Exploring Four-Dimensional Quantum Hall Physics

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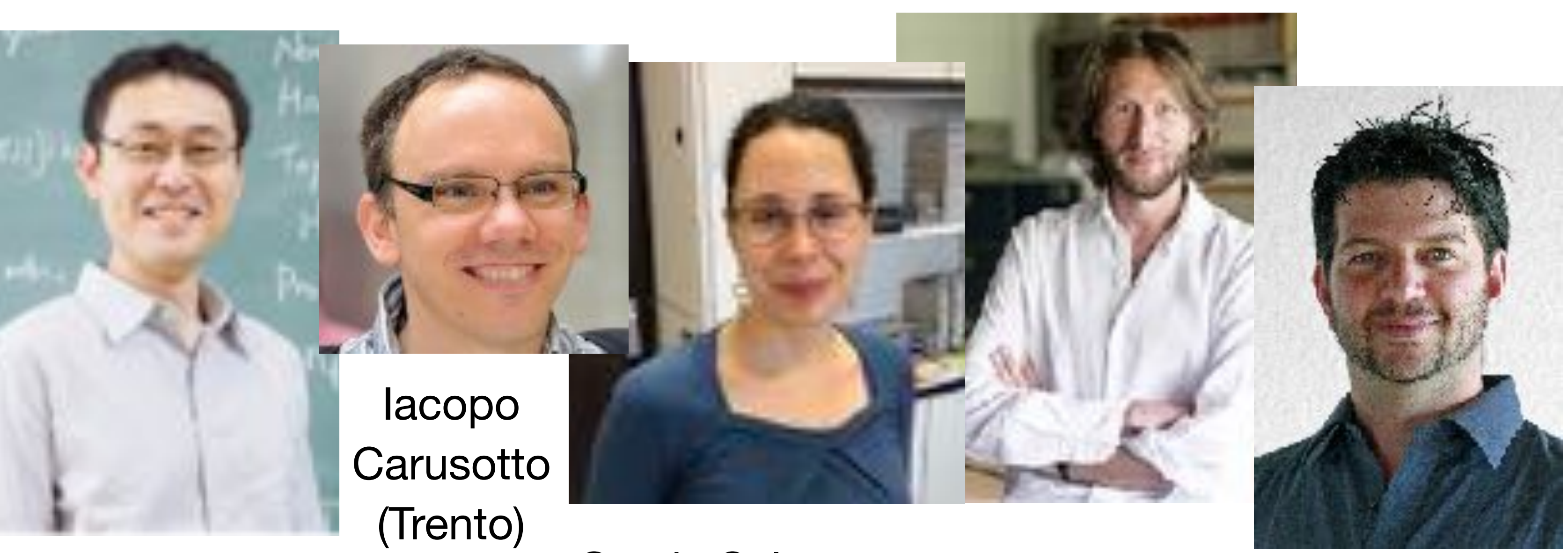

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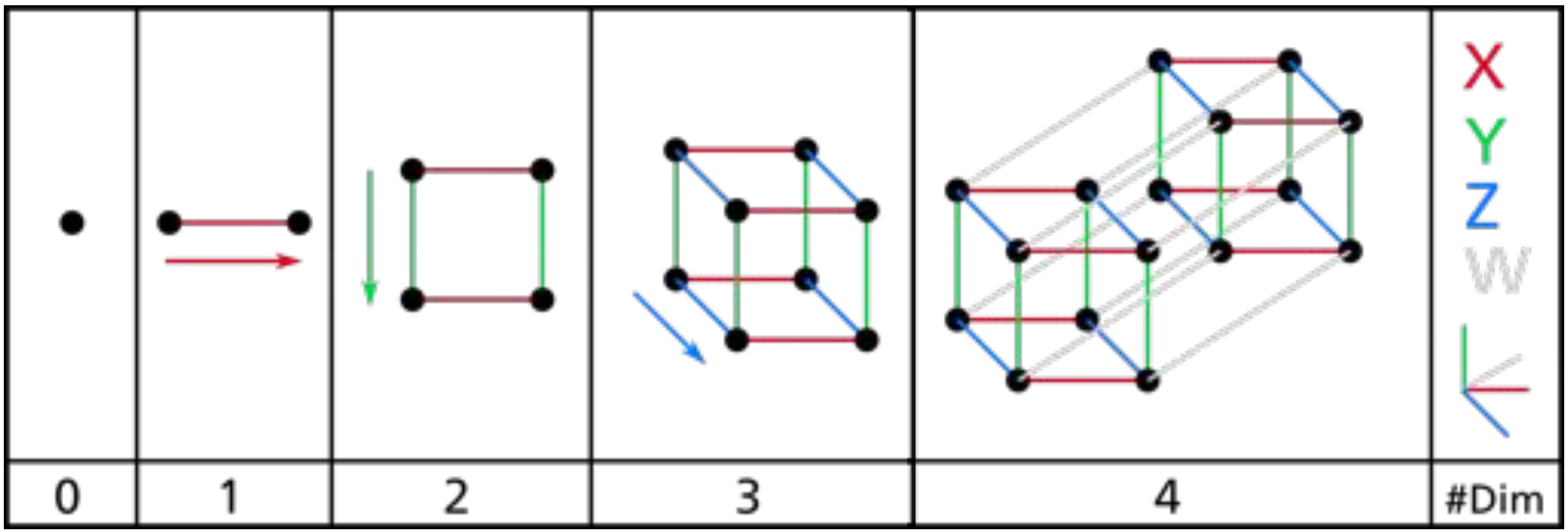
Overview

- Introduction to 4D Quantum Hall (QH) physics
- Using electrical circuits to realise a 4D QH model
- Superfluid vortices in four spatial dimensions

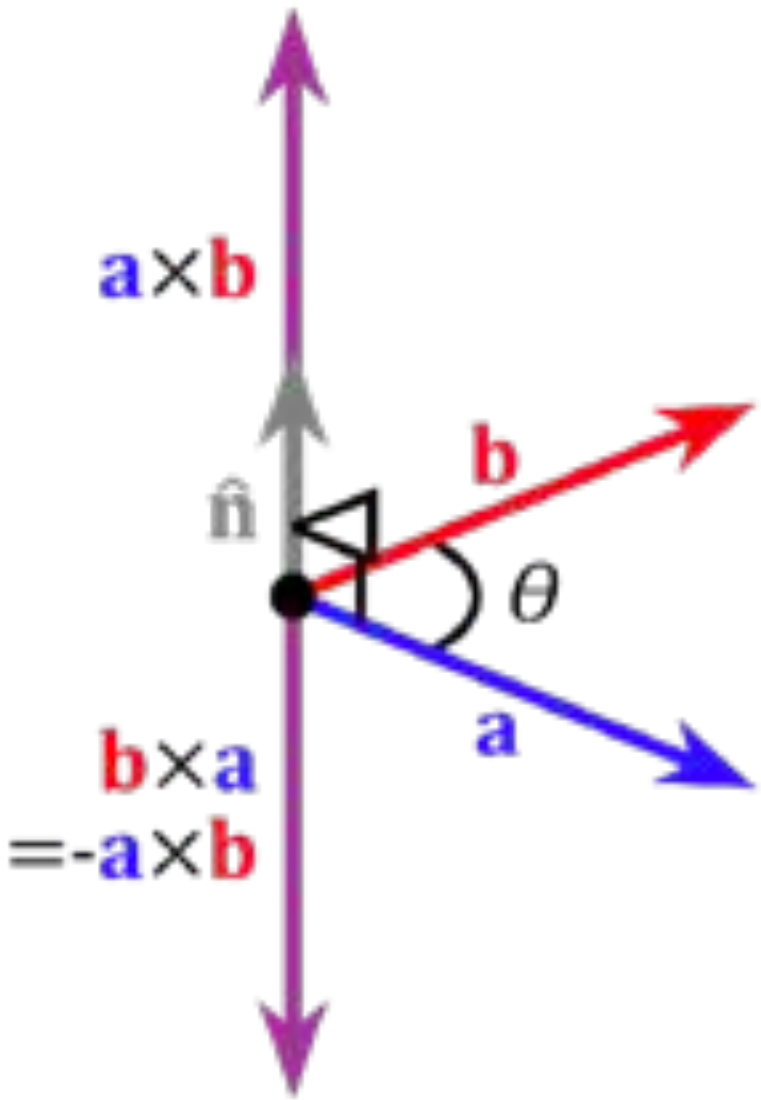
Four spatial dimensions



Some Key Differences in 4D

1. In 4D, avoid cross products

$$\mathbf{B} = \nabla \times \mathbf{A} \quad \longrightarrow \quad B_{\nu\mu} = \partial_\nu A_\mu - \partial_\mu A_\nu$$



In 2D,

$$B_{xy}$$

In 3D,

$$B_{xy}, B_{xz}, B_{yz}$$

(hence can treat like a 3D vector)

In 4D,

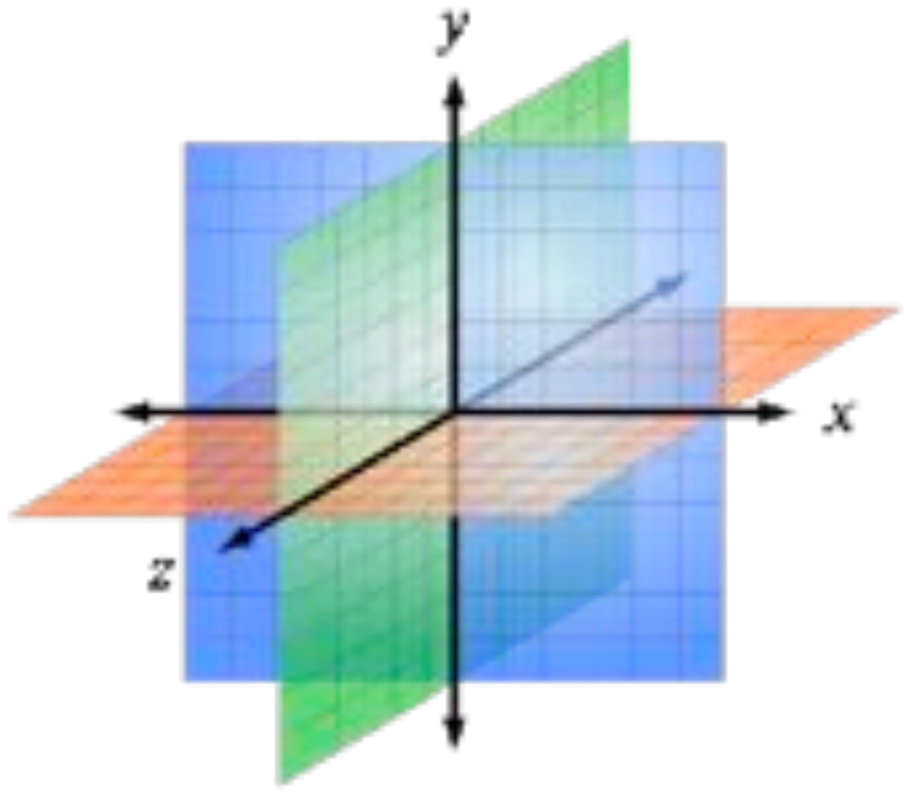
$$B_{xy}, B_{xz}, B_{xw}, B_{yz}, B_{yw}, B_{zw}$$

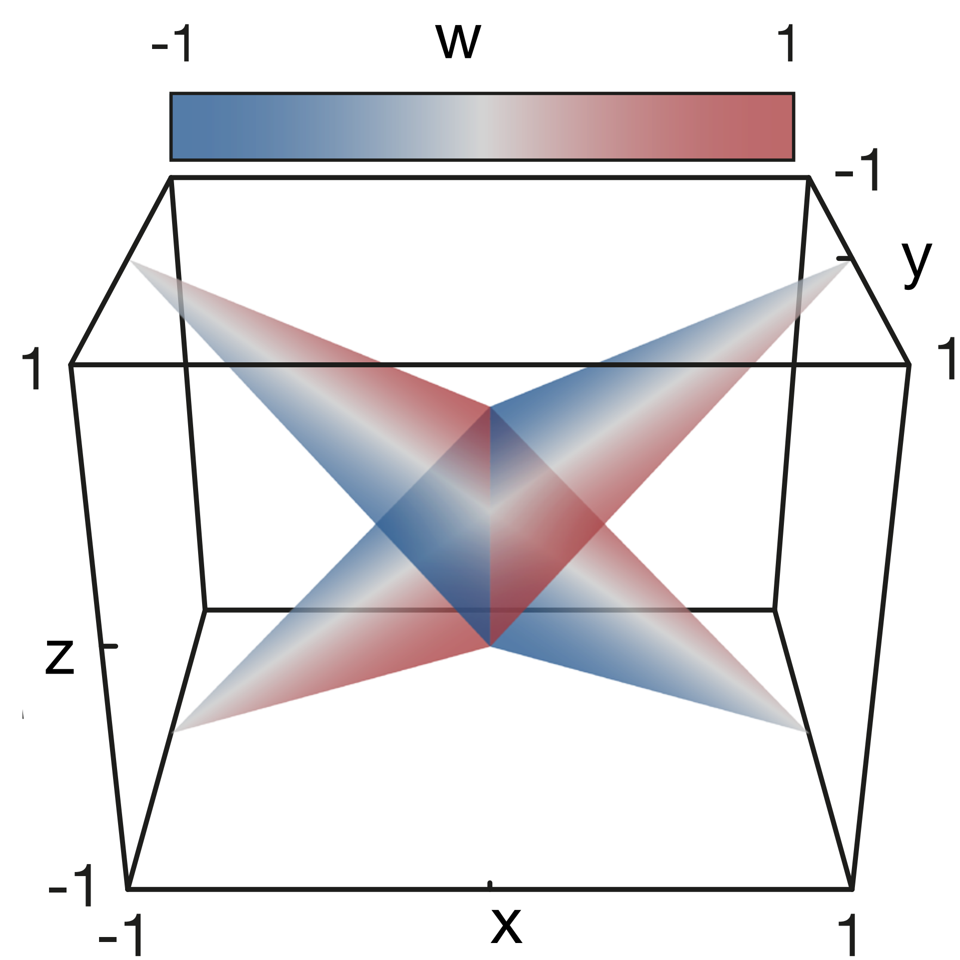
and similarly for Berry curvature…

Figure: User:Acdx, Public domain, via Wikimedia Commons

Some Key Differences in 4D

2. Intersections of orthogonal Cartesian planes





In 3D,

pairs of planes intersect at a line

$$xy, xz, zy$$

In 4D,

pairs of planes can intersect at a point

$$xy, xz, xw, zy, yw, zw$$

Figure: Lohse, Schweizer, Price, Zilberberg, Bloch, Nature 553, 55–58 (2018)

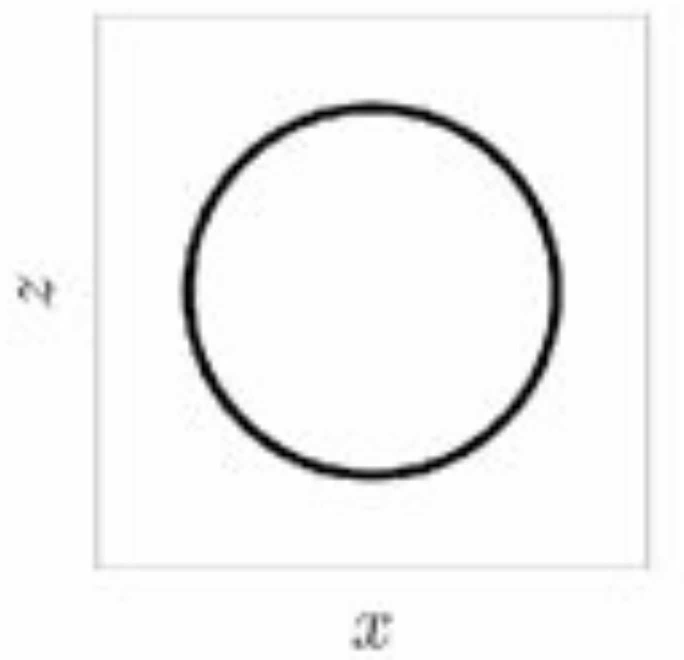
Classical Particle in a Magnetic Field

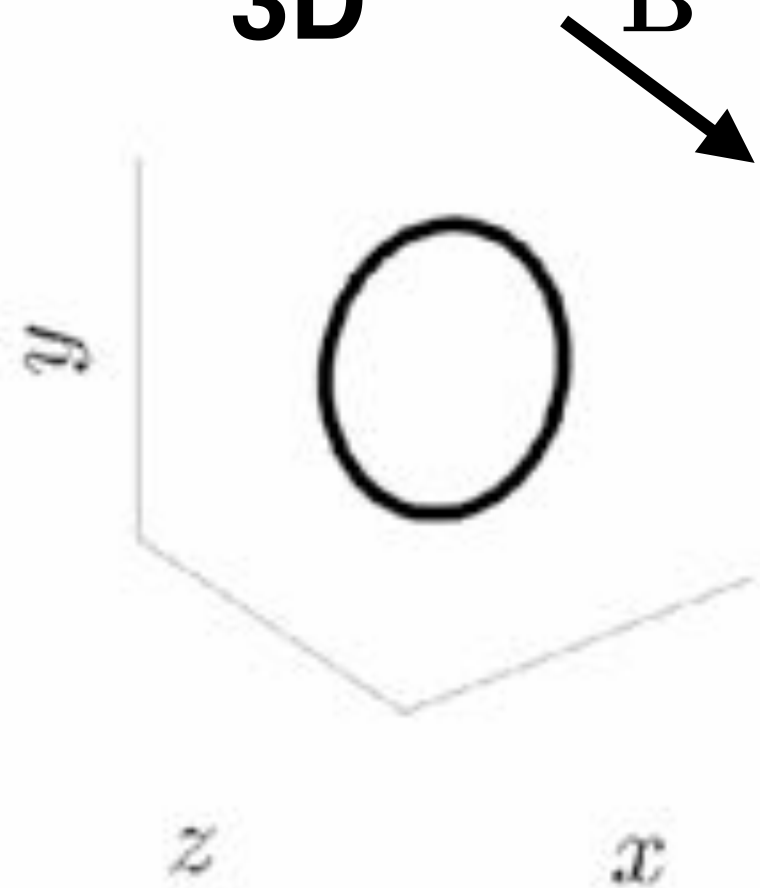
2D ⊗ B

$$F_\mu = q v_\nu B_{\mu\nu}$$

$$\omega = \frac{q|B|}{m}$$

3D B





$$B_{xz}$$

$$B_{xy}, B_{xz}, B_{yz} \to B_{x'z'}$$

$$x = \cos(\omega t), z = \sin(\omega t)$$

$$x' = \cos(\omega t), z' = \sin(\omega t)$$

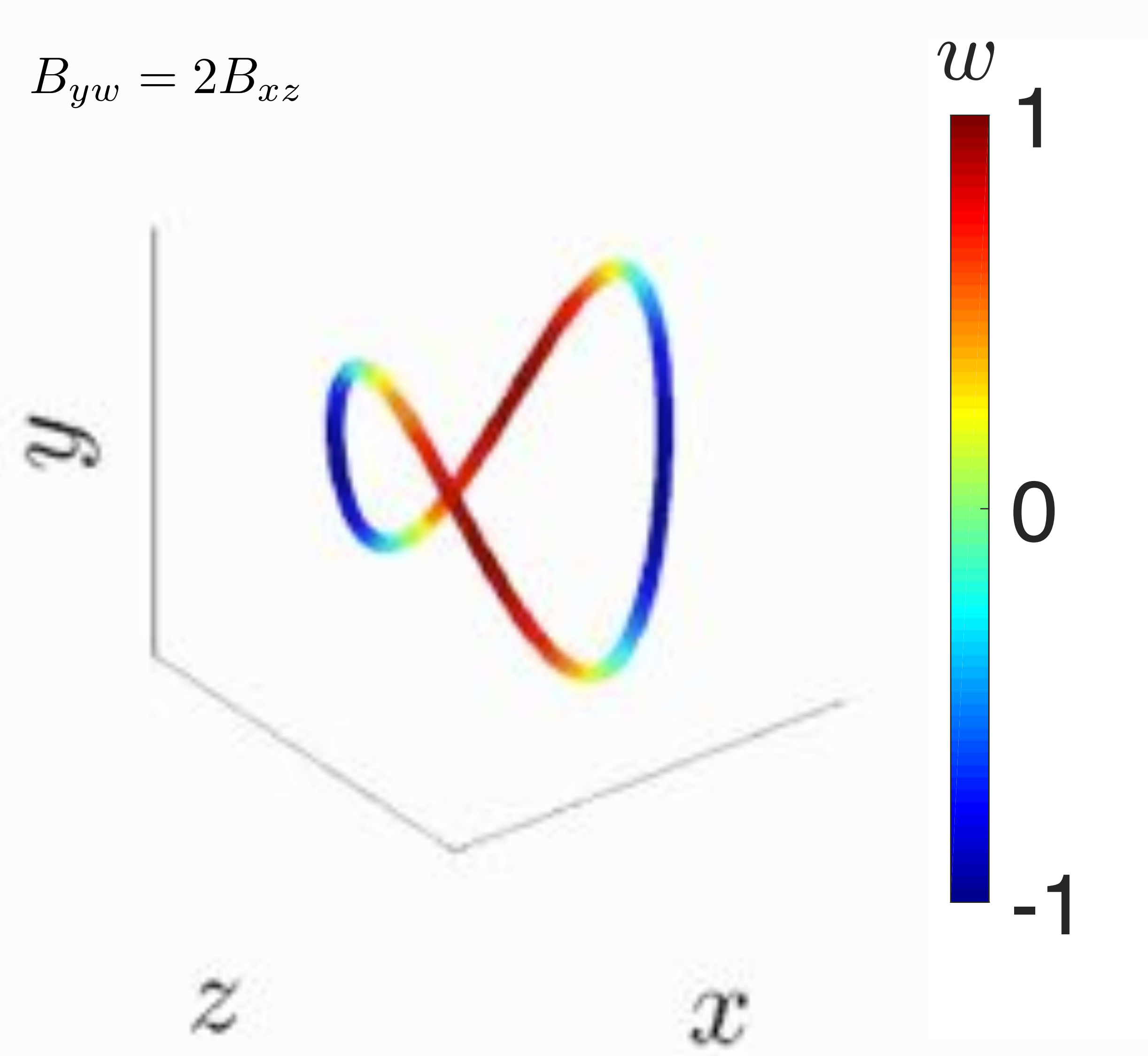
Classical Particle in a Magnetic Field

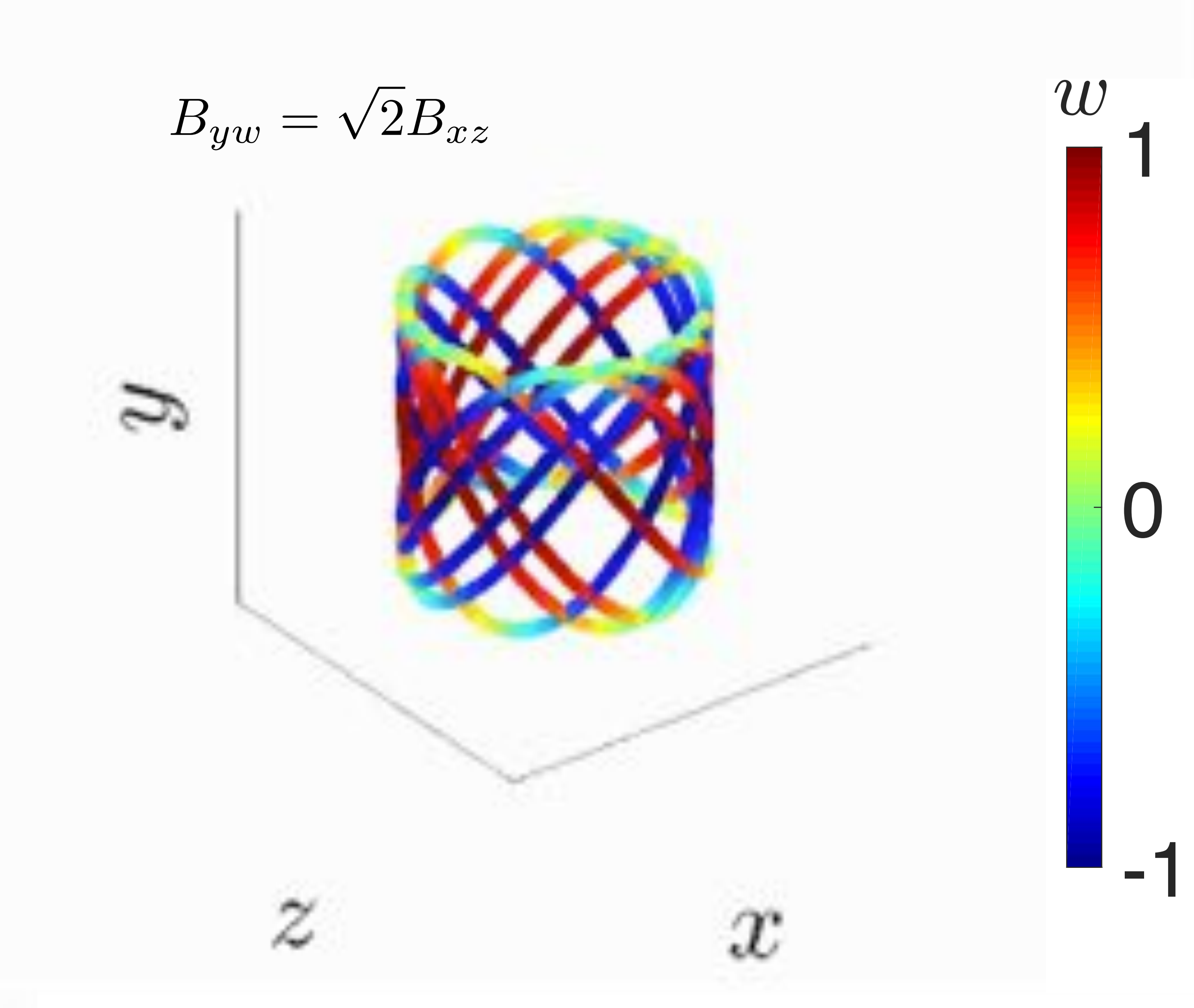
4D

e.g. $B_{xz}, B_{yw} \neq 0$

$$\omega = \frac{qB_{xz}}{m}, \quad \omega' = \frac{qB_{yw}}{m}$$

$x = \cos(\omega t), z = \sin(\omega t),$
$y = \cos(\omega' t), w = \sin(\omega' t)$





2D Quantum Hall Effect

$$\psi_{n,\mathbf{k}}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} u_{n,k}(\mathbf{r}) \qquad \hat{H}_{\mathbf{k}} u_{n,\mathbf{k}} = \mathcal{E}_n(\mathbf{k}) u_{n,\mathbf{k}}$$

Berry connection

$$\mathcal{A}_n(\mathbf{k}) = i\langle u_{n,\mathbf{k}}|\frac{\partial}{\partial \mathbf{k}}|u_{n,\mathbf{k}}\rangle$$

Berry curvature

$$\Omega_n(\mathbf{k}) = \nabla \times \mathcal{A}_n(\mathbf{k})$$

$$\Omega_n^{\mu\nu} = i\left[\langle\frac{\partial u_n}{\partial k_\mu}|\frac{\partial u_n}{\partial k_\nu}\rangle - \langle\frac{\partial u_n}{\partial k_\nu}|\frac{\partial u_n}{\partial k_\mu}\rangle\right]$$

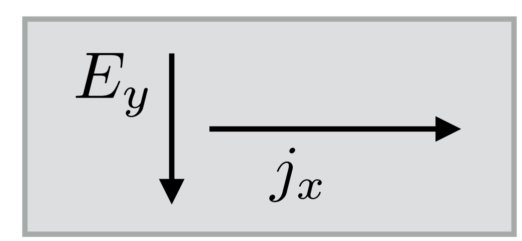
1st Chern Number (of a single non-degenerate band)

N.B. Always requires time-reversal symmetry-breaking (e.g. magnetic fields)

$$\nu_1^{\gamma\delta} = \frac{1}{2\pi}\int_{2\mathrm{DBZ}} \Omega^{\gamma\delta} dk_\gamma dk_\delta$$

Quantized response

$$j_\gamma = \frac{q^2}{h} E_\delta \nu_1^{\gamma\delta}$$



And then in 3D, can have a triad of first Chern numbers…

2nd Chern Number in 4D

2nd Chern Number (of a single non-degenerate band)

Avron et al, Phys. Rev. Lett. 61, 1329 (1988)….

$$\nu_2 = \frac{1}{32\pi^2} \int_{4\mathrm{DBZ}} \epsilon^{ijkl} \Omega^{ij} \Omega^{kl} d^4\mathbf{k}$$

N.B. Does not require time-reversal symmetry-breaking!

- Algorithm to calculate the 2nd Chern number

Mochol-Grzelak et al, Quantum Sci. and Tech. 4 (1), 014009 (2019)

- Dimensional reduction to get TIs

Qi et al, Phys. Rev. B 78, 195424 (2008)

- 2nd Chern Number and second-order TIs

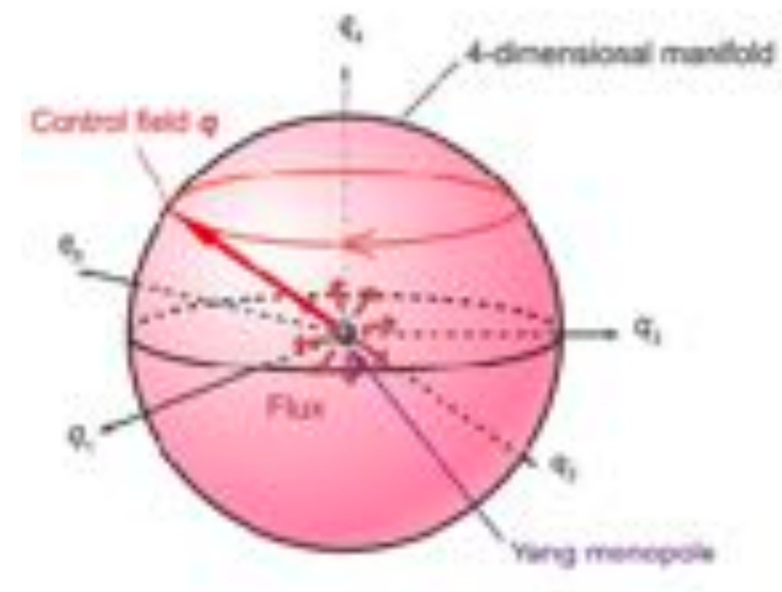
Petrides and Zilberberg, PRR. 2, 022049 (2020)

- 3rd Chern Number in 6D and so on…

Petrides, HMP, Zilberberg Phys. Rev. B 98, 125431 (2018) and references there-in

- Measuring 2nd Chern Number in a parameter space

Kolodrubetz, PRL. 117, 015301 (2016)
Cold atoms: Sugawa et al., Science 360,1429 (2018)



- Superconducting systems

Riwar et al, Nat. Comm., 7, 11167 (2016)
Weisbrich et al, PRX Quantum 2, 010310 (2021)

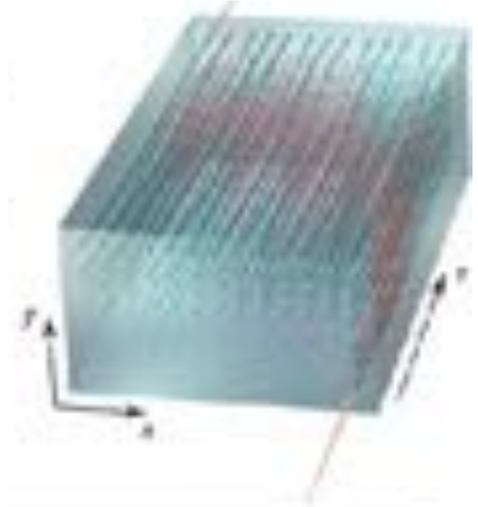
- Other types of 4D topology, e.g. 4D tensor monopoles

Palumbo and Goldman, PRL121, 170401 (2018)
Zhu et al, PRB 102, 081109 (2020)
Superconducting Qudits: Tan et al., PRL. 126, 017702 (2021)

2nd Chern Number in 4D

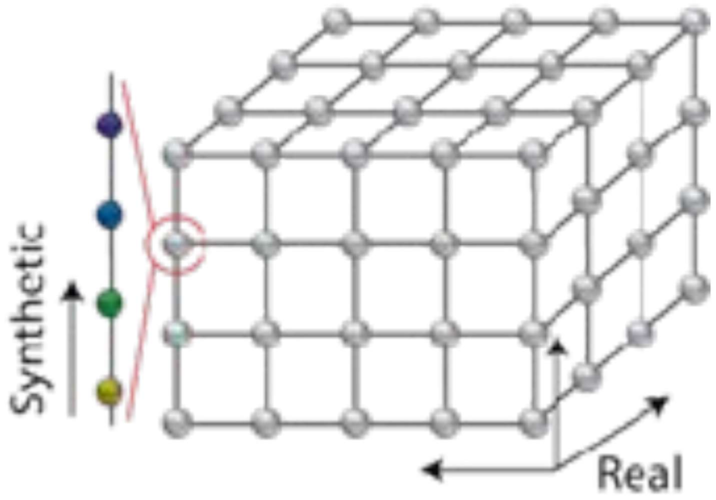
Quantized response $$j_\mu = \frac{q^3}{2h^2}\varepsilon^{\mu\gamma\delta\nu}E_\nu B_{\gamma\delta}\nu_2$$

Zhang et al, Science 294, 823 (2001),
Qi et al, Phys. Rev. B 78, 195424 (2008)….



- Observed signatures in topological pumping:

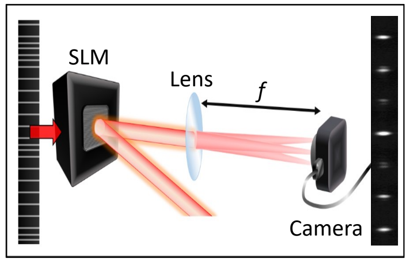
Kraus, Ringel, Zilberberg, PRL. 111, 226401 (2013)
Cold atoms: Lohse, Schweizer, HMP, Zilberberg, Bloch, Nature 553, 55 (2018)
Photonics: Zilberberg et al., Nature 553, 59 (2018)
Acoustics: Chen et al, Phys. Rev. X 11, 011016 (2021).



- Proposal for measurements with synthetic dimensions

HMP, Zilberberg, Ozawa, Carusotto & Goldman, PRL 115, 195303 (2015)
Ozawa, HMP, Goldman, Zilberberg, and Carusotto, PRA 93, 043827 (2016)
HMP, Zilberberg, Ozawa, Carusotto & Goldman, PRB 93, 245113 (2016)…

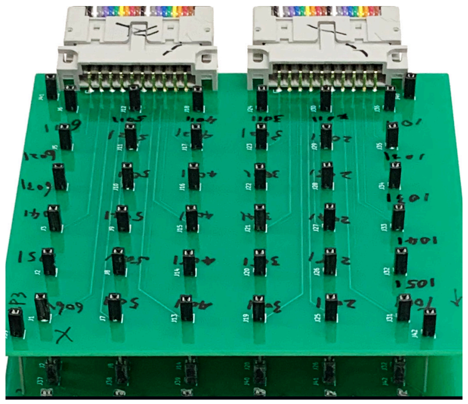
- Optical diffraction patterns



Di Colandrea et al, arXiv:2106.08837

- Surface states in 4D Electrical Circuits

M. Ezawa, Phys. Rev. B 100, 075423 (2019)
R. Yu, Y. X. Zhao, and A. P. Schnyder, Nat. Sci. Rev. (2020),
HMP, Phys. Rev. B 101, 205141 (2020)
Wang, HMP, Zhang, Chong, Nat. Comm. 11, 2356 (2020)
Zhang et al Phys. Rev. B 102, 100102 (2020)…



Overview

- Introduction to 4D Quantum Hall physics

- **Using electrical circuits to realise a 4D QH model**

Wang You, Baile Zhang, Yidong Chong

Singapore

- Superfluid vortices in four spatial dimensions

Electrical circuits for topological models

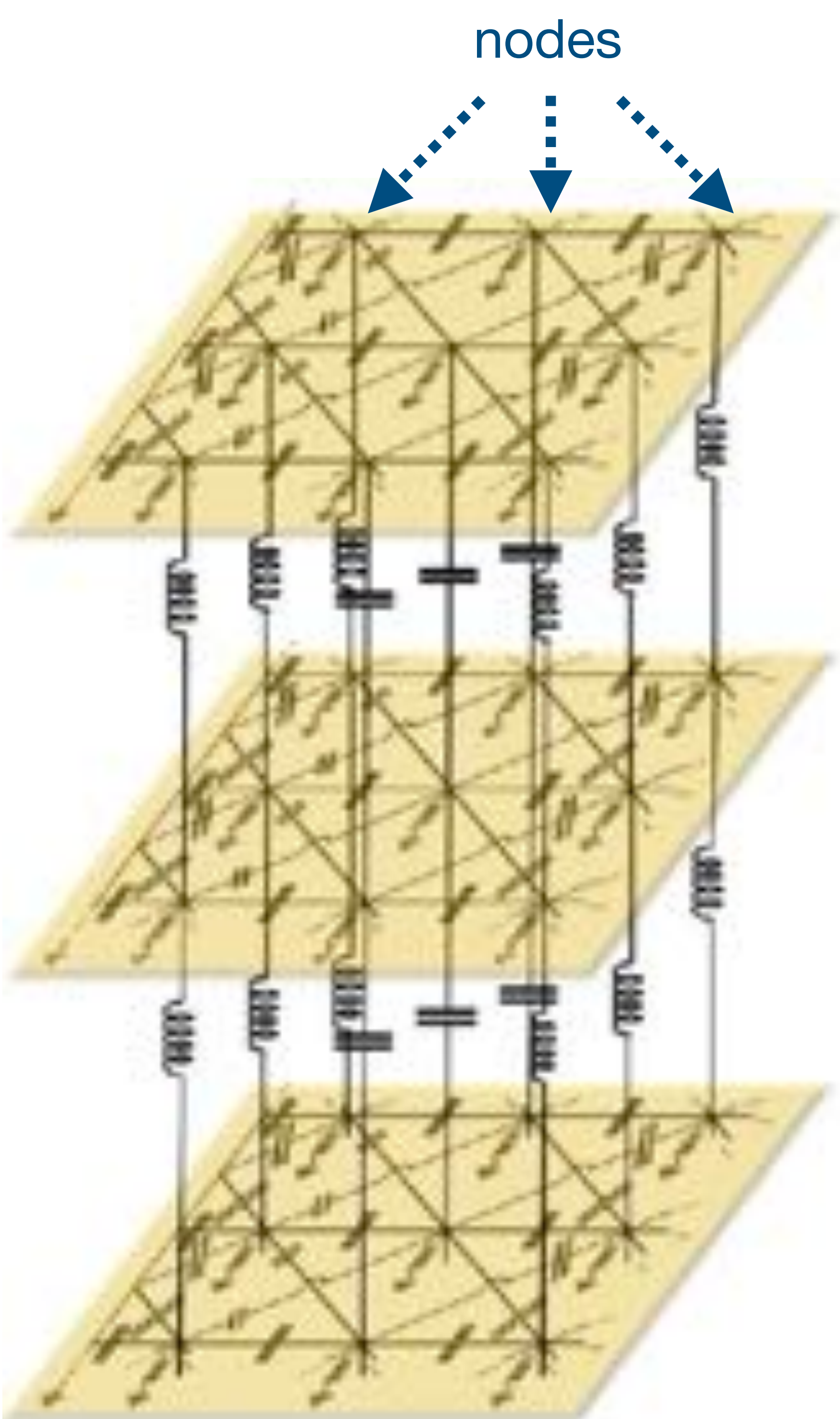


Figure from Lee et al, Communications Physics, Volume 1, 39 (2018)

Network of resistors, inductors, capacitors…

current at node i

voltage between node j and the ground

Kirchoff's law $$I_i \equiv \sum_j L_{ij} V_j$$

behaviour governed by the circuit Laplacian

which can be related to a
desired (topological) tight-binding Hamiltonian

Ningyuan et al Phys. Rev. X 5, 021031 (2015)
Albert et al, Phys. Rev. Lett. 114, 173902 (2015)
Lee et al, Communications Physics, Volume 1, 39 (2018)
Imhof et al, Nat Phys, 14, 925 (2018)
Ezawa, Phys. Rev. B **99**, 201411 (2019)
Dong et al, Phys. Rev. Research 3, 023056 (2021)…..

In more detail

current flowing to ground

current flowing to other nodes

Kirchoff's law $$I_i \equiv \sum_j L_{ij} V_j = (-D_{ii} + D'_{ii}) V_i + \sum_j D_{ij} (V_i - V_j)$$

maps to desired on-site terms

extra tuning terms

maps to desired hopping terms

Design the electrical conductances such that:

$$I_i(f_0) = -i\alpha \sum_j \Big[H_{ij}(f_0) - E\,\delta_{ij} \Big] V_j(f_0),$$

working frequency

target TB model

target energy

Lee et al, Communications Physics, Volume 1, 39 (2018)
Imhof et al, Nat Phys, 14, 925 (2018)….
Wang, HMP, Zhang, Chong, Nat. Comm. 11, 2356 (2020)

$D_{ij}(f) \equiv i\alpha H_{ij}(f).$

Impedance Measurements

Run a current through the circuit and measure the response

$$V_r = \sum_j (L^{-1})_{rj} I_j = Z_r I_r$$

grounding impedance

$$Z_r = \frac{i}{\alpha} \lim_{\epsilon \to 0} \sum_n \frac{|\psi_n(r)|^2}{E_n - E + i\epsilon}$$

$$\mathrm{Re}(Z_r) = \frac{1}{\pi\alpha} \sum_n \delta(E - E_n) |\psi_n(r)|^2$$

at the working frequency get

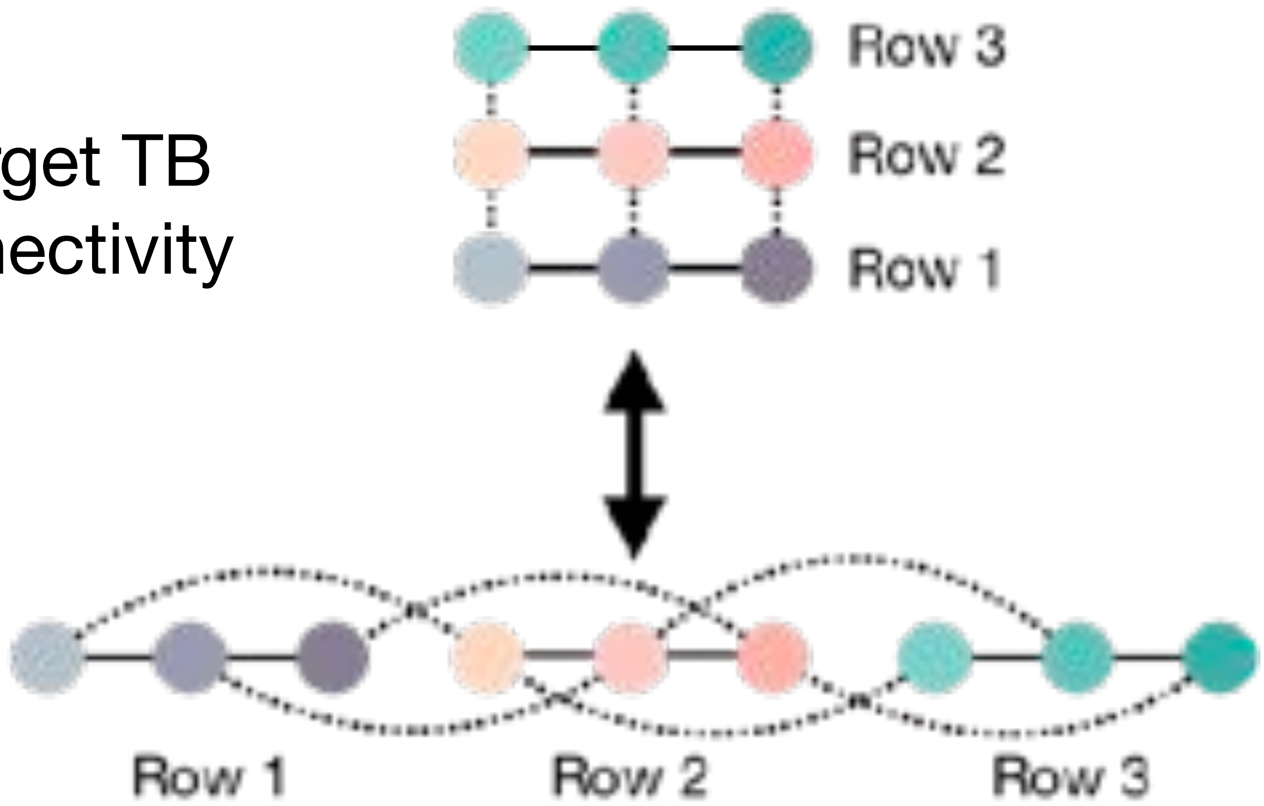
Local DOS of desired TB model at the target energy

e.g. edge states —> LDOS localised at edges of the system

Lee et al, Communications Physics, Volume 1, 39 (2018)
Imhof et al, Nat Phys, 14, 925 (2018)….
Wang, HMP, Zhang, Chong, Nat. Comm. 11, 2356 (2020)

Our goal

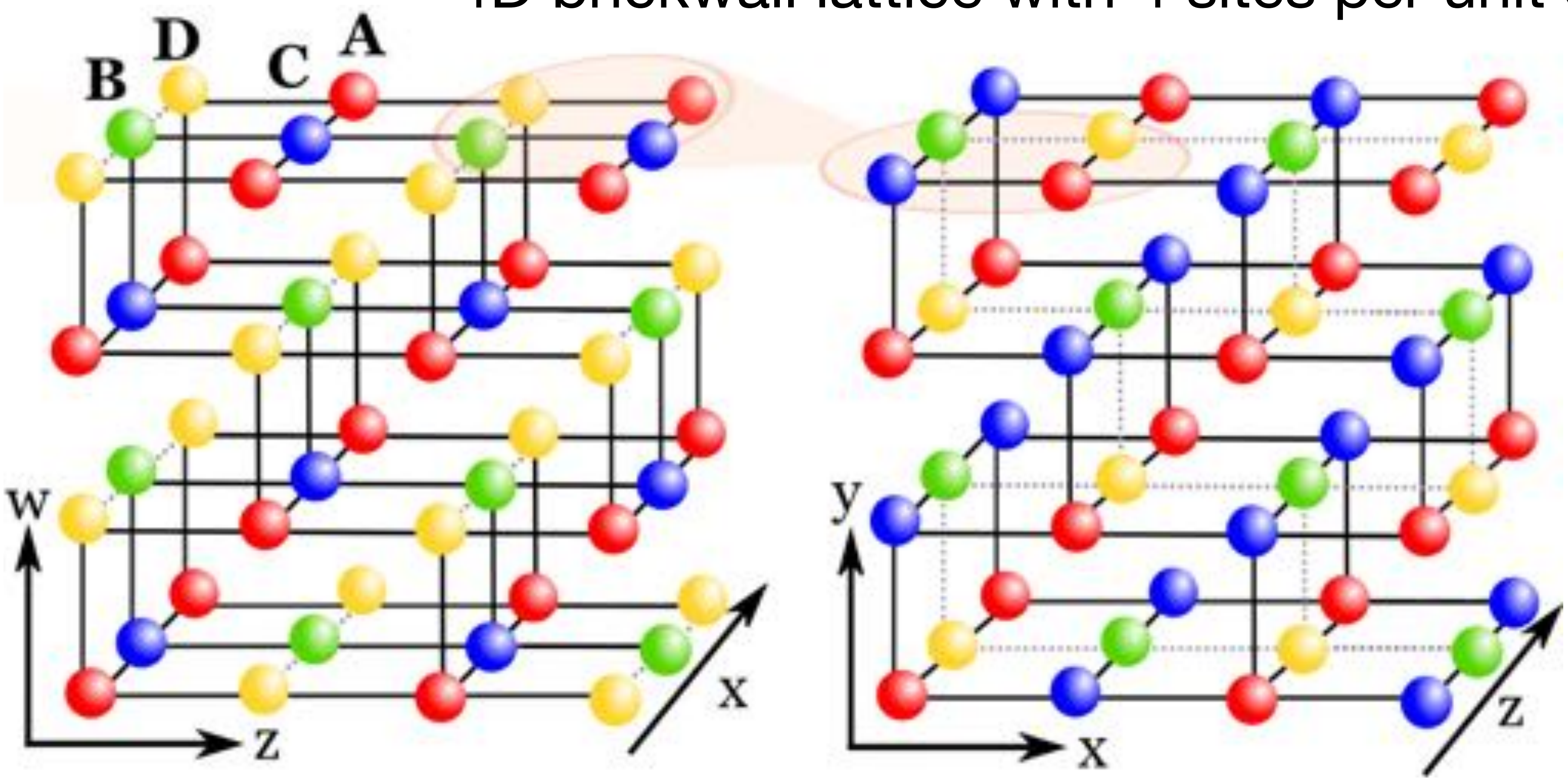
Make a 4DQH model our target TB model by exploiting the connectivity of an electrical circuit



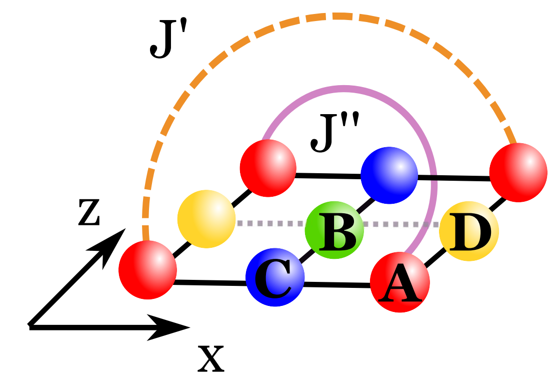
What sort of 4DQH model can we engineer easily in a circuit?

4D QH via connectivity

4D brickwall lattice with 4 sites per unit cell



+ some long-range hoppings e.g.:



··· negative hoppings
— positive hoppings

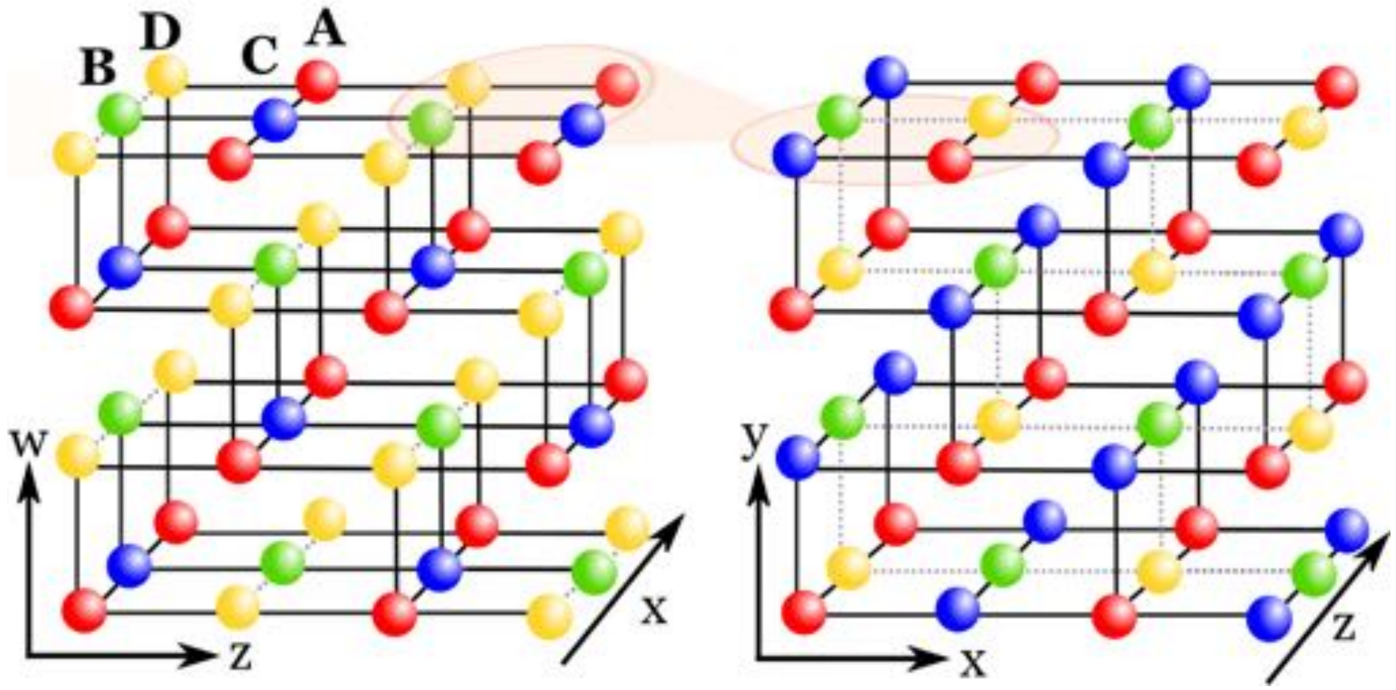
$$H(\mathbf{k}) = J\left[(2\cos k_x + \cos k_y)\Gamma_1 + \sin k_y \Gamma_2 + (2\cos k_z + \cos k_w)\Gamma_3 + \sin k_w \Gamma_4 + m\Gamma_5\right]$$
$$+ \left[2J' \cos(2k_x + 2k_z) + 2J'' \cos(2k_x - 2k_z)\right]\Gamma_5$$

$$\Gamma_1 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \Gamma_2 = \begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \\ i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix}, \Gamma_3 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \Gamma_4 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}, \Gamma_5 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix},$$

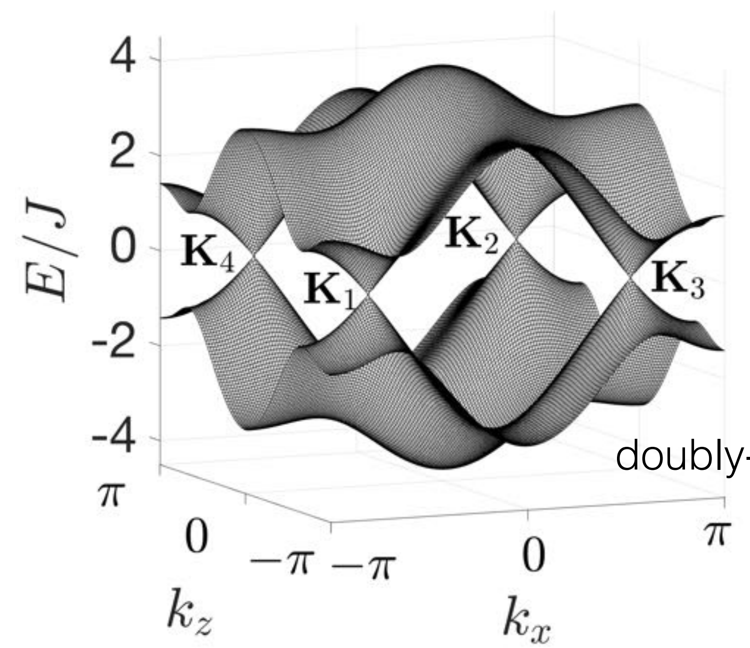
- spinless time-reversal symmetry
- *trivial* first Chern numbers
- *nontrivial* (even) second Chern number

HMP, Phys. Rev. B 101, 205141 (2020)
Wang, HMP, Zhang, Chong, Nat. Comm. 11, 2356 (2020)

Aside: 4D Dirac Cones

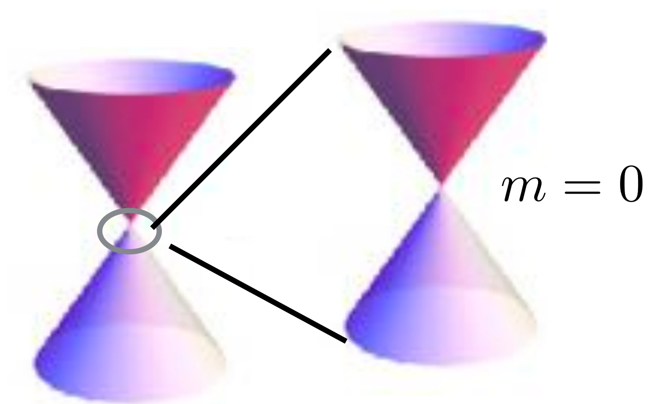


When there are no gap-opening terms with Γ_5



Around a single 4D Dirac cone $H \approx \mathbf{d}(\mathbf{q}) \cdot \mathbf{\Gamma}$

$$\mathbf{d}(\mathbf{q}) \approx (v_x q_x, v_y q_y, v_z q_z, v_w q_w, m)$$



$$\nu_2 = \frac{3}{8\pi^2} \int_{\mathrm{BZ}} d^4\mathbf{k} \epsilon^{abcde} \hat{d}_a \partial_{k_x} \hat{d}_b \partial_{k_y} \hat{d}_c \partial_{k_z} \hat{d}_d \partial_{k_w} \hat{d}_e$$

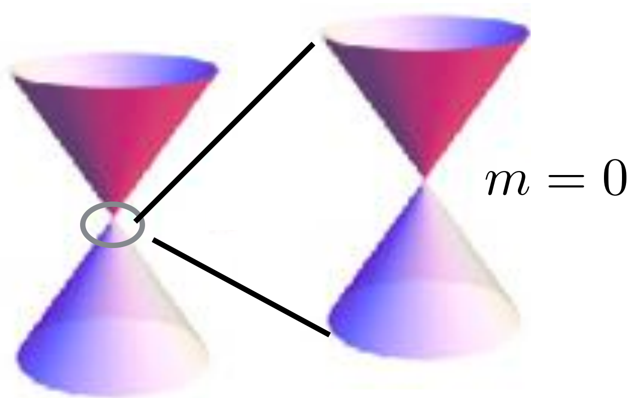
$\hat{\mathbf{d}} = \mathbf{d}/|\mathbf{d}|$

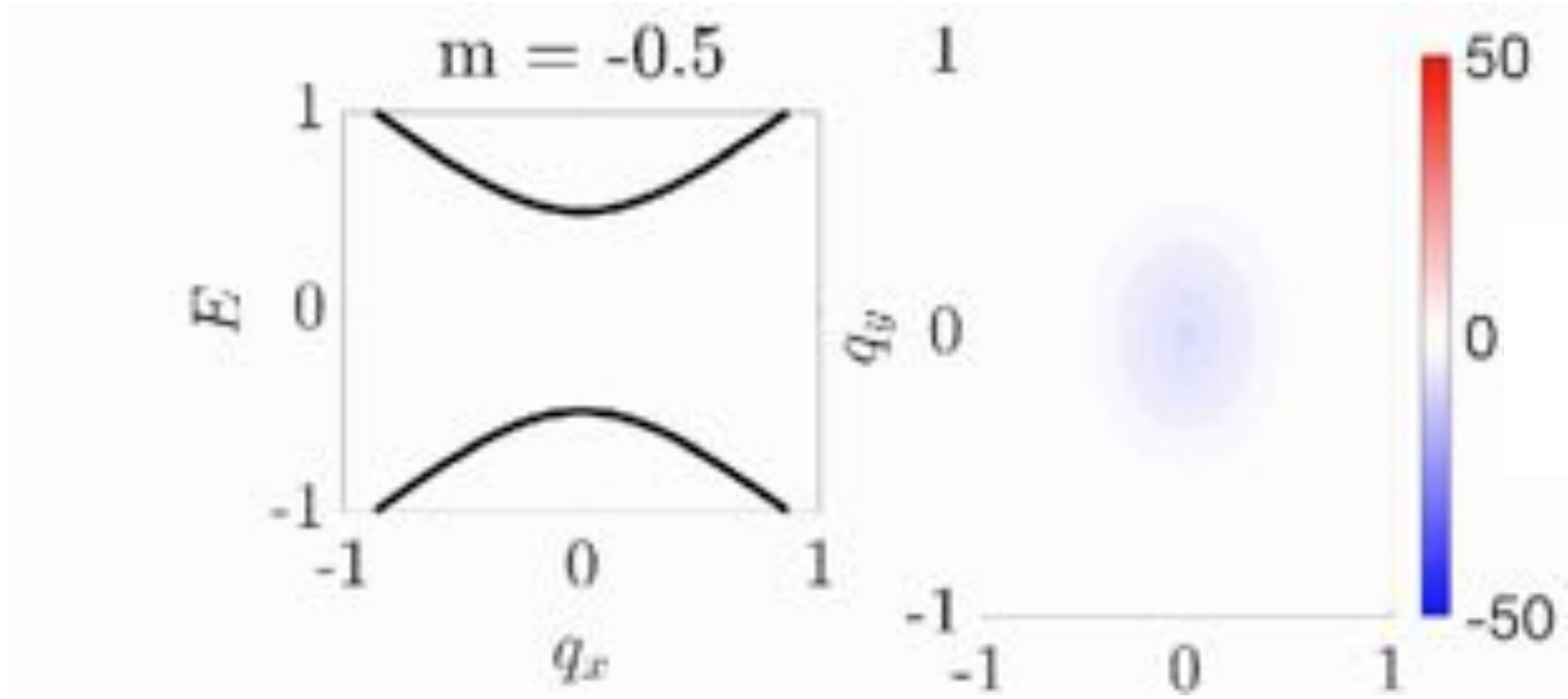
Qi et al, Phys. Rev. B 78, 195424 (2008)

Aside: 4D topological transitions

$$\nu_2 = \frac{3}{8\pi^2}\int_{\mathrm{BZ}} d^4\mathbf{k}\epsilon^{abcde}\hat{d}_a\partial_{k_x}\hat{d}_b\partial_{k_y}\hat{d}_c\partial_{k_z}\hat{d}_d\partial_{k_w}\hat{d}_e$$

integrand





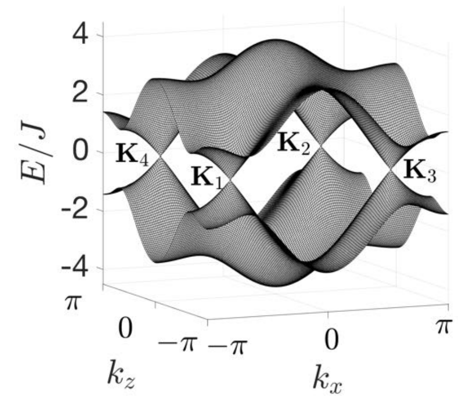
$$\mathbf{d}(\mathbf{q}) \approx (v_x q_x, v_y q_y, v_z q_z, v_w q_w, m)$$

$q_y = q_z = q_w = 0$ $\quad q_x$

when $d_5 = -m \to d_5 = m$

Type 1: d_1, d_2, d_3, d_4 even no/ minus signs —> ***increases*** integrand

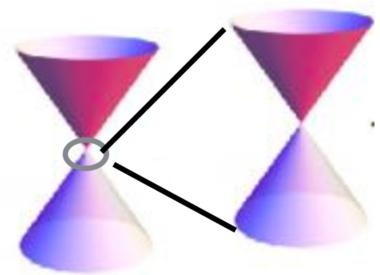
Type 2: d_1, d_2, d_3, d_4 odd no/ minus signs —> ***decreases*** integrand



No/ Type 1 Transitions = No/ Type 2 Transitions, then 2CN will be *trivial*

Aside: Time-reversal symmetry

Imagine we have a ***Type 1*** cone



$\mathbf{K}$

$(+q_x, +q_y, +q_z, +q_w)$

What about ***time-reversal symmetry***

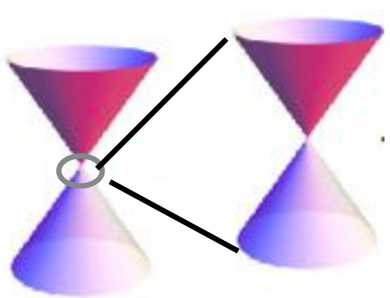
e.g, for spinless particles $H^*(\mathbf{k}) = H(-\mathbf{k})$

$$d_{1,3}(\mathbf{k}) = d_{1,3}(-\mathbf{k}), \quad d_{2,4}(\mathbf{k}) = -d_{2,4}(-\mathbf{k})$$

as

$$\Gamma^*_{1,3} = \Gamma_{1,3} \quad \Gamma^*_{2,4} = -\Gamma_{2,4}$$

Then the TRS-partner Dirac cone:



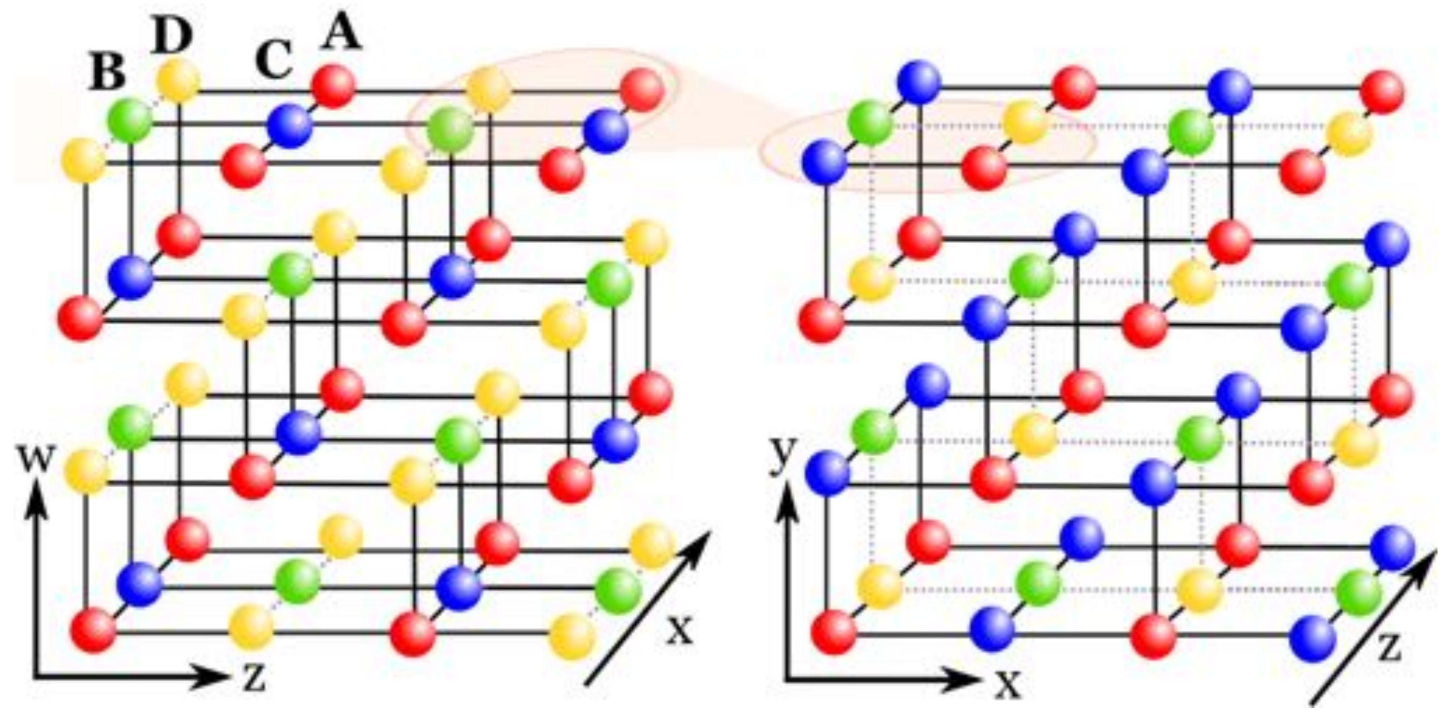
$-\mathbf{K}$

$(+q_x, -q_y, +q_z, -q_w)$ —> Also ***Type 1***

So can have <u>2nd</u> Chern number with TRS

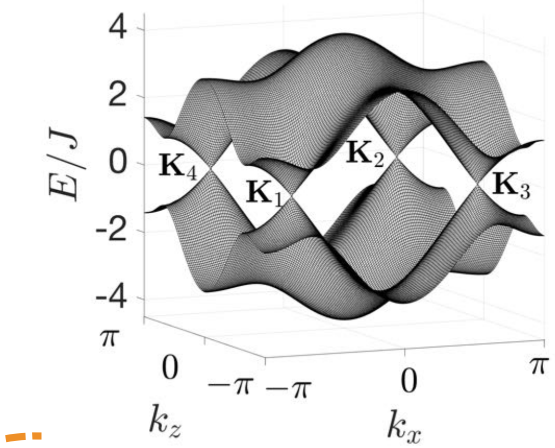
Qi et al, Phys. Rev. B 78, 195424 (2008)…
Price, Phys. Rev. B 101, 205141 (2020)

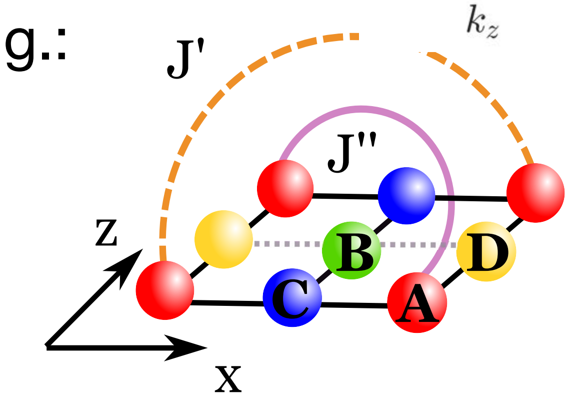
Back to our model

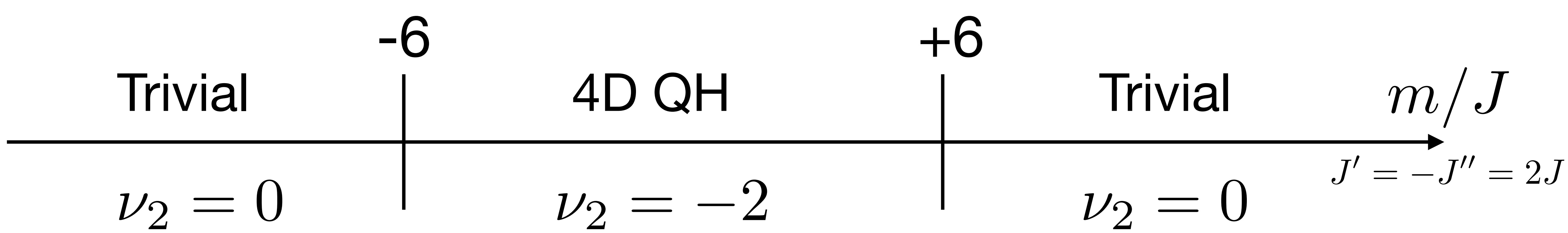


··· negative hoppings
— positive hoppings

+ some long-range hoppings e.g.:

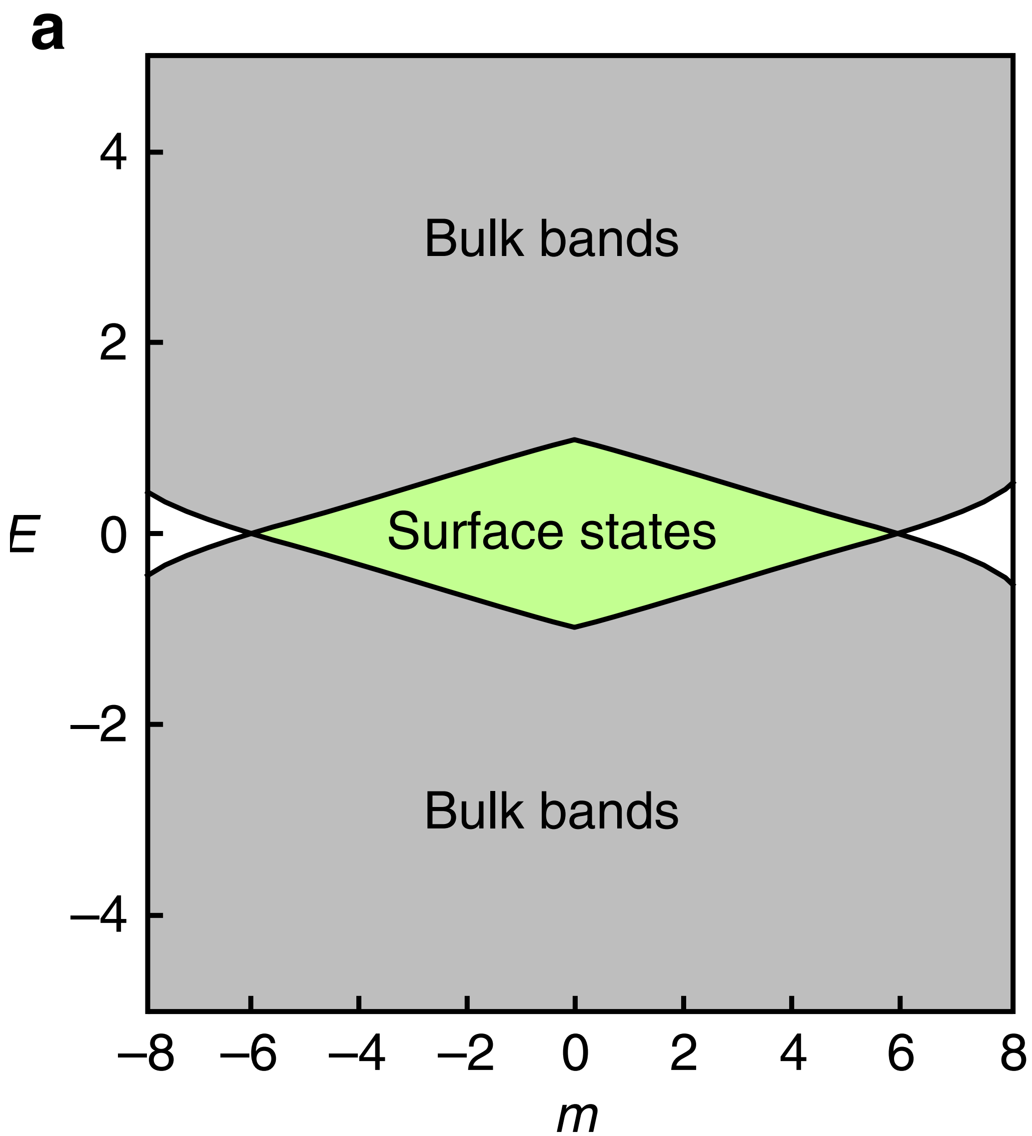






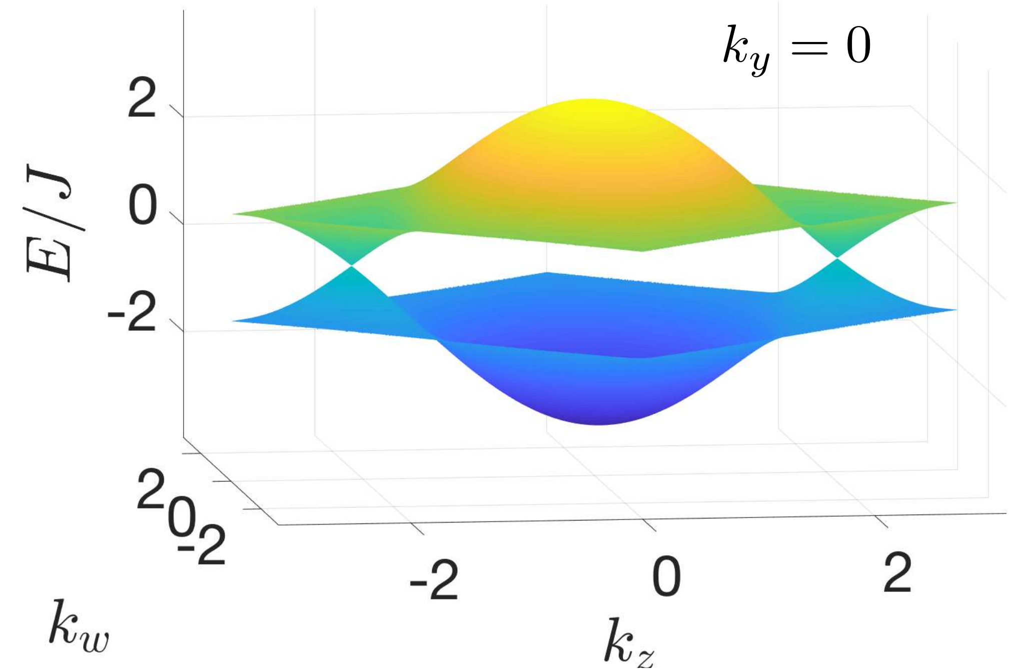
HMP, Phys. Rev. B 101, 205141 (2020)
Wang, HMP, Zhang, Chong, Nat. Comm. 11, 2356 (2020)

3D Surface States

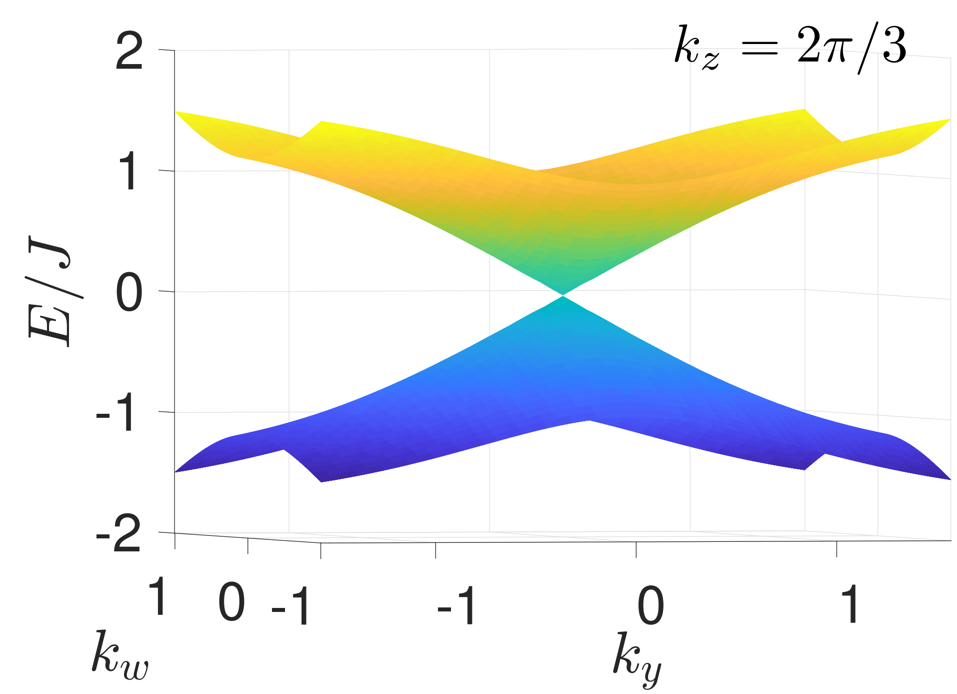


Surface state dispersion : **3D Weyl points** at

$$k_y = k_w = 0,\ k_z = \pm 2\pi/3$$



Open b.c. along x



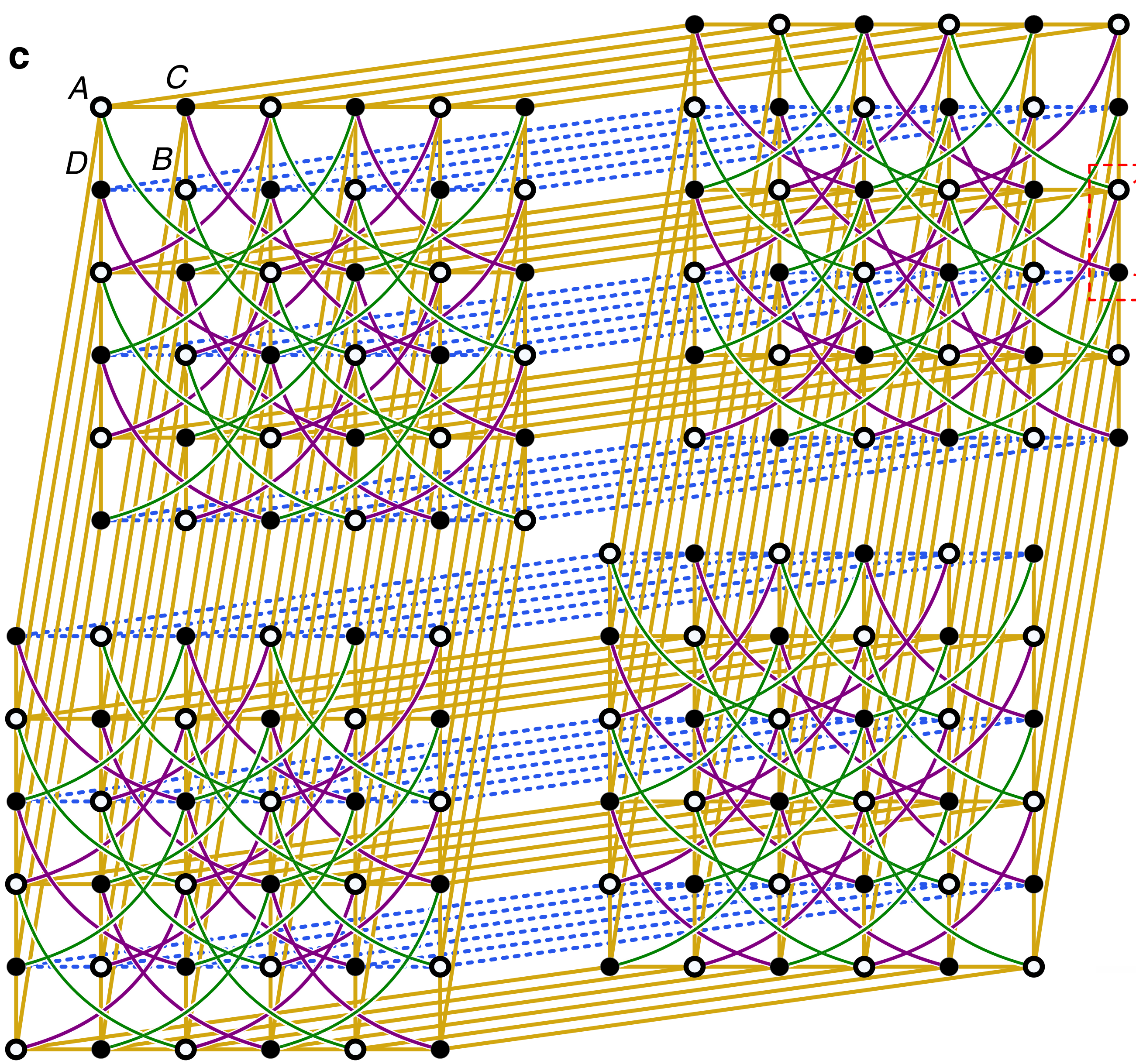
Aim: build this model in a circuit and observe these surface states in the LDOS (i.e. impedance measurements)

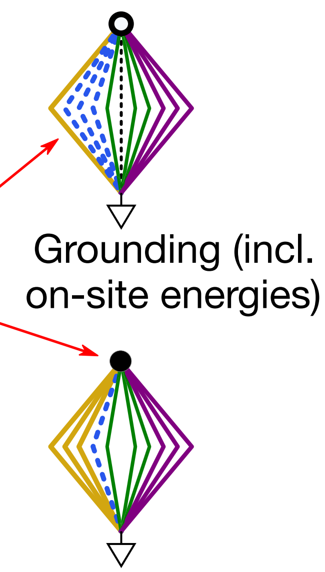
HMP, Phys. Rev. B 101, 205141 (2020)
Wang, HMP, Zhang, Chong, Nat. Comm. 11, 2356 (2020)

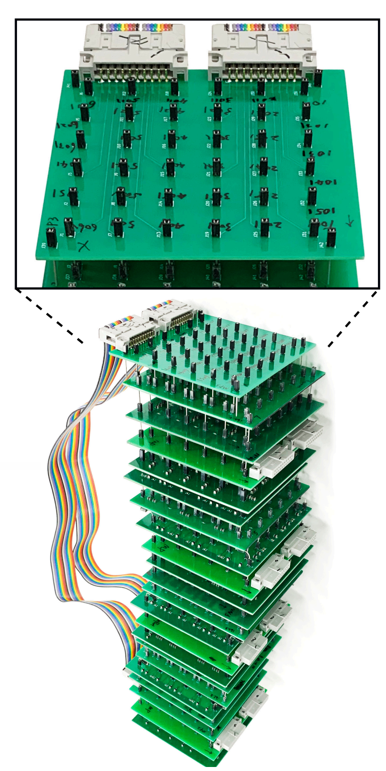
4D Circuit Design

$$D_{ij}(f_0) = i\alpha H_{ij}(f_0)$$

Positive (negative) values of the Hamiltonian correspond to capacitances (inductances)







— Positive NN hoppings

$$C_0 = 1\text{nF} \quad \leftrightarrow \quad J \;= 1$$

— Positive long-range hoppings

$$C' = 2C_0 \quad \leftrightarrow \quad J' \;= 2$$

··· Negative NN hoppings

$$L_0 = 2\text{mH} \quad \leftrightarrow -J \;= -1$$

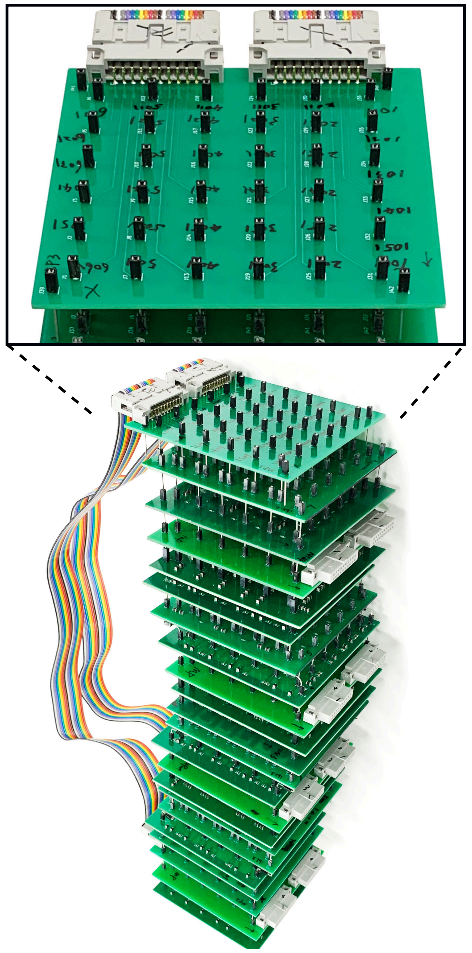
— Negative long-range hoppings

$$L' = L_0/2 \quad \leftrightarrow \quad J'' \;= -2$$

Wang, HMP, Zhang, Chong, Nat. Comm. 11, 2356 (2020)

$$2\pi f_0 = 1/\sqrt{L_0 C_0}, \qquad \alpha = 2\pi f_0 C_0$$

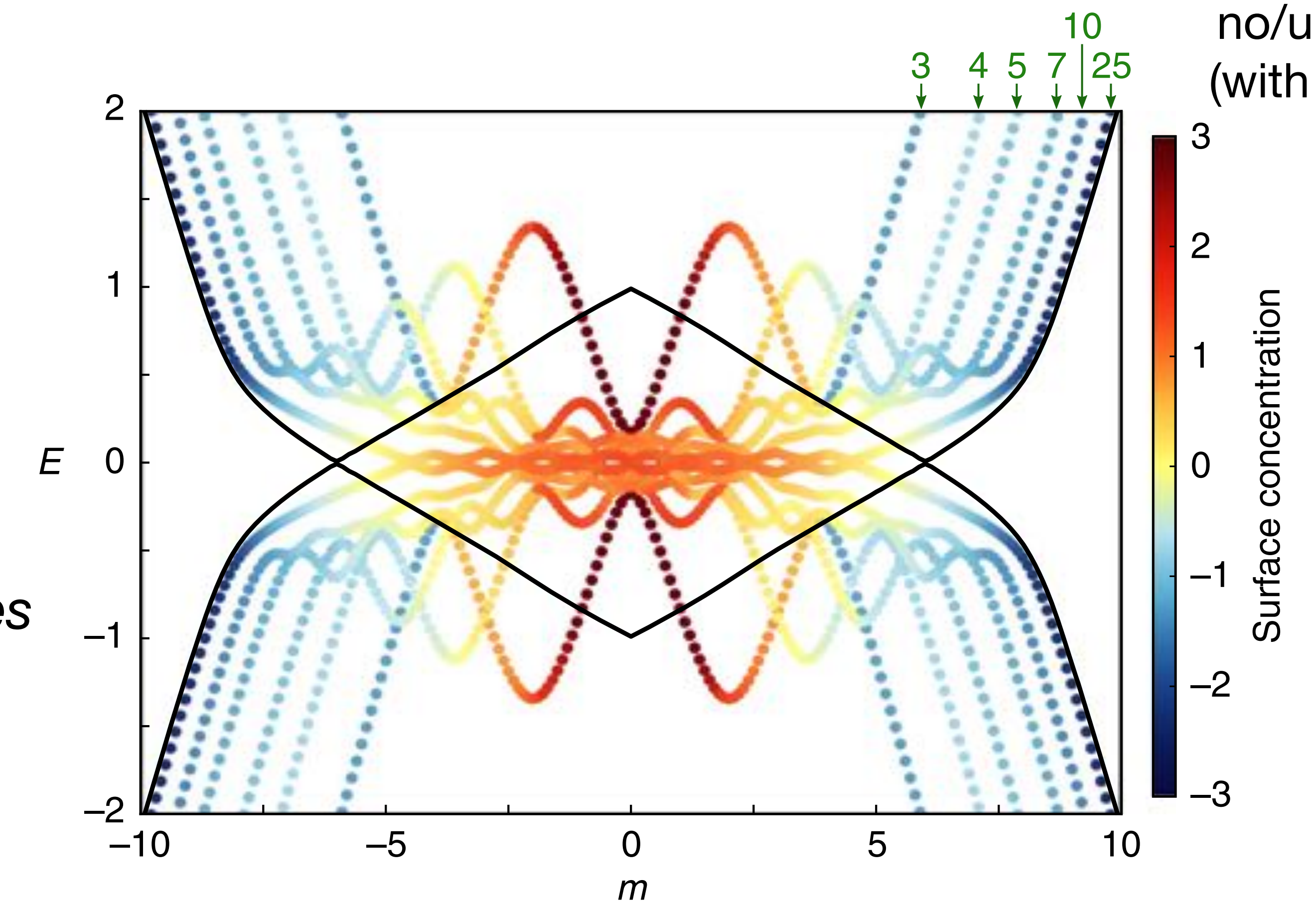
4D Circuit Experiment



144 sites
(6x2x6x2)

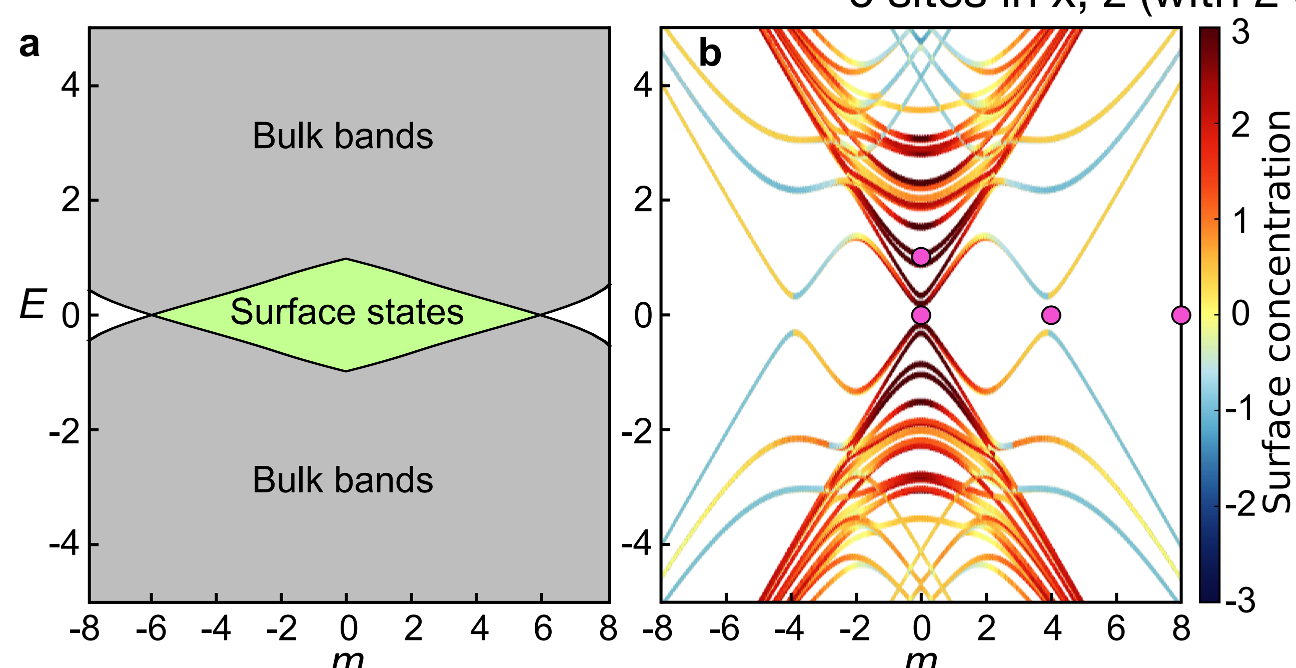
and with periodic bc
along “y” and “w”

middle two eigenenergies



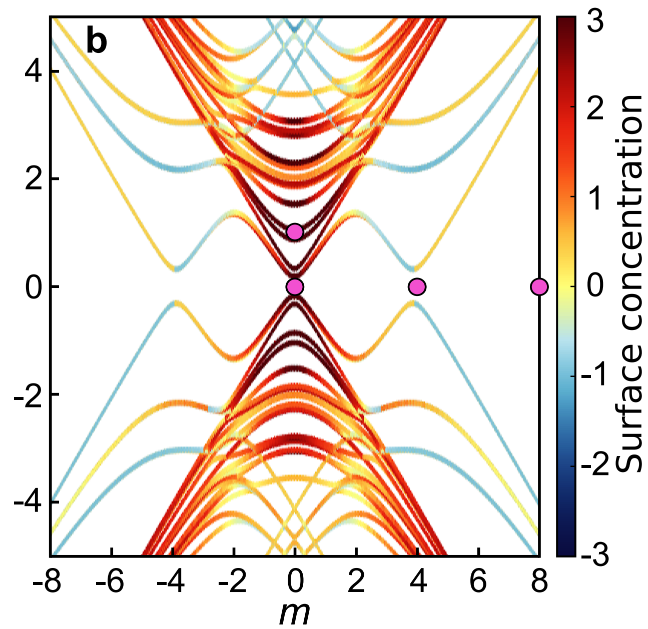
all eigenenergies

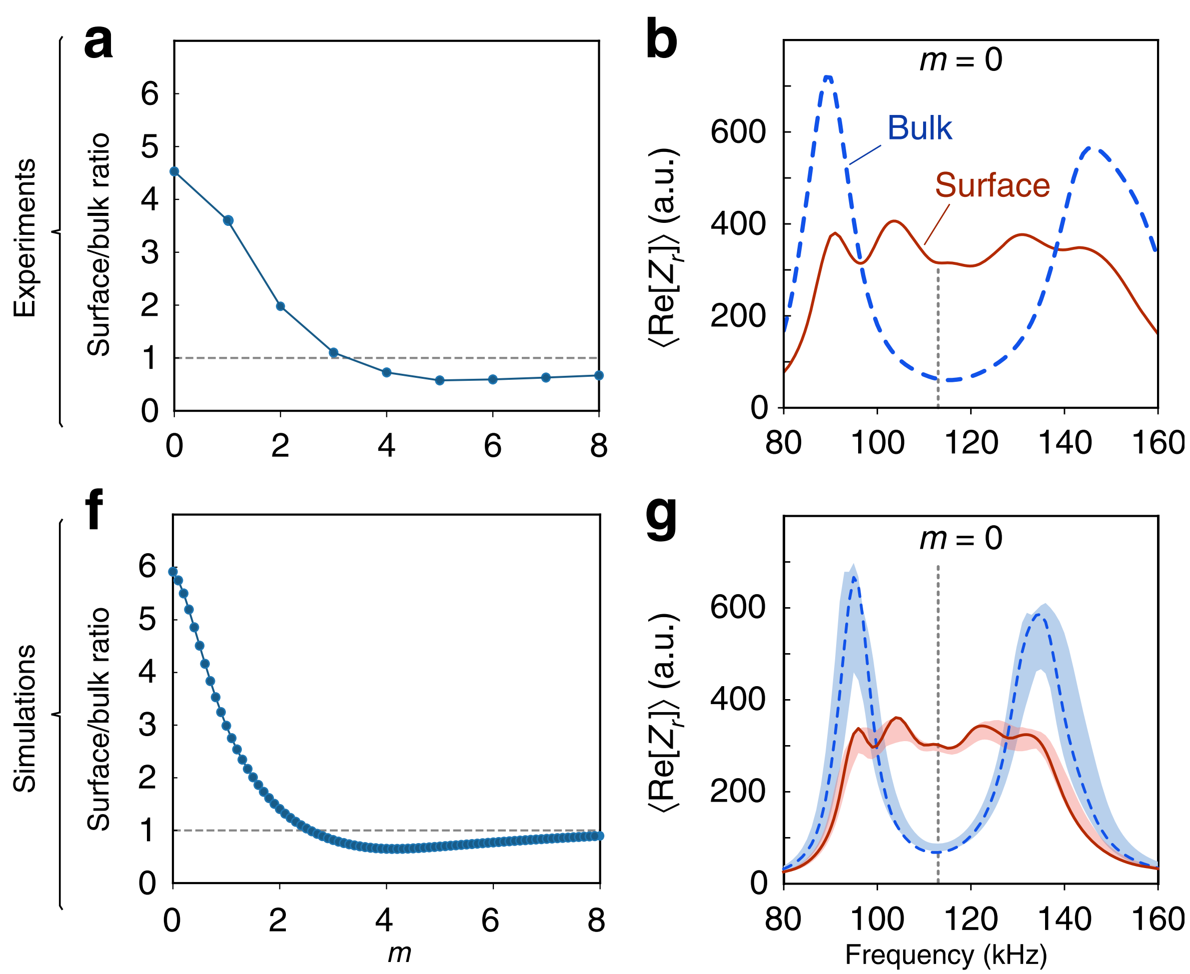
6 sites in x, z (with 2 sites in y, w)



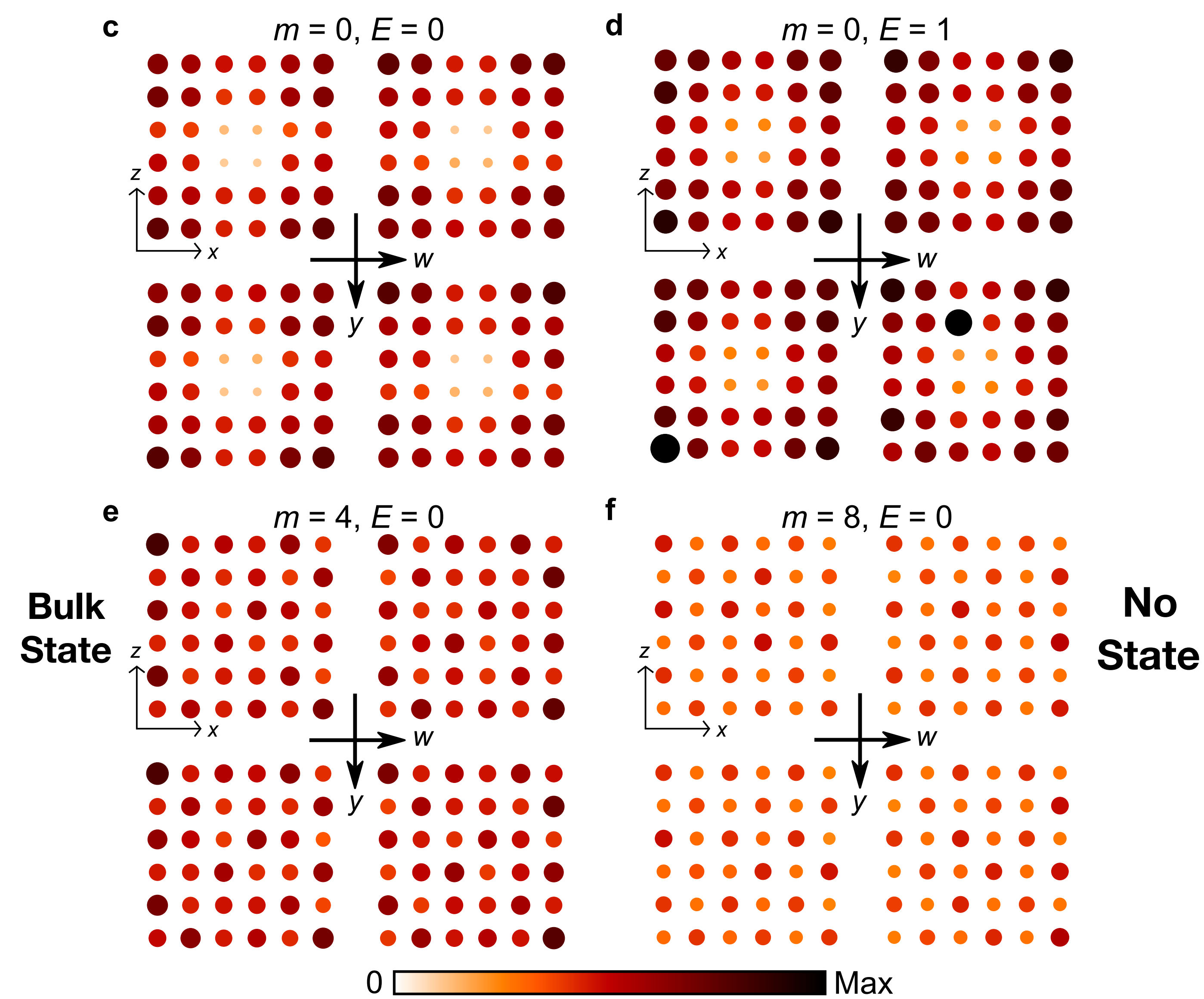
Wang, HMP, Zhang, Chong, Nat. Comm. 11, 2356 (2020)

Observing the 3D Surface States



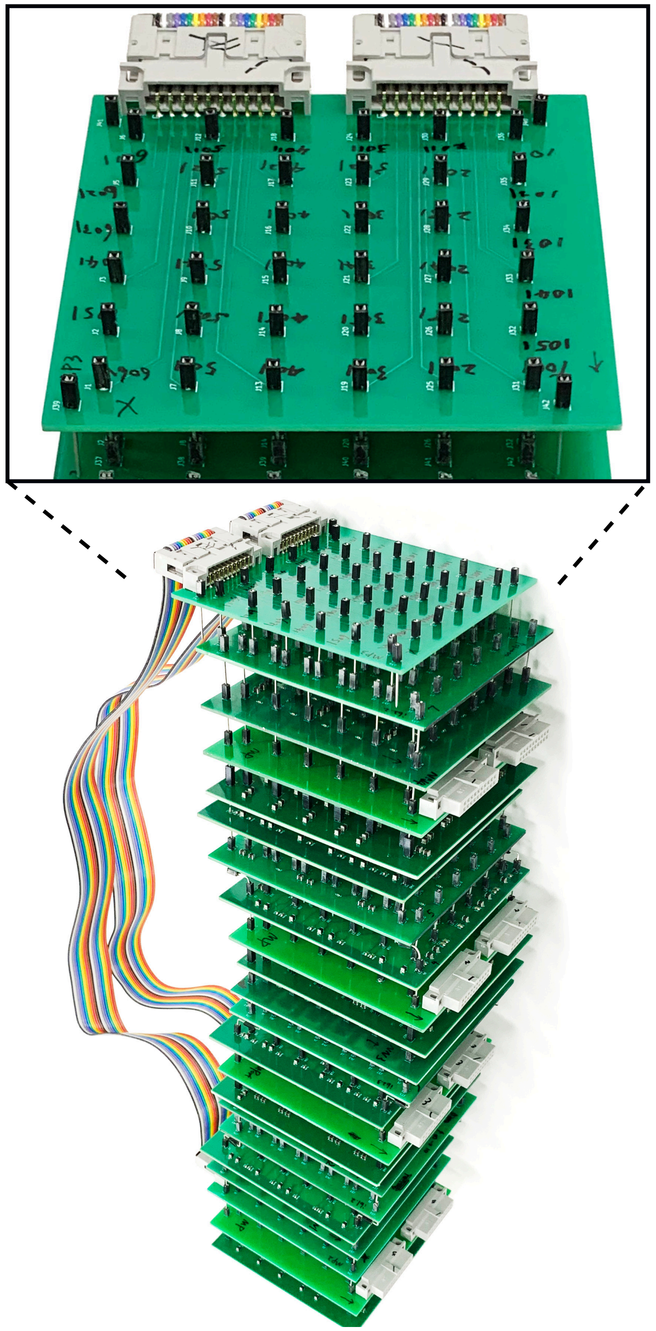


3D Surface states



Wang, HMP, Zhang, Chong, Nat. Comm. 11, 2356 (2020)

Conclusions (Part I)



- Topoelectric circuits!
- Simulation of 4D topological models in a circuit
- Observed 3D surface states due to 2nd Chern number

- Synthetic dimensions to see 4D QH response?
- Other higher-dimensional topological effects?

M. Ezawa, Phys. Rev. B 100, 075423 (2019)
R. Yu, Y. X. Zhao, and A. P. Schnyder, Nat. Sci. Rev. (2020),
HMP, Phys. Rev. B 101, 205141 (2020)
Y. Wang, HMP, B. Zhang, and Y. D. Chong, Nat. Comm. 11, 2356 (2020)
Zhang et al Phys. Rev. B 102, 100102 (2020)…

Overview

- Introduction to 4D Quantum Hall physics
- Using electrical circuits to realise a 4D QH model
- **Superfluid vortices in four spatial dimensions**

Ben McCanna

Motivation: 4DQH with magnetic fields

e.g. $B_{xz}, B_{yw} \neq 0$ 4D Landau levels

Then what happens to mean-field interacting bosons?

equivalent to:

Gross-Pitaevskii equation in doubly rotating frame

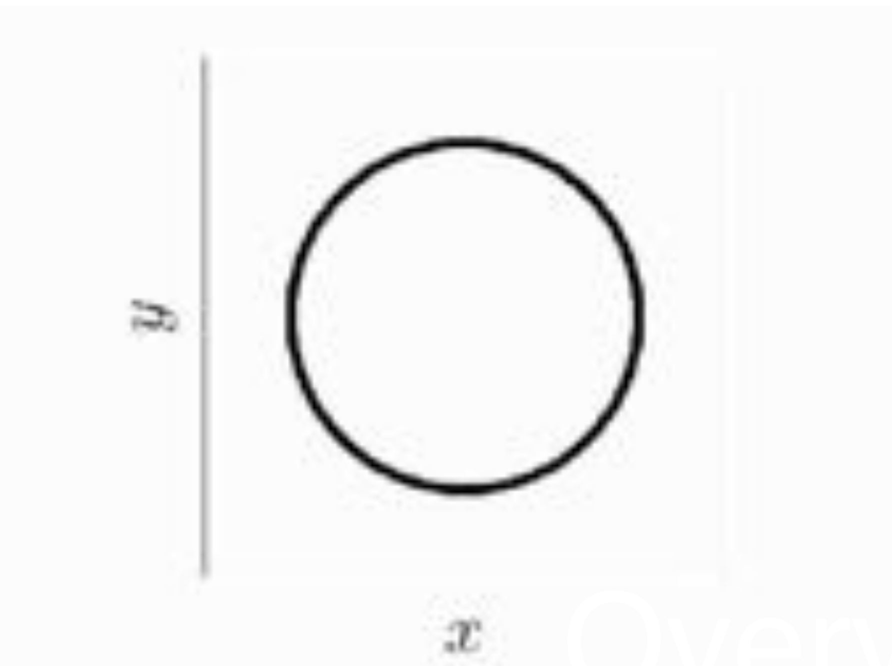
$$\left[-\frac{\hbar^2}{2m}\nabla^2 + g|\psi|^2 - \omega_{xy}L_{xy} - \omega_{zw}L_{zw}\right]\psi = \mu\psi,$$

rotation frequency in xy plane

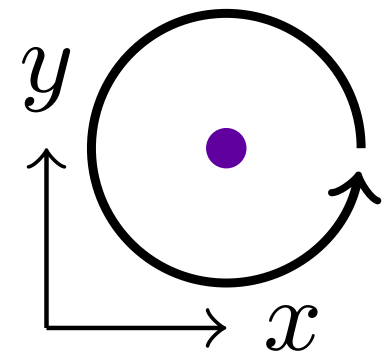
angular momentum in xy plane

Reminder: Vortices in 2D and 3D

Classical trajectories



Quantum vortex



$$\psi \to f_k(r)e^{ik\theta}$$

winding number

$f_k(0) = 0$

and profile from solving GPE

$$\psi = \sqrt{\rho}e^{iS}$$

$$\oint_C \mathbf{v} \cdot d\mathbf{r} = \frac{\hbar}{m}[\Delta S]_C$$

$$\mathbf{v} = \frac{\hbar}{m}\nabla S$$

$$[\Delta S]_C = 2\pi k$$

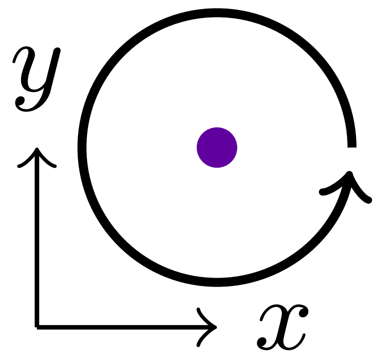
can be energetically stabilised by rotation/ magnetic field (e.g. in x-y plane)

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + g|\psi|^2 - \omega_{xy}L_{xy}\right]\psi = \mu\psi,$$

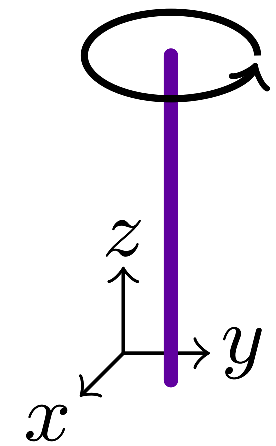
Pitaevskii & Stringari, Bose-Einstein Condensation (OUP Press)

Single 4D Vortex Plane

in **2D**, vortex core:
0D point



in **3D**, vortex core:
1D line



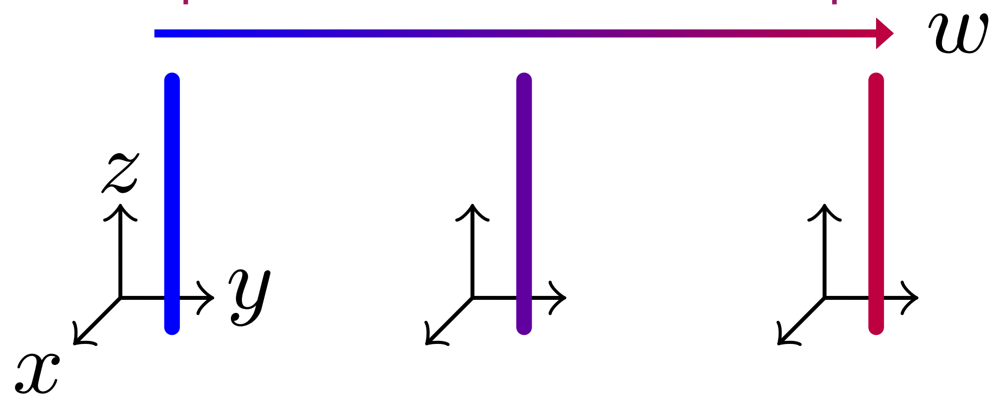
in **4D**, vortex core:
2D plane?

For $\omega_{xy} \neq 0, \omega_{zw} = 0$

NB this is a "simple rotation":

$$\begin{pmatrix} R(\alpha) & 0 \\ 0 & I \end{pmatrix}, \text{ where } R(\alpha) = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$$

Expect that core is entire z-w plane

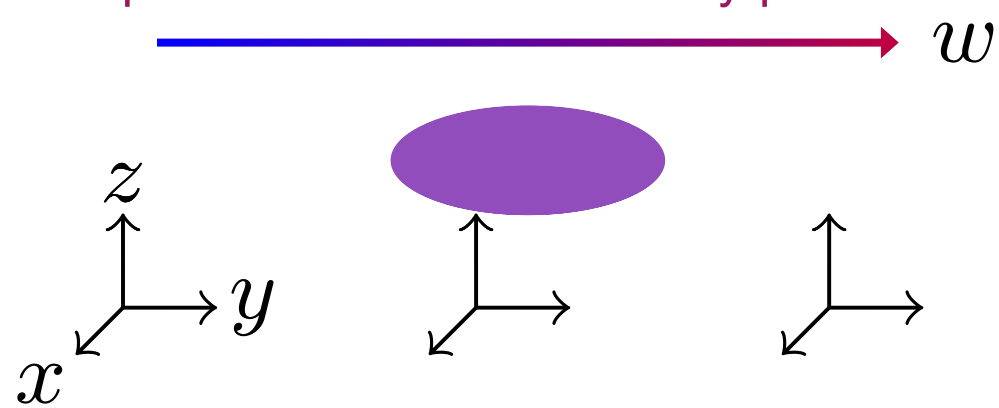


$$\psi \to f_{k_1}(r_1) e^{ik_1\theta_1}$$

And if instead had:

$$\omega_{xy} = 0, \omega_{zw} \neq 0$$

Expect that core is entire x-y plane



$$\psi \to f_{k_2}(r_2) e^{ik_2\theta_2}$$

$$(x, y, z, w) = (r_1 \cos\theta_1, r_1 \sin\theta_1, r_2 \cos\theta_2, r_2 \sin\theta_2),$$

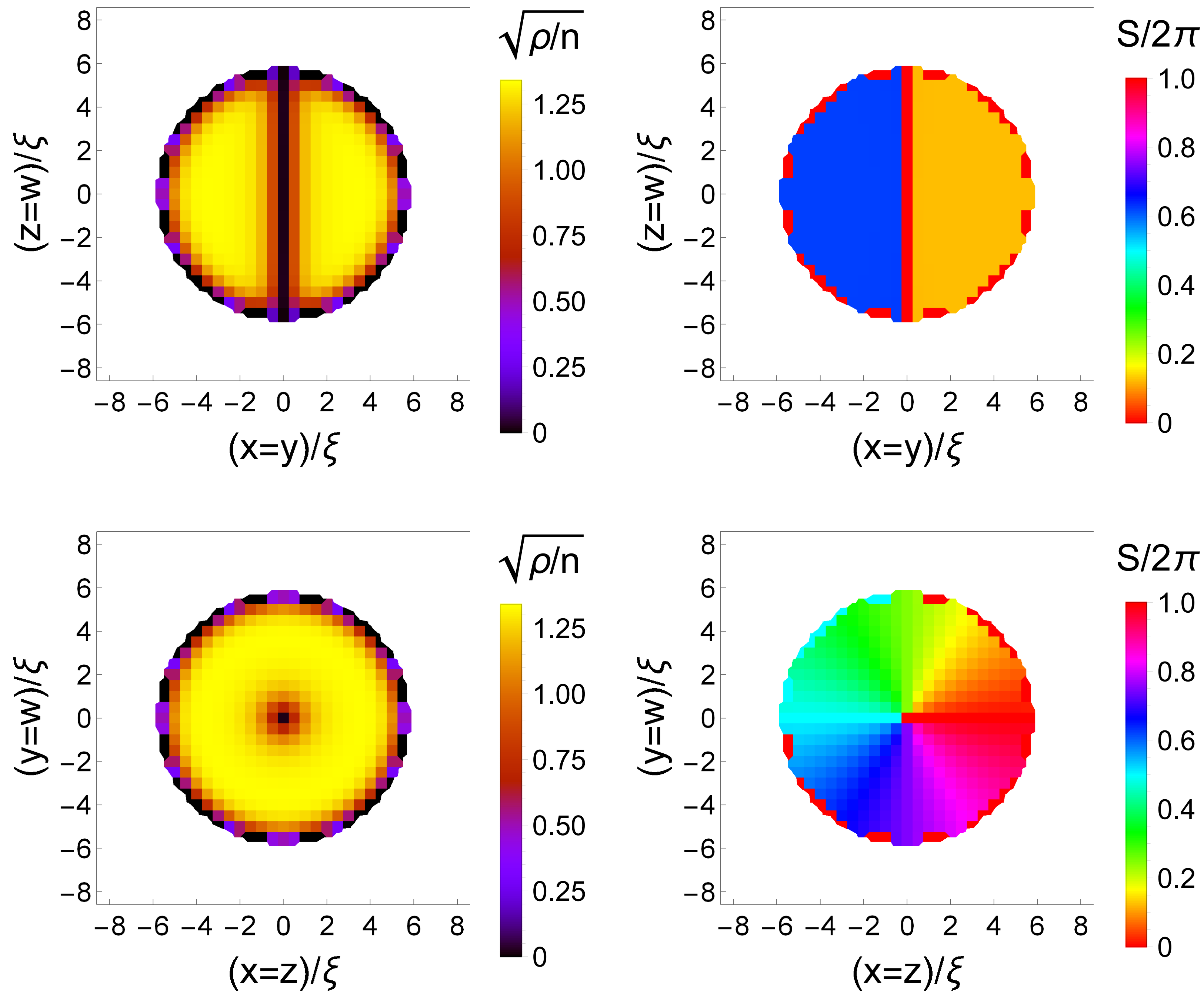
McCanna and HMP, Phys. Rev. Research 3, 023105 (2021)

Single 4D Vortex Plane

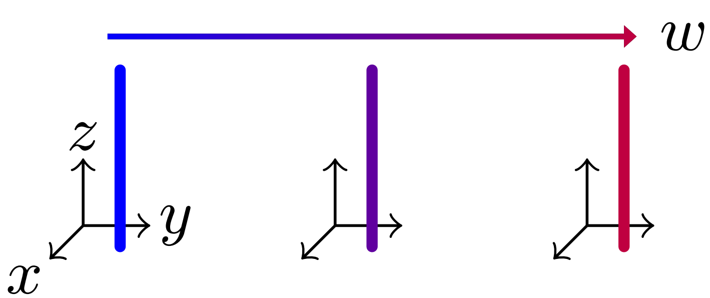
$\omega_{xy} \neq 0, \omega_{zw} = 0$

Solve the 4D GPE with imaginary time-evolution:

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + g|\psi|^2 - \omega_{xy} L_{xy}\right]\psi = \mu\psi,$$



remember expect $\psi \to f(r_1)e^{ik_1\theta_1}$



$\omega_{xy} = 2\omega_{\text{crit}}^{2D}$

McCanna and HMP, Phys. Rev. Research 3, 023105 (2021)

Intersecting 4D Vortex Planes?

What about?

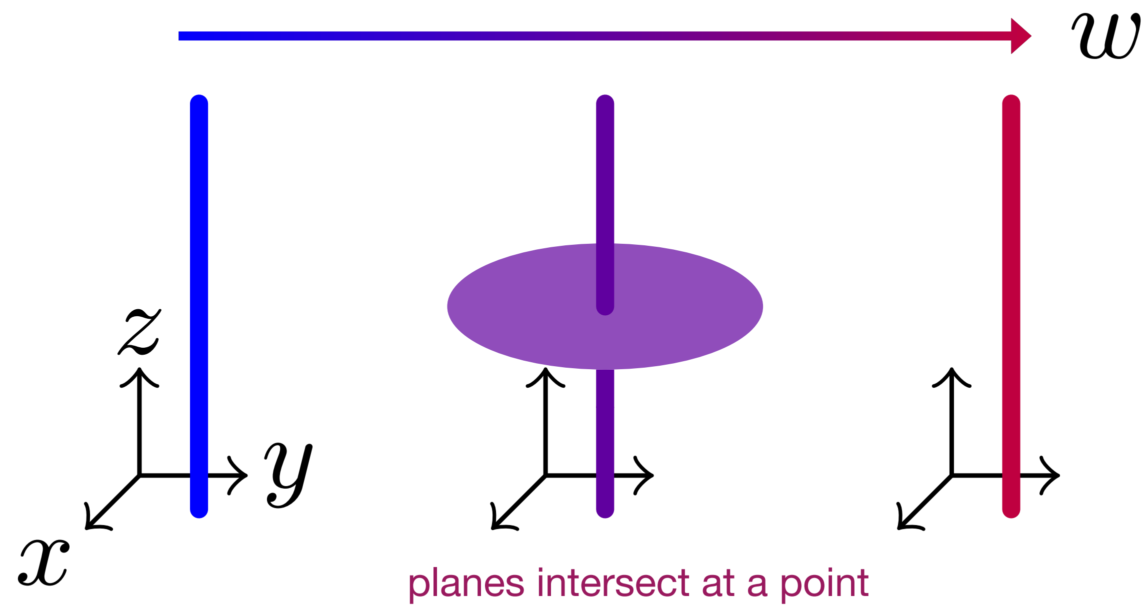
$$\omega_{xy} = \omega_{zw} \neq 0$$

(i.e. like 4D Landau levels)

NB this is a "double rotation":

$$\begin{pmatrix} R(\alpha) & 0 \\ 0 & R(\alpha) \end{pmatrix}, \text{ where } R(\alpha) = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$$

Vortex core could be e.g. entire z-w plane plus the entire x-y plane?



planes intersect at a point

McCanna and HMP, Phys. Rev. Research 3, 023105 (2021)

Intersecting 4D Vortex Planes

$\omega_{xy} = \omega_{zw} \neq 0$

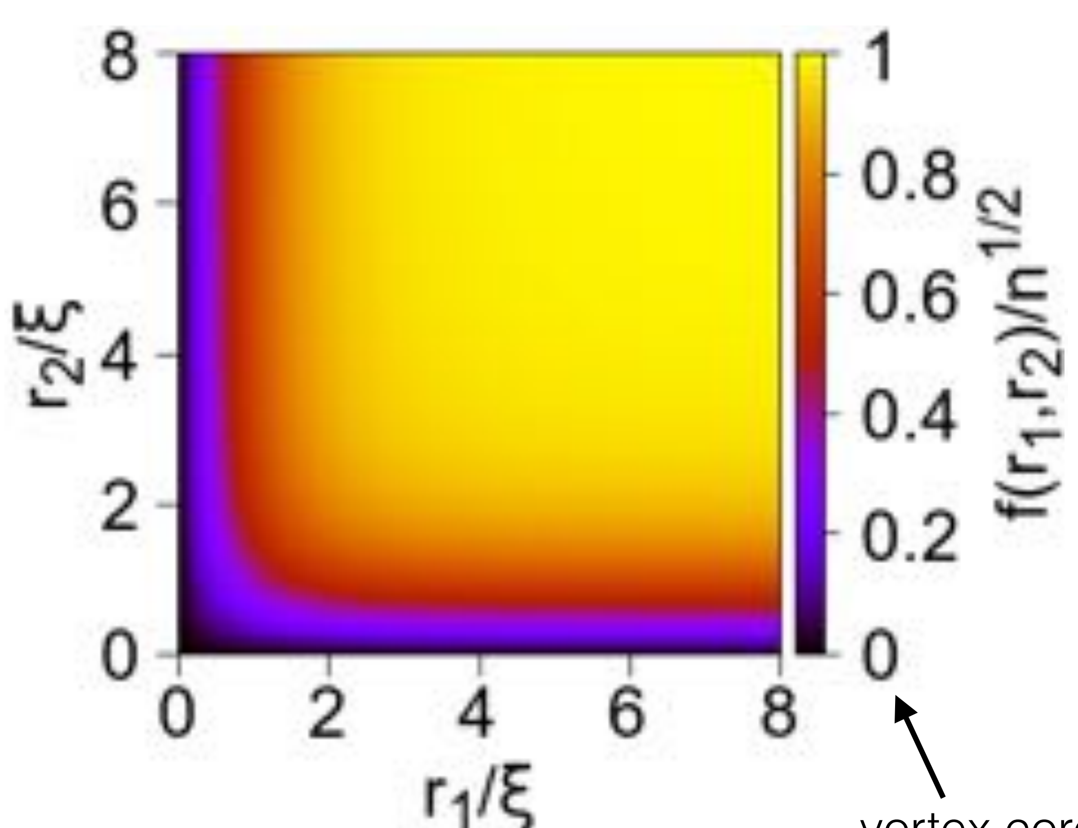
So is this simply a product of vortex planes? $\psi \approx f_1(r_1)e^{ik_1\theta_1} f_1(r_2)e^{ik_2\theta_2}$?

vortex in plane 1 (xy plane)

vortex in plane 2 (zw plane)

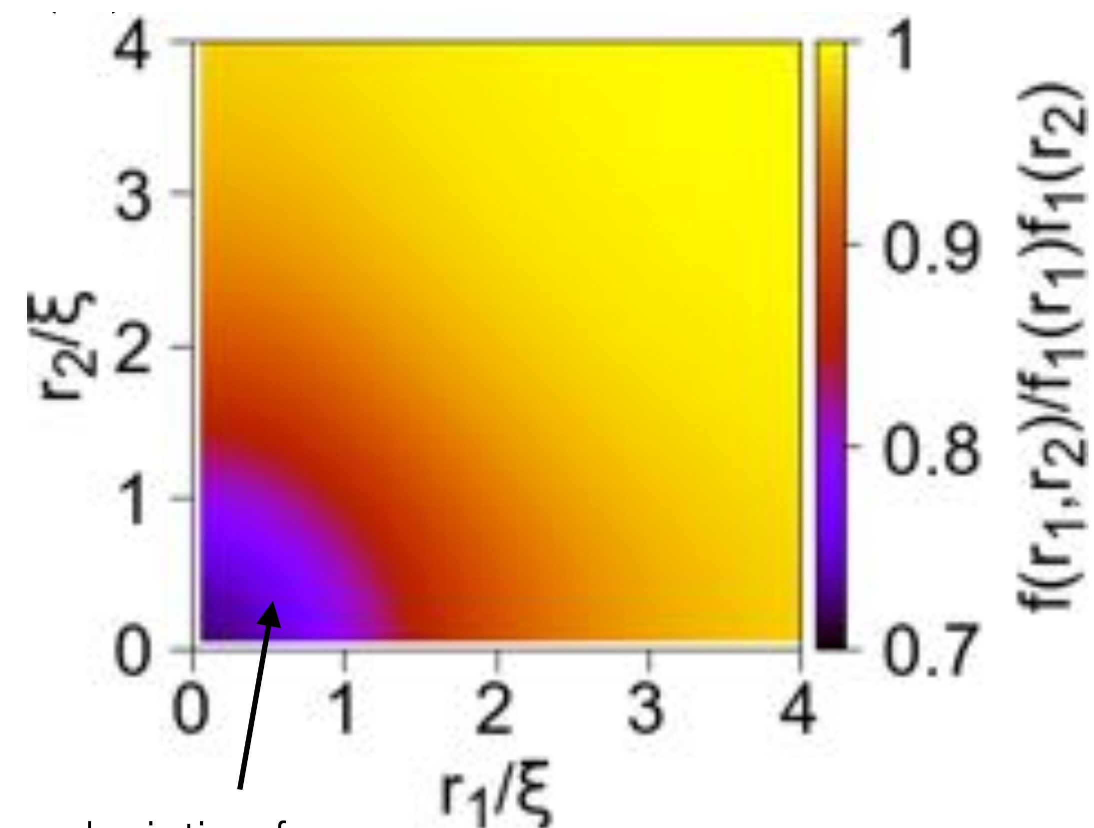
Solve 4D GPE with ansatz $\psi = f(r_1, r_2)e^{ik_1\theta_1 + ik_2\theta_2}$

Radial function



vortex core

Radial function compared to product



deviation from product of planes due to interactions

$(x, y, z, w) = (r_1 \cos\theta_1, r_1 \sin\theta_1, r_2 \cos\theta_2, r_2 \sin\theta_2),$

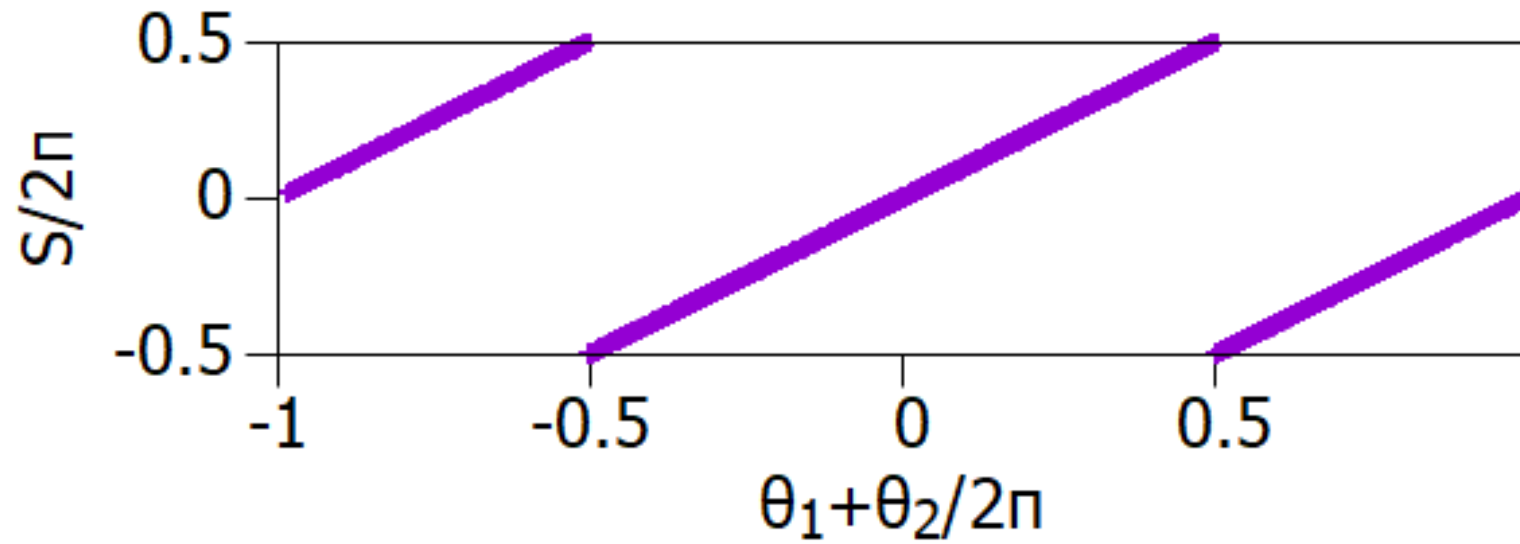
McCanna and HMP, Phys. Rev. Research 3, 023105 (2021)

Intersecting 4D Vortex Planes

$\omega_{xy} = \omega_{zw} \neq 0$

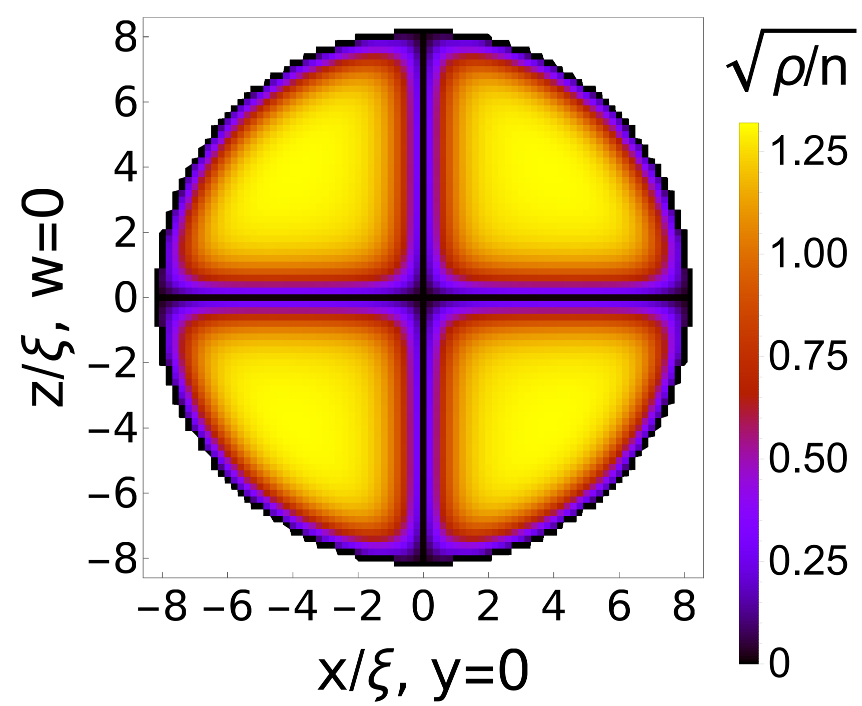
Full solutions of the 4D GPE with imaginary time-evolution:

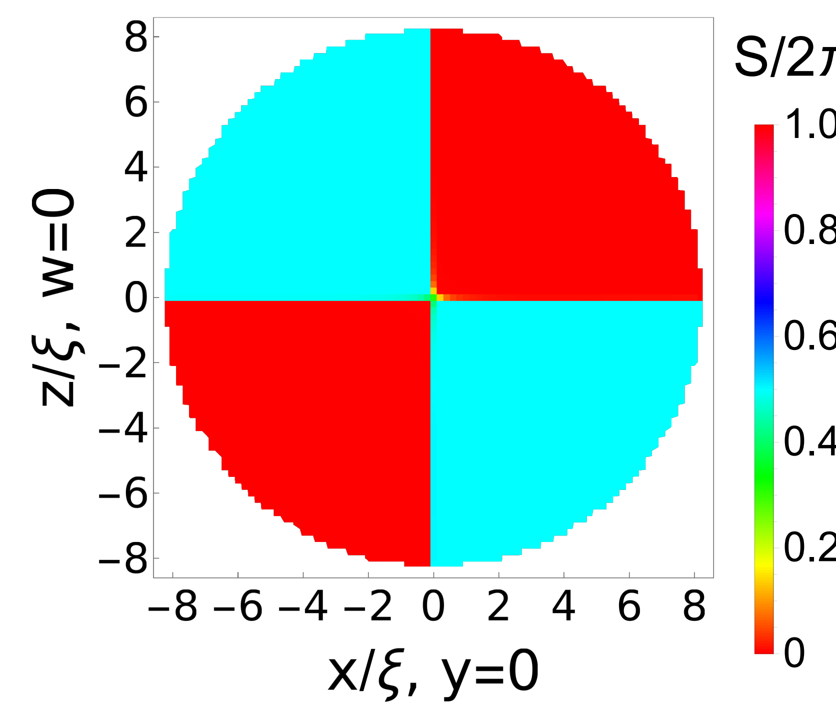
$$\left[-\frac{\hbar^2}{2m}\nabla^2 + g|\psi|^2 - \omega_{xy}L_{xy} - \omega_{zw}L_{zw}\right]\psi = \mu\psi,$$



we expect

$$\psi = f(r_1, r_2)e^{ik(\theta_1+\theta_2)}$$





$\omega_{xy} = \omega_{zw} \approx 2.5\omega_{\text{crit}}^{2D}$

Good agreement with expectations!

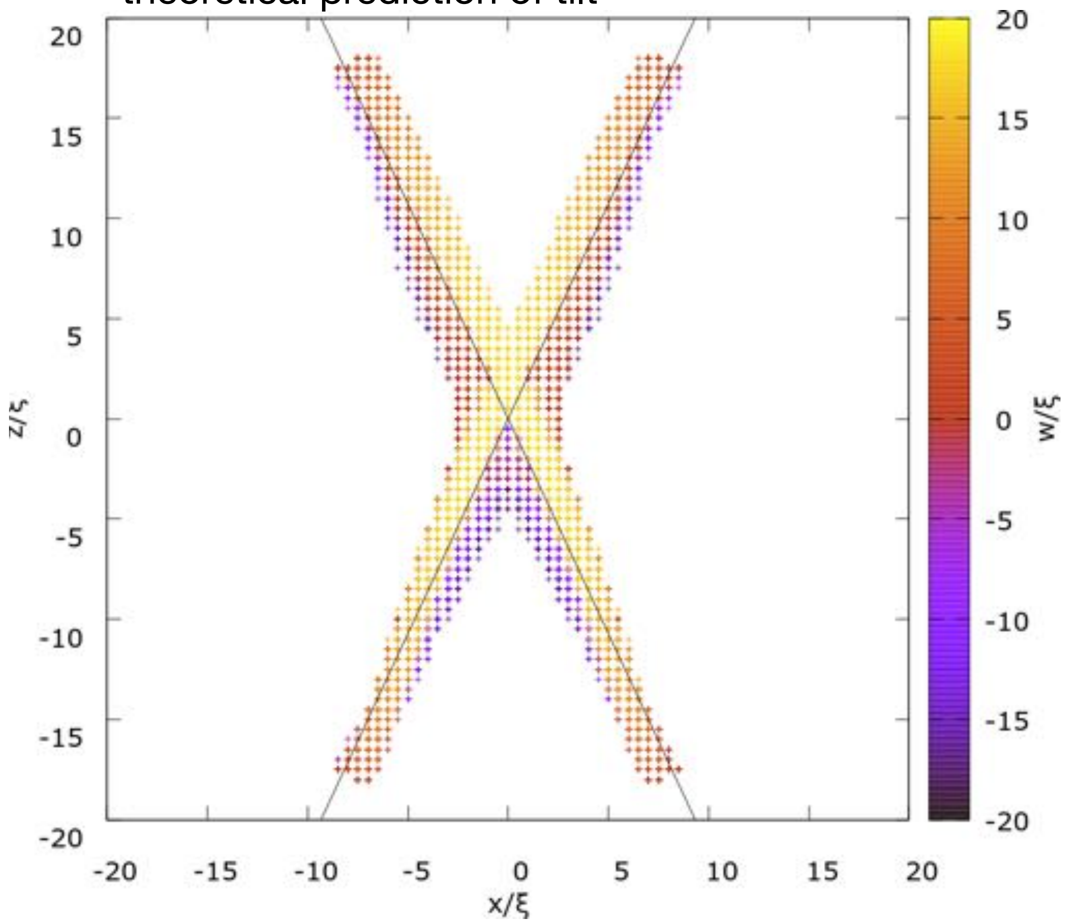
McCanna and HMP, Phys. Rev. Research 3, 023105 (2021)

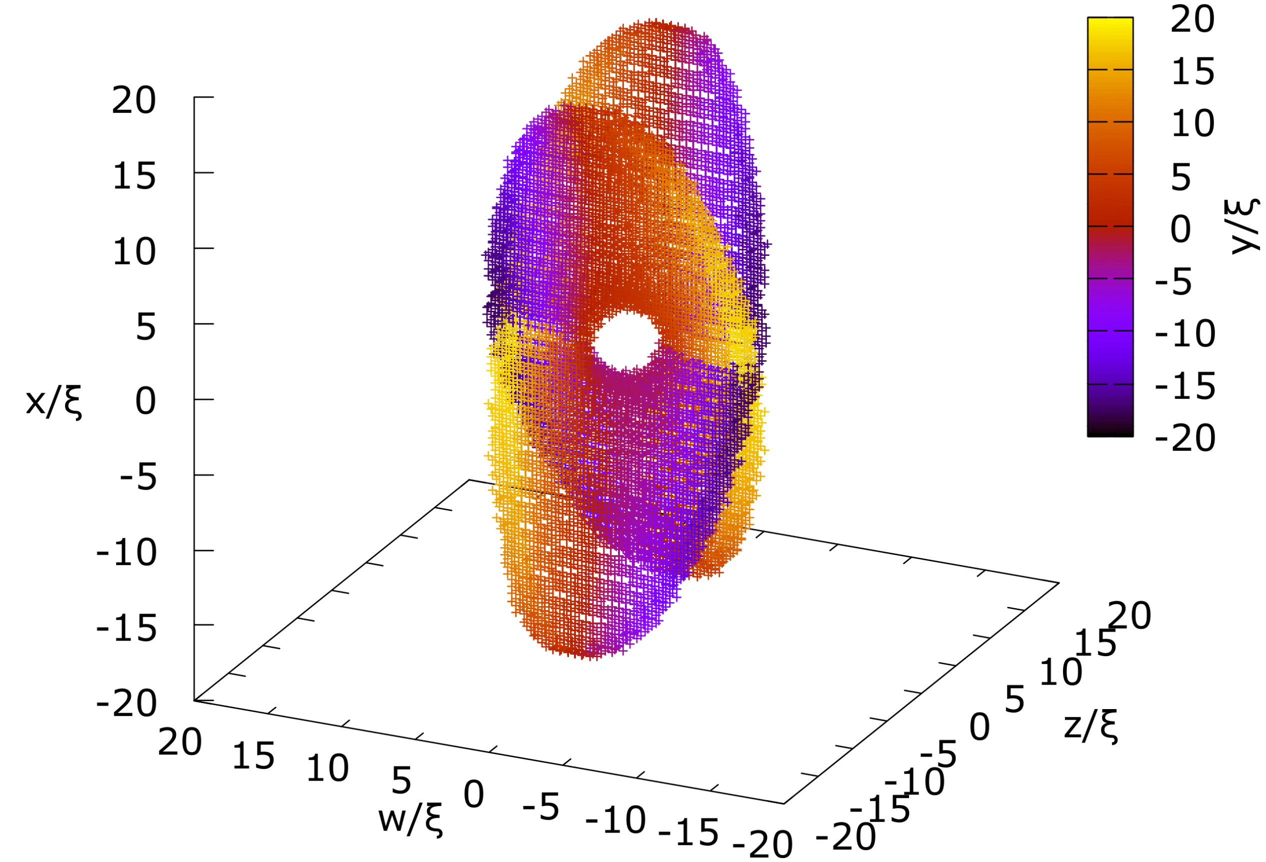
Unequal frequencies….

$\omega_{xy} > \omega_{zw}$

vortex planes begin to tilt towards zw plane, and reconnect at the intersection point!

theoretical prediction of tilt

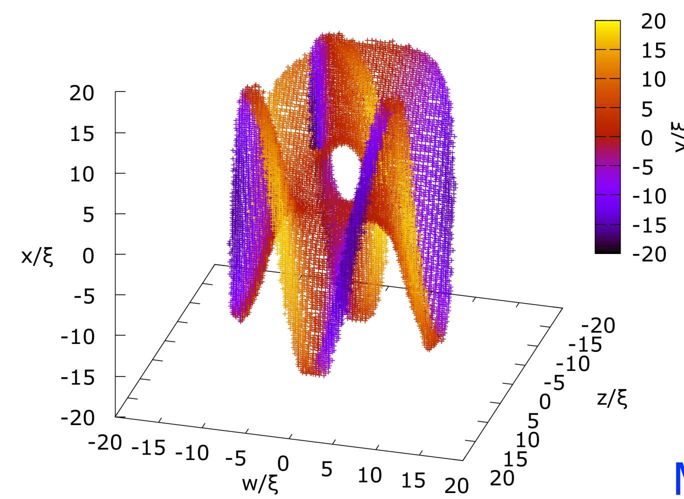




$\omega_{zw} = 2.5\omega^{2D}_{\text{crit}}, \quad \omega_{xy} \simeq 3.7\omega^{2D}_{\text{crit}}$

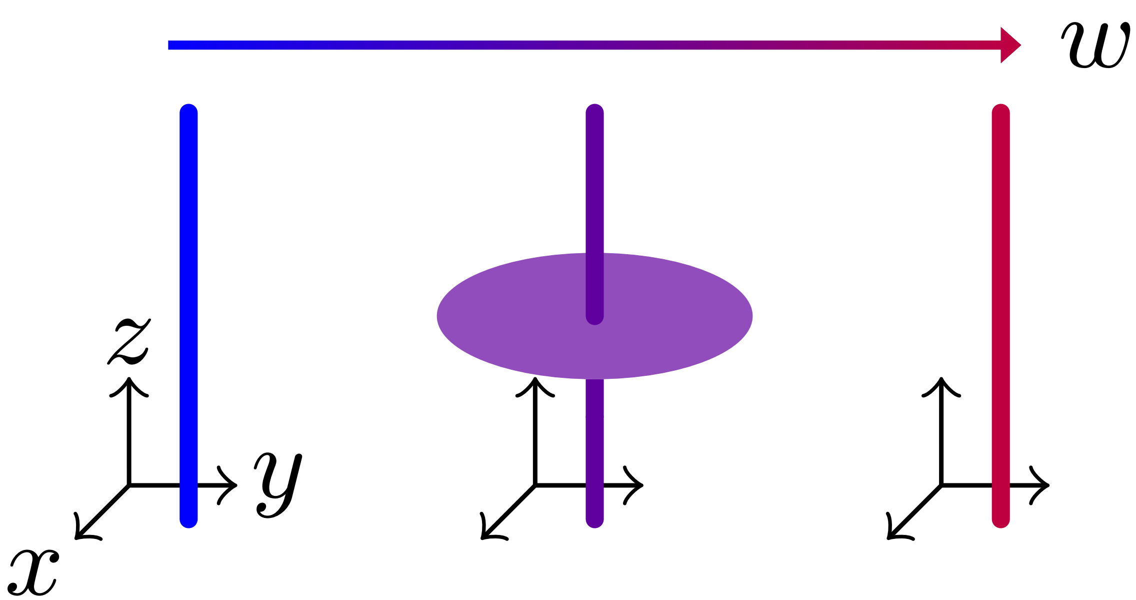
vortex cores from solving full 4D GPE with double rotation

but also other weirder states can happen…

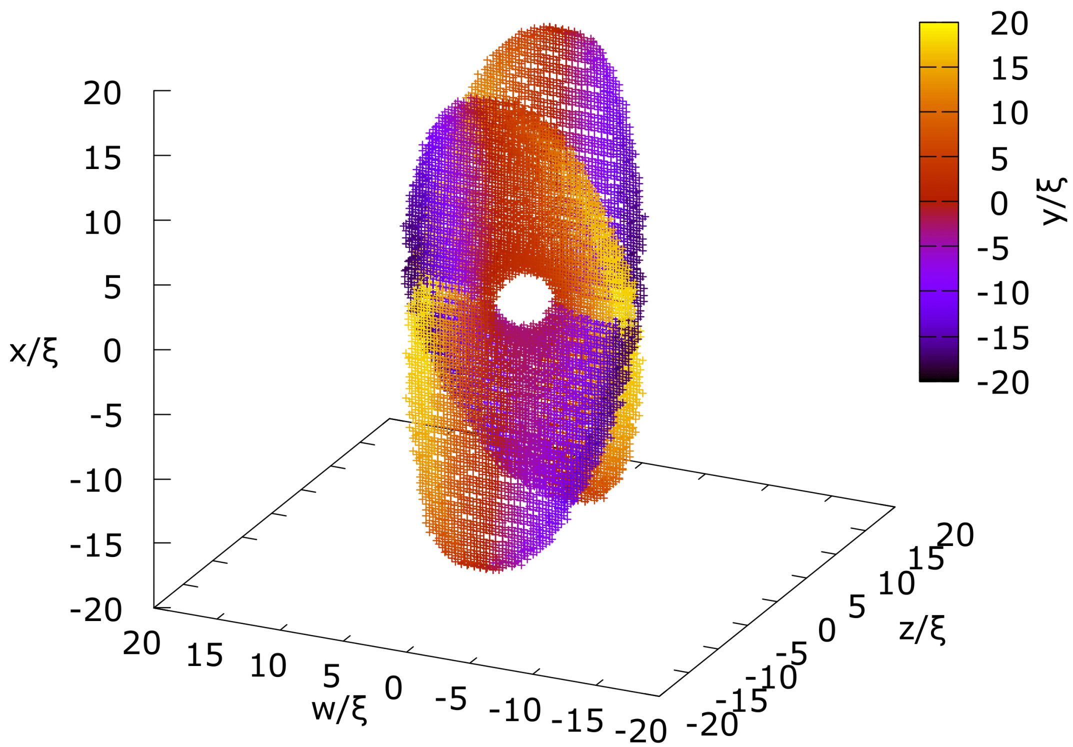


McCanna and HMP, unpublished

Conclusions (Part II)



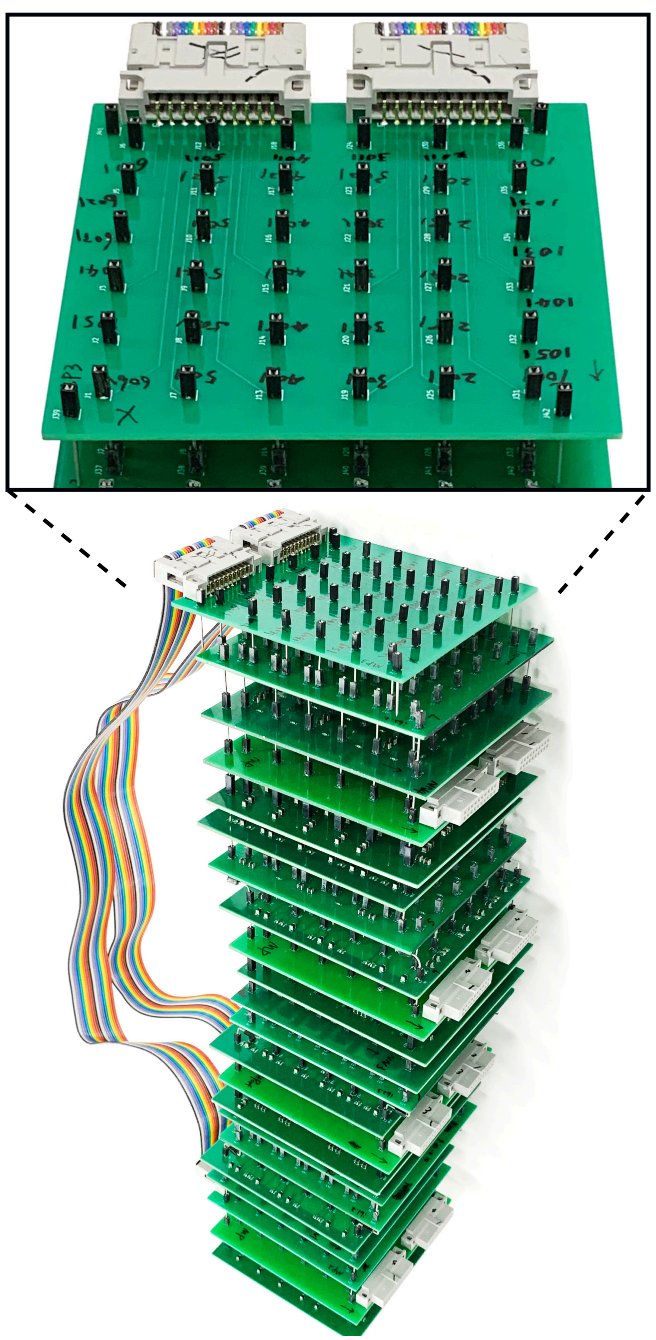
- 4D vortices!
- Equal frequency double rotation: orthogonal vortex planes intersecting at a point
- Unequal frequency double rotation: tilting planes, reconnections and more…



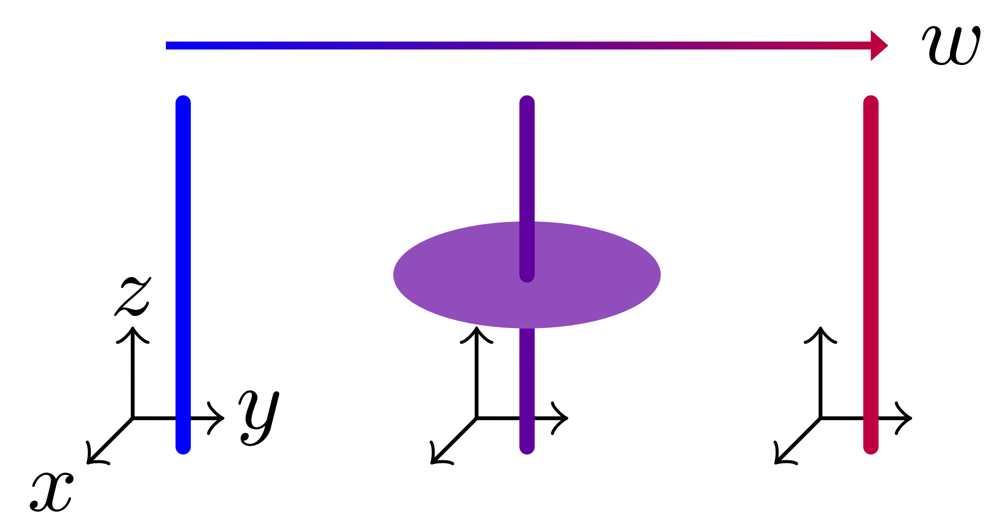
- Make connections to experiments on synthetic dimensions?
- Vortex lattices?
- Other types of topological excitations in higher dimensions?

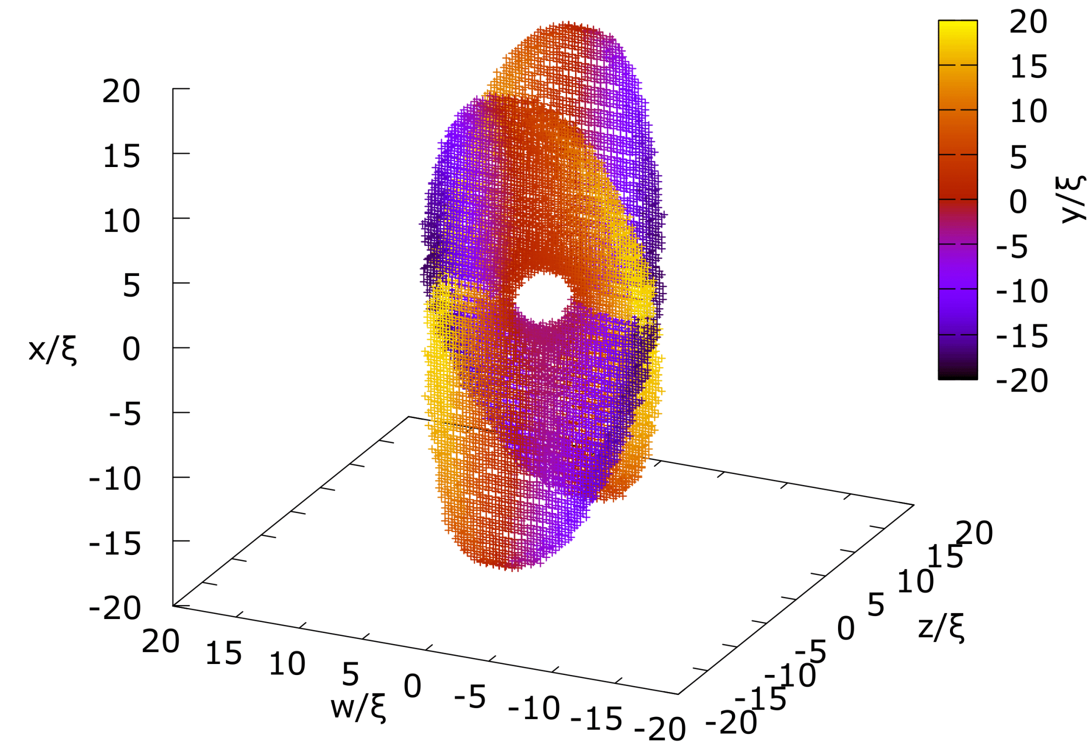
Summary

4D QH in a topoelectric circuit



4D superfluid vortices





And thanks again to:

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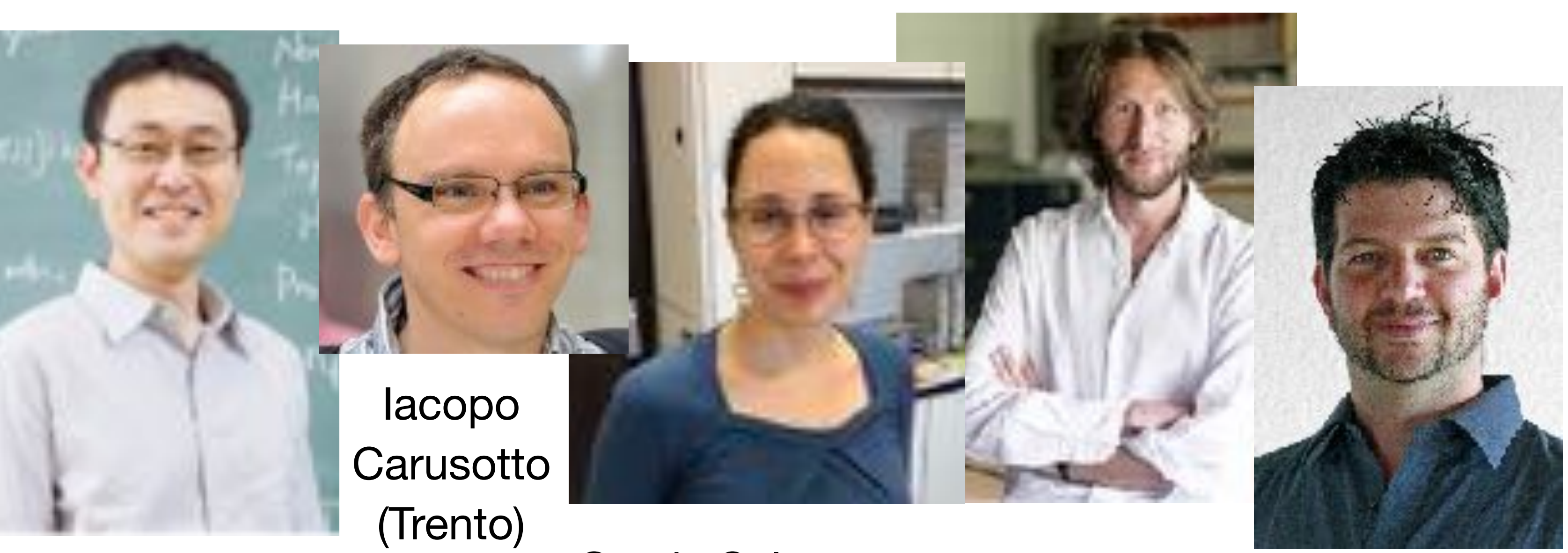

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Nathan Goldman (Brussels)

Oded Zilberberg (Zurich)

4D Quantum Hall Systems

Gapped phases of quadratic fermionic Hamiltonians without extra symmetries

Symmetry	Time-reversal	Particle-hole	Chiral	d=1	2	3	4	5	6	7	8
A	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AIII	0	0	1	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
AI	1	0	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
BDI	1	1	1	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
D	0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$
DIII	−1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0
AII	−1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$
CII	−1	−1	1	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
C	0	−1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
CI	1	−1	1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0

Dimensions d

← 2D Quantum Hall (row A, d=2)

← SSH Model (row BDI, d=1)

← Topological Superconductors (rows D, DIII)

← Topological Insulators/ quantum spin Hall (row AII)

4D Quantum Hall (column d=4)

Kitaev, arXiv:0901.2686
Ryu et al., New J. Phys. 12, 065010 (2010)
Chiu, et al., RMP 88, 035005, (2016)