Fragility of Time Reversal Symmetry-Protected Topological Phases

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Interacting Topological Matter: AMO Systems
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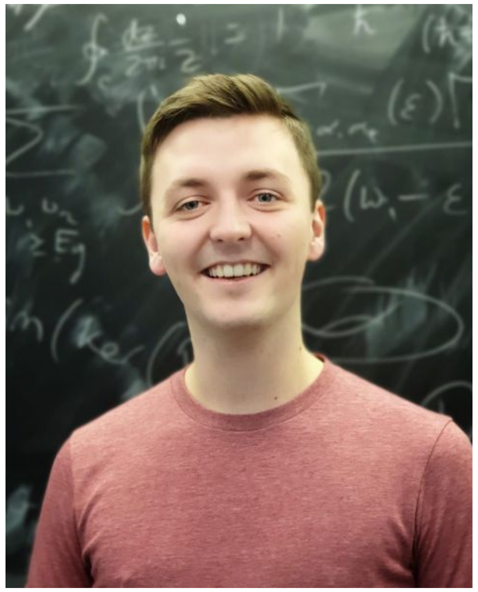

Work in collaboration with Max McGinley

[Max McGinley & NRC, PRL **121**, 090401 (2018); Nature Physics **16**, 1181 (2020)]

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SIMONS FOUNDATION

Time Reversal Symmetry (TRS)

- Classical dynamics $\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t} = \frac{\mathrm{d}H}{\mathrm{d}\boldsymbol{p}} \quad \frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = -\frac{\mathrm{d}H}{\mathrm{d}\boldsymbol{r}}$

  TRS if equations of motion invariant under

  $t \rightarrow -t \quad \boldsymbol{r} \rightarrow \boldsymbol{r} \quad \boldsymbol{p} \rightarrow -\boldsymbol{p}$ $\Rightarrow$ $H(\boldsymbol{r}, \boldsymbol{p}) = H(\boldsymbol{r}, -\boldsymbol{p})$

- Quantum dynamics (spinless particle) $\mathrm{i}\hbar \frac{\mathrm{d}}{\mathrm{d}t}\psi(\boldsymbol{r}, t) = H\psi(\boldsymbol{r}, t)$

  TRS if equations of motion invariant under

  $t \rightarrow -t \quad \psi(\boldsymbol{r}, t) \rightarrow K\psi(\boldsymbol{r}, t) \equiv \psi^*(\boldsymbol{r}, t)$ $\Rightarrow$ $KHK^{-1} = H \quad [K, H] = 0$

  $\boldsymbol{r} \rightarrow K\boldsymbol{r}K^{-1} = \boldsymbol{r}$

  $\boldsymbol{p} \rightarrow K\boldsymbol{p}K^{-1} = K\frac{\hbar}{\mathrm{i}}\frac{\mathrm{d}}{\mathrm{d}\boldsymbol{r}}K^{-1} = -\boldsymbol{p}$

  anti-unitary operator $|\Psi\rangle \rightarrow \hat{\mathcal{O}}|\Psi\rangle \quad (\hat{\mathcal{O}}\Psi, \hat{\mathcal{O}}\Phi) = (\Psi, \Phi)^*$

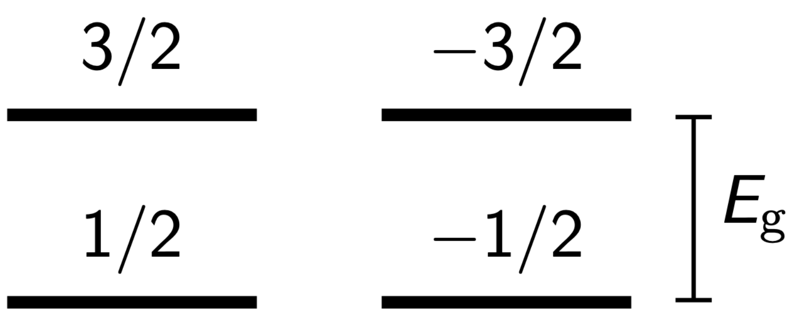
Time Reversal Symmetry-Protected Quantum Phenomena

$[\hat{\mathscr{O}}, \hat{\mathscr{H}}] = 0$ for anti-unitary time reversal operator $\hat{\mathscr{O}}$

- Kramers' Degeneracy (half-integer spin system $\hat{\mathscr{O}}^2 = -\hat{1}$)

$|\psi\rangle$ and $|\psi'\rangle = \hat{\mathscr{O}}|\psi\rangle$ are degenerate and orthogonal

e.g. Spin-3/2 $\hat{\mathscr{H}} = E_g \hat{S}_z^2/2$

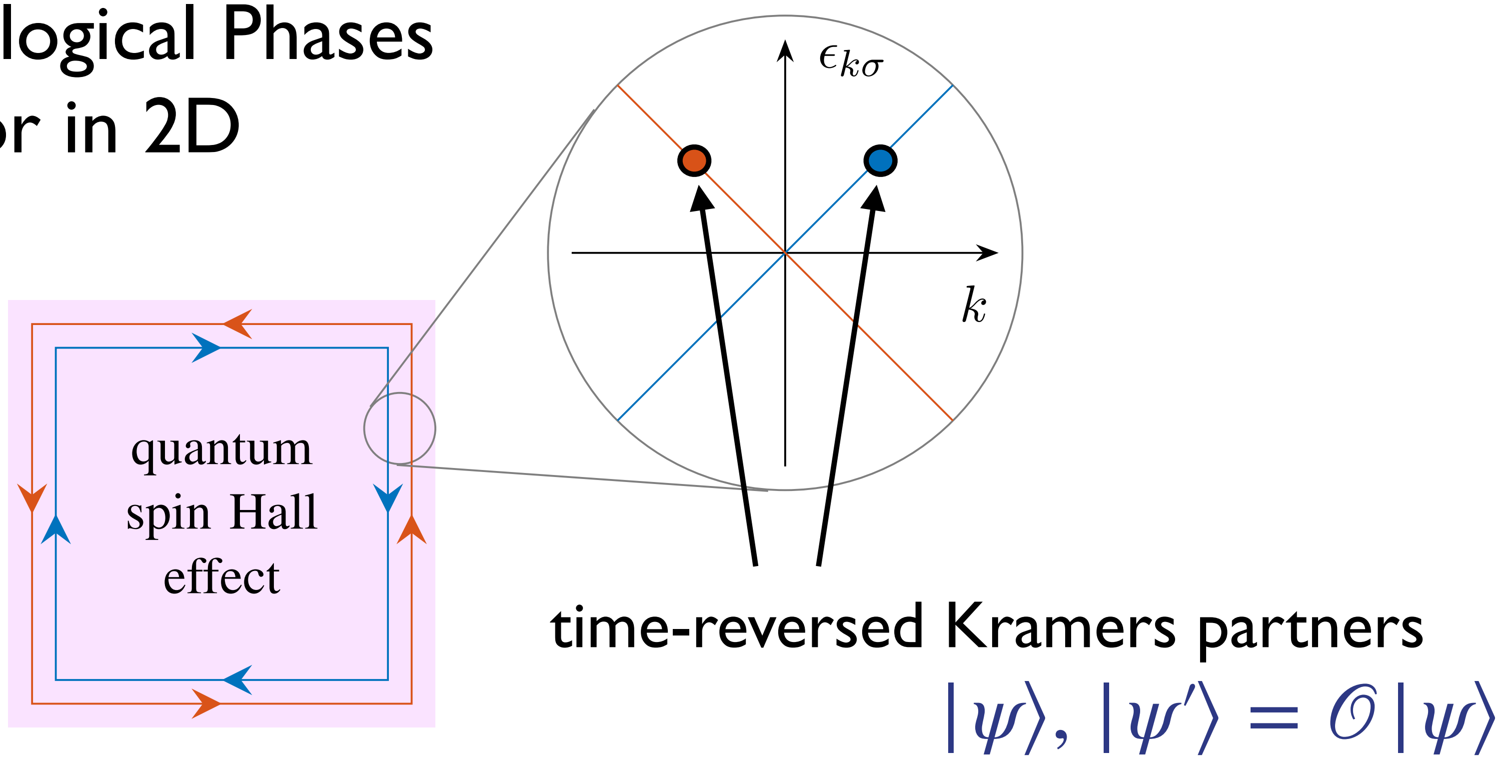


⇒ exact two-fold degeneracy for TRS-respecting $\hat{\mathscr{H}}$

Time Reversal Symmetry-Protected Quantum Phenomena

- Symmetry-Protected Topological Phases
  e.g. $\mathbb{Z}_2$ topological insulator in 2D

[Kane & Mele, PRL (2005)]



$\langle\psi'|\,\hat{A}\,|\psi\rangle = 0$ for any TRS-respecting observable $\hat{A}$ $\quad [\hat{\mathcal{O}}, \hat{A}] = 0$

⇒absence of elastic backscattering of helical edge states unless TRS broken (magnetic field, magnetic impurities)

Symmetry Protected Topological Phases

- Free-fermion topological insulators / superconductors

  [Altland-Zirnbauer: Time-reversal, charge-conjugation & sublattice symmetries]

[Schnyder, Ryu, Furusaki, Ludwig (2008); Kitaev (2009)]

	symmetries			spatial dimension							
Class	T	C	S	0	1	2	3	4	5	6	7
A	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
AIII	0	0	1	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AI	+	0	0	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
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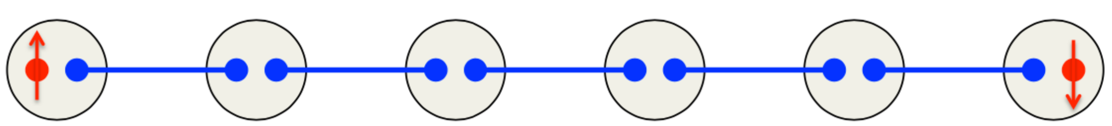
C	0	−	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
CI	+	−	1	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$

- Interacting symmetry-protected topological phases

[Chen, Gu, Wen, PRB (2010)]

  e.g. Haldane phase of spin-1 chain



[Image: Wierschem & Sengupta, Mod Phys Lett B (2015)]

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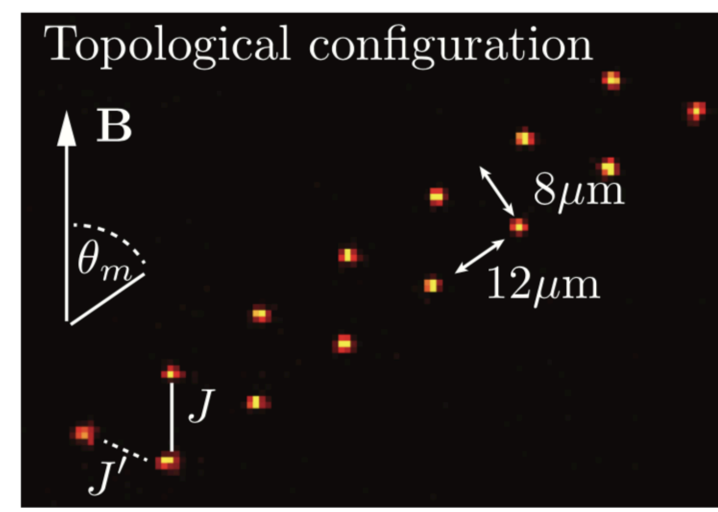
CI	+	−	1	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$

- Interacting symmetry-protected topological phases

  e.g. SPT phase of Rydberg atoms

  [S. de Léséleuc *et al.*, Science 2019]



+ crystalline symmetries, higher order topological phases…

⇒ Detailed classification of topological matter at equilibrium

Beyond Groundstate Topology

1) Non-equilibrium dynamics

Unitary evolution: $$|\Psi(t)\rangle = \mathscr{T}\exp\left[-\mathrm{i}\int_0^t \hat{\mathscr{H}}(t')\,\mathrm{d}t'\right]|\Psi(0)\rangle$$

— dynamical preparation of topological phases? (cold gases)

2) Open quantum systems

— effects of environmental couplings on topological edge modes

⇒Time reversal symmetry-protected phenomena are intrinsically fragile

Beyond Groundstate Topology

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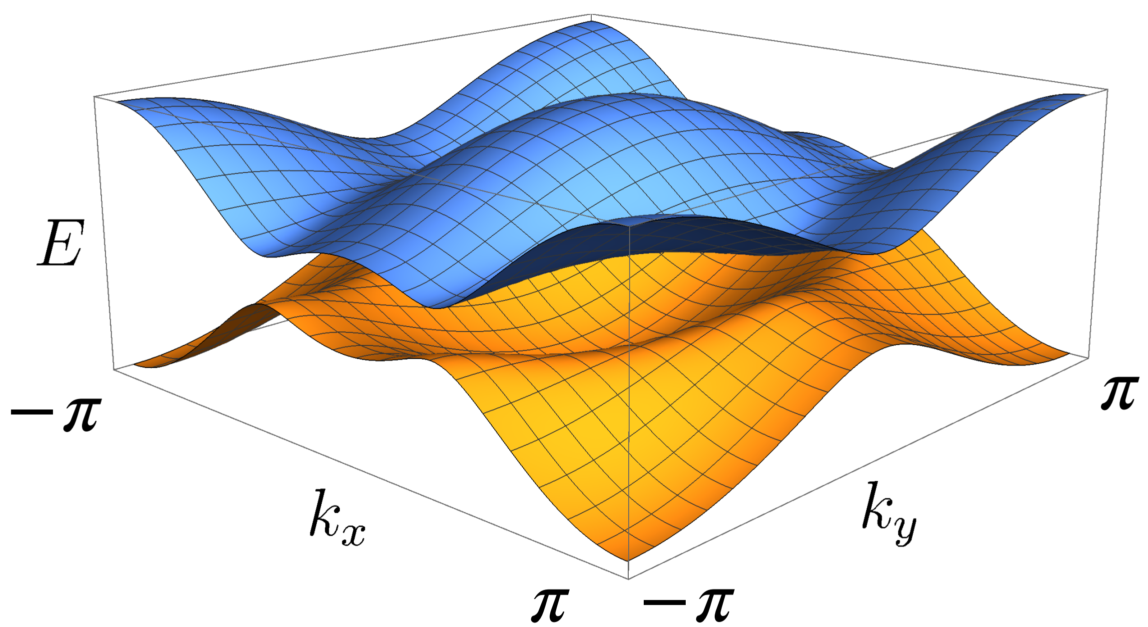
No symmetry protection: Chern Insulator (2D)

<u>2D Bloch Bands</u>

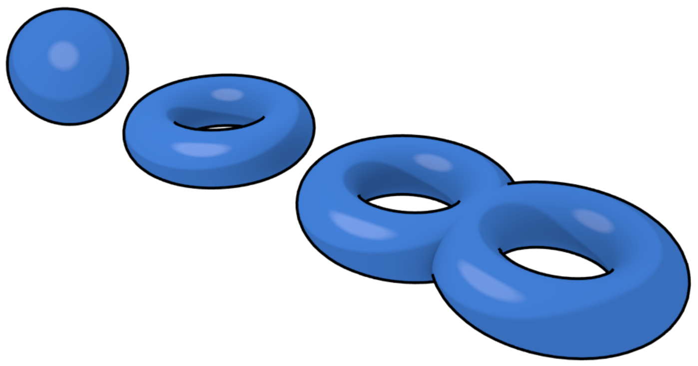
[Thouless, Kohmoto, Nightingale & den Nijs, PRL 1982]

Topological invariant: Chern number $C = \frac{1}{2\pi}\int_{\mathrm{BZ}} \mathrm{d}^2k\ \Omega(\boldsymbol{k}) \in \mathbb{Z}$



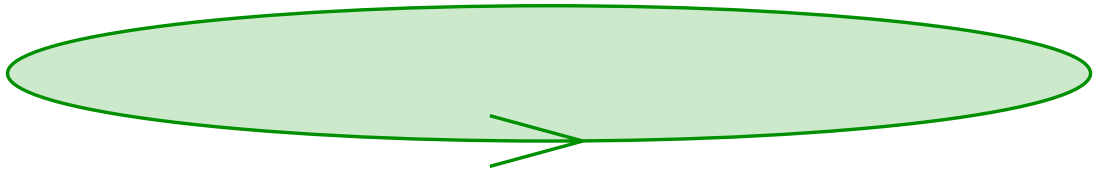
Berry curvature:

$$\Omega(\boldsymbol{k}) = -\,\mathrm{i}\nabla_{\boldsymbol{k}} \times \langle u(\boldsymbol{k})\,|\,\nabla_{\boldsymbol{k}} u(\boldsymbol{k})\rangle \cdot \hat{\boldsymbol{z}}$$

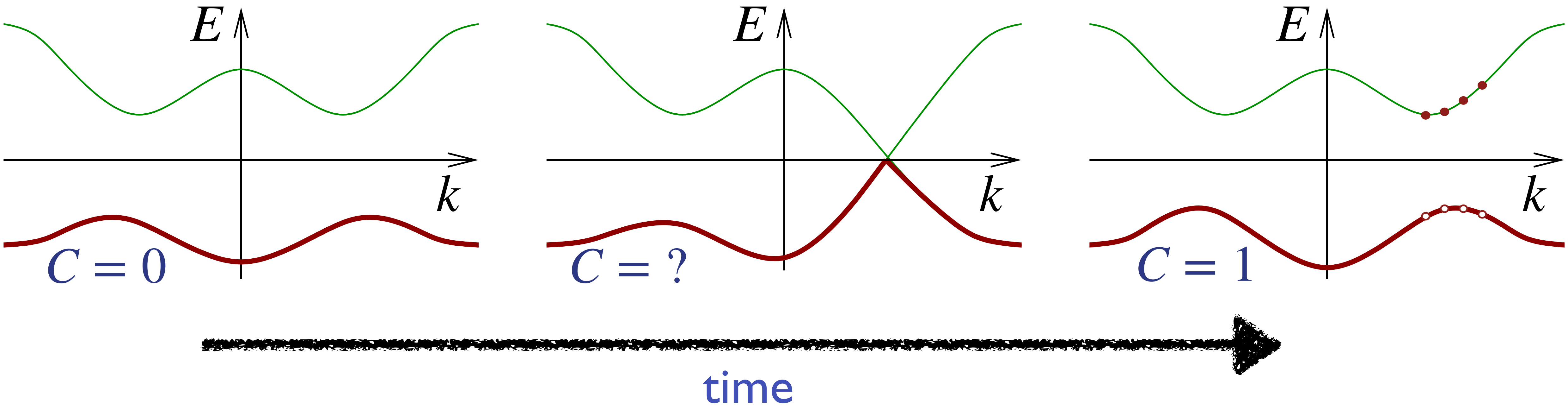


- C cannot change under smooth deformations

- Insulating bulk with C gapless edge states



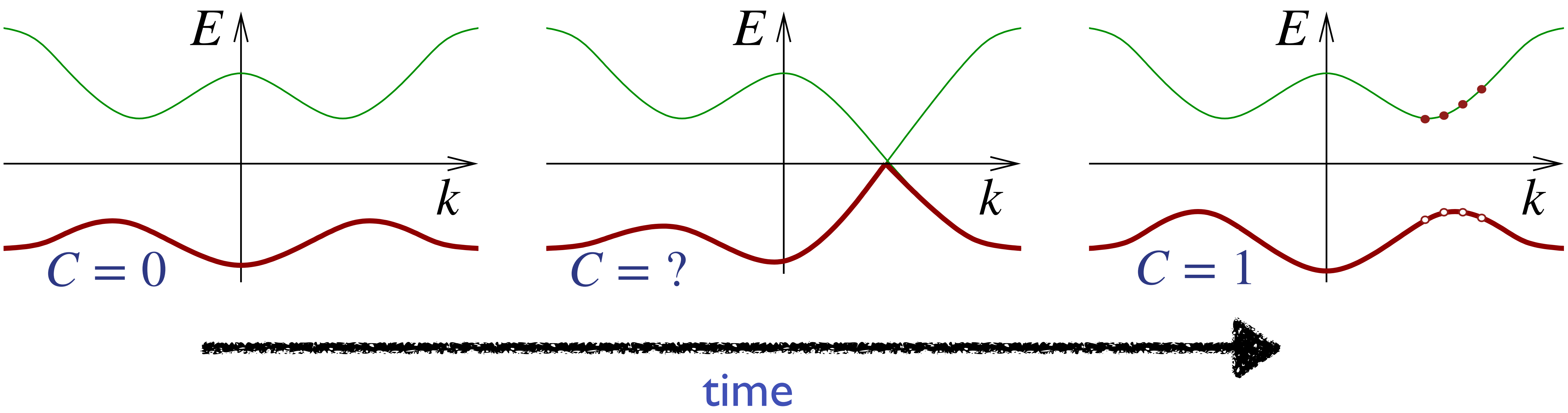
Non-Equilibrium Dynamics of Chern Insulator (2D)



What are the consequences for the topology of the system?

[Here no interactions, no disorder; results are more general]

Non-Equilibrium Dynamics of Chern Insulator (2D)



Quench: start in ground state of H^{i} then time evolve under H^{f}

Time-evolving Bloch state of particle at $\boldsymbol{k}$ $\quad |u(\boldsymbol{k},t)\rangle = \exp[-\mathrm{i}H^{\mathrm{f}}(\boldsymbol{k})t]\,|u(\boldsymbol{k},0)\rangle$

$$\Omega(\boldsymbol{k},t) = -\,\mathrm{i}\nabla_{\boldsymbol{k}} \times \langle u(\boldsymbol{k},t)\,|\,\nabla_{\boldsymbol{k}} u(\boldsymbol{k},t)\rangle \cdot \hat{z}$$

$\Rightarrow$ Chern number of the many-body state is preserved

[Foster, Dzero, Gurarie & Yuzbashan, PRB 2013 & PRL 2014;
D'Alessio & Rigol, Nat. Commun. 2015; Caio, NRC & Bhaseen, PRL 2015]

Non-Equilibrium Dynamics of Chern Insulator (2D)

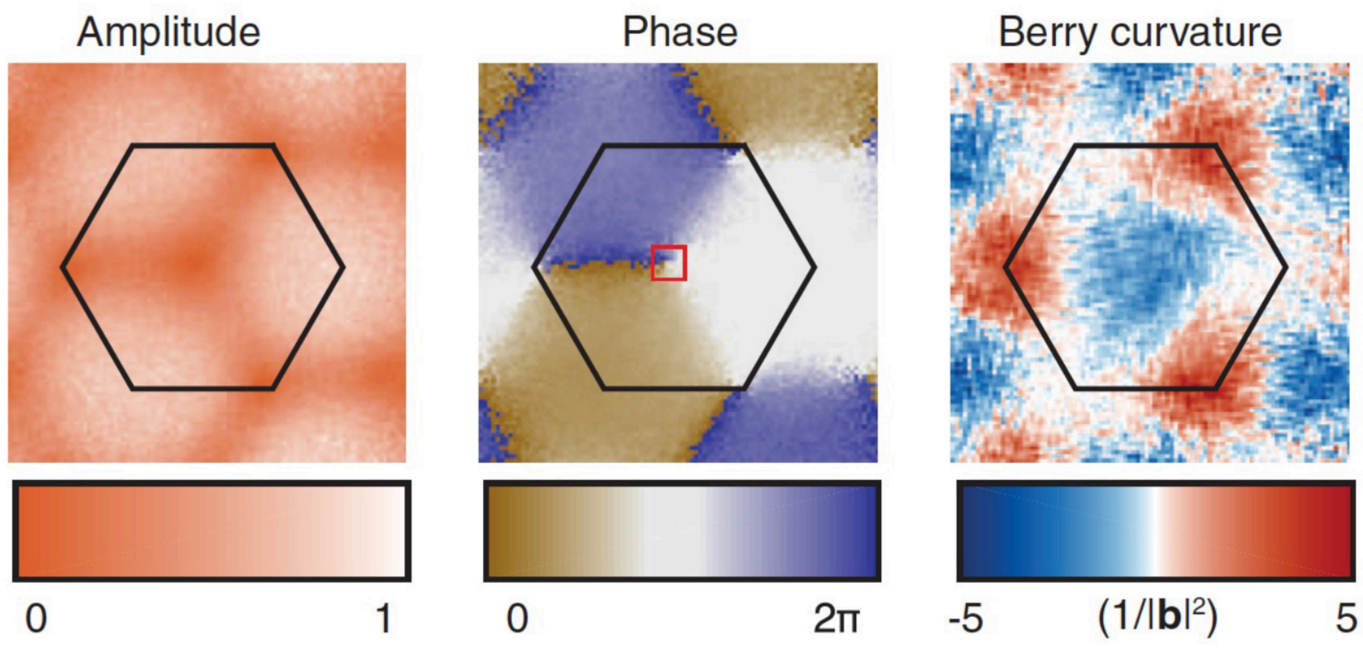
⇒ Obstruction to (fast) preparation of a state with differing Chern number

[for slow ramps, $\tau \gg L/v$, deviations can be small]

Direct experimental observation, by tomography of Bloch states

$$|u(\boldsymbol{k})\rangle \longrightarrow \begin{pmatrix} \cos[\theta(\boldsymbol{k})/2] \\ \sin[\theta(\boldsymbol{k})/2]e^{i\phi(\boldsymbol{k})} \end{pmatrix}$$

[Fläschner *et al.*, Science 2016]



Does the same hold for symmetry-protected topology?

Dynamics of Symmetry-Protected Topology

[Max McGinley & NRC, PRL 2018]

- Start in ground state of $\hat{\mathscr{H}}_{\rm i}$ then time evolve under $\hat{\mathscr{H}}_{\rm f}$

  Symmetry: $[\hat{\mathscr{O}}, \hat{\mathscr{H}}_{\rm i}] = 0$ $\Rightarrow$ some symmetry-protected topological invariant

- $\hat{\mathscr{H}}_{\rm f}$ breaks symmetry $\Rightarrow$ topological invariant lost

- What if $\hat{\mathscr{H}}_{\rm f}$ respects the symmetry? $[\hat{\mathscr{O}}, \hat{\mathscr{H}}_{\rm f}] = 0$

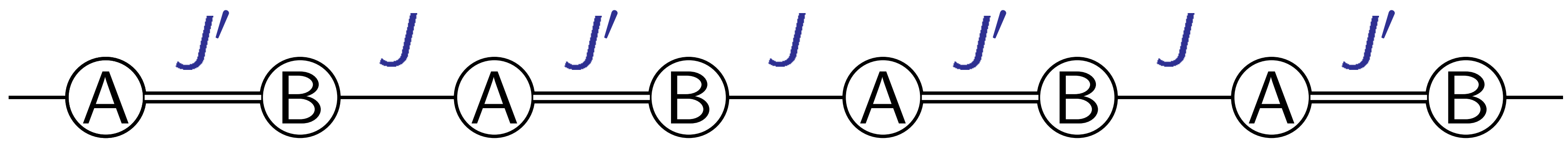
  Anti-unitary symmetries [$\hat{\mathscr{O}}{\rm i}\hat{\mathscr{O}}^{-1} = -\,{\rm i}$]

$$\hat{\mathscr{O}}{\rm e}^{-{\rm i}\hat{\mathscr{H}}_{\rm f}t}\hat{\mathscr{O}}^{-1} = {\rm e}^{+{\rm i}\hat{\mathscr{H}}_{\rm f}t}$$

Symmetry is lost in the time-evolved state

Topological "invariant" time-varying even if symmetries respected

Symmetry-Protected Topology: SSH Model



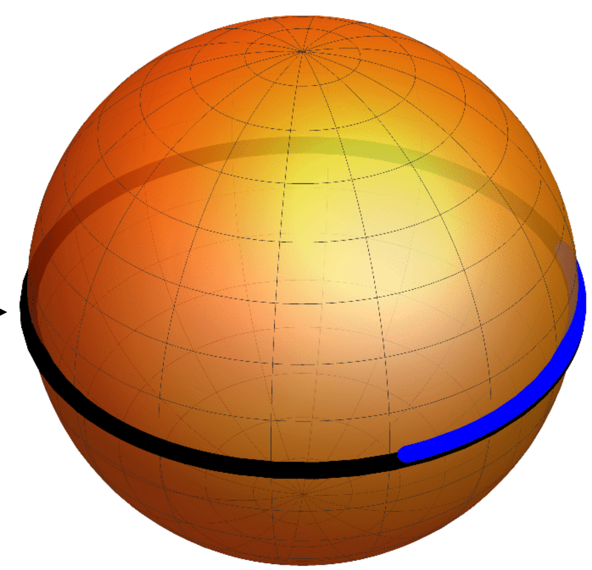
sublattice symmetry $\Rightarrow$ $H(k) = -\begin{pmatrix} 0 & J' + Je^{-ik} \\ J' + Je^{ik} & 0 \end{pmatrix}$ $\Rightarrow$ $|u(k)\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ e^{i\phi(k)} \end{pmatrix}$

$[\theta(k) = \pi/2]$

$J > J'$

topological

(gapless surface state)



$J < J'$

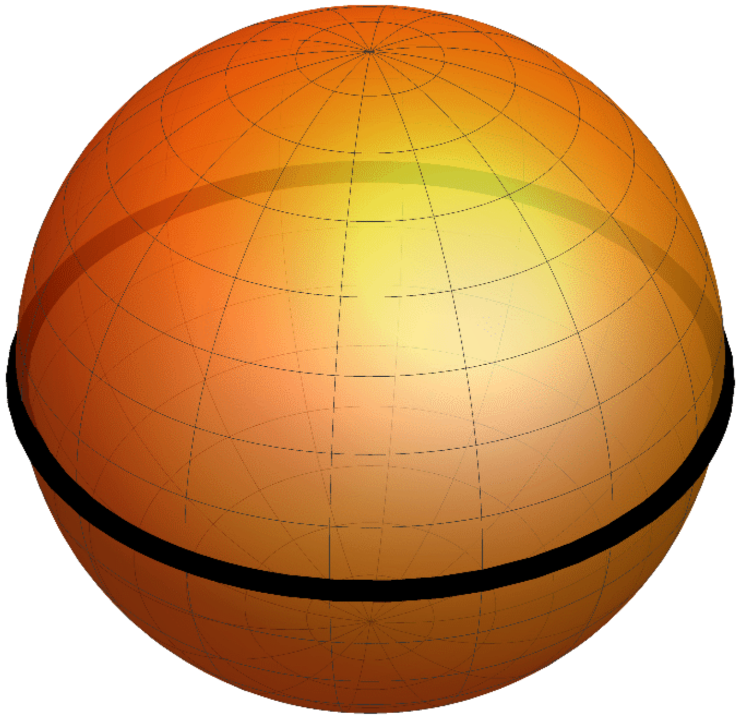
trivial

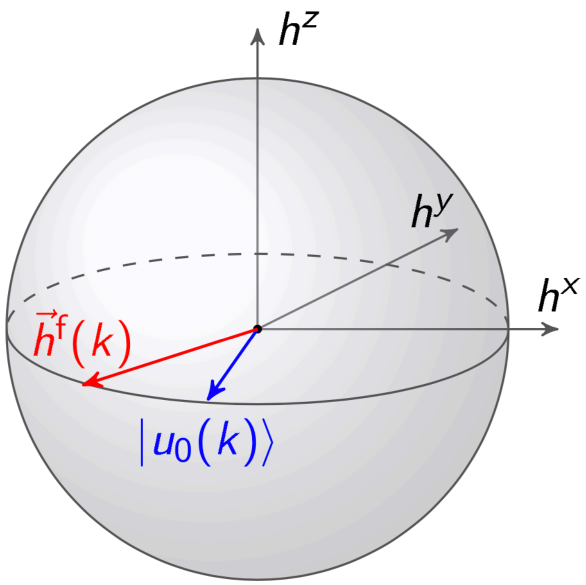
(no surface state)

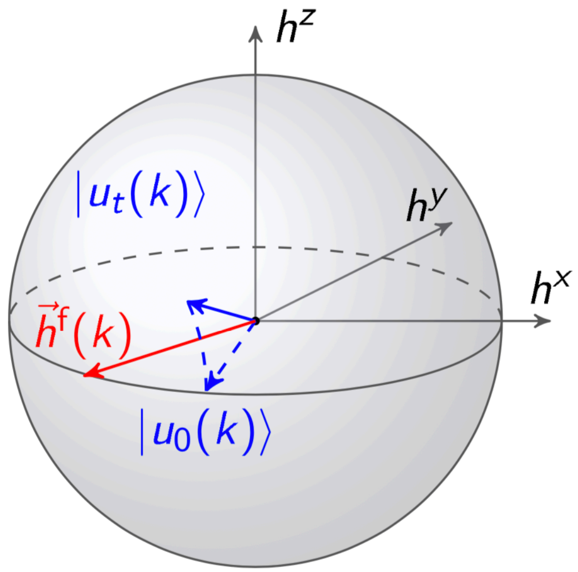
Symmetry-protected topological invariant:

$$\Phi_{\text{Zak}} = i \int \langle u(k) | \partial_k u(k) \rangle \, dk = \pi \times \text{winding number}$$

Example: Su-Schrieffer-Heeger Model







Topological "invariant" Φ_{Zak} time-varying even though Hamiltonian always respects symmetry

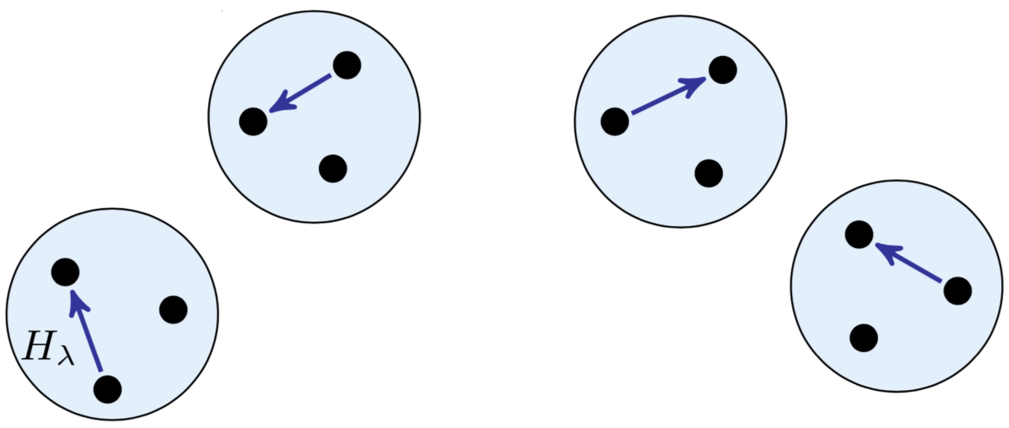
time-variations appear as a current: $I(t) = \frac{1}{2\pi}\frac{\mathrm{d}\Phi_{\mathrm{Zak}}}{\mathrm{d}t}$

Is there a topological classification of non-equilibrium systems?

Non-Equilibrium Topological Classification

<u>Equilibrium</u>:
smoothly connected under a gapped symmetric Hamiltonian with

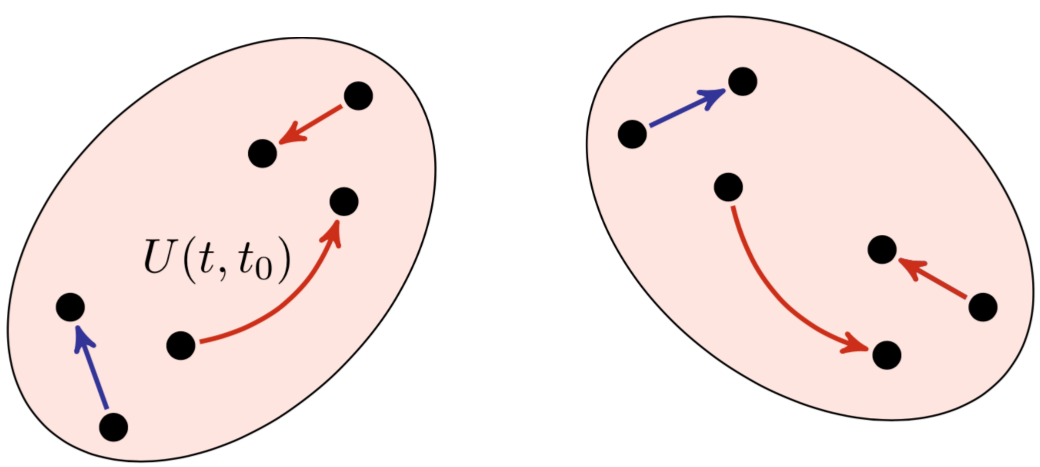
$$|\Psi_1\rangle \xrightarrow{H_\lambda} |\Psi_2\rangle$$



<u>Non-equilibrium</u>:
connected by (finite) time evolution governed by a symmetric Hamiltonian

[Max McGinley & NRC, PRB 2019; PRR 2019]

$$|\Psi_1\rangle \xrightarrow{e^{-\mathrm{i}Ht}} |\Psi_2\rangle$$



Non-Equilibrium Topological Classification

- Free-fermion topological insulators / superconductors

  [Altland-Zirnbauer: Time-reversal, charge-conjugation & sublattice symmetries]

[Max McGinley & NRC, PRB 2019]

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C	0	−	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
CI	+	−	1	0	0	0	$2\mathbb{Z} \to 0$	0	$\mathbb{Z}_2 \to 0$	$\mathbb{Z}_2 \to 0$	$\mathbb{Z} \to 0$

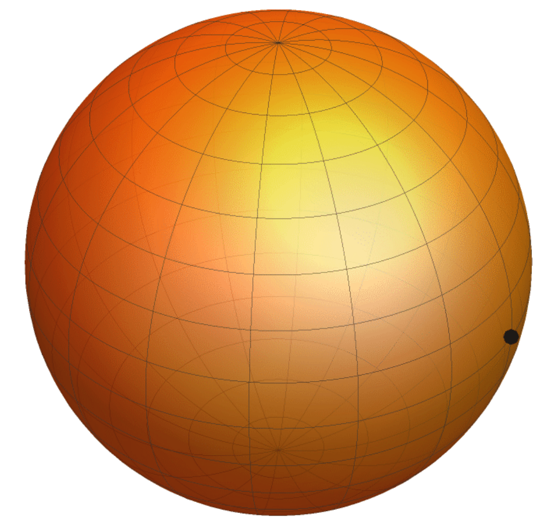
- Interacting symmetry-protected topological phases [Max McGinley & NRC, PRR 2019]

Non-Equilibrium Topological Classification: Physical Consequences

[Max McGinley & NRC, PRB 2019; PRR 2019]

1) Preparation of topological states

Determines which states can be quickly interconverted dynamically by symmetry-respecting Hamiltonians

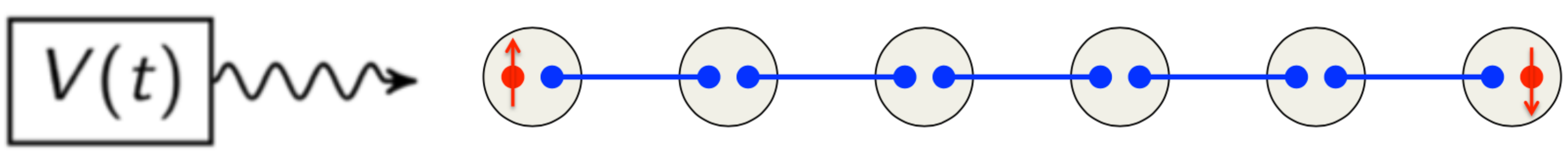


e.g. Su-Schrieffer-Heeger model (class BDI in 1D): $\mathbb{Z} \rightarrow \mathbb{Z}_2$

Topological "invariant" can be changed by even integers

2) Stability of "topologically protected" surface states

External symmetry-respecting noise



[Image: Wierschem & Sengupta, Mod Phys Lett B (2015)]

Determines which symmetry-protected topological quantum registers decohere

TRS broken by time-history / preparation sequence of non-equilibrium state

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— dynamical preparation of topological phases? (cold gases)

2) Open quantum systems

— effects of environmental couplings on topological edge modes

Open Quantum Systems

Arrow of Time

Can emerge from time-reversal symmetric microscopic laws

$$\Delta S > 0$$

Quantum Approach: System + Environment

Reduced density matrix

$$\rho_S = \mathrm{Tr}_E[\rho_{\mathrm{total}}]$$

typically shows increasing entropy (due to growth of entanglement)

Open Quantum Systems: Symmetries

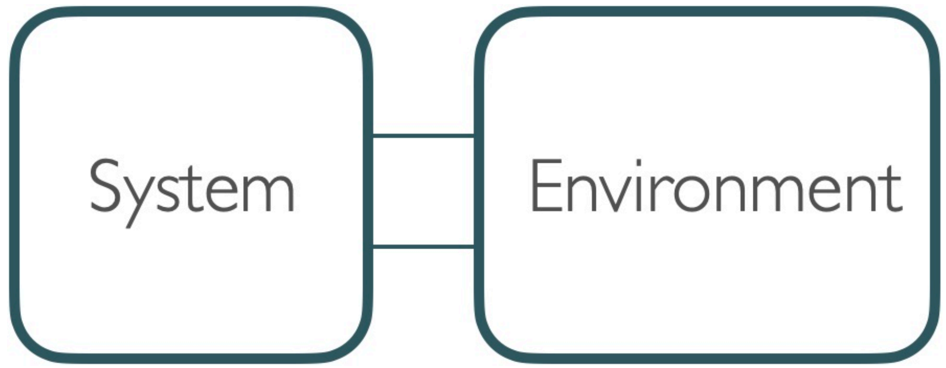
[Max McGinley & NRC, Nature Physics **16**, 1181 (2020)]

$$\hat{\mathscr{H}} = \hat{\mathscr{H}}_S + \hat{\mathscr{H}}_E + \hat{\mathscr{H}}_{SE}$$

Symmetric system

$$[\hat{\mathscr{O}}, \hat{\mathscr{H}}_S] = 0$$

with symmetry protected features



Symmetric environment and coupling

$$[\hat{\mathscr{O}}, \hat{\mathscr{H}}] = 0$$

$$\hat{\mathscr{H}}_{SE} = \sum_\alpha \hat{A}_\alpha \otimes \hat{B}_\alpha \qquad (\hat{A}_\alpha = \hat{A}_\alpha^\dagger \,, \hat{B}_\alpha = \hat{B}_\alpha^\dagger)$$

Impose *strongest possible* symmetry condition: $[\hat{\mathscr{O}}, \hat{A}_\alpha] = [\hat{\mathscr{O}}, \hat{B}_\alpha] = 0$

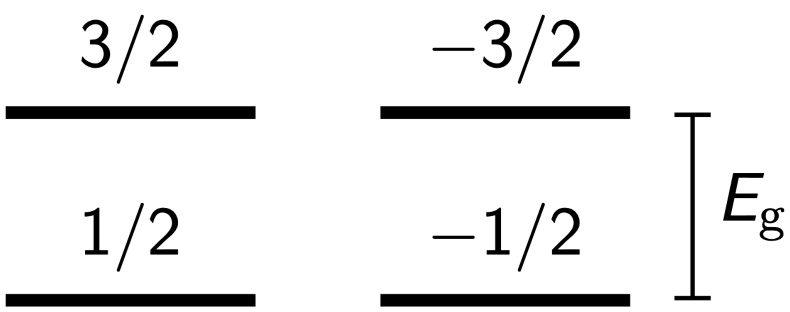
e.g. charge conservation: $[\hat{\mathscr{O}}, \hat{A}_\alpha \otimes \hat{B}_\alpha] = 0$ $\Rightarrow$ charge consv. of S+E

$[\hat{\mathscr{O}}, \hat{A}_\alpha] = 0$ $\Rightarrow$ charge consv. of S

TRS: $[\hat{\mathscr{O}}, \hat{A}_\alpha] = [\hat{\mathscr{O}}, \hat{B}_\alpha] = 0$ $\Rightarrow$ both $\hat{A}_\alpha$ and $\hat{B}_\alpha$ TR-even

Simple Example (No topology!)

Spin-3/2 $\hat{\mathscr{H}}_S = E_g \hat{S}_z^2/2$



Two-fold degenerate ground state preserved if symmetry maintained

— e.g. TRS (Kramers degeneracy) $\hat{\mathscr{O}} = e^{-i\pi \hat{S}_y} \hat{K}$ (antiunitary)

— or dihedral group formed by $\hat{\mathscr{O}}_{x,y,z} = e^{i\pi \hat{S}_{x,y,z}}$ (unitary)

⇒ “qubit” encoded in ground states remains coherent

$$|\psi\rangle_S = \alpha|1/2\rangle + \beta|-1/2\rangle$$

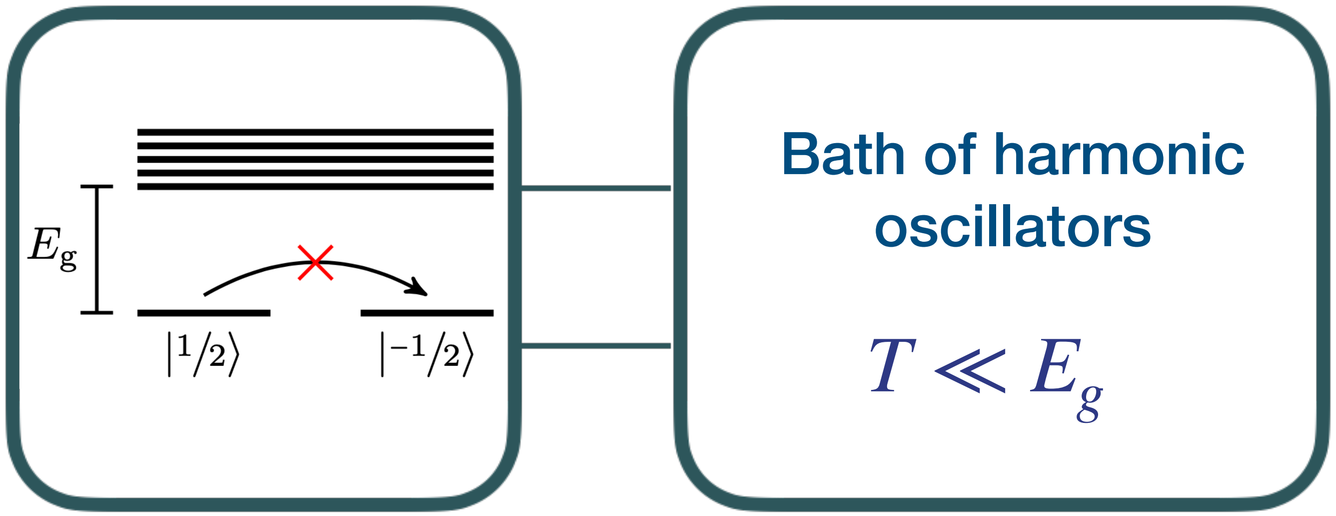
Environment couples only via symmetry-preserving $\hat{A}_\alpha$ $[\hat{\mathscr{O}}, \hat{A}_\alpha] = 0$

Can this environment cause decoherence?

Simple Example: Results

$$\hat{\mathscr{H}} = \hat{\mathscr{H}}_S + \sum_{\alpha=1}^{M} \hat{A}_\alpha \otimes \hat{B}_\alpha + \hat{\mathscr{H}}_E$$

Direct transition forbidden to lowest order in $V = \|\hat{\mathscr{H}}_{SE}\|$ by strong symmetry $[\hat{\mathscr{O}}, \hat{A}_\alpha] = 0$

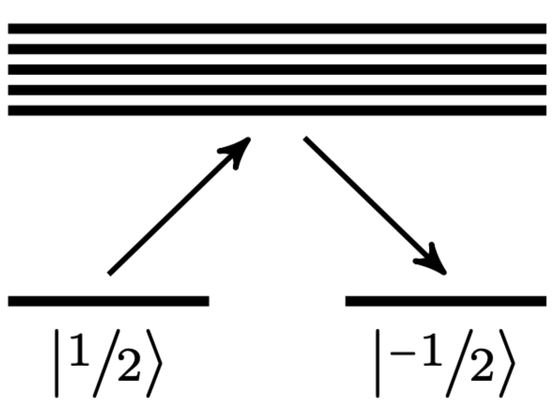


[Caldeira & Leggett, PRL 1981]

Unitary symmetry: direct transition vanishes to all orders in $V = \|\hat{\mathscr{H}}_{SE}\|$

$\Rightarrow$ decoherence rate $\sim e^{-E_g/T}$

Antiunitary symmetry: transition at second order in V



$\sim \dfrac{V^2}{E_g}$

$\Rightarrow$ decoherence rate $\sim \dfrac{V^4 T^{2s+3}}{E_g^4 \omega_c^{2s+2}}$

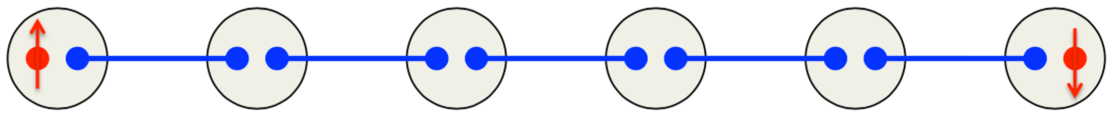
[spectral density $J_{\alpha\beta}(\omega) \sim V^2 \omega^s \omega_c^{-s-1} e^{-\omega/\omega_c}$]

TRS cannot be "factorized" between system and environment

Open Quantum Systems: Physical Consequences

Decoherence of Kramers degenerate pairs or zero modes

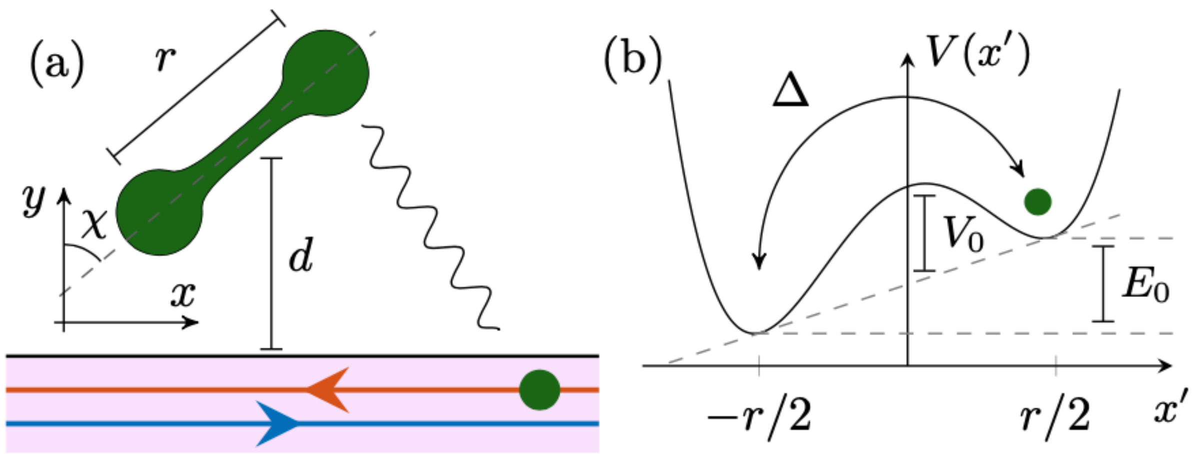
e.g. end spin-1/2 modes of Haldane phase stabilised by time-reversal symmetry



[Image: Wierschem & Sengupta, Mod Phys Lett B (2015)]

Backscattering of helical edge states

[Max McGinley & NRC, PRB **103**, 235164 (2021)]



Coulomb interaction with 2-level system

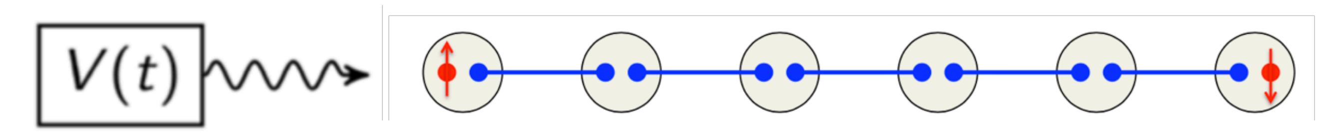
$$\hat{H}_{\text{int}} = \int d\mathbf{r}\, \hat{\rho}_{\text{el}}(\mathbf{r}) \otimes \left[\hat{\sigma}_x V_x(\mathbf{r}) + \hat{\sigma}_z V_z(\mathbf{r})\right]$$

⇒ Non-quantized conductance down to low T without magnetism/exchange

Summary: Fragility of TRS-protected phenomena

1) Non-equilibrium dynamics [Max McGinley & NRC, PRL **121**, 090401 (2018)]

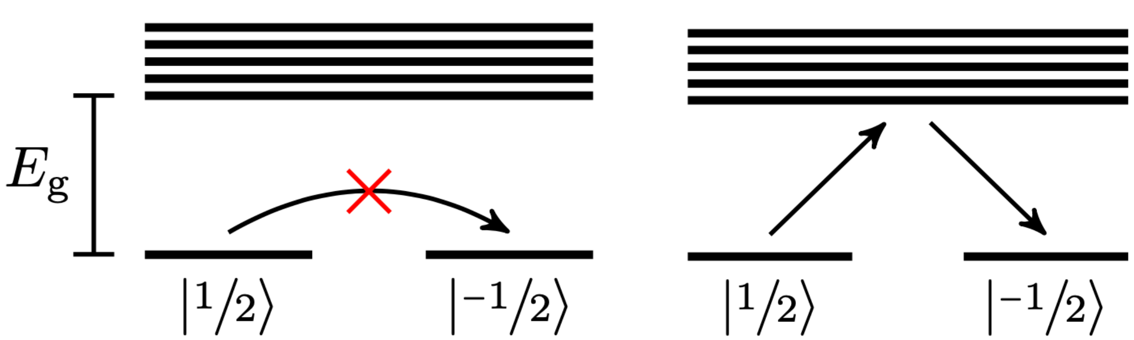
- Fast interconversion of (certain) symmetry-protected topological states by symmetry-respecting Hamiltonians
- Sensitivity of topologically protected boundary modes to external noise



TRS broken by time-history / preparation sequence of non-equilibrium state

2) Open quantum systems [Max McGinley & NRC, Nature Physics **16**, 1181 (2020)]

- Decoherence of topological protected boundary modes



Unitarity symmetry: $\Gamma \sim e^{-E_g/T}$

Antiunitarity symmetry: $\Gamma \sim V^4 T^{2s+3}$

TRS cannot be factorized between system and environment