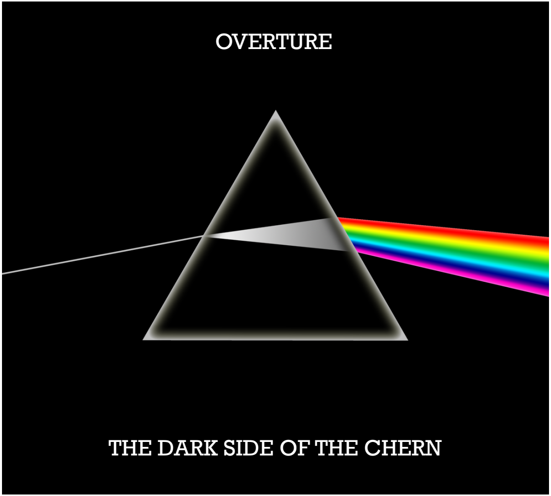
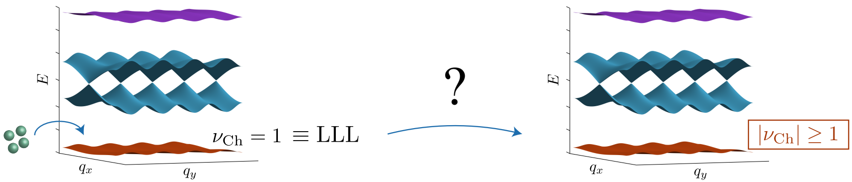
Detecting fractional quantum Hall states of few bosons in an optical lattice

Nathan Goldman

UNIVERSITÉ LIBRE DE BRUXELLES, UNIVERSITÉ D'EUROPE

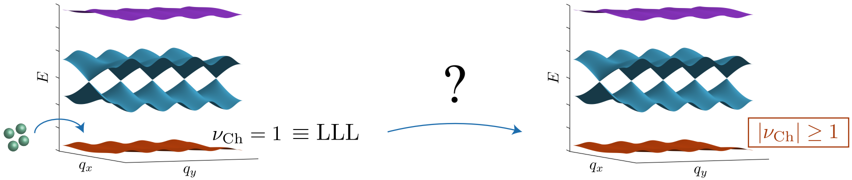

OVERTURE
THE DARK SIDE OF THE CHERN

- **Topic :** Engineering flat Bloch bands with higher Chern number



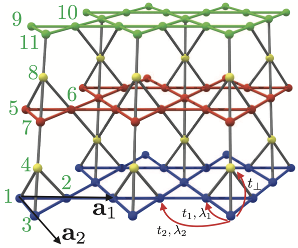
Review : Bergholtz and Liu, Int. J. Mod. Phys. B ’13

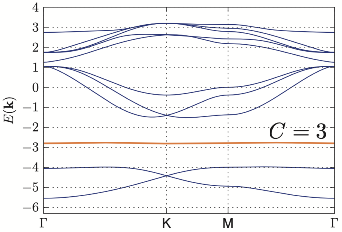
- **Topic :** Engineering flat Bloch bands with higher Chern number



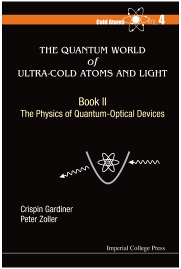
Review : Bergholtz and Liu, Int. J. Mod. Phys. B ’13

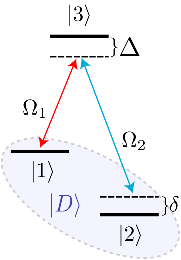
- **Standard approach :** Multilayer lattice models





- **Our approach :** Use the notion of **dark state** (quantum optics)





In the limit $\delta \to 0$:

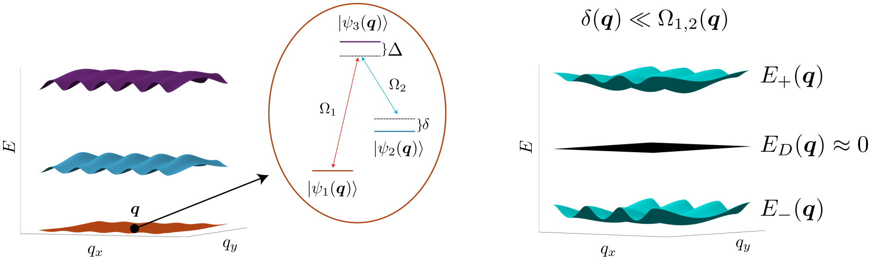
Dark state : $|D\rangle \sim (\Omega_1|2\rangle - \Omega_2|1\rangle)$ $\qquad E_D = 0$

Bright states : $|B_\pm\rangle \sim \left(\frac{\Omega_1^*}{E_\pm}|1\rangle + \frac{\Omega_2^*}{E_\pm}|2\rangle + |3\rangle\right)$ $\qquad E_\pm = f(\Omega_{1,2}, \Delta)$

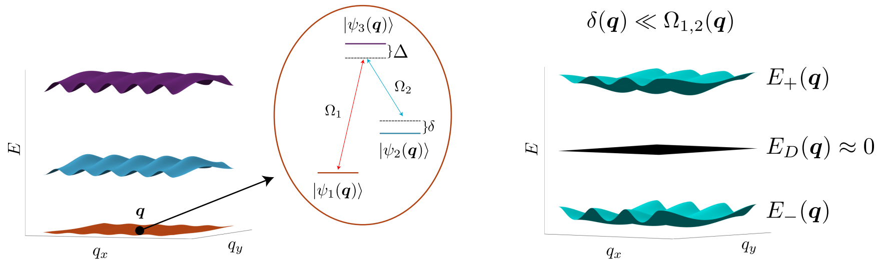
- **Here :** Couple three **Bloch bands** to create a **dark band** ...

Ref : Łącki, Zakrzewski, NG, SciPost Phys. ’21

- **Coupling Bloch bands :** A collection of Lambda systems (one at each $\boldsymbol{q} \in$ FBZ)



- **Coupling Bloch bands :** A collection of Lambda systems (one at each $\boldsymbol{q} \in$ FBZ)



- **The sum rule :** The Chern number of the **dark band** is

$$C_D = C_1 + C_2 - C_3 \qquad C_{1,2,3} \text{ : Chern nb in bare bands}$$

$\longrightarrow$ **Flat band** with **predictable** and **tunable** Chern number !

Ref : Łącki, Zakrzewski, NG, SciPost Phys. '21

- **Details can be found in :**

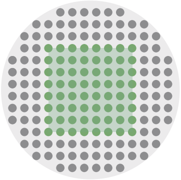
A dark state of Chern bands : Designing flat bands with higher Chern number

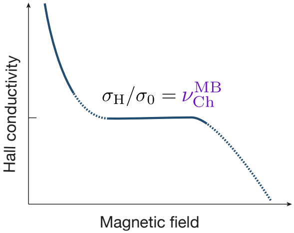
Mateusz Łącki, Jakub Zakrzewski, NG, SciPost Phys. **10,** 112 (2021)

- **Proof of the sum rule**
- **Concrete scheme (Hofstadter bands) and numerical simulations for $C_D = 2$**
- **Loading scheme**
- **Probing scheme**

End of the dark overture

Detecting fractional quantum Hall states of few bosons in an optical lattice



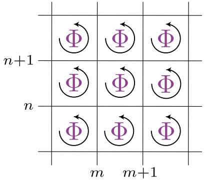


Cécile Repellin, Julian Léonard and NG, Phys. Rev. A **102,** 063316 (2020)

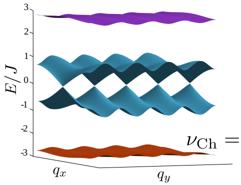
I. Scope and motivations

Quantum-engineered systems and the Harper-Hofstadter model

artificial magnetic flux



Chern bands

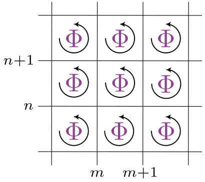


$\nu_{\mathrm{Ch}} = 1 \equiv \mathrm{LLL} \longrightarrow$ Quantum Hall physics !

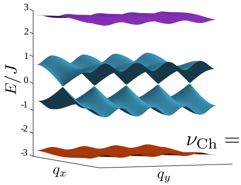
Cold atoms : Cooper, Dalibard, Spielman, RMP ’19
Photonics : Ozawa et al., RMP ’19

- **Quantum-engineered systems and the Harper-Hofstadter model**

artificial magnetic flux



Chern bands



$\nu_{\mathrm{Ch}} = 1 \equiv \mathrm{LLL} \longrightarrow$ Quantum Hall physics !

Cold atoms : Cooper, Dalibard, Spielman, RMP ’19
Photonics : Ozawa et al., RMP ’19

- **A setting for fractional quantum Hall physics**

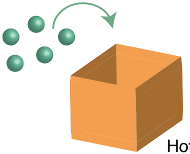
Bosons with strong repulsive interactions at filling factor $\nu = 1/2$

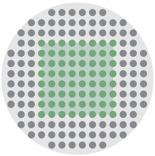
$\longrightarrow$ fractional Chern insulator (**FCI**) akin to the **Laughlin state**

Theory : Sørensen, Demler and Lukin, PRL **94** 086803 (2005)
Scaffidi and Möller, PRL **109** 246805 (2012)

Experimental strategy

Prepare $N \sim 3 - 10$ interacting bosons in few sites (**box** potential)

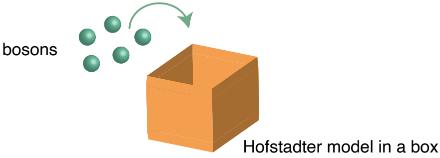


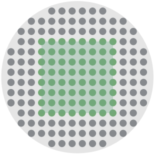


$N = 2$ interacting atoms : Greiner's group, Nature **546** 519 (2017)
$N = 2$ interacting photons : Martinis' group, Nat. Phys. **13** 146 (2017)

- **Experimental strategy**

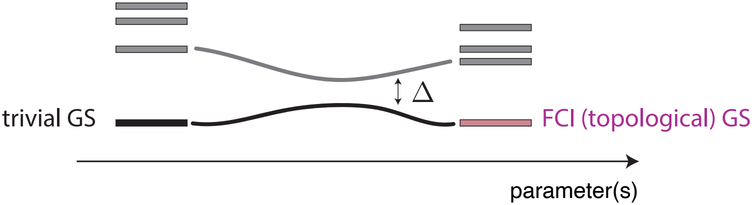
Prepare $N \sim 3-10$ interacting bosons in few sites (**box** potential)





$N=2$ interacting atoms : Greiner's group, Nature **546** 519 (2017)
$N=2$ interacting photons : Martinis' group, Nat. Phys. **13** 146 (2017)

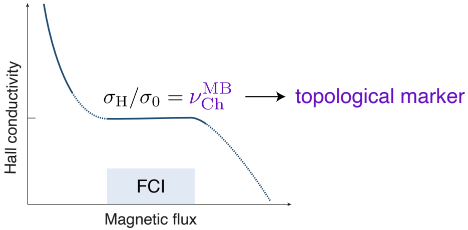
Adiabatic quantum state engineering :



Theory : He, Grusdt et al. PRB **96** 201103 (2017) ; Motruk and Pollmann PRB **96** 165107 (2017)

- **Question :** Phase diagram of this setting ?

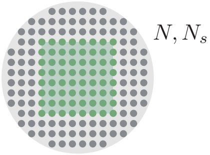
- **Question :** FCI state revealed by **Hall plateau** ?

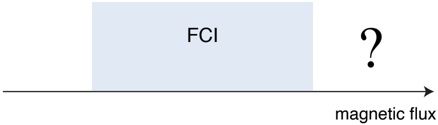


- **Question :** Practical methods to **extract the Hall response** ?

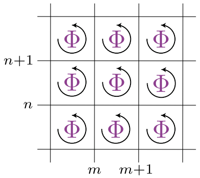
Ref : Repellin, Leonard, NG, PRA **102** 063316 (2020)

II. Phase diagram





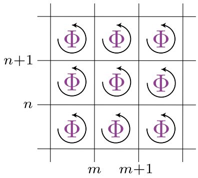
- **Model :** Harper-Hofstadter model



N hardcore bosons in N_s sites ⟶ $\rho = N/N_s$: particle density

α : flux quanta per cell (flux density) ⟶ $\nu = \rho/\alpha$: filling factor

- **Model :** Harper-Hofstadter model



N hardcore bosons in N_s sites $\longrightarrow$ $\rho = N/N_s$: particle density

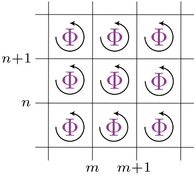
α : flux quanta per cell (flux density) $\longrightarrow$ $\nu = \rho/\alpha$: filling factor

- **Torus (PBC) :** $\nu = \rho/\alpha = 1/2 \longrightarrow$ **FCI** with $\nu_{\text{Ch}}^{\text{MB}} = 1/2$

Optimal flux window : $\alpha \approx 0.2 - 0.25 \rightarrow$ maximal MB gap

Theory : Hafezi, Sørensen, Demler and Lukin PRA **76** 023613 (2007)

- **Model :** Harper-Hofstadter model



N hardcore bosons in N_s sites $\longrightarrow$ $\rho = N/N_s$: particle density

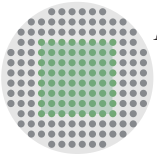
α : flux quanta per cell (flux density) $\longrightarrow$ $\nu = \rho/\alpha$: filling factor

- **Torus (PBC) :** $\nu = \rho/\alpha = 1/2 \longrightarrow$ **FCI** with $\nu_{\text{Ch}}^{\text{MB}} = 1/2$

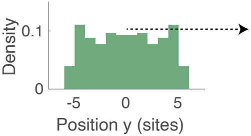
Optimal flux window : $\alpha \approx 0.2 - 0.25 \rightarrow$ maximal MB gap

Theory : Hafezi, Sørensen, Demler and Lukin PRA **76** 023613 (2007)

- **Box (OBC) :** Filling factor in the bulk $\nu_{\text{bulk}} = \rho_{\text{bulk}}/\alpha$



N, N_s



$\rho_{\rm bulk} = \rho_{\rm bulk}(\alpha)$

$\neq N/N_s$

$\longrightarrow$ difficult to predict FCI regime for small systems !

see also Raciunas, Unal, Anisimovas, Eckardt, PRA **98**, 063621 (2018)

Illustration using $N = 4$ bosons in $N_s = 60$ lattice sites :

- **Naive** approach (neglect edge) : the total density is $\rho = N/N_s \approx 0.07$
  $\nu = \rho/\alpha \approx 1/2 \Longrightarrow \alpha \approx 0.13 \longrightarrow$ far from optimal flux window $\alpha \sim 0.2 - 0.25$
  (a priori, not an excellent choice !)

Illustration using $N = 4$ bosons in $N_s = 60$ lattice sites :

- **Naive** approach (neglect edge) : the total density is $\rho = N/N_s \approx 0.07$

  $\nu = \rho/\alpha \approx 1/2 \Longrightarrow \alpha \approx 0.13 \longrightarrow$ far from optimal flux window $\alpha \sim 0.2\text{–}0.25$

  (a priori, not an excellent choice !)

- An **intuitive** approach : analogy with FQH effect in Landau levels (continuum)

  Laughlin state of N bosons at $\nu = 1/2 \longrightarrow$ occupies $N_o = 2N - 1$ LLL orbitals

  How many orbitals in the lowest Hofstadter band for $\alpha \approx 0.2$ and $N_s = 60$ sites ?

Illustration using $N = 4$ bosons in $N_s = 60$ lattice sites :

- **Naive** approach (neglect edge) : the total density is $\rho = N/N_s \approx 0.07$

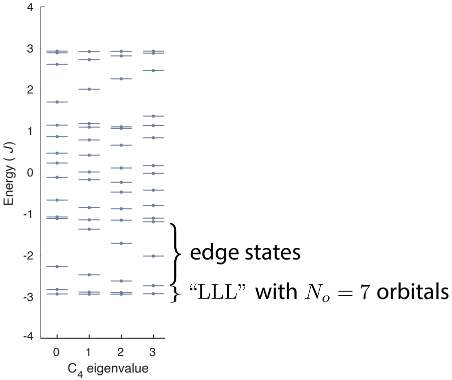
  $\nu = \rho/\alpha \approx 1/2 \Longrightarrow \alpha \approx 0.13 \longrightarrow$ far from optimal flux window $\alpha \sim 0.2$–0.25

  (a priori, not an excellent choice !)

- An **intuitive** approach : analogy with FQH effect in Landau levels (continuum)

  Laughlin state of N bosons at $\nu = 1/2 \longrightarrow$ occupies $N_o = 2N - 1$ LLL orbitals

  How many orbitals in the lowest Hofstadter band for $\alpha \approx 0.2$ and $N_s = 60$ sites ?



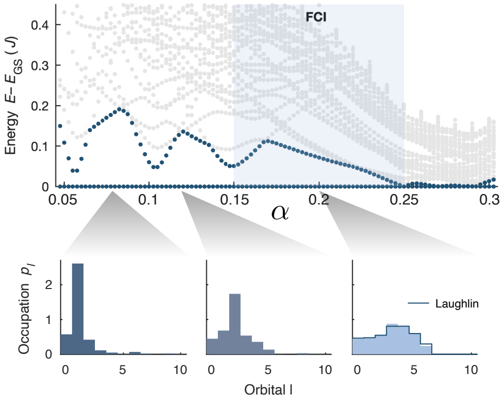
Setting (analogy with Laughlin):

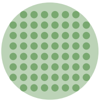
$$N_o = 7 = 2N - 1$$

$$\longrightarrow N = 4 \text{ bosons}$$

it is a **good candidate** after all!

- **Ground state analysis** for $N = 4$ bosons in $N_s = 60$ lattice sites



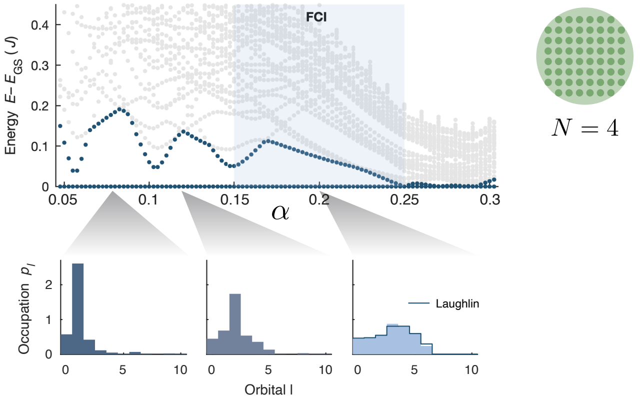


$N = 4$

- Avoided crossings : finite-size signatures of phase transitions

- In the range $\alpha = 0.15 - 0.25$: phase compatible with **FCI**

- **Ground state analysis** for $N = 4$ bosons in $N_s = 60$ lattice sites



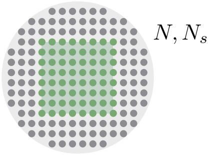
- Avoided crossings : finite-size signatures of phase transitions

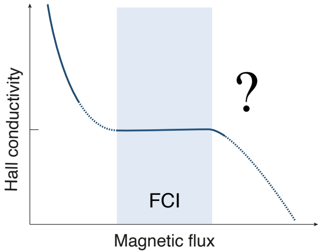
- In the range $\alpha = 0.15 - 0.25$: phase compatible with **FCI**

- **Topology of FCI candidate** : confirmed by particle entanglement spectrum

Ref : Repellin, Leonard, NG, PRA **102** 063316 (2020)

III. Quantized Hall response





- **Hall response** in small atomic system ?

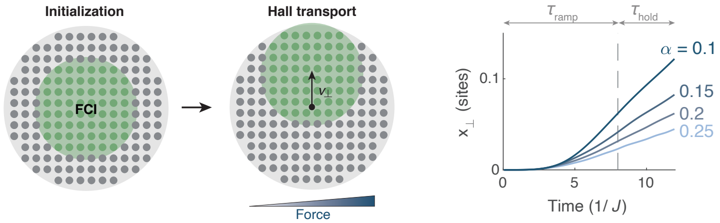
  Issues : local currents fluctuate, edge effects, ...

- **Hall response** in small atomic system ?

  Issues : local currents fluctuate, edge effects, ...

$\Longrightarrow$ monitor **center-of-mass** Hall drift upon **release** into larger lattice

Cold atoms : Dauphin and NG, PRL **111** 135302 (2013)
Photonics : Ozawa and Carusotto, PRL **112** 133902 (2014)

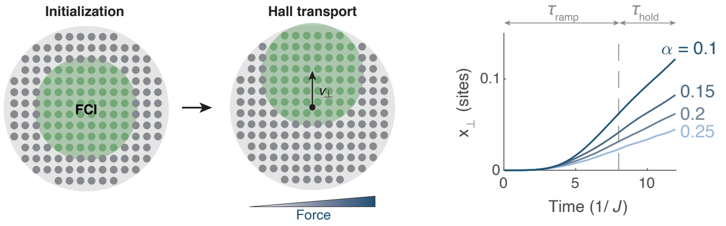


- **Hall response** in small atomic system ?

  Issues : local currents fluctuate, edge effects, ...

$\Longrightarrow$ monitor **center-of-mass** Hall drift upon **release** into larger lattice

Cold atoms : Dauphin and NG, PRL **111** 135302 (2013)
Photonics : Ozawa and Carusotto, PRL **112** 133902 (2014)



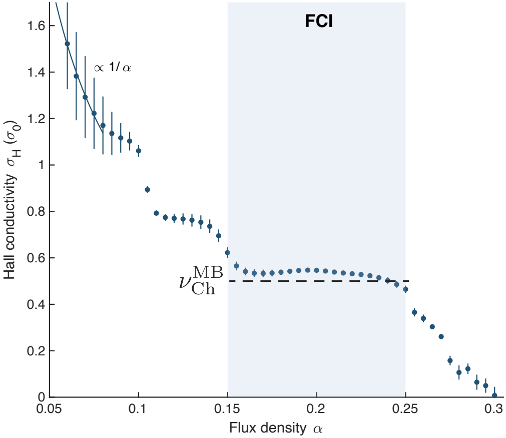
- **Transport equation**

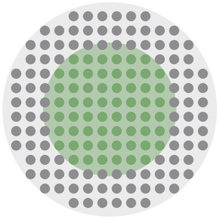
$$\sigma_{\mathrm{H}}/\sigma_0 = (2\pi\rho_{\mathrm{bulk}}/F)\, v_\perp , \qquad \sigma_0 = 1/2\pi$$

$\longrightarrow$ Measure σ_{H} from **Hall drift ($v_\perp$)** and **bulk density (ρ_{bulk})**

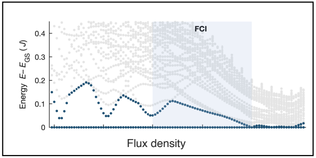
Ref : Repellin, Leonard, NG, PRA **102** 063316 (2020)

- **Hall conductivity from Hall drift** ($N = 4$ bosons in $N_s = 60$ sites)





Ground-state analysis

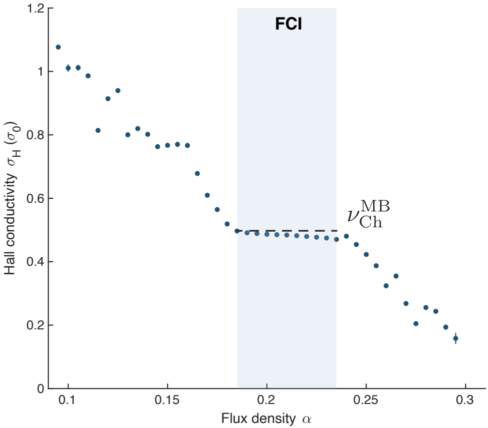


- **Hall plateau** at $\sigma_{\rm H}/\sigma_0 \approx 1/2 \rightarrow$ **topological marker for FCI** at $\nu = 1/2$
- Width of the plateau compatible with GS properties (many-body gap, PES)

Ref : Repellin, Leonard, NG, PRA **102** 063316 (2020)

- **Convergence of Hall plateau** at $\sigma_{\mathrm{H}}/\sigma_0 \approx 1/2$

DMRG results for $N = 10$ bosons in $N_s = 120$ lattice sites :



Hall drift : practical method to estimate **many-body Chern number** of few-boson FCIs

Ref : Repellin, Leonard, NG, PRA **102** 063316 (2020)
see also Motruk and Na, PRL **125**, 236401 (2020)

- **Density response** to magnetic perturbations

  Streda's formula : $\sigma_{\rm H}/\sigma_0 = \frac{\partial \rho_{\rm bulk}}{\partial \alpha}$ within an incompressible phase

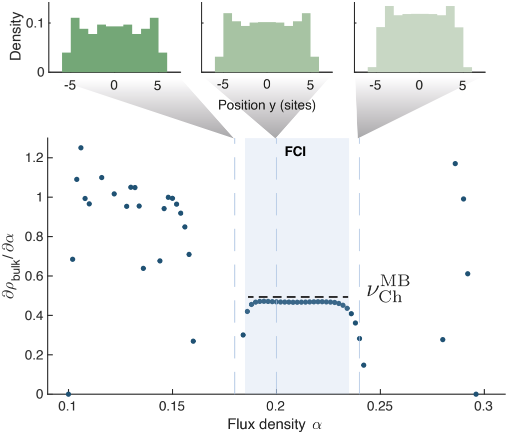
  $\longrightarrow$ measuring **bulk density** $\rho_{\rm bulk}(\alpha)$ **reveals FCI !**

- **Density response** to magnetic perturbations

  Streda's formula : $\sigma_{\mathrm{H}}/\sigma_0 = \frac{\partial \rho_{\mathrm{bulk}}}{\partial \alpha}$ within an incompressible phase

  $\longrightarrow$ measuring **bulk density** $\rho_{\mathrm{bulk}}(\alpha)$ **reveals FCI !**

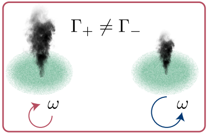
- DMRG results for $N = 10$ bosons in $N_s = 120$ lattice sites

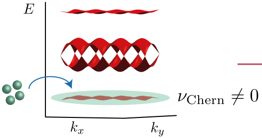


- The "**Streda plateau**" perfectly matches the Hall-drift plateau !

Ref : Repellin, Leonard, NG, PRA **102** 063316 (2020)

- **Other recent proposals** :
    - **Circular dichroism** :

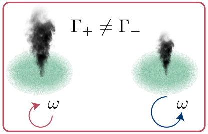


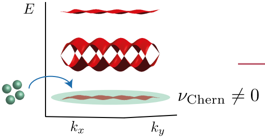


$$\int \Gamma_+ - \Gamma_- \, d\omega \;\sim\; \nu_{\rm Ch}^{\rm MB}$$ **quantized**

Th : Repellin and NG, PRL **122**, 166801 (2019)
Exp (non-int.) : Asteria et al., Nat. Phys. **15**, 449 (2019)

- **Other recent proposals** :
  - **Circular dichroism** :





$$\int \Gamma_+ - \Gamma_- \, d\omega \;\sim\; \nu_{\mathrm{Ch}}^{\mathrm{MB}}$$ **quantized**

Th : Repellin and NG, PRL **122**, 166801 (2019)
Exp (non-int.) : Asteria et al., Nat. Phys. **15**, 449 (2019)

  - **Interferometry with impurities bound to quasiholes**

Ref : Grusdt, Yao, Abanin, Fleischhauer, Demler, Nat. Comm. **7**, 11994 (2016)

  - **Fractional charge pumping (local flux insertion)**

Ref : Raciunas, Unal, Anisimovas, Eckardt, PRA **98**, 063621 (2018)

  - **Signatures of anyonic statistics in time-of-flight**

Ref : Umucalilar, Macaluso, Comparin, Carusotto, PRL **120**, 230403 (2018)

  - **Statistical correlation between randomized measurements**

Ref : Cian, Dehghani, Elben, Vermersch, Zhu, Barkeshli, Zoller, Hafezi, PRL (2021)

- **Take-home message 1** :

  Center-of-mass Hall drift reveals quantized plateaus in few-boson settings

$\longrightarrow$ Hall drift provides a **practical topological signature**

- **Take-home message 2** :

  Density response (Streda) yields clear Hall plateau for $N \gtrsim 10$ bosons

$\longrightarrow$ Measuring $\rho_{\mathrm{bulk}}(\alpha)$ provides a **practical topological signature**

Reference : Repellin, Leonard, NG, PRA **102** 063316 (2020)

Slides are available upon request

Appendices

- **Topological marker** : the particle entanglement spectrum (**PES**)

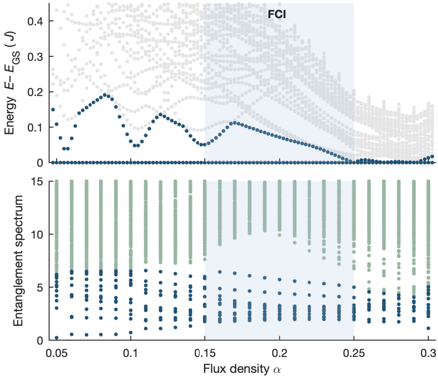
Ref : Sterdyniak, Regnault, Bernevig, PRL '11

- One sets a bi-partition $N = N_A + N_B$, while keeping geometry fixed

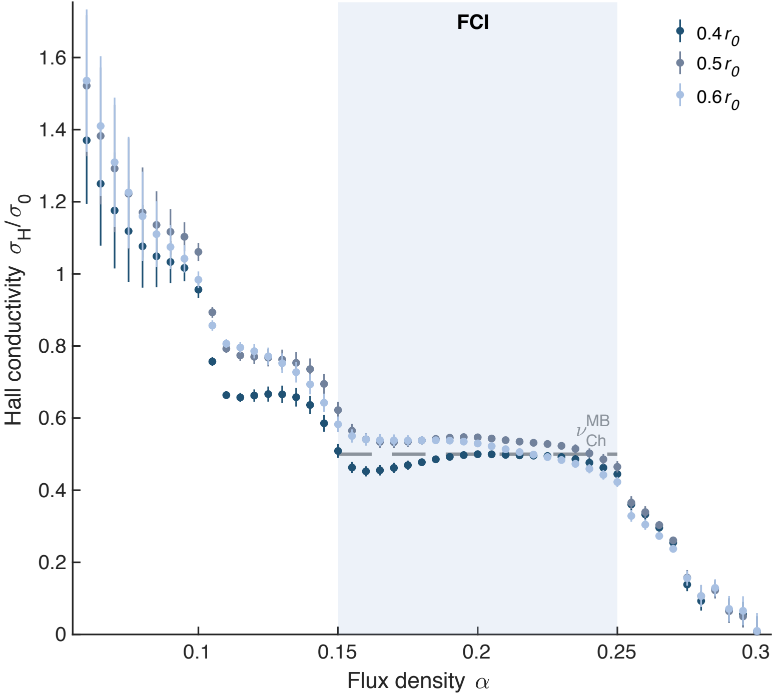
  and one considers the reduced density matrix

$$\hat{\rho}_A = \mathrm{Tr_B}\, \hat{\rho} = \exp(-\mathrm{H}) \text{ , where } \hat{\rho} = |\Psi_{\mathrm{GS}}\rangle\langle\Psi_{\mathrm{GS}}|$$

- The PES is defined as the spectrum of H



- Nb of states below the PES gap = nb of available quasi-hole states which is set by Haldane's exclusion principle (fractional statistics)
- For $N_A = 2$ in $N_o = 7$ orbitals, $\#\mathrm{qh{-}states} = 15 \longrightarrow$ **FCI** confirmed



FCI
0.4 r0
0.5 r0
0.6 r0
Hall conductivity σH/σ0
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0.1
0.15
0.2
0.25
0.3
Flux density α
ν MB Ch