Gauge theories of gapless fractons

Boulder

Center for Theory of Quantum Matter

CTQM

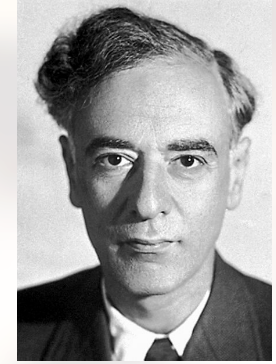
M. Pretko and L.R., PRLs 2018
Z. Zhai and L.R., PRB 2019
L.R. and M. Hermele, PRL 2020
L.R., PRL 2020
Z. Zhai and L.R., AOP 2021

$: Simons Investigator, NSF-MRSEC

SMRC

Interacting Topological Matter, KITP, 15 June, 2021

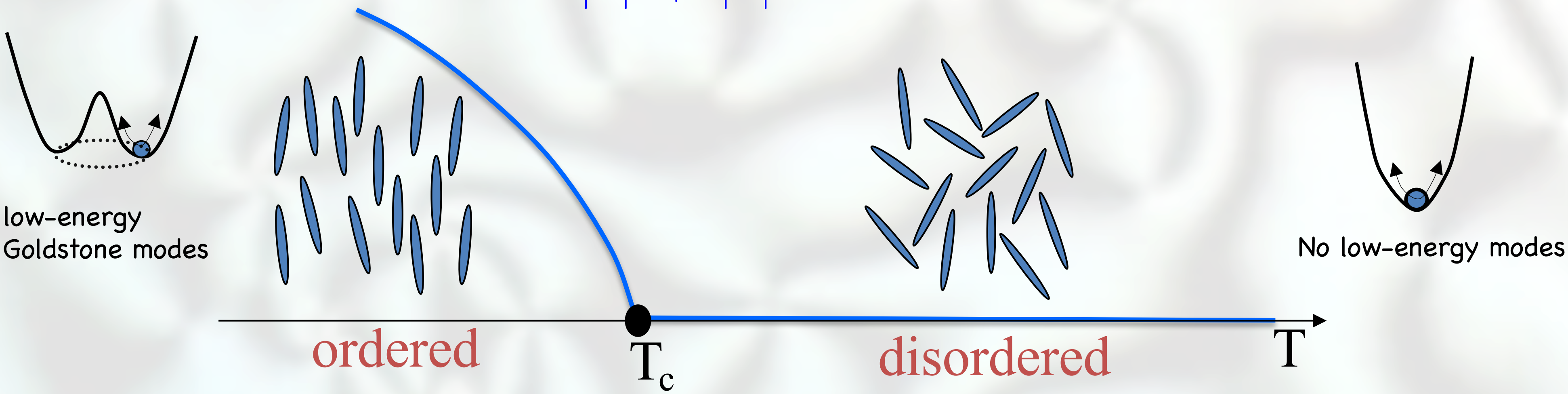
States of (bosonic) matter: Landau paradigm



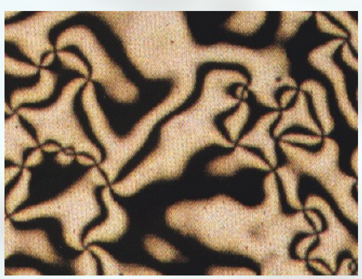
Lev Landau

- "conventional" *ordered* states, *e.g., AFM, SF, liquid crystals…*
  - *local* order parameter, *S(r)*
  - classified by patterns of *spontaneously broken symmetry*
  - *short*-range entangled

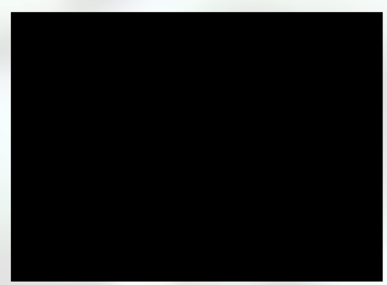
$$H = a|\vec{S}|^2 + b|\vec{S}|^4$$



(ferromagnet, nematic)



(paramagnet, isotropic)

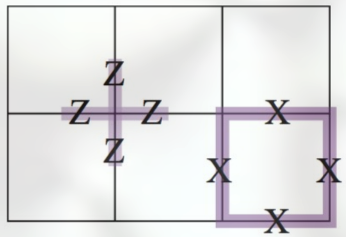


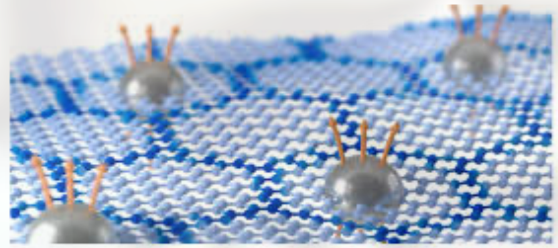
Anderson
Laughlin
Wen
Kitaev
Sachdev
Fisher
…

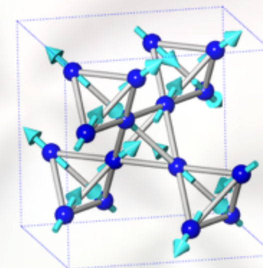
States of quantum matter

(beyond symmetry breaking)

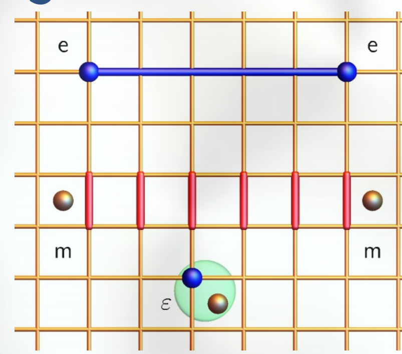
"conventional" quantum *'liquid'* states, e.g., FQHE, spin ice, toric code, …







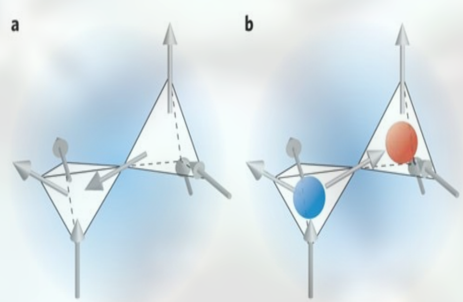
– Non-local, fractionalized bulk excitations as ends of strings: anyons – *free to move but with statistical "interaction"*



– Topological order with O(1) gs degeneracy

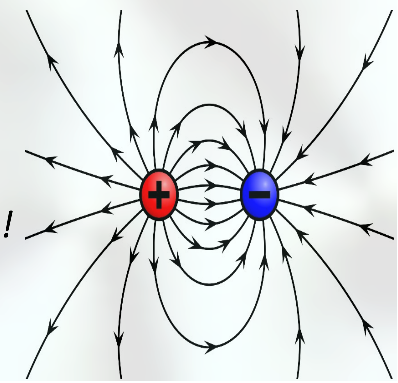
– Long-range entangled

Geometric frustration (e.g. "spin-ice rules")



– Gauge theory (Z_2, U(1), …)

Emergent electromagnetism!



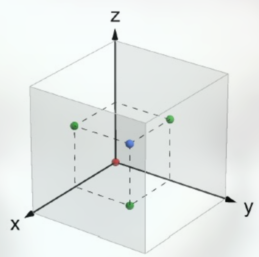
Outline

- New type of quantum 'liquids': "***fractons***"
- Symmetric *tensor* gauge theories
- Elasticity
- Duality
- Fractons from *vector* gauge theories
- Symmetry-enriched fractons from a *supersolid*
- Melting into *super-hexatic, super-smectic*

C. Chamon, 2005
A. Rasmussen, et al., 2016
J. Haah, 2011, '13
S. Bravyi, et al., 2011
B. Yoshida, 2013
S. Vijay, L. Fu, 2015, '16
...

Fracton quantum matter

- *new class of* quantum *'liquids': Z_2 fractons*, e.g., Haah's code, X-cube, lattice rotors,...

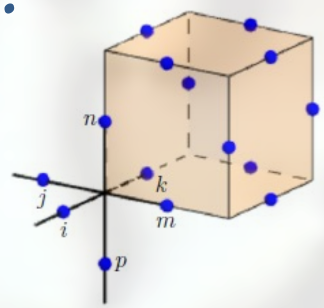


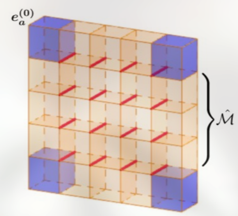
$$A_c = \prod_{n \in \partial c} \sigma_n^x$$

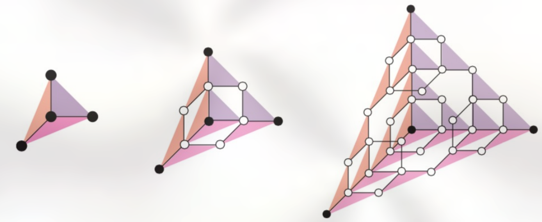
$$B_s^{(xz)} = \sigma_i^z \sigma_p^z \sigma_n^z \sigma_k^z$$

$$B_s^{(xy)} = \sigma_i^z \sigma_j^z \sigma_k^z \sigma_m^z$$

$$B_s^{(yz)} = \sigma_j^z \sigma_n^z \sigma_m^z \sigma_p^z$$



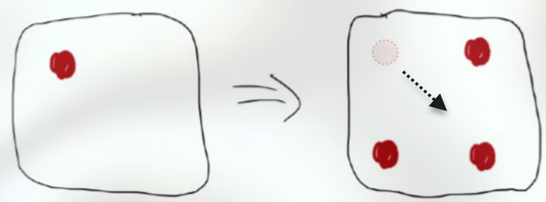




Fractal operator

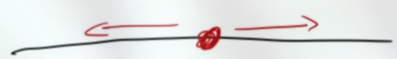
  - Non-local, fractionalized excitations with restricted mobility and exponential topological degeneracy, beyond TQFT description,...
    - -> at corners of extended objects: fractons – *immobile* in isolation

fracton



    - -> at ends of undeformable string: dipoles - *subdimensional*

planons



lineons

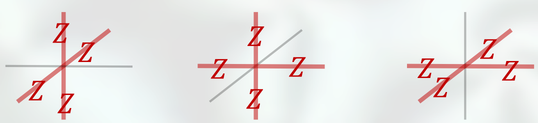
Vijay, Haah, Fu, PRB (2016)

X-Cube Model

ground state: all *A = B = +1*

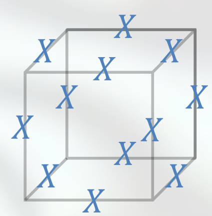
2^{6L-3} *degenerate on 3 torus*

$$\mathcal{H}_{\mathrm{X-cube}} = -\sum_{v,\mu} A_v^{\mu} - \sum_{c} B_c$$

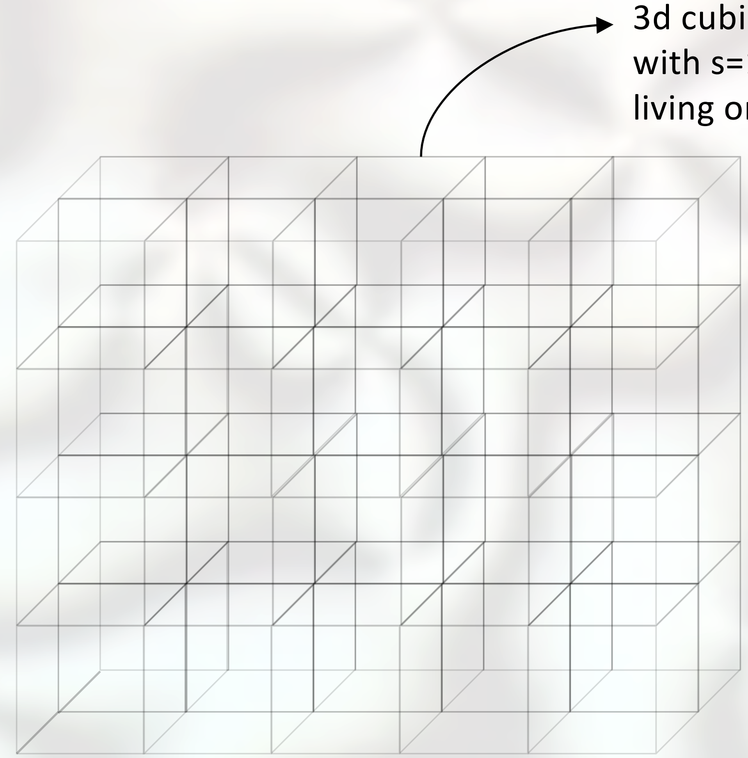


A_x A_y A_z

"vertex" terms



"cube" term



3d cubic lattice
with s=1/2 spins
living on its links

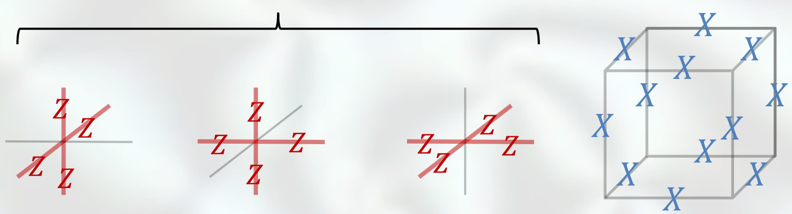
Vijay, Haah, Fu, PRB (2016)

X-Cube Model

(four) fracton excitations: $Z_l\,|gs\rangle$ -> four $B_l = -1$

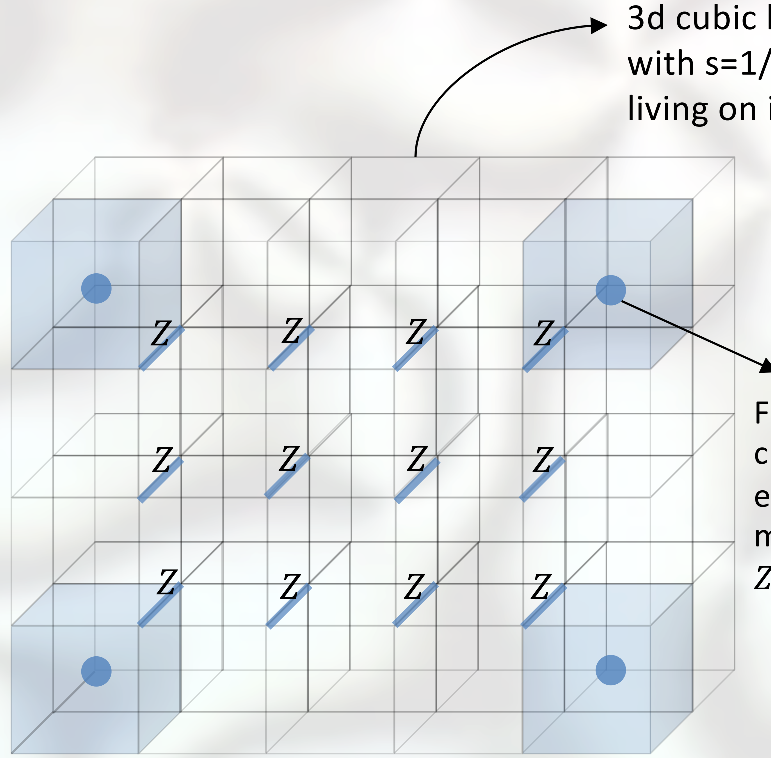
cannot move

$$\mathcal{H}_{\text{X-cube}} = -\sum_{v,\mu} A_v^{\mu} - \sum_{c} B_c$$



A_x A_y A_z

"vertex" terms "cube" term



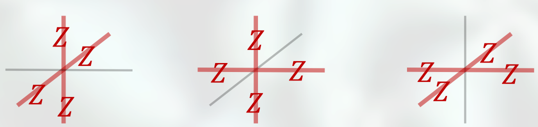
Vijay, Haah, Fu, PRB (2016)

X-Cube Model

(two) planeon excitations: $Z_l |gs\rangle$ -> four $B_l = -1$

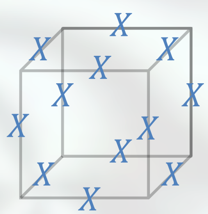
dipole moves in a ⊥ plane

$$\mathcal{H}_{\mathrm{X-cube}} = -\sum_{v,\mu} A_v^{\mu} - \sum_{c} B_c$$



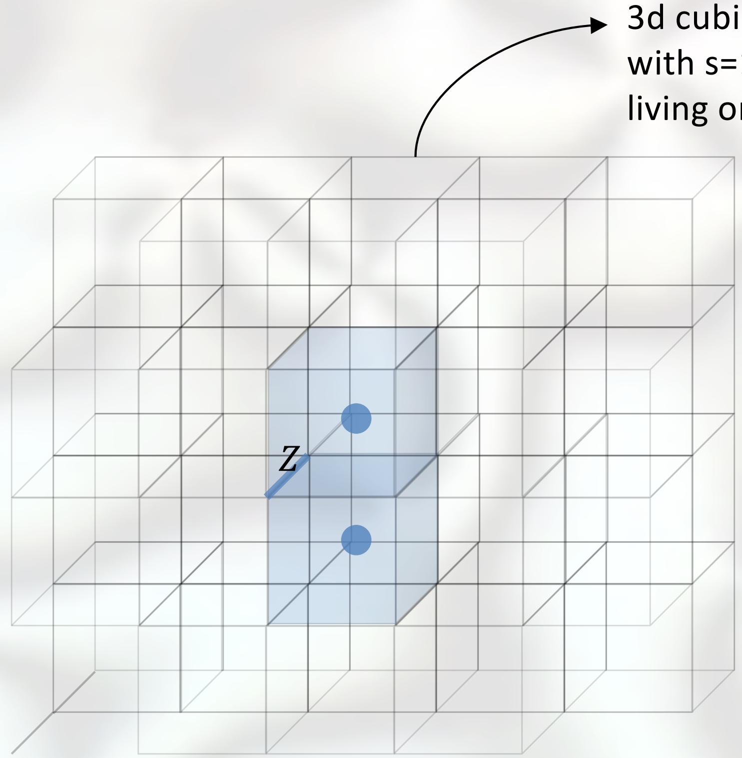
A_x A_y A_z

"vertex" terms



"cube" term

3d cubic lattice
with s=1/2 spins
living on its links



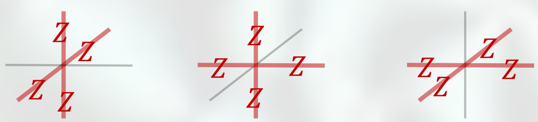
Vijay, Haah, Fu, PRB (2016)

X-Cube Model

(four) lineon excitation: X_l |gs> -> four $A_l = -1$

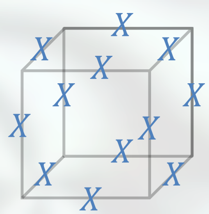
moves along a line, cannot turn

$$\mathcal{H}_{\text{X-cube}} = -\sum_{v,\mu} A_v^\mu - \sum_c B_c$$



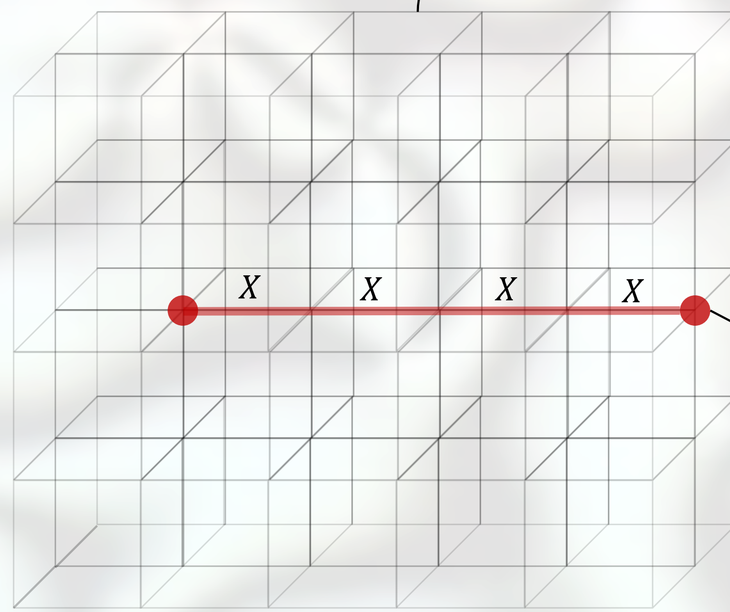
A_x A_y A_z

"vertex" terms



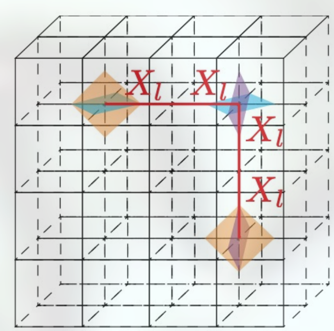
"cube" term

3d cubic lattice with s=1/2 spins living on its links



X-directed 1d particle

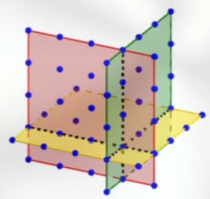
turn creates gapped excitations



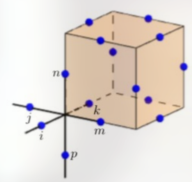
S. Vijay, et al., 2015, '16
M. Pretko, 2016, '17
T. Hsieh, et al., 2017
K. Slagle, Y. B. Kim, 2017
H. Ma, et al., 2017
X. Chen, et al., 2017, '18

Fracton developments

- *"gauging" global Z_2 subdimensional symmetry spin model*



Planar



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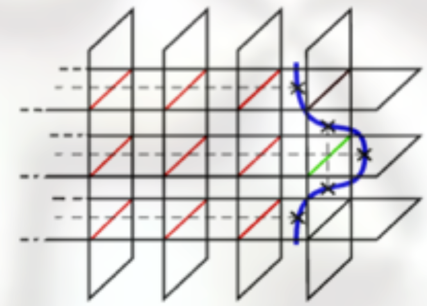
$B_s^{(yz)} = \sigma_j^z \sigma_n^z \sigma_m^z \sigma_p^z$

X-Cube Model

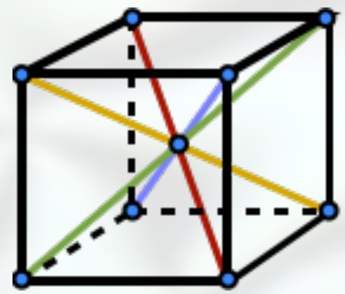
S. Vijay, J. Haah, L. Fu, 2016

- *Coupled-layers construction*



Ma, Lake, Chen, Hermele, 2017

- *Coupled-chains construction*



Halasz, Hsieh, et al., 2017

- *Parton construction*

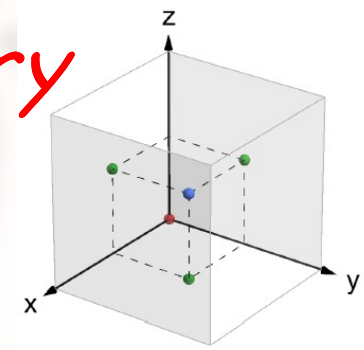
- *Higher rank tensor gauge theory* $\partial_i \partial_j E_{ij} = \rho_f$

M. Pretko, 2016

Rasmussen, You, C. Xu, 2016

Pretko, 2016

*Fractons via **tensor** gauge theory*



- U(1) symmetric tensor gauge theory (2+1D):

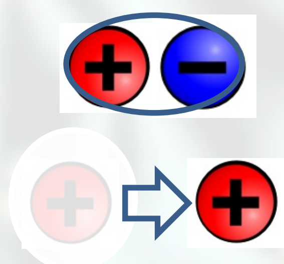
$$\mathcal{H} = \frac{1}{2}E_{ij}E_{ij} + \frac{1}{2}B_iB_i \quad [E^{ij}, A_{ij}] = i\delta^{(2)}(\mathbf{x}) \quad B^i = \epsilon_{jk}\partial^j A^{ki}$$

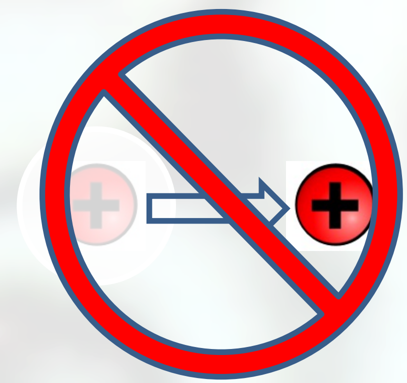
- Gauss' law: $\partial_i\partial_j E^{ij} = \rho$

- Conservation of charges and of *dipoles* ---> *fracton* phenomenology!

-> *moving charge changes dipole moment -> forbidden by dipole conservation*

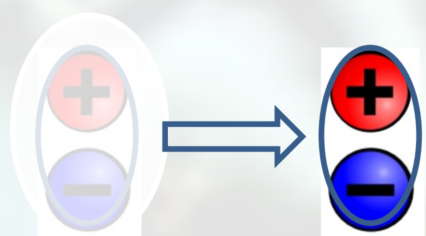
- *immobile*





-> *dipole motion constrained*

- *subdimensional*



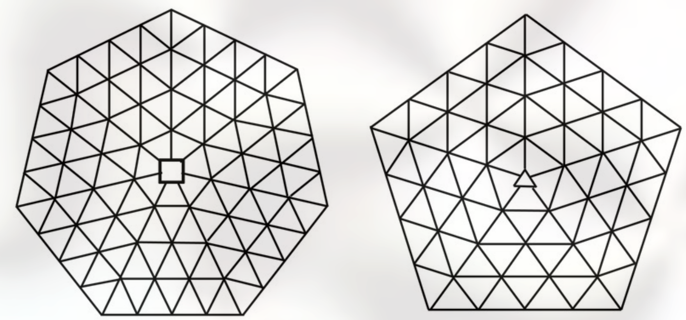
Fractons

¿ any physical realizations ?

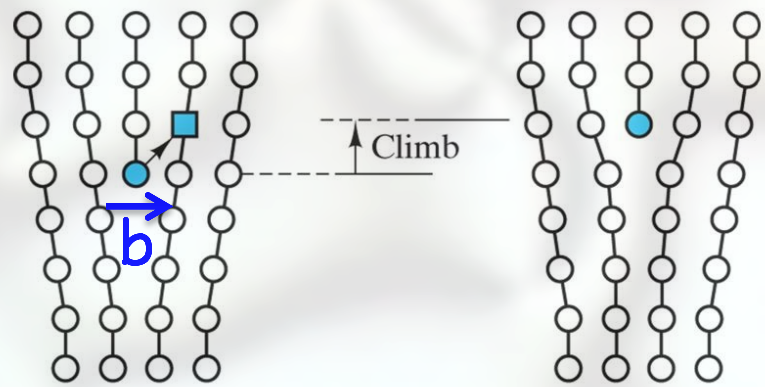
***YES*: 2D quantum crystal!**

Topological defects in a crystal

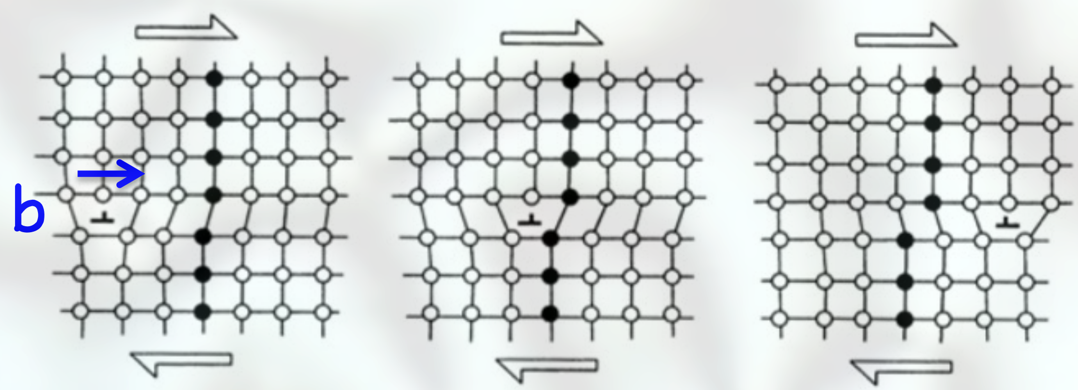
- *Disclination:*
  immobile



- *Dislocation climb:*
  constrained by
  v/i diffusion



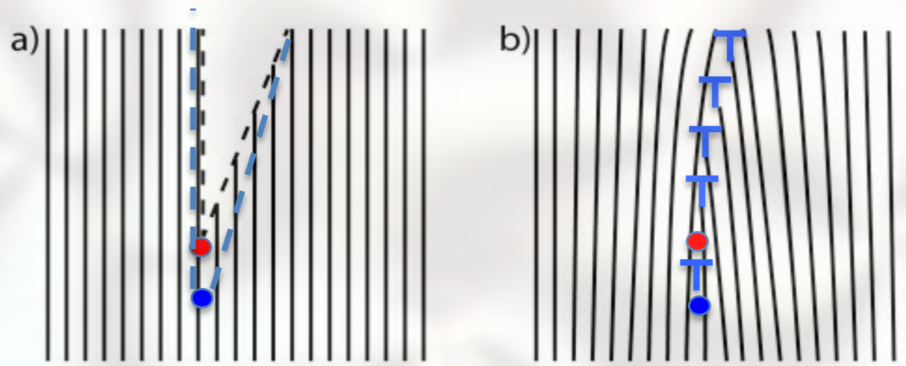
- *Dislocation glide:*
  subdimension (d-1)
  motion



L. R., 2016
classical duality

Topological defects in a crystal

- ***Disclination:***
  immobile



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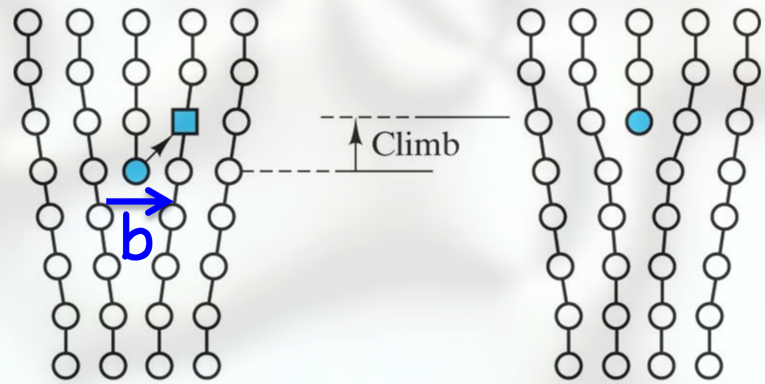

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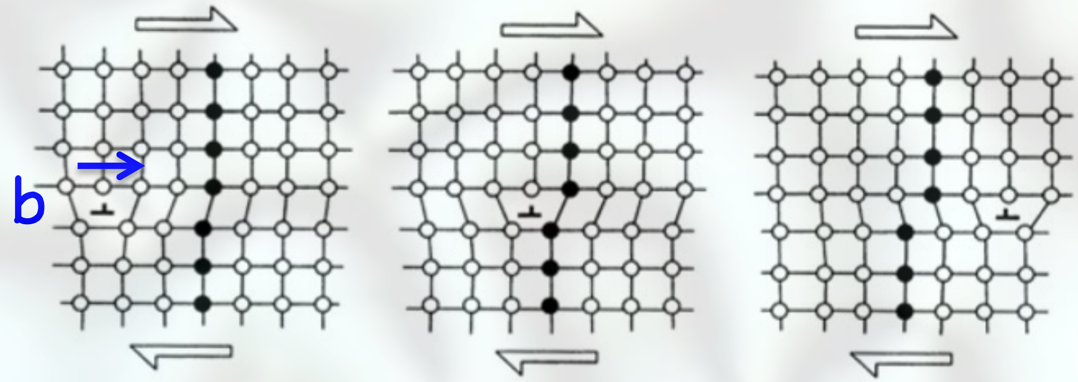
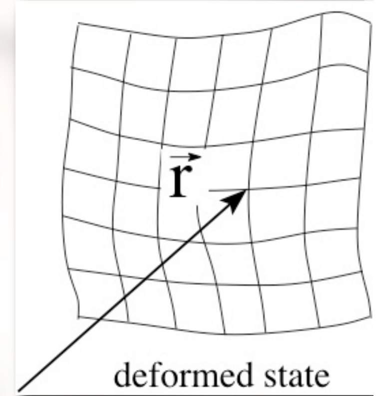
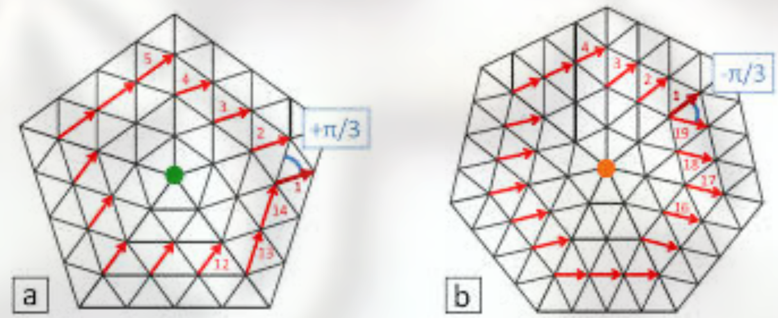


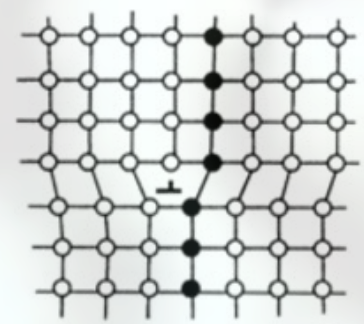
Elasticity theory and defects



- Eulerian phonons: $\vec{r} = \vec{R} + \vec{u}(\vec{r})$
- Strain: $u_{ij} = \frac{1}{2}(\partial_i \vec{R} \cdot \partial_j \vec{R} - \delta_{ij}) \approx \frac{1}{2}(\partial_i u_j + \partial_j u_i)$
- Hamiltonian: $\mathcal{H} = \frac{1}{2}\pi^2 + \frac{1}{2} C_{ij,kl} u_{ij} u_{kl}$
- Topological defects
  - Disclinations: $\nabla \times \nabla \theta = s\, \delta^2(\vec{r}) \equiv s(\vec{r})$
    (bond angle: $\theta = ½\, \varepsilon_{ij} \partial_i u_j$)

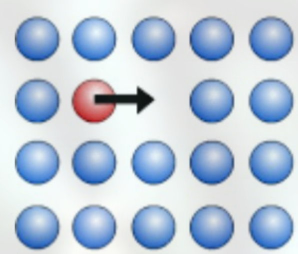


  - Dislocations: $\nabla \times \nabla u_i = b_i \delta^2(\vec{r}) \equiv b_i(\vec{r})$.



  - Vacancies/interstitials:

$$n_d = n_v - n_i$$

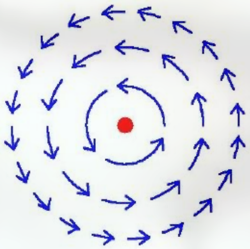


Dasgupta, Halperin
Fisher, Lee

Boson-vortex duality

<u>Superfluid</u>

vortices



topological winding

$$\nabla \times \mathbf{j} = \rho$$

$\mathbf{j} = \nabla\phi$

<u>Maxwell Gauge Theory (with matter)</u>

particles

Gauss's law:

$$\nabla \cdot \mathbf{E} = \rho$$

Goldstone mode

$$H = \frac{1}{2}\int d^2x \left[|\nabla\phi|^2 + n^2\right]$$

$$[n, \phi] = i$$

photon

$$H = \frac{1}{2}\int d^2x \left[|\mathbf{E}|^2 + (\nabla \times \mathbf{A})^2\right]$$

$$[A_i, E_j] = i\delta_{ij}$$

M. Pretko, L. R.,2017
PRLs 2018

Fracton-elasticity duality

see also
Zaanen, et al
other contexts

- Elastic Lagrangian: $\mathcal{L} = \frac{1}{2}(\partial_t u_i)^2 - \frac{1}{2}u_{ij}^2$
- Disclinicity: $\partial_i^* \partial_j^* u_{ij} = s(\mathbf{x}) + \hat{\mathbf{z}} \cdot \nabla \times \mathbf{b}(\mathbf{x})$
- Momentum conservation (Newton) constraint: $\partial_t \pi^i - \partial_j \sigma^{ij} = 0$
- "Electric", "magnetic" fields: $B^i = \epsilon^{ij}\pi_j \qquad E_\sigma^{ij} = \epsilon^{ik}\epsilon^{j\ell}\sigma_{k\ell}$

-> Faraday law: $\partial_t B^i + \epsilon_{jk}\partial^j E_\sigma^{ki} = 0$

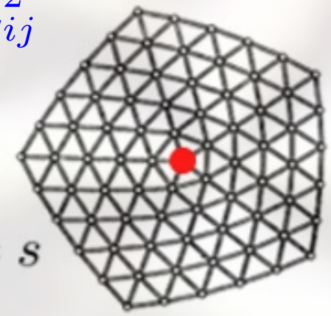
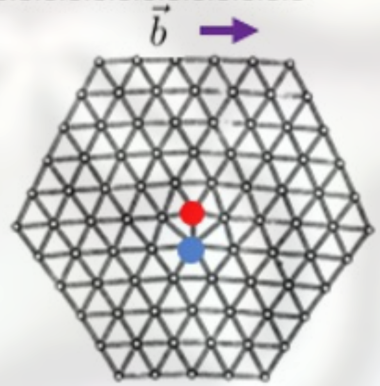
-> Gauge fields: $B^i = \epsilon_{jk}\partial^j A^{ki} \qquad E_\sigma^{ij} = -\partial_t A^{ij} - \partial_i \partial_j \phi$

-> Gauge freedom: $A_{ij} \to A_{ij} + \partial_i \partial_j \alpha \qquad \phi \to \phi + \partial_t \alpha$

-> Peach-Koehler force: $F_i = E_{ij}\, p_j$

M. Pretko, L. R., 2017
PRLs 2018

Fracton-elasticity duality

$\mathcal{H} = \frac{1}{2}B_i^2 + \frac{1}{2}E_{ij}^2$	$\mathcal{H} = \frac{1}{2}\pi_i^2 + \frac{1}{2}u_{ij}^2$
Fracton $\partial_i \partial_j E^{ij} = \rho$	Disclination $\epsilon^{ik}\epsilon^{j\ell}\partial_i\partial_j u_{k\ell} = s$ 
Dipole	Dislocation 
Gauge Modes	Phonons
Electric Field E_{ij}	Strain Tensor u_{ij}
Magnetic Field B_i	Lattice Momentum π_i

$$\partial_t B^i + \epsilon_{jk}\partial^j E_\sigma^{ki} = 0. \quad \longleftrightarrow \quad \partial_t \pi^i - \partial_j \sigma^{ij} = 0$$

Faraday <-> Newton

L. R., 2019

Fractons via ***vector*** *gauge theory ?*

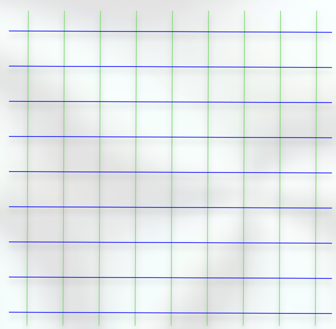
- *Flavored* xy model -> vector gauge duality *(no fractons)*

$$\mathcal{H} = \frac{1}{2}n_k^2 + \frac{1}{2}|\nabla\phi_k|^2 \quad \longrightarrow \quad \tilde{\mathcal{H}} = \frac{1}{2}(\nabla\times\mathbf{A}_k)^2 + \frac{1}{2}|\mathbf{E}_k|^2$$

- Reformulate elasticity into coupled xy models: $u_{ik} \longrightarrow \partial_i u_k$

$$\mathcal{H} = \frac{1}{2}\pi_k^2 + \frac{1}{2}Cu_{ik}^2 \quad \longrightarrow \quad \mathcal{H} = \frac{1}{2}\pi_k^2 + \frac{1}{2}C(\partial_i u_k - g\theta\epsilon_{ik})^2 + \frac{1}{2}L^2 + \frac{1}{2}K(\nabla\theta)^2$$

- Target space rotational symmetry:



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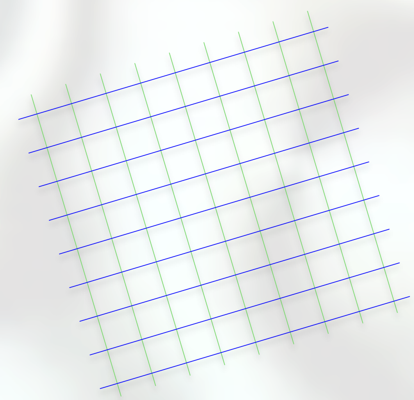


-> $u_x = x(\cos\theta - 1) + y\sin\theta$

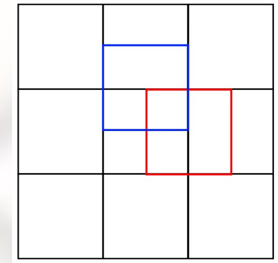
-> $u_y = -x\sin\theta + y(\cos\theta - 1)$

L. R., Hermele '19

Fractons via **vector** gauge theory !

- Reformulate elasticity into flavored coupled xy models:

$$\mathcal{H} = \frac{1}{2}\pi_k^2 + \frac{1}{2}C(\partial_i u_k - g\theta\epsilon_{ik})^2 + \frac{1}{2}L^2 + \frac{1}{2}K(\nabla\theta)^2$$



- Dualize to a coupled vector gauge theory: $(A_a = \epsilon_{ik}A_{ik})$

$$\tilde{\mathcal{H}} = \frac{1}{2}C|\mathbf{E}_k|^2 + \frac{1}{2}(\nabla\times\mathbf{A}_k)^2 + \frac{1}{2}K|\mathbf{e}|^2 + \frac{1}{2}(\nabla\times\mathbf{a} + gA_a)^2 - \mathbf{A}_k\cdot\mathbf{J}_k^b - \mathbf{a}\cdot\mathbf{j}^s$$

- Gauss' law: $\nabla\cdot\mathbf{e} = s$ $\quad \nabla\cdot\mathbf{E}_k = \tilde{p}_k$ $\quad (\tilde{p}_k = p_k - e_k)$

- Gauge redundancy: $\mathbf{A}_k \to \mathbf{A}_k + \nabla\chi_k, \quad A_{0k} \to A_{0k} + \partial_t\chi_k$

$$a_k \to a_k + \partial_k\phi - \chi_k, \quad a_0 \to a_0 + \partial_t\phi$$

- fractons:

gauge invariance demands $\partial_t p_k + \nabla\cdot\mathbf{J}_k = j_k \longrightarrow \mathbf{j} = 0$

L. R., Hermele PRL 2020

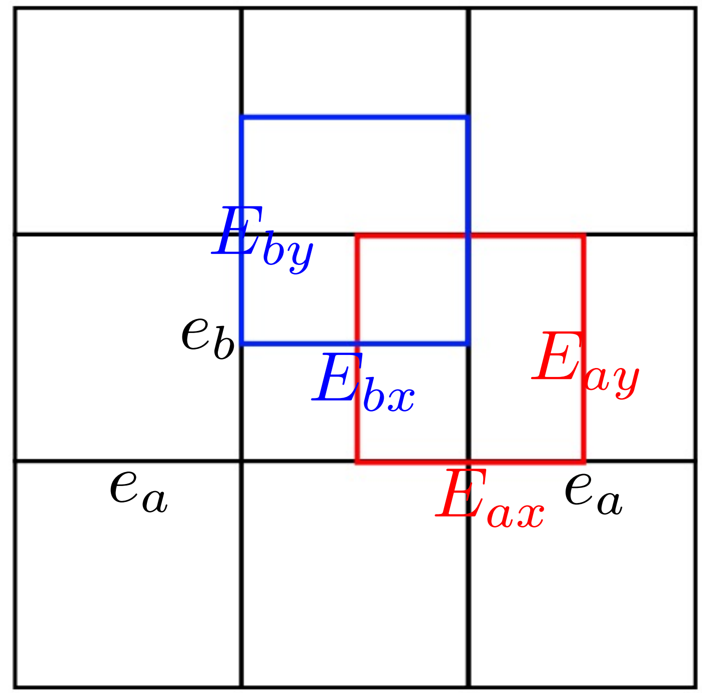
Fractons via *vector* gauge theory

- Lattice fractonic *vector* gauge theory:

E_a E_b e

$$[\hat{A}_{ik}, \hat{E}_{jk'}] = -i\delta_{ij}\delta_{kk'}\delta^2(\mathbf{x}-\mathbf{x}'),$$

$$[\hat{a}_i, \hat{e}_j] = -i\delta_{ij}\delta^2(\mathbf{x}-\mathbf{x}')$$



– Gauss' law:

$$\nabla \cdot \mathbf{e} = s \qquad \nabla \cdot \mathbf{E}_k = e_k$$

(k = a, b)

$$\tilde{\mathcal{H}} = \frac{1}{2}C|\mathbf{E}_k|^2 + \frac{1}{2}(\nabla \times \mathbf{A}_k)^2 + \frac{1}{2}K|\mathbf{e}|^2 + \frac{1}{2}(\nabla \times \mathbf{a} - A_{[ij]})^2 - \mathbf{A}_k \cdot \mathbf{J}_k^b - \mathbf{a} \cdot \mathbf{j}^s$$

gauge invariance demands $\partial_t p_k + \nabla \cdot \mathbf{J}_k = j_k \longrightarrow \mathbf{j} = 0$

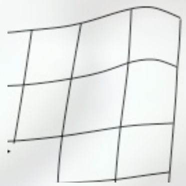
"Fractons via higher-form symmetry breaking"
Qi, L. R., Hermele, AOP 2020

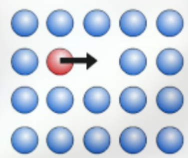
M. Pretko,
L. R., PRL2018

2D *Bosonic crystal*

- Coupled elasticity and bosonic vacancies/interstitials:

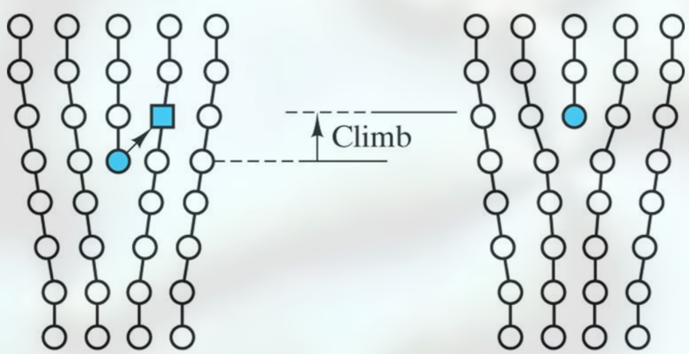
$$\hat{\mathcal{H}} = \underbrace{\frac{1}{2}\hat{\pi}^2 + \frac{1}{2}\hat{u}_{ij}^2}_{\text{elasticity}} + \underbrace{\frac{1}{2}(\nabla\hat{\phi})^2 + \frac{1}{2}\hat{n}^2}_{\text{vacancies/interstitials}} + \underbrace{\nabla\hat{\phi}\cdot\hat{\pi} + \hat{n}\hat{u}_{ii}}_{\text{coupling}}$$





-> ***Commensurate*** (Mott-insulating) crystal

-> ***Incommensurate*** (supersolid) crystal



$$\partial_t n_d + \partial_i J^i_d = -J^i_{\ i}$$

(→ Ampere's law)

Marchetti, L.R. 1998

M. Pretko, L. R., PRL2018

2D *Symmetry-enriched fractons*

Kumar Potter, 2018

- Hybrid U(1) vector-tensor gauge duality

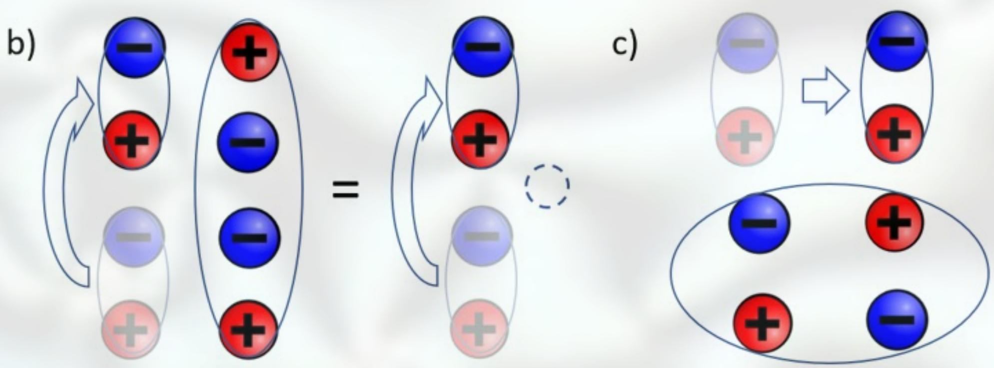
$$\mathcal{H} = \underbrace{\frac{1}{2}(E_{ij}^2 + B_i^2)}_{\text{elasticity}} + \underbrace{\frac{1}{2}(\mathbf{e}^2 + b^2)}_{\text{bosons}} + \underbrace{g(\mathbf{B}\cdot\mathbf{e} + E_{ii}b)}_{\text{"axion" coupling}} + \underbrace{J^{\mu\nu}A_{\mu\nu} + j^\mu a_\mu}_{\text{charges}}$$

-> supersolid -> "fracton superfluid" (mobile dipoles) F

-> normal crystal -> "fracton Mott insulator" (confined dipoles) $F_{U(1)}$

$\hat{a}_{\mathbf{r}}\hat{b}^\dagger_{x,\mathbf{r}+\hat{\mathbf{y}}}\hat{b}_{x,\mathbf{r}}e^{iA_{yy}}$

$E_{yy} \neq 0$



$\hat{b}^\dagger_{x,\mathbf{r}+\hat{\mathbf{x}}}\hat{b}_{x,\mathbf{r}}e^{iA_{xy}}$

$E_{xy} \neq 0$

- Vortex condensation: F --> $F_{U(1)}$

-> fracton dipole dimensional confinement

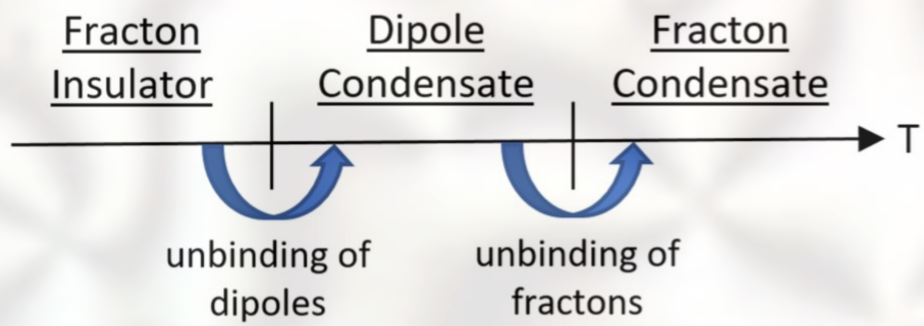
-> superfluid to Mott-insulating fracton transition

Zhai, L. R., 2018

Fracton condensation transition

2D

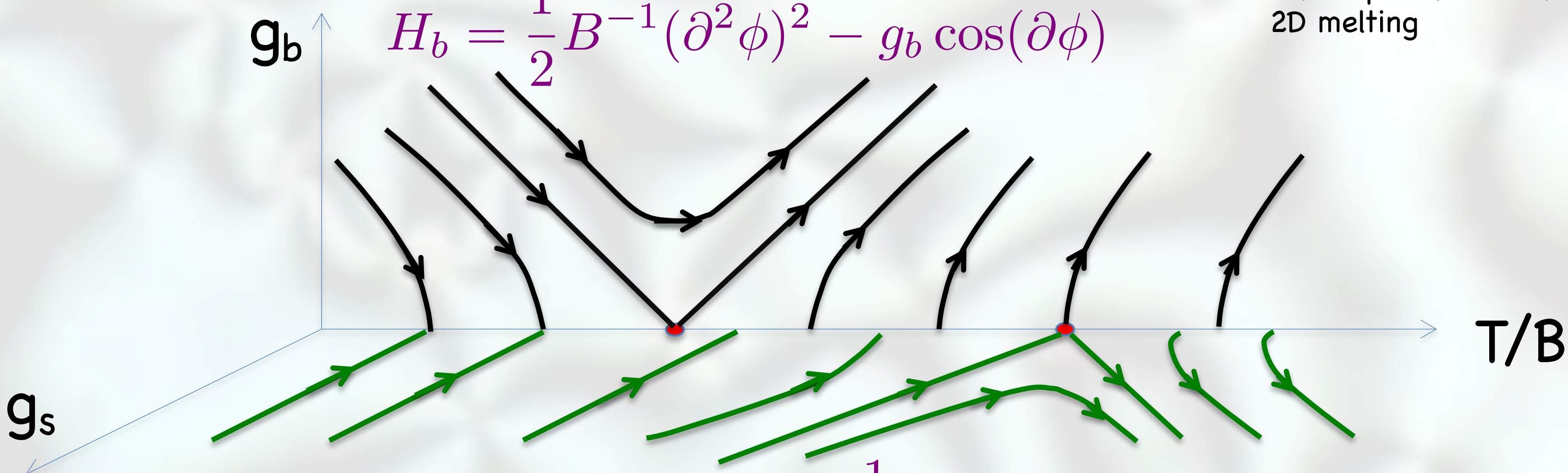
2D scalar fracton model:



$$\tilde{\mathcal{H}} = \frac{1}{2}B^{-1}(\nabla^2\phi)^2 \underbrace{- g_s \cos\left(\frac{2\pi}{6}\phi\right)}_{\text{charges}} - g_b \sum_{n=1,2,3} \underbrace{\cos\left(\mathbf{b}_n \cdot \hat{z} \times \nabla\phi\right)}_{\text{dipoles}}$$

KT, Halperin, Nelson, Young '79
2D melting

$$H_b = \frac{1}{2}B^{-1}(\partial^2\phi)^2 - g_b\cos(\partial\phi)$$



$$H_s = \frac{1}{2}g_b(\partial\phi)^2 - g_s\cos(\phi)$$

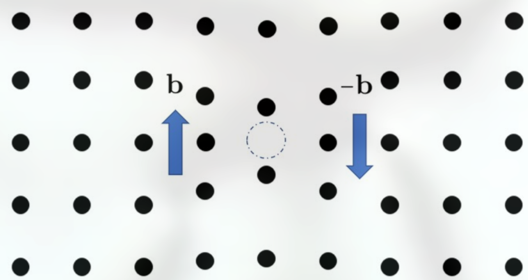
M. Pretko, L. R., 2018

2D *Quantum melting fractons*

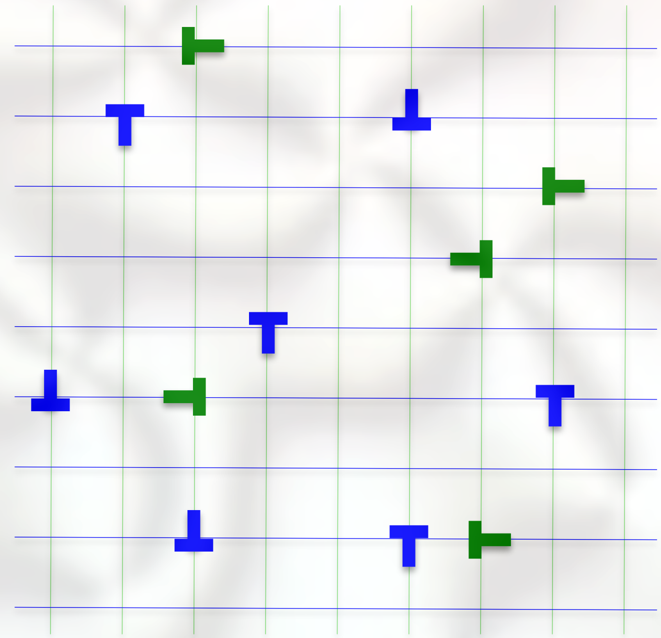
Kumar Potter, 2018

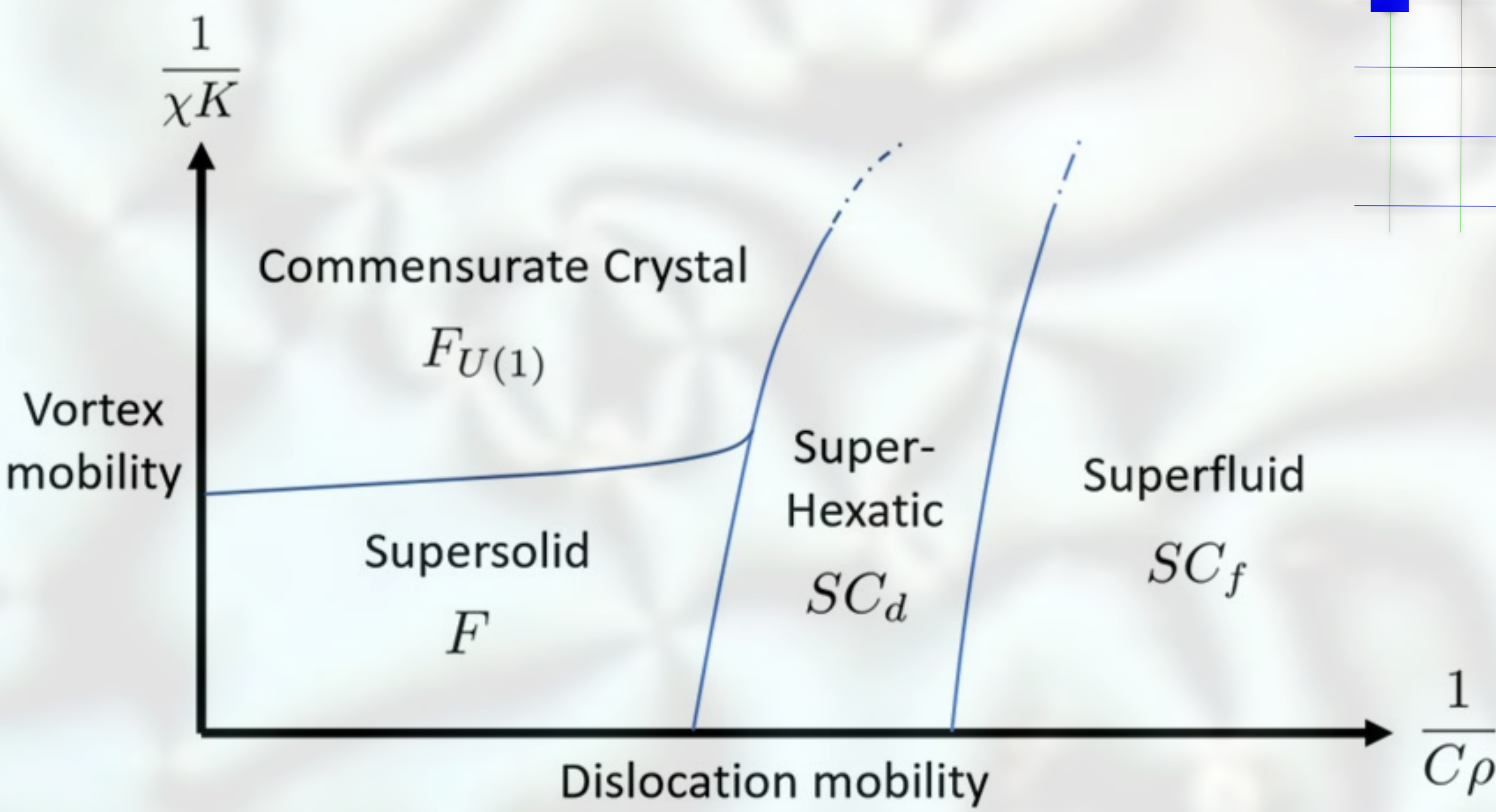
- Fracton dipoles $b_n = \sqrt{n_d}e^{i\theta_n}$ condense: -> ***super**-hexatic*

$$\mathcal{L} = \frac{1}{2}E_{ij}^2 - \frac{1}{2}B_i^2 - \cos(\partial_t\theta - A_0) + g\cos(\partial_i\partial_j\theta - A_{ij})$$



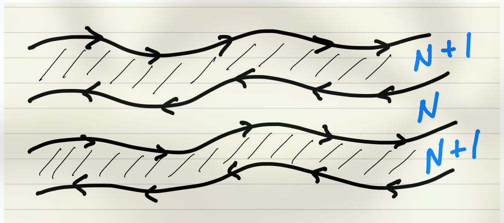
$$\hat{b}^\dagger_{\mathbf{b}}\hat{b}^\dagger_{-\mathbf{b}}\hat{a}$$

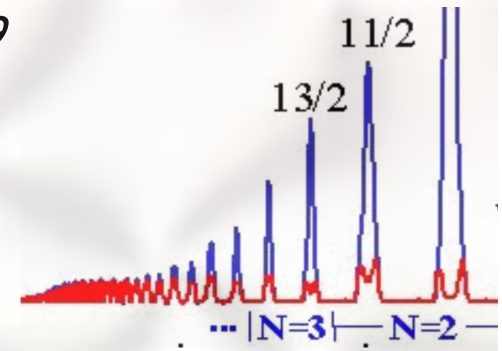




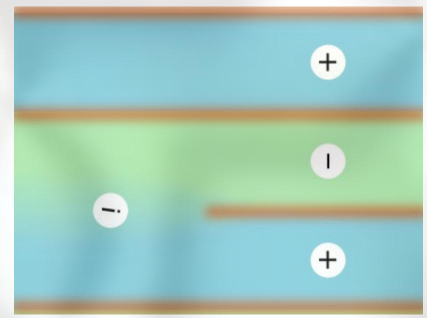
Quantum liquid crystals

- Quantum Hall *Fogler, et al. '96, Moessner, Chalker '96, Fradkin, Kivelson '99, MacDonald, Fisher '99, L.R., Dorsey '02,…Eisenstein, et al. '99*

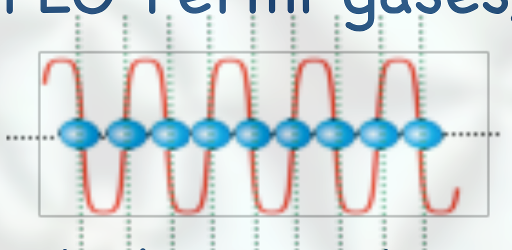




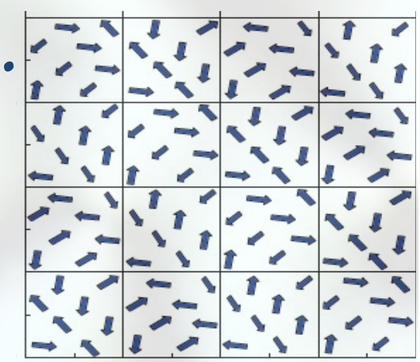
- SDW, CDW, PDW in doped Mott insulators *Tranquada, et al., '97, Kivelson, Fradkin, Emery '98, Sachdev,…*



- Imbalanced FFLO Fermi gases, SOC Bose gases, *L.R. et al. '09,'11, Zhai '15*



- Helical, frustrated magnets, e.g., MnSi, FeGe, AB_2X_4,… *Pfleiderer, et al.' 09, Bergman, et al. '07*

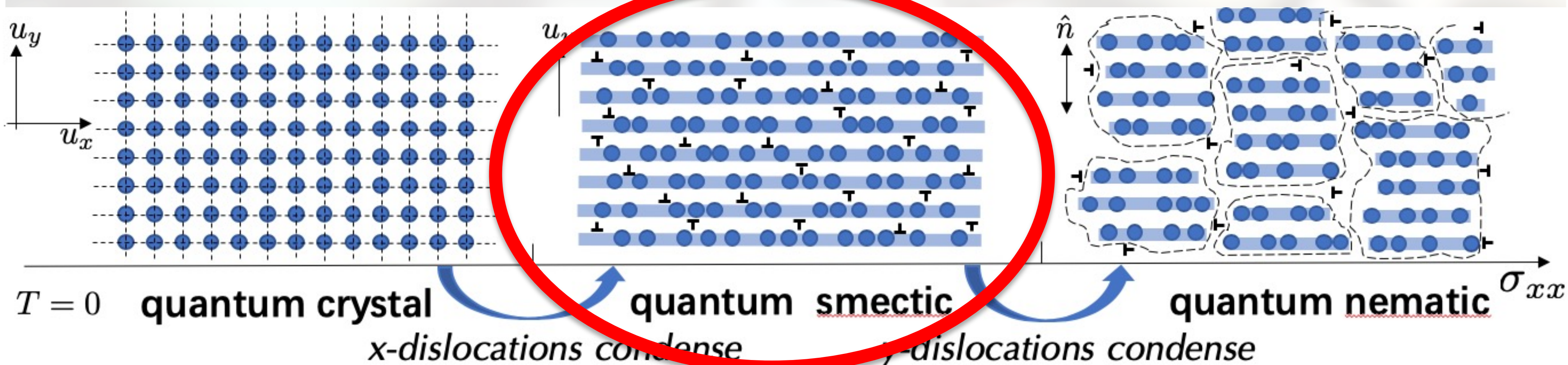


L.R PRL 2020
L.R., Z. Zhai
AOP 2021

Anisotropic quantum melting

Ostlund, Halperin '81

- Crystal: $\mathcal{H}_{cr} = \frac{C}{2} u_{ij}^2 + \frac{1}{2}\pi^2$
- Condense x-dislocations: $b_x = \sqrt{n_d} e^{i\theta_x}$ -> ***super***-*smectic*

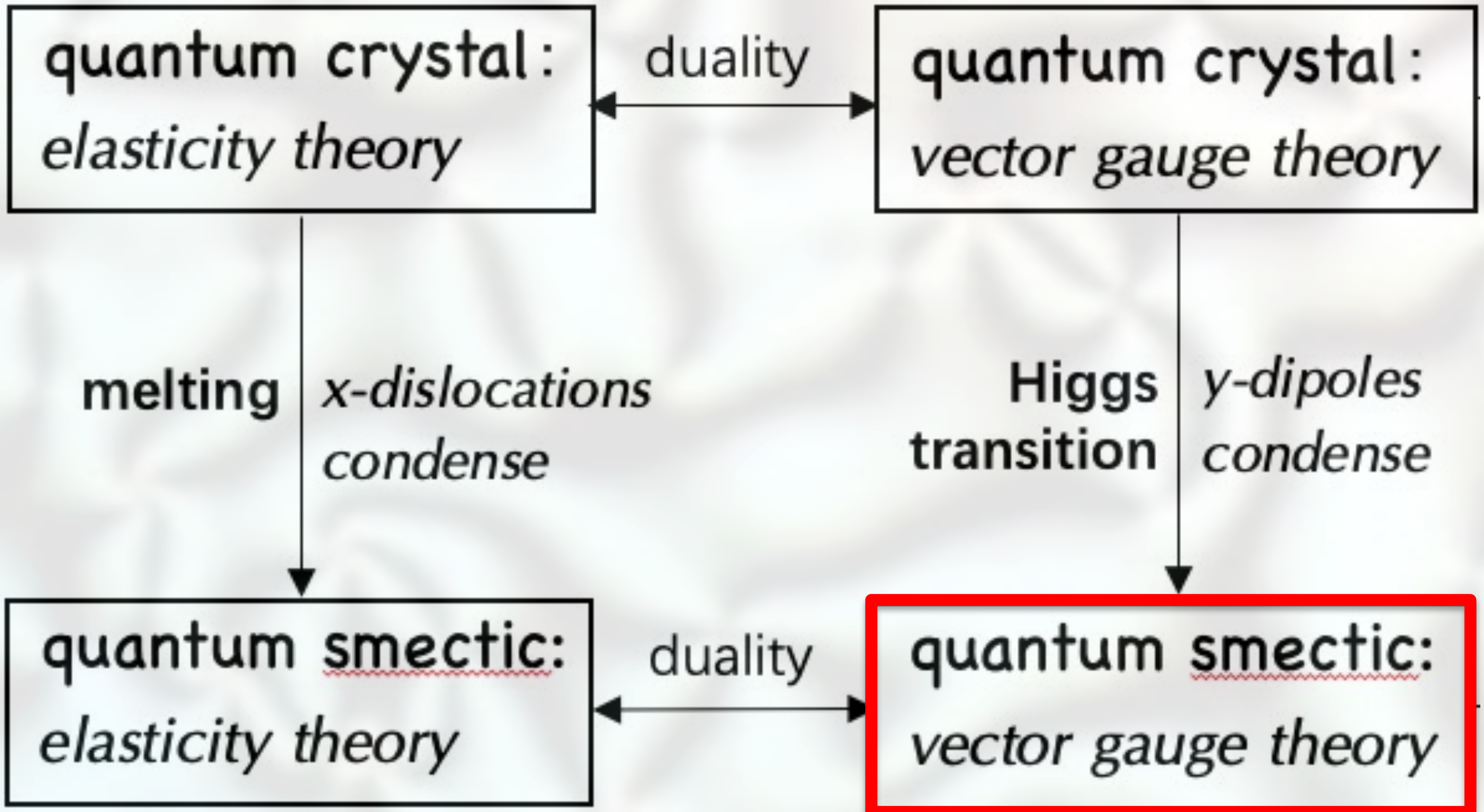


- Super-smectic:

$$\mathcal{H}_{sm} = \underbrace{\frac{C}{2}(\nabla u_y - \theta\hat{\mathbf{x}})^2 + \frac{K}{2}(\nabla\theta)^2 + \frac{1}{2}\pi^2 + \frac{1}{2}L^2}_{\text{quantum smectic elasticity}} + \underbrace{\frac{1}{2}(\nabla\phi_s)^2 + \frac{U}{2}n^2}_{\text{bosonic atoms}}$$

L.R PRL 2020
L.R., Z. Zhai
AOP 2021

Crystal – smectic – gauge duality



L.R arXiv2009
L.R., Z. Zhai 2020

Smectic elasticity – gauge duality

- Super-smectic:

$$\mathcal{H}_{sm} = \underbrace{\frac{C}{2}(\nabla u_y - \theta\hat{\mathbf{x}})^2 + \frac{K}{2}(\nabla\theta)^2 + \frac{1}{2}\pi^2 + \frac{1}{2}L^2}_{\text{quantum smectic elasticity}} + \underbrace{\frac{1}{2}(\nabla\phi_s)^2 + \frac{U}{2}n^2}_{\text{bosonic atoms}}$$

- Dualize to U(1) vector gauge theory:

$$\tilde{\mathcal{H}}_{sm} = \frac{1}{2}C\mathbf{E}^2 + \frac{1}{2}(\nabla\times\mathbf{A})^2 + \frac{1}{2}K\mathbf{e}^2 + \frac{1}{2}(\nabla\times\mathbf{a} - \hat{\mathbf{y}}\cdot\mathbf{A})^2 - \mathbf{A}\cdot\mathbf{J} - \mathbf{a}\cdot\mathbf{j}$$

- Gauss' law: $\nabla\cdot\mathbf{E} = p + \hat{\mathbf{x}}\cdot\mathbf{e}$ $\quad\nabla\cdot\mathbf{e} = n_s$

- Gauge redundancy: $\mathbf{A} \to \mathbf{A} + \nabla\chi, \quad A_0 \to A_0 + \partial_t\chi$

  $\mathbf{a} \to \mathbf{a} + \nabla\phi - \chi\hat{\mathbf{x}}, \quad a_0 \to a_0 + \partial_t\phi$

- restricted mobility:

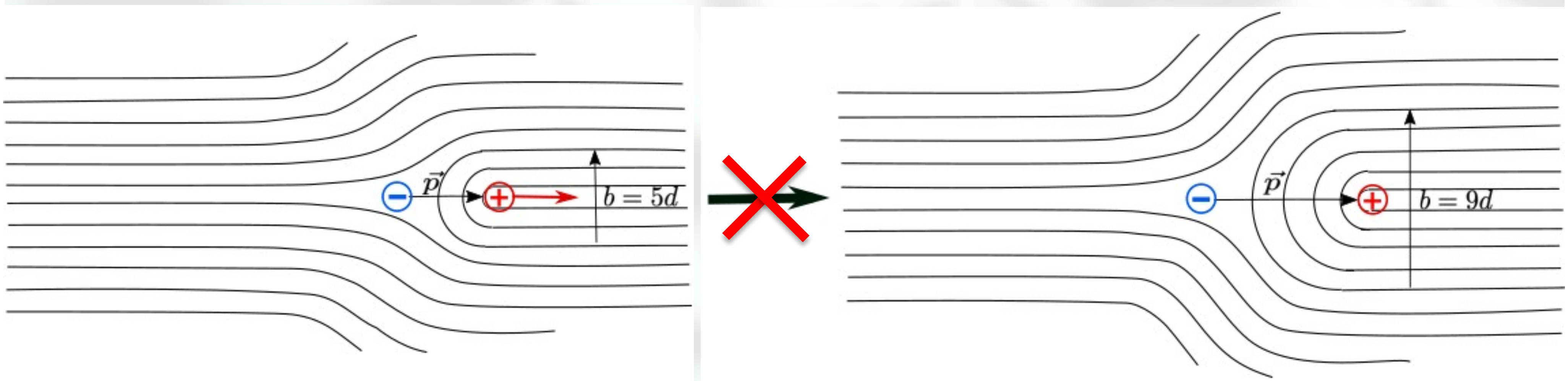
  gauge invariance demands $\partial_t p + \nabla\cdot\mathbf{J} = \hat{\mathbf{x}}\cdot\mathbf{j} \longrightarrow j_x = 0$

L.R PRL 2020
L.R., Z. Zhai
AOP 2021

Restricted disclination mobility

gauge invariance demands $\partial_t p + \nabla \cdot \mathbf{J} = \hat{\mathbf{x}} \cdot \mathbf{j} \longrightarrow j_x = 0$

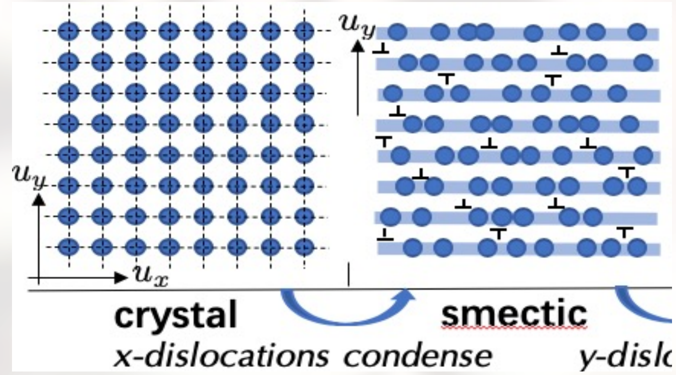
- Fractonic restricted dynamics *via disclination microscopics:*



requires a nonlocal process of adding a pair smectic half-layer per lattice constant of disclination separation

L.R PRL 2020
L.R., Z. Zhai
AOP 2021

Higgs'ing crystal gauge dual -> smectic gauge dual



- Crystal gauge dual:

$$\tilde{\mathcal{H}} = \frac{1}{2}C|\mathbf{E}_k|^2 + \frac{1}{2}(\nabla\times\mathbf{A}_k)^2 + \frac{1}{2}K|\mathbf{e}|^2 + \frac{1}{2}(\nabla\times\mathbf{a} - A_{[ij]})^2 - \mathbf{A}_k\cdot\mathbf{J}_k^b - \mathbf{a}\cdot\mathbf{j}^s$$

$$\tilde{\mathcal{H}}_{cr} = |(i\nabla - p_k\mathbf{A}_k)\psi_k|^2 + V(\psi_k) + \tilde{\mathcal{H}}_{Max}[\mathbf{A}_k, \mathbf{E}_k]$$

  - Higgs transition - condensation of y-dipoles (x-dislocations)

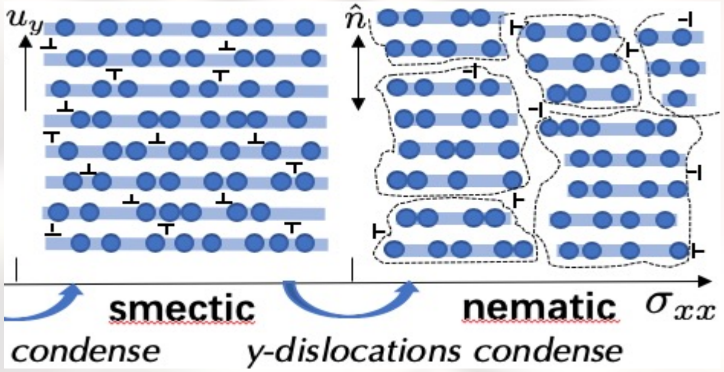
$$\psi_x = 0, \quad \psi_y \neq 0 \quad \rightarrow \quad \mathbf{A}_y \approx 0 \ \text{ gapped}$$

- Smectic gauge dual: $\tilde{\mathcal{H}}_{sm}[\mathbf{A}^x, \mathbf{a}] \approx \tilde{\mathcal{H}}_{cr}[\mathbf{A}^x, \mathbf{A}^y = 0, \mathbf{a}]$

$$\tilde{\mathcal{H}}_{sm} = \frac{1}{2}C\mathbf{E}^2 + \frac{1}{2}(\nabla\times\mathbf{A})^2 + \frac{1}{2}K\mathbf{e}^2 + \frac{1}{2}(\nabla\times\mathbf{a} - \hat{\mathbf{y}}\cdot\mathbf{A})^2 - \mathbf{A}\cdot\mathbf{J} - \mathbf{a}\cdot\mathbf{j}$$

L.R arXiv2009
L.R., Z. Zhai
2020

Higgs'ing smectic gauge dual -> nematic gauge dual



- Smectic gauge dual:

$$\tilde{\mathcal{H}}_{sm} = \frac{1}{2}C\mathbf{E}^2 + \frac{1}{2}(\nabla\times\mathbf{A})^2 + \frac{1}{2}K\mathbf{e}^2 + \frac{1}{2}(\nabla\times\mathbf{a} - \hat{\mathbf{y}}\cdot\mathbf{A})^2 - \mathbf{A}\cdot\mathbf{J} - \mathbf{a}\cdot\mathbf{j}$$

$$\tilde{\mathcal{H}}_{sm} = |(i\nabla - p\mathbf{A})\psi|^2 + V(\psi) + \tilde{\mathcal{H}}_{Max}[\mathbf{A},\mathbf{E},\mathbf{a},\mathbf{e}]$$

  - Higgs transition - condensation of x-dipoles (y-dislocations)

$$\psi_x \neq 0 \quad \rightarrow \quad \mathbf{A} \approx 0 \ \ \text{gapped}$$

- Nematic gauge dual: $\tilde{\mathcal{H}}_{nem}[\mathbf{a}] \approx \tilde{\mathcal{H}}_{sm}[\mathbf{A}^x = 0, \mathbf{a}]$

$$\tilde{\mathcal{H}}_{nem} = \frac{1}{2}K\mathbf{e}^2 + \frac{1}{2}(\nabla\times\mathbf{a})^2 - \mathbf{a}\cdot\mathbf{j}$$

$$\tilde{H}_{nem} = \frac{1}{2}K(\nabla\theta)^2 + \frac{1}{2}L^2$$

L. R., 2018
Pretko, Zhai, L.R. 2018

2D *T-R breaking crystal*

Kumar
Potter, 2018

- Wigner crystal in B-field elasticity (also vortex lattice)

$$\hat{\mathcal{H}} = \frac{1}{2} C^{ijk\ell} \hat{u}_{ij} \hat{u}_{k\ell} \qquad [u_x(\mathbf{r}), u_y(\mathbf{r}')] = i\ell^2 \delta^2(\mathbf{r} - \mathbf{r}').$$

$$\mathcal{L} = \frac{1}{2\ell^2} \mathbf{u} \times \partial_t \mathbf{u} - \frac{1}{2} C^{ijk\ell} u_{ij} u_{k\ell}$$

- T-R breaking fracton phase

$$\hat{\mathcal{L}} = \frac{1}{2} \mathbf{B} \times \partial_t \mathbf{B} - \frac{1}{2} C^{ijk\ell} E_{ij} E_{k\ell}$$

$$\rightarrow \quad \omega \sim q^2$$

L. R., 2018

Nguyen, et al, 2020

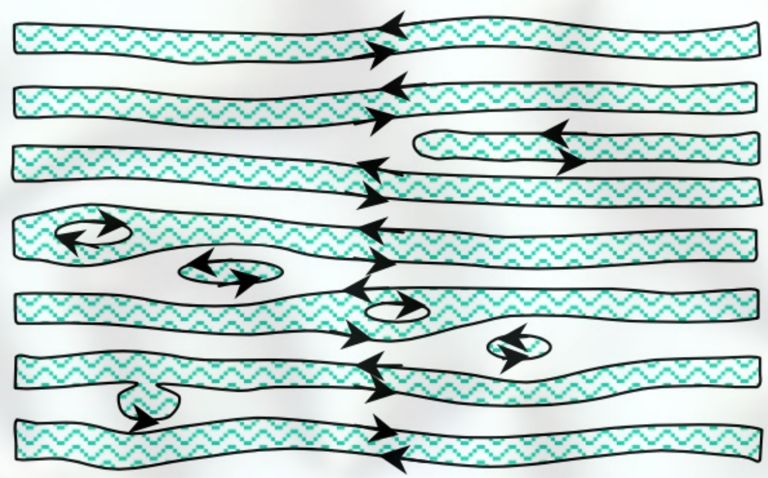
2D *T-R breaking **smectic***

- melt Wigner crystal in B-field -> QH smectic stripes

$$[u_x(\mathbf{r}), u_y(\mathbf{r}')] = i\ell^2\delta^2(\mathbf{r}-\mathbf{r}').$$

$$\mathcal{L} = \frac{1}{\ell^2}u_x\partial_t u_y - \frac{1}{2}(\partial_x u_x)^2 - \frac{1}{2}(\partial_y u_y)^2 - \frac{1}{2}K(\partial_y^2 u)^2 + \ldots$$

- Smectic liquid
  - *vacancies+interstitials*
  - *dislocations*
  - *compressible vs incompressible?*

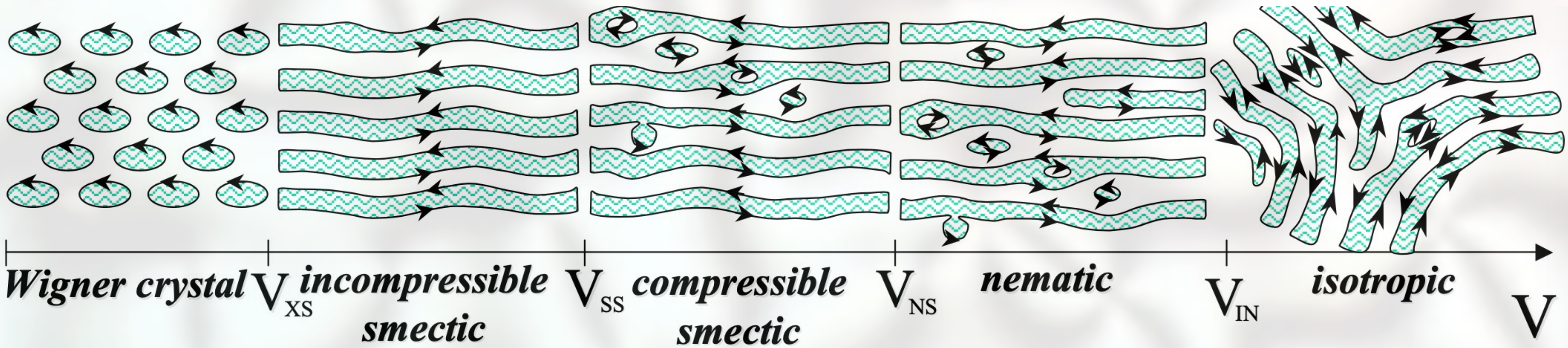


L. R., 2018

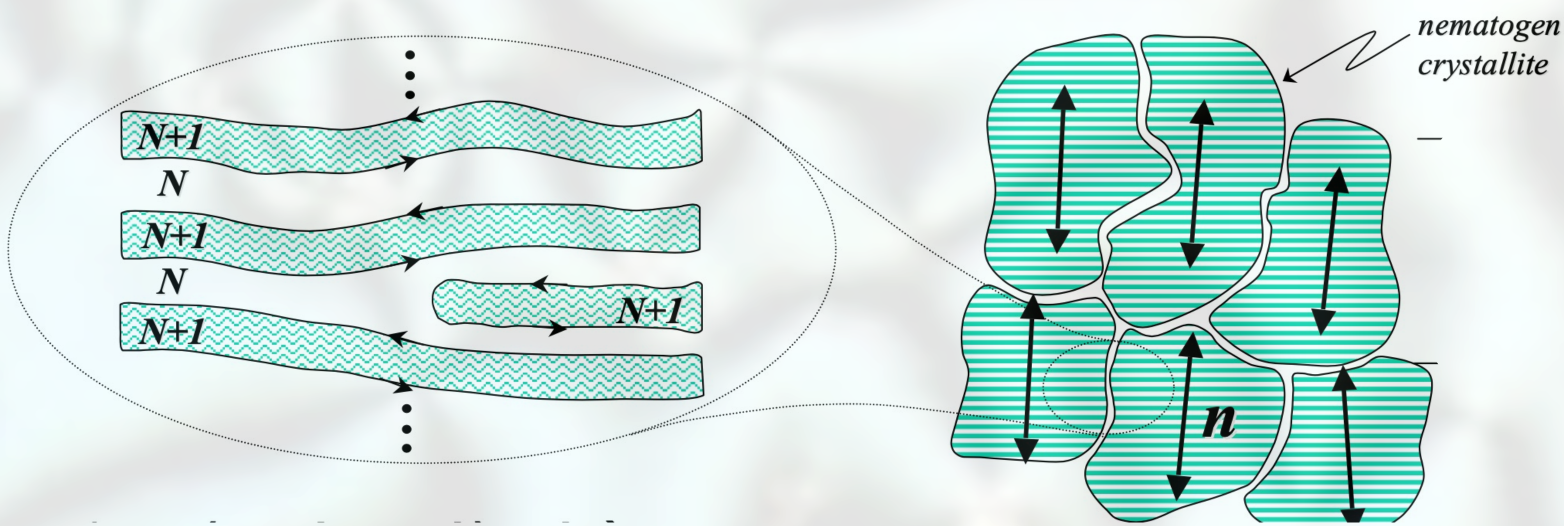
2D *T-R breaking* ***smectic***

Koulakov, et al, 1996
Moesner, et al., 1006
MacDonald, Fisher, 2000
L.R., Dorsey, 2002

- Wigner crystal B-field -> QH smectic -> QH nematic -> HLR



- QH nematic and compressible isotropic liquid

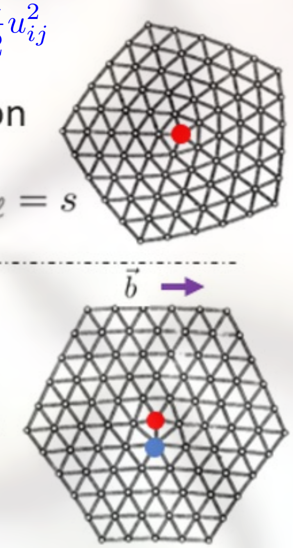


Summary and conclusions

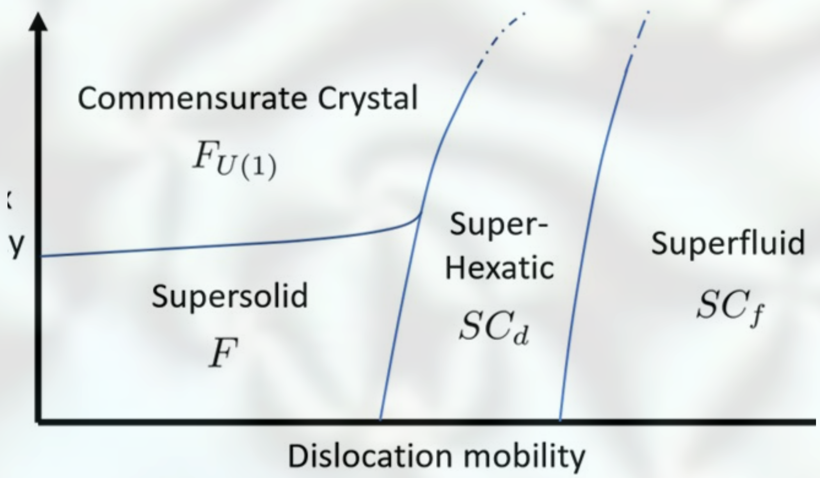
- *New class of fractonic quantum liquids*
  excitations w/ restricted/fractionalized mobility

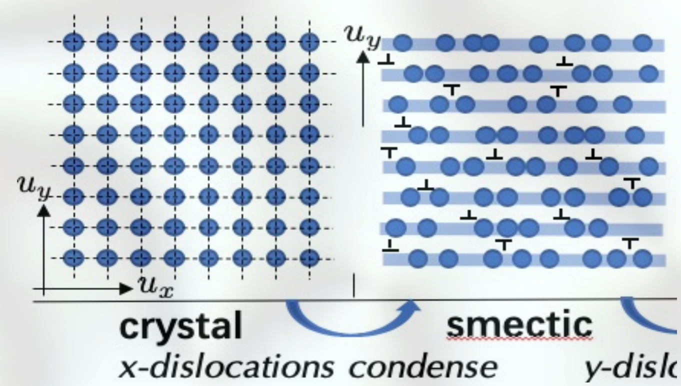
- *Fractons – elasticity duality*
  realized as defects in quantum crystal

$\mathcal{H} = \frac{1}{2}B_i^2 + \frac{1}{2}E_{ij}^2$	$\mathcal{H} = \frac{1}{2}\pi_i^2 + \frac{1}{2}u_{ij}^2$
Fracton $\partial_i\partial_j E^{ij} = \rho$	Disclination $\epsilon^{ik}\epsilon^{j\ell}\partial_i\partial_j u_{k\ell} = s$
Dipole	Dislocation
Gauge Modes	Phonons
Electric Field E_{ij}	Strain Tensor u_{ij}
Magnetic Field B_i	Lattice Momentum π_i



- *Fractonic phases and transitions:*





- *Quantum melting criticality?*
- *QH smectic ? (anisotropic melting, via Higgs transitions, LR 'PRL20)*
- *Elastic nonlinearities ?*
- *Classification, relation to Z_2 models, higher form symmetries, …?*
- *Animation dynamics and editing of tessellated surfaces ?*

Thank you