Floquet-engineering counterdiabatic protocols in quantum many-body systems

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1. Counterdiabatic driving
2. Approximate counterdiabatic driving
3. Floquet engineering
4. Application in dynamical polarization
5. Conclusion
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Counterdiabatic driving - Setting

**Goal:** Prepare system in quantum state with high fidelity
Counterdiaabatic driving - Setting

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\[ |\psi_A\rangle \xrightarrow{\lambda} \mathcal{H}(\lambda_A) \xrightarrow{\lambda} \mathcal{H}(\lambda_B) \xrightarrow{\psi_B} \]

- **Solution:** Suppress excitations by including auxiliary driving terms
- **Problem:** No experimental access to long time scales
- **Adiabatic theorem:** System remains in an instantaneous eigenstate if \( \lambda \) is varied slowly enough
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Counterdiabatic driving

Adiabatic

Diabatic
Counterdiabatic driving

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Counterdiabatic/Transitionless
Counterdiabatic driving

Counterdiabatic driving = counteract diabatic excitations
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\[ i\partial_t |\psi(t)\rangle = \mathcal{H}(\lambda) |\psi(t)\rangle \]
Counterdiabatic driving

Counterdiabatic driving = counteract diabatic excitations

- Adiabatic control: change control parameter $\lambda(t)$ with $\dot{\lambda} \ll 1$

$$i\partial_t |\psi(t)\rangle = \mathcal{H}(\lambda) |\psi(t)\rangle$$

- Counterdiabatic control: add velocity-dependent term

$$i\partial_t |\psi(t)\rangle = \left(\mathcal{H}(\lambda) + \dot{\lambda} A_\lambda\right) |\psi(t)\rangle$$

Counterdiabatic driving

**Counterdiabatic driving = counteract diabatic excitations**

- **Adiabatic control:** change control parameter $\lambda(t)$ with $\dot{\lambda} \ll \frac{\hbar}{\text{energy scale}}$

\[ i\dot{\psi}(t) = \mathcal{H}(\lambda) |\psi(t)\rangle \]

- **Counterdiabatic control:** add velocity-dependent term

\[ i\dot{\psi}(t) = \left( \mathcal{H}(\lambda) + \dot{\lambda} \mathcal{A}_\lambda \right) |\psi(t)\rangle \]

- **System remains in instantaneous eigenstate** $\mathcal{H}(\lambda) |n\rangle = \epsilon_n |n\rangle$

provided

\[ \langle m | \mathcal{A}_\lambda | n \rangle = i \langle m | \partial_\lambda n \rangle \]

Single-spin example

- Counterdiabatic term $\langle m | A_\lambda | n \rangle = i \langle m | \partial_\lambda n \rangle$

- Example: **Single spin**

\[ \mathcal{H}(\theta) = \Delta \left[ \cos(\theta) \sigma^z + \sin(\theta) \sigma^x \right] \]
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Bloch sphere
Single-spin example

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- Example: **Single spin**

\[
\mathcal{H}(\theta) = \Delta \left[ \cos(\theta) \sigma^z + \sin(\theta) \sigma^x \right] \quad \Rightarrow \quad A_\theta = \frac{\sigma_y}{2}
\]
Single-spin example

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- Counterdiabatic term
  - \( = \text{Rotation around } y\text{-axis} \)
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- Example: **Single spin**

  $\mathcal{H}(\theta) = \Delta [\cos(\theta)\sigma^z + \sin(\theta)\sigma^x]$  
  \[ A_\theta = \frac{\sigma_y}{2} \]

Counterdiabatic term

= Rotation around y-axis

Counterdiabatic driving

$\mathcal{H}_{CD} = \mathcal{H}(\theta) + \frac{\dot{\theta}}{2} \sigma_y$
Many-body problem

\[ \langle m | A_\lambda | n \rangle = -i \langle m | \partial_\lambda n \rangle \]

- Many-body systems
  - Involves **full Hilbert space**
  - **Divergent** in thermodynamic limit
  - **Nonlocal** \( \sim \) No clear 'rotation axis'

\[ \varepsilon_n \]
Many-body problem

\[ \langle m | A_\chi | n \rangle = -i \langle m | \partial_\chi n \rangle \]

- Many-body systems
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**ETH**: States cannot be distinguished using local operators
Need for approximate counterdiabatic driving

+ Can perform exact adiabatic evolution...

- ... if we know state beforehand

- ... and can experimentally realize counterdiabatic term
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Approximate counterdiabatic driving

1. Variational principle
2. Efficient local ansatz
1. Counterdiabatic driving
2. Approximate counterdiabatic driving
3. Floquet engineering
4. Application in dynamical polarization
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Approximate counterdiabatic driving

- Variational principle

\[ \chi = A_\lambda \text{ minimizes } || \partial_\lambda \mathcal{H} + i[\chi, \mathcal{H}] ||^2 \]

\[ \implies \text{Minimize action} \]
Approximate counterdiabatic driving

- Variational principle
  \[ \chi = A_\chi \text{ minimizes } \| \partial_\chi \mathcal{H} + i[\chi, \mathcal{H}] \|^2 \]
  \[ \Rightarrow \text{ Minimize } \text{action} \]

- Efficient ansatz
  \[ \chi = \sum_k \chi_k [\mathcal{H}, \ldots, [\mathcal{H}, \partial_\chi \mathcal{H}]] ] \]
  \[ \Rightarrow \text{ Minimize for coefficients } \chi_k \]
Approximate counterdiabatic driving

- Variational principle

\[ \chi = A_\lambda \text{ minimizes } \left| \left| \partial_\lambda \mathcal{H} + i[\chi, \mathcal{H}] \right| \right|^2 \]

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- Efficient ansatz

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Quick check: qubit \[ [\mathcal{H}, \partial_\lambda \mathcal{H}] \propto [\sigma_x, \sigma_z] \propto \sigma_y \]
Many-body example

**Example:** Ising model, quantum simulation w/ Trotterization

\[ \mathcal{H}(\lambda) = (1 - \lambda) \sum_{j=1}^{L} \hbar x S_j^x + \lambda \sum_{j=1}^{L} (\hbar^j S_j^z + J S_s^z S_{j+1}^z) \]

![Graph showing fidelity versus number of qubits with three curves: 2 CD terms, 1 CD term, and UA.]
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Need for Floquet-engineering

- Approximate counterdiabatic potential allows for approximate counterdiabatic driving

\[
H_{CD}(t) = H(\lambda) + i\dot{\lambda} \sum_{k=1}^{\ell} \alpha_k \left[ H, [H, \ldots [H, \partial_\lambda H]] \right]_{2k-1}
\]

Boyers, Pandey, ..., Sushkov (2018), Petiziol et al. (2018),...
Need for Floquet-engineering

- Approximate counterdiabatic potential allows for approximate counterdiabatic driving

\[ \mathcal{H}_{CD}(t) = \mathcal{H}(\lambda) + i\lambda \sum_{k=1}^{\ell} \alpha_k \left[ \mathcal{H}, \left[ \mathcal{H}, \ldots \left[ \mathcal{H}, \partial_\lambda \mathcal{H} \right] \ldots \right] \right]_{2k-1} \]

- Already expressed in 'local' operators

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\[ \mathcal{H}_{CD}(t) = \mathcal{H}(\lambda) + i\dot{\lambda} \sum_{k=1}^{\ell} \alpha_k \left[ \mathcal{H}, \underbrace{\mathcal{H}, \ldots \mathcal{H}}_{2k-1}, \partial_{\lambda}\mathcal{H} \right] \]

- Already expressed in 'local' operators
- Limited amount of variables
- Can be systematically improved

Boyers, Pandey, ..., Sushkov (2018), Petiziol et al. (2018),...
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- Approximate counterdiabatic potential allows for approximate counterdiabatic driving

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\mathcal{H}_{CD}(t) = \mathcal{H}(\lambda) + i \lambda \sum_{k=1}^{\ell} \alpha_k \left[ \mathcal{H}, [\mathcal{H}, \ldots [\mathcal{H}, \partial_\lambda \mathcal{H}]_{2k-1} \right]
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- Already expressed in 'local' operators
- Limited amount of variables
- Can be systematically improved
  - ... Interactions within commutators not necessarily accessible

Boyers, Pandey, ..., Sushkov (2018), Petiziol et al. (2018),...
Need for Floquet-engineering

- Approximate counterdiabatic potential allows for **approximate counterdiabatic driving**

\[ \mathcal{H}_{CD}(t) = \mathcal{H}(\lambda) + i \lambda \sum_{k=1}^{\ell} \alpha_k \left[ \mathcal{H}, [\mathcal{H}, \ldots [\mathcal{H}, \partial_\lambda \mathcal{H}]_{2k-1} \right] \]

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Construct counterdiabatic Hamiltonian as **effective Floquet Hamiltonian**

Boyers, Pandey, ..., Sushkov (2018), Petiziol et al. (2018),...
Floquet-engineering counterdiabatic driving

Consider a protocol oscillating $\mathcal{H}(\lambda)$ and $\partial_\lambda \mathcal{H}(\lambda)$

$$\mathcal{H}_{FE}(t) = \left[ 1 + \frac{\omega}{\omega_0} \cos(\omega t) \right] \mathcal{H}(\lambda) + \dot{\lambda} \left[ \sum_{k=1}^{\infty} \beta_k \sin((2k - 1)\omega t) \right] \partial_\lambda \mathcal{H}(\lambda),$$
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Fourier coefficients
Floquet-engineering counterdiabatic driving

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Fourier coefficients

- Leads to a Floquet Hamiltonian

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\mathcal{H}_F = \mathcal{H}(\lambda) + \dot{\lambda} \mathcal{A}_F
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Floquet-engineering counterdiabatic driving

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Fourier coefficients

- Leads to a Floquet Hamiltonian

$$\mathcal{H}_F = \mathcal{H}(\lambda) + \lambda \mathcal{A}_F$$

- Additional gauge term

$$\langle m | \mathcal{A}_F | n \rangle = \sum_{k=1}^{\infty} \beta_k J_k \left( \frac{\epsilon_m - \epsilon_n}{\omega_0} \right) \langle m | \partial_\lambda \mathcal{H} | n \rangle$$
Floquet-engineering counterdiabatic driving

- Consider a protocol oscillating $\mathcal{H}(\lambda)$ and $\partial_{\lambda} \mathcal{H}(\lambda)$

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$$

Reproduces structure of counterdiabatic protocol
Overview

- Commutator expansion

\[ \langle m| \mathcal{A}_\lambda^\ell |n \rangle = i \sum_{k=1}^{\ell} \alpha_k (\epsilon_m - \epsilon_n)^{2k-1} \langle m| \partial_\lambda \mathcal{H} |n \rangle \]
Overview

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- Floquet-engineering

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- Floquet-engineering \rightarrow \text{Match coefficients}

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\[ \mathcal{H}(t) = \left[ 1 + \frac{\omega}{\omega_0} \cos(\omega t) \right] \mathcal{H}(\lambda) \]

\[ + 2 \dot{\lambda} \left[ \alpha_1 \omega_0 \sin(\omega t) + (24 \alpha_2 \omega_0^3 + 3 \alpha_1 \omega_0) \sin(3\omega t) \right] \partial_\lambda \mathcal{H}(\lambda) \]
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\]

\[
\mathcal{H}_F = \mathcal{H} + i \dot{\lambda} \alpha_1 [\mathcal{H}, \partial_\lambda \mathcal{H}] + i \dot{\lambda} \alpha_2 [\mathcal{H}, [\mathcal{H}, \partial_\lambda \mathcal{H}]] + \mathcal{O}(\omega_0^{-2}).
\]
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**Dynamic polarization**

- **Bath spins**
- **Control spin**

**Goal:** Polarize spin bath
Dynamic polarization

- **Goal**: Polarize spin bath
- Relevant in NMR, NV centers in diamond, ...

\[ \mathcal{H}(\lambda) = \lambda S_0^z + B \sum_j S_j^z + \sum_j g_j (S_0^+ S_j^- + S_0^- S_j^+) \]
Schematic eigenspectrum
Schematic eigenspectrum

\[ \epsilon_n \]

\[ \lambda \]
Bands of bright states
...adiabatically connect states with different central spin polarization

Band of dark states
...fixed central spin polarization

\[ |\psi_n\rangle = |\downarrow\rangle_0 \otimes |B\rangle \]
\[ \epsilon_n = -\lambda/2 \]
Polarization protocols

- Two-step protocols
  
  i) **Reset** polarization of control spin to $|\downarrow\rangle$
    
    Nonadiabatic, e.g. rapid optical pulse
Polarization protocols

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    Nonadiabatic, e.g. rapid optical pulse
  ii) **Sweep** magnetic field: transfer control spin polarization to bath
    Adiabatic $\rightarrow$ Can be sped up!
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<table>
<thead>
<tr>
<th>Magnetization $M$</th>
<th>Magnetization $M - 1$</th>
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</thead>
<tbody>
<tr>
<td><img src="image1" alt="Diagram" /></td>
<td><img src="image2" alt="Diagram" /></td>
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Approximate counterdiabatic driving

- Improve **transfer efficiency** with a single commutator

Evolve with $\mathcal{H}(\lambda) + \dot{\lambda} \alpha_1 [\mathcal{H}, S_0^z]$

where $[\mathcal{H}, S_0^z] = i \sum_j g_j (S_0^x S_j^y - S_0^y S_j^x)$
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Transfer efficiency $\eta_T = \text{fraction of polarization successfully transferred}$
Approximate counterdiabatic driving

- Improve transfer efficiency with counterdiabatic terms

Transfer efficiency $\eta_T = \text{fraction of polarization successfully transferred}$
Floquet-engineering counterdiabatic control

- Experimentally realize single commutator by introducing high-frequency oscillations in $\lambda$
  - Only requires access to control (magnetic) field!
Floquet-engineering counterdiabatic control

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- Illustration for single-spin system
  
  (Everything expressed in commutators: extends to many-body systems)
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  (Everything expressed in commutators: extends to many-body

\[ H = \lambda \tilde{S}^z + \Delta \tilde{S}^x \]
Floquet-engineering counterdiabatic control

- Experimentally realize single commutator by introducing high-frequency oscillations in $\lambda$
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- Illustration for single-spin system
  - (Everything expressed in commutators: extends to many-body)

\[
\mathcal{H} = \lambda \tilde{S}^z + \Delta \tilde{S}^x
\]

\[
\mathcal{H}_{CD} = \lambda \tilde{S}^z + \Delta \tilde{S}^x + \alpha \dot{\lambda} \tilde{S}^y
\]
High-frequency expansion

- **Goal**: find an effective Floquet Hamiltonian

\[ \mathcal{H}_{CD} = \lambda \tilde{S}^z + \Delta \tilde{S}^x + \alpha \lambda \tilde{S}^y \]

\[ \text{Const.} \]
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- Single **high-frequency** control term, \( \omega \gg \)

\[ \mathcal{H}_{FE}(t) = \gamma(t) \tilde{S}^z + \Delta \tilde{S}^x \]

\[ + \left[ \beta(t) \omega \sin(\omega t) + \dot{\beta}(t) (1 - \cos(\omega t)) \right] \tilde{S}^z \]

Slowly-varying fields \( \gamma(t), \beta(t) \)
High-frequency expansion

- **Goal:** find an effective Floquet Hamiltonian

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H_{CD} = \lambda \tilde{S}^z + \Delta \tilde{S}^x + \alpha \lambda \tilde{S}^y
\]

Const.

- Single **high-frequency** control term, \( \omega \gg \)

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H_{FE}(t) = \gamma(t) \tilde{S}^z + \Delta \tilde{S}^x
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+ \left[ \beta(t)\omega \sin(\omega t) + \dot{\beta}(t)(1 - \cos(\omega t)) \right] \tilde{S}^z
\]

Slowly-varying fields \( \gamma(t), \beta(t) \)

- **Returns** Floquet Hamiltonian

\[
H_F = \gamma \tilde{S}^z + J_0(\beta) \Delta \left[ \cos(\beta) \tilde{S}^x + \sin(\beta) \tilde{S}^y \right]
\]
Rescaling time

- We want

\[ \mathcal{H}_{CD} = \lambda \tilde{S}^z + \Delta \tilde{S}^x + \alpha \lambda \tilde{S}^y \]

\[ \text{Const.} \]
Rescaling time

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  Either both constant or both time-dependent
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Rescale Hamiltonian
Rescaling time

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Rescale Hamiltonian

\[ i \partial_t |\psi\rangle = \mathcal{H}_F |\psi\rangle = G(t) \mathcal{H}_{CD} |\psi\rangle \]
Rescaling time

- We want
  \[ \mathcal{H}_{CD} = \lambda \tilde{S}^z + \Delta \tilde{S}^x + \alpha \lambda \tilde{S}^y \]
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- We have
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**Rescale** Hamiltonian
\[ i \partial_t |\psi\rangle = \mathcal{H}_F |\psi\rangle = G(t) \mathcal{H}_{CD} |\psi\rangle \]

**Counterdiabatic control in rescaled time** \( S \)
\[ \partial_s = G(t) \partial_t \]
Floquet protocols

- Immediately extends to many-body situation
- ... largely system-agnostic
Floquet protocols

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Floquet protocols

- Immediately extends to many-body situation
- .... largely system-agnostic
- Floquet protocol mimics counterdiabatic control

Significant increase in transfer efficiency
Quantum speed limit

- Significant increase in transfer efficiency

- **But**: quantum speed limit $\tau_{SL}$
  
  $\tau > \tau_{SL}$: FE can realize counterdiabatic control
  
  $\tau < \tau_{SL}$: Construction fails!
Quantum speed limit

- Significant increase in transfer efficiency

- **But**: quantum speed limit $\tau_{SL}$
  
  $\tau > \tau_{SL}$: FE can realize counterdiabatic control
  
  $\tau < \tau_{SL}$: Construction fails!

- Rescaling of time only sensible if both run forward

\[
\partial_s = G(t) \partial_t = J_0(\beta) \cos(\beta) \partial_t
\]

| Should be positive! |
Quantum speed limit

- Significant increase in transfer efficiency

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\[
1/\tau_{SL} \approx \sqrt{L \bar{g}}
\]
1. Counterdiabatic driving
2. Approximate counterdiabatic driving
3. Floquet engineering
4. Application in dynamical polarization
5. Conclusion
Conclusions

THANK YOU FOR YOUR ATTENTION

Approximate counterdiabatic driving can be systematically realized...
... and naturally implemented through Floquet engineering with speeding up dynamical polarization as one promising application

Conclusions

- **Approximate counterdiabatic driving** can be systematically realized...

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