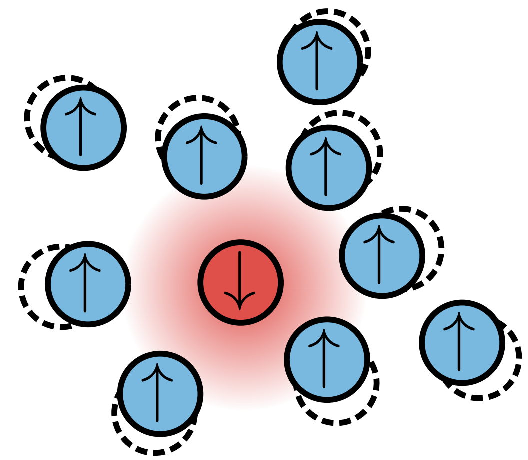
↓- impurity in a Fermi sea of ↑-majorities

+ interactions

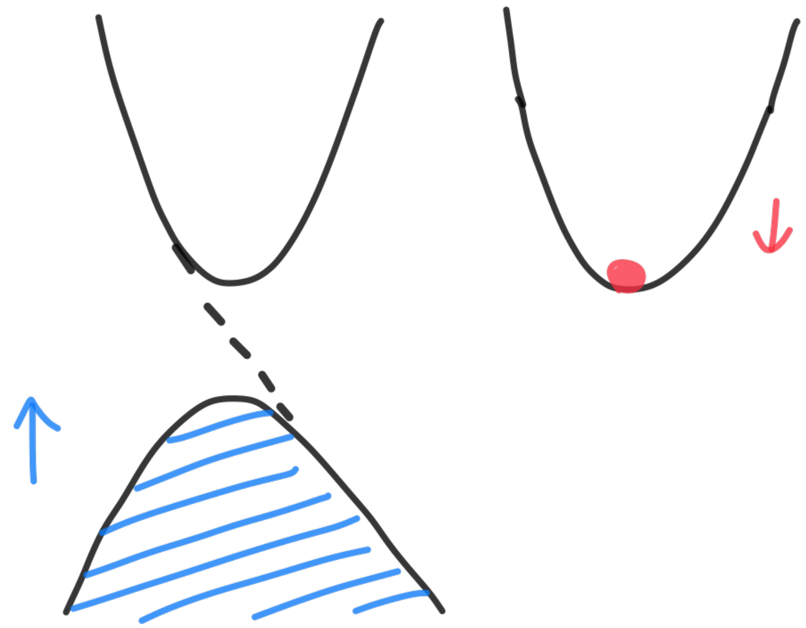


Realization:
- Doped semiconductors — Smolka et al., Science 346 (2014)...
- Ultracold atoms — Schirotzek et al., PRL 33 (2009)...

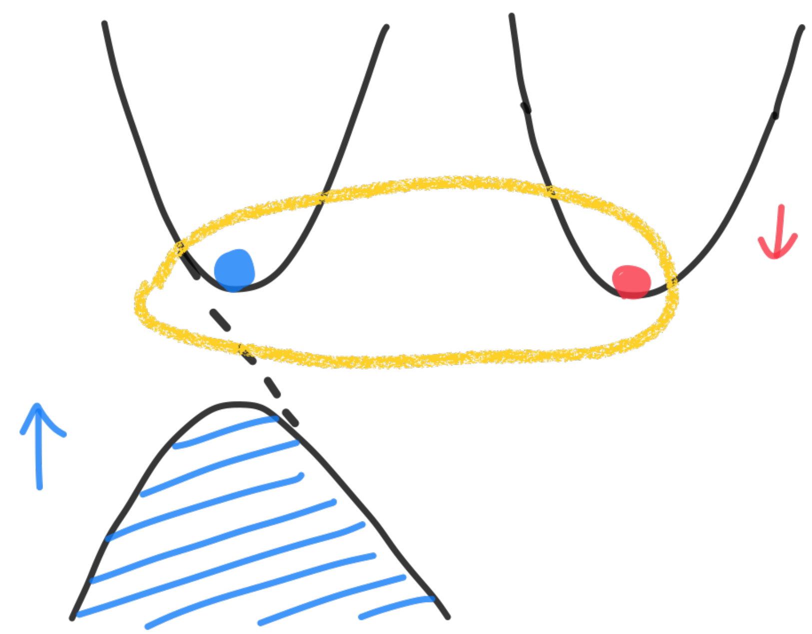
Typical questions:
- Ground state (bound state – yes or no?)
- Impurity spectrum (Edge singularity)
- Transport: Impurity drag — Cotlet, Pientka et al., PRX 9 (2019)...

Local kinematic properties of impurity are modified

Modification of impurity topology by non-trivial medium?



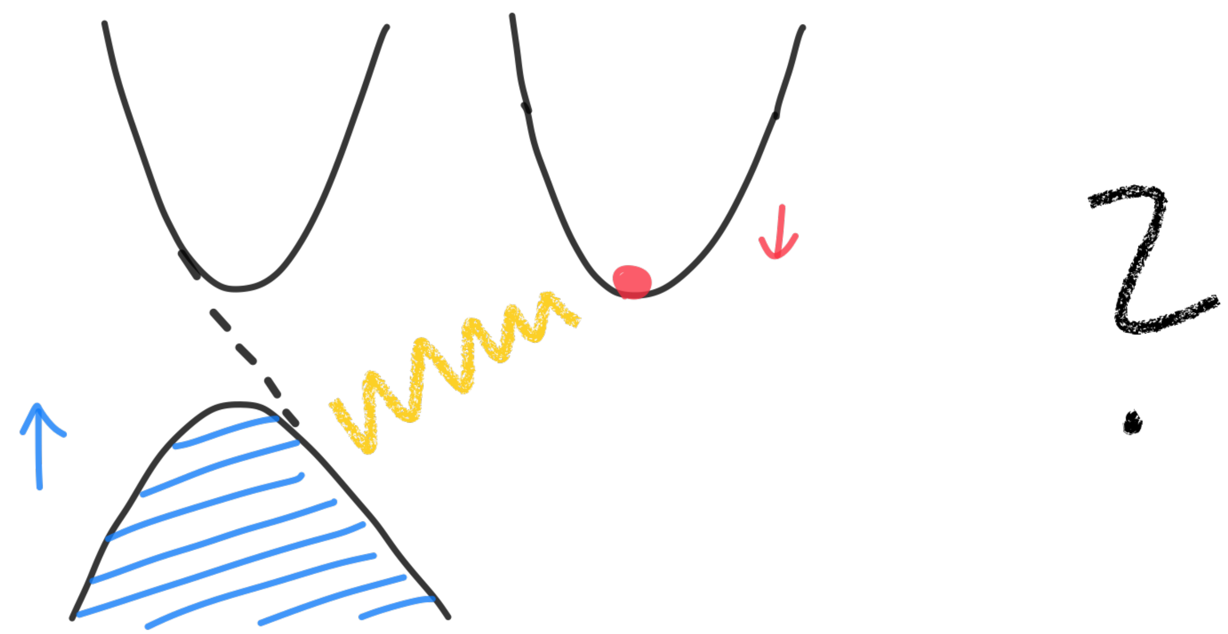
Modification of impurity topology by non-trivial medium?



Strong coupling limit: Grusdt et al., Nature Com. 7 (2016), PRB 100 (2019)

- impurity binds to topological excitation of majority medium
- inherits its topological properties

Modification of impurity topology by non-trivial medium?



What about weak coupling?

Camacho-Guardian et al., PRB 99 (R) (2019),
D.P. et al., PRB 103 (2021)

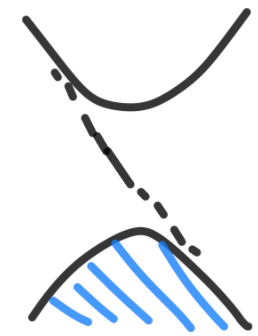
Controlled combination of ultracold gas expertise on polarons & topology

Collaborators: Arturo Camacho-Guardian (Cambridge)
Georg Bruun (Aarhus)
Pietro Massignan (Barcelona)
Nathan Goldman (Brussels)
Moshe Goldstein (Tel Aviv)

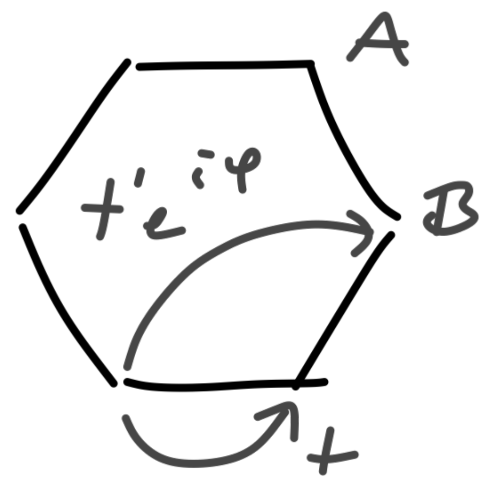
Topological majorities

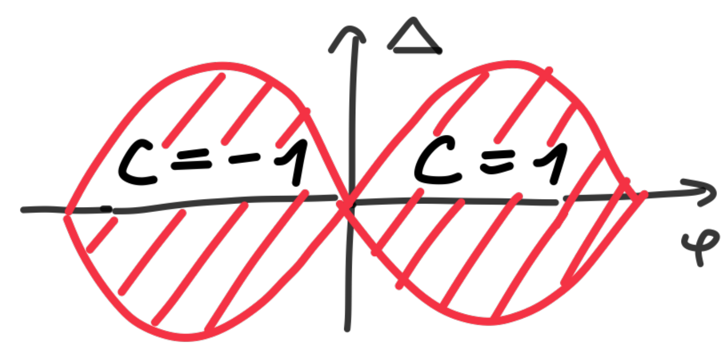
2D Chern insulator
(no symmetries enforced)



$$H_\uparrow(\boldsymbol{k}) = h_i(\boldsymbol{k})\sigma_i \quad \mathcal{C} \in \mathbb{Z}$$

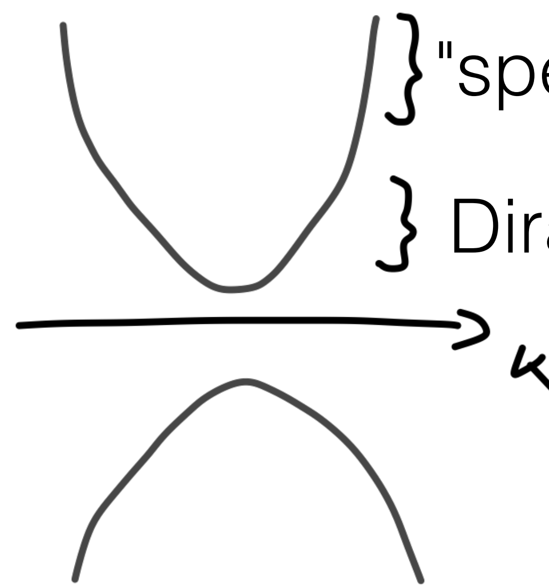
- Haldane model





- Gapped Dirac cone with quadratic part – continuum model

C quantized (not half-quantized)
due to spectator fermions



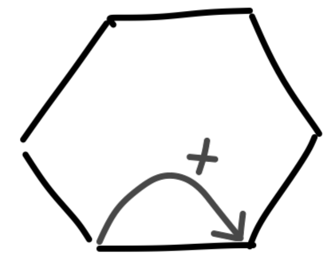
"spectator fermions"

Dirac fermions

Setup

Trivial impurity

- Graphene



- Trivial quadratic band $H_{\downarrow}(\boldsymbol{k}) = k^2/2M$

Short-ranged interaction

$$H_{\text{int}} = \frac{g}{A_0} \sum_{\ell=A,B} \sum_{\boldsymbol{k},\boldsymbol{p},\boldsymbol{q}} c^{\dagger}_{\uparrow,\ell}(\boldsymbol{k}+\boldsymbol{q}) c_{\uparrow,\ell}(\boldsymbol{k}) c^{\dagger}_{\downarrow}(\boldsymbol{p}-\boldsymbol{q}) c_{\downarrow}(\boldsymbol{p}) =$$

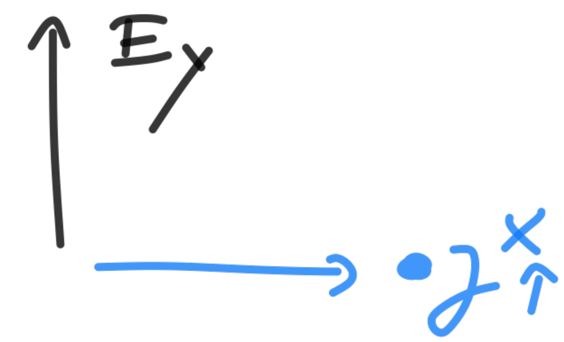
$$\frac{g}{A_0} \sum_{\boldsymbol{k},\boldsymbol{p},\boldsymbol{q}} c^{\dagger}_{\uparrow,\alpha}(\boldsymbol{k}+\boldsymbol{q}) c_{\uparrow,\beta}(\boldsymbol{k}) c^{\dagger}_{\downarrow}(\boldsymbol{p}-\boldsymbol{q}) c_{\downarrow}(\boldsymbol{p}) W_{\alpha\beta}(\boldsymbol{k},\boldsymbol{q})$$

↙ projectors on band basis

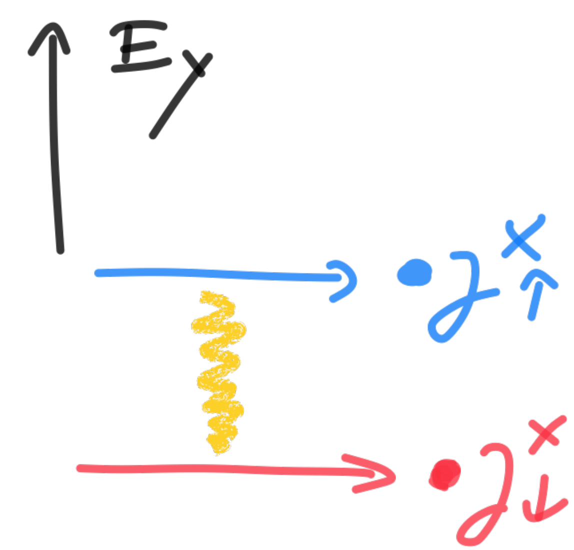
Drag Transconductivity

- Topology related to transport: $\sigma_{xy} = -\dfrac{\mathcal{C}}{2\pi}$

- Force induces transversal majority current



- Topology related to transport: $\sigma_{xy} = -\frac{\mathcal{C}}{2\pi}$
- Force induces transversal majority current
- Does majority drag impurity behind?



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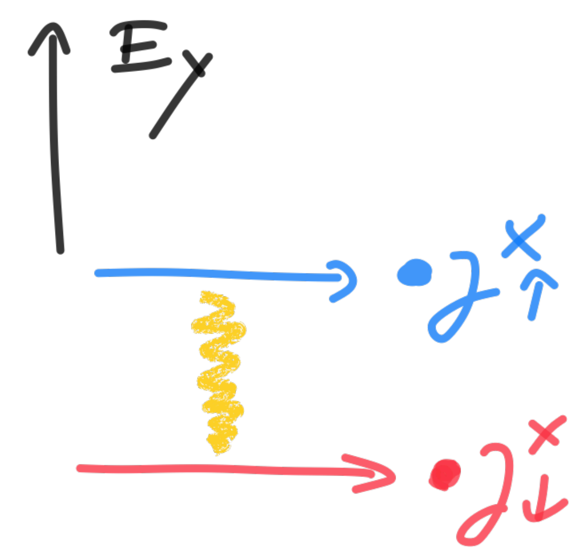


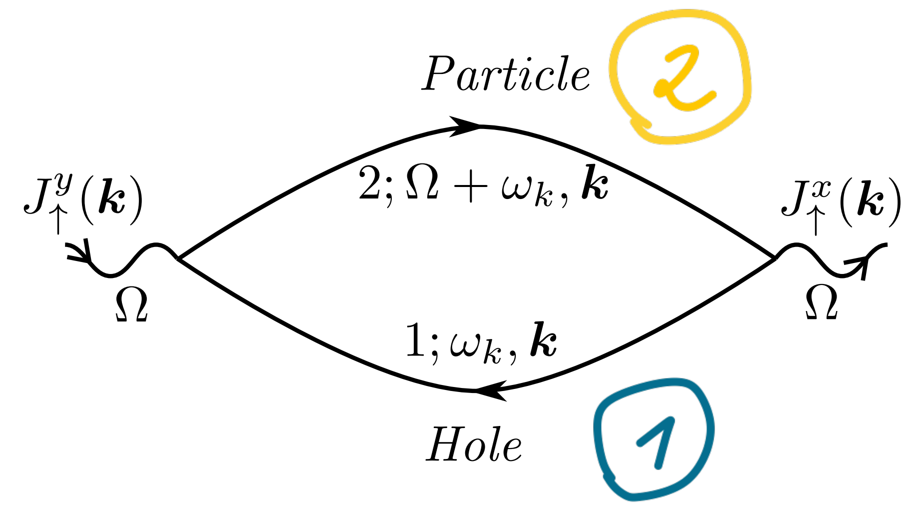
- Drag transconductivity $\sigma_{\downarrow\uparrow} \equiv \lim_{\omega\to 0} \frac{1}{-i\omega A_0}[- \langle \hat{J}^x_\downarrow \hat{J}^y_\uparrow \rangle \, (i\Omega)|_{i\Omega\to\omega+i0^+}].$

  Not quantized (impurity band is not filled), but does it follow σ_{xy}?
  Leading contribution: $O(g^2)$

- Impurity transconductivity $\sigma_{xy,\downarrow} \sim \langle J^x_\downarrow J^y_\downarrow \rangle$ starts at $O(g^4)$

Diagram for σ_{xy}

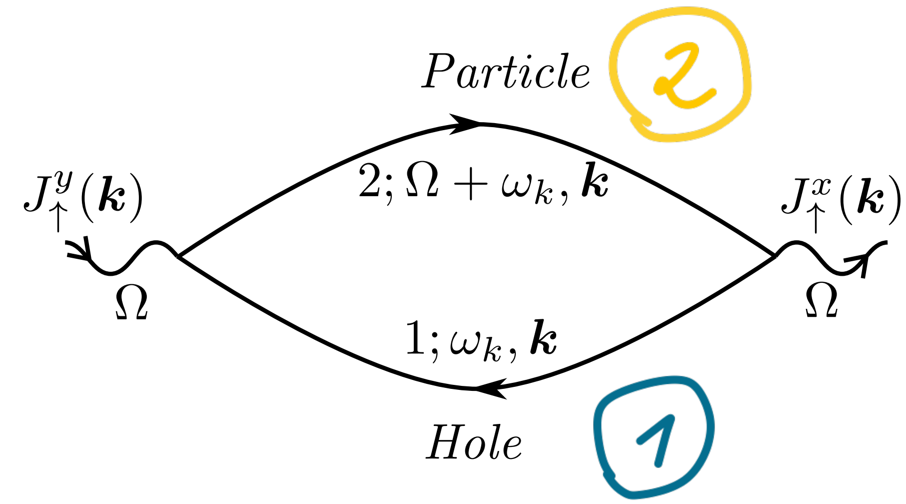
Current carried by virtual particles and holes



Feynman diagrams

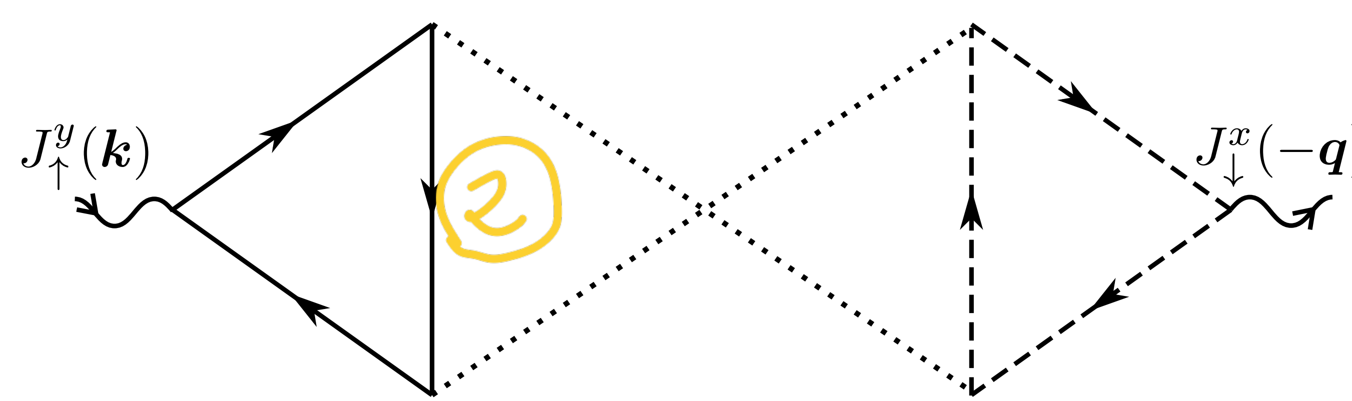
Diagram for σ_{xy}

Current carried by virtual particles and holes



Diagrams for $\sigma_{\uparrow\downarrow}$

Impurity scatters with particles or holes



Drag cancellation

- Particles and holes drag impurity in opposite directions

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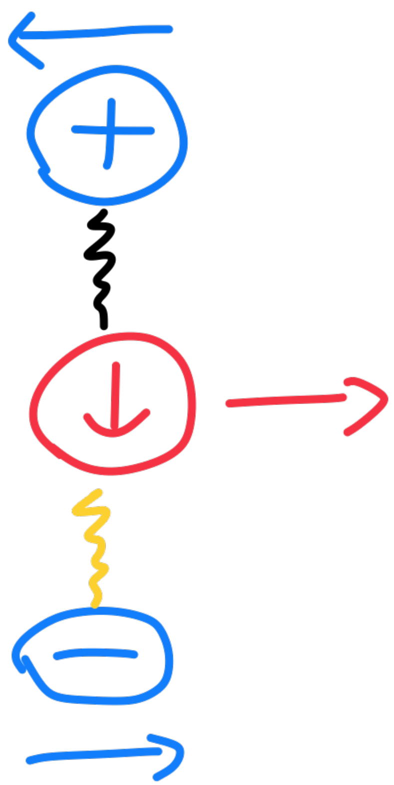
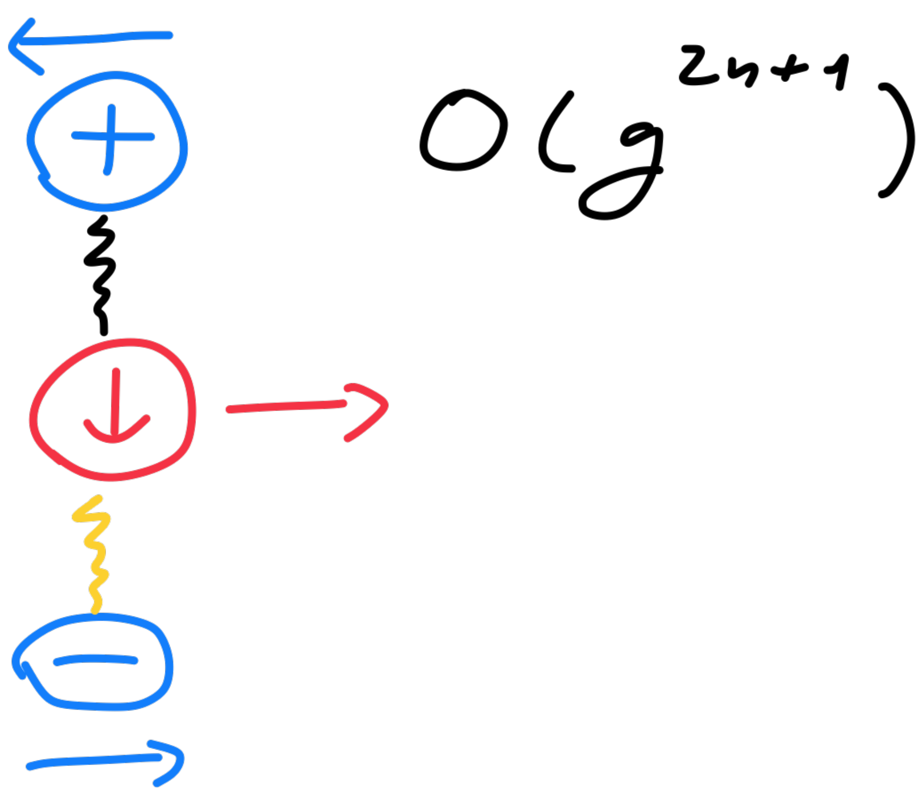
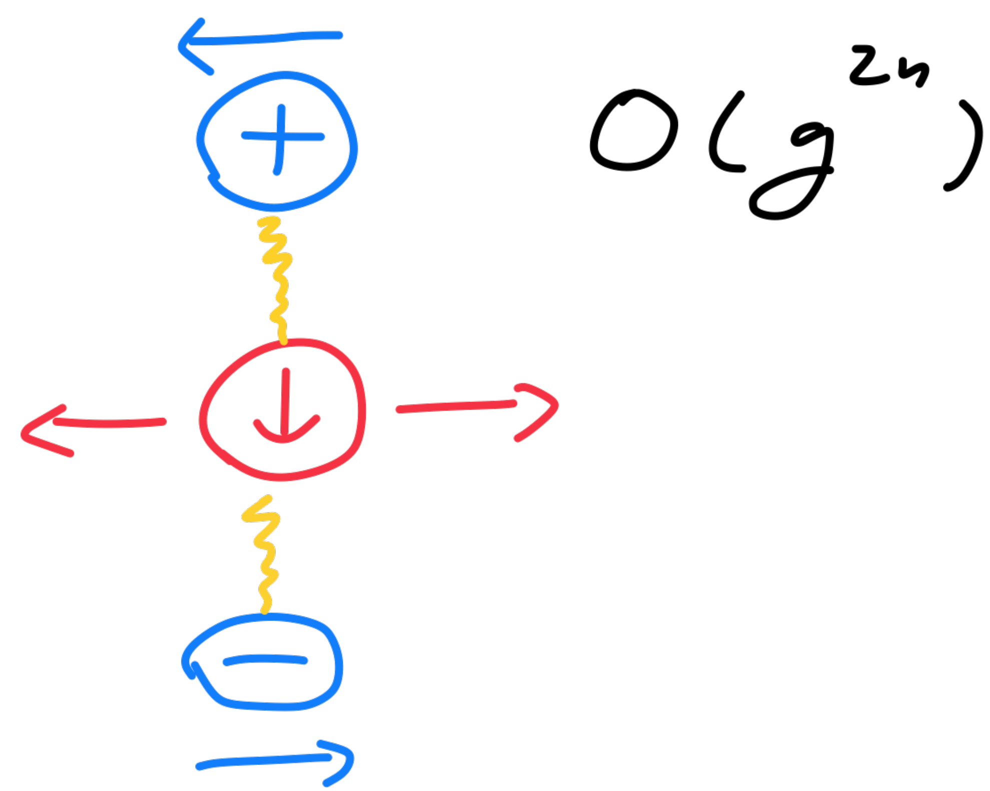


$O(g^{2n+1})$

Drag cancellation

- Particles and holes drag impurity in opposite directions





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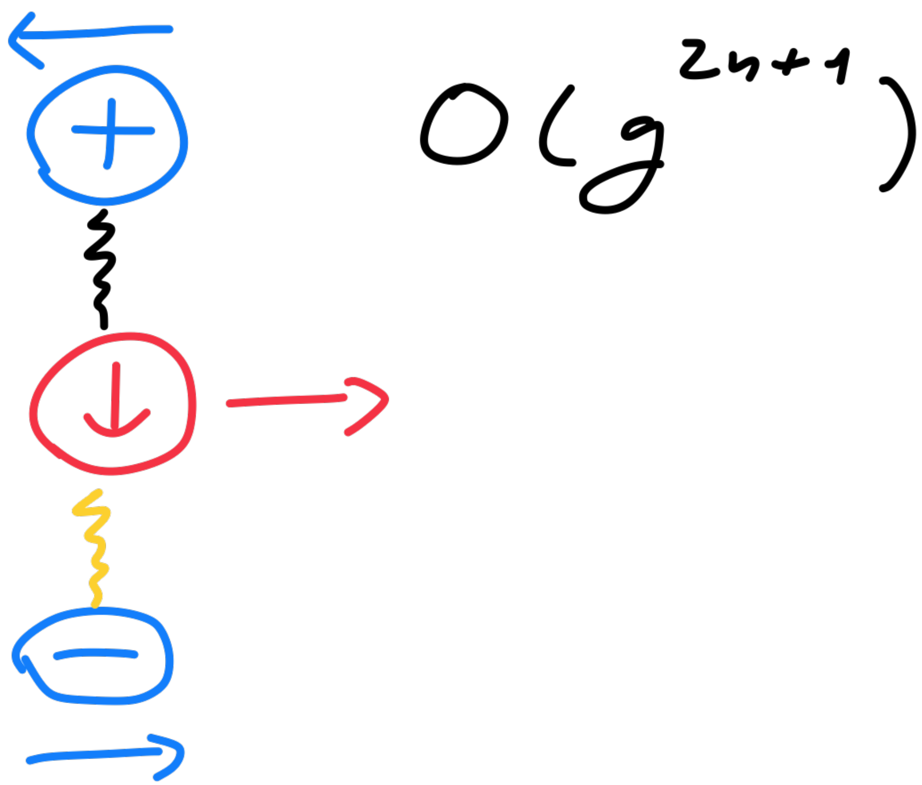
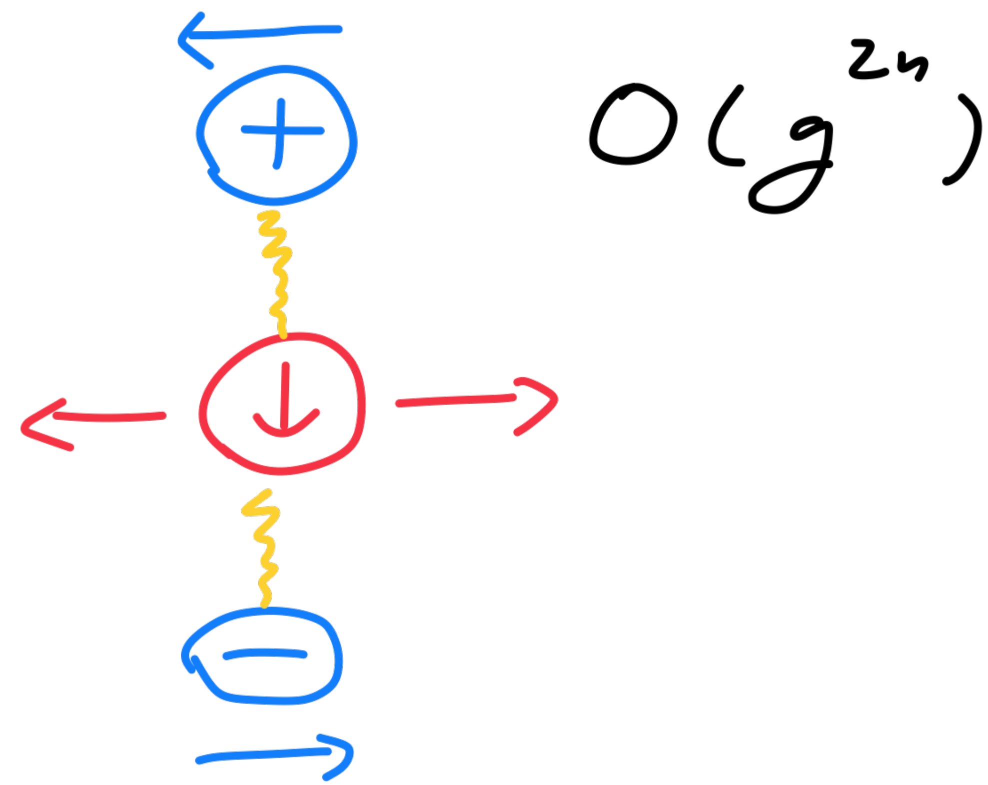




- Cancellation in particle-hole symmetric case, as for Coulomb drag in two-layer systems

Kamenev and Oreg, PRB 52 (1995)

- Haldane model

$\phi = \pm\pi/2, \Delta = 0$

$$\sigma_{\downarrow\uparrow}(g) = -\sigma_{\downarrow\uparrow}(-g)$$

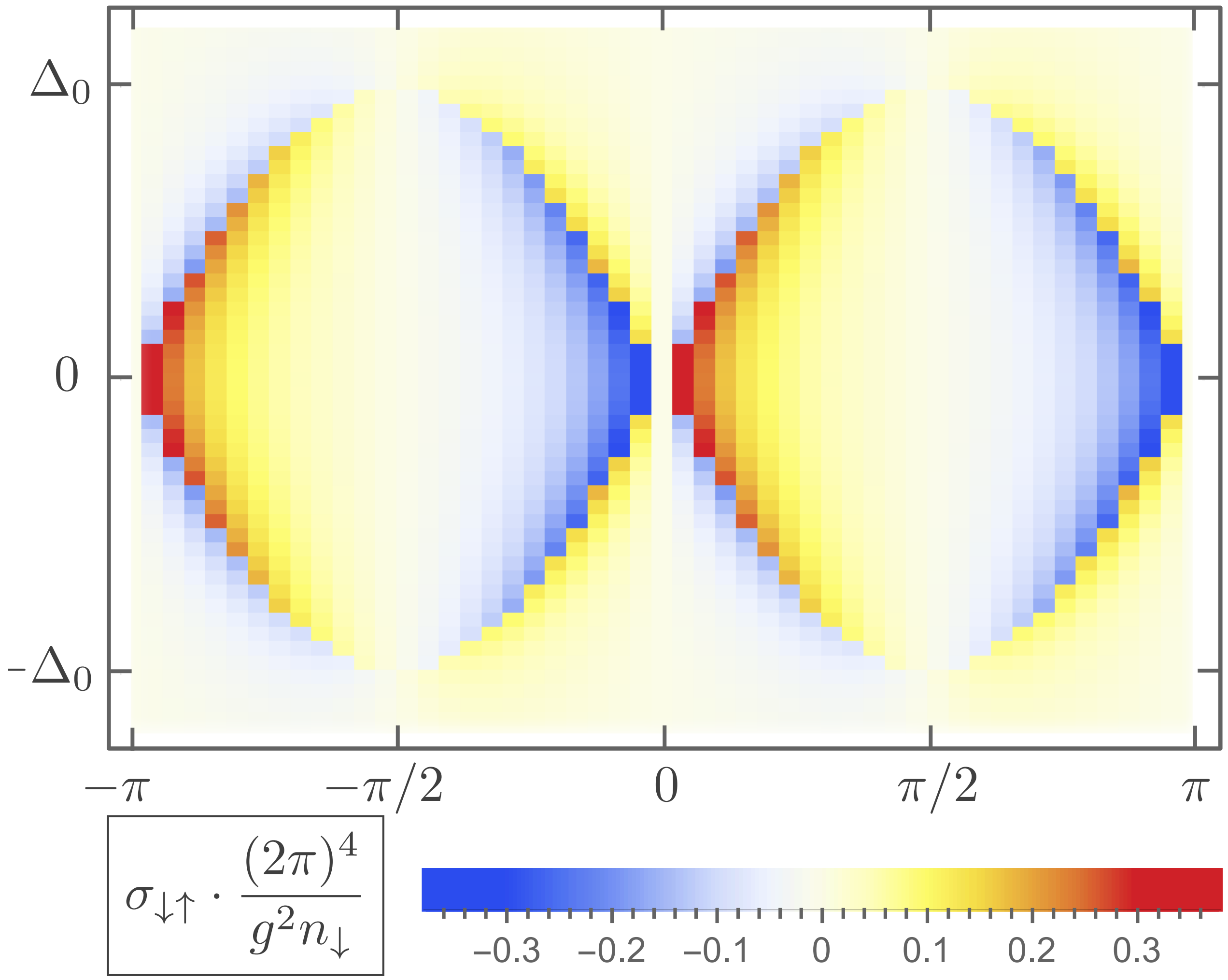
to all orders in g

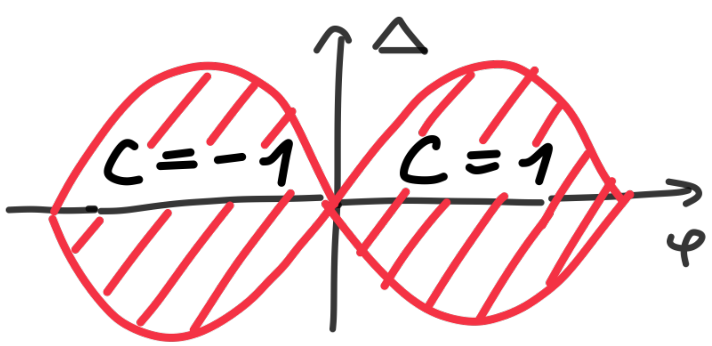
Drag in the Haldane model

- Drag vanishes at p-h symmetric lines $\phi = \pm\pi/2$

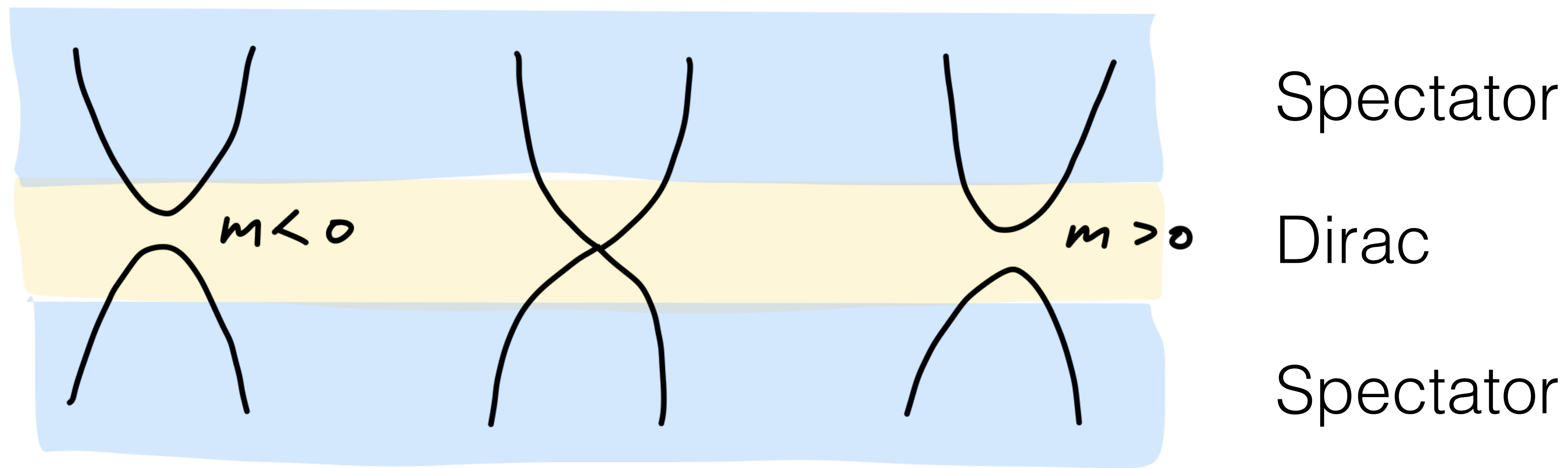
- Drag does not vanish in trivial phase (time-reversal broken everywhere)

- Phase boundaries clearly visible due to sharp jump of $\sigma_{\downarrow\uparrow}$





- Drag comes from Dirac- and spectator majorities



- Dirac contribution changes sign across topological phase transition & becomes singular (Berry curvature = $\frac{1}{2}\mathrm{sign}(m)\delta(\boldsymbol{k})$)

$$\Delta\sigma_{\uparrow\downarrow} = \Delta\mathcal{C} \times g^2 n_{\downarrow} \int d\boldsymbol{q}\, f(t', \phi; \boldsymbol{q})$$

- Spectator fermions yield smooth background contribution
- Exact analytical expressions for continuum model

Hall Drag Measurement

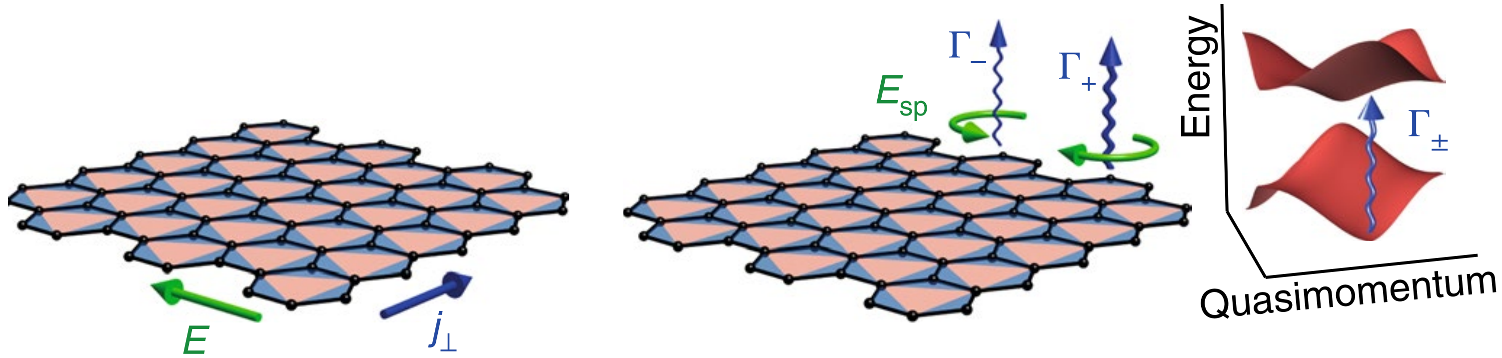
"Simply" measure current:

- in-situ observation of atomic cloud Aidelsburger et al., Nat. Phys. 11 (2015)
- state-dependent time-of-flight Cheuk et al., PRL 109 (2012)
- Raman spectroscopy Ness et al., PRX 10 (2020)

Alternative: Circular dichroism Tran et al., Sci. Adv. 3 (2017)

- Shine system with left- and right- polarized fields (lattice shaking), measure differential depletion

$$\Delta\Gamma_{\uparrow}(\omega) = \Gamma_{\uparrow,+}(\omega) - \Gamma_{\downarrow,-}(\omega) \qquad A_0 E^2 \mathcal{C} = -\int_0^{\infty} d\omega \Delta\Gamma_{\uparrow}(\omega) \;.$$



Asteria et al.,
Nat. Phys. 15 (2019)

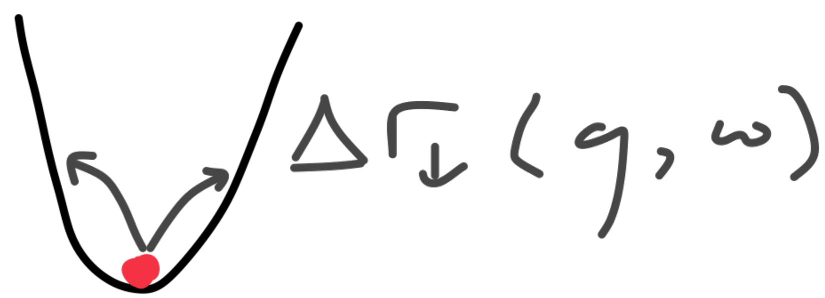
Hall Drag Measurement

- For trivial impurity: without coupling to majority: $\Delta\Gamma_{\downarrow}(\omega) = 0$

- With coupling

$$\sigma_{\downarrow\uparrow} = \frac{1}{4\pi A_0 E^2}\int_0^{\infty} d\omega \Delta\Gamma_{\downarrow}(\omega)$$

$$\Delta\Gamma_{\downarrow}(\omega) = \sum_{\boldsymbol{q}>0} \Delta\Gamma_{\downarrow}(\boldsymbol{q},\omega)$$



- Compare $\Delta\sigma_{\uparrow\downarrow} = \Delta\mathcal{C} \times g^2 n_{\downarrow} \int d\boldsymbol{q} f(t', \phi; \boldsymbol{q})$

  Independent Chern number estimate at every q

Summary

- Impurity weakly interacting with Chern insulator
- Hall Drag sensitively depends on particle-hole symmetry
- Hall Drag jumps across phase transition $\sim \Delta\mathcal{C}$
- Detection: circular Dichroism

To Do:

- Strong coupling – bound state formation
- Genuine many-body effects in bound state topology?

Thanks for your attention!