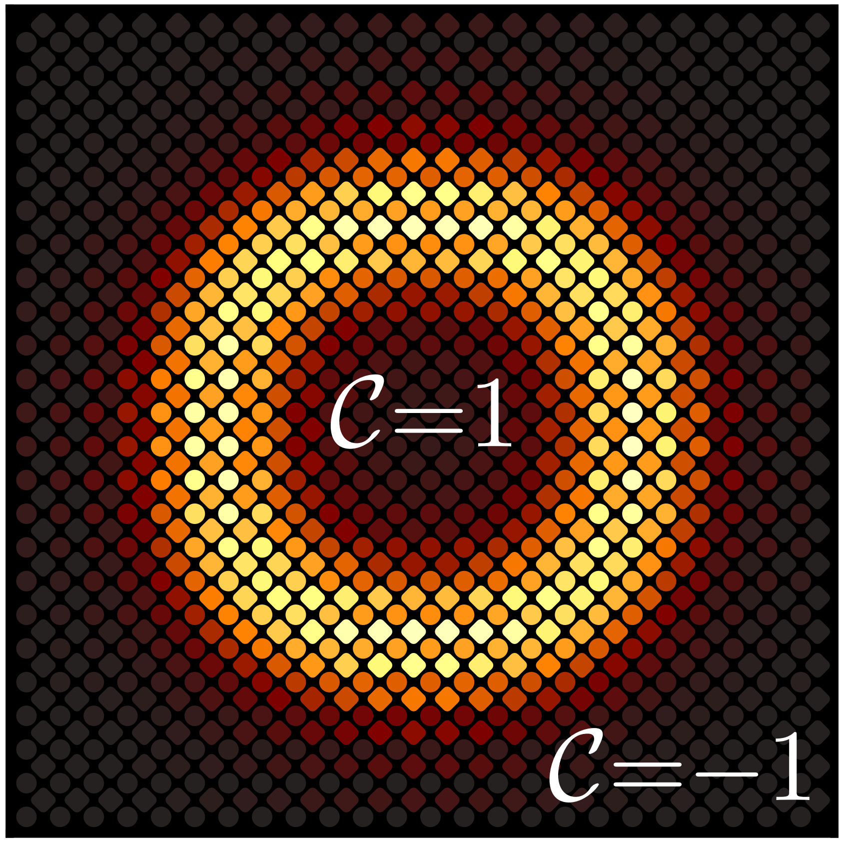
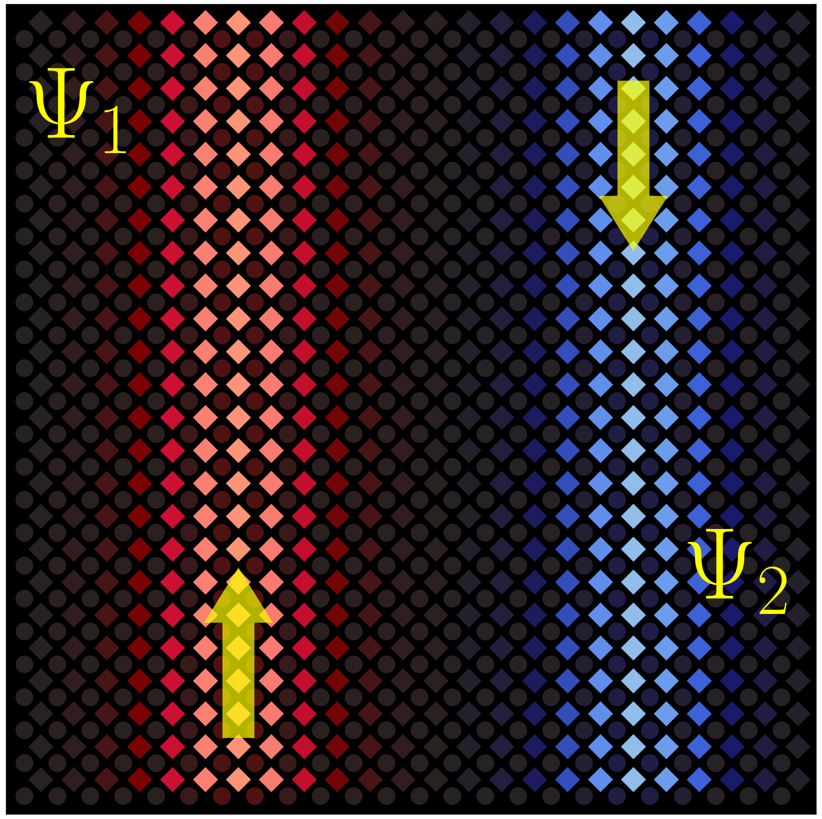
Self-Trapped Polarons and Topological Defects in a Topological Mott Insulator

Alexandre Dauphin

KITP Program: Interacting Topological Matter: Atomic, Molecular and Optical Systems

19 July 2021





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I am currently working as a la Caixa Junior Leader fellow at ICFO (Barcelona, Spain).

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PHYSICAL REVIEW LETTERS **125,** 240601 (2020)

Editors' Suggestion

Self-Trapped Polarons and Topological Defects in a Topological Mott Insulator

Sergi Julià-Farré,[1,*] Markus Müller,[2,3] Maciej Lewenstein,[1,4] and Alexandre Dauphin[1,†]

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Maciej Lewenstein
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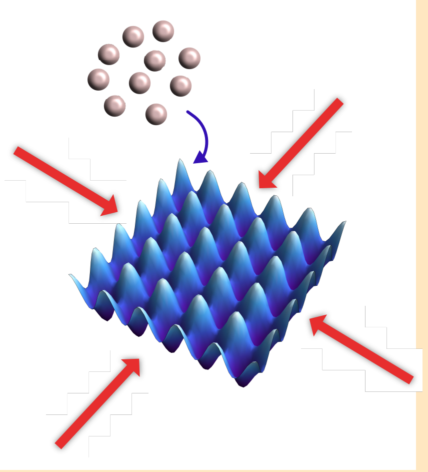
Markus Müller
Aachen /Jülich
(Germany)

Outline

Outline

1. Introduction



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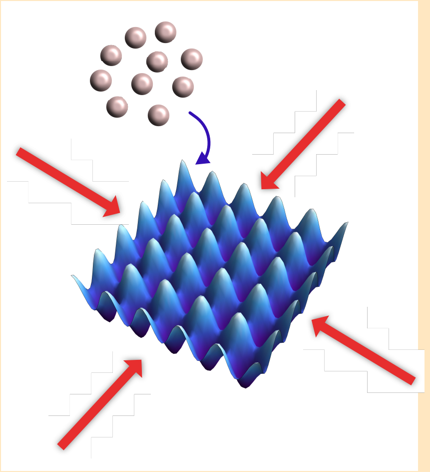
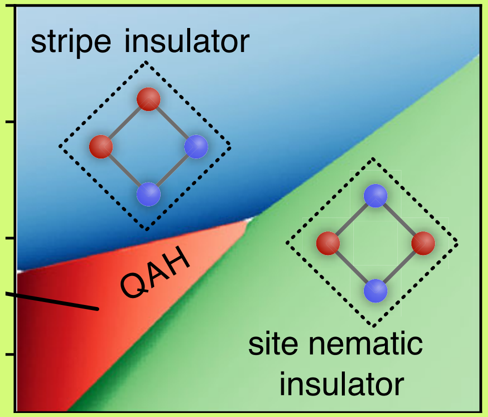


2. Topological Mott insulator in the checkerboard lattice



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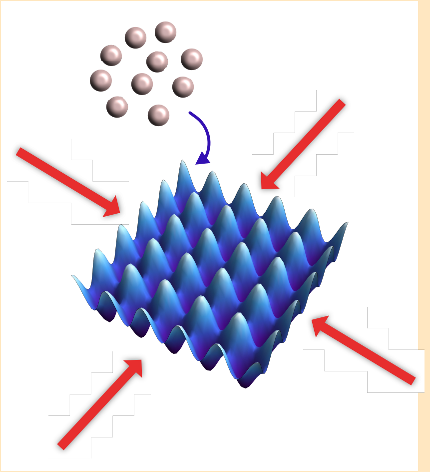

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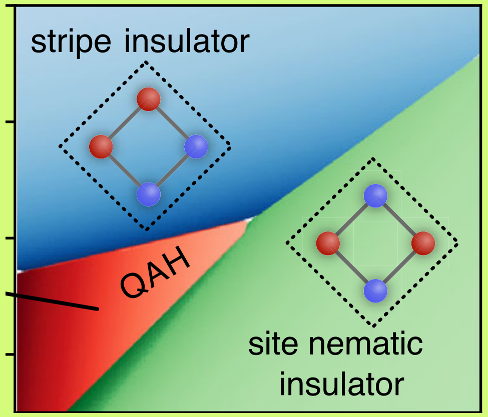
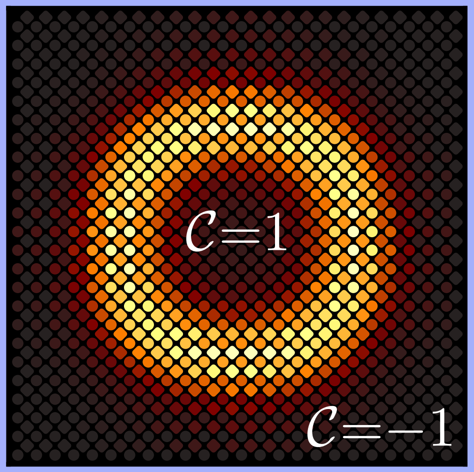


3. Beyond half filling



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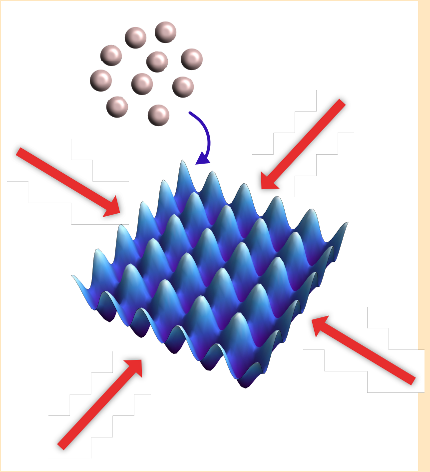

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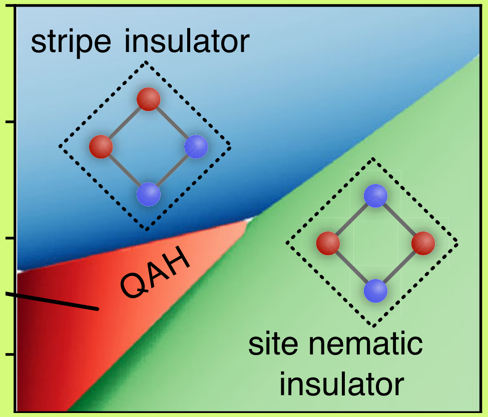

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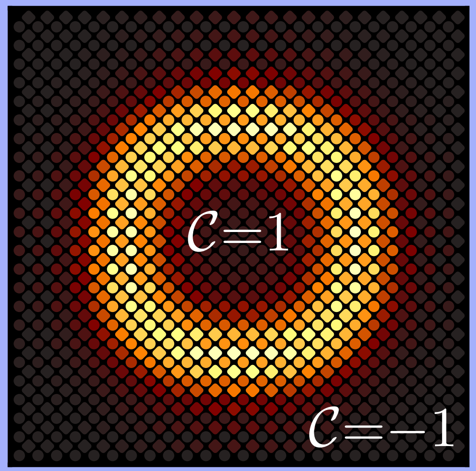
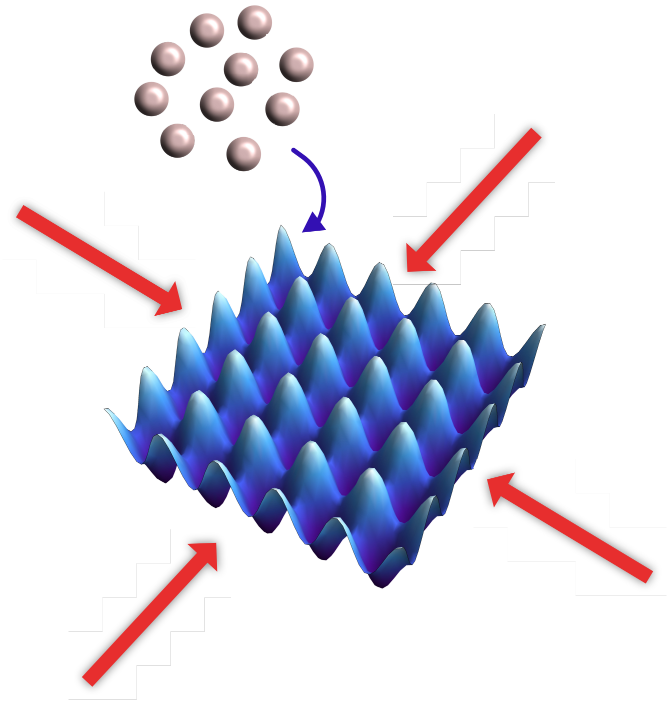


4. Conclusions and Outlook

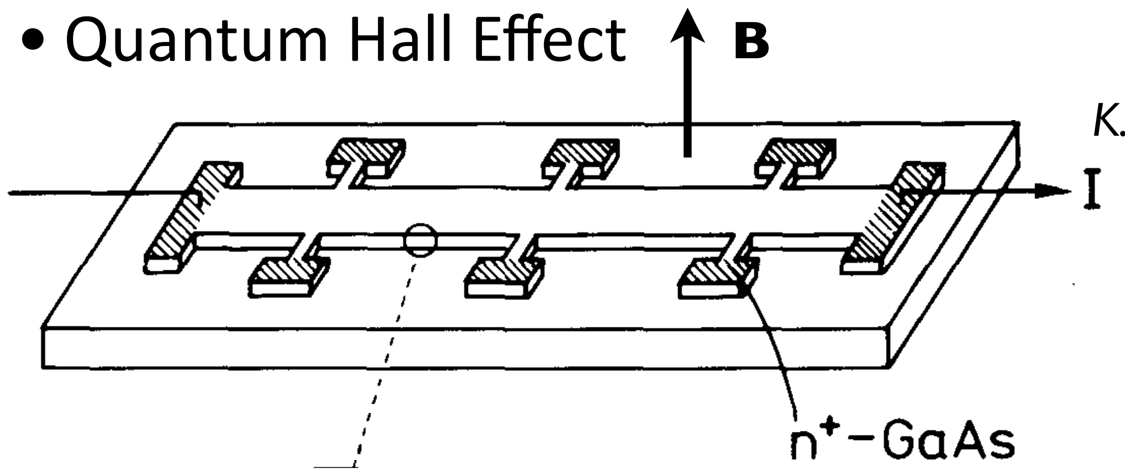
1. Introduction



Topological Insulators

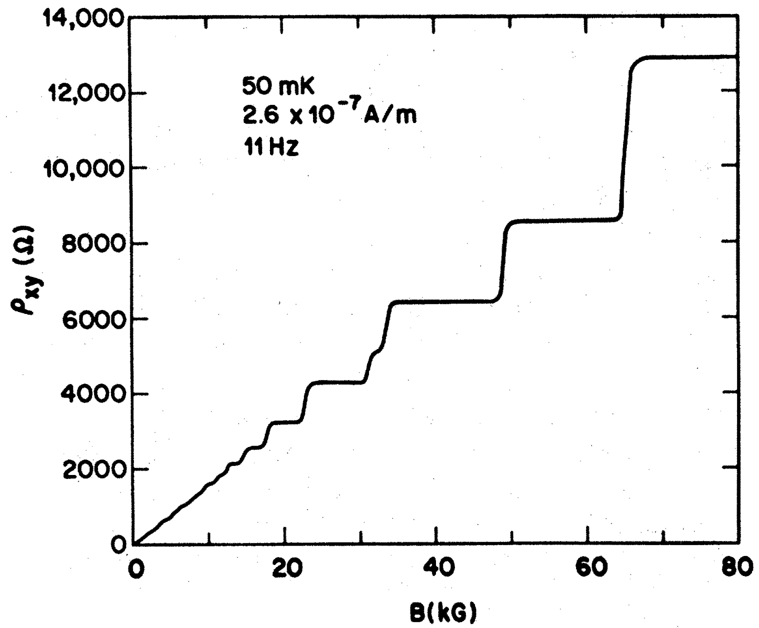
1980

- Quantum Hall Effect



K.Von Klitzing, Nobel lecture

M. A. Paalanen et al., Phys. Rev. B 25 5566 (1982)



- *Magnetic field ⟶ Time reversal symmetry breaking*
- *Quantized conductivity*
- *Robust conductivity against disorder and interactions*
- *TKNN formula ⟶ Link with the topological invariant*
- *Characterized by a global topological property*

Thouless et al., Phys. Rev. Lett. 49, 405(1982).

2005

- Topological insulators

- *Time reversal symmetry conserved*
- *Bulk insulators ⟶ gap in the energy spectrum*
- *Non trivial topological invariant ⟶ quantized conductivity*
- *In geometry with border, conducting edge states protected by the topology.*

Realization of Topological Insulators : Solid State Physics

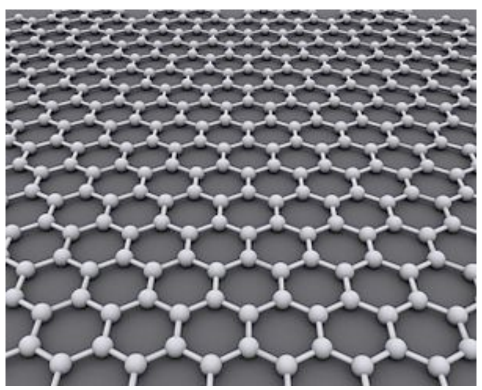
2005

- 2D topological insulators

Quantum Spin Hall effect

Theoretically proposed in graphene:
C. L. Kane and E. J. Mele,
Phys. Rev. Lett. 95, 226801 (2005)

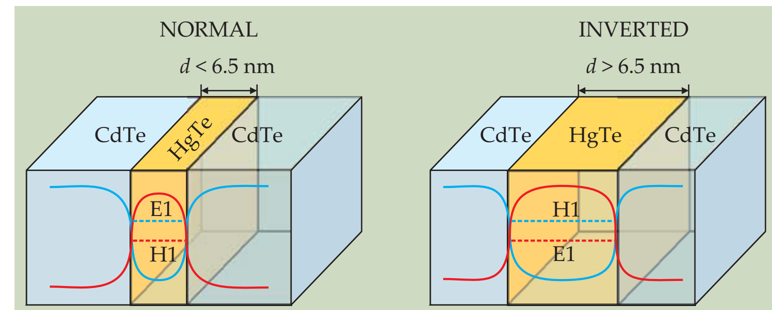
Conjecture: spin-orbit coupling converts graphene into a Q. spin Hall insulator



2006

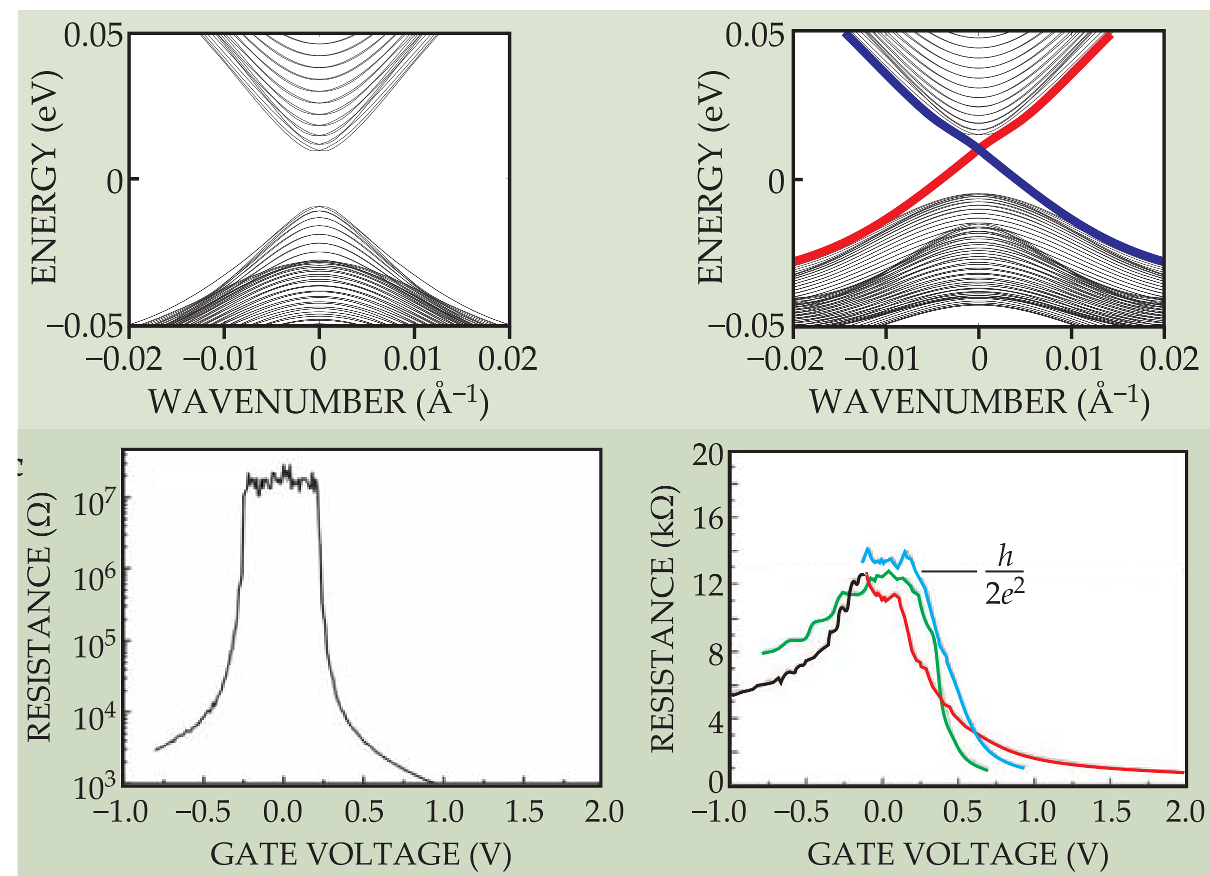
... effect turned out to be too small.

Theoretical proposal in CdTe/HgTe nanowell structure
B. A. Bernevig, T. A. Hughes, and S. C. Zhang,
Science 314, 1757 (2006)



Experiment
M. König, et al.,
Science 318, 766 (2007)

Insulator **Top. Insulator**



Qi and Zhang, Physics Today 63, 33 (2010).

2007

- 3D topological insulators

Natural generalization of the Quantum spin Hall effect in 3D.

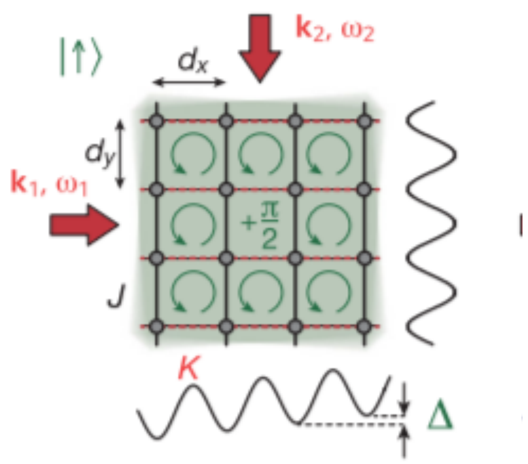
Theoretically predicted in real materials :

Fu and Kane, Phys. Rev. B 76, 045302 (2007).

1st experiment reported:

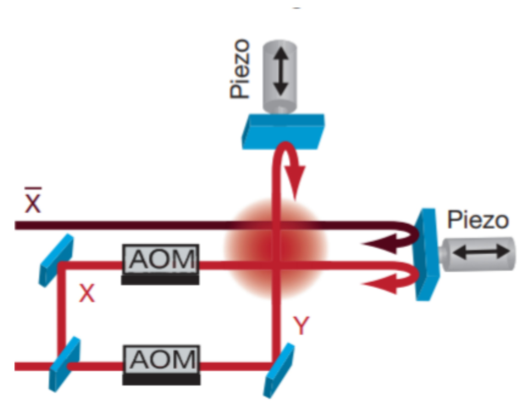
Hsieh et al., Nature 452, 970 (2008)

Realization of Topological Insulators : Quantum Simulation



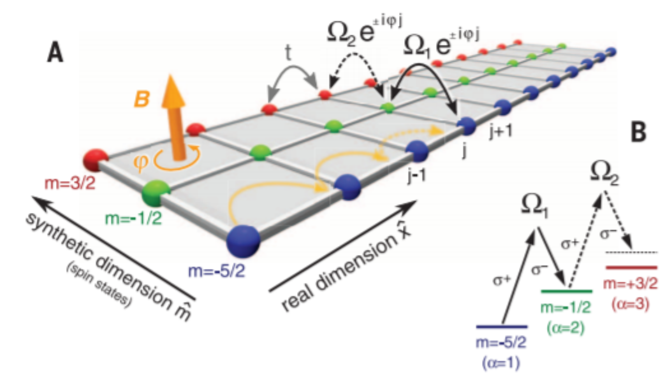
Laser Assisted tunneling

[M. Aidelsburger et al.,*Phys. Rev. Lett.* 2013]



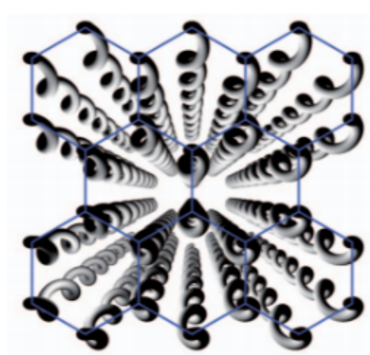
lattice shaking

[G. Jotzu et al., *Nature* (2014)]



Synthetic dimensions

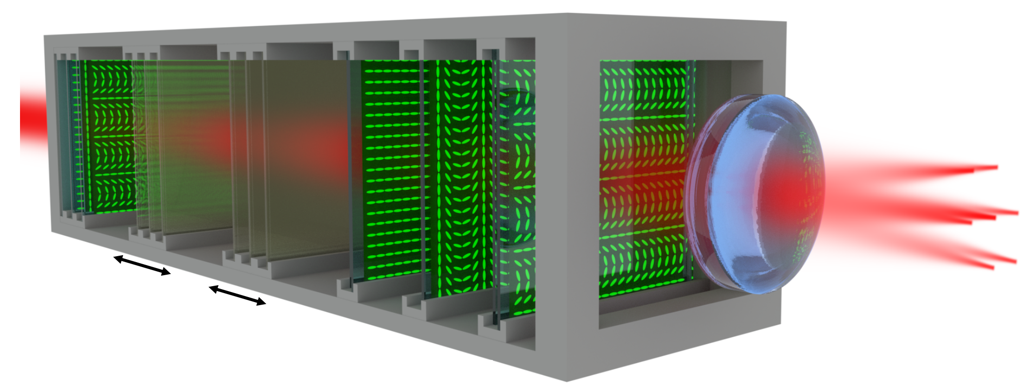
[Mancini et al., *Science* (2015)]



M.C.Rechtsman et al.,
Nature (2013)

Array of wave guides

[M.C.Rechtsman et al., Nature (2013)]



Quantum walk of twisted photons

[A D'errico et al., Optica (2020)]

Classification and Beyond

- Classification of topological insulators in terms of the discrete symmetries they break, the celebrated periodic table of topological insulators and superconductors.

Chiu et al., Rev. Mod. Phys. 88, 035005 (2017).

- Beyond the periodic table: crystalline topological insulators, Weyl semimetals, topological Mott insulator, Floquet topological insulators, HOTIs, etc.

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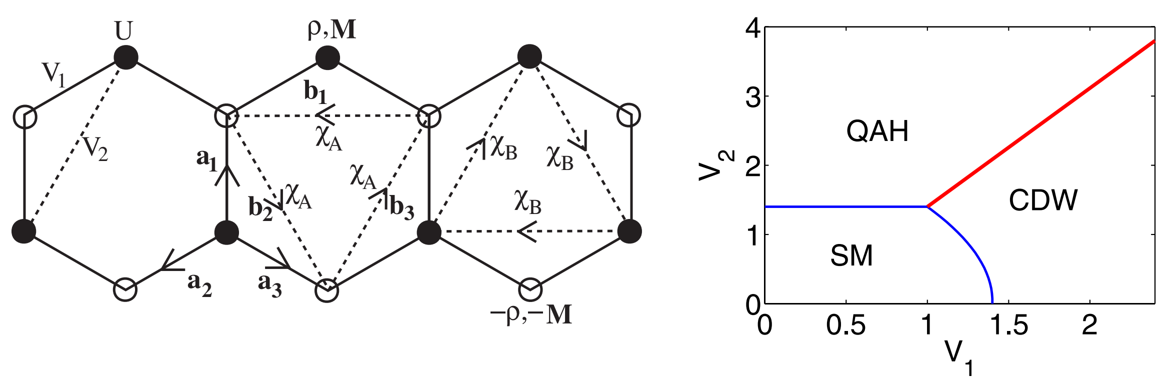
Interactions in Chern Insulators

- Interactions can destroy topological phases.
- They can induce a fractional QAH effect.

    T. Neupert et al. PRL 106, 236804 (2011)

- Even more, they can even induce the topology.

S. Raghu et al. ,PRL **100,** 156401 (2008)



Chern insulators due to spontaneous breaking of time-reversal symmetry driven by interactions

Review: S. Rachel, Rep. Prog. Phys. **81**, 116501 (2018)

Chern Insulators VS SSB Chern insulators

	Gauge field	**Spontaneous symmetry breaking**
Time reversal symmetry breaking?	✓	✓
Topology mechanism	Externally induced	From system interactions
Local order parameter?	✗	✓
Ground-state degeneracy?	✗	✓

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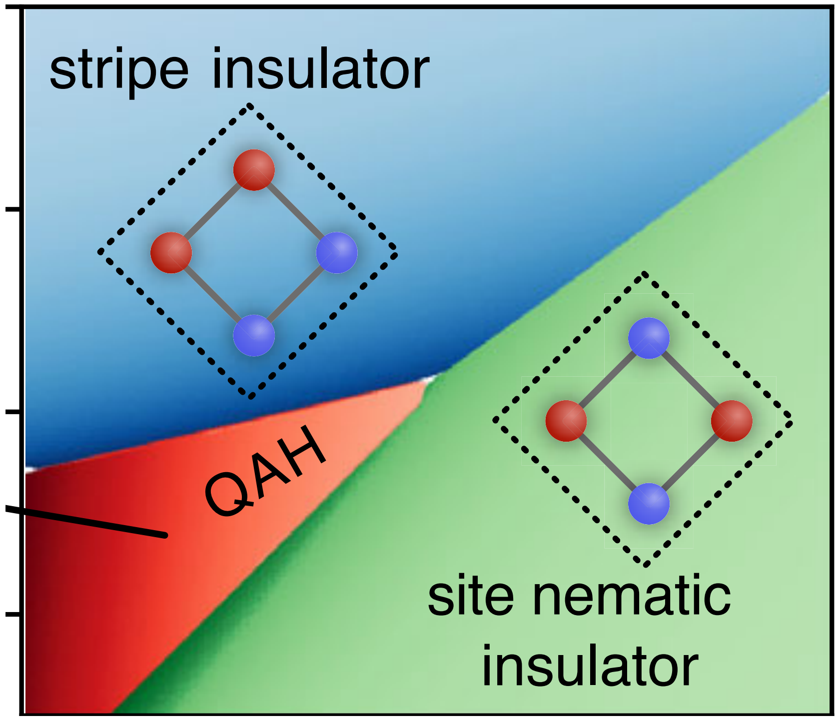


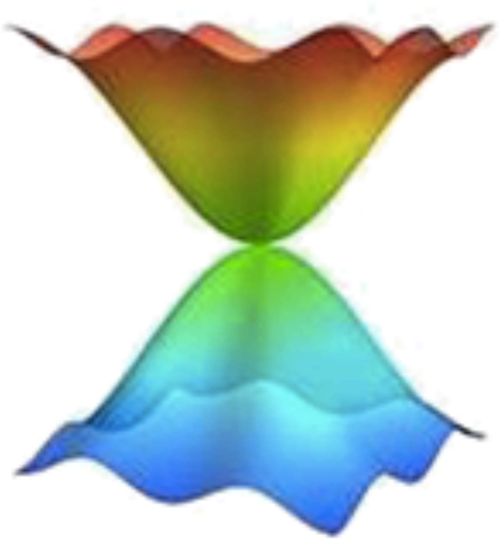
Topological Mott Insulator on the Checkerboard lattice

- Extended Fermi-Hubbard model of spinless fermions on the checkerboard lattice with a pi-flux.

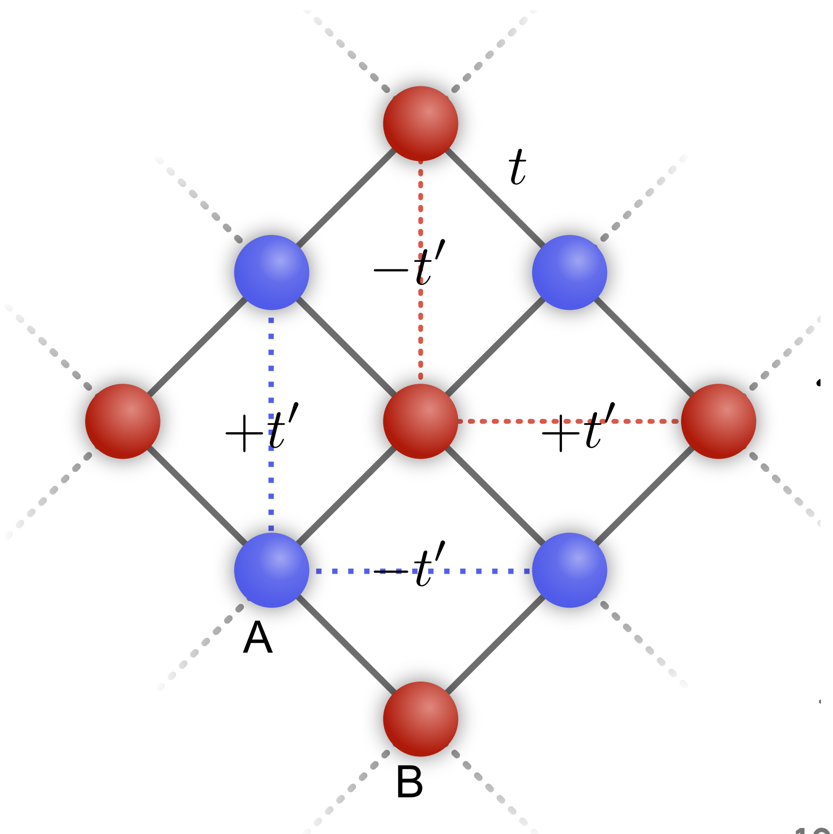
  K. Sun et al. PRL **103**, 046811 (2009)

$$\hat{H} = \sum_{ij} t_{ij} \hat{c}_i^\dagger \hat{c}_j$$

$E(\mathbf{k})$



Zeng et al, NPJ quant mat 2019



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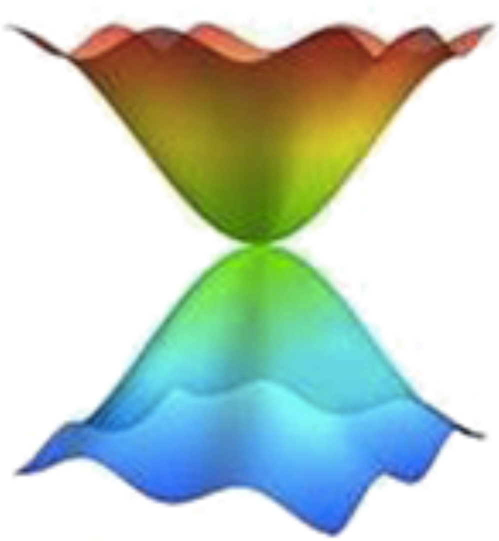

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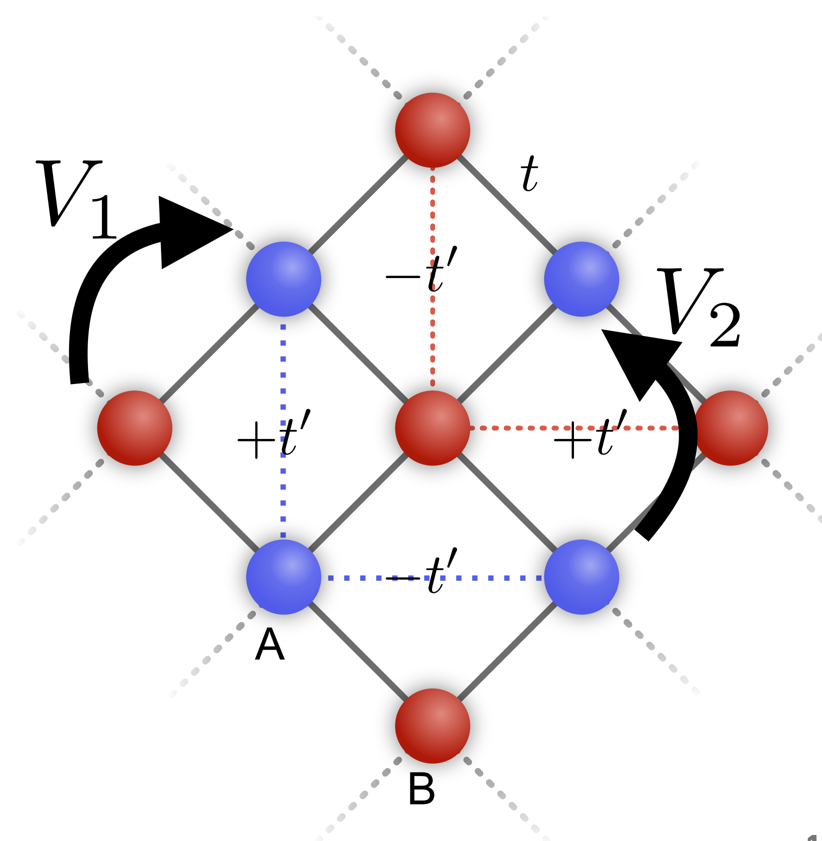

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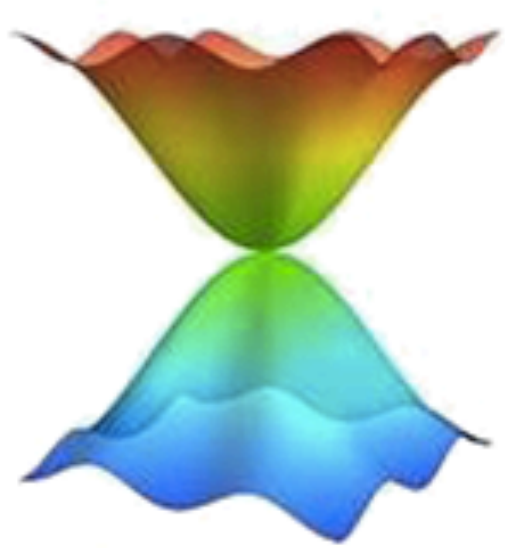

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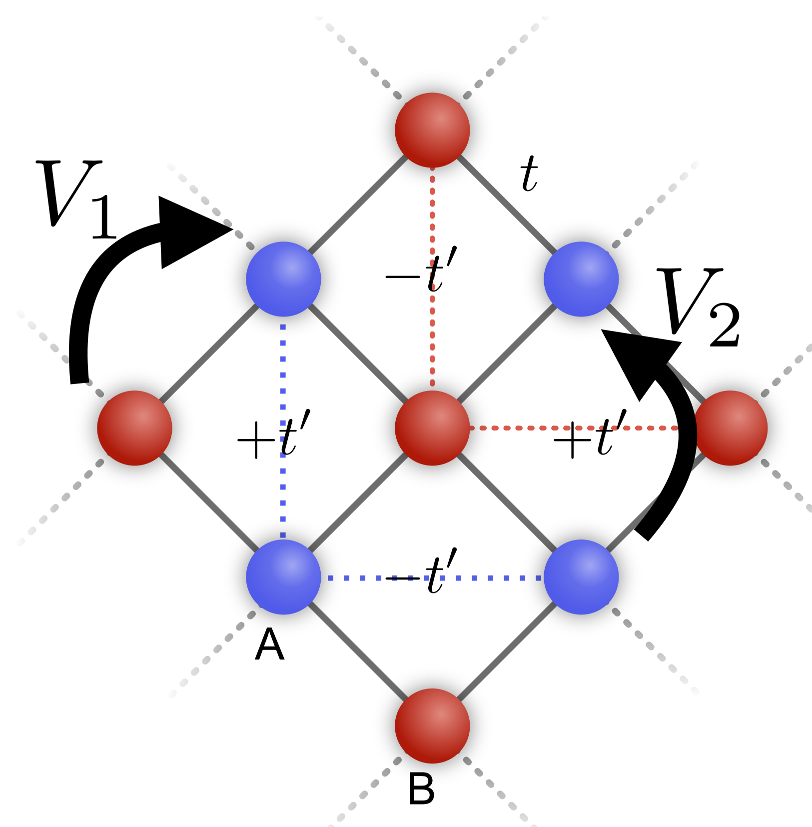

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mimicks Haldane's imaginary hopping

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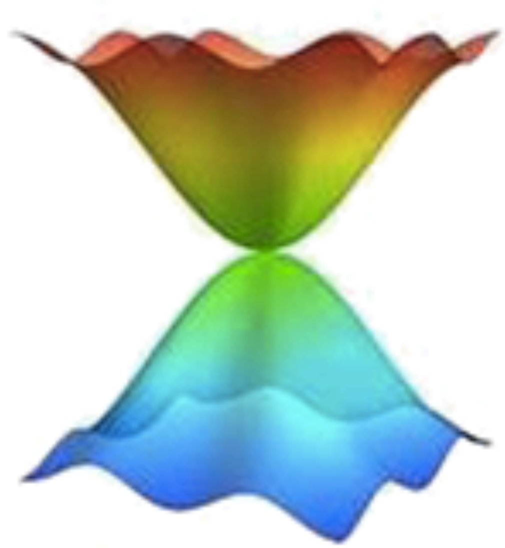
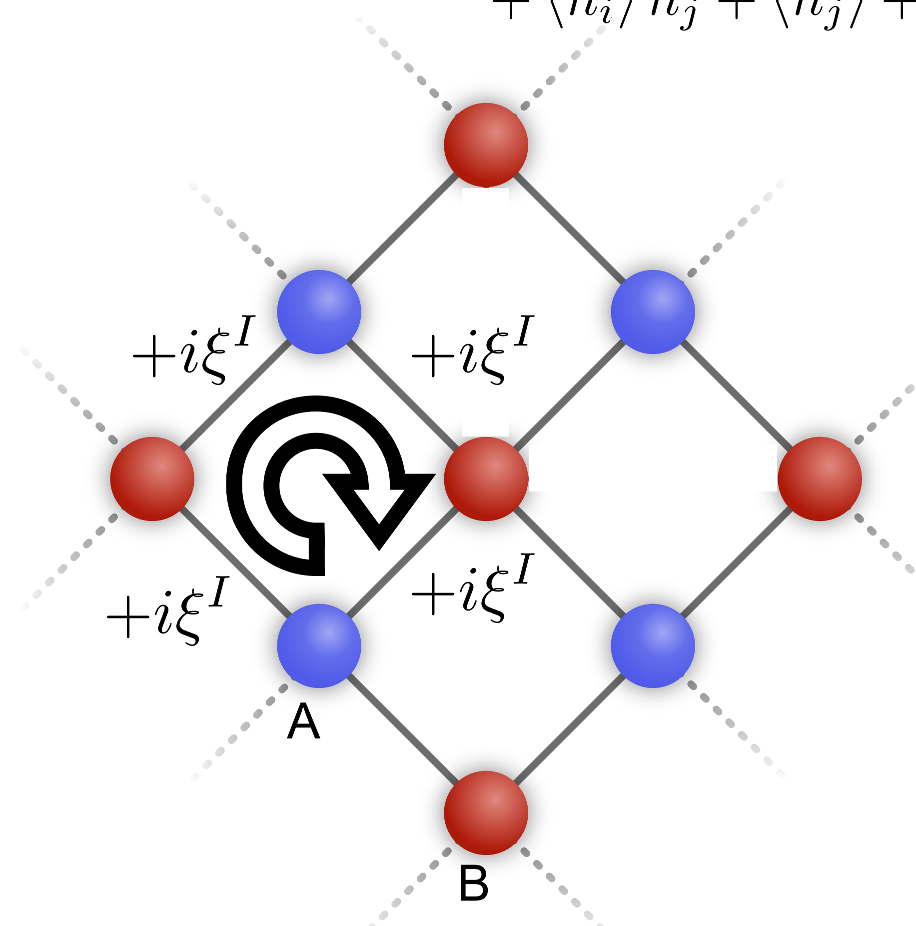


Zeng et al, NPJ quant mat 2019



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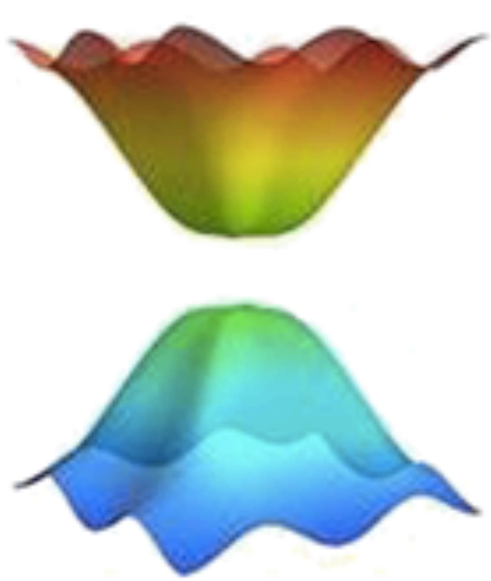
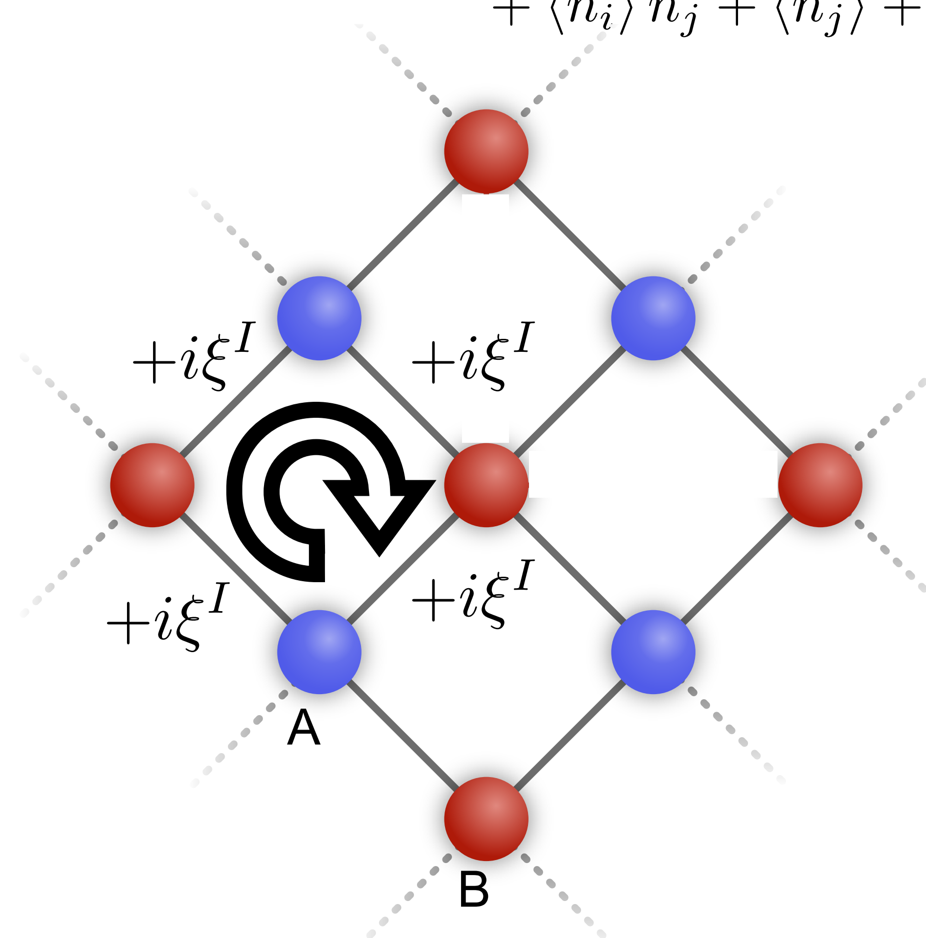
$E_{int}(\mathbf{k})$

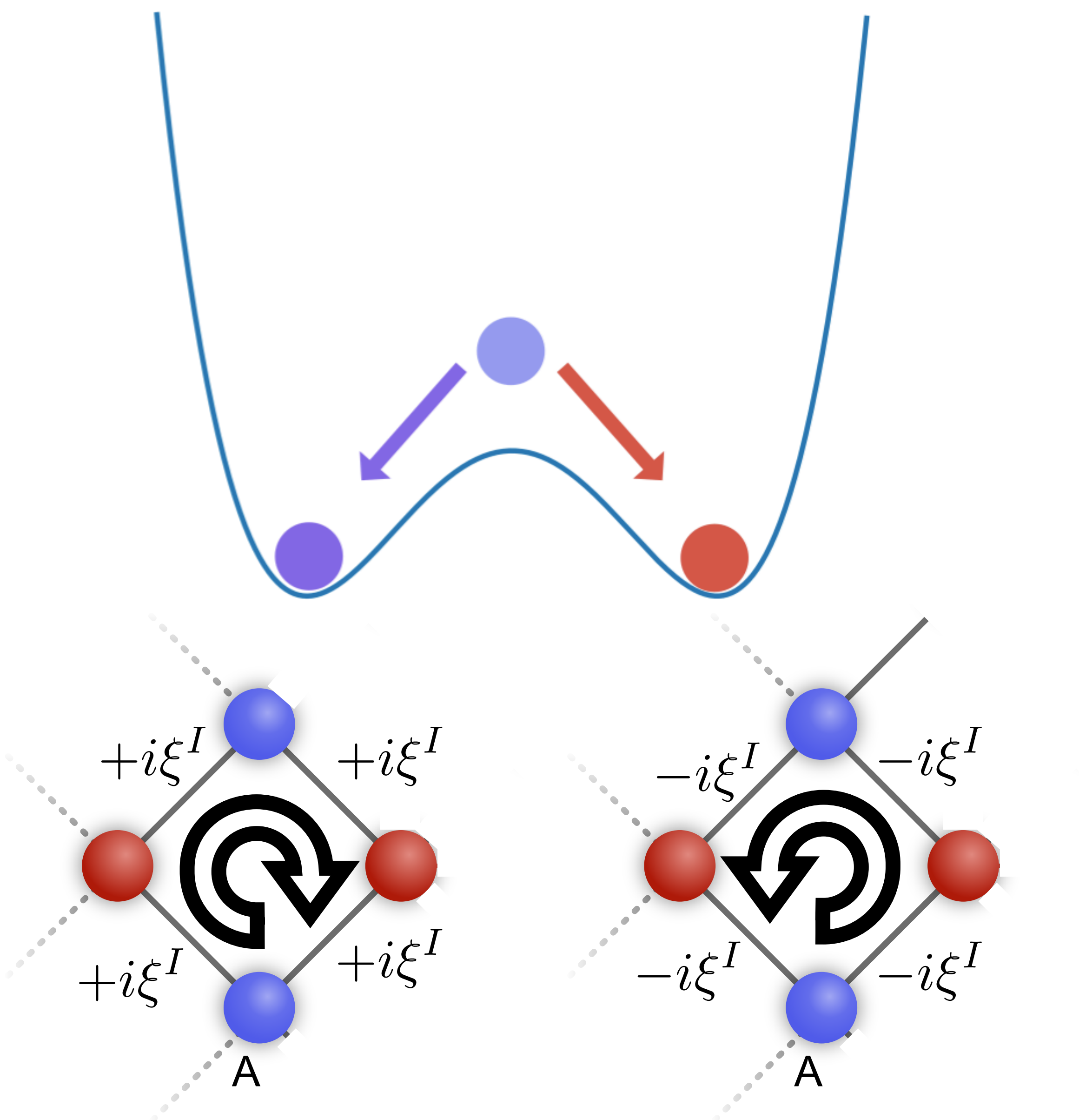


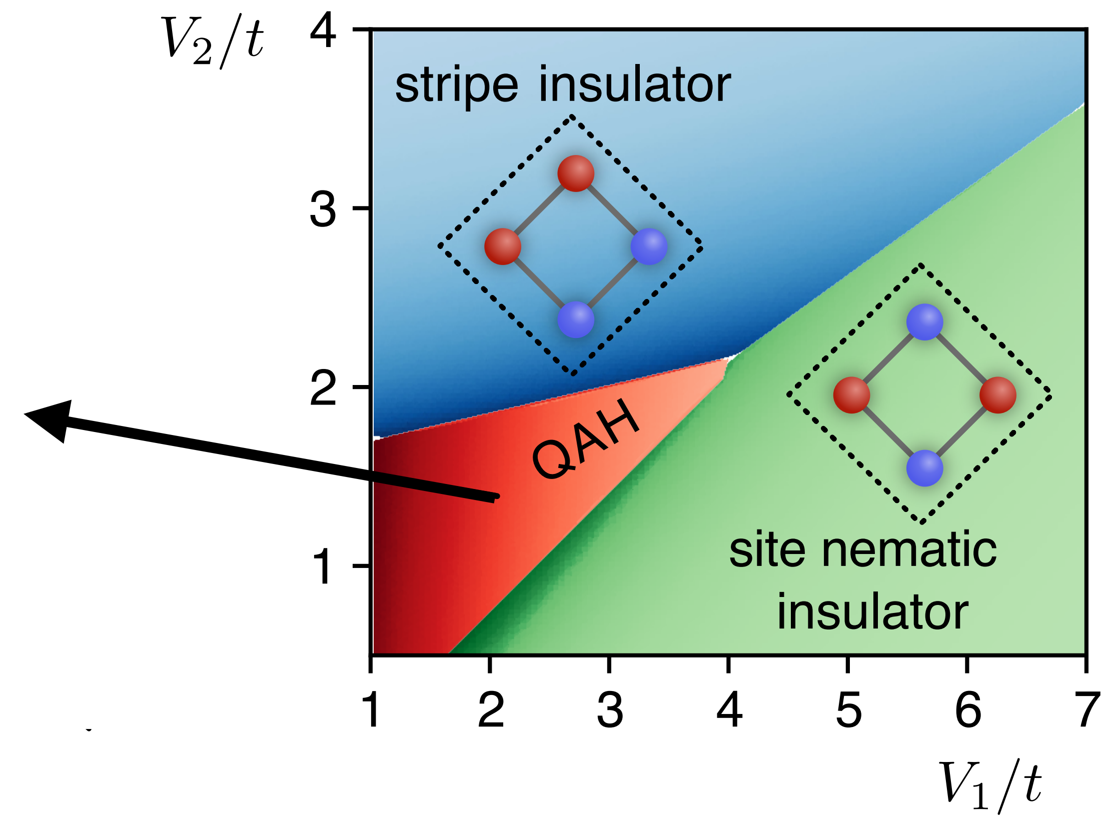
Zeng et al, NPJ quant mat 2019



Phase diagram at half filling

- Quantum Anomalous Hall phase (two ground states)





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translationally invariant ansatz

Quantum Anomalous Hall phase and Chern number

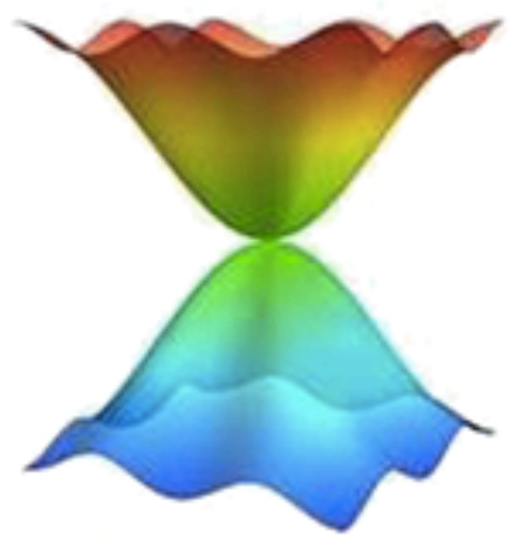
- The QAH can salo be captured with a two-atom unit cell ansatz

$$H_{\text{eff}} = \alpha(k)\,1 + \vec{v}(k) \cdot \vec{\sigma}$$

Pauli vector

- One can show that the system is actually gapped in the QAH
- This phase is not insulating (gap) but also topological.
- Topological invariant: Chern number

$E(\mathbf{k})$



$$\vec{n} = \frac{\vec{v}}{|\vec{v}|} \qquad C = \frac{1}{4\pi}\int_{\text{BZ}} \vec{n} \cdot (\partial_{kx}\vec{n} \times \partial_{ky}\vec{n})$$

Qi et al., Phys. Rev. B **78**, 195424 (2008)

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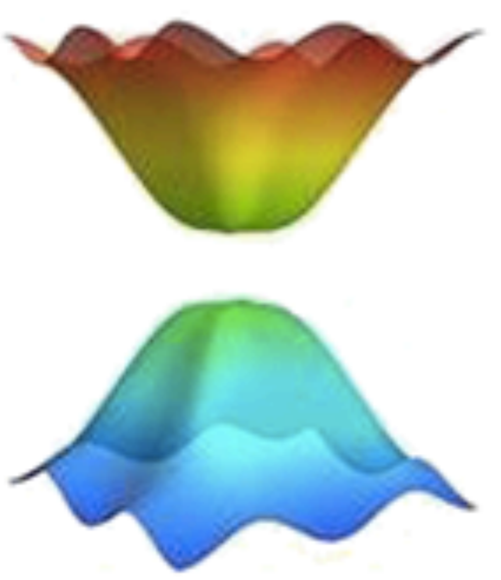
$E_{int}(\mathbf{k})$



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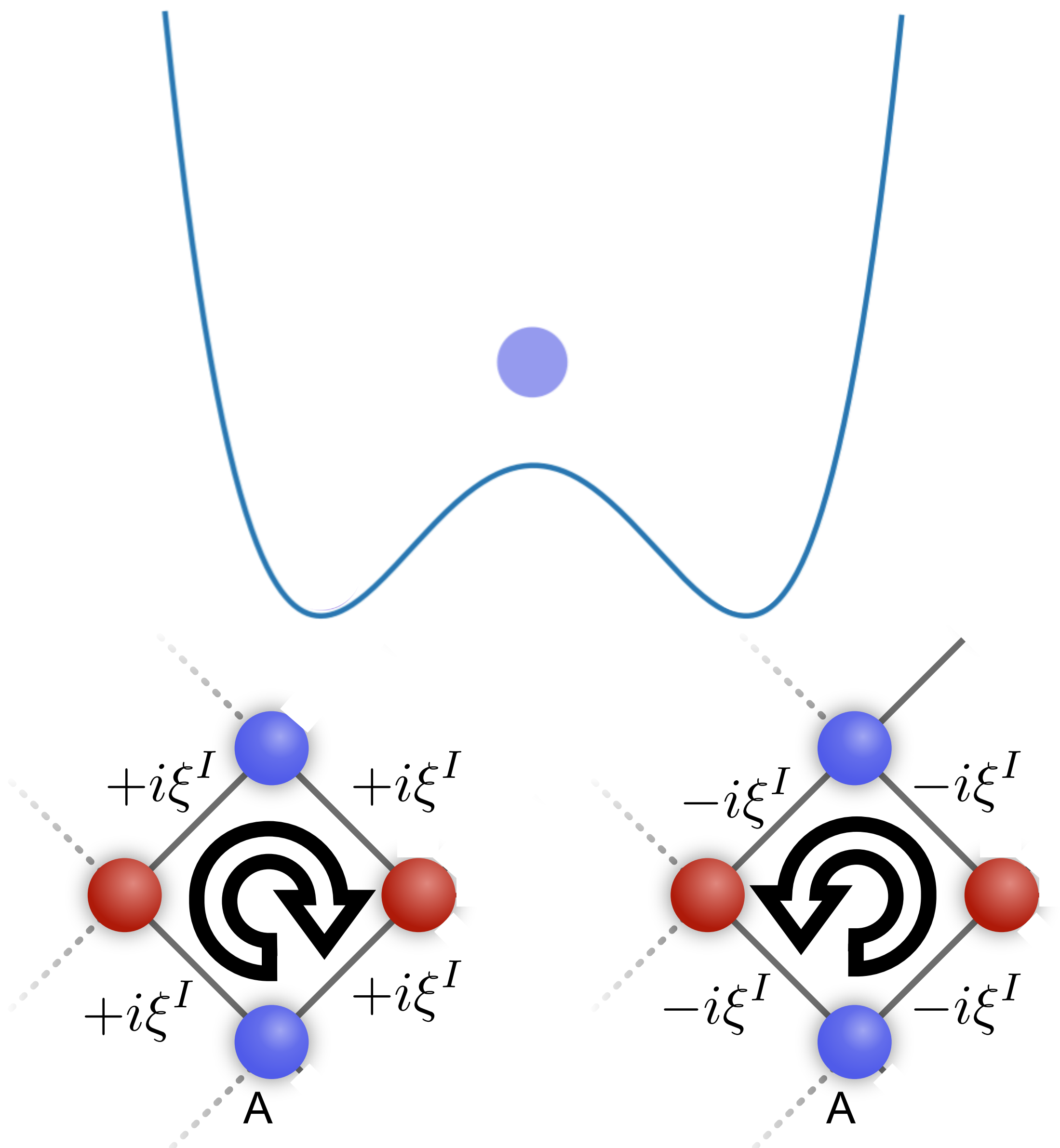
Qi et al., Phys. Rev. B **78**, 195424 (2008)

Quantum Anomalous Hall phase and Chern number

- The two SSB ground states have an opposite Chern number



$$C = 1 \qquad C = -1$$

Quantum Anomalous Hall phase and Chern number

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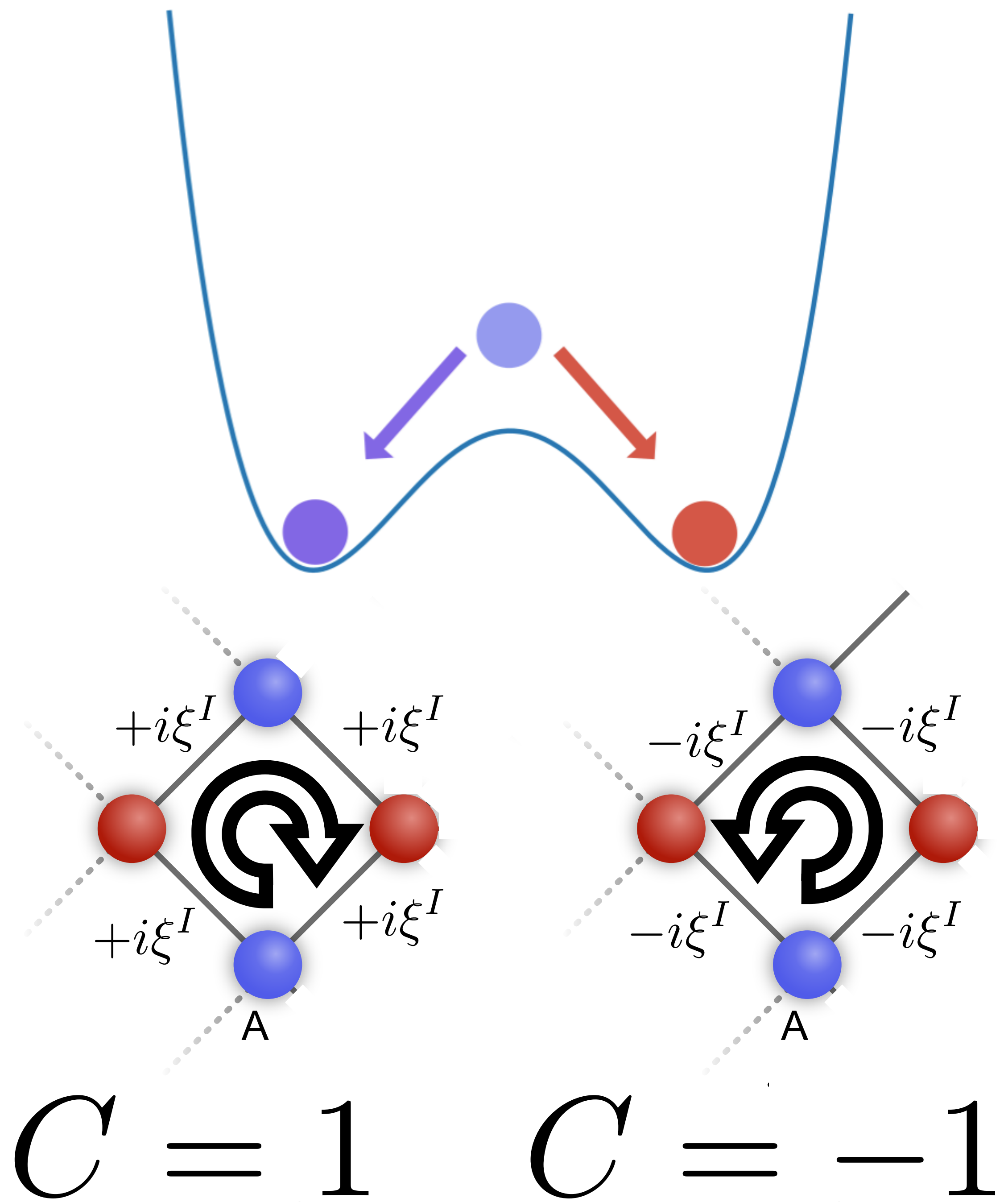

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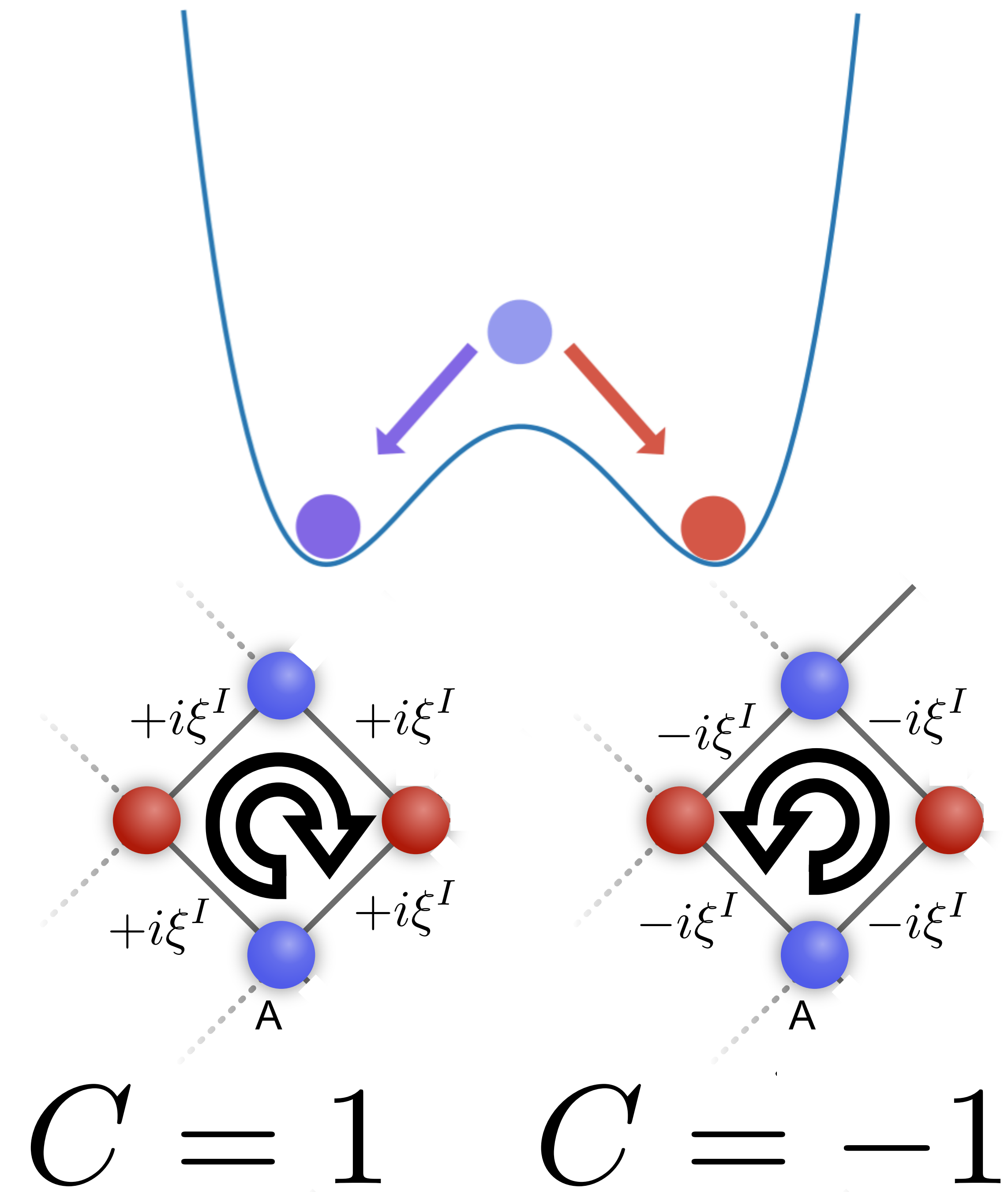

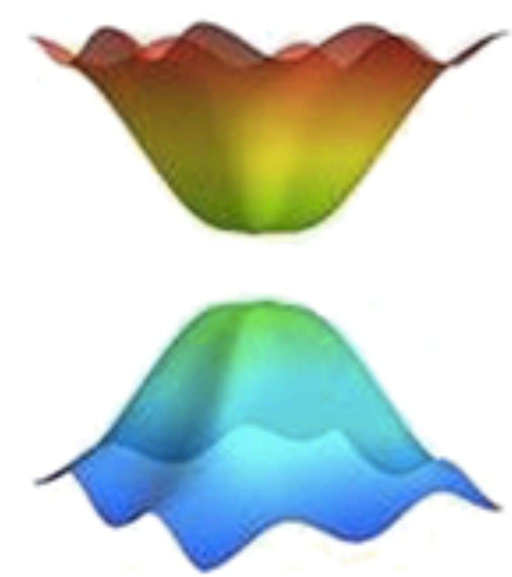

Fate of the TMI beyond mean field

- Recent Density Renormalization group studies.
- The authors also found the QAH phase.

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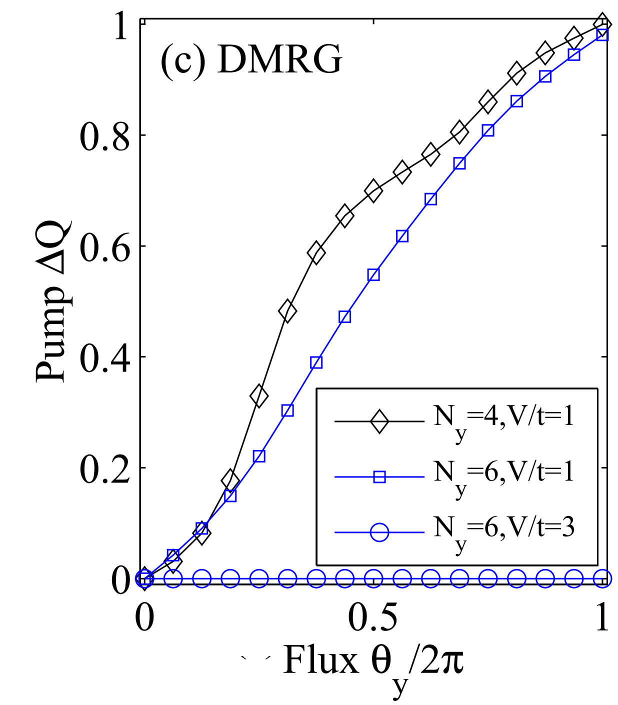
ARTICLE OPEN

Tuning topological phase and quantum anomalous Hall effect by interaction in quadratic band touching systems

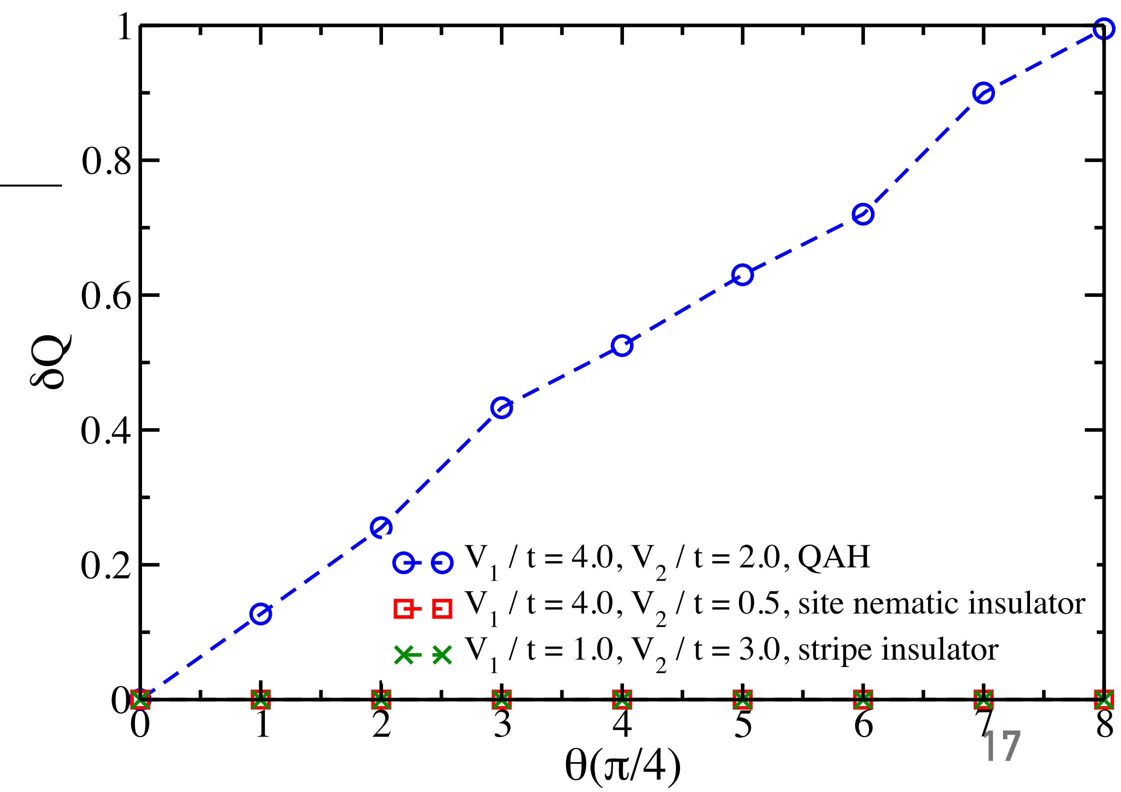
Tian-Sheng Zeng[1], Wei Zhu[2,3] and Donna Sheng[4]



PHYSICAL REVIEW B **98**, 125144 (2018)

Quantum anomalous Hall insulator stabilized by competing interactions

Shouvik Sur,[1] Shou-Shu Gong,[2,3,*] Kun Yang,[1] and Oskar Vafek[1,†]



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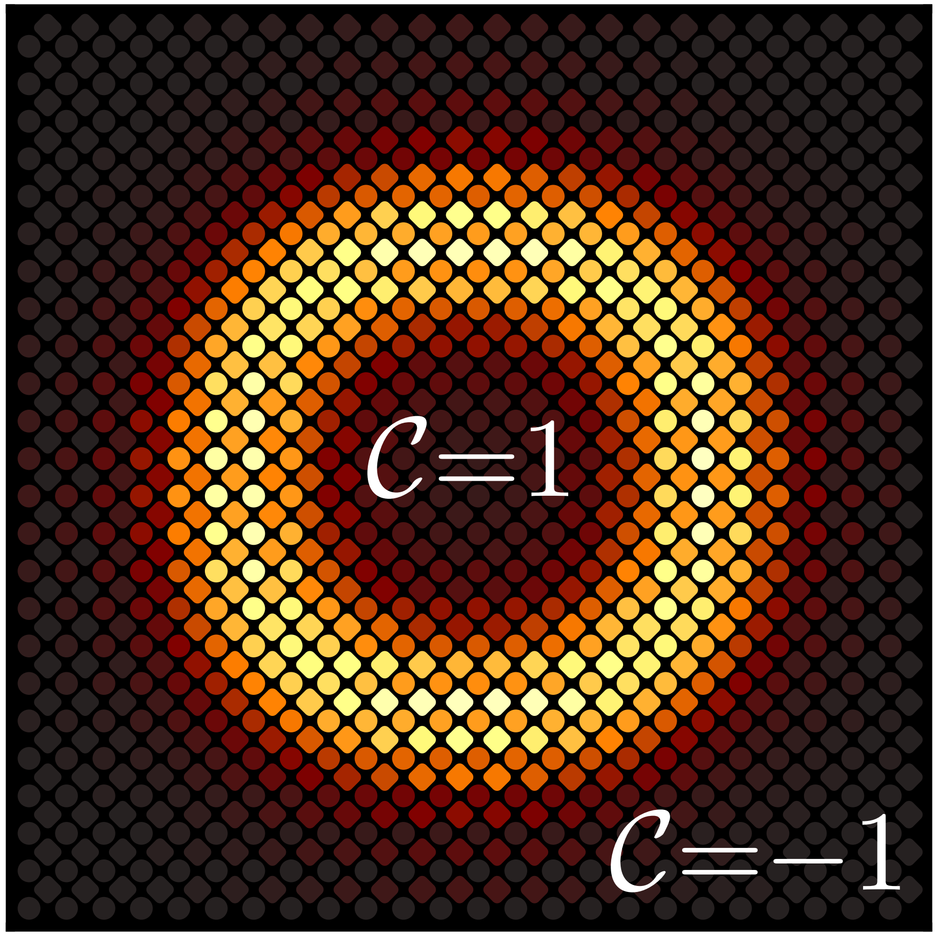
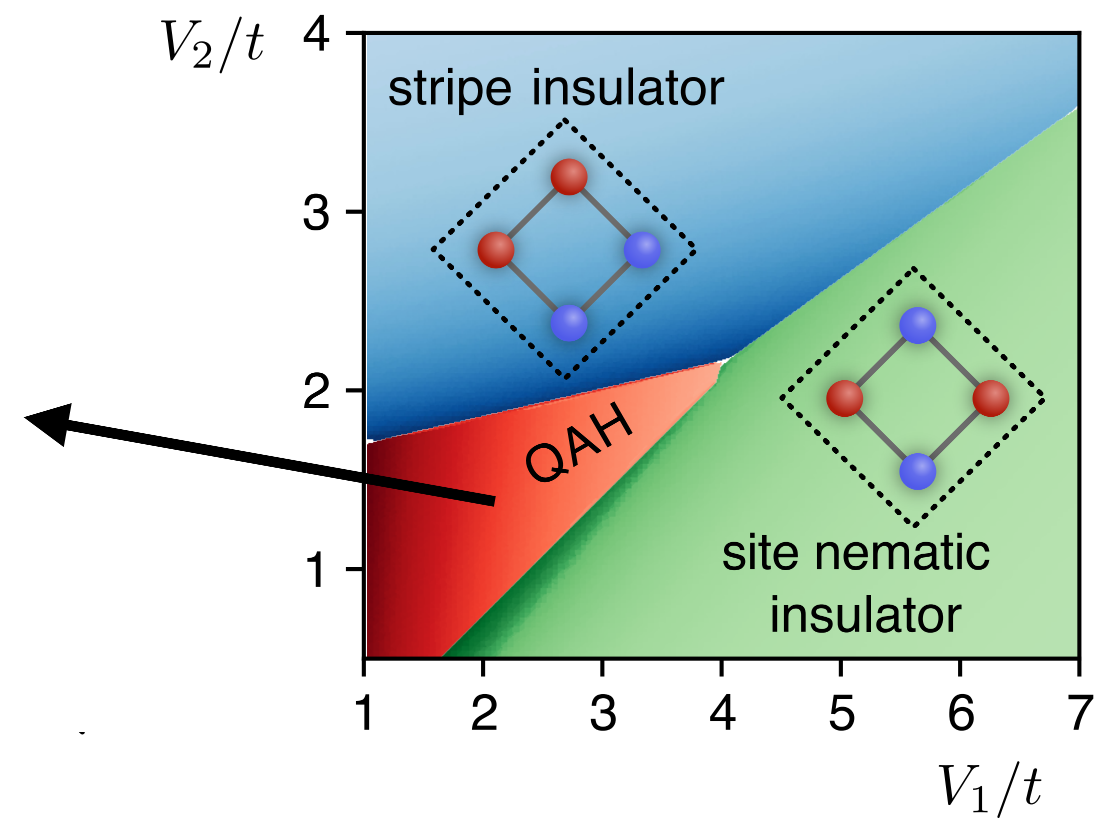
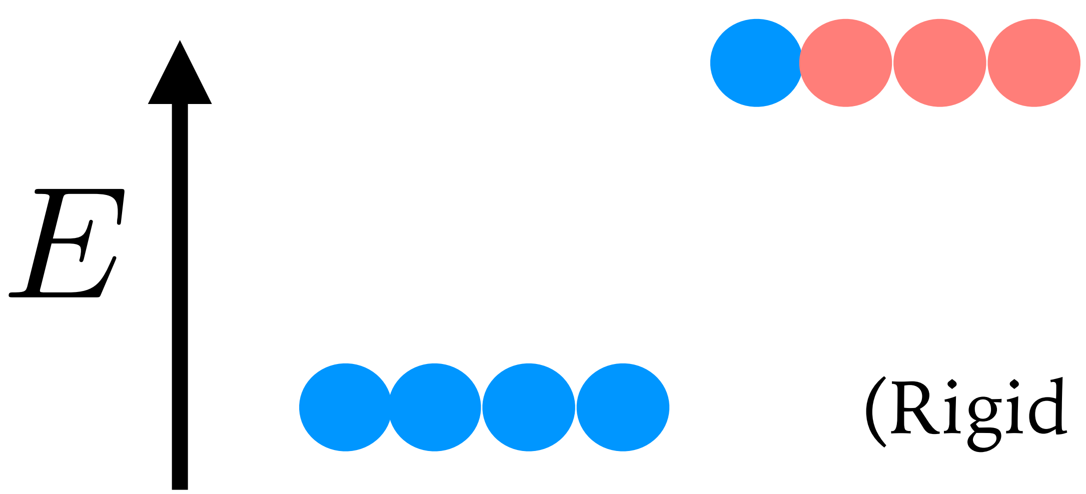


Phases beyond half filling

- We fix V1/t = 2.5, V2/t= 1.5 .
- What is the behavior with particle doping?
- Two possibilities:





(Rigid band scenario)

Phases beyond half filling

- We fix V1/t = 2.5, V2/t= 1.5 .
- What is the behavior with particle doping?
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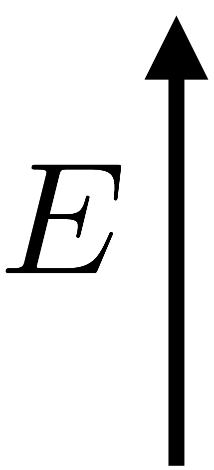

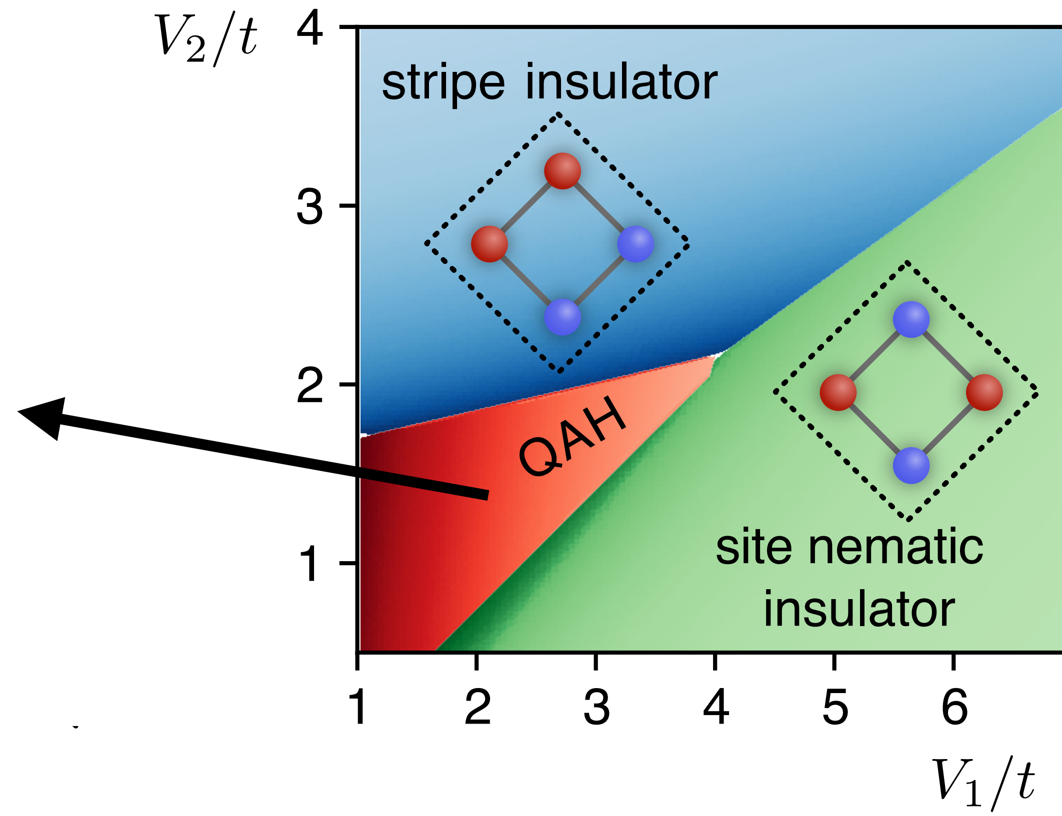

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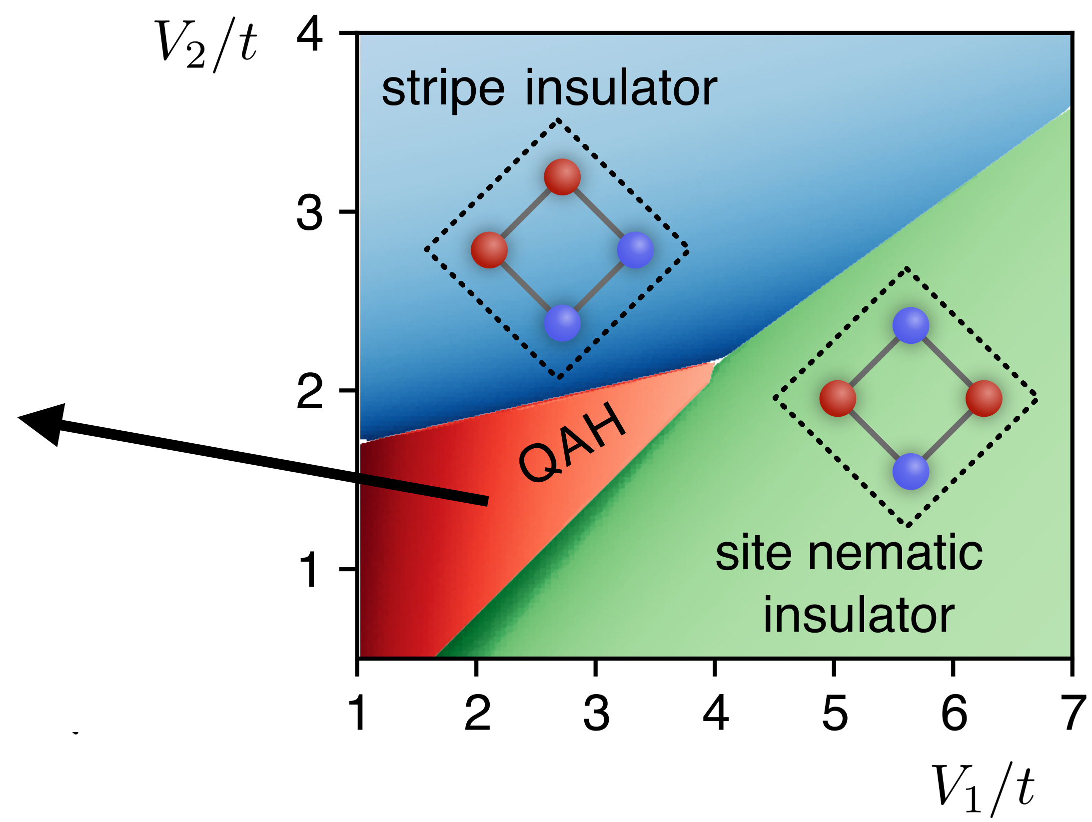

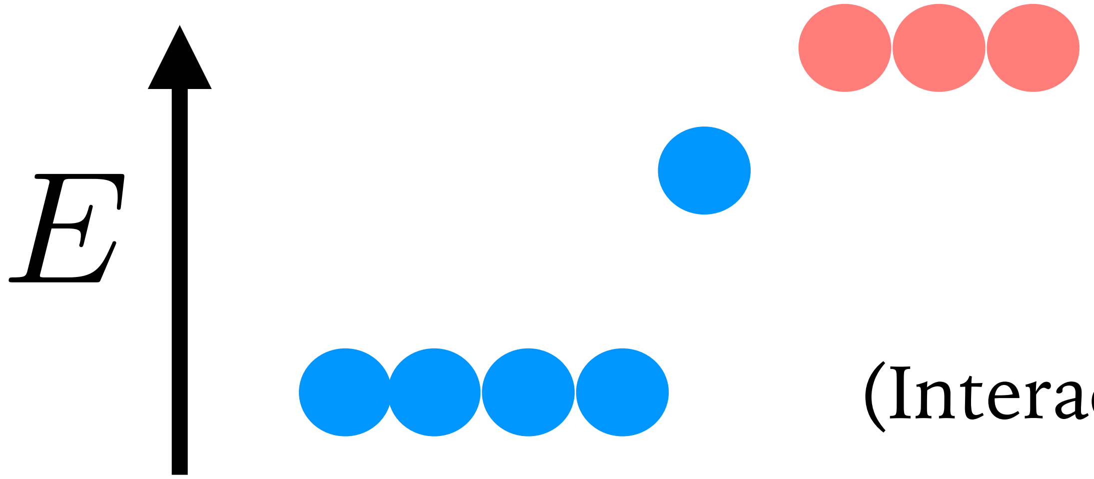

(Interacting band scenario)

Phases beyond half filling

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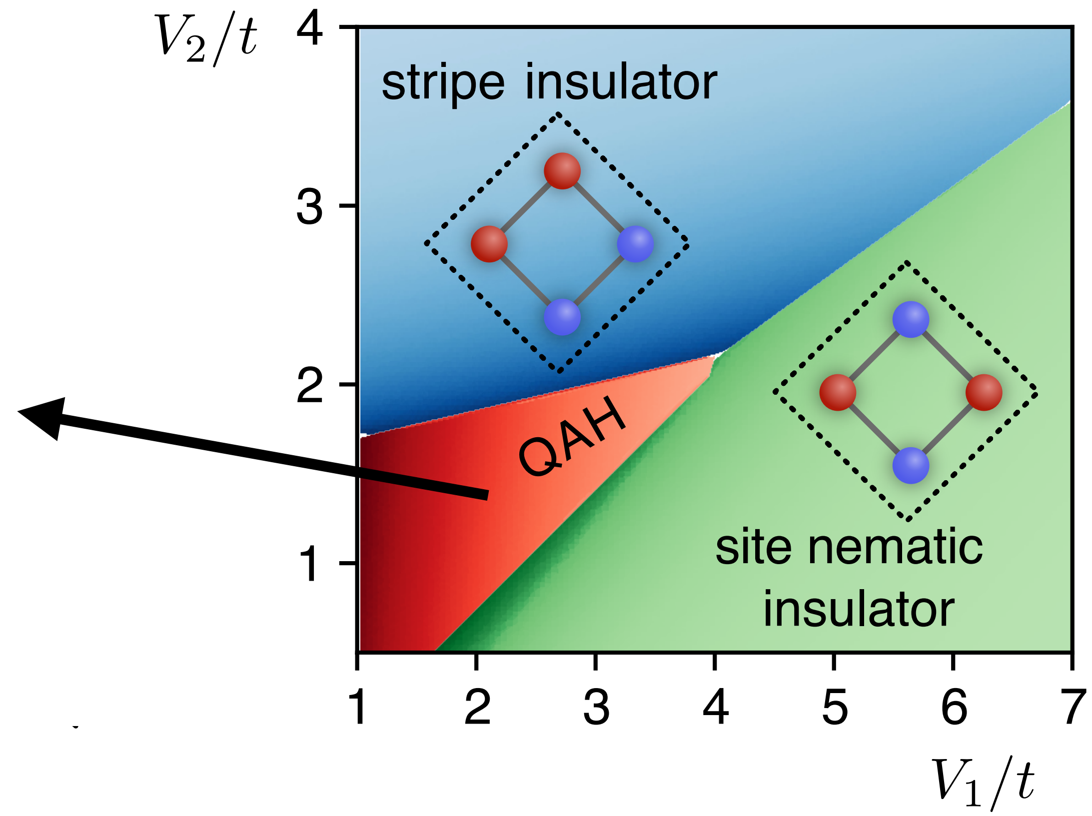
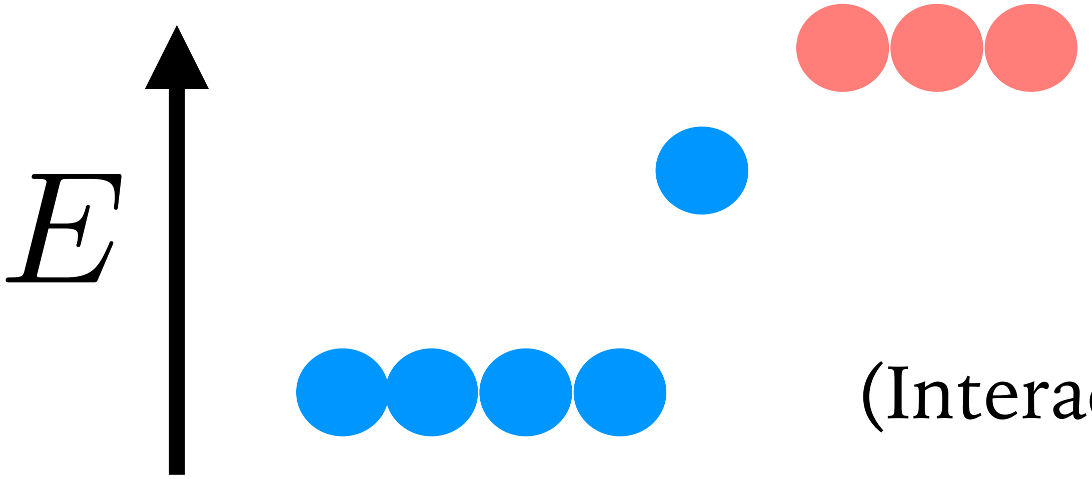




(Interacting band scenario)

- Localized solutions inside the gap break also translational invariance

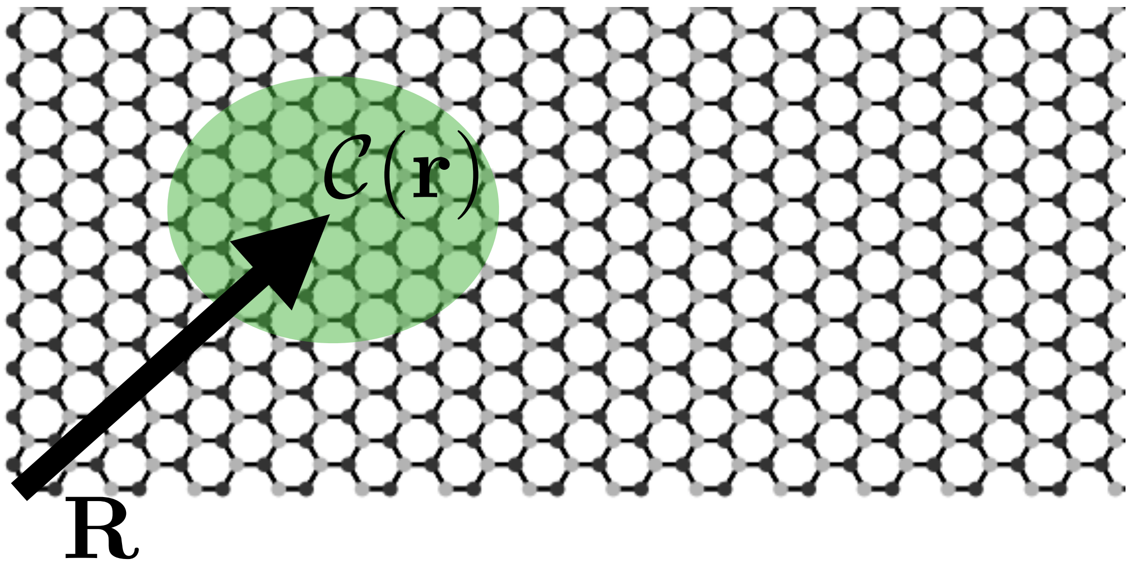
$$\hat{n}_i\hat{n}_j \simeq -\langle \hat{c}_i^\dagger \hat{c}_j\rangle \hat{c}_j^\dagger \hat{c}_i - \langle \hat{c}_j^\dagger \hat{c}_i\rangle \hat{c}_i^\dagger \hat{c}_j \\ + \langle \hat{n}_i\rangle \hat{n}_j + \langle \hat{n}_j\rangle + \langle \ldots\rangle \langle \ldots\rangle$$

- How do we study the topology in this case?

Local Chern number

- The definition of the Chern number relies on some kind of boundary conditions (periodic or twisted) giving a notion of k-space.
- But the single-particle insulating ground state is characterized by

$$P = \sum_{i \in \mathrm{occc}} |\Phi_i\rangle \langle\Phi_i| \qquad P(\mathbf{r}, \mathbf{r}') \sim e^{-\alpha|\mathbf{r}-\mathbf{r}'|}$$

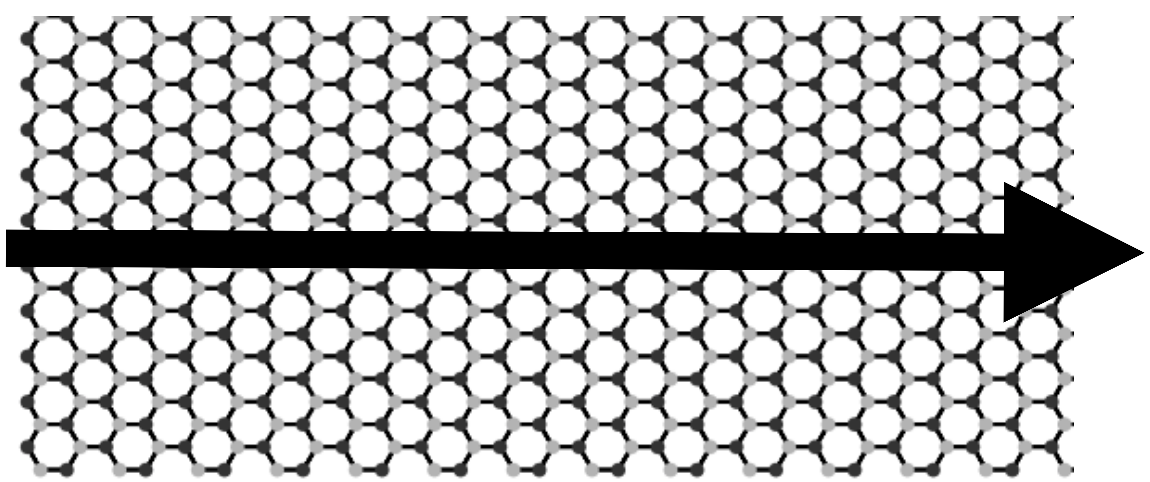


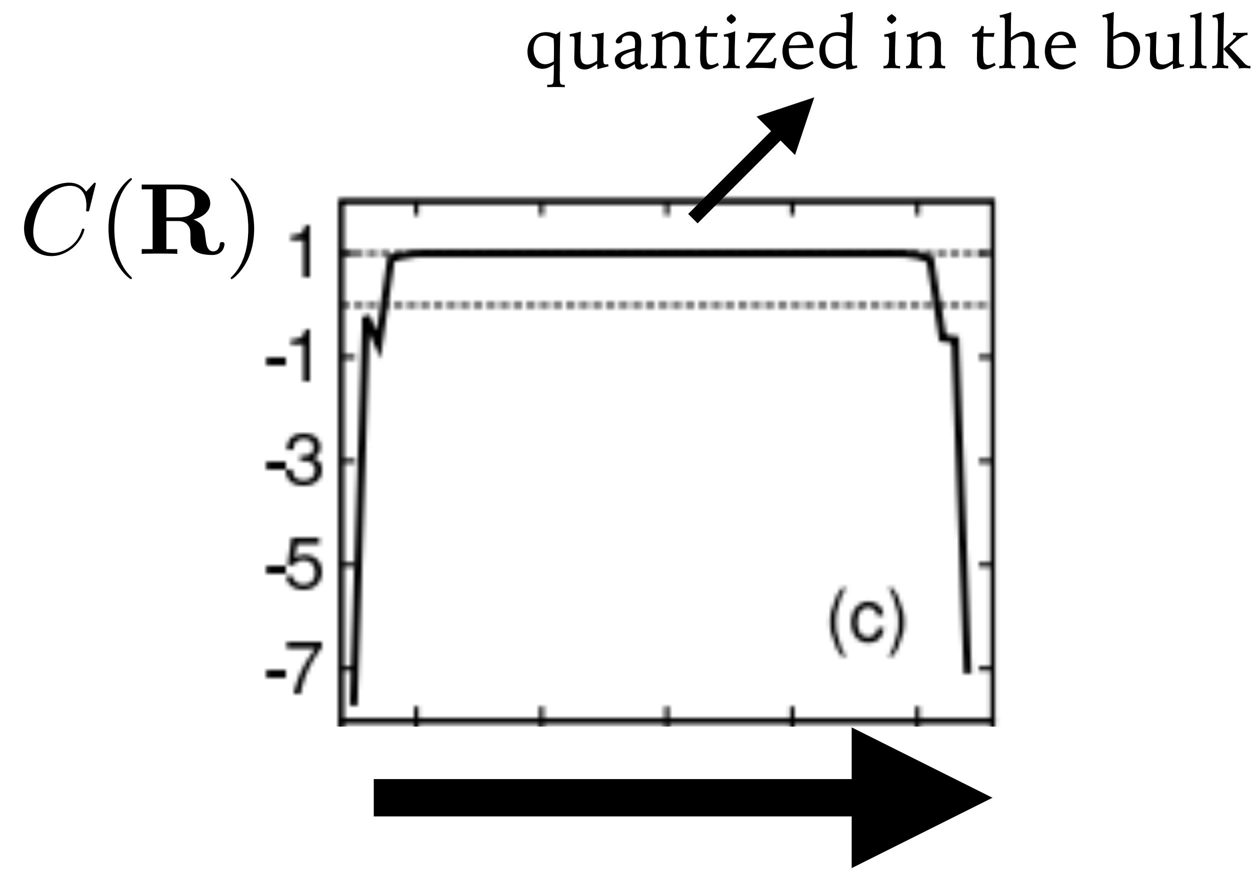
$$\Longrightarrow \quad C(\mathbf{R}) = \frac{1}{A} \sum_{\mathbf{r}} \mathcal{C}(\mathbf{r})$$

R. Bianco and R. Resta, Phys. Rev. B **84**, 241106(R) (2011)

Local Chern number

- Extremely useful in non-homogeneous phases, to detect different topological regions.





R. Bianco and R. Resta, Phys. Rev. B **84**, 241106(R) (2011)

Local order parameter and global topological invariant

- Local order parameter of the time reversal symmetry breaking

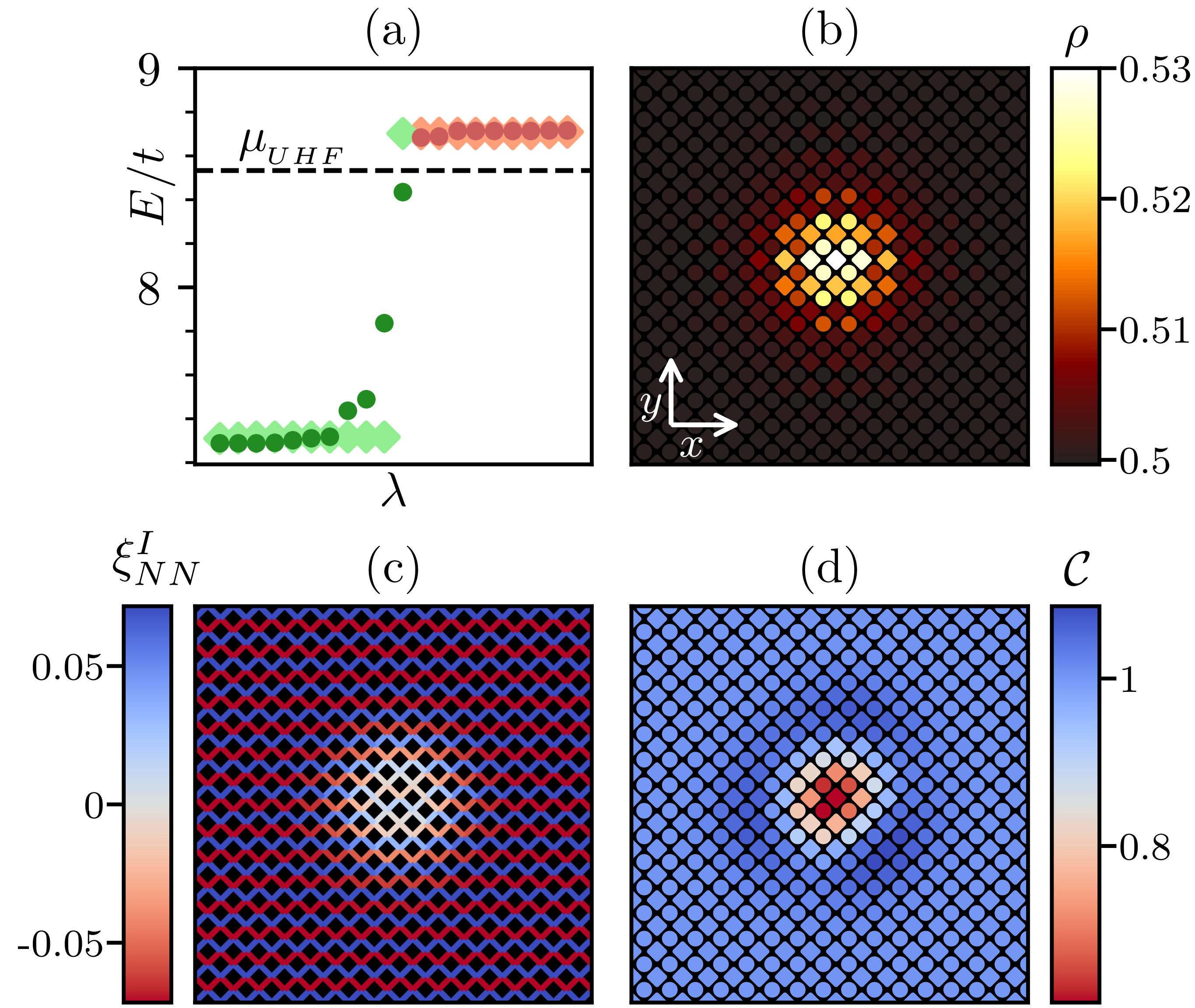
$$\langle \hat{c}_i^\dagger \hat{c}_j \rangle \equiv \xi_{ij}^R + i\xi_{ij}^I$$ (~Haldane's imaginary hopping)

- Local Chern number (not a local order parameter, identifies global topology)

$$C(\mathbf{R}) = \frac{1}{A}\sum_{\mathbf{r}} \mathcal{C}(\mathbf{r})$$

One extra particle: Self-trapped polaron

- State localized in the gap
- Density cloud in real space
- Reduction of the local order parameter
- Even sign inversion (change of the SSB sector)
- Local Chern number not quantized inside

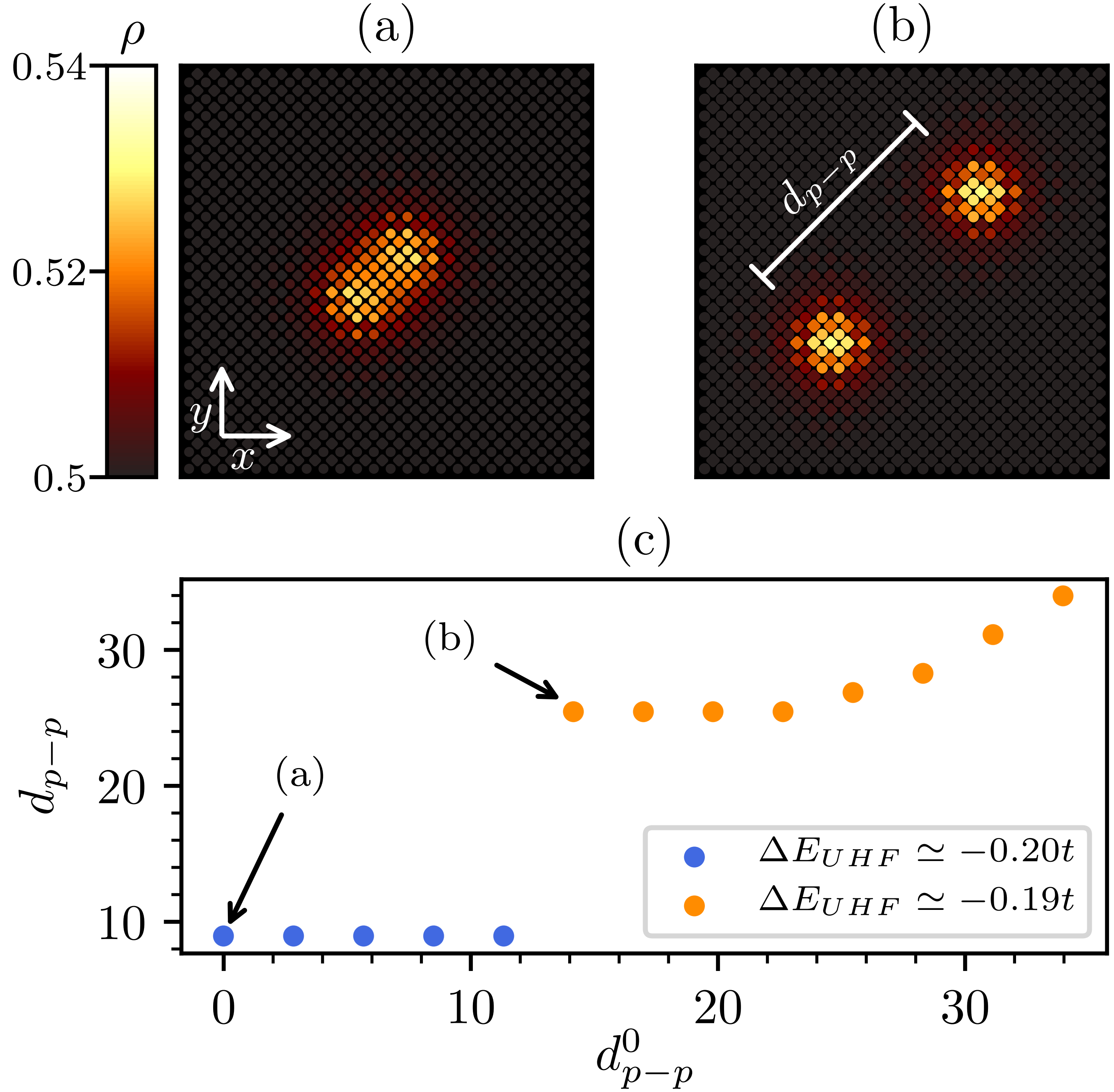


Similar behavior in self-trapped polarons in 2D fermi-hubbard model (but no topology).

J. A. Verges et al., Phys. Rev. B 43, 6099 (1991)

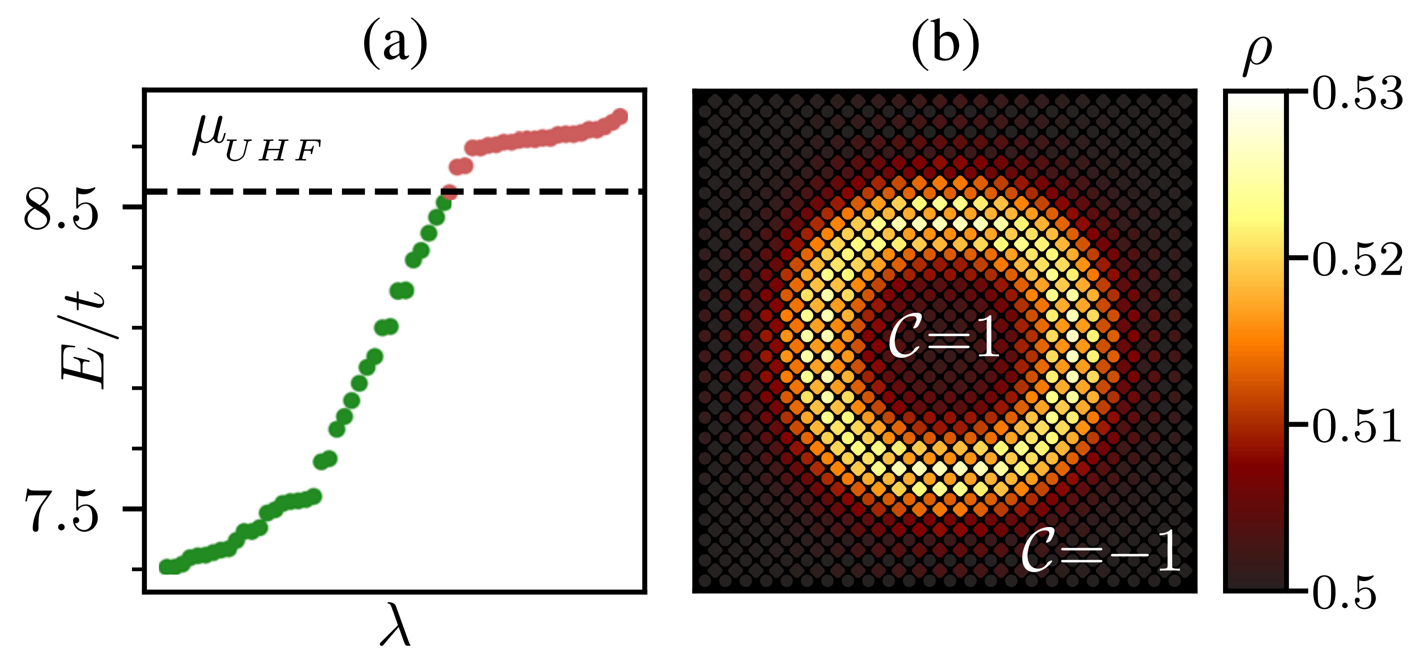
Two Extra Particles: Bi-Polaron States

- Separated polarons are metastable solutions.
- There is a collapse radius.
- Bipolaron state.



Eight Extra Particles: Ring Domain Walls

- Single domain wall.
- Bulk solution (independent of boundaries)
- Coexistence of the two ground states
- Appearance of chiral edge states.



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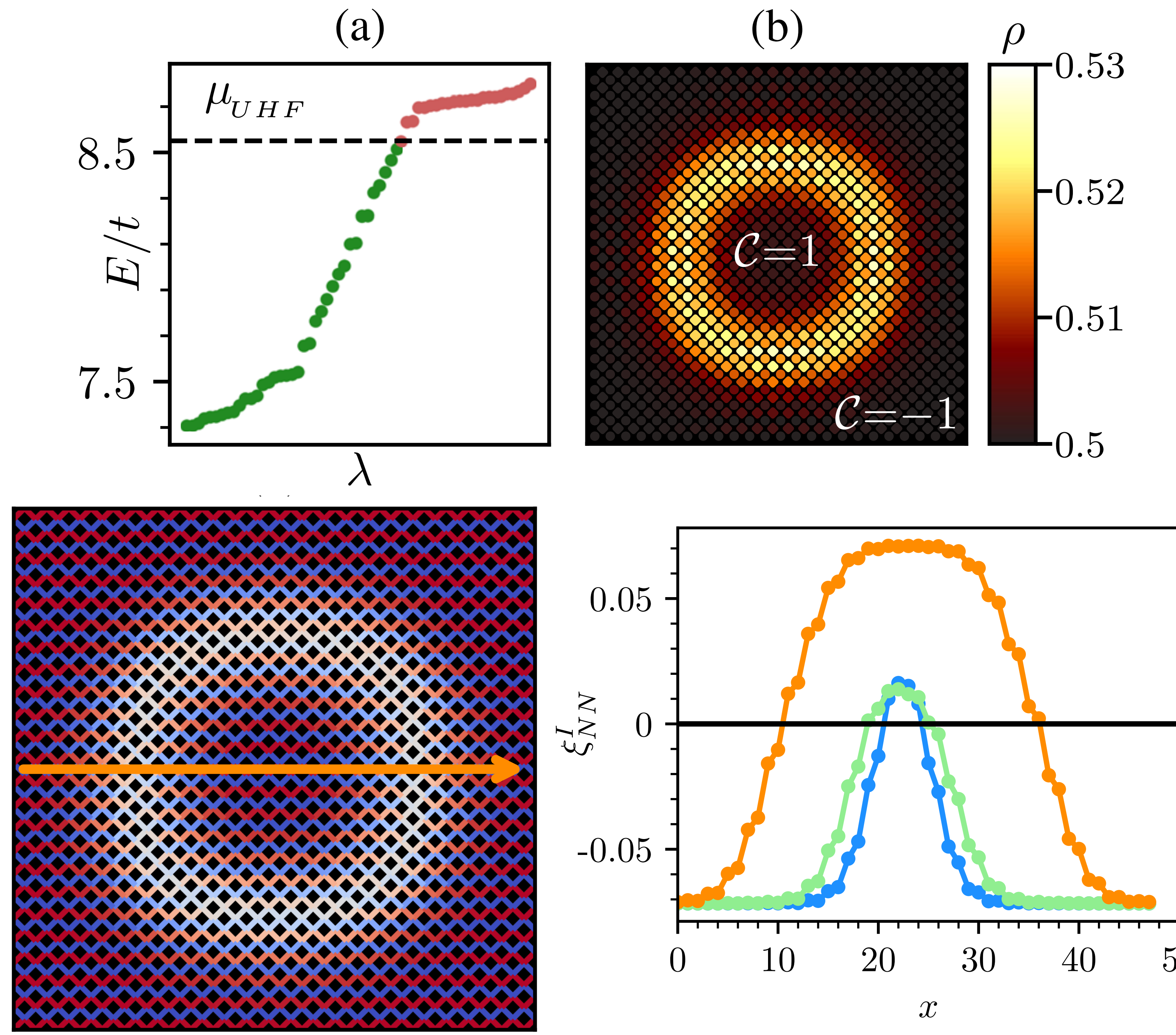

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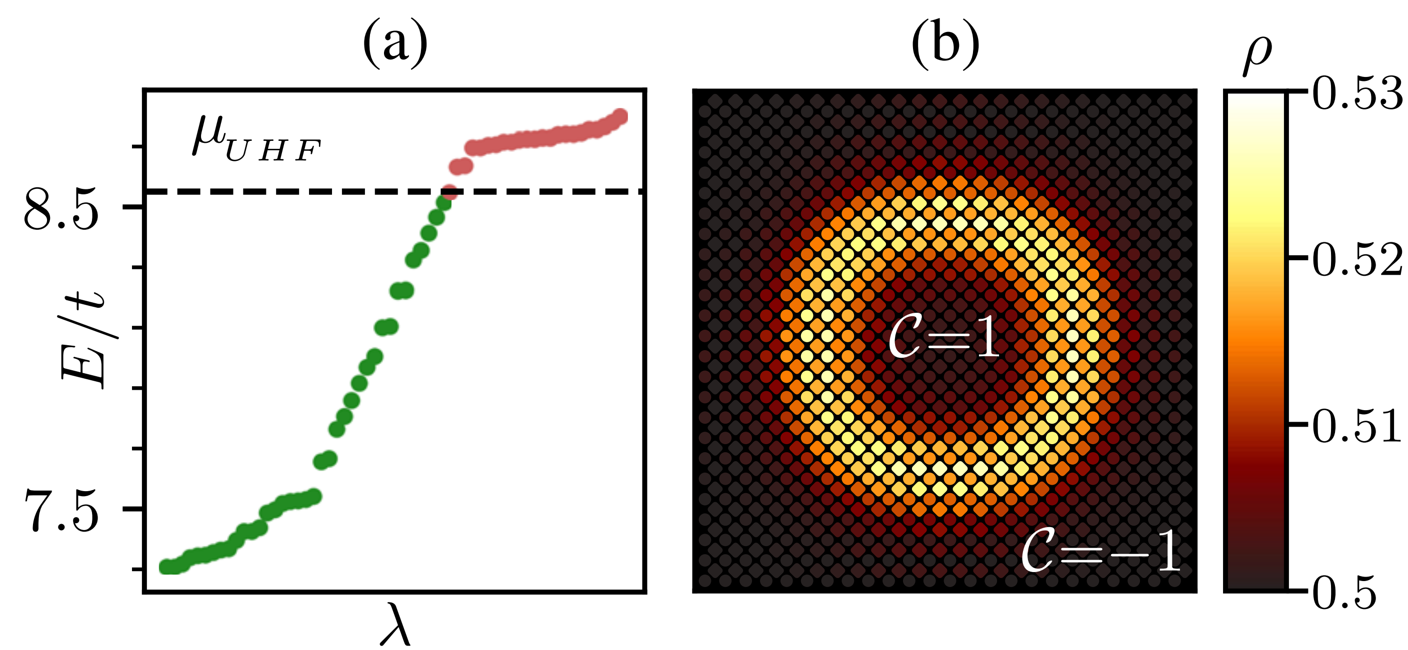

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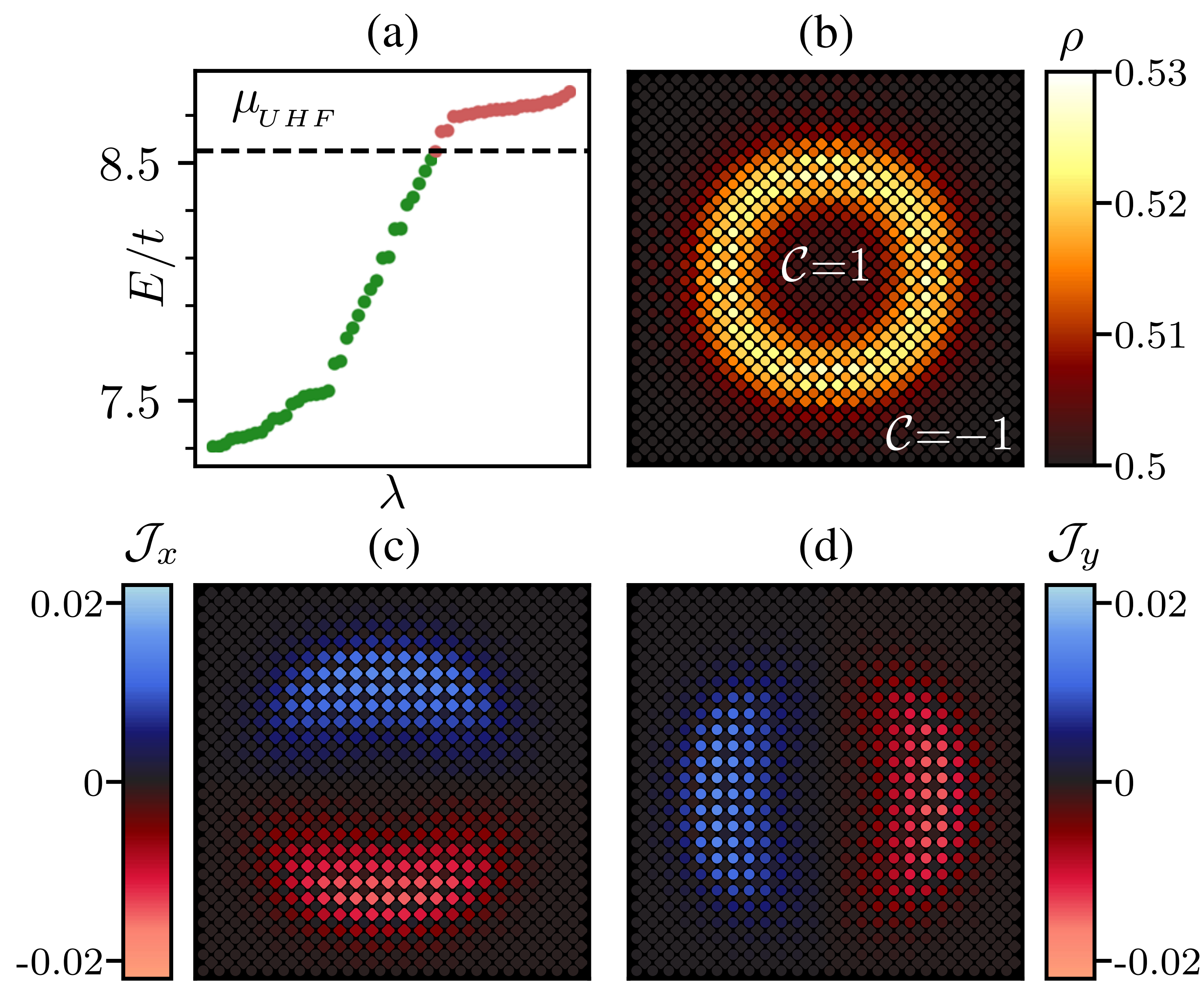
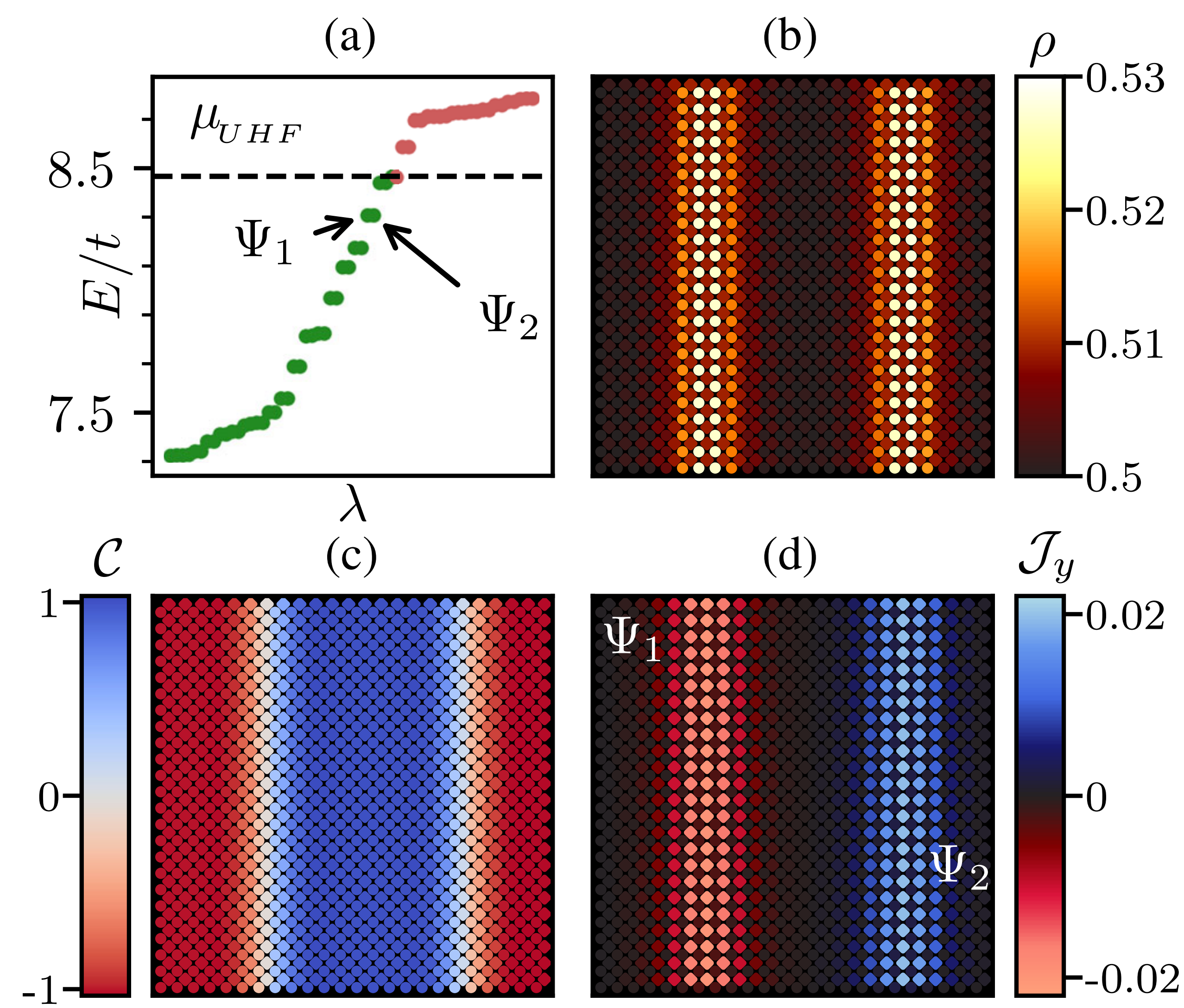


Eight Extra Particles: Metastable State

LINEAR DOMAIN WALLS

- Density accumulation in linear regions (DW).
- Appearance of chiral edge states.



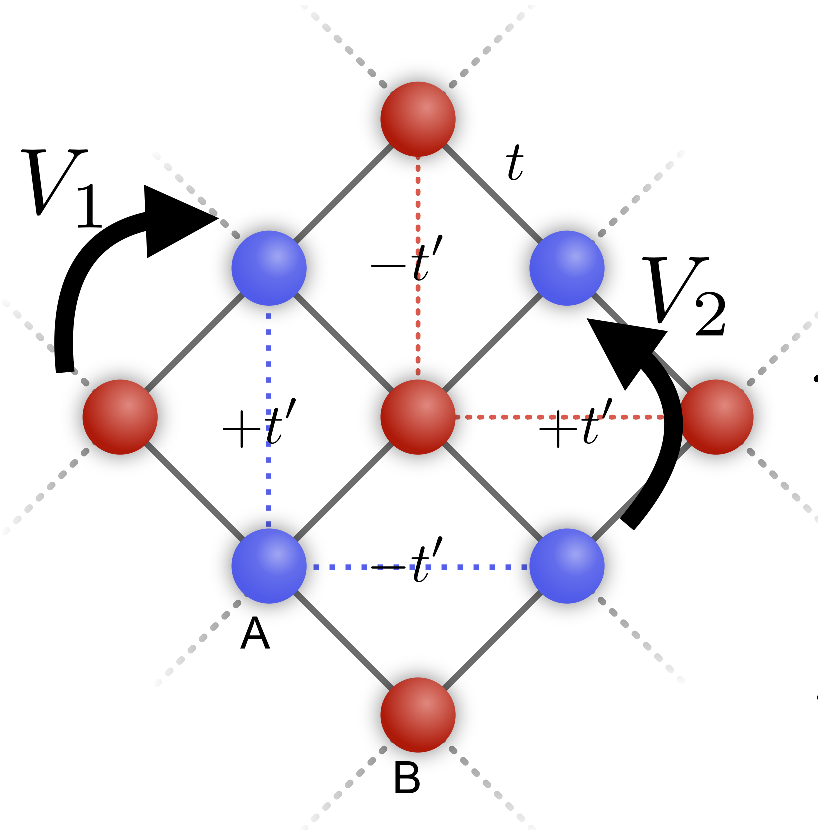
Quantum simulation with Dressed Rydberg atoms

- Simulation Topological Mott insulator in other lattices:

  A. Dauphin et al, Phys. Rev. A **86**, 053618 (2012)

  A. Dauphin et al, Phys. Rev. A **93**, 043611 (2016)

- pi-flux can be achieved with laser assisted tunneling.
- V1 and V2 interactions ~ t can be achieved with dressed Rydberg atoms due to a far detuned laser field.



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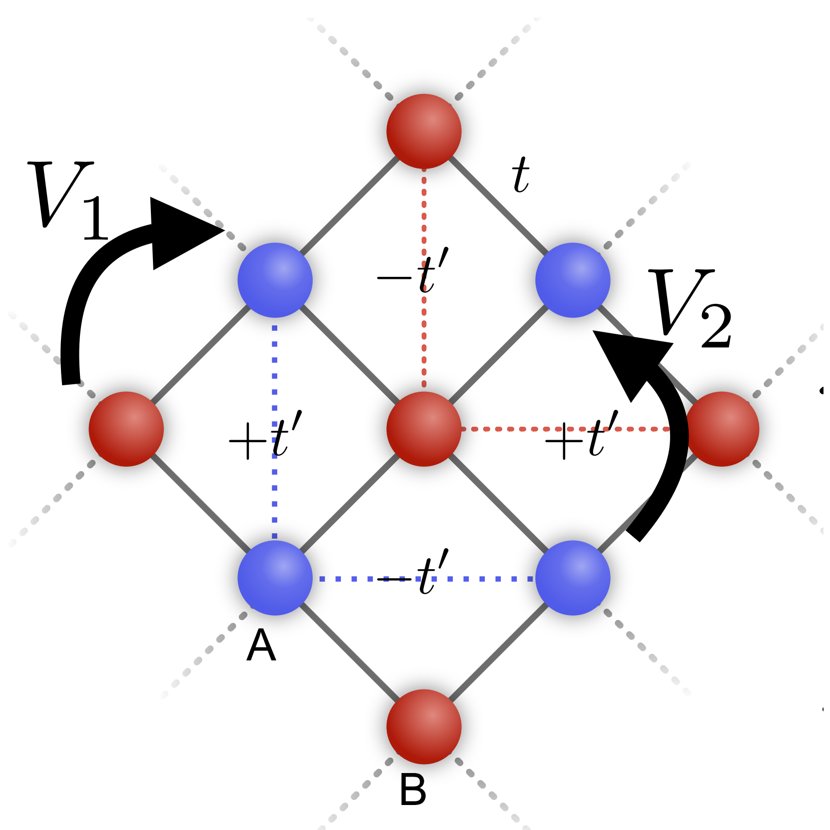


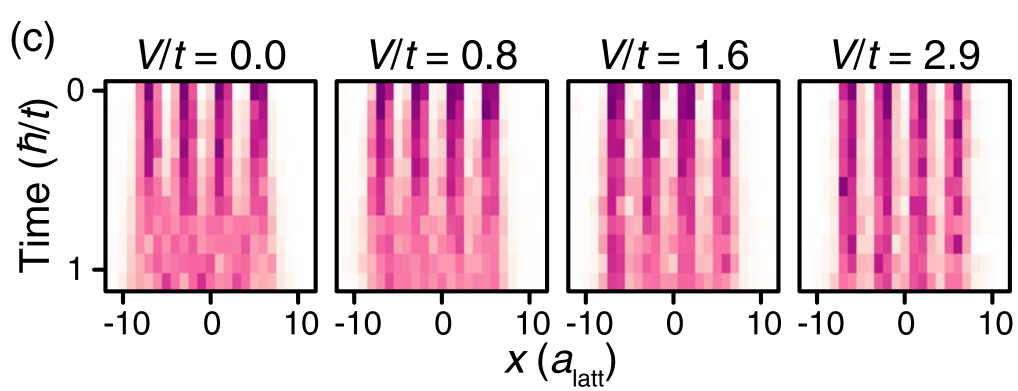
PHYSICAL REVIEW X **11**, 021036 (2021)

Featured in Physics

Quench Dynamics of a Fermi Gas with Strong Nonlocal Interactions

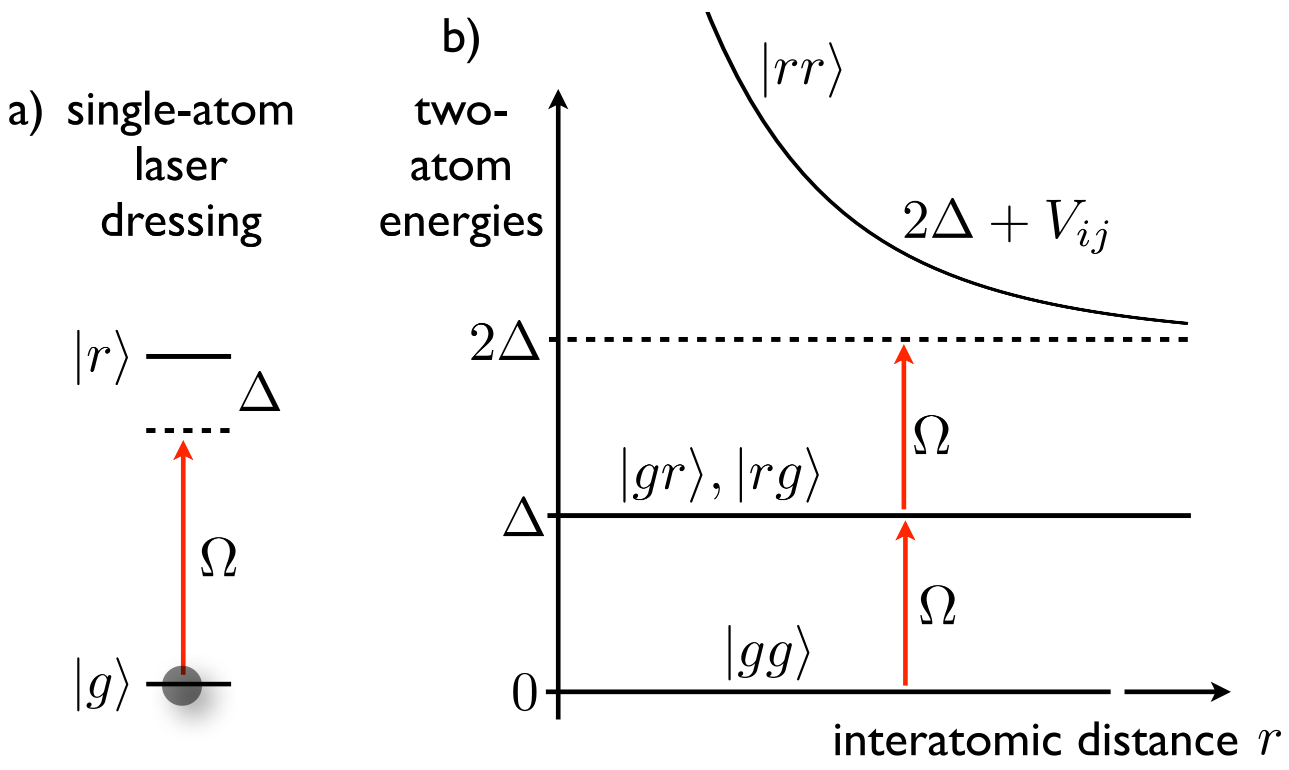
Elmer Guardado-Sanchez,[1] Benjamin M. Spar,[1] Peter Schauss,[2] Ron Belyansky,[3,4] Jeremy T. Young,[3,5,6] Przemyslaw Bienias,[3,4] Alexey V. Gorshkov,[3,4] Thomas Iadecola,[7] and Waseem S. Bakr[1,*]

$$\hat{H} = -t\sum_{\langle i,j\rangle_v}(\hat{c}_i^\dagger \hat{c}_j + \text{H.c.}) + \sum_{i\neq j}\frac{V_{ij}}{2}\hat{n}_i\hat{n}_j. \qquad (2)$$



The implementation scheme

- off-resonant laser coupling to the Rydberg state + vdW- Rydberg-Rydberg interactions



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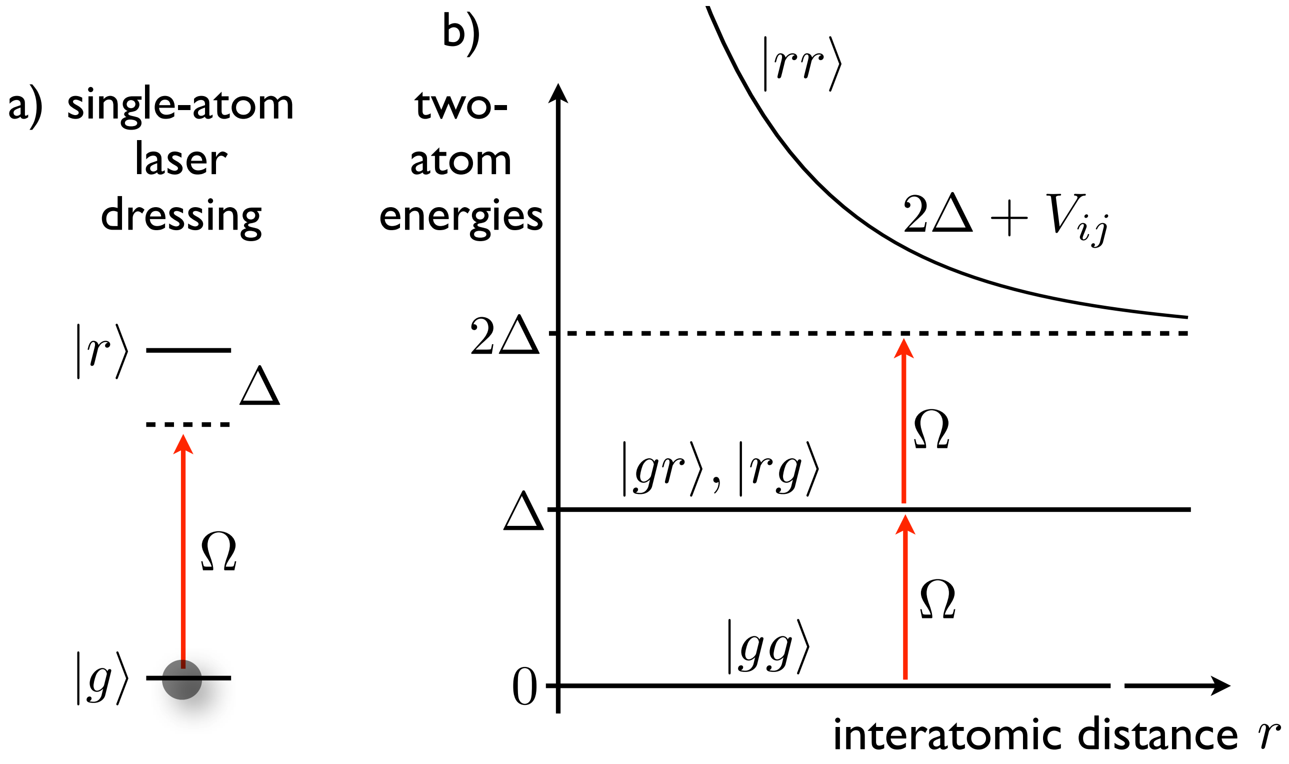

- Perturbation theory up to the fourth order in small parameter $\Omega/\Delta \ll 1$

note: treatment is non-perturbative in the interaction strength V_{ij}

$$(\Delta E)_{|gg\rangle} = 2\frac{\Omega^4}{\Delta^3}\left[1 + \frac{2\Delta}{V_{ij}}\right]^{-1}$$

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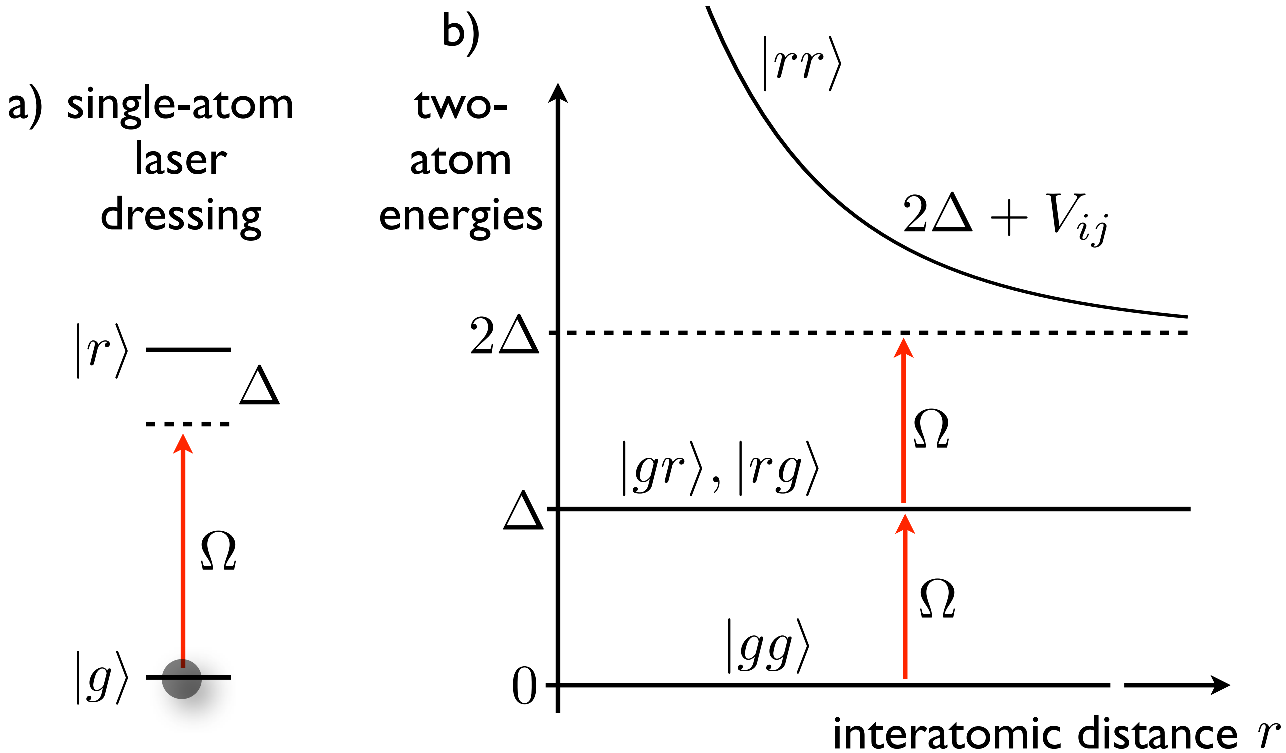

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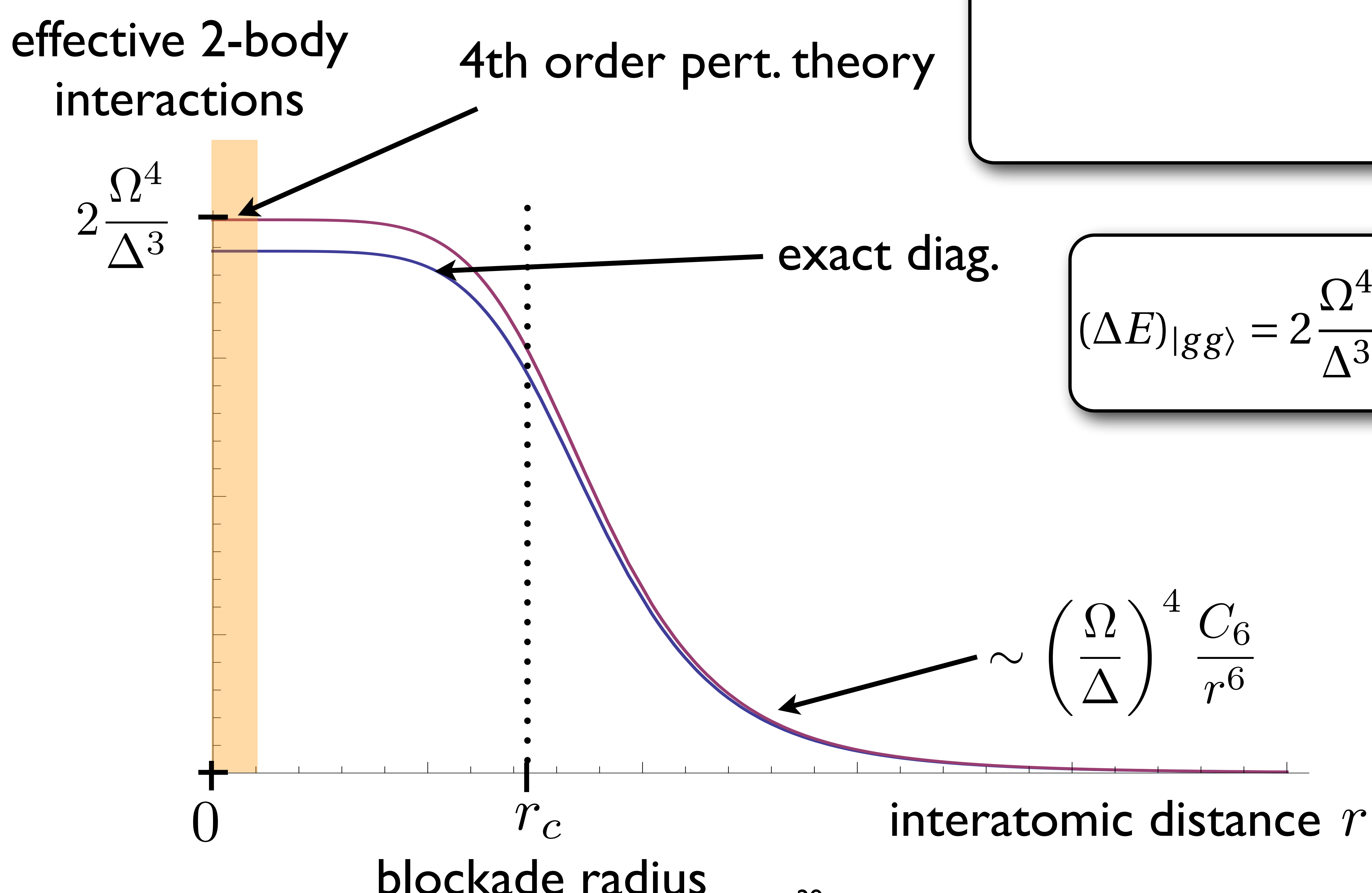
critical Rydberg blockade radius: $2\Delta \approx V_{ij}$

$$r_c = (C_\alpha/2\Delta)^{1/\alpha}$$

Choice of the parameters

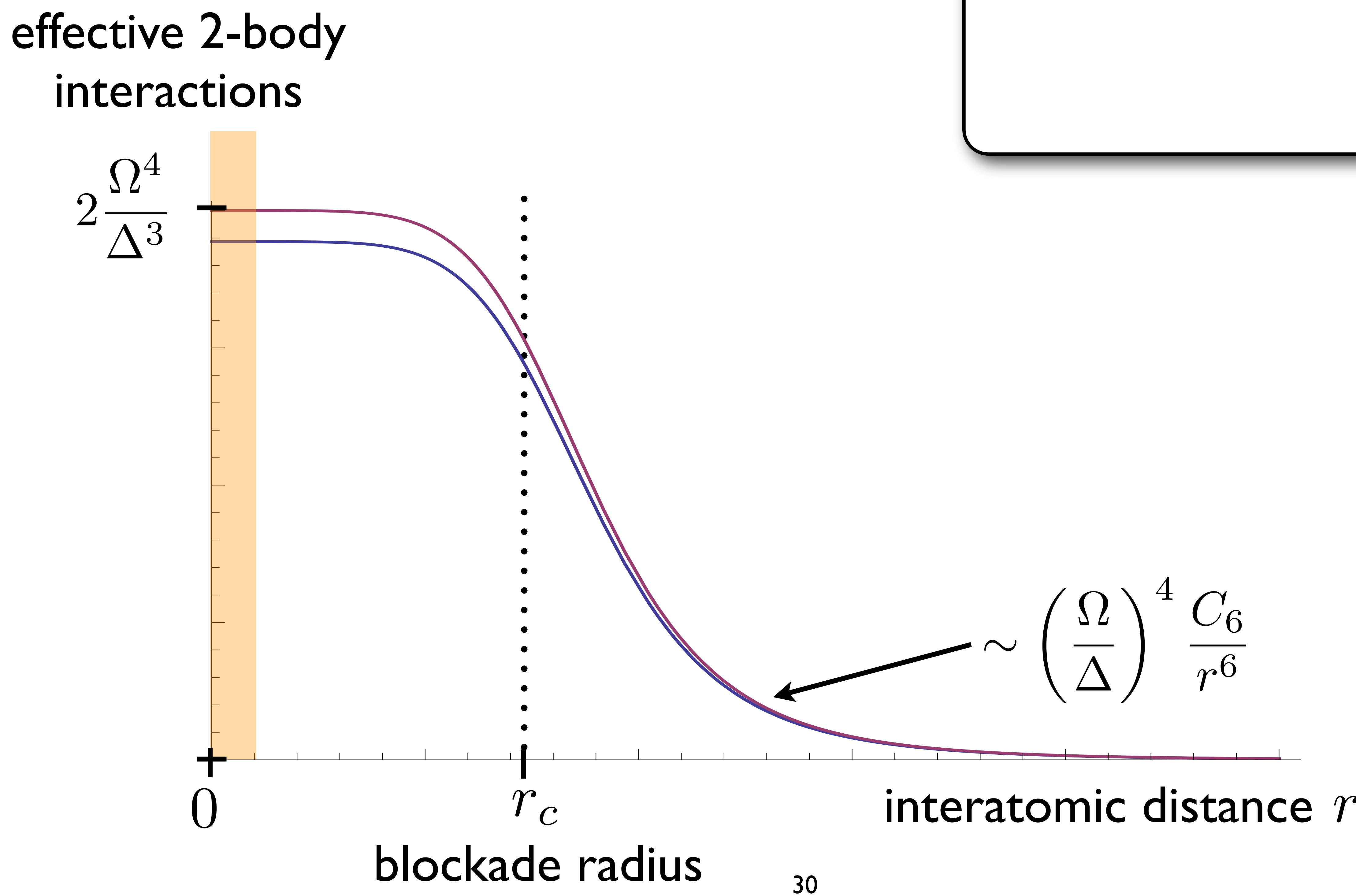
▶ interactions tunable completely independently from hopping t



$$(\Delta E)_{|gg\rangle} = 2\frac{\Omega^4}{\Delta^3}\left[1+\frac{2\Delta}{V_{ij}}\right]^{-1}$$

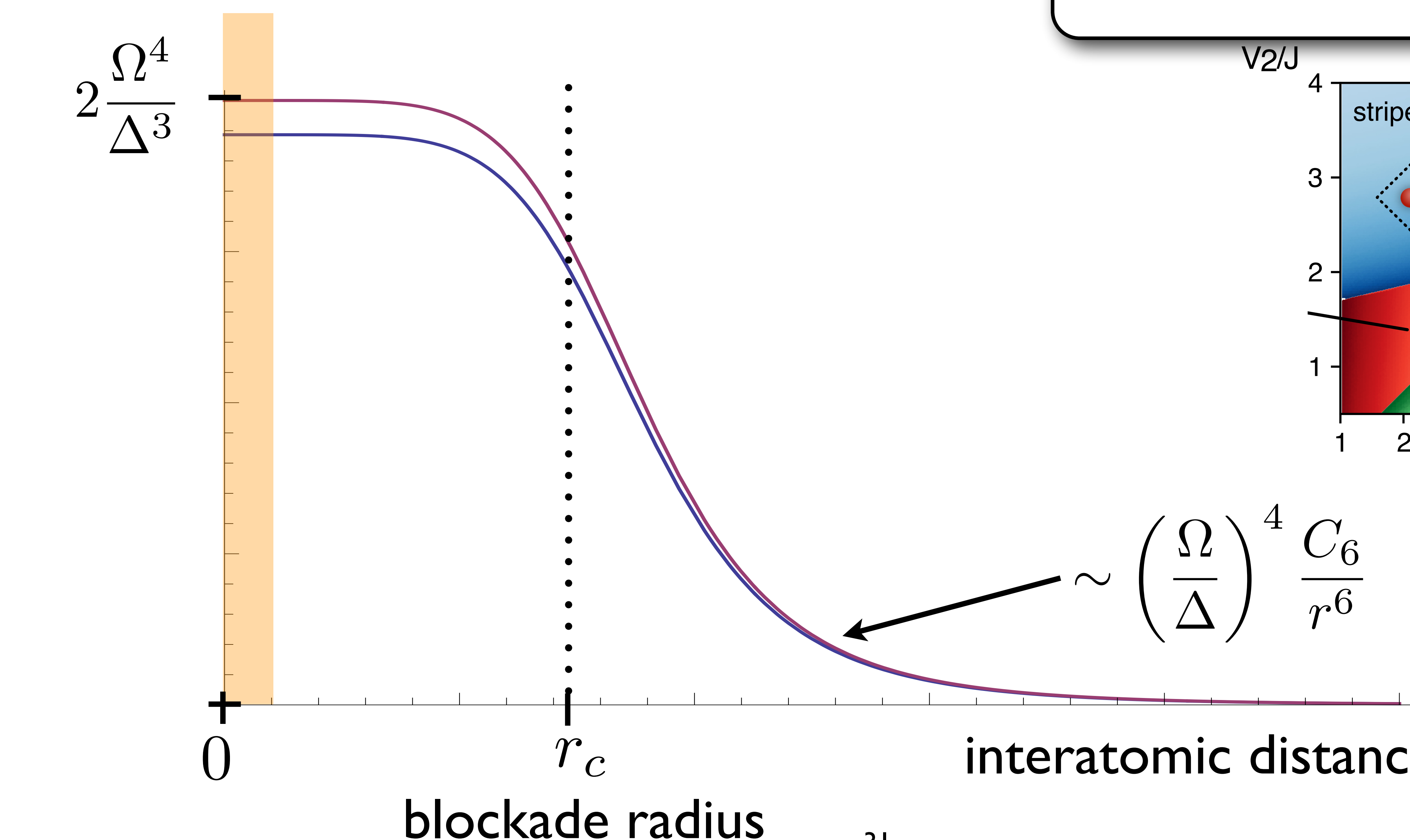
Choice of the parameters

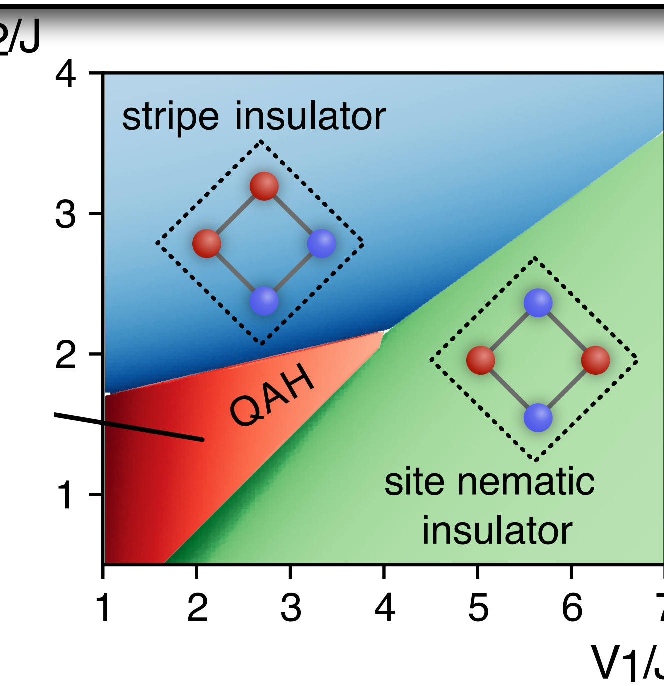
- interactions tunable completely independently from hopping t
- strong V_1 and V_2 interactions



Choice of the parameters

- interactions tunable completely independently from hopping t
- strong and almost equally large V_1 and V_2 interactions
- significantly weaker long-range interactions $V_3, V_4, \ldots$

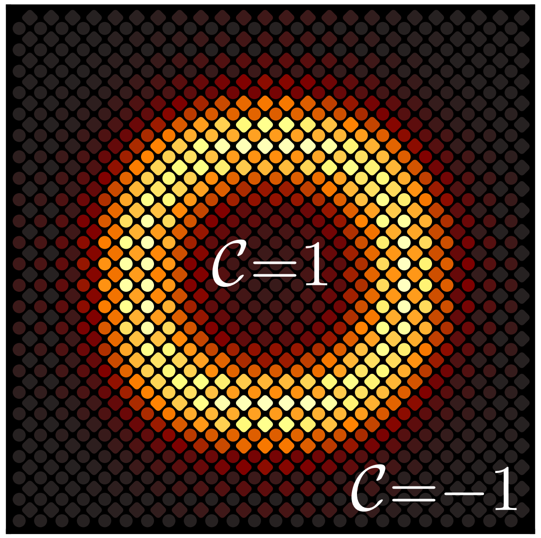




4. Conclusions and Outlook

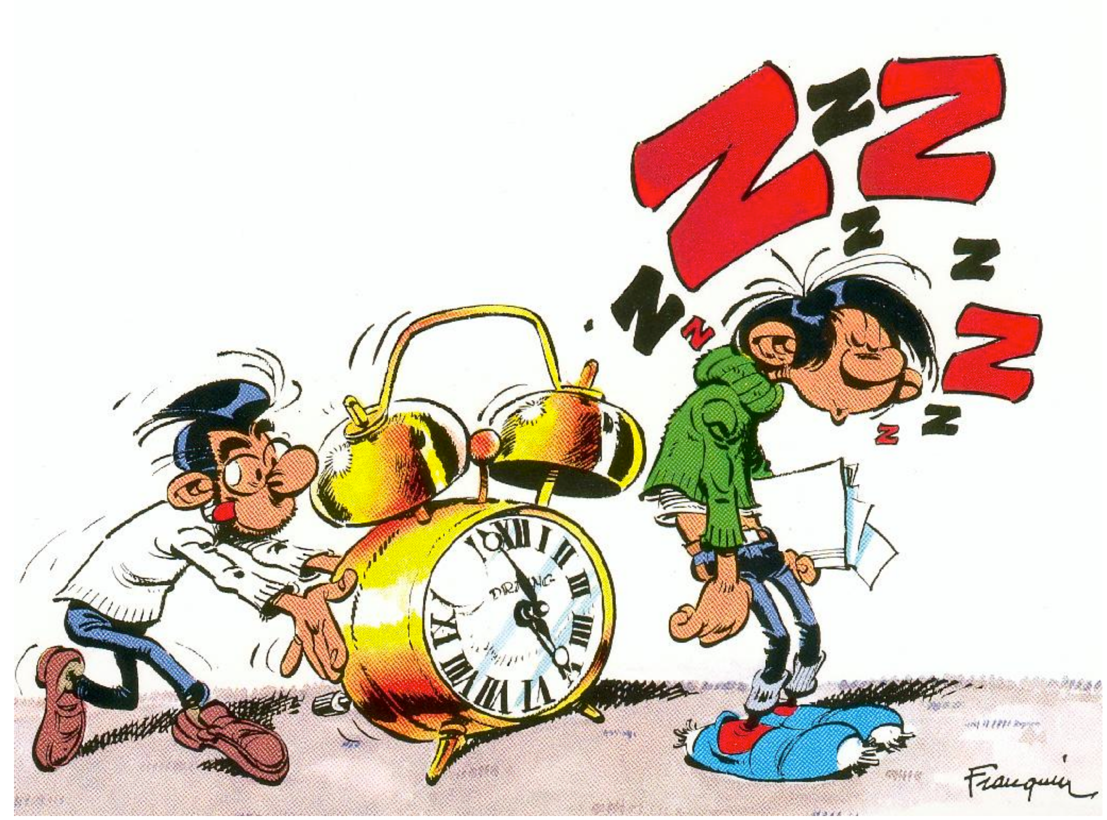
Conclusions and Outlook

- Interaction-induced topological insulators are richer than topological insulators with external gauge fields.
- Away from half filling, the system prefer to deform the lattice and create in gap states than becoming metallic.
- Interplay SSB and topology leads to interaction induced domain walls (Edge states in the bulk)



- Effect of the temperature
- More realistic treatment of the truncation of the interactions.
- Beyond Mean field study: DMRG?

Thank you for
your attention!



Beyond Mean Field: Configuration Interaction

- The polaron solution spontaneously breaks translational invariance.
- Configuration interaction (CI) method:

One hybridizes all the polaron solutions to restore translational invariance and decrease the ground state energy.

$$|\Psi^{CI}\rangle = \sum_{\alpha} \alpha \, |\psi_{\text{polaron}}(\alpha)\rangle$$

Polaron band structure