UNIVERSITY OF CAMBRIDGE
Cavendish Laboratory

Floquet-engineering counterdiabatic protocols in quantum many-body systems

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Phys. Rev. B 103, 075118 (2021)
Phys. Rev. Res. 2, 032052 (2020)
Phys. Rev. Lett. 123, 090602 (2019)

1. Counterdiabatic driving
2. Approximate counterdiabatic driving
3. Floquet engineering
4. Application in dynamical polarization
5. Conclusion

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2. Approximate counterdiabatic driving

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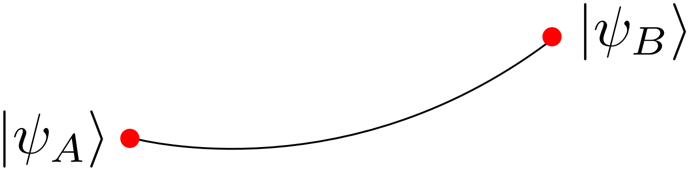
Counterdiabatic driving - Setting

Goal: Prepare system in quantum state with high fidelity



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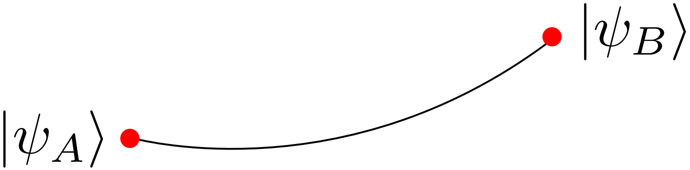

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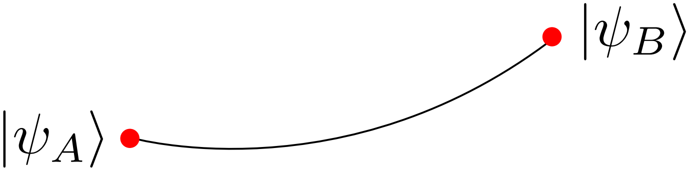

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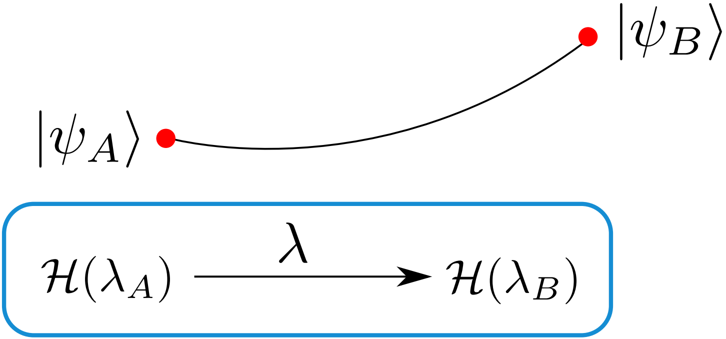

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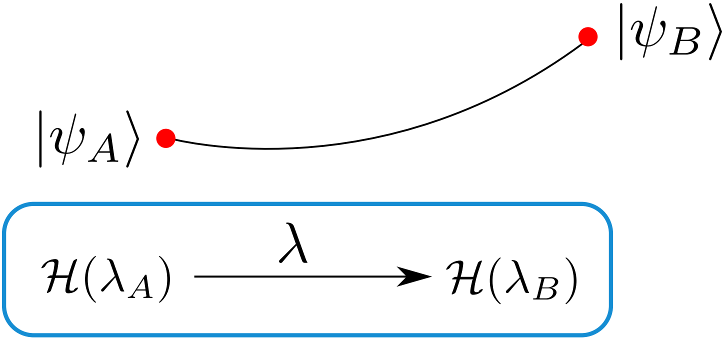

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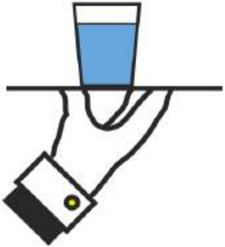
Problem: No experimental access to long time scales

Solution: Suppress excitations by including auxiliary driving terms

Counterdiabatic driving

Adiabatic



Diabatic

Counterdiabatic driving

Adiabatic | Diabatic | Counterdiabatic/ Transitionless

Counterdiabatic driving

Counterdiabatic driving = counteract diabatic excitations

Demirplak & Rice (2013, 2015), Berry (2009), del Campo et al. (2015)

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- Adiabatic control: change control parameter $\lambda(t)$ with $\dot{\lambda} \ll$

$$i\partial_t |\psi(t)\rangle = \mathcal{H}(\lambda) |\psi(t)\rangle$$

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- System remains in instantaneous eigenstate $\mathcal{H}(\lambda) |n\rangle = \epsilon_n |n\rangle$

provided $$\langle m|\mathcal{A}_\lambda|n\rangle = i \langle m|\partial_\lambda n\rangle$$

Demirplak & Rice (2013, 2015), Berry (2009), del Campo et al. (2015)

Single-spin example

- Counterdiabatic term $\langle m|\mathcal{A}_\lambda|n\rangle = i\,\langle m|\partial_\lambda n\rangle$
- Example: **Single spin**

$$\mathcal{H}(\theta) = \Delta\left[\cos(\theta)\sigma^z + \sin(\theta)\sigma^x\right]$$

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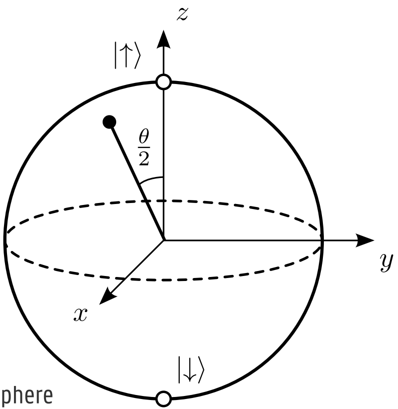

Bloch sphere

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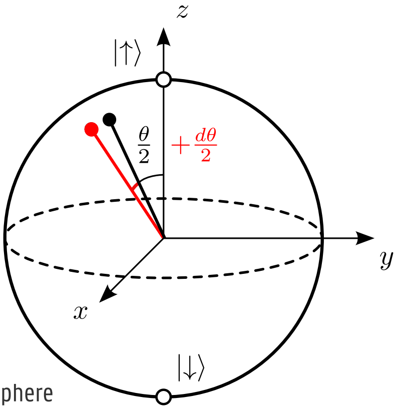

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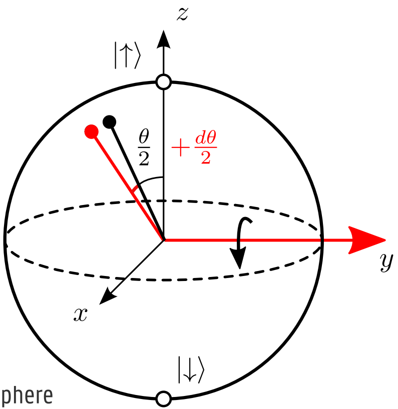

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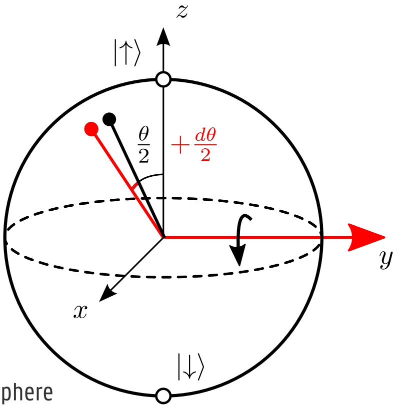

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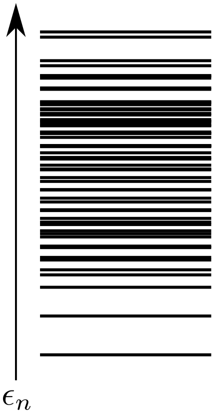
- Counterdiabatic driving

$$\mathcal{H}_{CD} = \mathcal{H}(\theta) + \frac{\dot{\theta}}{2}\sigma_y$$

Many-body problem

$$\langle m|\mathcal{A}_\lambda|n\rangle = -i\,\langle m|\partial_\lambda n\rangle$$



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  - **Divergent** in thermodynamic limit
  - **Nonlocal** $\sim$ No clear 'rotation axis'

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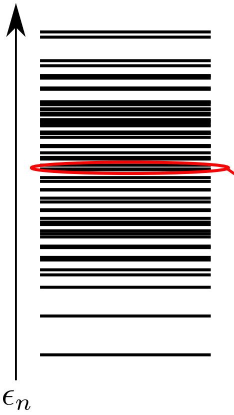

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ETH: States cannot be distinguished using local operators

Need for approximate counterdiabatic driving

- + Can perform exact adiabatic evolution...
- − ... if we know state beforehand
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$$\Downarrow$$

Approximate counterdiabatic driving

1. Variational principle
2. Efficient local ansatz

Approximate counterdiabatic driving

- Variational principle

$$\chi = \mathcal{A}_\lambda \text{ minimizes } ||\partial_\lambda \mathcal{H} + i[\chi, \mathcal{H}]||^2$$

$\Longrightarrow$ Minimize **action**

PRL 123, 090602 (2019) & PNAS 114, E3909 (2017)

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- Efficient ansatz

$$\chi = \sum_k \chi_k \underbrace{[\mathcal{H}, \ldots, [\mathcal{H}}_{k}, \partial_\lambda \mathcal{H}]]]$$

$\Longrightarrow$ Minimize for **coefficients** χ_k

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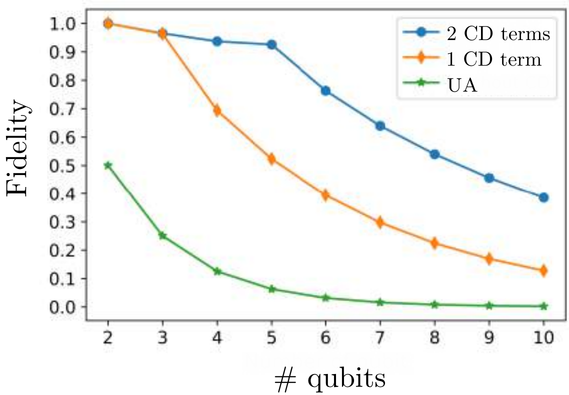
Quick check : qubit $[\mathcal{H}, \partial_\lambda \mathcal{H}] \propto [\sigma_x, \sigma_z] \propto \sigma_y$

PRL 123, 090602 (2019) & PNAS 114, E3909 (2017)

Many-body example

- **Example**: Ising model, quantum simulation w/ Trotterization

$$\mathcal{H}(\lambda) = (1-\lambda)\sum_{j=1}^{L} h_x S_j^x + \lambda \sum_{j=1}^{L}(h_z^j S_j^z + J S_s^z S_{j+1}^z)$$



Phys. Rev. Applied 15, 024038 (2021)

Need for Floquet-engineering

- Approximate counterdiabatic potential allows for **approximate counterdiabatic driving**

$$\mathcal{H}_{CD}(t) = \mathcal{H}(\lambda) + i\dot{\lambda} \sum_{k=1}^{\ell} \alpha_k \underbrace{[\mathcal{H}, [\mathcal{H}, \ldots [\mathcal{H},}_{2k-1} \partial_\lambda \mathcal{H}]]$$

Boyers, Pandey, ..., Sushkov (2018), Petiziol et al . (2018),...

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→ Construct counterdiabatic Hamiltonian as **effective Floquet Hamiltonian**

Boyers, Pandey, ..., Sushkov (2018), Petiziol et al . (2018),...

Floquet-engineering counterdiabatic driving

- Consider a protocol oscillating $\mathcal{H}(\lambda)$ and $\partial_\lambda \mathcal{H}(\lambda)$

$$\mathcal{H}_{FE}(t) = \left[1 + \frac{\omega}{\omega_0}\cos(\omega t)\right]\mathcal{H}(\lambda) + \dot{\lambda}\left[\sum_{k=1}^{\infty}\beta_k \sin\left((2k-1)\omega t\right)\right]\partial_\lambda \mathcal{H}(\lambda),$$

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➡ Reproduces structure of **counterdiabatic protocol**

Overview

- Commutator expansion

$$\langle m|\mathcal{A}_\lambda^\ell|n\rangle = i\sum_{k=1}^{\ell}\alpha_k(\epsilon_m-\epsilon_n)^{2k-1}\,\langle m|\partial_\lambda\mathcal{H}|n\rangle$$

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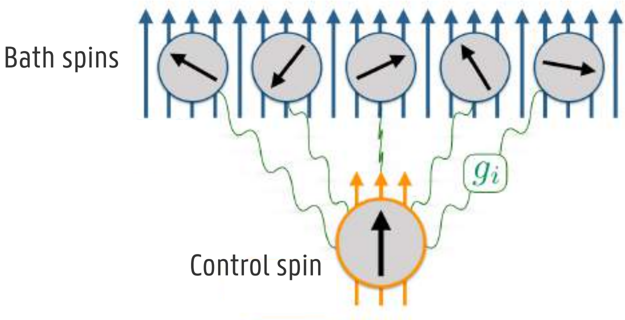
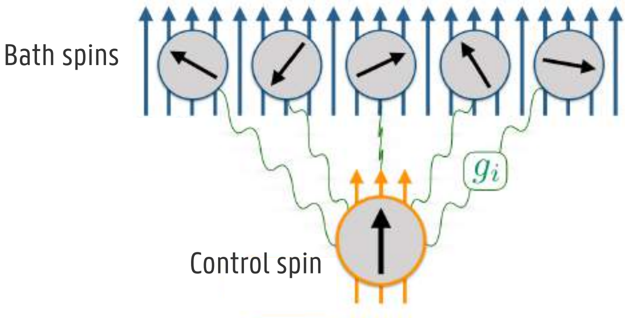
Dynamic polarization



- **Goal**: Polarize spin bath

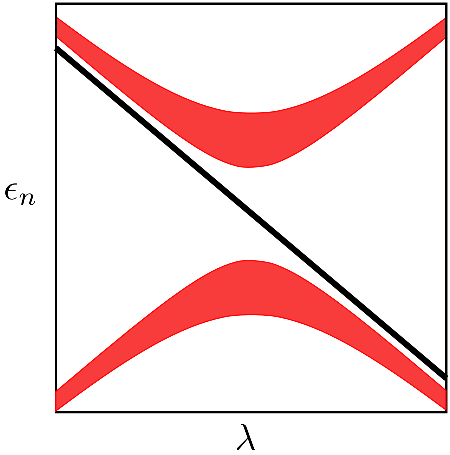
Dynamic polarization



- **Goal**: Polarize spin bath
- Relevant in NMR, NV centers in diamond,...

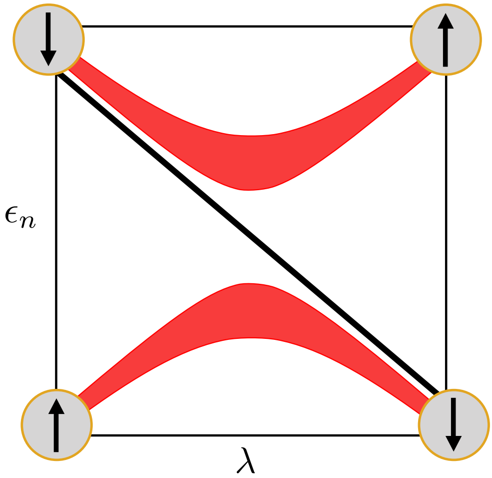
$$\mathcal{H}(\lambda) = \lambda S_0^z + B \sum_j S_j^z + \sum_j g_j \left(S_0^+ S_j^- + S_0^- S_j^+ \right)$$

Schematic eigenspectrum



Phys. Rev. Res. 2, 032052 (2020)

Schematic eigenspectrum



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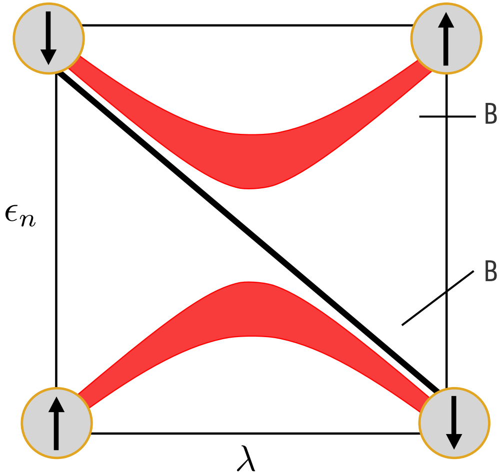

Bands of **bright states**

...adiabatically connect states with different central spin polarization

Band of **dark states**

...fixed central spin polarization

$$|\psi_n\rangle = |{\downarrow}\rangle_0 \otimes |B\rangle$$

$$\epsilon_n = -\lambda/2$$

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Polarization protocols

- Two-step protocols

i) **Reset** polarization of control spin to $|\downarrow\rangle$

Nonadiabatic, e.g. rapid optical pulse

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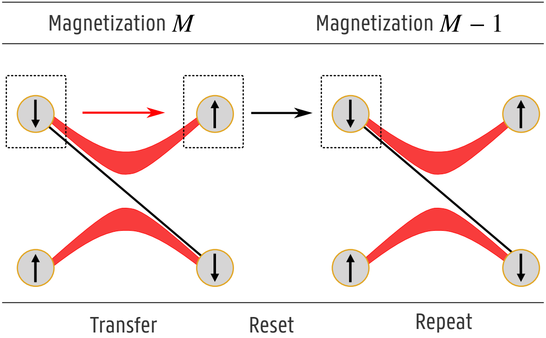

Approximate counterdiabatic driving

- Improve **transfer efficiency** with a single commutator

  Evolve with $\mathcal{H}(\lambda) + \dot{\lambda}\alpha_1[\mathcal{H}, S_0^z]$

  where $[\mathcal{H}, S_0^z] = i\sum_j g_j \left(S_0^x S_j^y - S_0^y S_j^x\right)$

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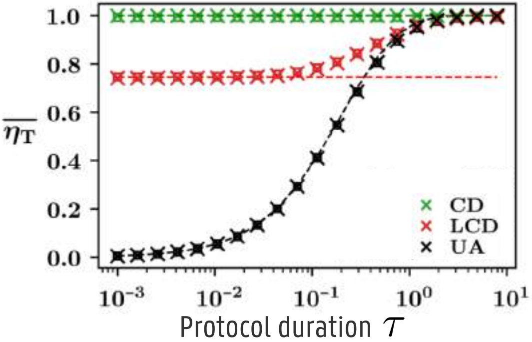
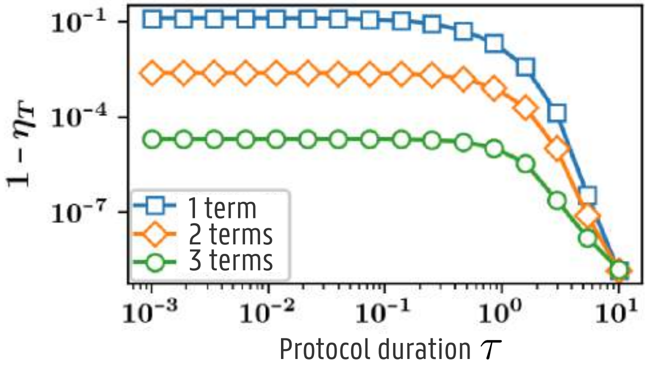


Transfer efficiency η_T = fraction of polarization successfully transfered

Approximate counterdiabatic driving

- Improve transfer efficiency with counterdiabatic terms



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Floquet-engineering counterdiabatic control

- Experimentally realize single commutator by introducing **high-frequency oscillations** in λ

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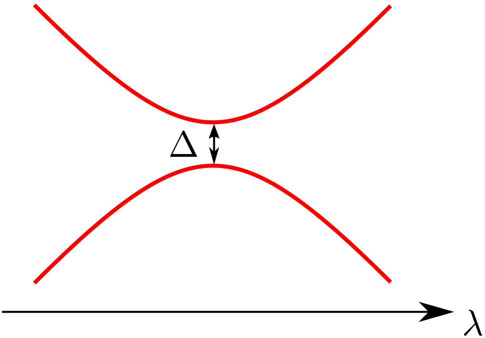

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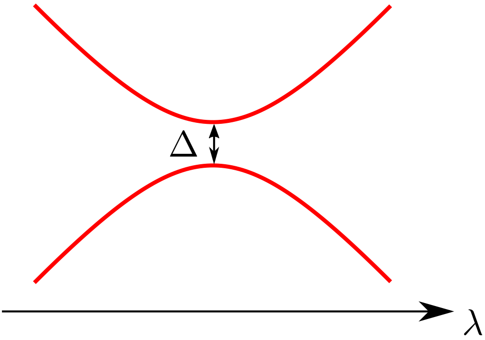

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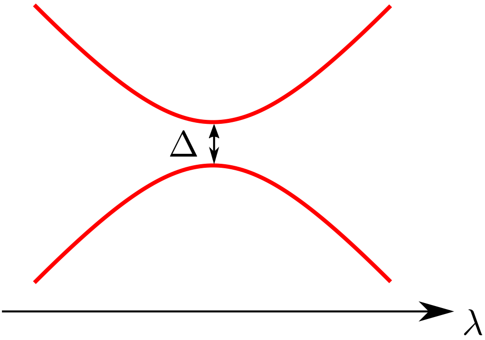

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- **Goal**: find an effective Floquet Hamiltonian

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- Returns **Floquet Hamiltonian**

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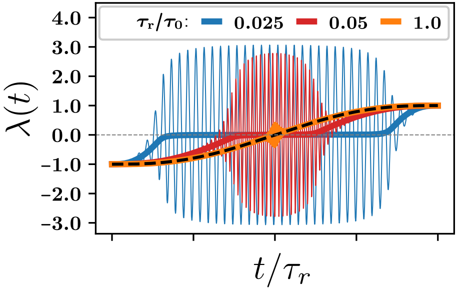
→ Counterdiabatic control in **rescaled time** s

$$\partial_s = G(t)\partial_t$$

Floquet protocols



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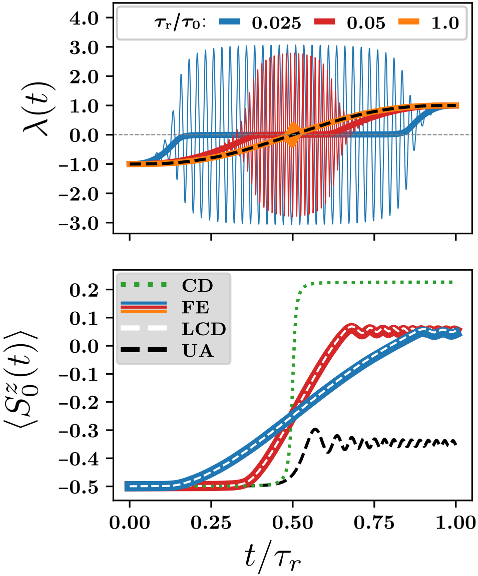

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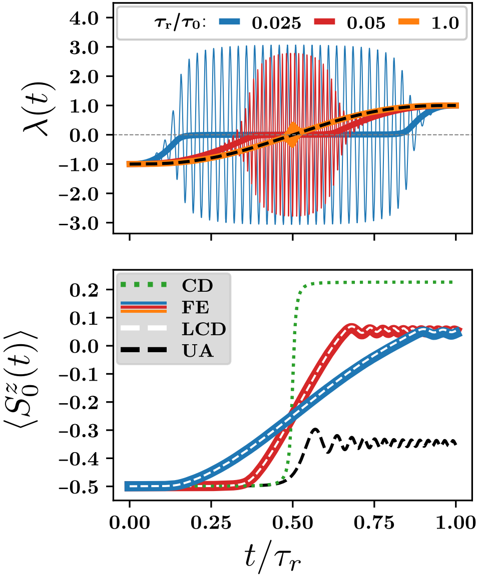

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- Floquet protocol mimics counterdiabatic control

↓

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$$\Rightarrow \quad 1/\tau_{SL} \approx \sqrt{L}\,\overline{g}$$

1. Counterdiabatic driving
2. Approximate counterdiabatic driving
3. Floquet engineering
4. Application in dynamical polarization
5. **Conclusion**

Conclusions

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THANK YOU FOR YOUR ATTENTION

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