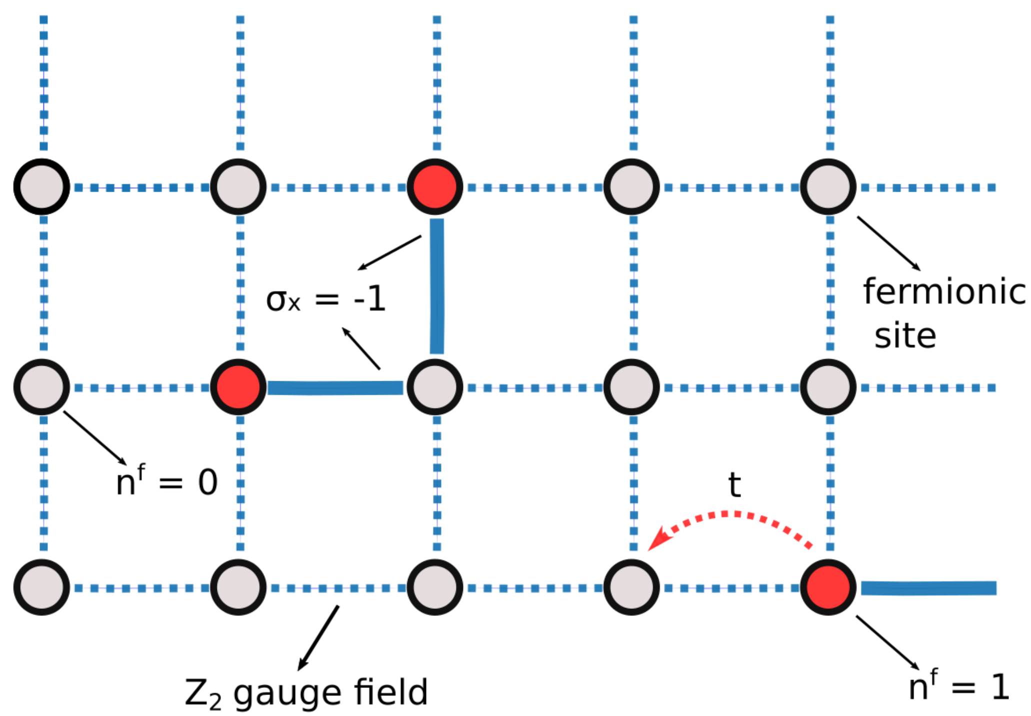
Z_2 gauge theory coupled to fermion matter:
from topological order to confinement and fractons



TECHNISCHE UNIVERSITÄT MÜNCHEN

MCQST

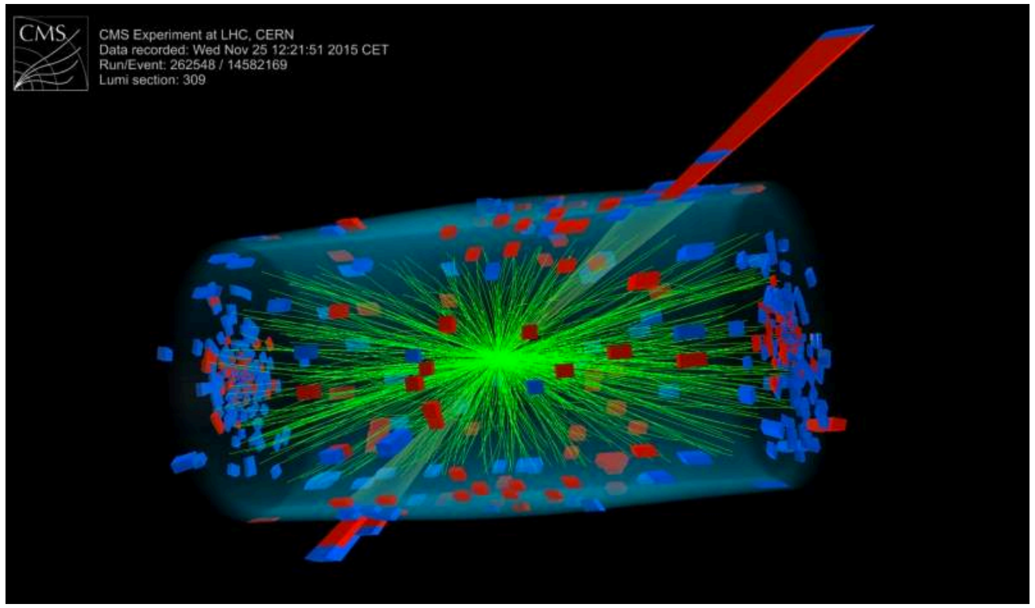
Emmy Noether-Programm
Deutsche Forschungsgemeinschaft
DFG

together with
Umberto Borla, Bhilahari Jeevanesan and Frank Pollmann

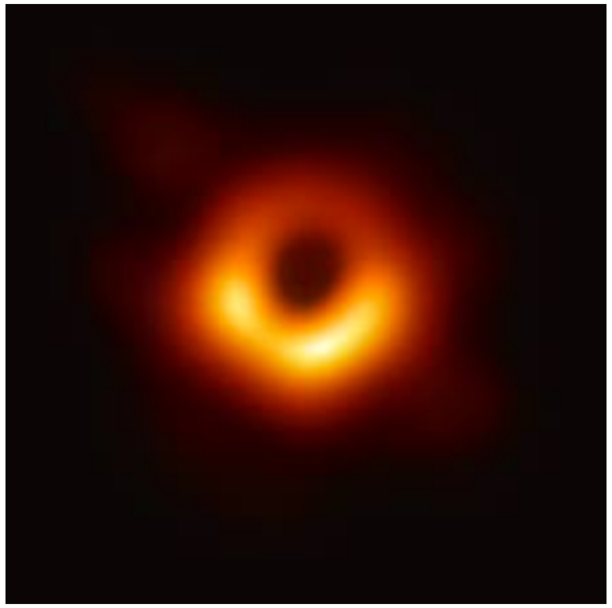
arXiv:2012.08543

Gauge theories

- General theory of relativity
- Standard model of particle physics

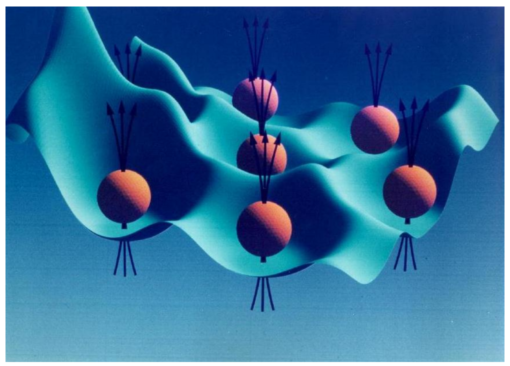


CMS, LHC

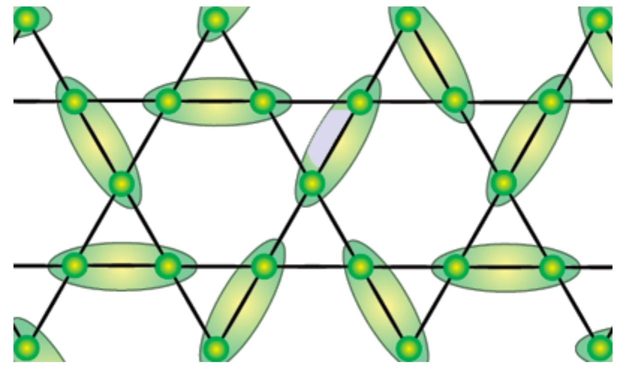


Event Horizon Telescope Team

Emergent gauge theories

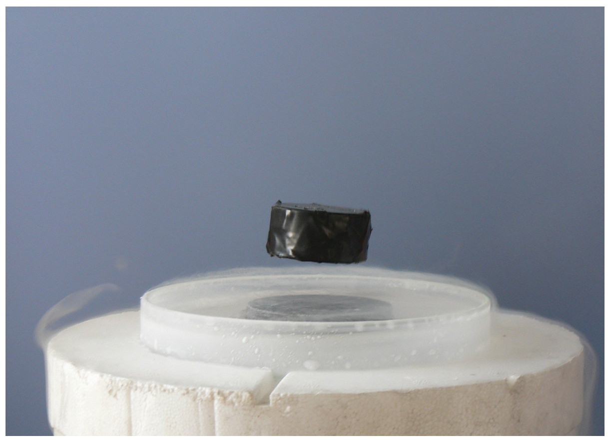


fractional quantum Hall fluids



L. Clark

quantum spin liquids

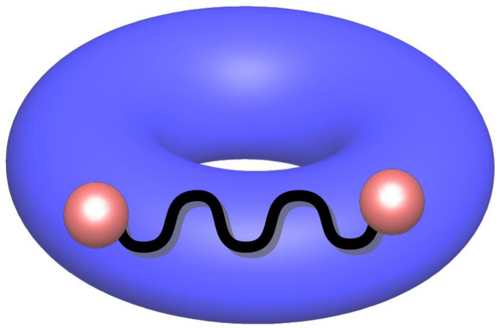


Wikipedia

superconductors

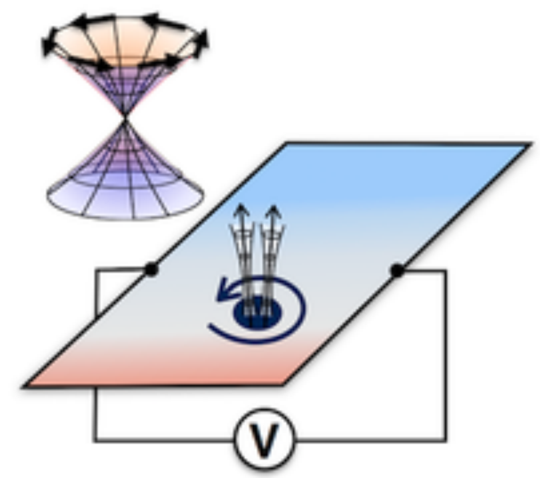
- Anyons
- Ground state degeneracy
- Long-range entanglement

Wen
Kitaev

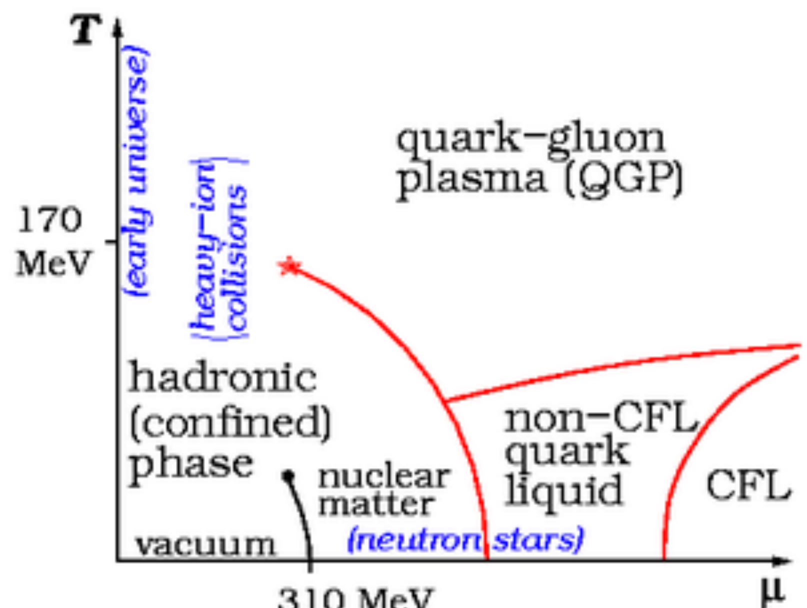


Gauge theories coupled to fermion matter

- Half-filled Landau level

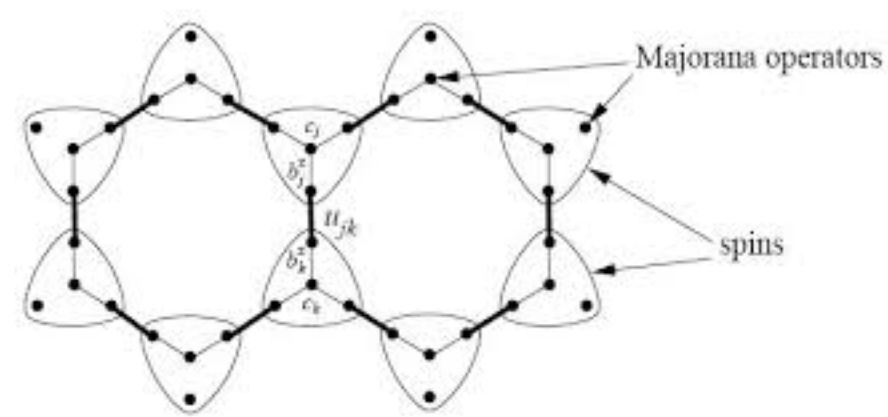


- QCD



Wikipedia

- Kitaev spin liquids



Kitaev2006

Z_2 gauge theory

JOURNAL OF MATHEMATICAL PHYSICS VOLUME 12, NUMBER 10 OCTOBER 1971

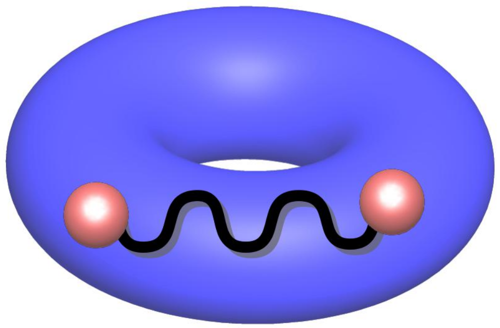
Duality in Generalized Ising Models and Phase Transitions without Local Order Parameters*

Franz J. Wegner†
Department of Physics, Brown University, Providence, Rhode Island 02912
(Received 29 March 1971)

It is shown that any Ising model with positive coupling constants is related to another Ising model by a duality transformation. We define a class of Ising models M_{dn} on d-dimensional lattices characterized by a number $n = 1, 2, \ldots, d$ ($n = 1$ corresponds to the Ising model with two-spin interaction). These models are related by two duality transformations. The models with $1 < n < d$ exhibit a phase transition without local order parameter. A nonanalyticity in the specific heat and a different qualitative behavior of certain spin correlation functions in the low and the high temperature phases indicate the existence of a phase transition. The Hamiltonian of the simple cubic dual model contains products of four Ising spin operators. Applying a star square transformation, one obtains an Ising model with competing interactions exhibiting a singularity in the specific heat but no long-range order of the spins in the low temperature phase.

Simplest gauge theory we can define on a lattice

Z_2 gauge theory



Discrete cousin of electrodynamics

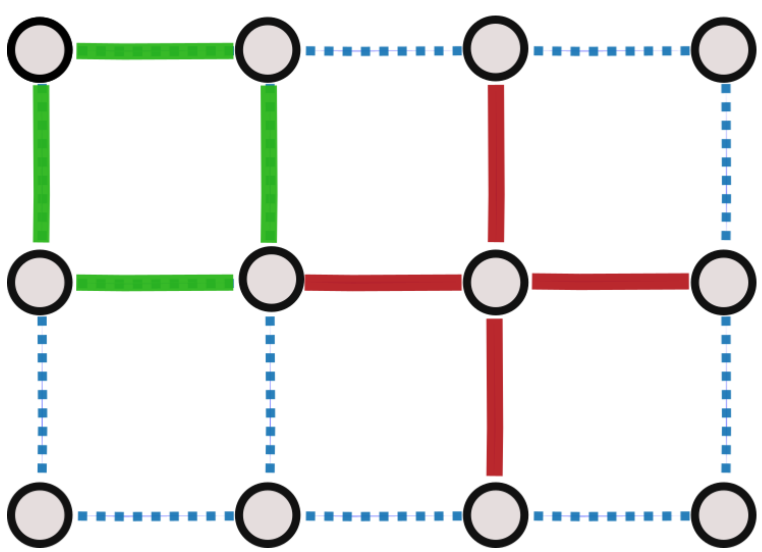
Wegner 1971
Kogut 1979

$$H = -J \sum_{\mathbf{r}*} \prod_{b \in \square_{\mathbf{r}*}} \sigma_b^z - h \sum_{\mathbf{r},\eta} \sigma_{\mathbf{r},\eta}^x$$

Correspondence

$$\sigma^z \sim e^{iA}$$

$$\sigma^x \sim e^{iE}$$



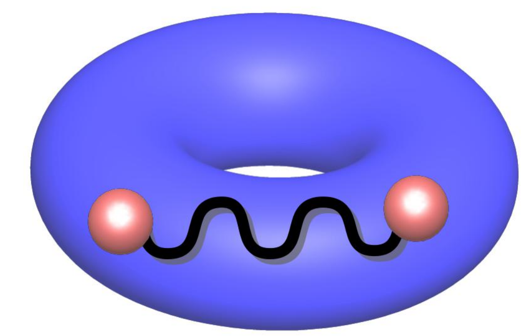
Z_2 gauge transformations

$$G_{\mathbf{r}} = \prod_{b \in +_{\mathbf{r}}} \sigma_b^x$$

Gauss' law: $G_{\mathbf{r}} = 1$

no static charges

Z_2 gauge theory



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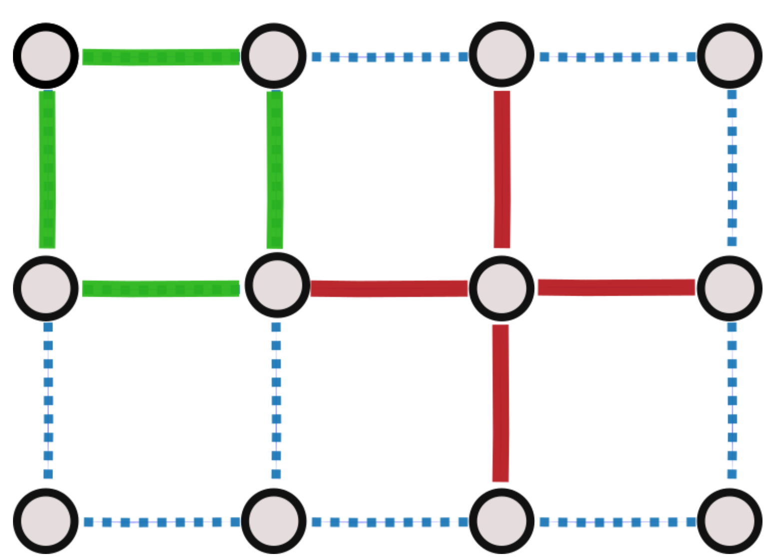
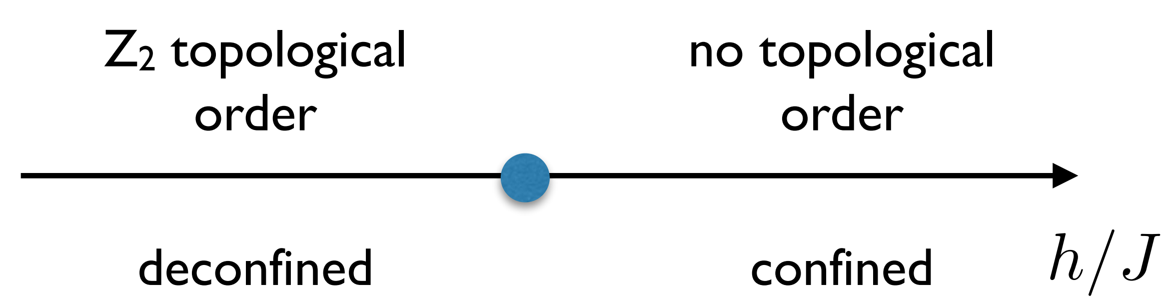
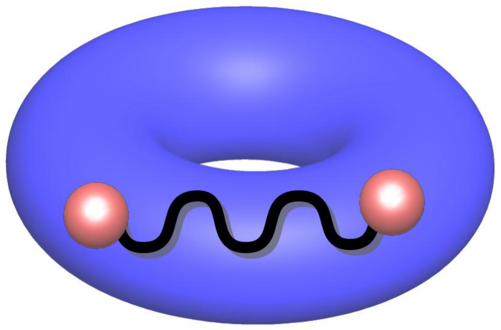


Phase transition without local order parameter



Z_2 gauge theory



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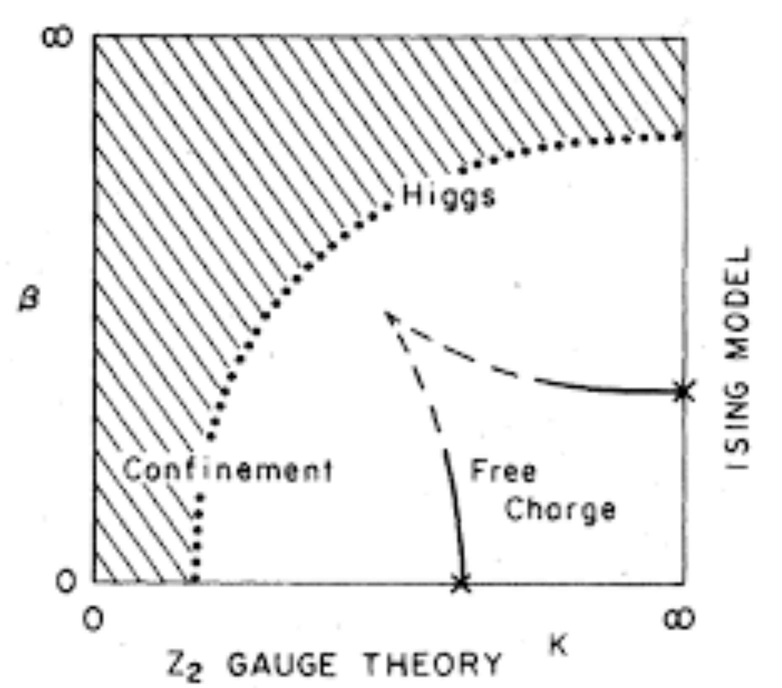
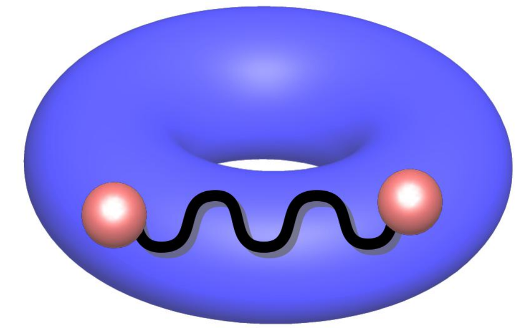
Adding dynamical
Ising matter



Fradkin&Shenker 1979

Z_2 gauge theory



Discrete cousin of electrodynamics

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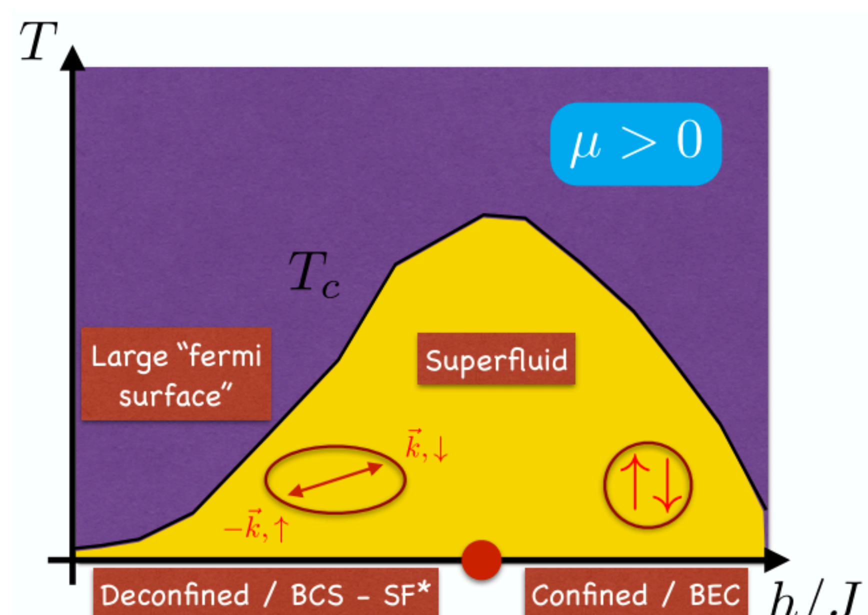
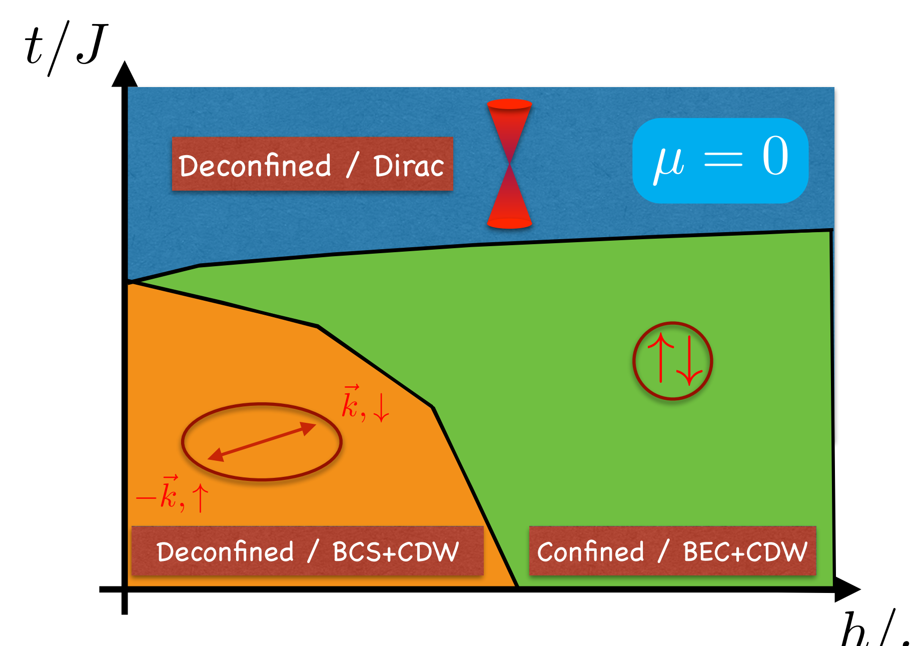
Correspondence

$$\sigma^z \sim e^{iA}$$
$$\sigma^x \sim e^{iE}$$

Adding dynamical two-component fermion matter

Senthil&Fisher 2000
Assad&Grover 2016
Gazit et al 2017, 18, 20
...



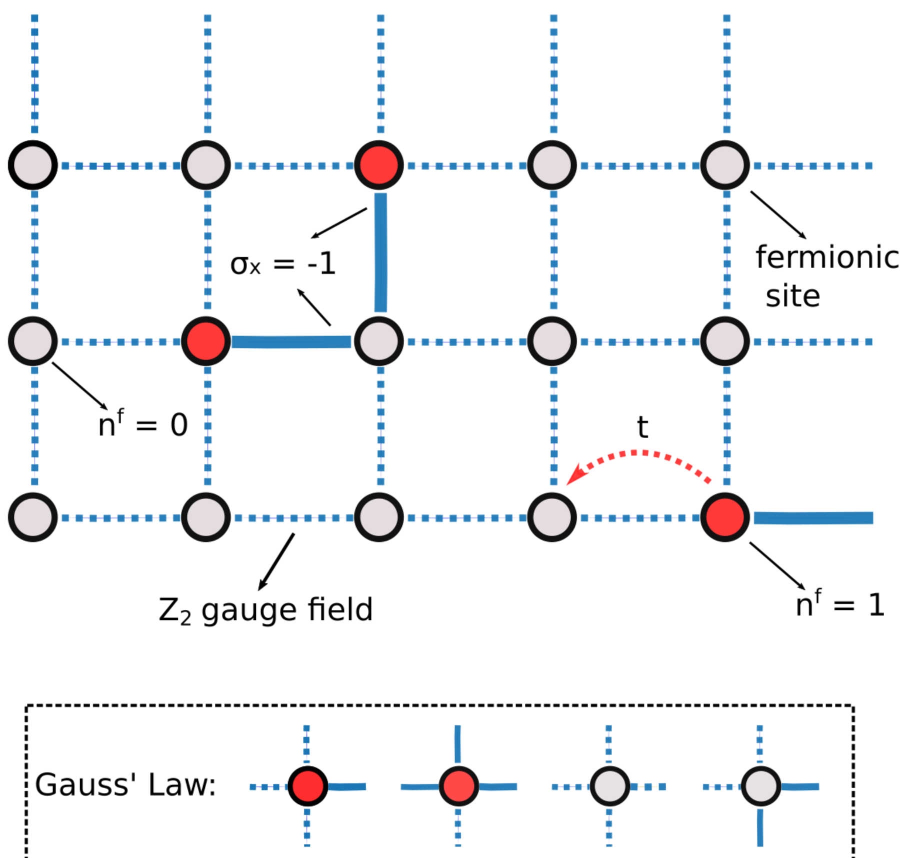


sign-problem-free QMC

Adding spinless fermion matter

Borla, Jeevanesan, Pollmann, Moroz 2012.08543

$$H_f = -t\sum_{\mathbf{r},\eta}\left(c^\dagger_{\mathbf{r}}\sigma^z_{\mathbf{r},\eta}c_{\mathbf{r}+\eta} + \text{h.c.}\right) - \mu\sum_{\mathbf{r}} c^\dagger_{\mathbf{r}}c_{\mathbf{r}}$$



Z2 gauge transformations modified

$$G_{\mathbf{r}} = (-1)^{n_{\mathbf{r}}}\prod_{b\in +_{\mathbf{r}}}\sigma^x_b$$

We will set everywhere

$$\prod_{b\in +\mathbf{r}}\sigma^x_b = (-1)^{n_{\mathbf{r}}}$$

Gauging fermion parity

Any fermionic model has an unbreakable Z_2 symmetry

$$P = \prod_{\mathbf{r}} (-1)^{n_{\mathbf{r}}} = (-1)^N$$

We gauge this symmetry

$$\prod_{b \in +\mathbf{r}} \sigma_b^x = (-1)^{n_{\mathbf{r}}}$$

After gauging the model becomes bosonic

$$P = +1$$

Adding spinless fermion matter

Borla, Jeevanesan, Pollmann, Moroz 2012.08543

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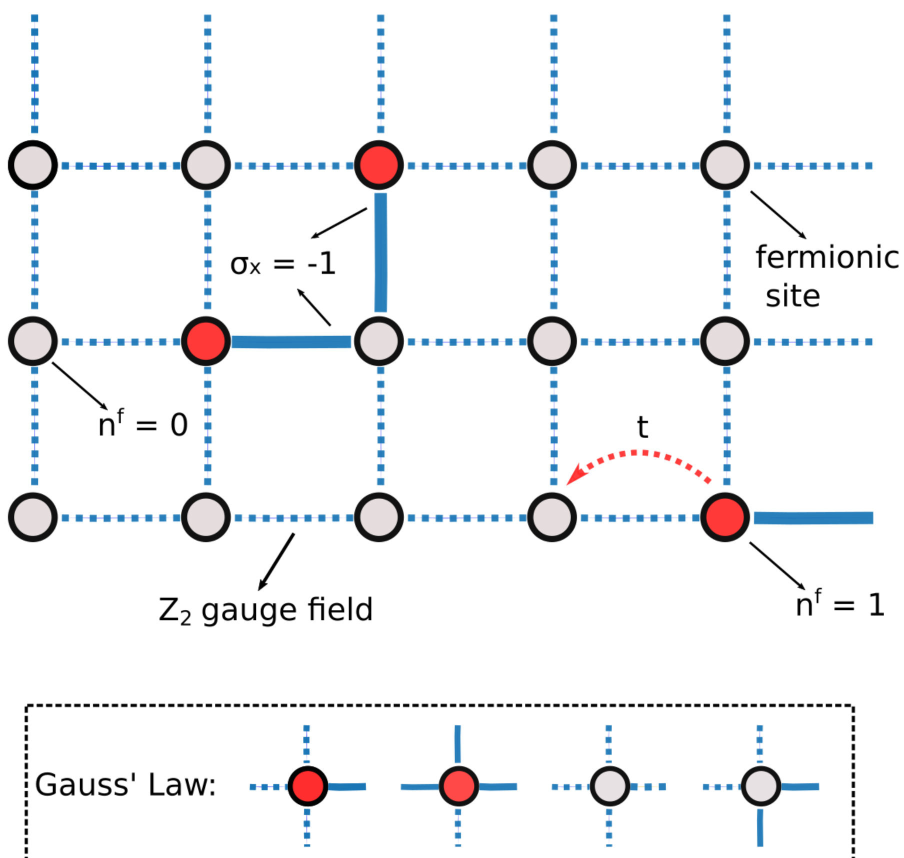


U(1) global symmetry

$$c_{\mathbf{r}} \to e^{i\alpha} c_{\mathbf{r}}$$

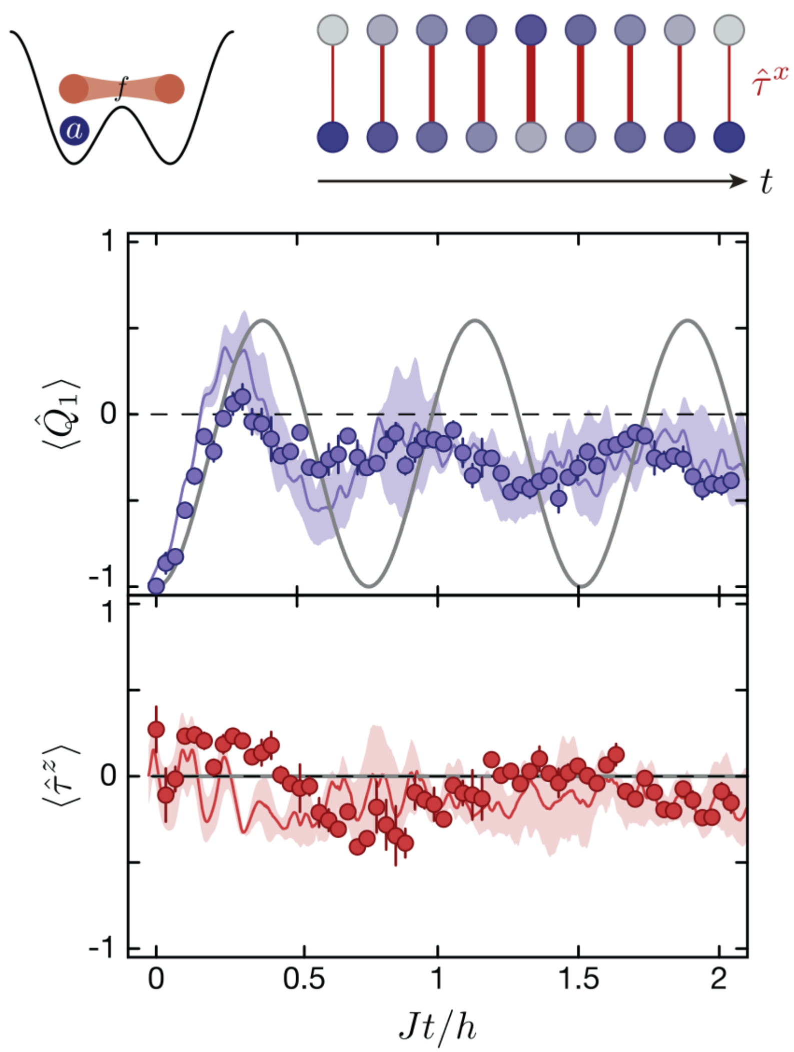
Time-reversal symmetry

No sign-problem-free QMC known

related cross-linked ladder study
Gonzales-Cuadra et al 2020
A. Bermudez talk this Thursday

Motivations

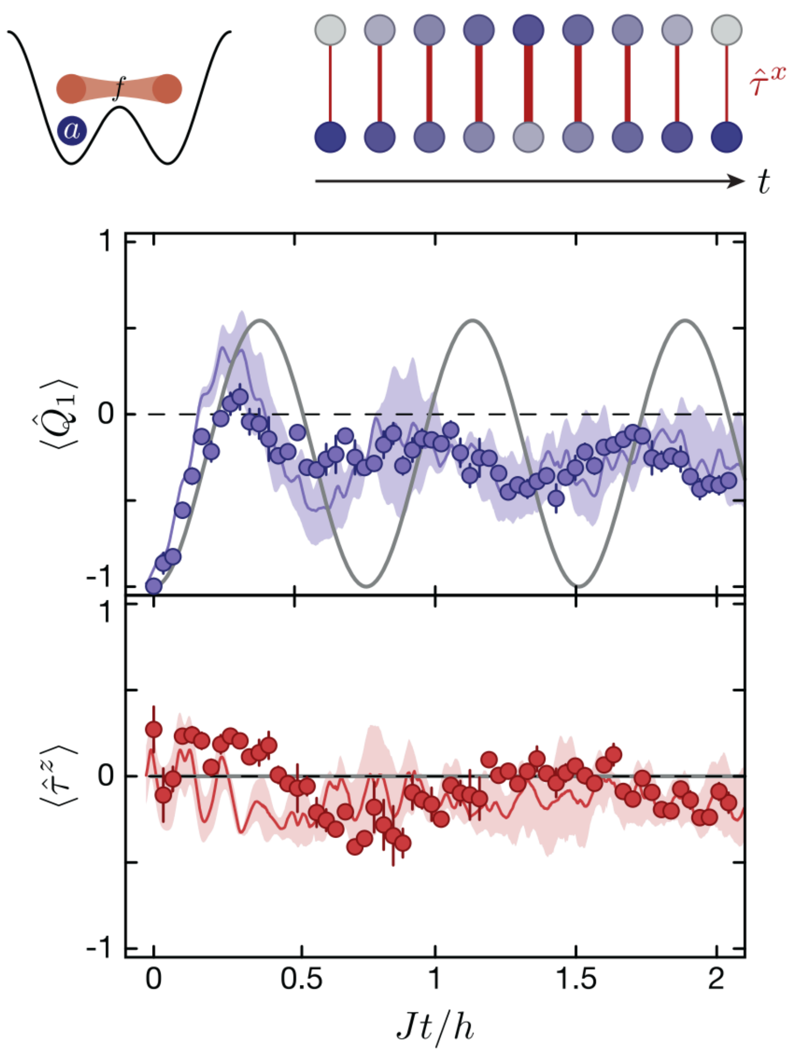
Cold atom
Floquet engineering



Bermudez&Porras 2015
Barbiero et al 2018
Goerg et al 2019
Schweizer et al 2019

Motivations

Cold atom
Floquet engineering



Bermudez&Porras 2015
Barbiero et al 2018
Goerg et al 2019
Schweizer et al 2019

Other proposals:

- Digital simulations

Zohar et al 2017

- Superconducting qubits

Homeir et al 2020

- Rydberg dressing

Kebric et al 2021

Motivations

Generalized Kitaev models

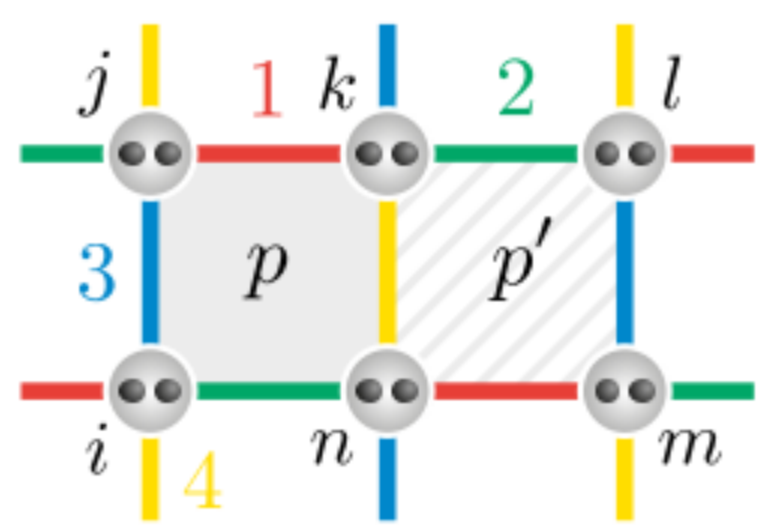
Chulliparambil et al
PRB 2020, 2021

$$\mathcal{H}_J^{(\nu)} = -\sum_{\langle ij\rangle_\gamma} J_\gamma \left(\Gamma_i^\gamma \Gamma_j^\gamma + \sum_{\beta=\gamma_{\mathrm{m}}+1}^{2q+3} \Gamma_i^{\gamma\beta} \Gamma_j^{\gamma\beta} \right)$$

For nu=2
spin-orbital
Hamiltonian

$$\mathcal{H}_J^{(2)} = -\sum_{\langle ij\rangle_\gamma} J_\gamma \left(\sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y \right) \otimes \tau_i^\gamma \tau_j^\gamma$$

expressed as
Z_2 gauge theory



Rest of my talk

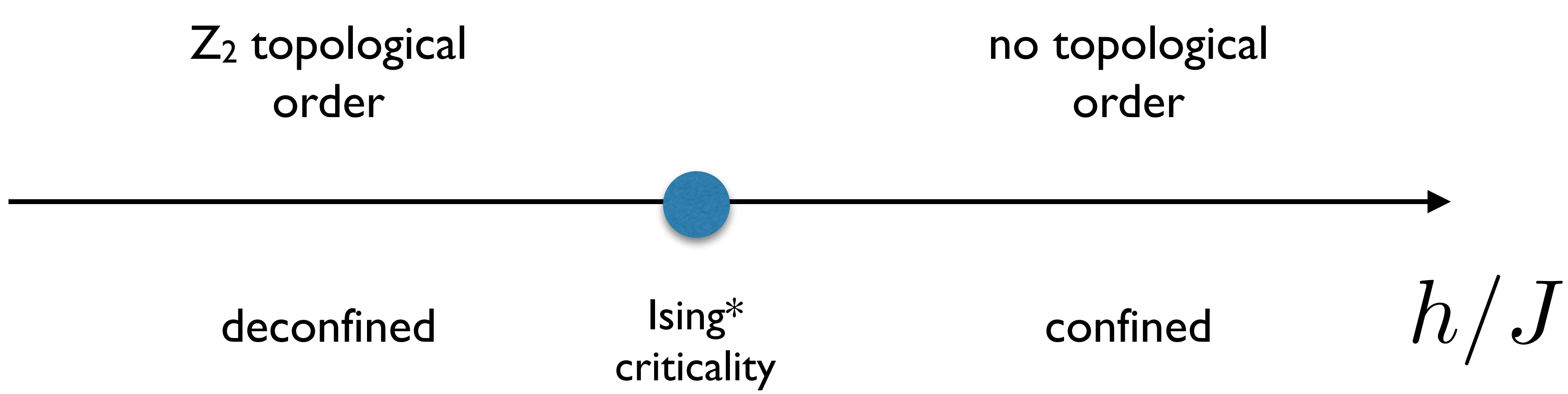
- Four limiting cases
- Eliminating Z_2 gauge redundancy
- iDMRG results

Pure gauge theory limits

Zero fermion filling for $\mu \to -\infty$

even pure gauge theory

$$\prod_{b \in +_{\mathbf{r}}} \sigma_b^x = +1$$

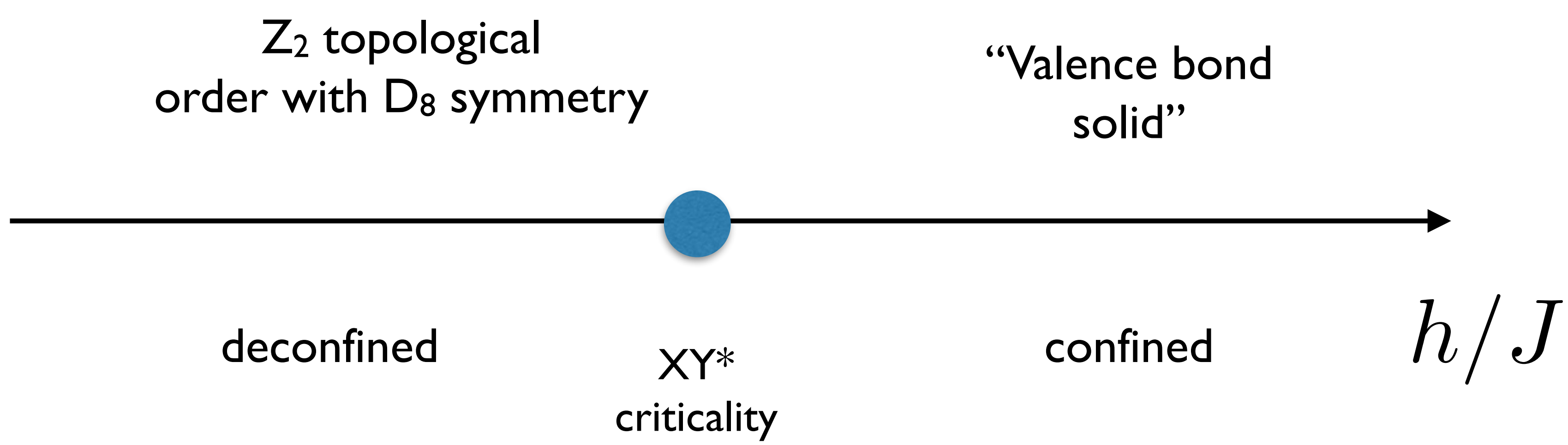


Pure gauge theory limits

Unit fermion filling for $\mu \to +\infty$

odd pure gauge theory

$$\prod_{b \in +_{\mathbf{r}}} \sigma_b^x = -1$$



Sachdev 2018

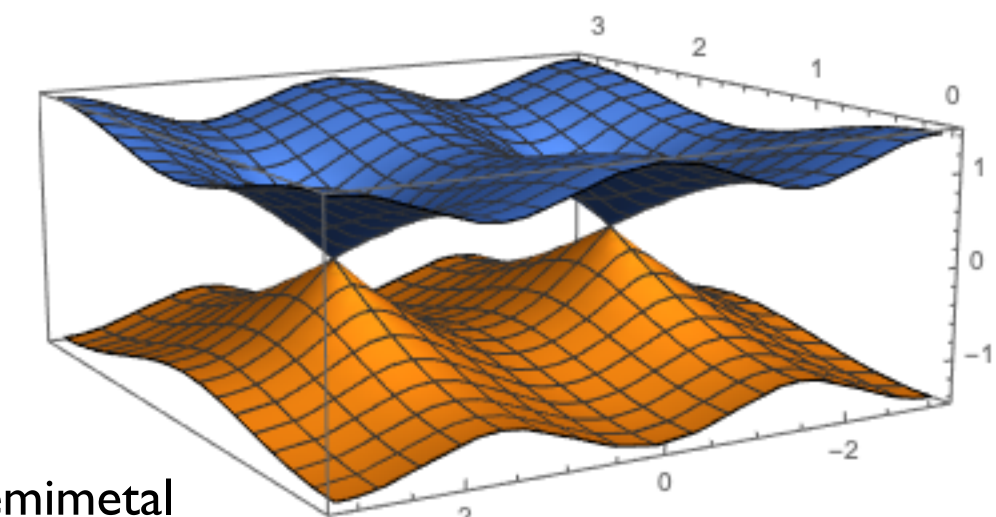
Zero string tension limit

At h=0, gauge fields are static,
fermions in background magnetic flux
which flux is realized in the ground state?

<u>Half filling</u>

$t \gg J$

Lieb theorem:
pi-flux is favored

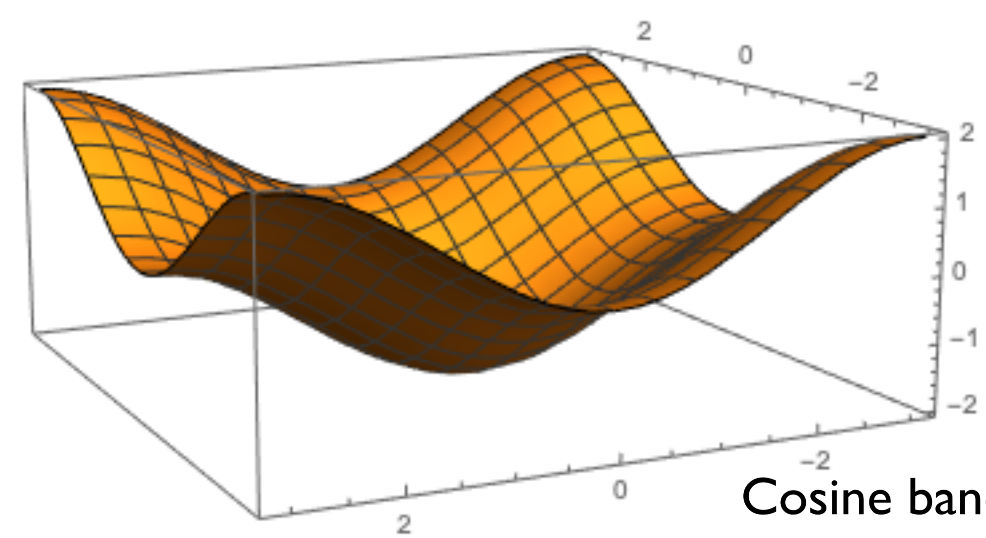


Dirac semimetal
with deconfined
Z_2 gauge fields

t vs J
competition studied in
Prosko et al 2017
König et al 2020

$t \ll J$

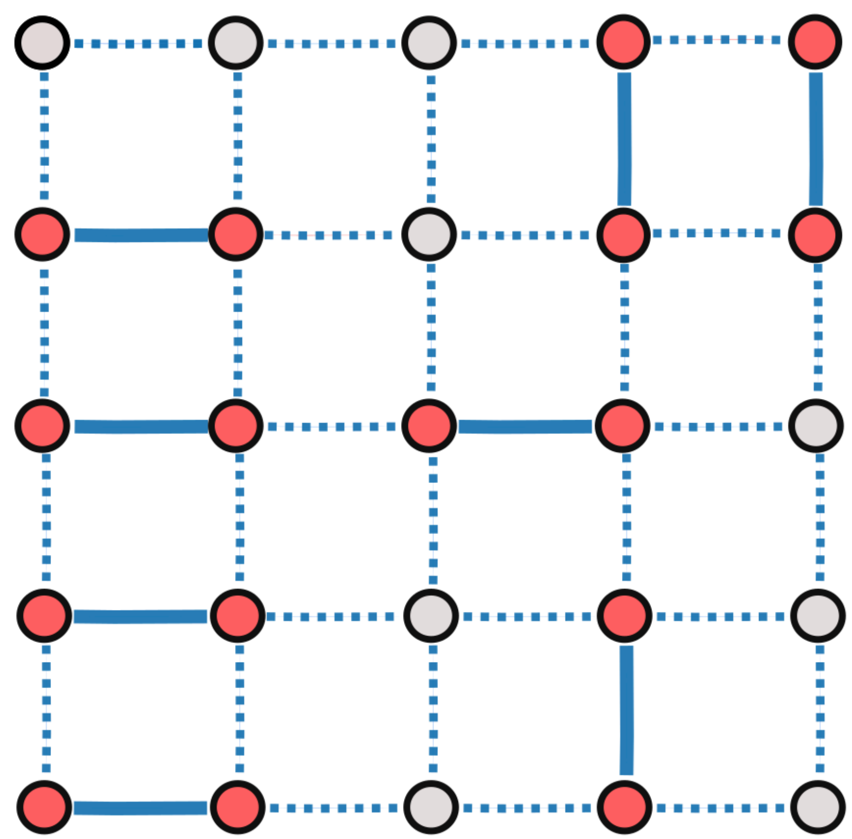
Plaquette term
favors 0-flux



Cosine band
with deconfined
Z_2 gauge fields

Strong tension limit

Fermions pair into dimers with shortest electric strings



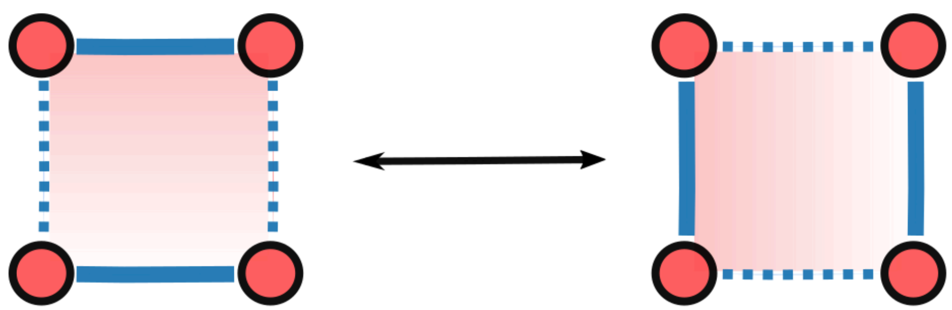
$$h \sim \mu \gg t, J$$

Hard-core condition: Not more than one dimer attached to site

We do degenerate perturbation theory

Clustering of dimers

Magnetic plaquette term $$-J \sum_{\mathbf{r}*} \prod_{b \in \square_{\mathbf{r}*}} \sigma_b^z$$



first order degenerate perturbation theory

Short-range attraction between two dimers

$$H_d^{\text{res}} = -J \sum \left(|\,\|\;\|\,\rangle\langle\,=\,| + \text{h.c.} \right)$$

Rokhsar&Kivelson 1988

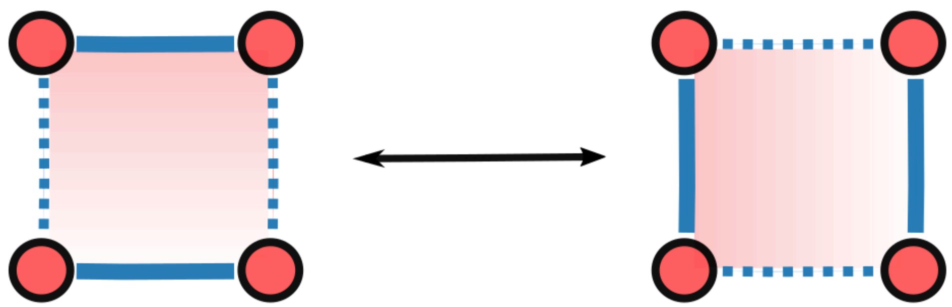
- Full filling- columnar VBS state

Wenzel et al 2012

- Partial filling- clustering of dimers

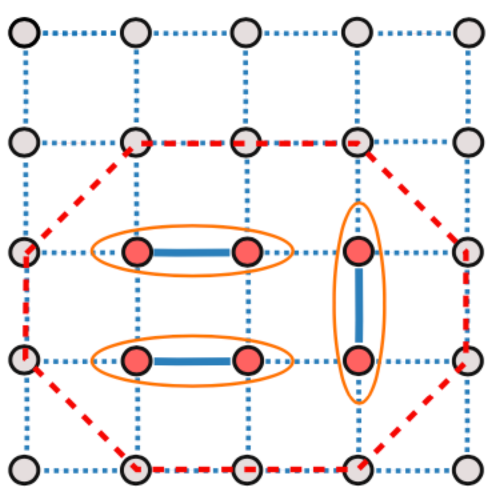
Clustering of dimers

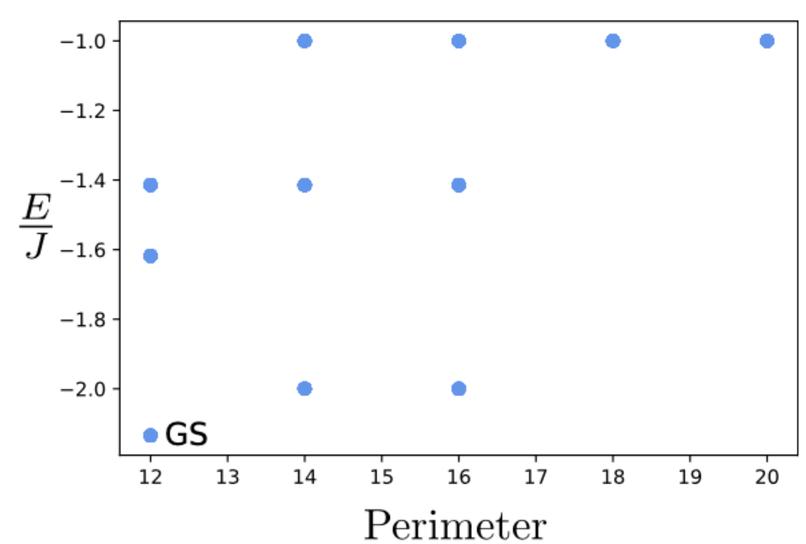
Magnetic plaquette term $-J\sum_{\mathbf{r}*}\prod_{b\in\Box_{\mathbf{r}*}}\sigma_b^z$



first order degenerate perturbation theory

Clusters with smallest perimeter have lowest energy



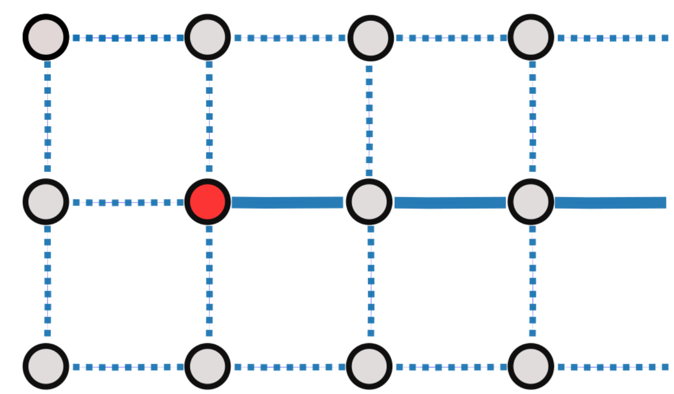


four dimers on 5x5 lattice

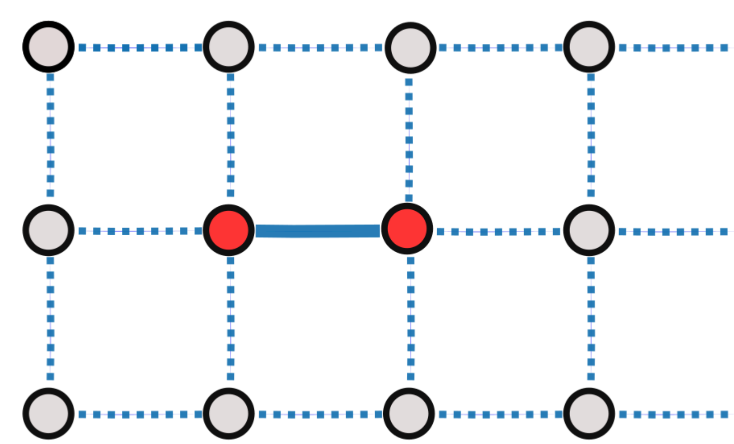
Emergent fractons

Within second order perturbation theory in t:

Isolated fermions
are immobile

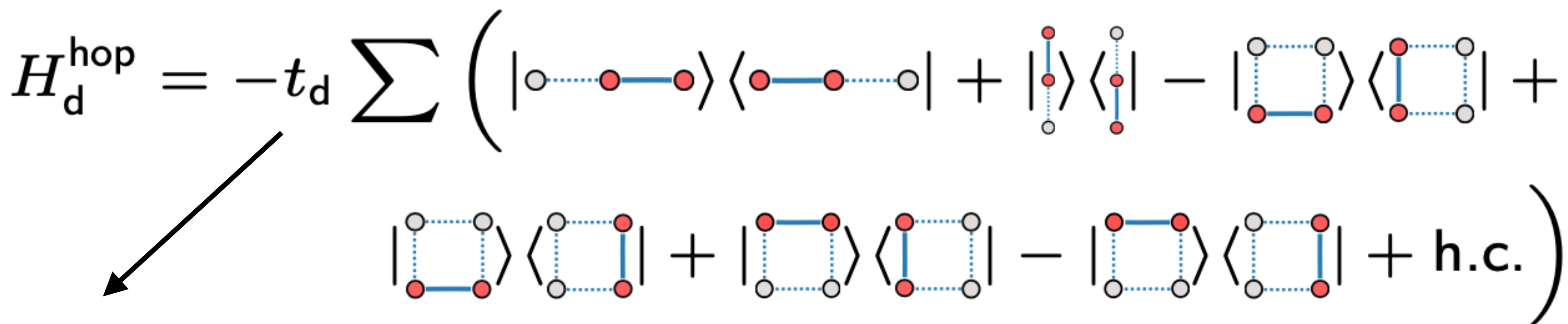


Dimers exhibit
restricted mobility



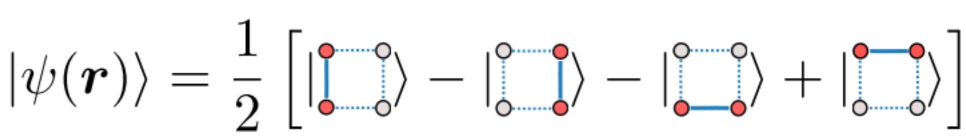
Second order hopping

Anisotropic hopping of dimers

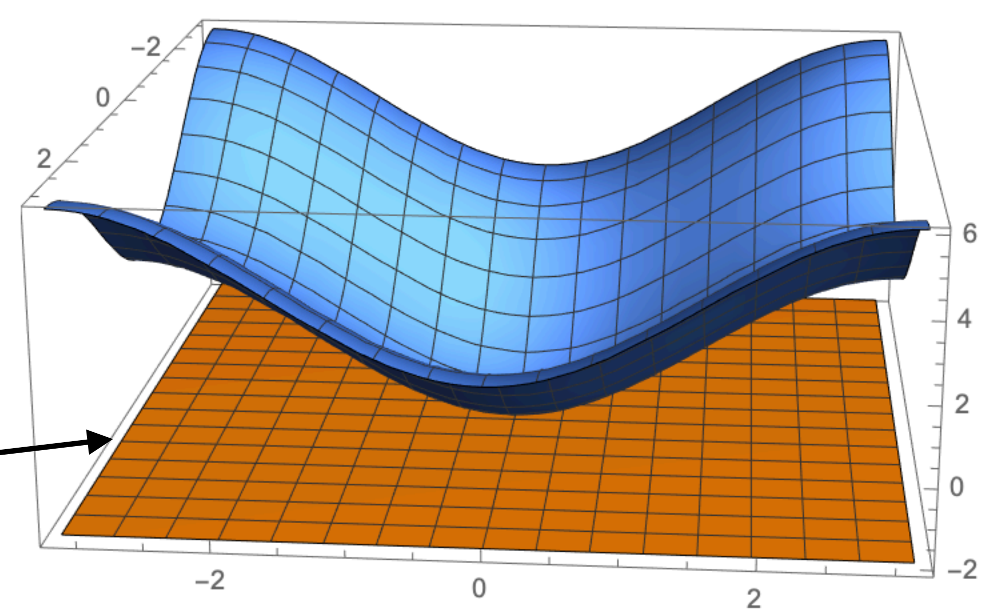


$t^2/2h$

Single dimer dispersion:



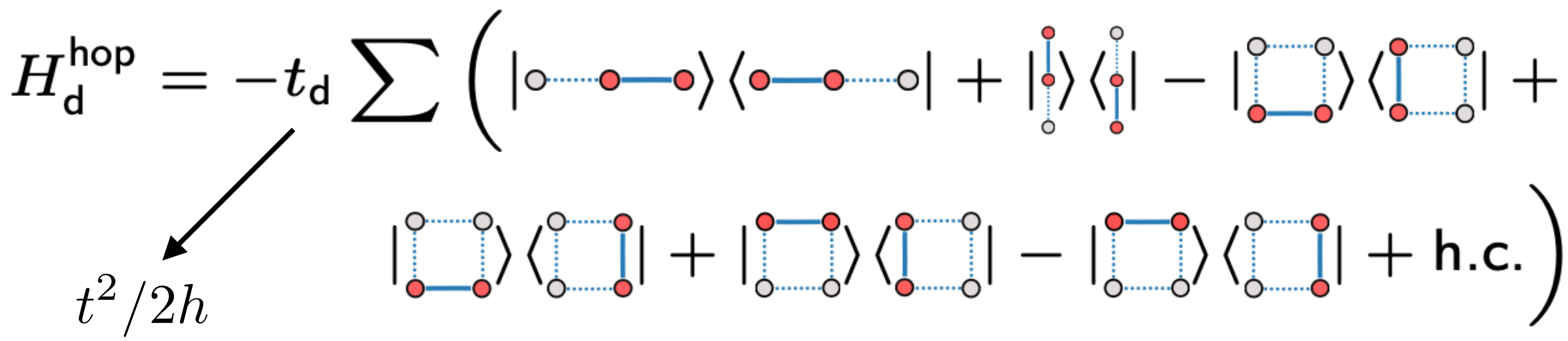
frozen dimer states



Many-body physics with frozen states…

Second order processes

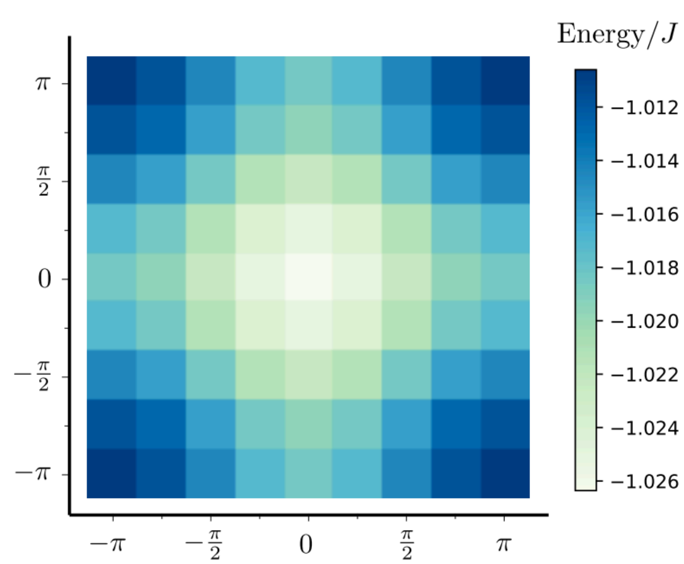
Anisotropic hopping of dimers- fracton phenomenology

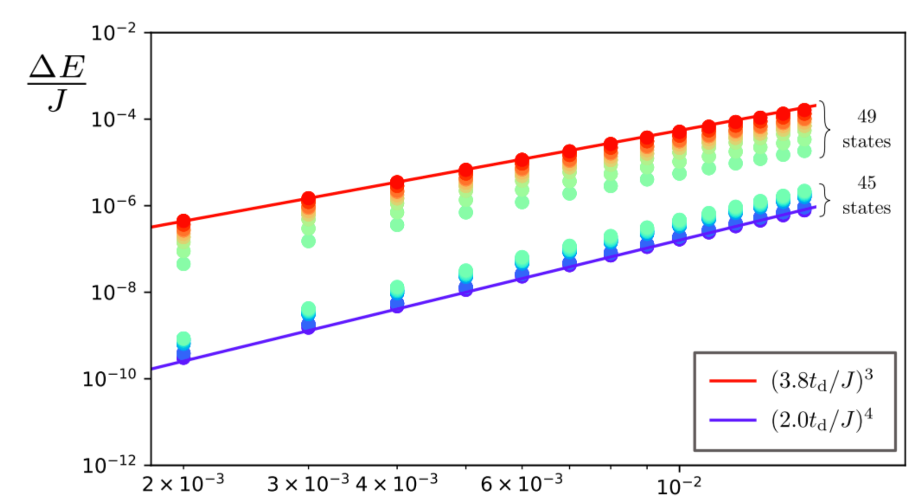


Clusters hopping is exponentially small in their size

two-dimer cluster hopping

t_{d}^2/J





three-dimer cluster hopping

$t_{\mathsf{d}}^3/J^2, t_{\mathsf{d}}^4/J^3$

Eliminating Z_2 gauge redundancy

Introduce Majorana fermions

$$\gamma_{\mathbf{r}} = c^\dagger_{\mathbf{r}} + c_{\mathbf{r}}$$
$$\tilde{\gamma}_{\mathbf{r}} = i(c^\dagger_{\mathbf{r}} - c_{\mathbf{r}})$$

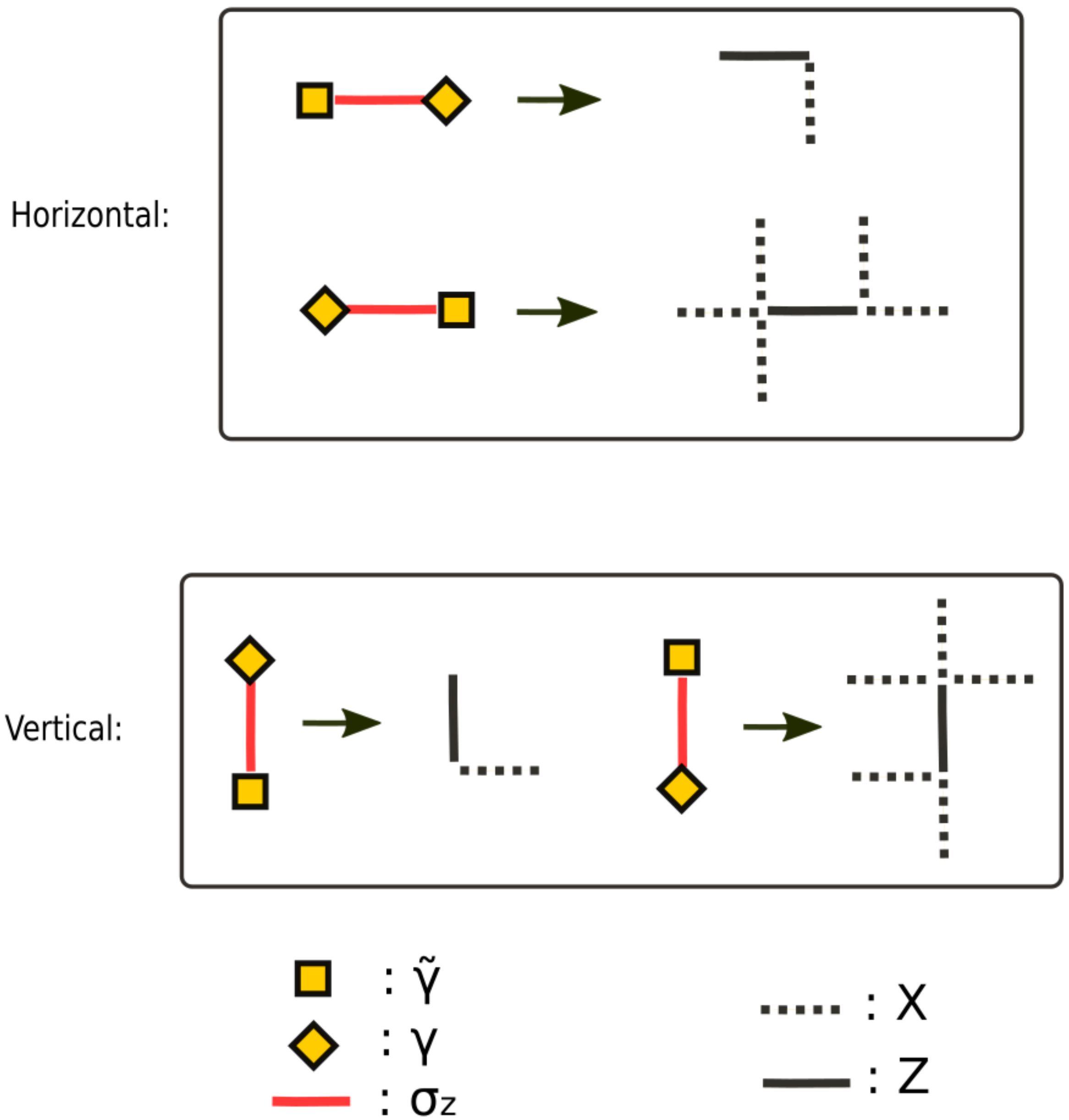
Gauge-invariant Pauli operators

related mappings:
Wosiek 1982
Chen et al 2018
...

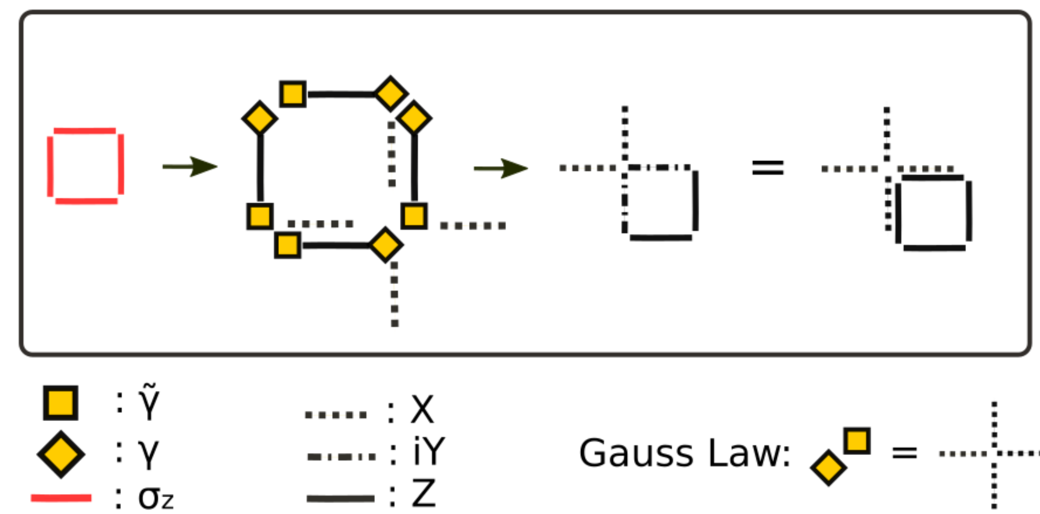
$$X_{\mathbf{r},\eta} = \sigma^x_{\mathbf{r},\eta}$$
$$Z_{\mathbf{r},\hat{x}} = -i\tilde{\gamma}_{\mathbf{r}}\sigma^z_{\mathbf{r},\hat{x}}\gamma_{\mathbf{r}+\hat{x}}\sigma^x_{\mathbf{r}+\hat{x},-\hat{y}}$$
$$Z_{\mathbf{r},\hat{y}} = -i\tilde{\gamma}_{\mathbf{r}}\sigma^z_{\mathbf{r},\hat{y}}\gamma_{\mathbf{r}+\hat{y}}\sigma^x_{\mathbf{r},\hat{x}}$$

Physical Hilbert space: spins 1/2 on links of the lattice

Examples



Hopping



Plaquette

2d spin model

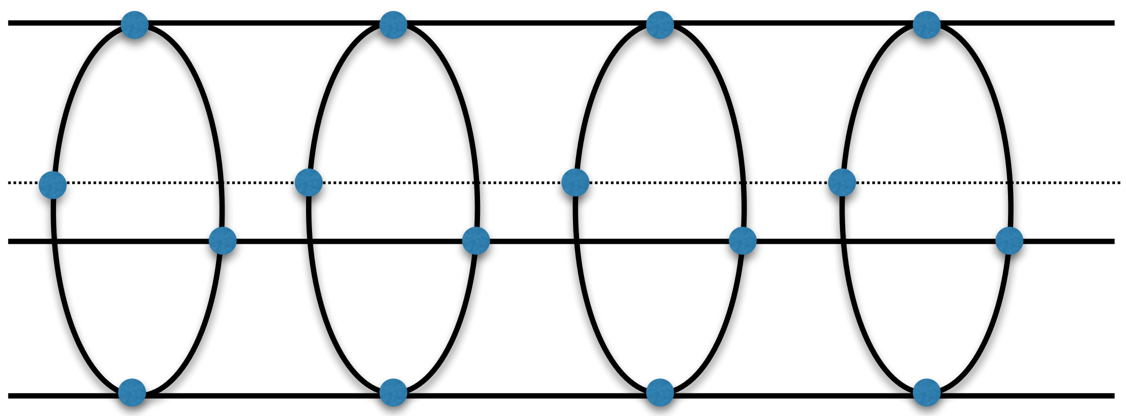
Local model of gauge-invariant spins 1/2 defined on links

$$
\begin{aligned}
H = &- t \sum_{\mathbf{r}} \left(Z_{\mathbf{r},\hat{x}}\, X_{\mathbf{r}+\hat{x},-\hat{y}}\, \mathcal{P}_{\mathbf{r},\hat{x}} + Z_{\mathbf{r},\hat{y}}\, X_{\mathbf{r},\hat{x}}\, \mathcal{P}_{\mathbf{r},\hat{y}} \right) \\
&- \frac{\mu}{2} \sum_{\mathbf{r}} \left(1 - \prod_{b \in +_{\mathbf{r}}} X_b \right) \\
&- J \sum_{\mathbf{r}^*} \prod_{b \in \square_{\mathbf{r}^*}} Z \prod_{b \in +_{\mathbf{r}}} X - h \sum_{\mathbf{r},\eta} X_{\mathbf{r},\eta}
\end{aligned}
$$

We use iDMRG to map out
quantum phase diagram

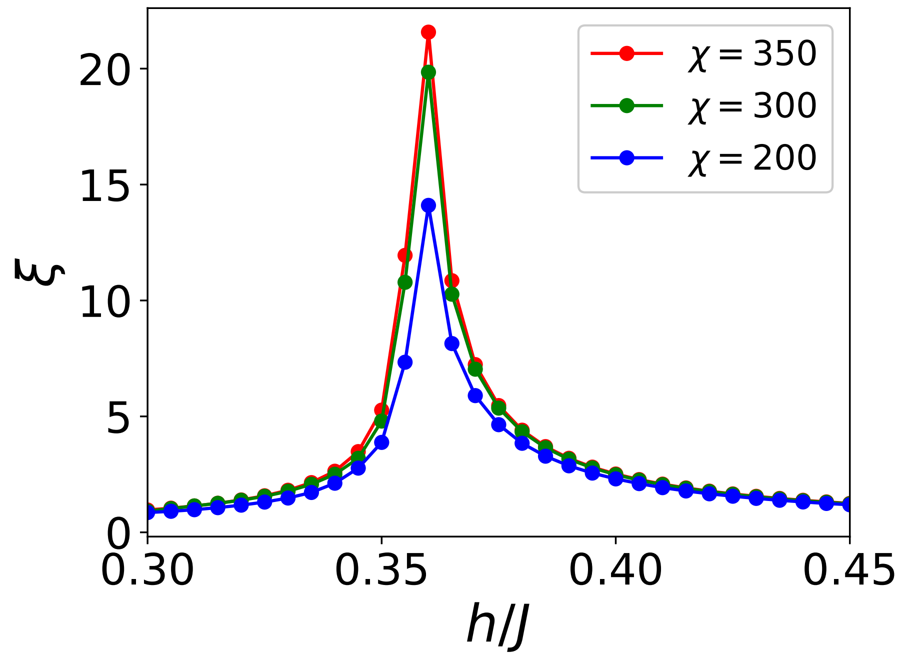
iDMRG results

Infinite cylinder geometry



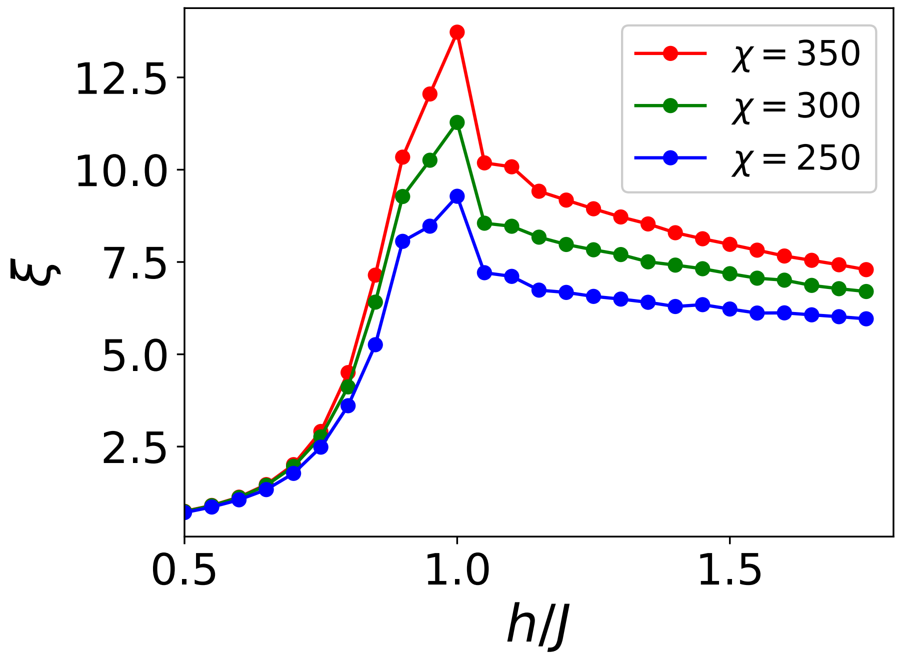
Confinement transitions:

even Z_2



$\mu \to -\infty$

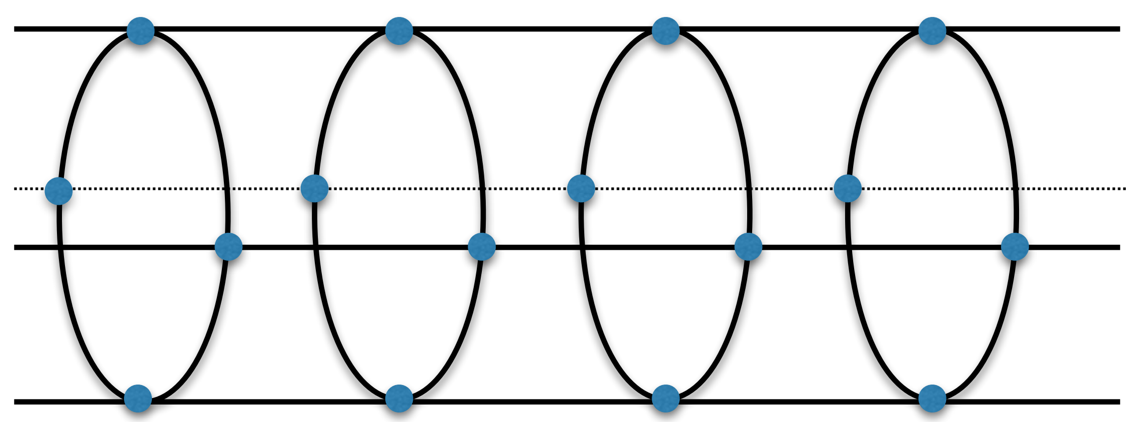
odd Z_2



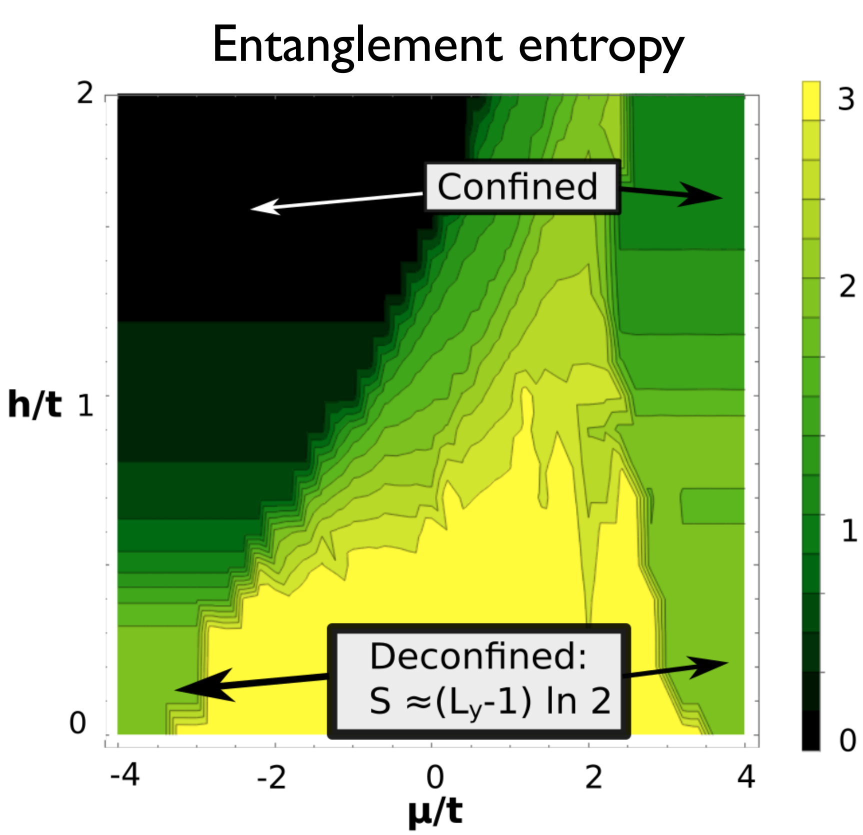
$\mu \to +\infty$

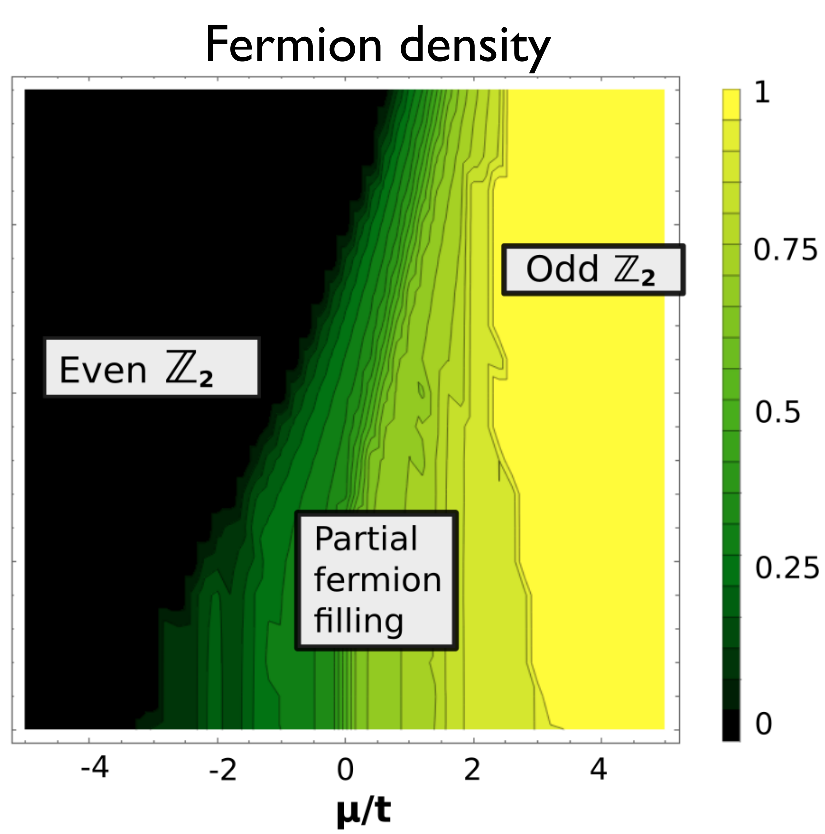
iDMRG results

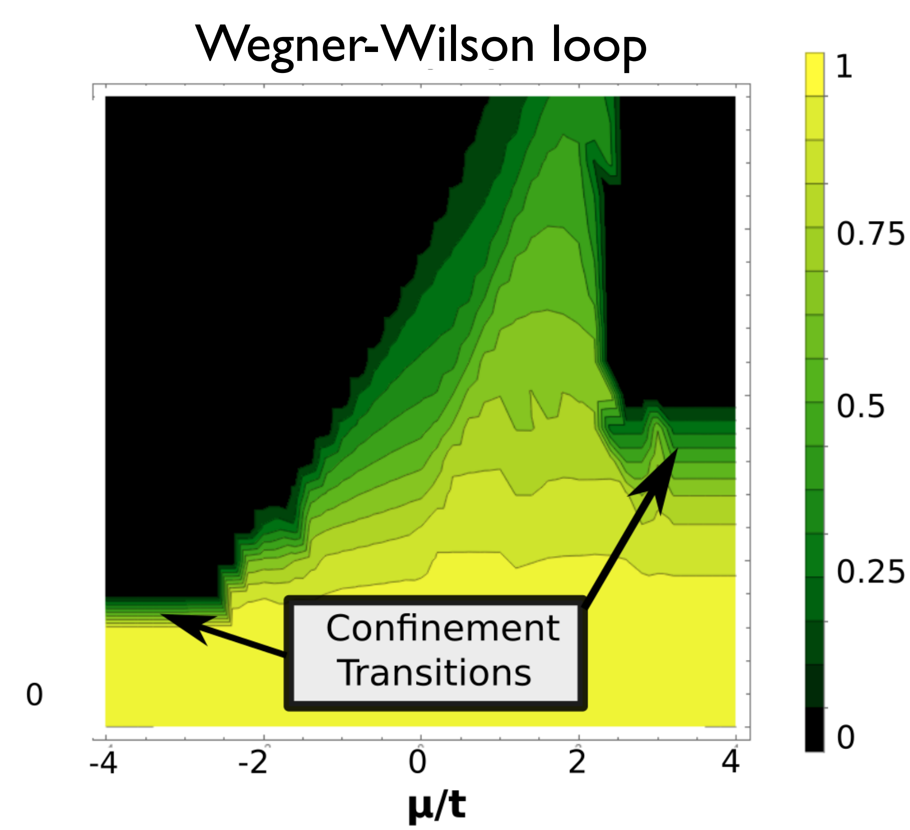
Infinite cylinder geometry



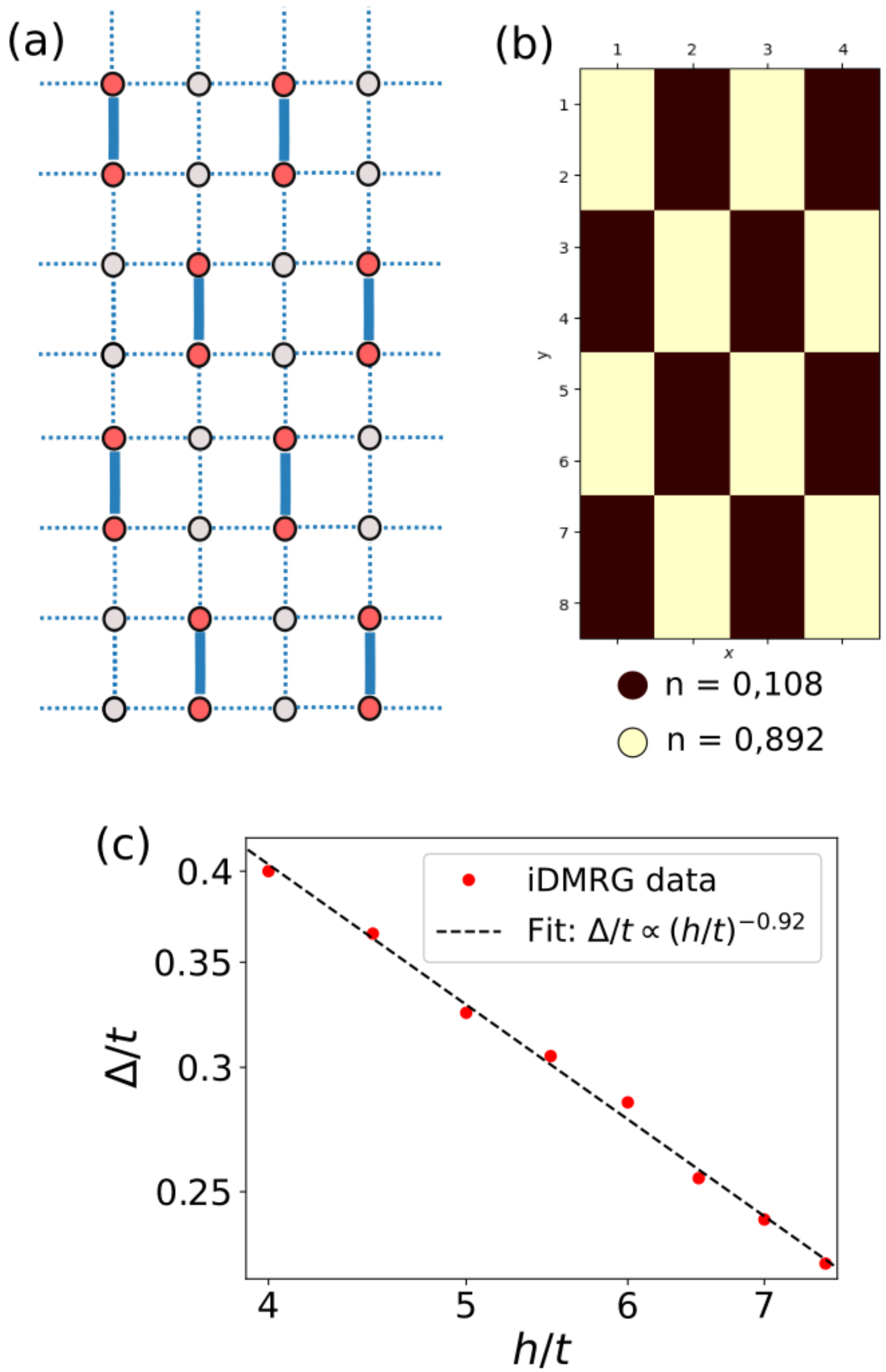
Set J=t:







Dimer Mott state for J=0



<u>Half filling</u>

At h>>t Mott state stabilized by NNN repulsion

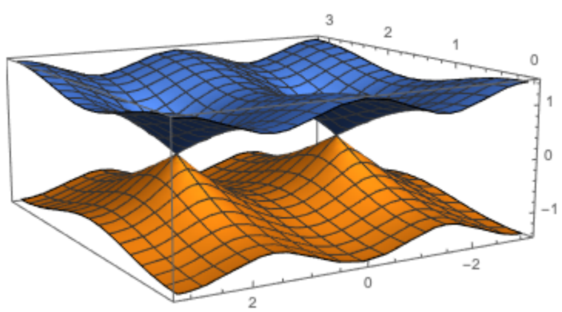
iDMRG results for L_y=8

Mott gap

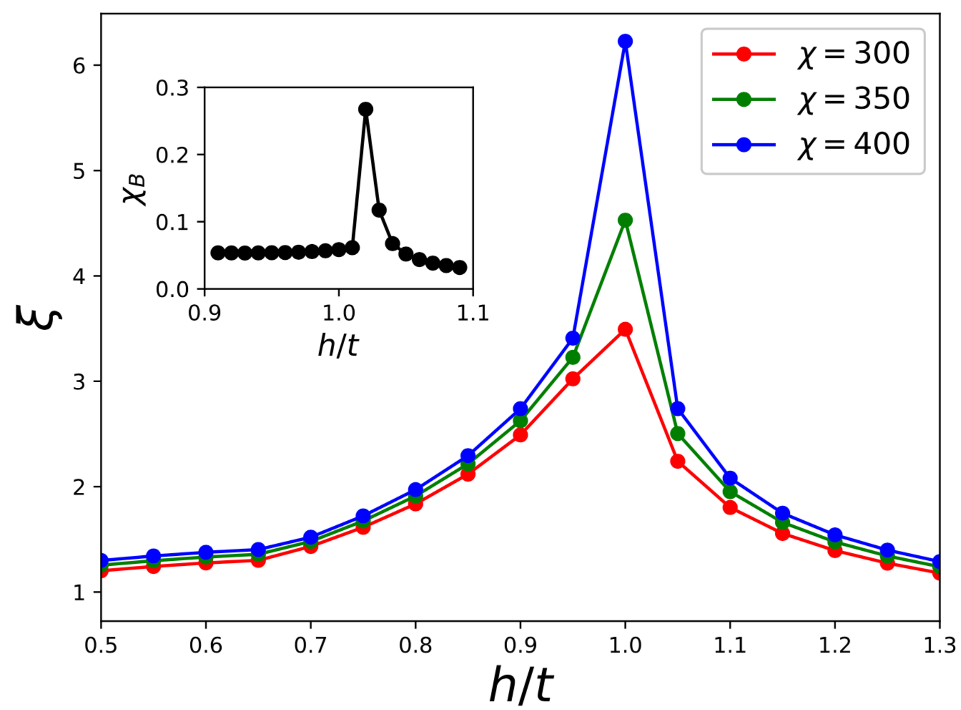
$$\Delta \propto \frac{t^2}{h}$$

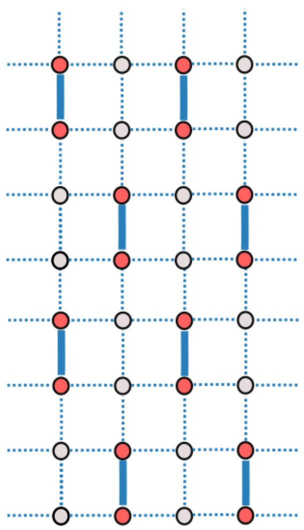
Dirac semimetal-Mott transition for J=0

Half filling



Dirac semimetal with deconfined Z_2 gauge fields





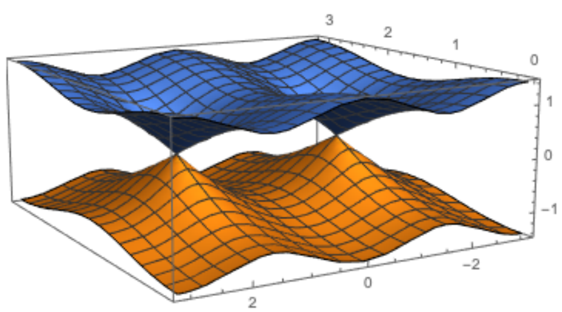
Mott phase confined Z_2 gauge fields

Two diagnostics: DMRG correlation length ξ

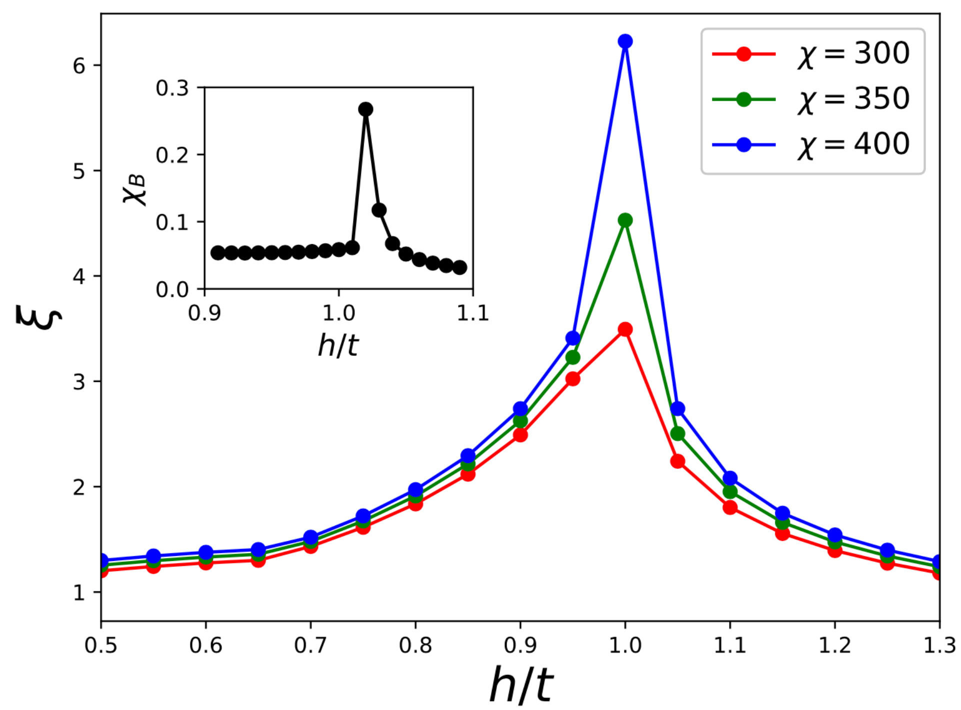
magnetic flux susceptibility $\chi_B = \partial\overline{\langle P_{\mathbf{r}^*}\rangle}/\partial h$

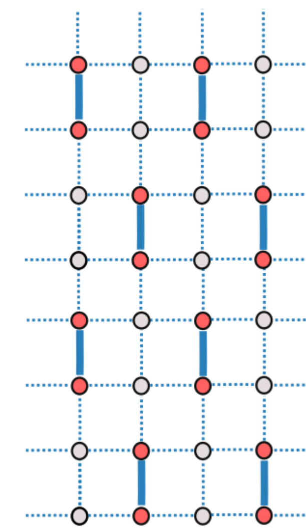
Dirac semimetal-Mott transition for J=0

Half filling



Dirac semimetal with deconfined Z_2 gauge fields





Mott phase confined Z_2 gauge fields

Does confinement transition coincide with translation SSB?

Second order phase transition?

Outlook

- Fate of U(1) global symmetry, p+ip superfluidity?
- Edge physics: role of fermion parity at the edge
- Quantum thermalization, quantum scars?
- Search for better ways to simulate this problem:

QMC, iPEPS, digital simulations, …

Extra slides

Gauge principle

Description of Nature <u>does not</u> depend how we calibrate our measurement equipment

Gauging: from symmetry to redundancy

Particle in 1d periodic potential:

$$\mathcal{H} = \{|x\rangle\}$$

$$H = \frac{1}{2m}p^2 + V\cos(2\pi x/L)$$

Global translation symmetry:

$$T_L^\dagger H T_L = H, \qquad T_L|x\rangle = |x+L\rangle$$

orthogonal states

Gauging: from symmetry to redundancy

Gauging translations: particle on a circle

$$\mathcal{H} = \{|\psi\rangle, T_L|\psi\rangle = |\psi\rangle\}$$

$$H = \frac{1}{2m}p^2 + V\cos(2\pi x/L)$$

- New Hilbert space is gauge-invariant
- Gauge transforms are do-nothing transformations
- Global symmetry is lost

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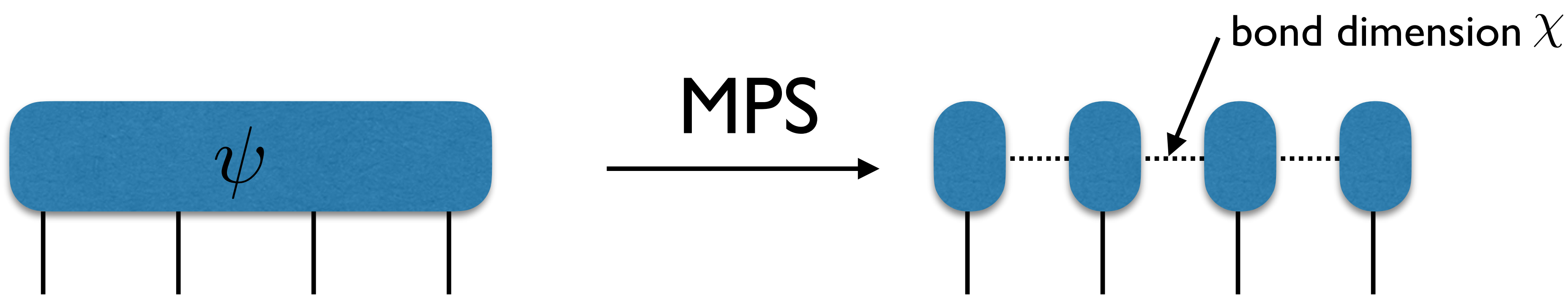
For some reason Nature likes the gauge principle

MPS based DMRG

Our problem: two-dimensional local Hilbert space



exponential

linear

Ideal for gapped 1d, but also useful beyond that

- guess for wave-function in MPS form
- quantum Hamiltonian

Hauschild&Pollmann 2018

- GS in MPS form
- Schmidt decomposition
- Entanglement entropy and spectrum
- …

Deconfined Dirac semimetal

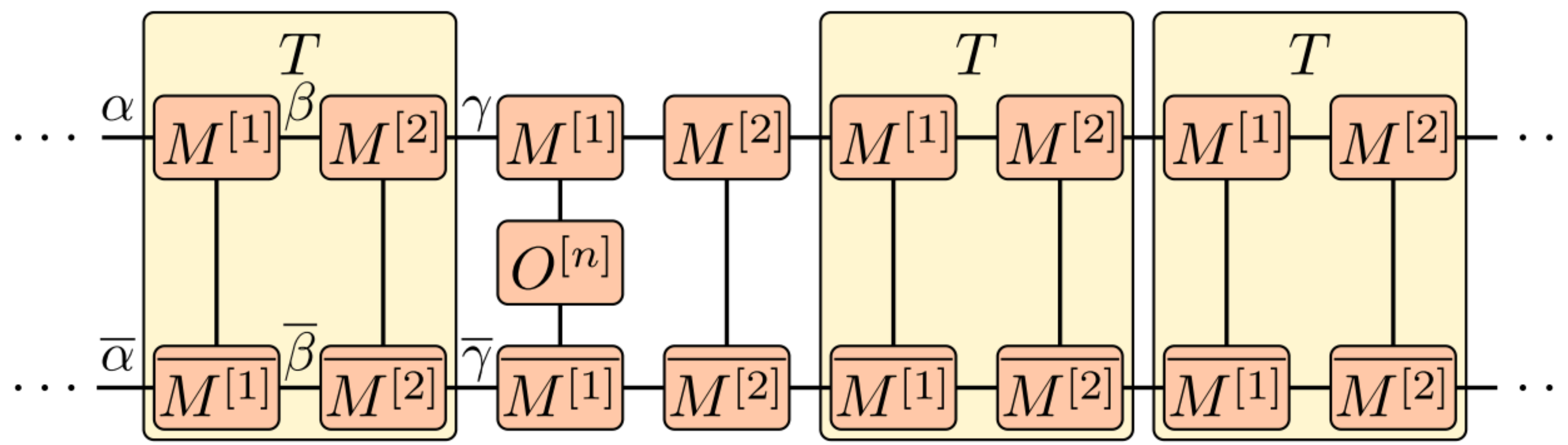
Entanglement entropy at J=h=0

L_y	χ	$S_f + S_{\mathbb{Z}_2}$	S	Rel. Error
2	400	1.03972	1,03972	0.00
4	1000	3.04080	3.03225	$\approx 0.28\%$
6	2000	5.05664	4.93008	$\approx 2.5\%$

MPS correlation length

Example for length two unit cell

Hauschild&Pollmann 2018



Transfer matrix:

$$T_{\alpha\overline{\alpha},\gamma\overline{\gamma}} = \sum_{j_1,j_2,\beta,\overline{\beta}} M^{[1]j_1}_{\alpha\beta}\,\overline{M^{[1]j_1}_{\overline{\alpha}\overline{\beta}}}\,M^{[2]j_2}_{\beta\gamma}\,\overline{M^{[2]j_2}_{\overline{\beta}\overline{\gamma}}}$$

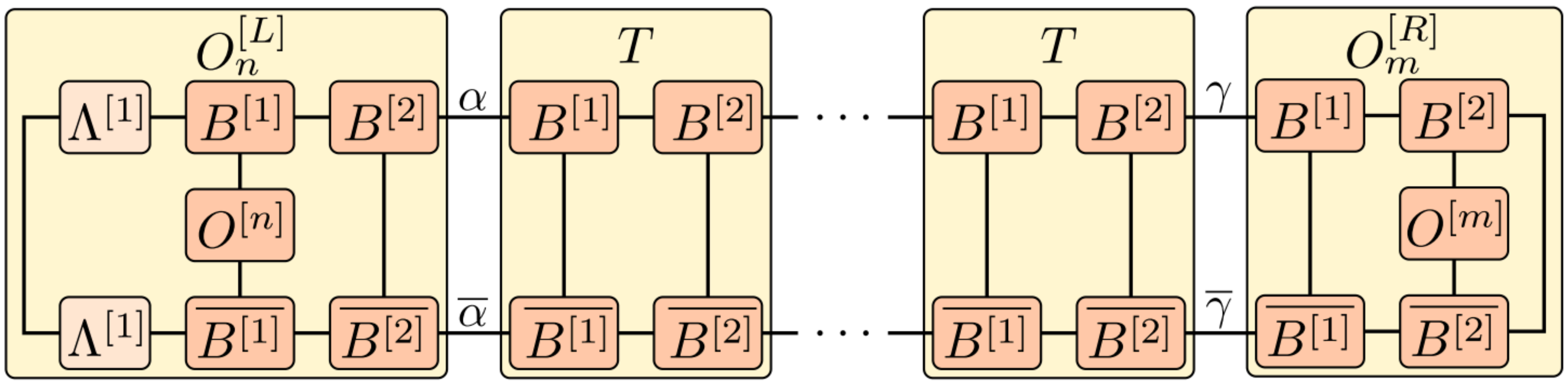
Normalize T such that its largest eigenvalue is unity.

subleading eigenvalues: $\eta_2, \eta_3, \ldots$

MPS correlation length

Two-point correlation function

Hauschild&Pollmann 2018



$$\langle\psi|O_n O_m|\psi\rangle = \langle\psi|O_n|\psi\rangle\,\langle\psi|O_m|\psi\rangle + (\eta_2)^N C_2 + (\eta_3)^N C_3 + \cdots$$

$$C_i = (O_n^{[L]}\eta_i^{[R]})(\eta_i^{[L]}O_n^{[R]})$$

Second largest eigenvalue determines the MPS correlation length

$$\xi = -\frac{L}{\log|\eta_2|}$$