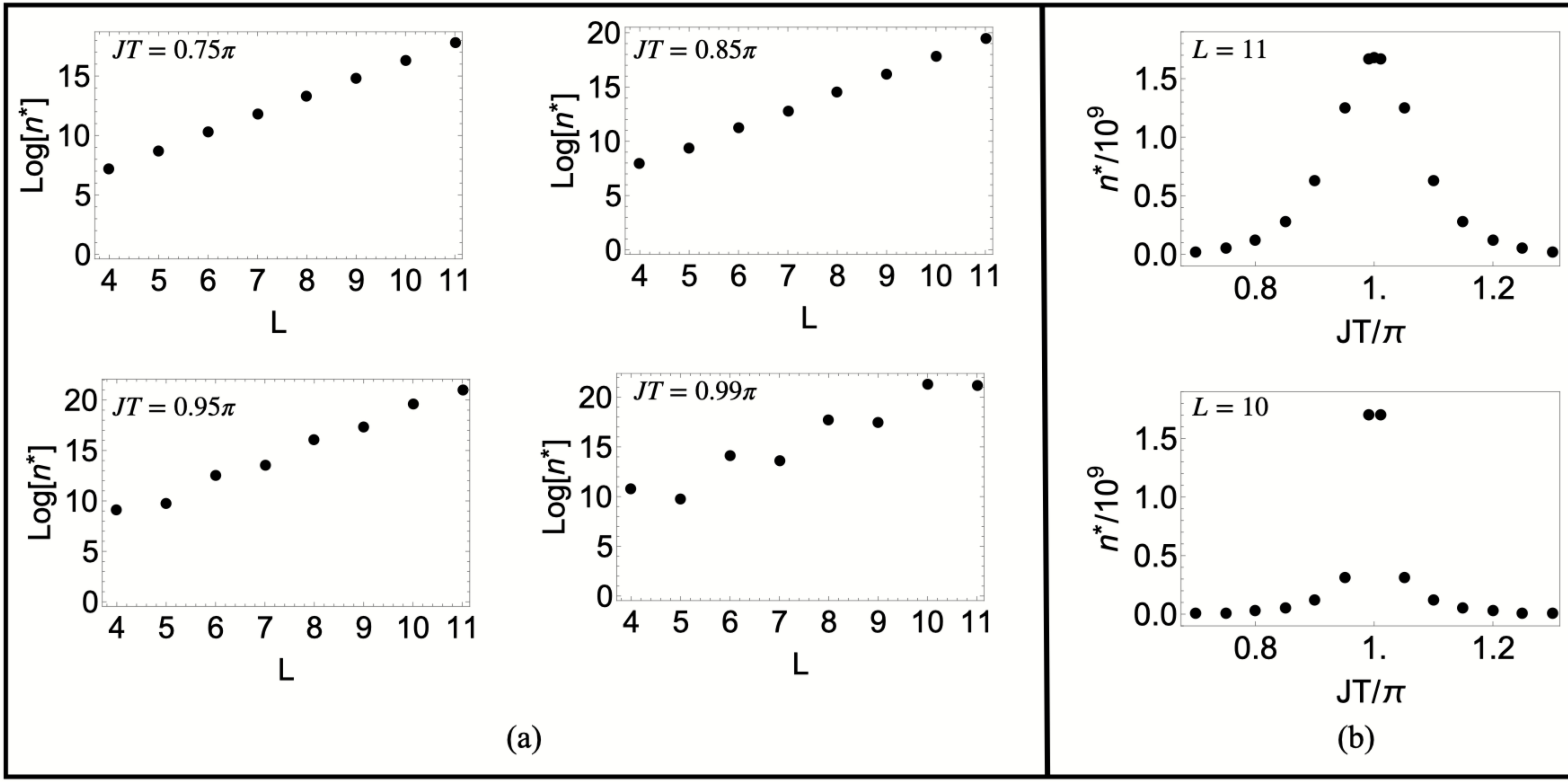

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The DTC lifetime is maximum when $JT = \pi$

The DTC lifetime shows a significant enhancement for even size chains when $JT \sim \pi$. Why?

An Eternal Time Crystal

Our model possesses a time reflection symmetry when $JT = \pi$:

$$RU(T)R^\dagger = \exp(-iL\pi/2)U(T), \text{ where } R = \prod_{i=1}^{L}\sigma_i^x \prod_{j=1}^{L}\sigma_j^z.$$

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As a consequence of this symmetry, there are at least $2^{L/2}$ states exactly at quasi-energies 0 and π for even size systems. (Iadecola and Hsieh, Phys. Rev. Lett. **120**, 210603 (2018)).

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Due to the existence of such "π-spectral paired" states, this system can exhibit infinitely long lived period doubling oscillations for certain initial states.

An Eternal Time Crystal

Let us consider a pair of such states whose quasi-energies offer by π: $|\psi_0\rangle$ and $|\psi_\pi\rangle$.

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Perfect Revival $\Rightarrow$ **Eternal Period-doubling oscillations**

Caveat: In this model and eternal DTC can be realized for very few initial states.

Question: Can we overcome this issue?

An Eternal Time Crystal

PHYSICAL REVIEW RESEARCH **2**, 033070 (2020)

Eternal discrete time crystal beating the Heisenberg limit

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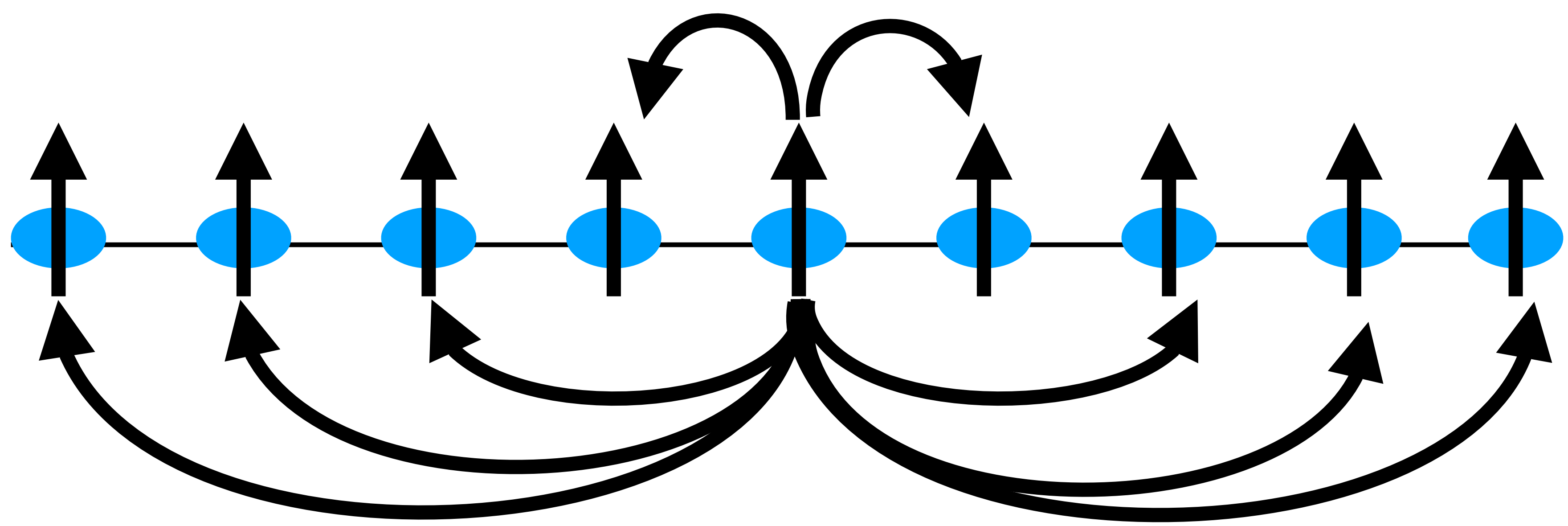
A discrete time crystal (DTC) repeats itself with a rigid rhythm, mimicking a ticking clock set by the interplay between its internal structures and an external force. Discrete time crystals promise profound applications in precision timekeeping and other quantum techniques. However, it has been facing a grand challenge of thermalization. The periodic driving supplying the power may ultimately bring DTCs to thermal equilibrium and destroy their coherence. Here we show that an all-to-all interaction delivers a DTC that evades thermalization and maintains quantum coherence and quantum synchronization regardless of spatial inhomogeneities in the driving field and the environment. Moreover, the sensitivity of this DTC scales with the total particle number to the power of 3/2, realizing a quantum device of measuring the driving frequency or the interaction strength beyond the Heisenberg limit. Our work paves the way for designing nonequilibrium phases with long coherence time to advance quantum metrology.

Infinite-Range Interactions



$$H(t) = 2J\sum_i \sum_{j\neq i} S_i^z S_j^z + \sum_i (\pi - \epsilon) S_i^x \delta(t - nT)$$

An infinite range interacting Ising model can give rise to a perfect revival after every 2 pulses for any value of ϵ!

Conditions: (1) $JT = \pi$ and

(2) The number of particles is even

A Perfect Eternal Time-Crystal

Imperfect π Pulse, $\epsilon \neq 0$

$$|\uparrow\uparrow\ldots\uparrow\uparrow\rangle \xrightarrow{\cos(\epsilon)^L} |\downarrow\downarrow\ldots\downarrow\downarrow\rangle \xrightarrow{\cos(\epsilon)^{2L}} |\uparrow\uparrow\ldots\uparrow\uparrow\rangle$$

A Perfect Eternal Time-Crystal

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$$|\uparrow\uparrow\ldots\uparrow\uparrow\rangle \xrightarrow{-\cos(\epsilon)^{L-1}\sin(\epsilon)} |\downarrow\downarrow\ldots\uparrow\ldots\downarrow\downarrow\rangle \xrightarrow{e^{i\phi_1}\cos(\epsilon)^{2L-2}\sin(\epsilon)^{2}} |\uparrow\uparrow\ldots\uparrow\uparrow\rangle$$

A Perfect Eternal Time-Crystal

Imperfect π Pulse, $\epsilon \neq 0$

$$
\begin{tikzcd}[column sep=huge]
\lvert \uparrow\uparrow \ldots \uparrow\uparrow \rangle \arrow[r, "\cos(\epsilon)^L"] \arrow[dr, "-\cos(\epsilon)^{L-1}\sin(\epsilon)"'] \arrow[ddr, "(-1)^m\cos(\epsilon)^{L-m}\sin(\epsilon)^m"'] & \lvert \downarrow\downarrow \ldots \downarrow\downarrow \rangle \arrow[r, "\cos(\epsilon)^{2L}"] & \lvert \uparrow\uparrow \ldots \uparrow\uparrow \rangle \\
 & \lvert \downarrow\downarrow \ldots \uparrow \ldots \downarrow\downarrow \rangle \arrow[ur, "e^{i\phi_1}\cos(\epsilon)^{2L-2}\sin(\epsilon)^2"'] & \\
 & \bullet\ \bullet\ \bullet \arrow[uur, "e^{i\phi_m}\cos(\epsilon)^{2L-2m}\sin(\epsilon)^{2m}"'] &
\end{tikzcd}
$$

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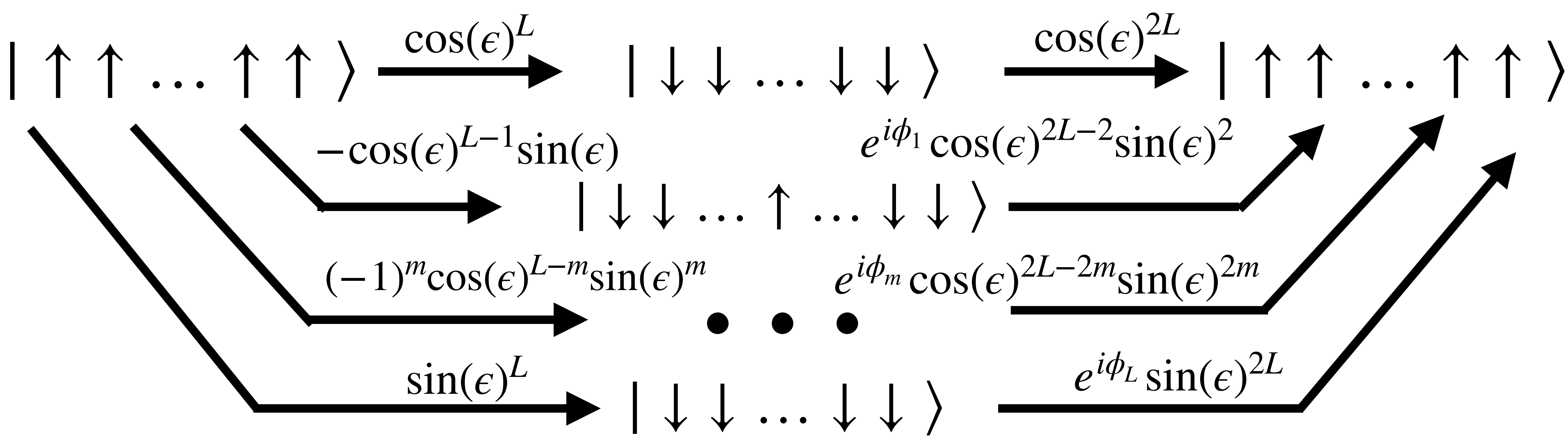

A Perfect Eternal Time-Crystal

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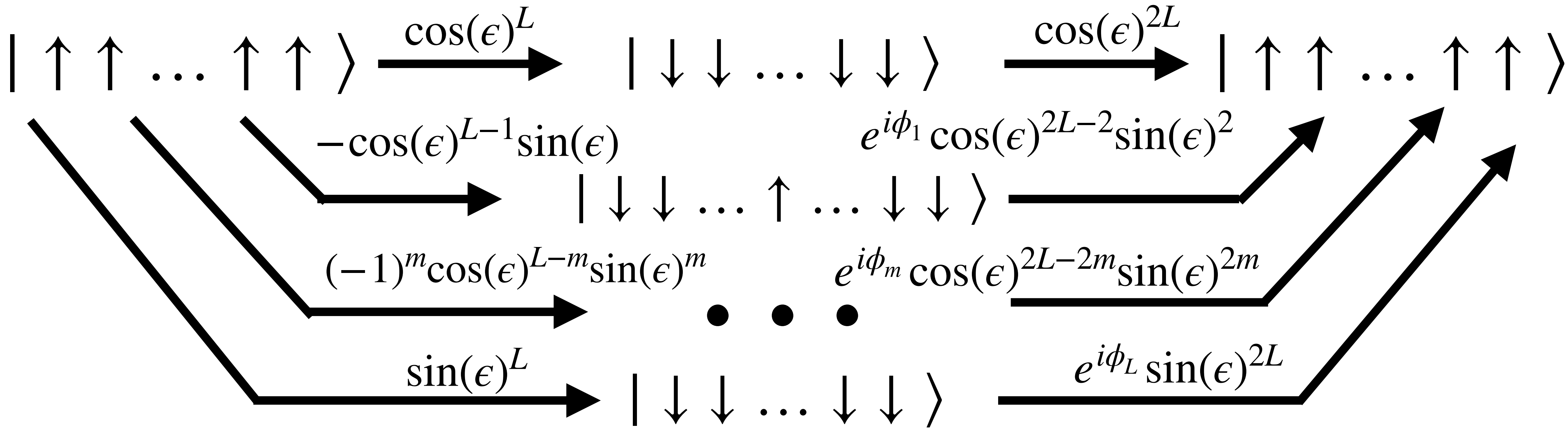
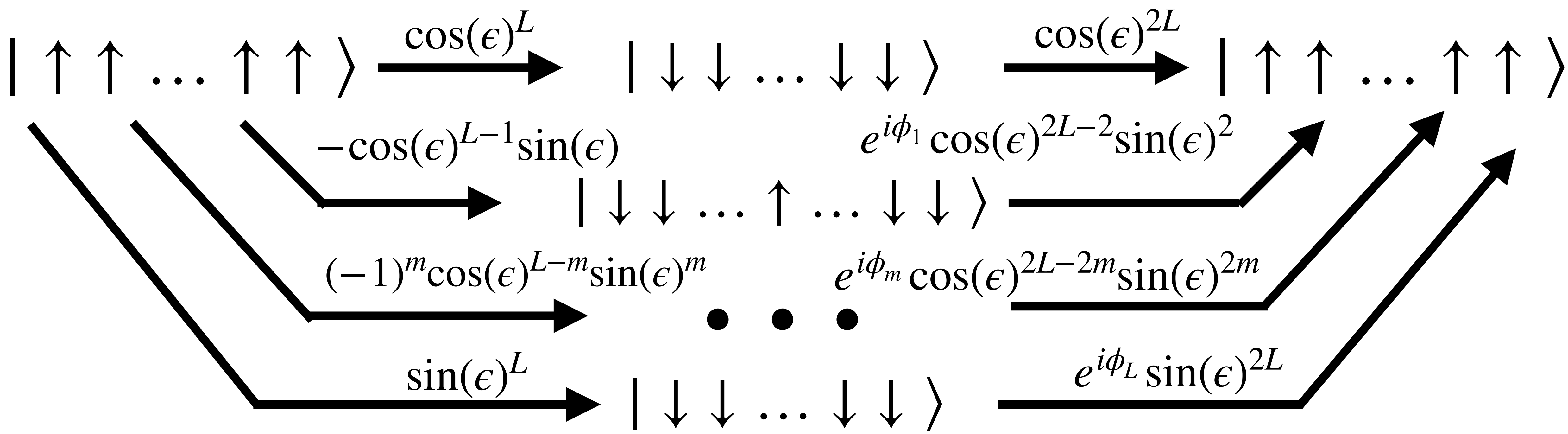


Here, the dynamical phase $\phi_m = m(L-m)JT$

A Perfect Eternal Time-Crystal

Imperfect π Pulse, $\quad \epsilon \neq 0$



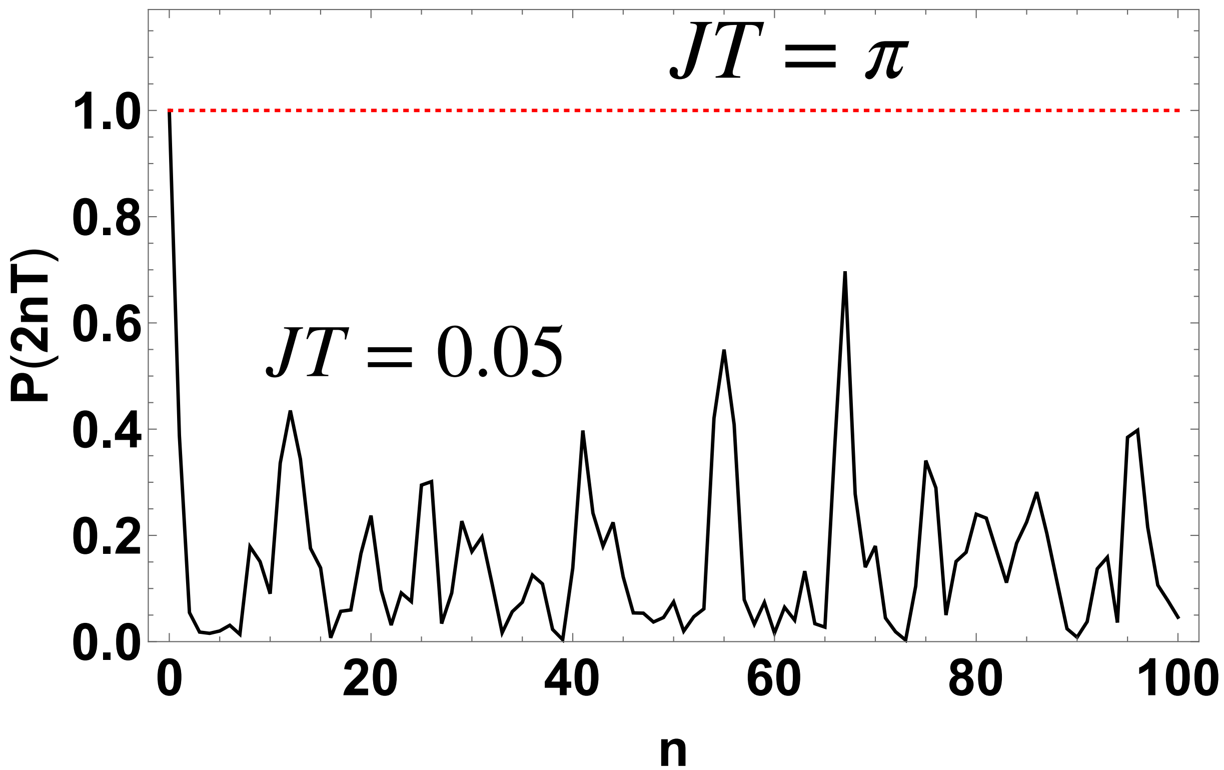
Here, the dynamical phase $\phi_m = m(L-m)JT$

When $JT = \pi$, there is a perfect many-body constructive interference after every even pulse for any value of ϵ!

Results

Return Probability

$$P(2nT) = |\langle \psi(2nT) | \psi(0) \rangle|^2$$



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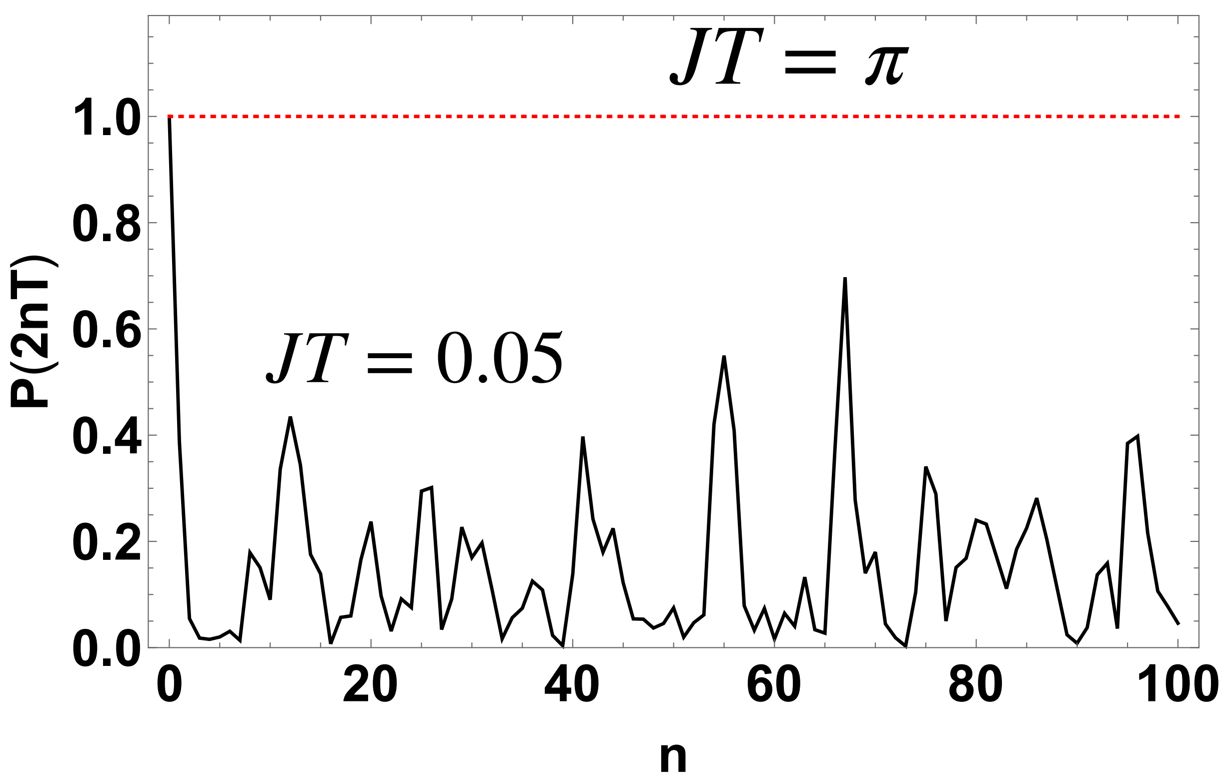
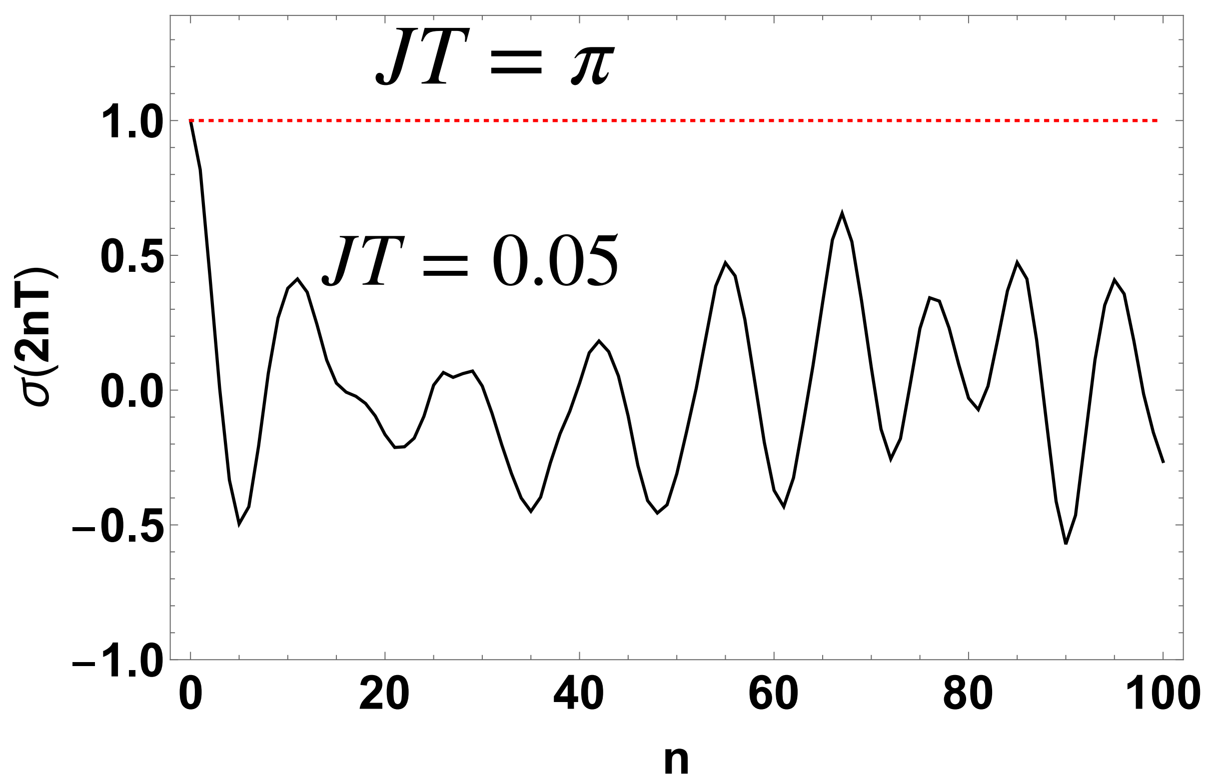


Local Magnetization

$$\sigma(2nT) = \frac{1}{L}\langle \psi(2nT) | \sigma_i^z | \psi(2nT) \rangle$$



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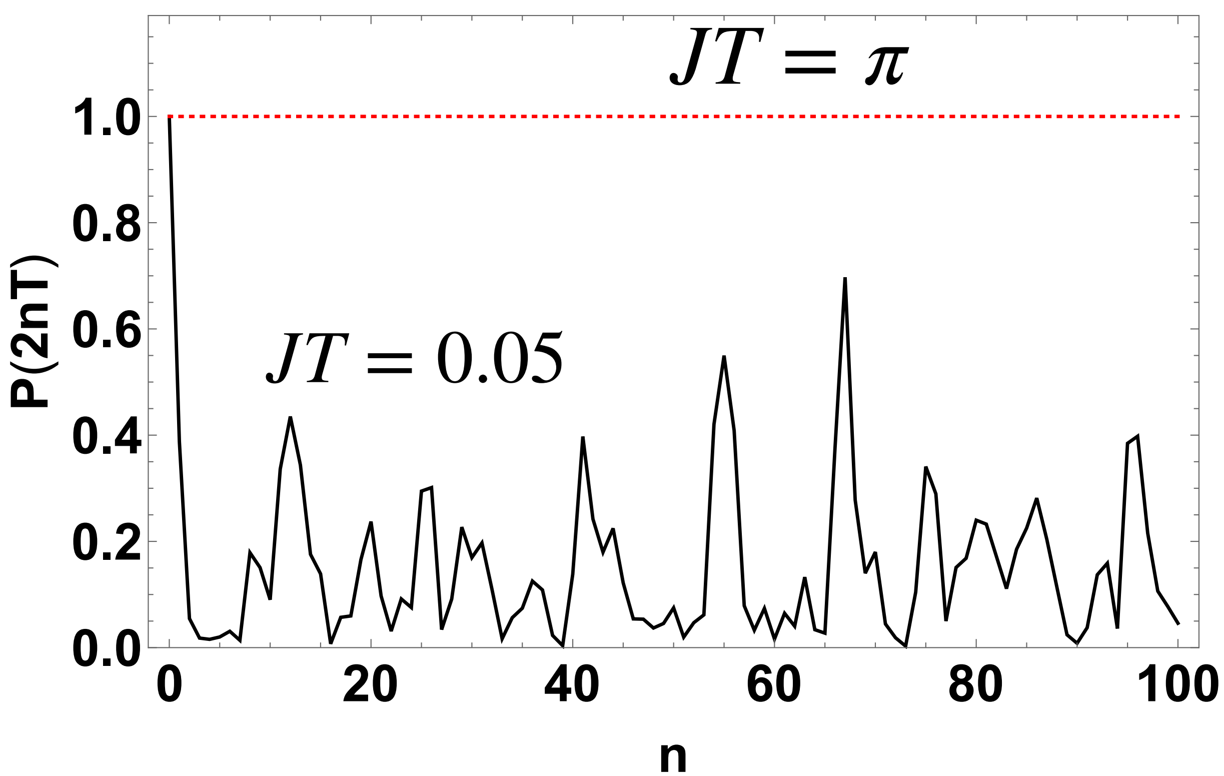
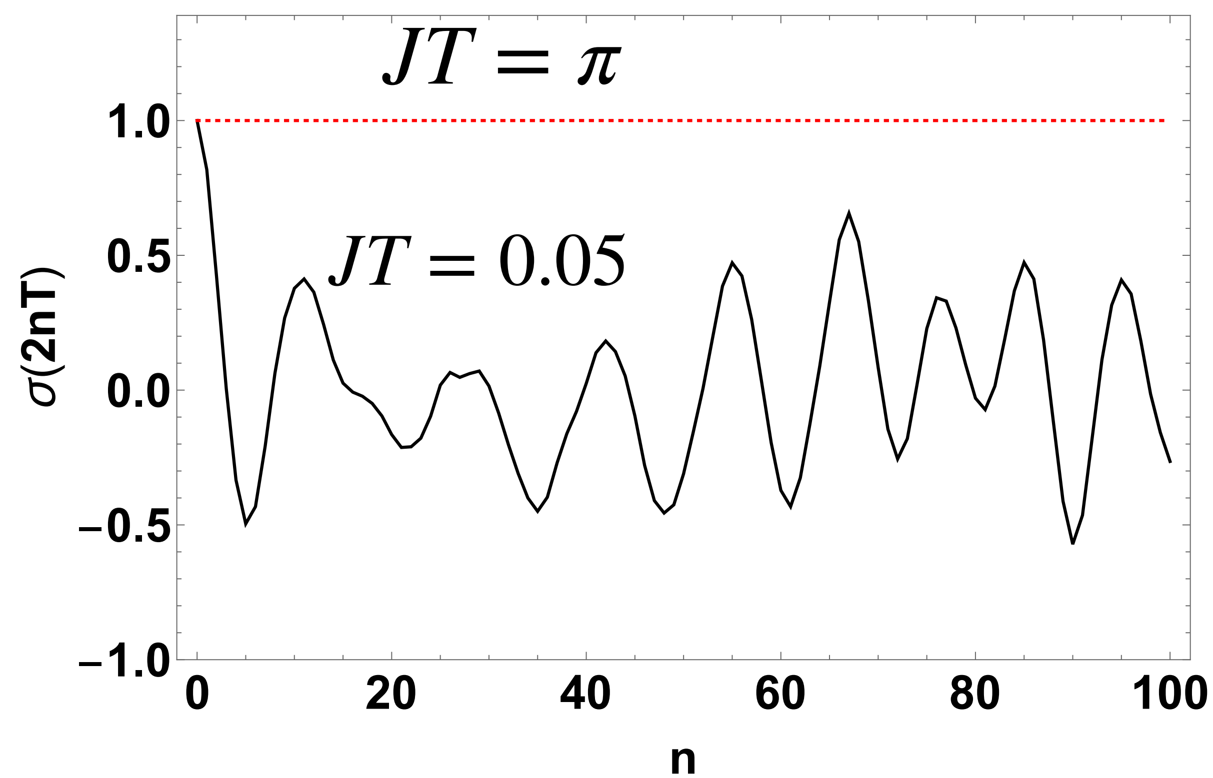
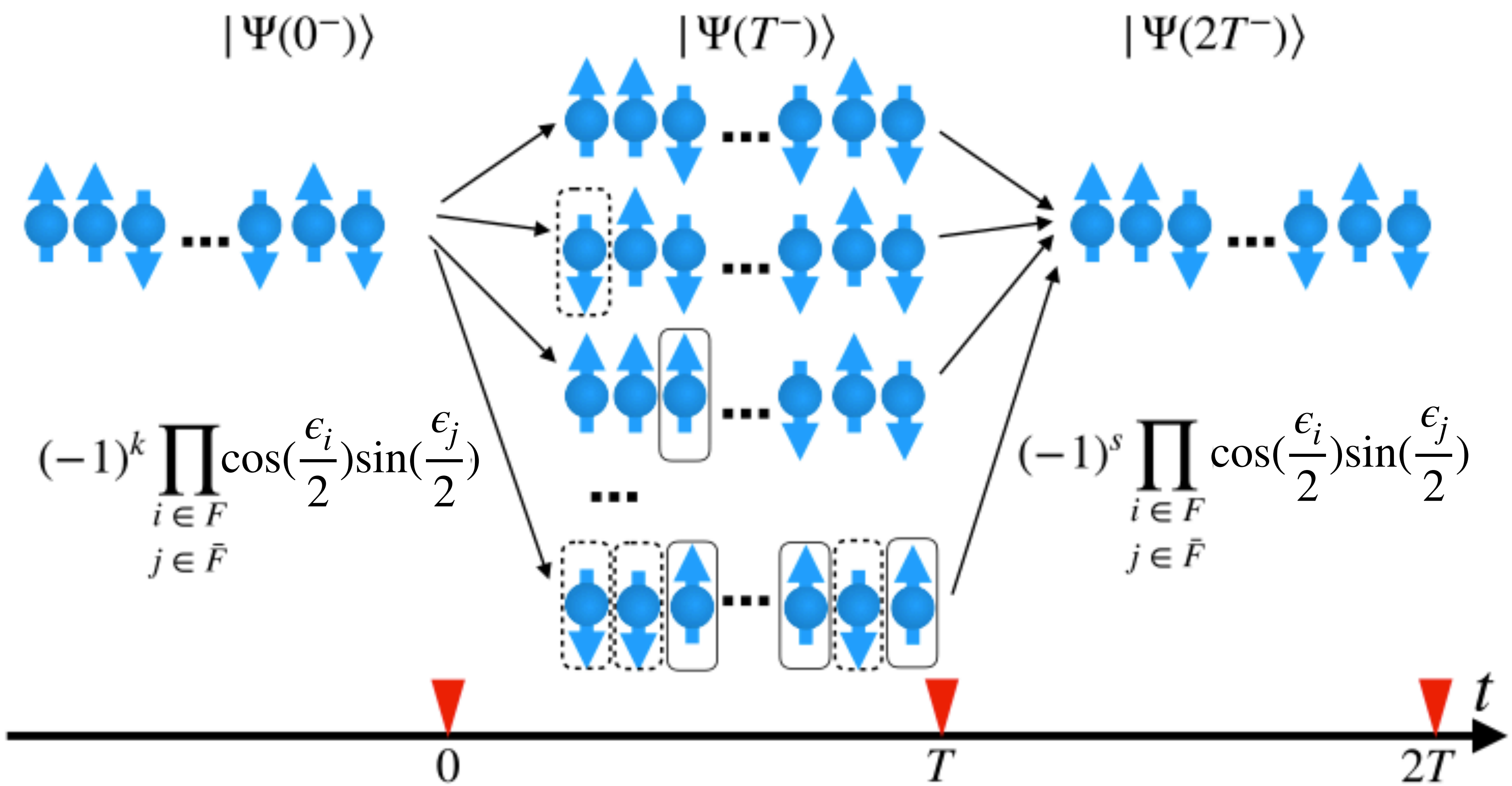




A perfect revival of the initial state occurs when $JT = \pi$

Robustness of the DTC



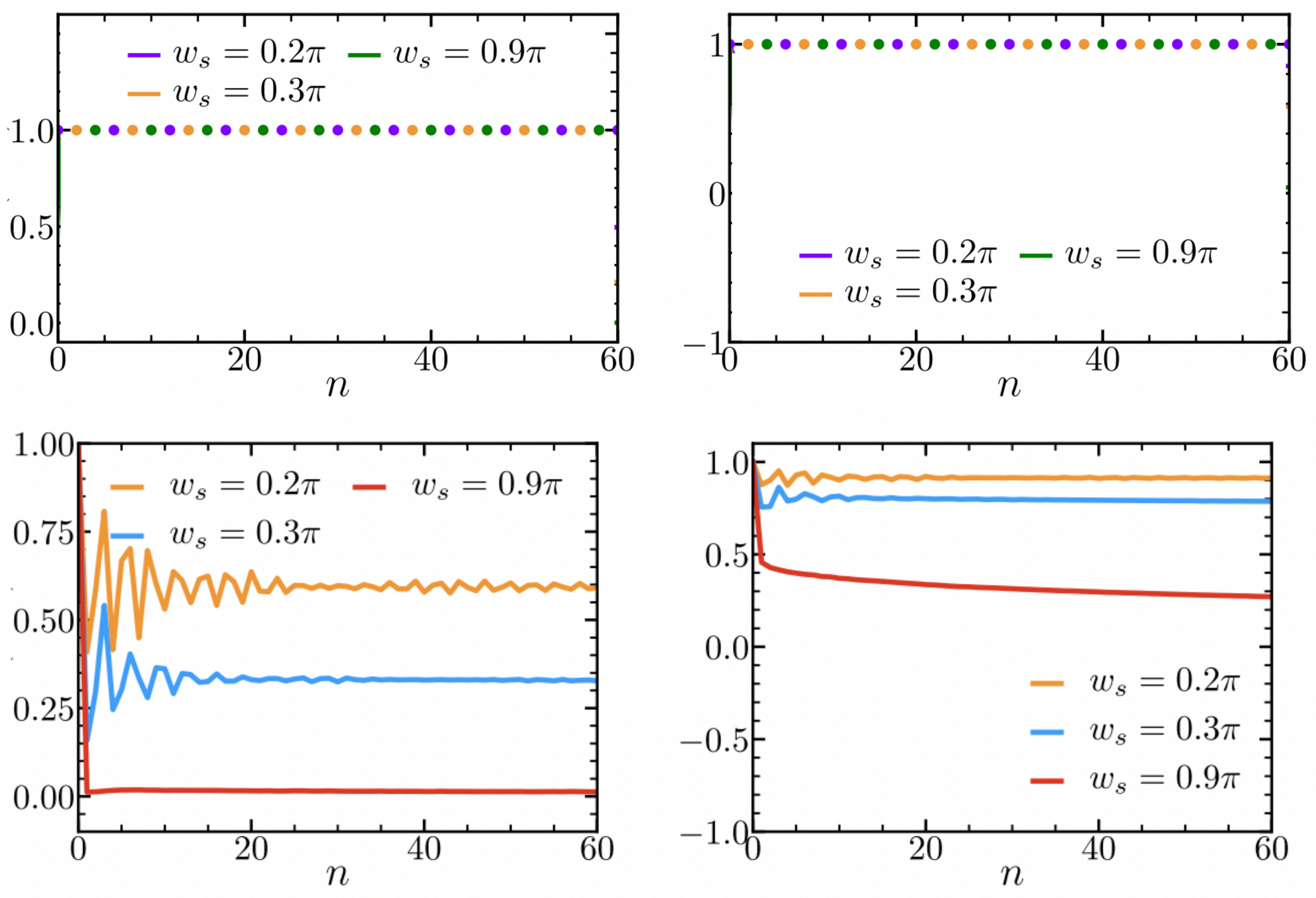
These eternal period doubling oscillations are observed for:
(A) Any $\mathbb{Z}_2$ symmetry breaking initial state.
(B) Even if ϵ varies spatially

Comparison with MBL: Spatially Inhomogeneous Pulse

$P(2nT)$ $\sigma(2nT)$

Our Model

MBL



$\epsilon \in [-w_s, w_s]$

Comparison with MBL: Spatially Inhomogeneous Pulse

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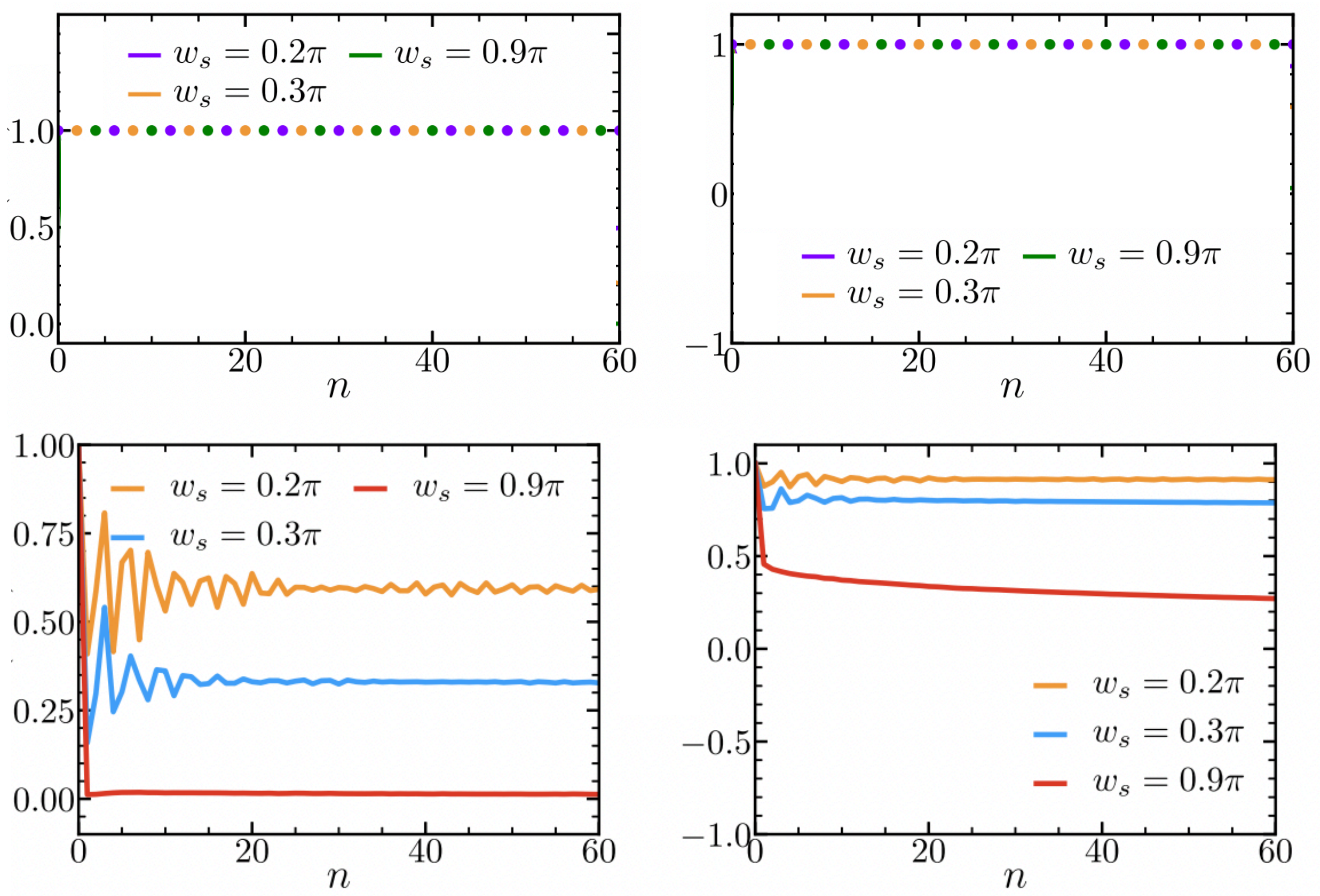
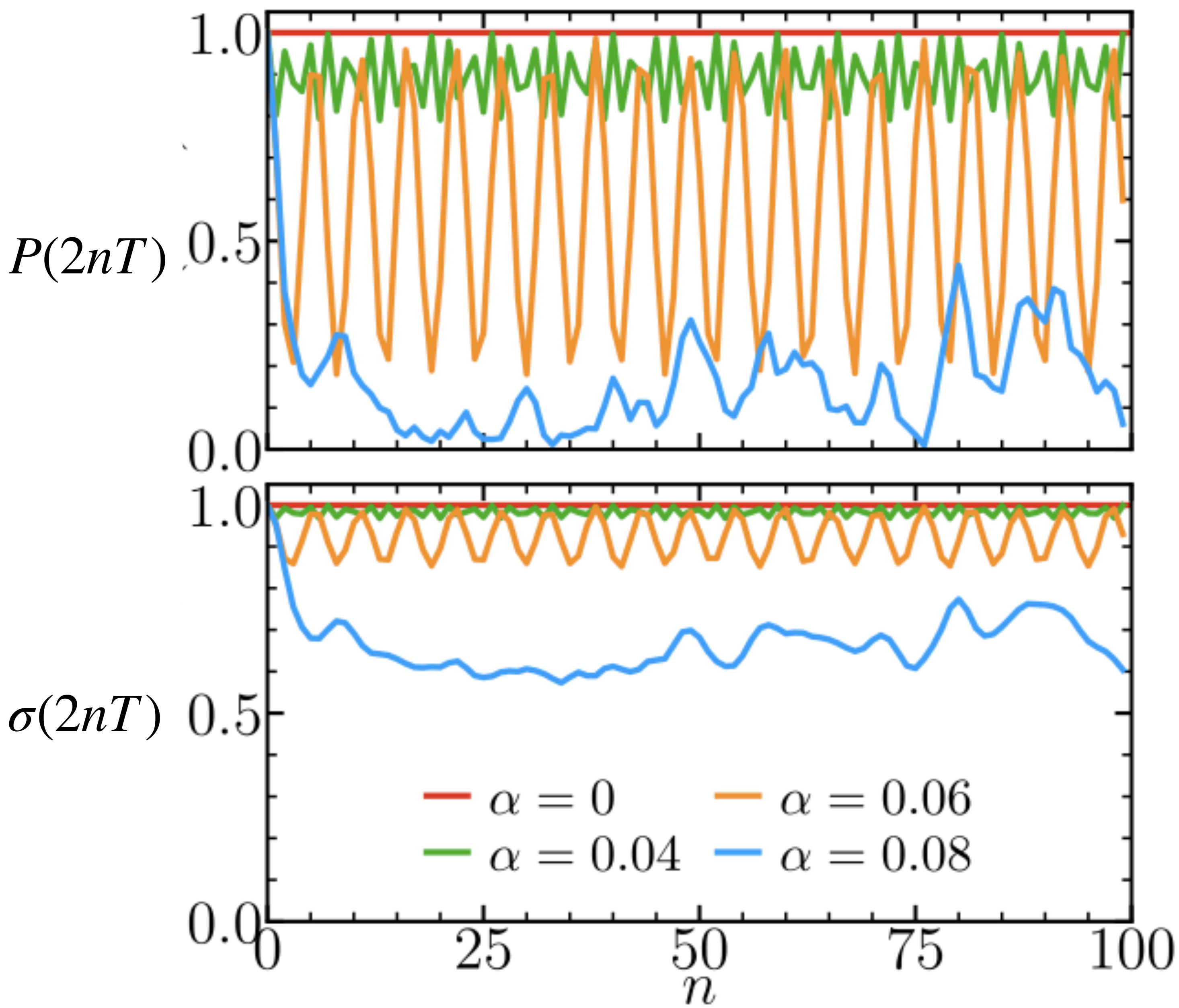


Our model exhibits eternal period doubling oscillations

$\epsilon \in [-w_s, w_s]$

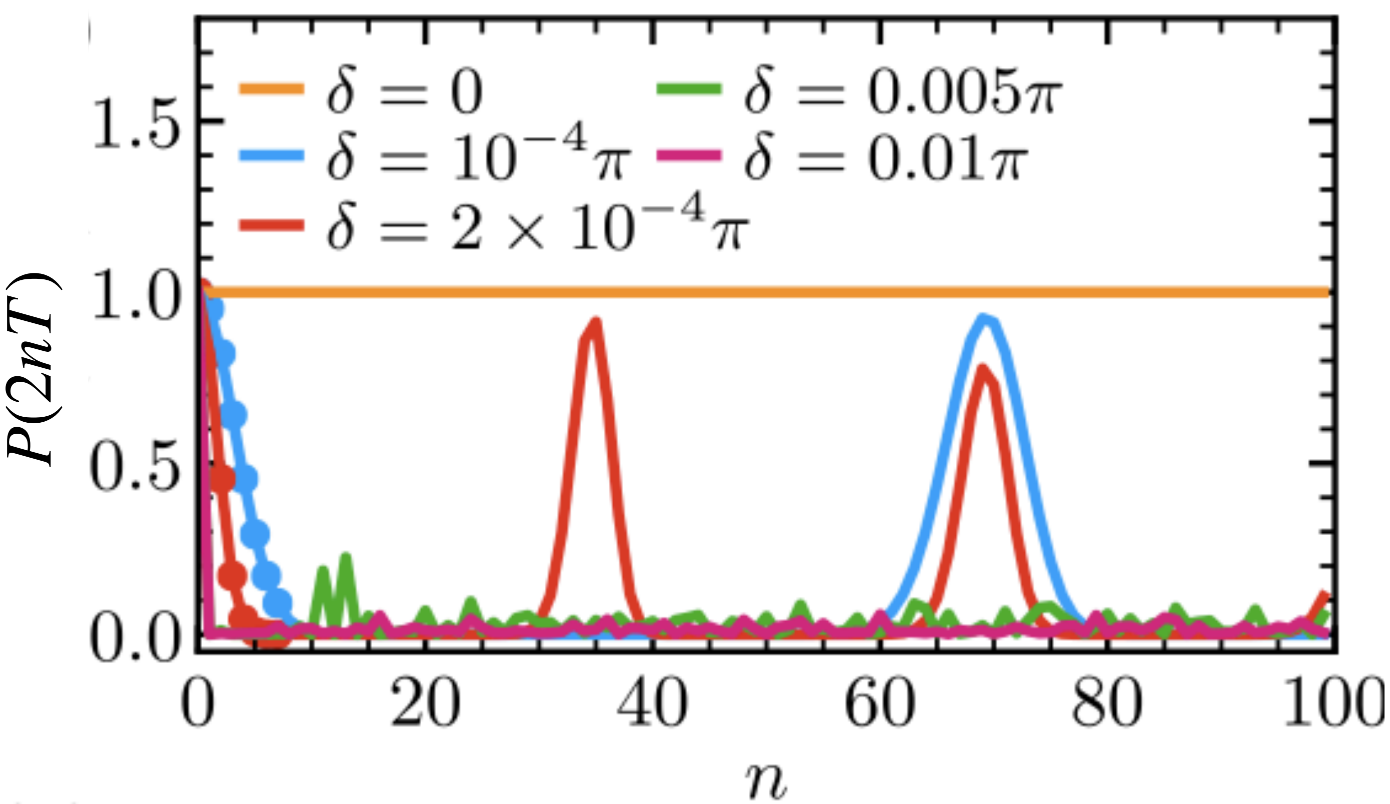
What if Interactions are not Infinite Range

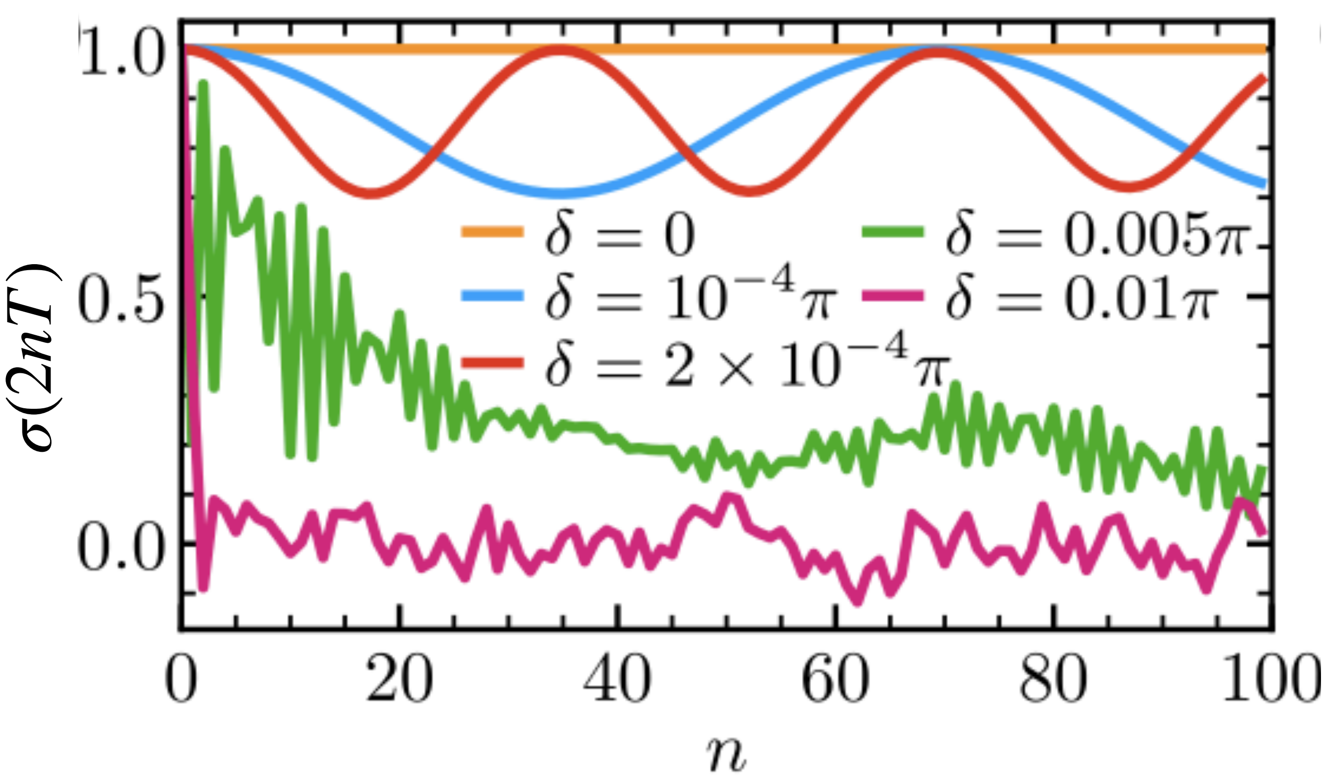


$$H(t) = \sum_i \sum_{j \neq i} \frac{2J_0}{|j-i|^\alpha} S_i^z S_j^z + \sum_i (\pi - \epsilon) S_i^x \delta(t - nT)$$

An Important Caveat

The perfect revival disappears when $JT \neq \pi$.





$\delta = |JT - \pi|$ $\qquad$ $\epsilon = \pi/4$ $\qquad$ $L = 200$

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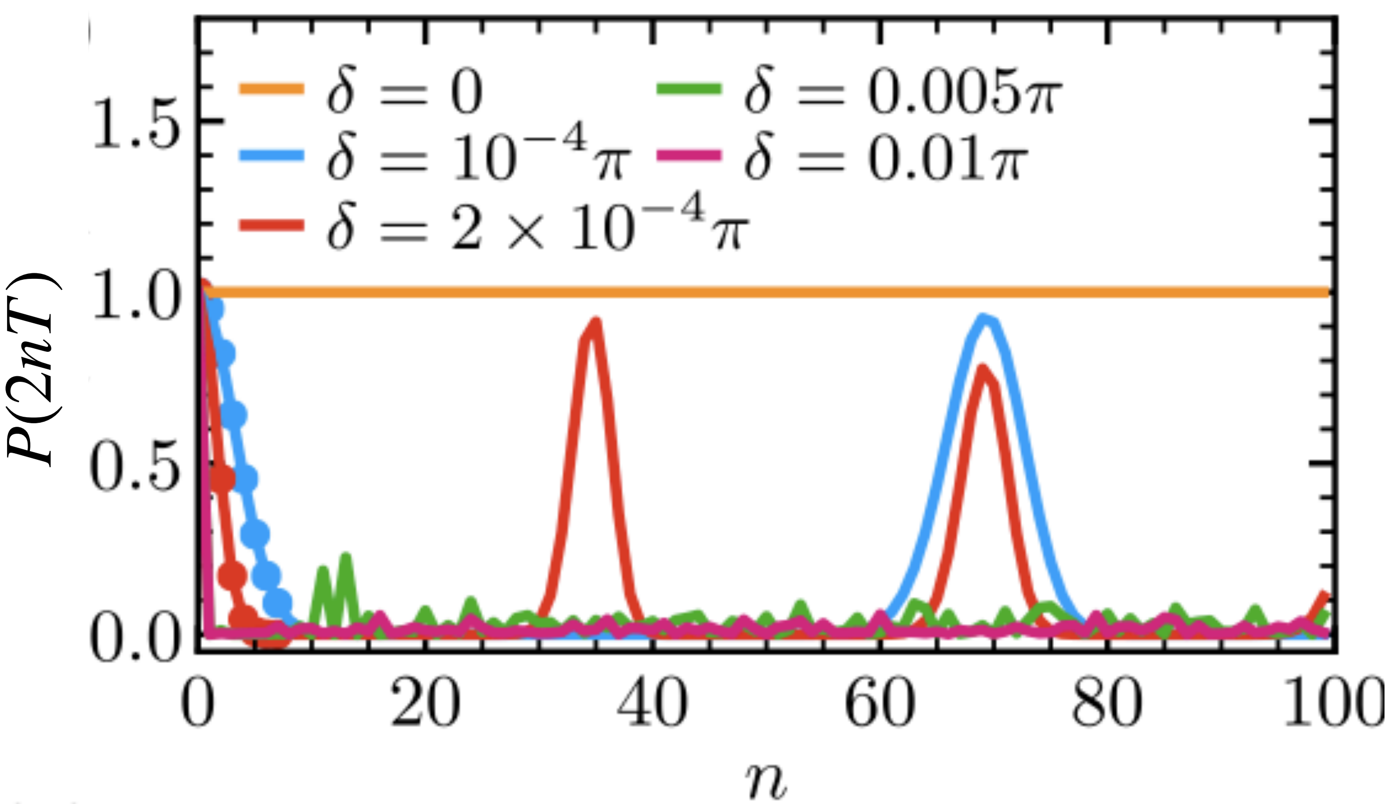
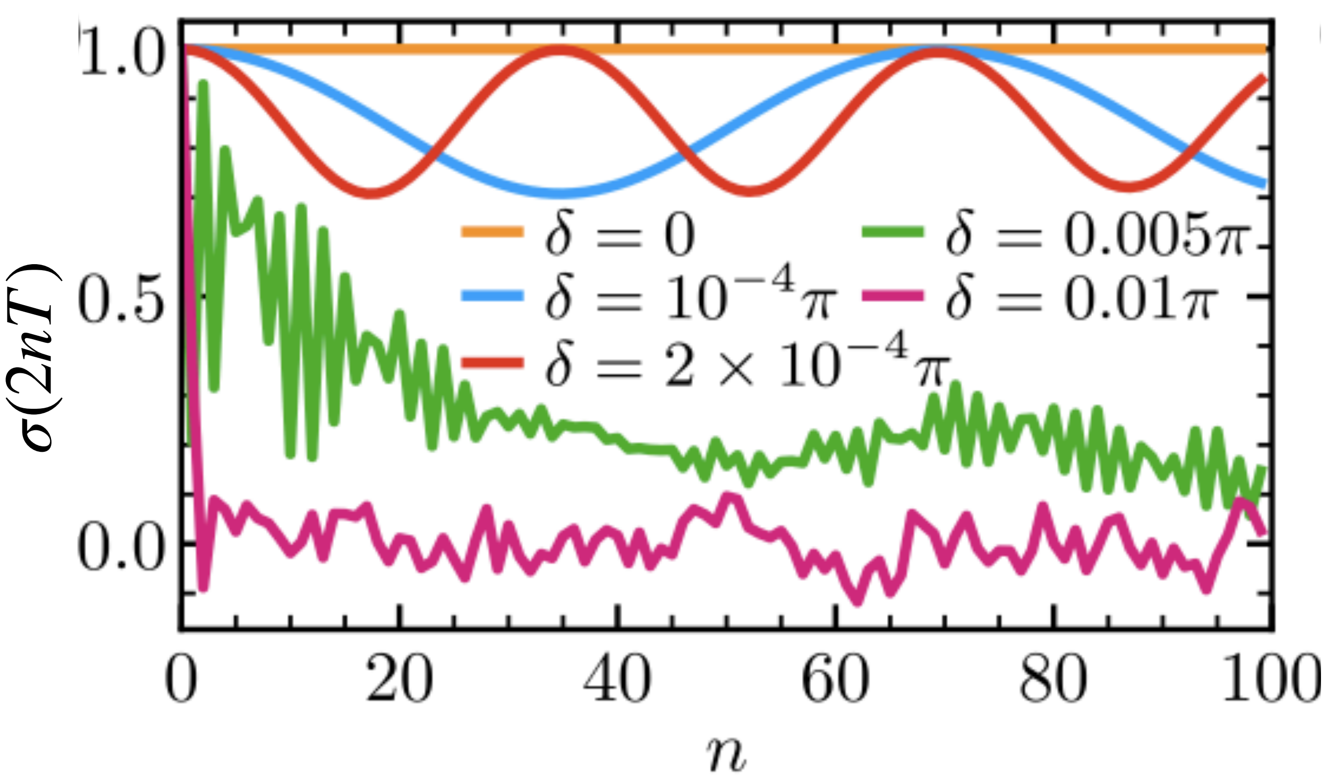
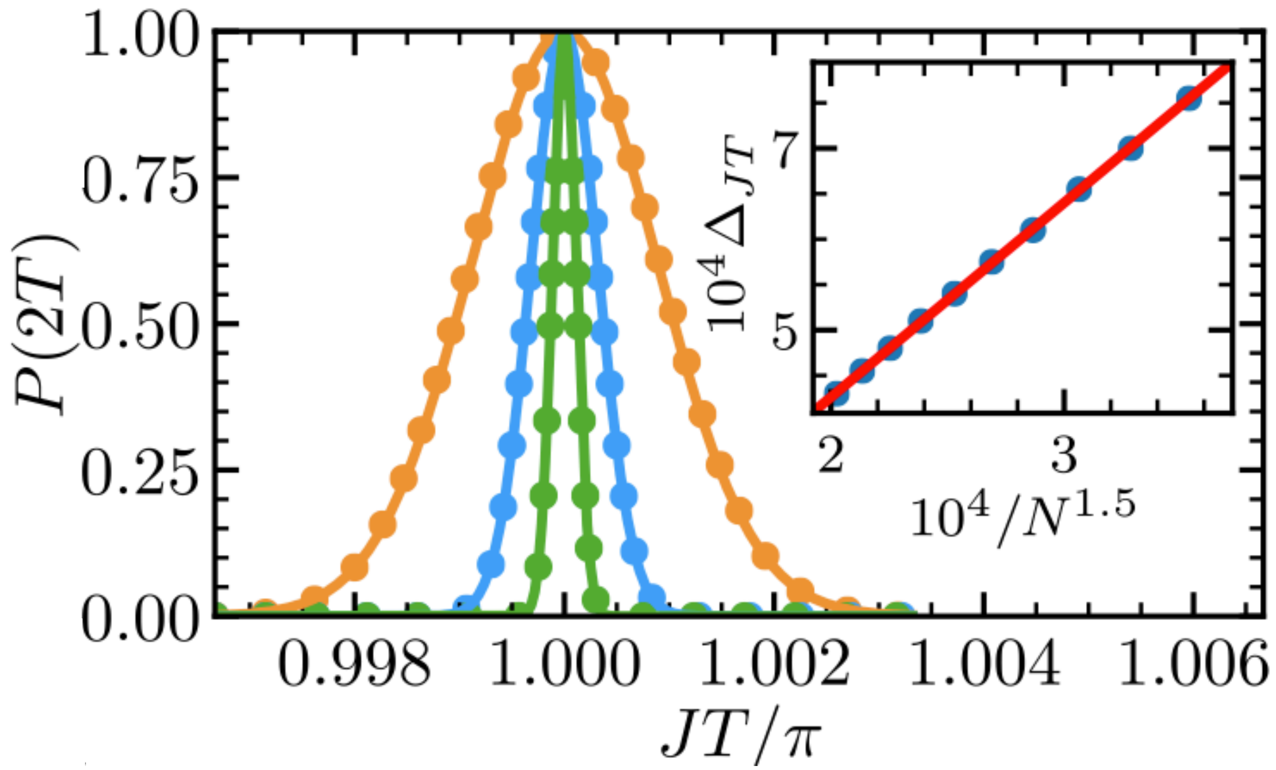


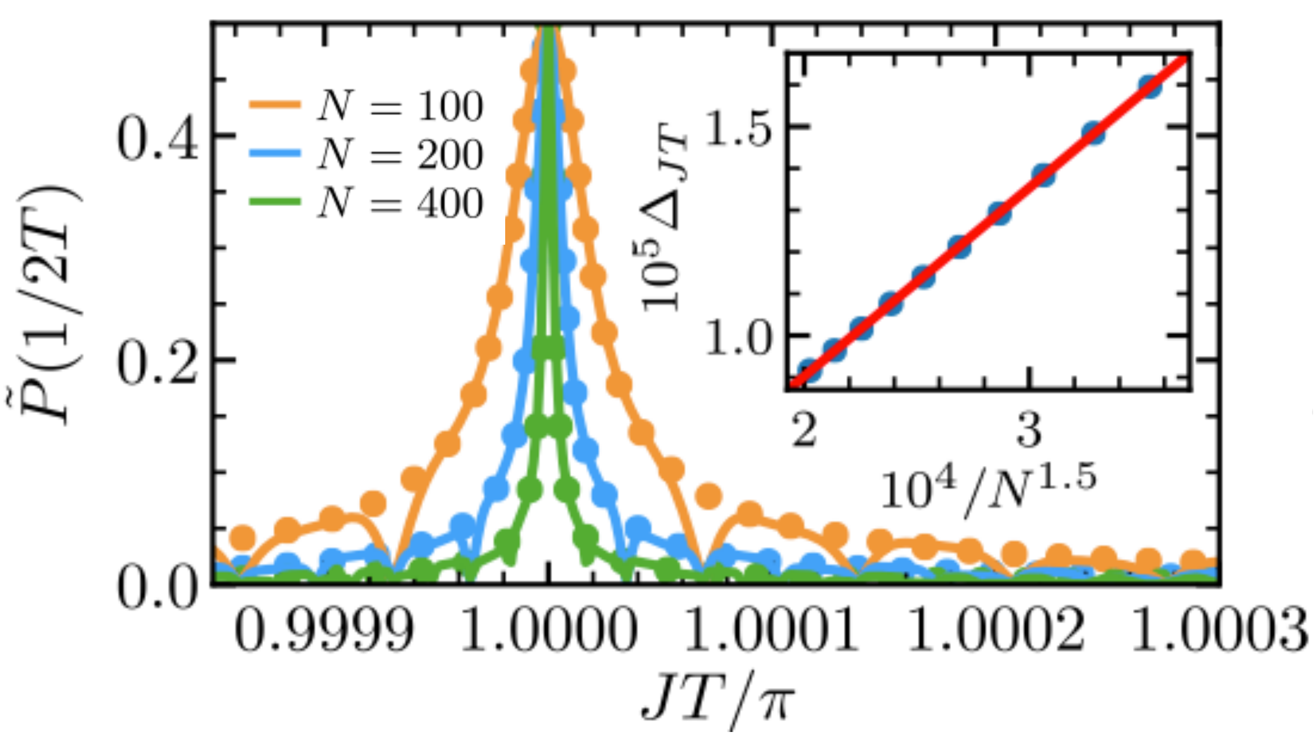


$\delta = |JT - \pi|$ $\epsilon = \pi/4$ $L = 200$

This feature can actually be useful!

Use in Metrology





$\Delta_{JT} =$ FWHM

$\Delta_\pi \propto 1/N^{1.5}$

This beats the Heisenberg Limit!

Use in Metrology

The Quantum Fisher Information, I_{JT} is given by:

$$I_{JT}(2nT) = \lim_{\epsilon \to 0} 4 \frac{1 - F_\epsilon}{\epsilon^2},$$

$$F_\epsilon = |\langle \Psi(0^-)| U_{JT}(2nT) U_{JT+\epsilon}(-2nT) |\Psi(0^-)\rangle|^2$$

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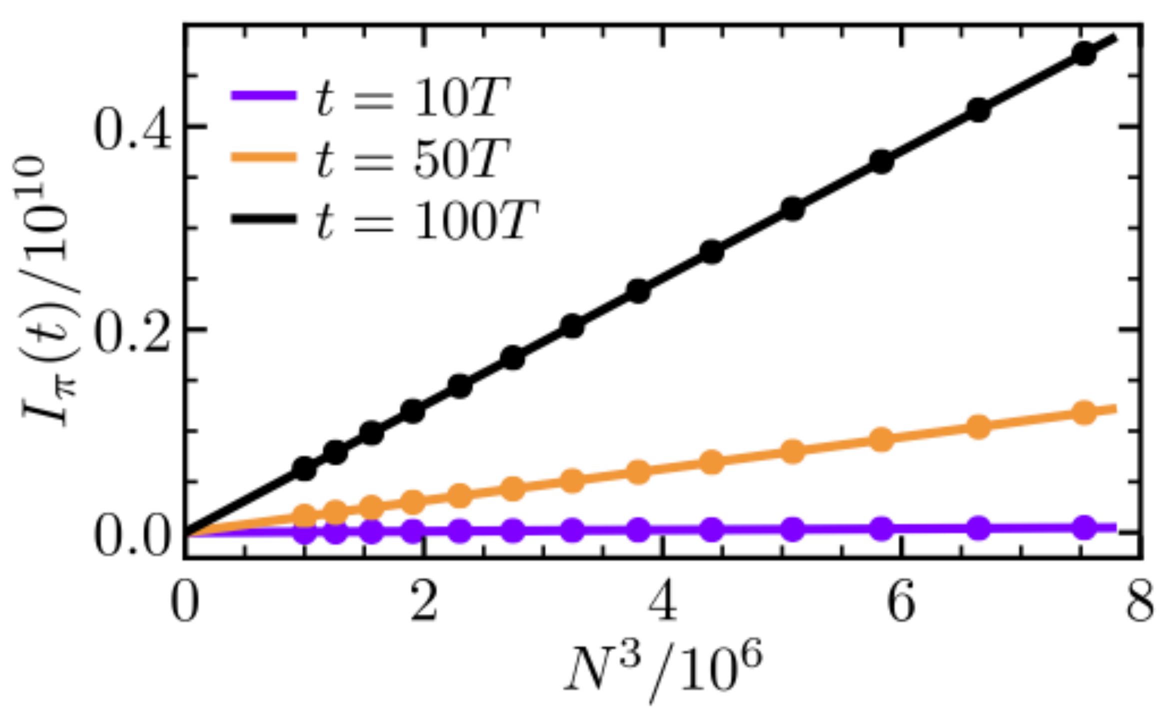
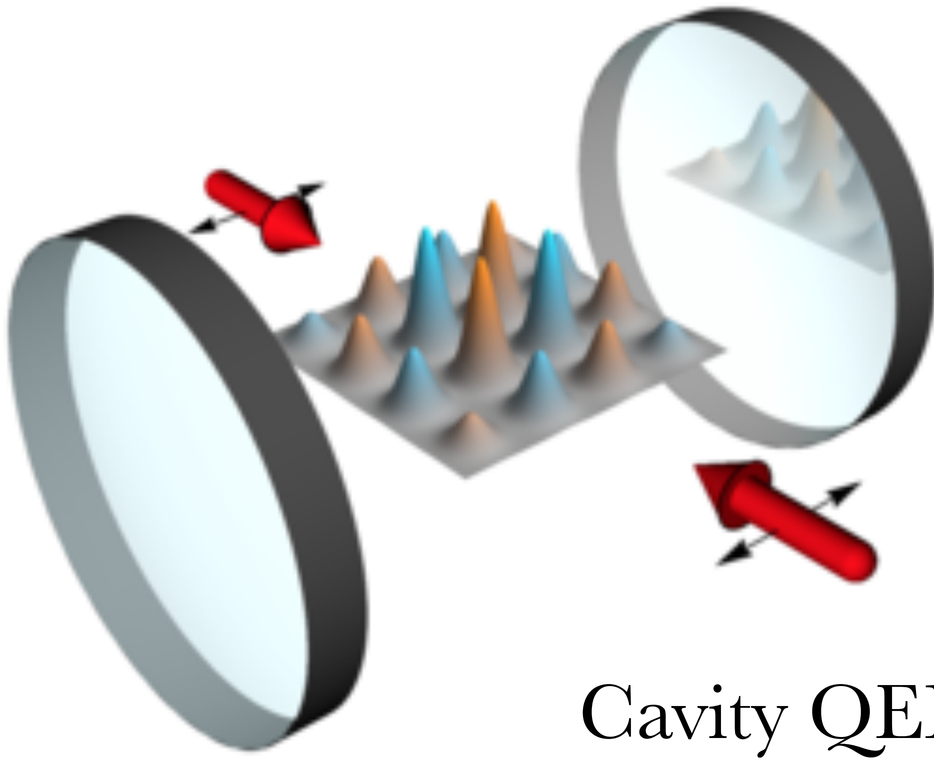
This beats the Linear Heisenberg Limit!

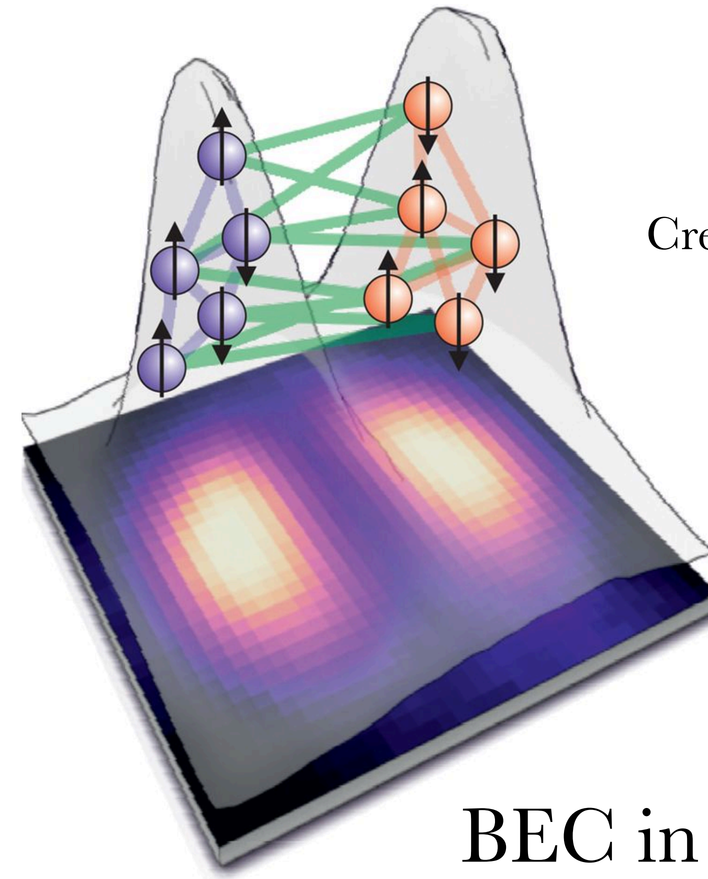


Experimental Realization



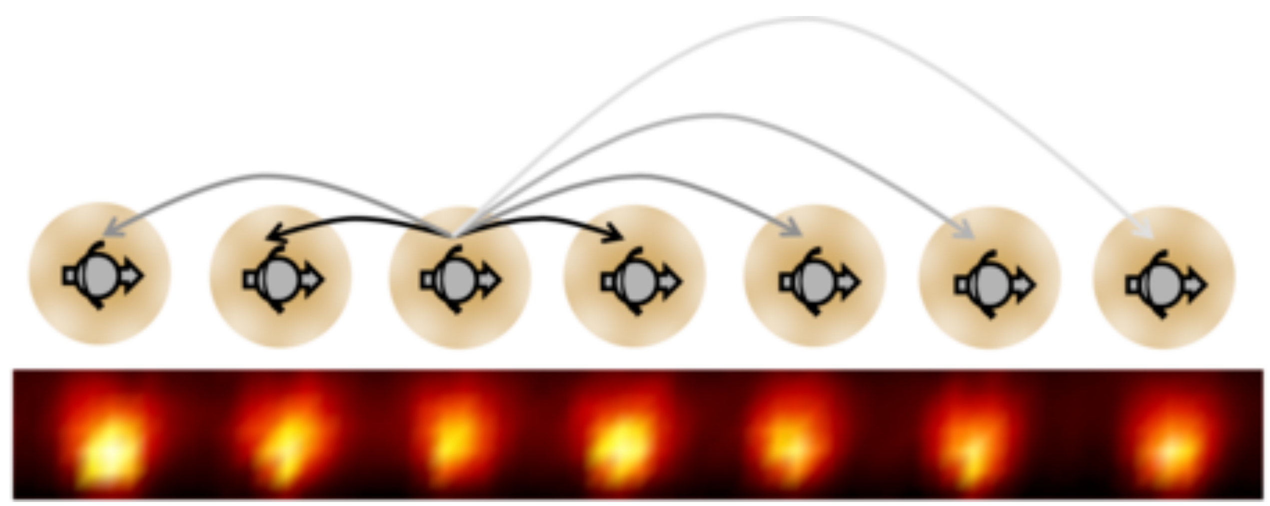
Cavity QED

Credit: Tilman Esslinger Group



Credit: Carsten Klempt Group

BEC in a Double-well



Ion Trap

Credit: Norman Yao Group

Further Connections

nature physics LETTERS
https://doi.org/10.1038/s41567-019-0537-1

Quantum simulation of Unruh radiation

Jiazhong Hu*, Lei Feng, Zhendong Zhang and Cheng Chin

PHYSICAL REVIEW A **102**, 011301(R) (2020)
Rapid Communications

Many-body echo

Yang-Yang Chen,[1,2] Pengfei Zhang,[3] Wei Zheng,[4] Zhigang Wu,[1,*] and Hui Zhai[3,5,†]

PHYSICAL REVIEW X **11,** 011057 (2021)

Suppressing Dissipation in a Floquet-Hubbard System

Konrad Viebahn,[*] Joaquín Minguzzi, Kilian Sandholzer, Anne-Sophie Walter, Manish Sajnani,
Frederik Görg, and Tilman Esslinger

Coming Soon!!

Self-ordered Time Crystals in a Quasiperiodically Driven Spin Chain

Sayan Choudhury[1,*] and W. Vincent Liu[1,2,3,4,†]

[1] *Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, PA 15260, USA*
[2] *Wilczek Quantum Center, School of Physics and Astronomy and T. D. Lee Institute, Shanghai Jiao Tong University, Shanghai 200240, China*
[3] *Shanghai Research Center for Quantum Sciences, Shanghai 201315, China*
[4] *Shenzhen Institute for Quantum Science and Engineering and Department of Physics, Southern University of Science and Technology, Shenzhen 518055, China*

(Dated: July 21, 2021)

Recent work has demonstrated that quasiperiodically driven many-body localized systems can host a rich array of non-equilibrium quantum phases of matter. Motivated by the question of whether such phases can arise in the absence of disorder, we investigate the dynamics of a Lipkin-Meshkov-Glick model under quasiperiodic kicking. Intriguingly, we find that this infinite range interacting spin chain can exhibit long-lived periodic oscillations when the kicking amplitudes are drawn from the Thue-Morse sequence (TMS). We dub this phase a "self-ordered time crystal" (SOTC) and establish that it is prethermal in nature. We demonstrate that this system can host at least two qualitatively distinct kinds of SOTC phases, and trace the origin of these phases to the recursive structure of the TMS. Furthermore, we demonstrate the robustness of the SOTCs under different kinds of perturbations. Our results suggest that quasiperiodic driving protocols can provide a promising route for realizing novel non-equilibrium phases of matter in long-range interacting systems.

Conclusion

- We have proposed a scheme for extending the lifetime of discrete time-crystals in disorder free systems by optimizing the interaction strength (or the driving frequency) for both short and long-range interacting systems.

- We have shown that for an infinite range interacting spin chain, we can create an eternal time crystal that would show perfect revival after an even number of pulses, due to a perfect many-body constructive interference.

- This eternal time crystal can be useful for performing precision measurements.

- Our scheme can be easily realized in various quantum simulator platforms.

Thank You