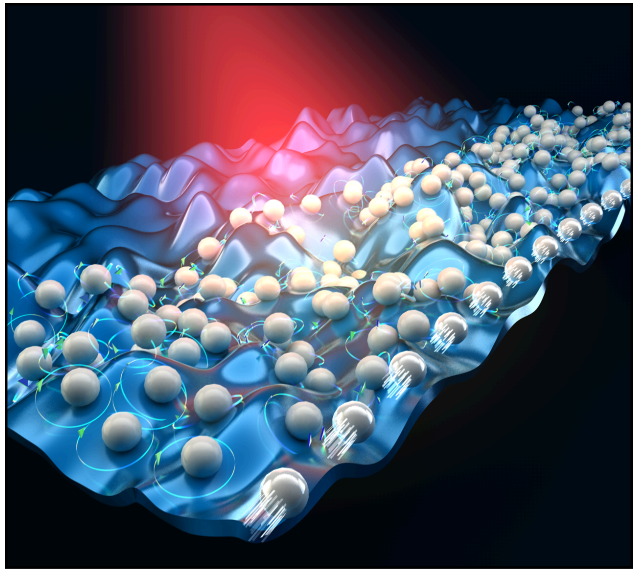
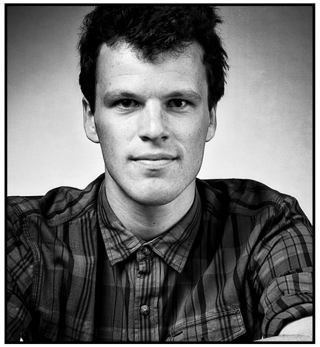
Interaction-enabled topological phases of Floquet systems

Mark Rudner

Niels Bohr Institute/University of Washington



Raffael Gawatz



Frederik Nathan

Prethermalization and universal transport in quantum pumps

Lindner, Berg, and MR, PRX (2017).

Gulden, Berg, MR, and Lindner, SciPost (2020).

Gawatz, Balram, Lindner, Berg, and MR, arXiv:2103.15831.

Support provided by:

erc

VILLUM FONDEN

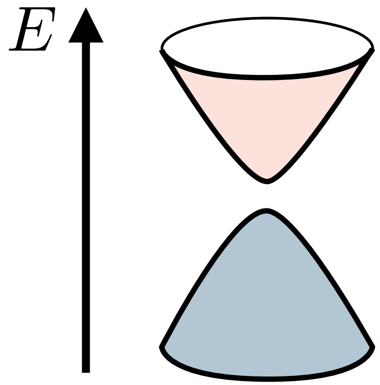
Correlation-induced anomalous Floquet insulators

MR, Lindner, Berg, Levin, PRX (2013).

Nathan, Abanin, Berg, Lindner, and MR, PRB (2019).

Nathan, Abanin, Lindner, Berg, and MR, SciPost (2021).

What/how can topological phenomena be realized out of equilibrium?



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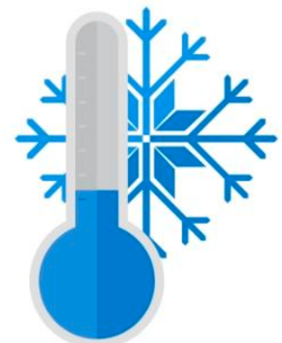

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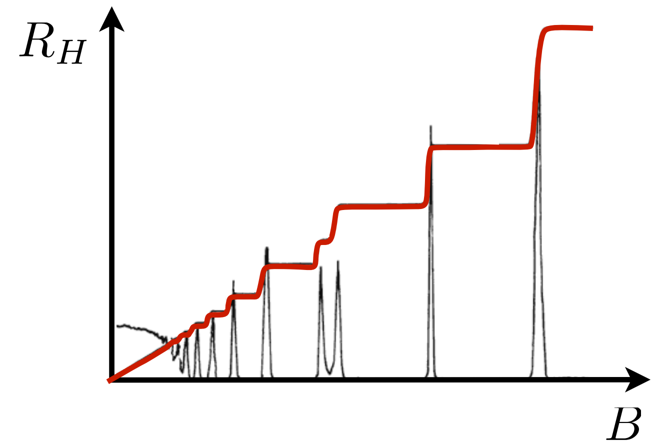


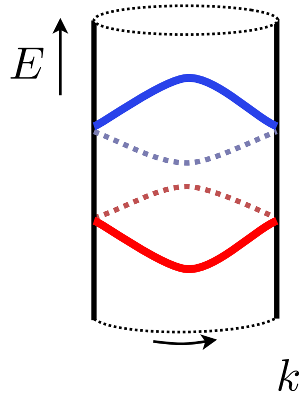
vs.

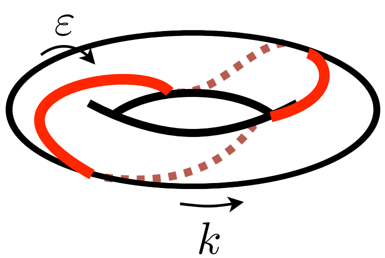
$\longrightarrow$

?

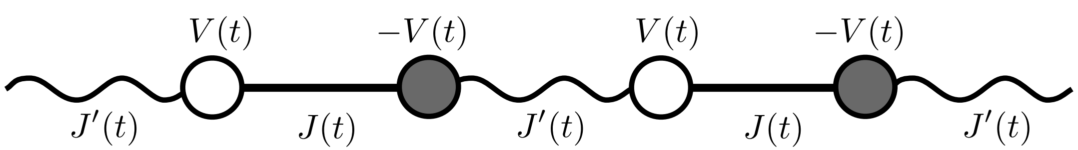
The Plan

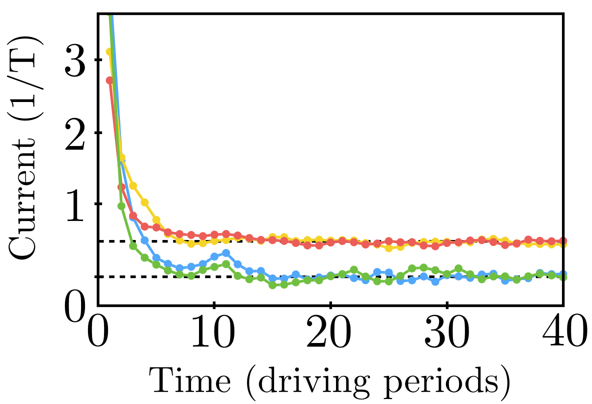
I. Topology in non-interacting periodically-driven (Floquet) systems



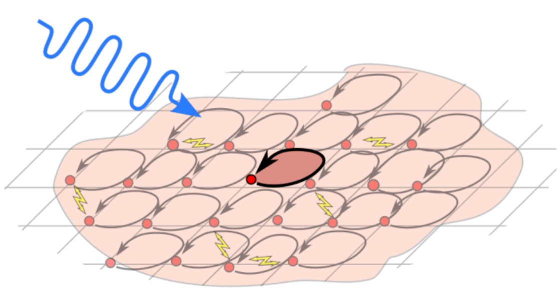


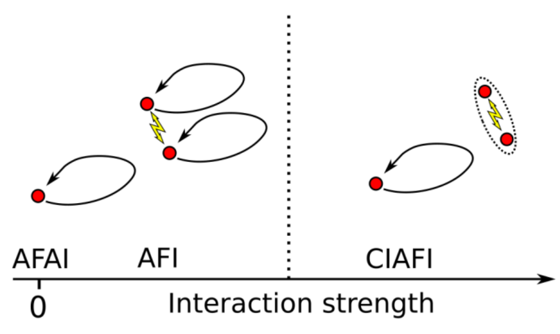
II. Prethermalization and universal transport in non-adiabatic quantum pumps





III. Correlation-induced 2D anomalous Floquet insulators



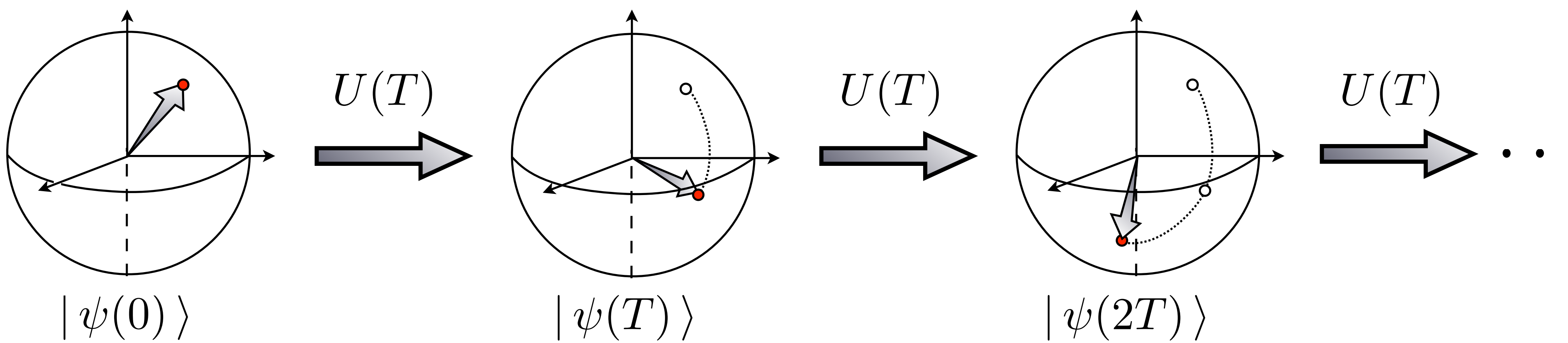


Floquet operator propagates state over driving period

$$U(T) = \mathcal{T} e^{-i \int_0^T H(t) dt}$$

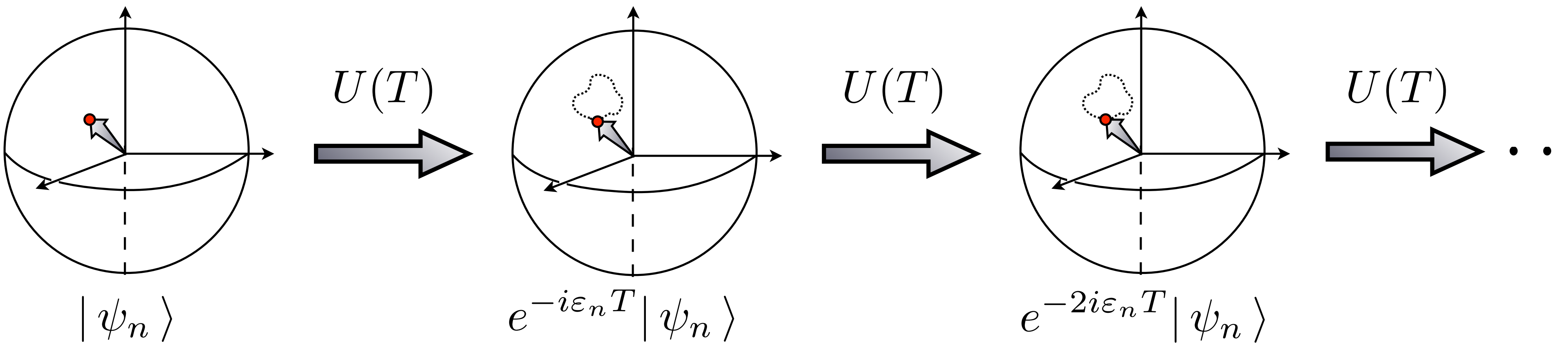
$$H(t+T) = H(t)$$

periodic driving



Quasienergy spectrum defined via eigenstates/eigenvalues of Floquet operator

$$U(T)|\psi_n\rangle = e^{-i\varepsilon_n T}|\psi_n\rangle$$



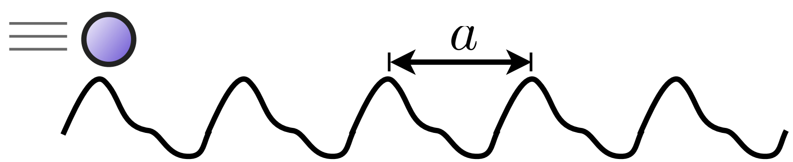
$$|\psi_n(t)\rangle = e^{-i\varepsilon_n t}|\Phi_n(t)\rangle, \qquad |\Phi_n(t+T)\rangle = |\Phi_n(t)\rangle$$

periodic micromotion

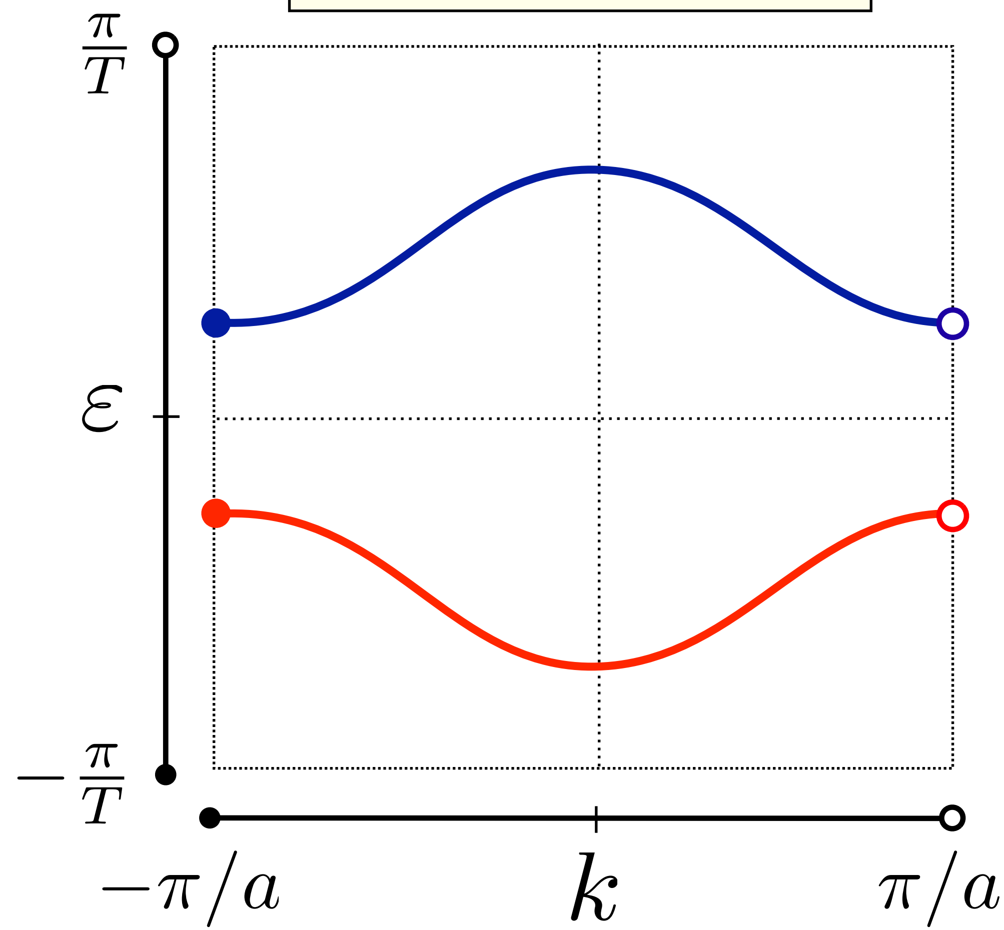
Eigenvalue invariant under $\varepsilon_n \to \varepsilon_n + 2\pi N/T$: quasi-energy lives on a <u>circle</u>

On a lattice, spectrum forms Floquet-Bloch band structure

$$U(T) \equiv e^{-iH_{\mathrm{eff}}T}$$



$$V(x+a) = V(x)$$



Suggests to realize analogues of topological phenomena from static systems in driven systems

Early works on synthetic gauge fields for cold atoms (theory):

Jaksch and Zoller, NJP (2003).

Mueller, PRA (2004).

Sørensen, Demler, and Lukin, PRL (2005).

⋮

Solid state, general topological classification (theory):

Oka and Aoki, PRB (2009).

Kitagawa, Berg, MR, and Demler, PRB (2010).

Lindner, Refael, and Galitski, Nature Physics (2011).

⋮

Early experiments in cold atoms:

Aidelsburger, et al., Nature Physics (2014).

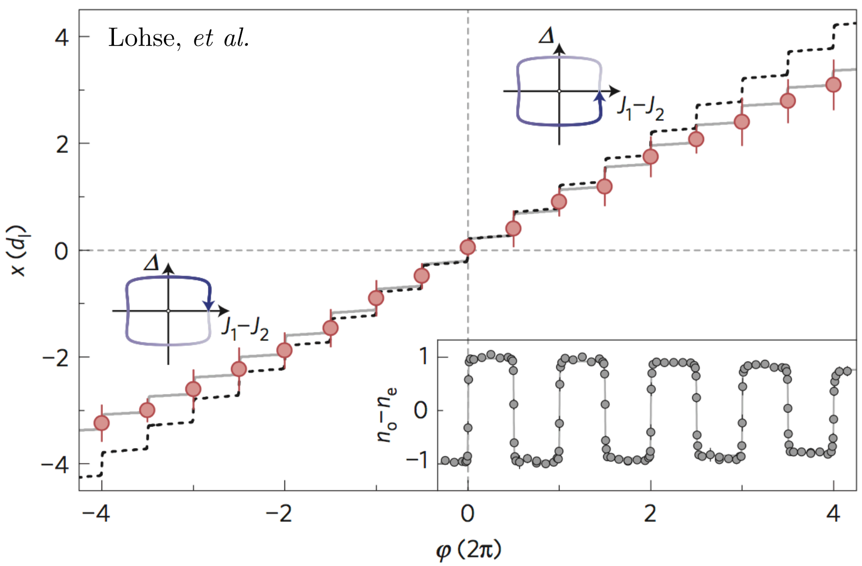
Jotzu, et al., Nature (2014).

Fläschner, et al., Science (2016).

⋮

Periodically-driven systems may host new types of topological phenomena, with no equilibrium analogues

Quantized pumping



Original theory (1D):

Thouless, PRB (1983)

Experiments(1D):

Lohse *et al.*, Nature Physics (2015)

Nakajima *et al.*, Nature Physics (2015)

Extensions to 3D (theory):

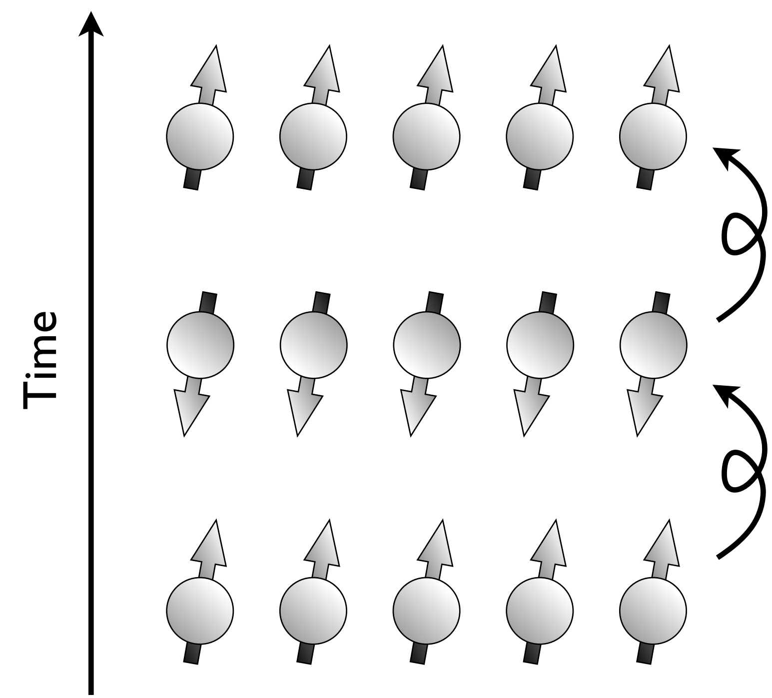
Kitagawa, Berg, MR, Demler, PRB (2010)

Sun, Xiao, Bzdusek, Zhang, Fan, PRL (2018).

Higashikawa, Nakagawa, Ueda, PRL (2019).

⋮

Micromotion phases



Examples in 1D ("time crystals"):

Khemani, Lazarides, Moessner, Sondhi, PRL (2016)

Else, Bauer, Nayak, PRL (2016)

Examples in 2D (quantized magnetization, edge transport):

MR, Lindner, Berg, Levin, PRX (2013)

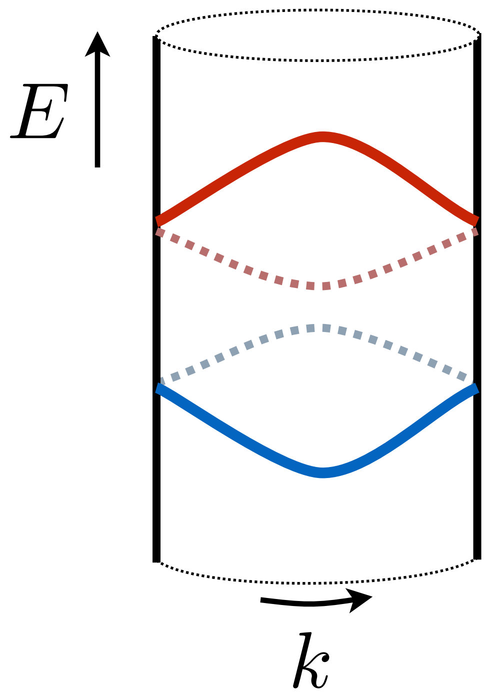
Po, Fidkowski, Morimoto, Potter, Vishwanath, PRX (2016)

Nathan, MR, Lindner, Berg, Refael, PRL (2017)

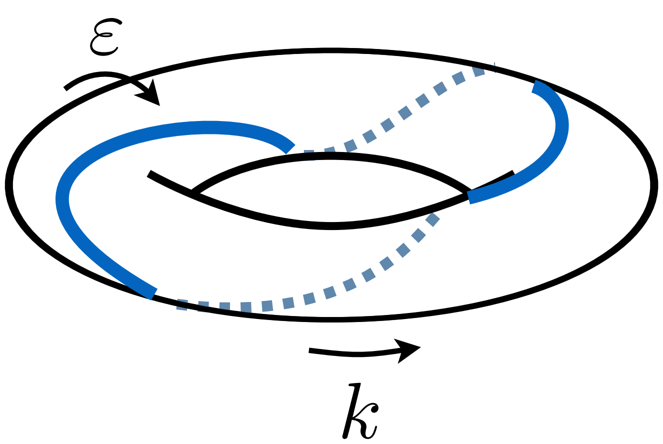
⋮

Floquet bands may "wind" around the quasienergy zone

"Usual" band structure: cylinder



Floquet band structure: torus



A fully-filled chiral Floquet mode carries a *quantized* current

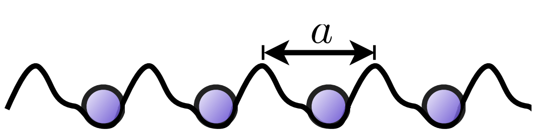
Average group velocity quantized

$$\bar{v}_g = \overline{\frac{d\varepsilon_k}{dk}} = a/T$$

Average current:

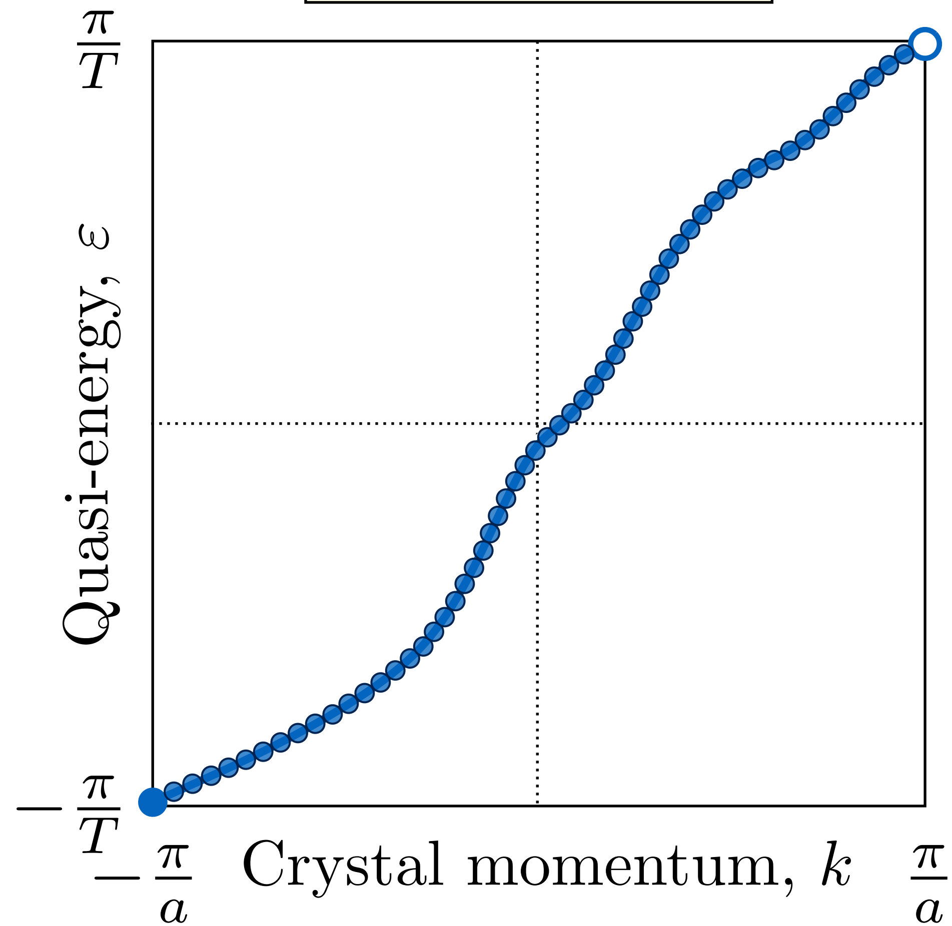
$$I = \rho\, \bar{v}_g = 1/T$$

particle density: $\rho = 1/a$



$$V(x+a) = V(x)$$

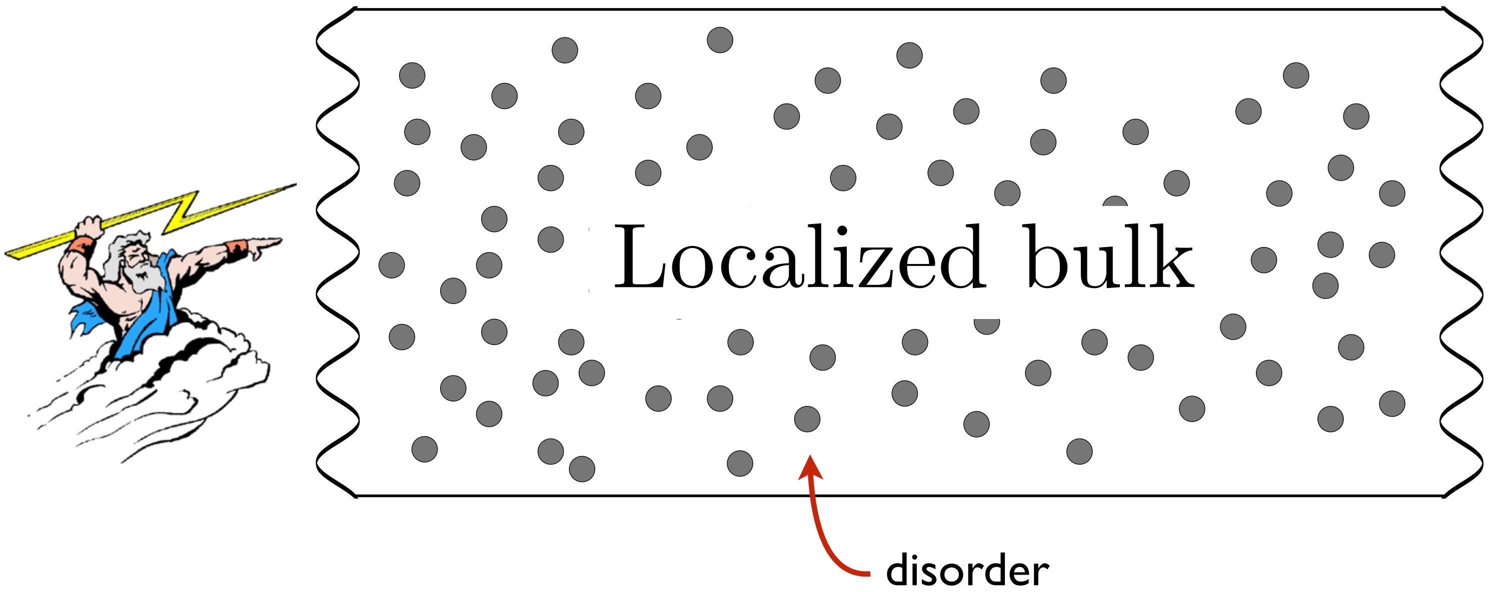
$$U(T) \neq e^{-iH_{\text{eff}}T}$$



Thouless, PRB (1983)

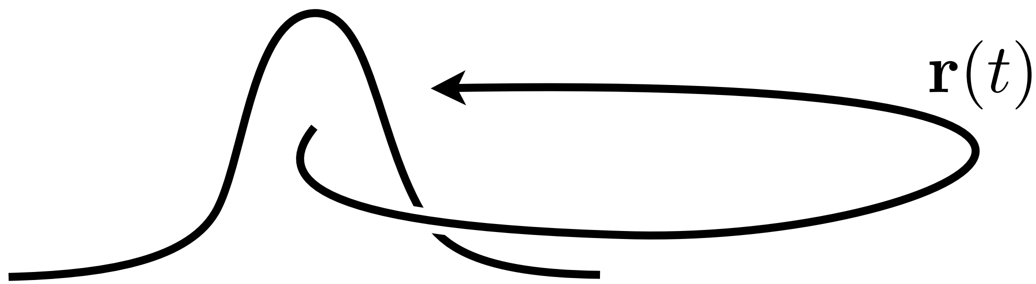
Kitagawa, Berg, MR, Demler, PRB (2010)

Micromotion in 2D most clearly exposed for *localized* states



Magnetization (orbital moment) characterizes micromotion of localized Floquet states

$$M(t) = \frac{1}{2}\mathbf{r}(t) \times \partial_t \mathbf{r}(t)$$



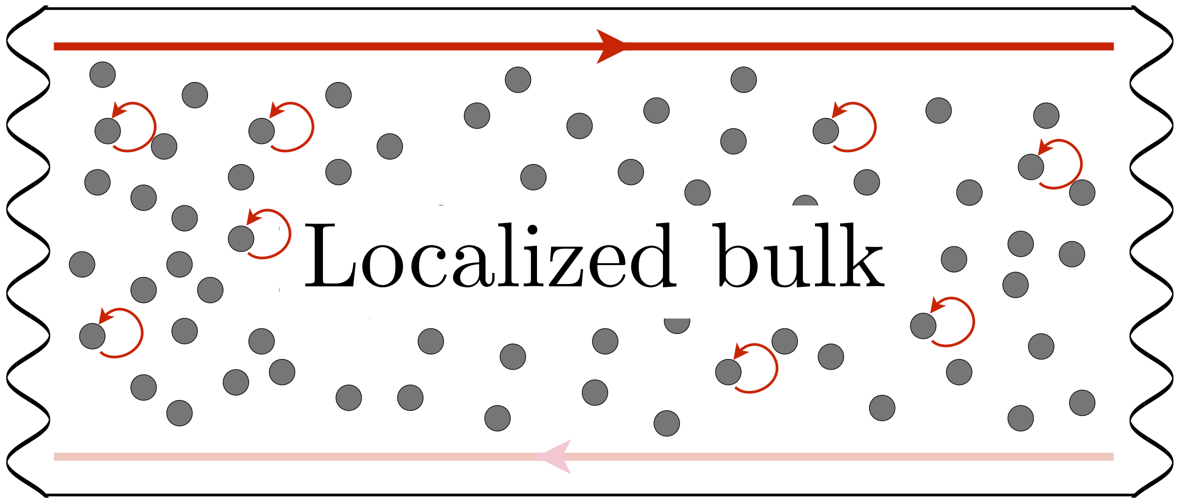
Time-averaged magnetic momentation:

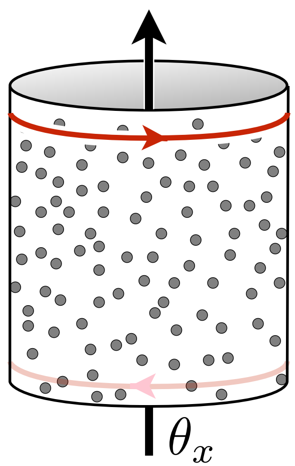
$$\langle M \rangle_T^{(n)} \equiv \frac{1}{T}\int_0^T dt \, \langle \, \psi_n(t) \, | \, M(t) \, | \, \psi_n(t) \, \rangle = -\frac{\partial \varepsilon_n}{\partial B}$$

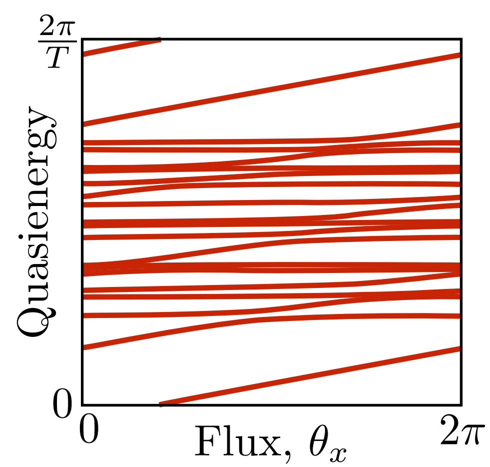
localized Floquet eigenstate

Nathan, MR, Lindner, Berg, and Refael, PRL (2017).

Anomalous Floquet Insulator: disorder fully localizes bulk, chiral edge states persist at all quasienergies







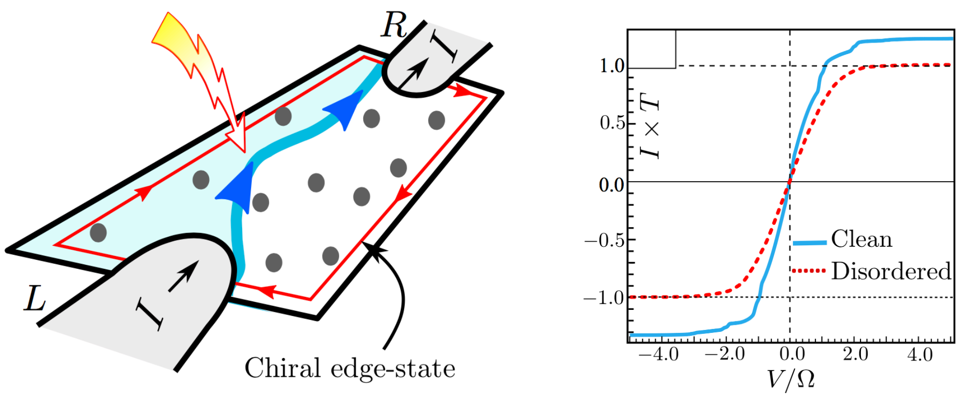
Titum, Berg, MR, Refael, and Lindner, PRX (2016).

Realizations in photonics and cold atoms:

Hu, et al., PRX (2015). Mukherjee, et al., Nature Comm. (2017). Maczewsky, et al., Nature Comm. (2017).
Wintersperger, et al., Nature Physics (2020).

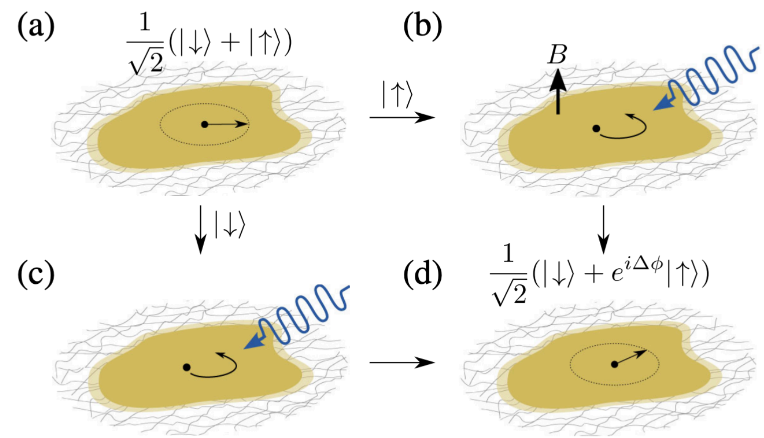
AFAI exhibits robust topological responses

Quantized current at large source-drain bias



Kundu, MR, Berg, and Lindner, PRB (2020).

Quantized magnetization of filled droplets

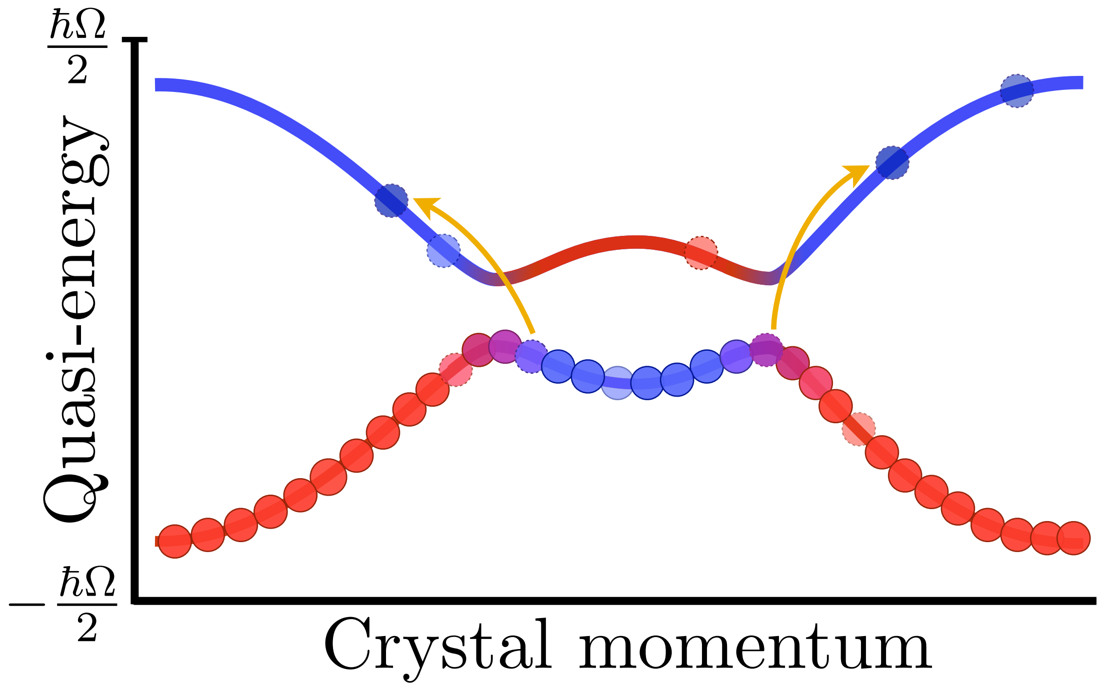


Nathan, MR, Lindner, Berg, and Refael, PRL (2017).

Part II

Prethermalization and universal transport in non-adiabatic quantum pumps

Closed, interacting, periodically driven systems generically absorb energy, tend to high entropy density states



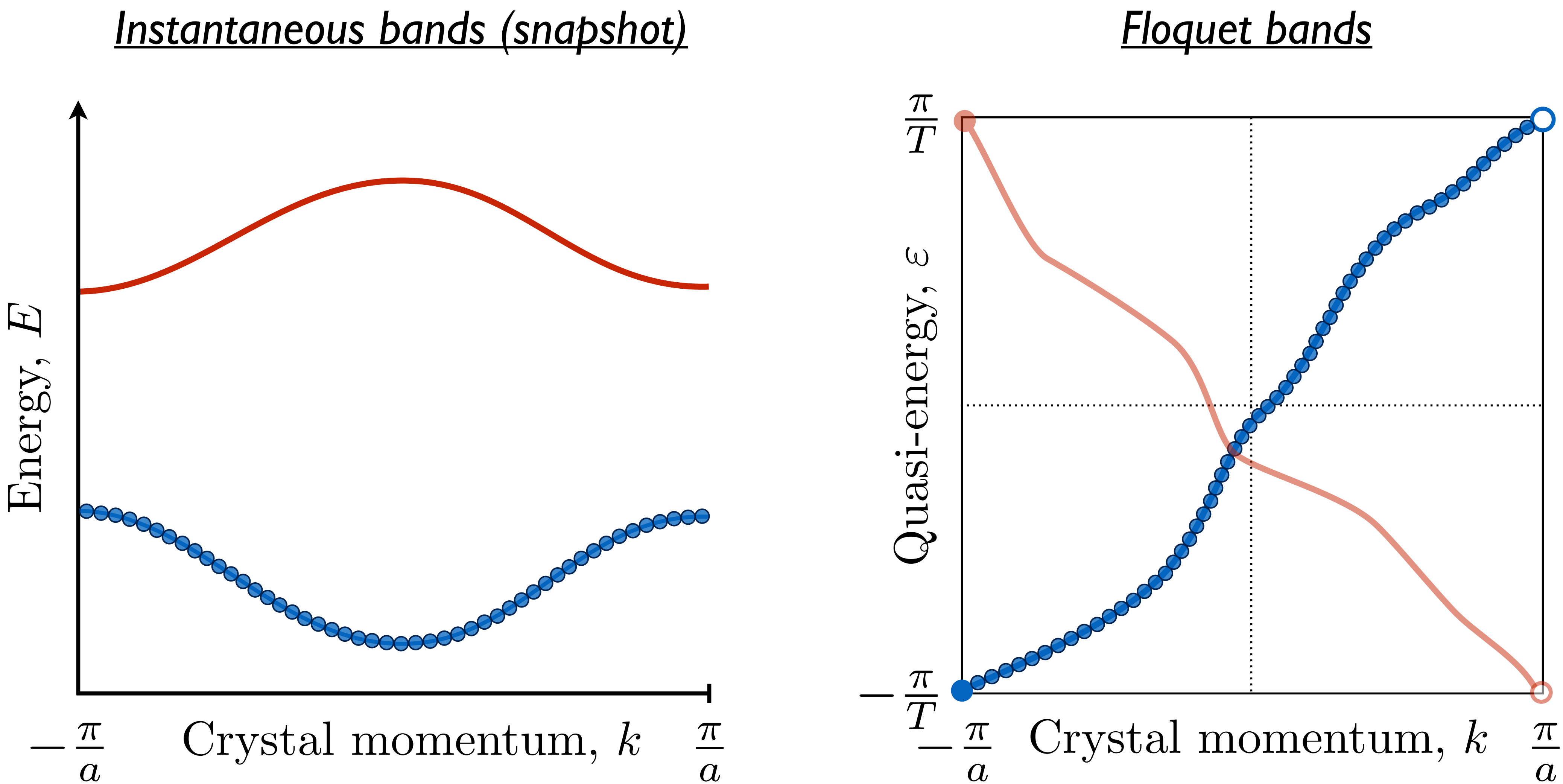
See for example:

D'Alessio and Rigol, PRX (2014).

Lazarides, Das, and Moessner, PRE (2014).

Ponte, Chandran, Papic, and Abanin, Ann. of Phys. (2015).

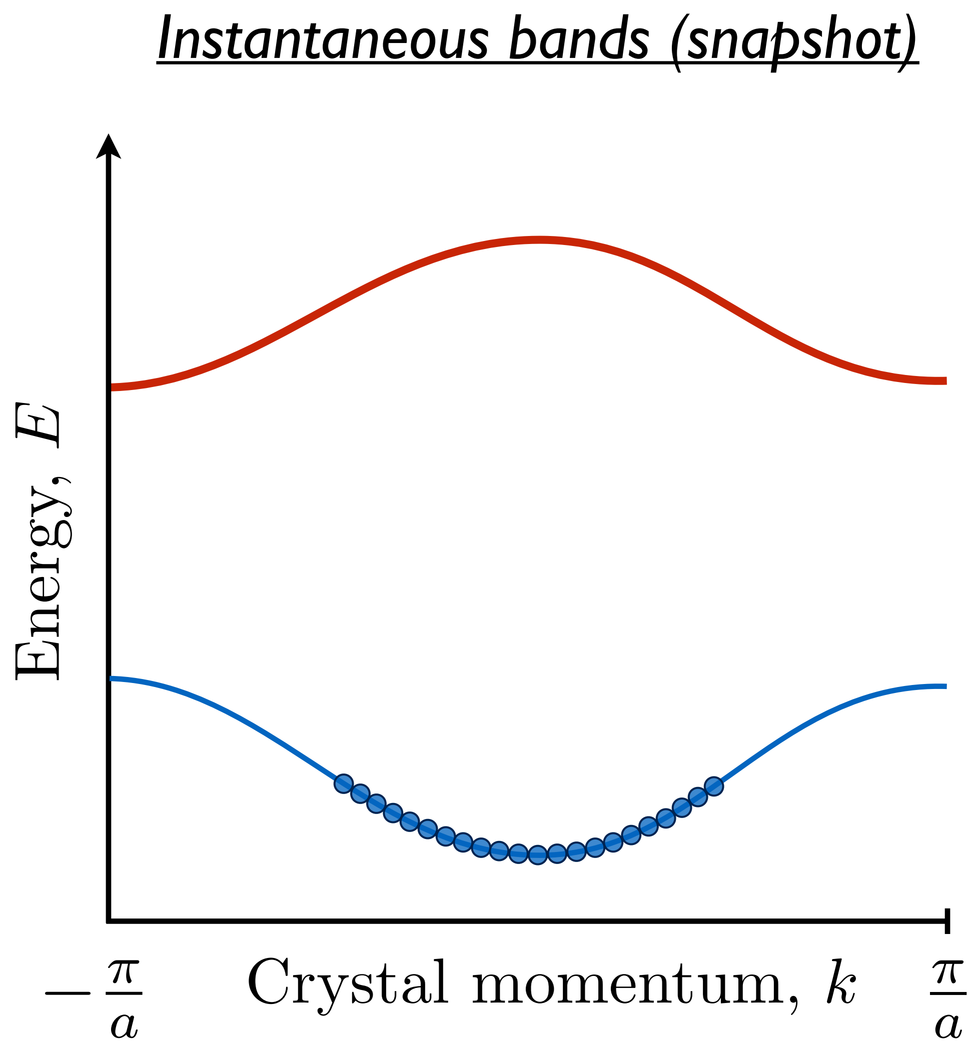
Adiabatically pumped charge is quantized for (band) insulator



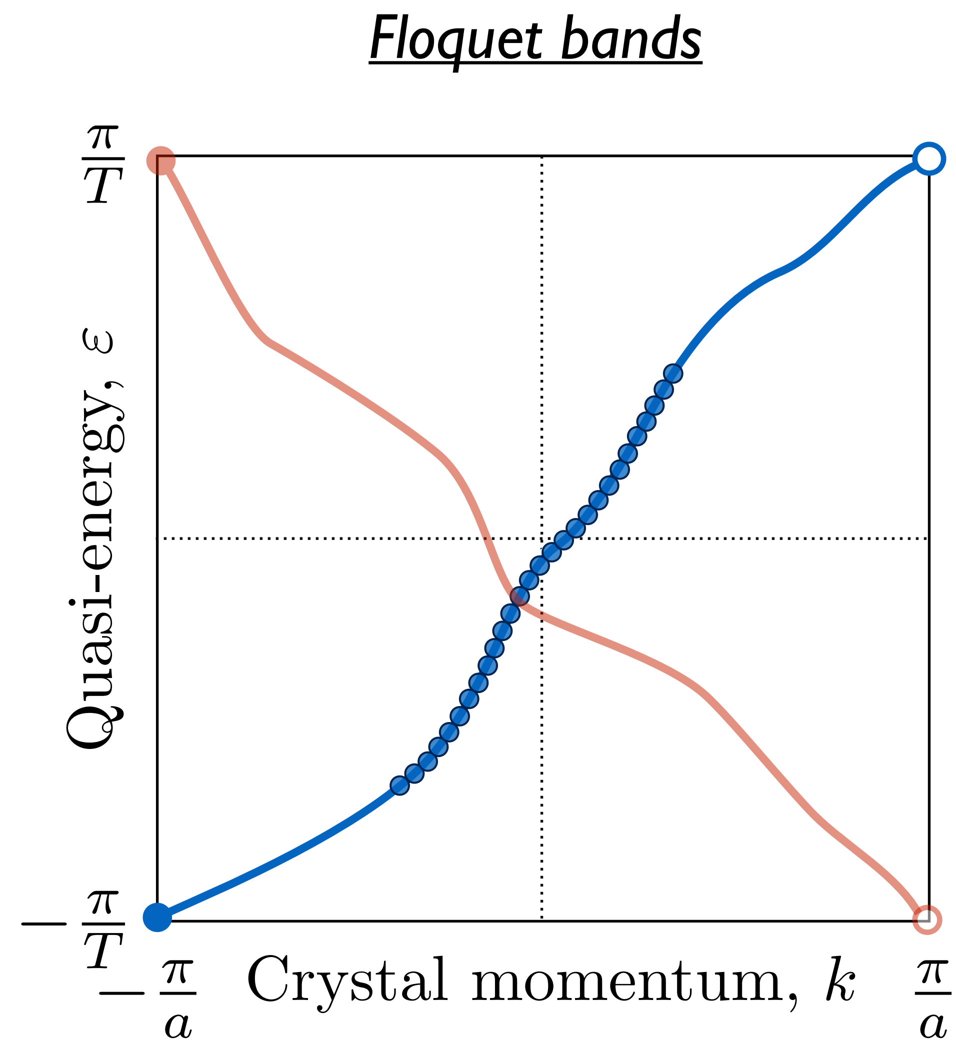
Thouless, PRB (1983)

Kitagawa, Berg, MR, Demler, PRB (2010)

System gapless for partial filling; pumped current non-universal

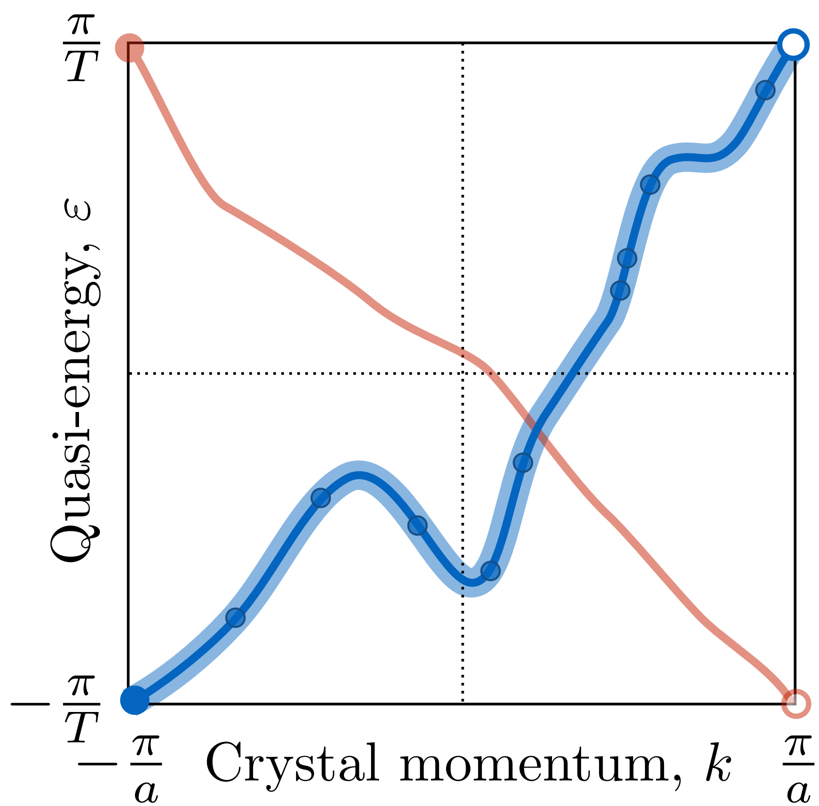


Thouless, PRB (1983)



Kitagawa, Berg, MR, Demler, PRB (2010)

Band-projected infinite temperature state yields uniform averages, restores universality

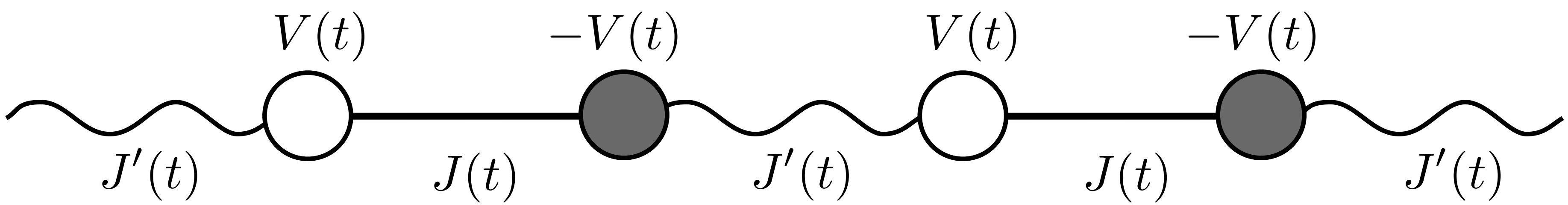


* Universal quantized pumping coefficient (!?)

$$(\text{current}) = (1/T) \times (\text{density})$$

Lindner, Berg, and MR, PRX **7**, 011018 (2017).

Universal transport revealed in modulated two-band model



Modulated hopping and sublattice potential:

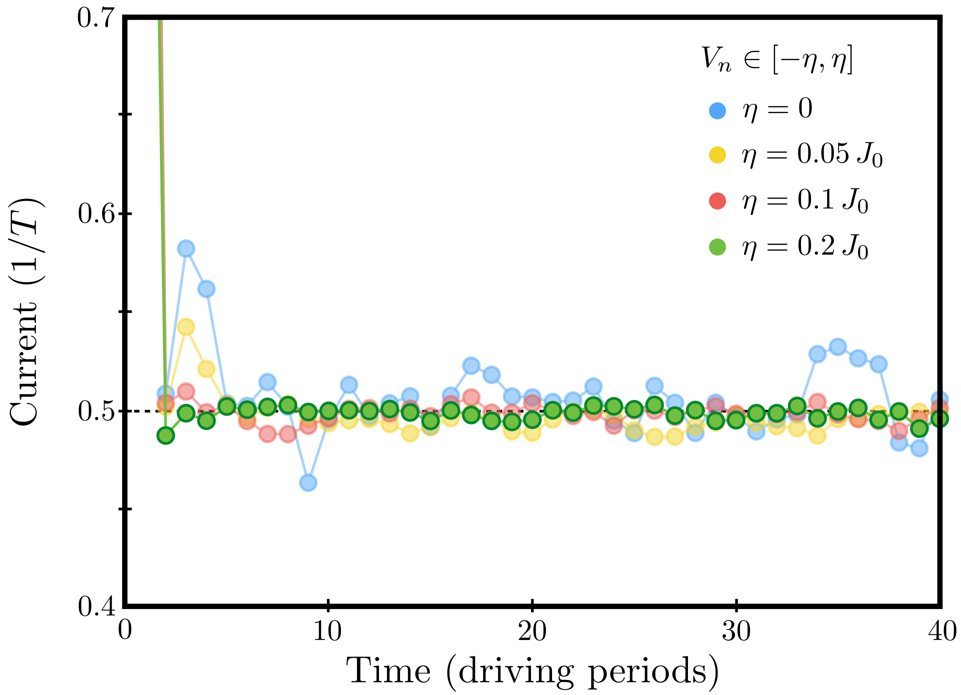
$$J(t) = J_0 + J_1 \cos\omega t \qquad J'(t) = J_0 - J_1 \cos\omega t \qquad V(t) = V_0 + V_1 \sin\omega t$$

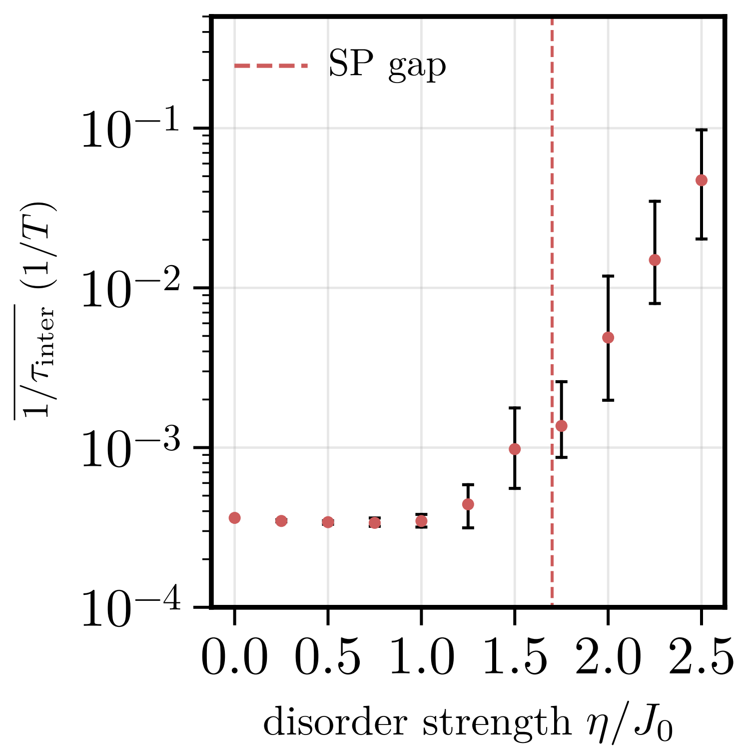
Intra-cell interaction strength: U

Random on-site disorder potential: $\delta V_i \in [-\eta, \eta]$

Simulations show rapid equilibration to quasisteady state with universal current, long lifetime, and robustness to disorder

Numerics: 8 fermionic particles, 32 sites (16 unit cells)





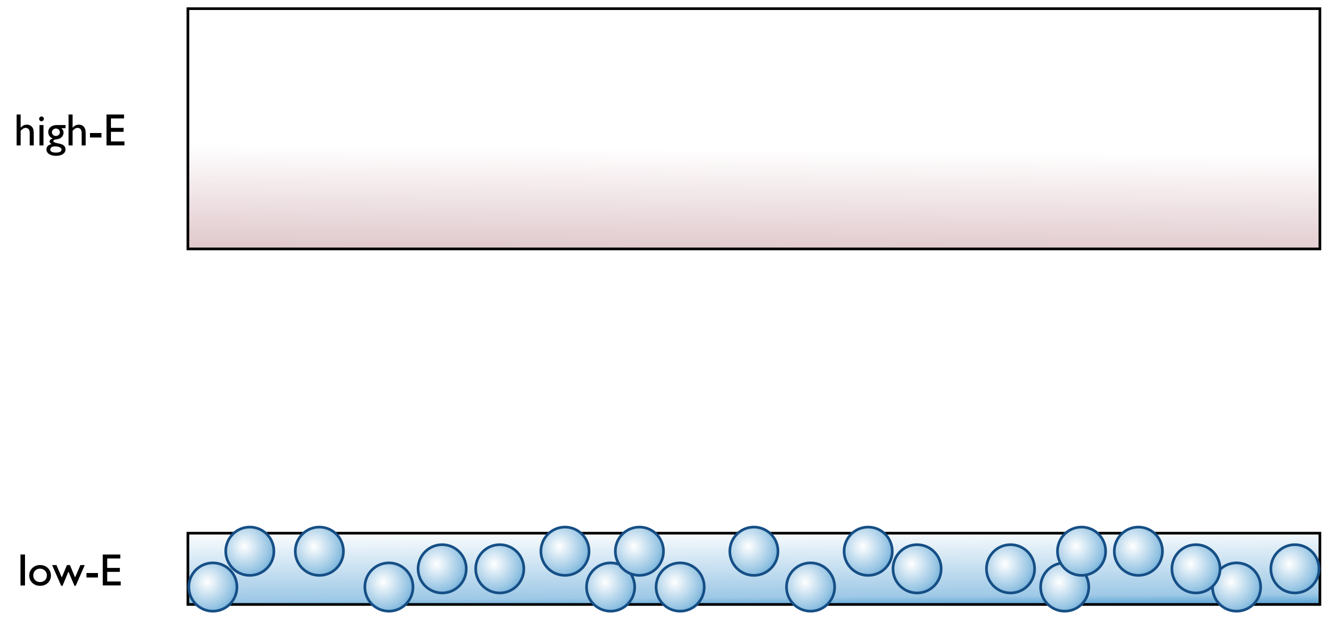
Gawatz, Balram, Lindner, Berg, and MR, arXiv:2103.15831.

General result and open questions (part I)

* Restricted heating of low energy degrees of freedom reveals universal (topological) features of low-energy sector

* How "quantized" is transport in the quasisteady state?

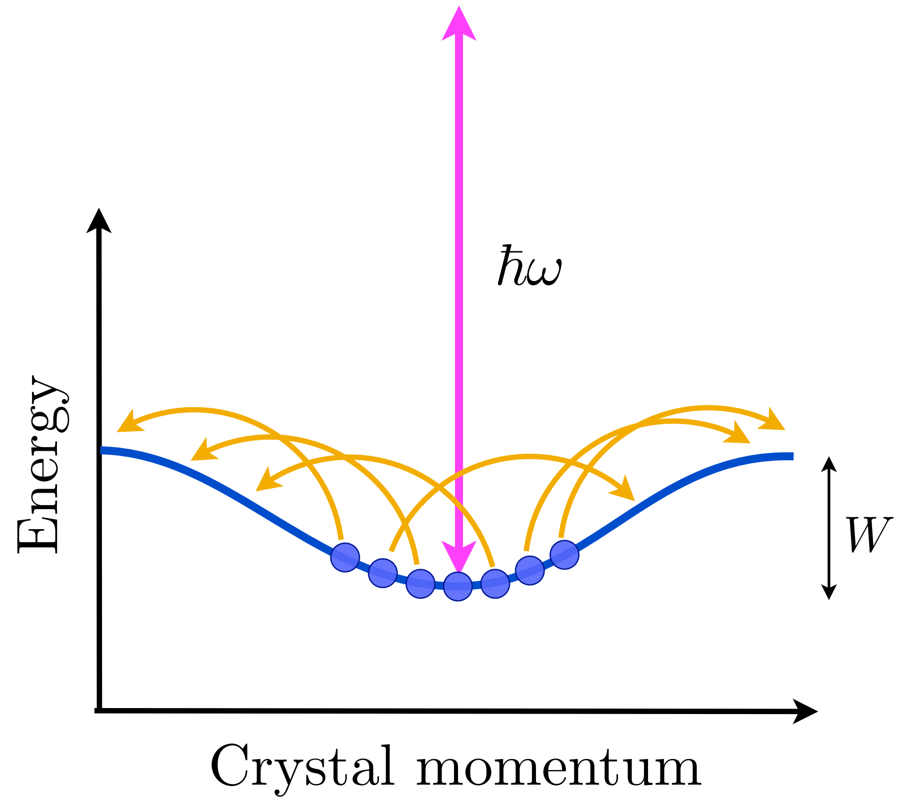
* What is the invariant that captures the topology of the quasisteady state?



Part III

(Correlation-induced) Anomalous Floquet Insulators

Energy absorption exponentially suppressed at high frequency



System *prethermalizes* with respect to a static, effective Hamiltonian

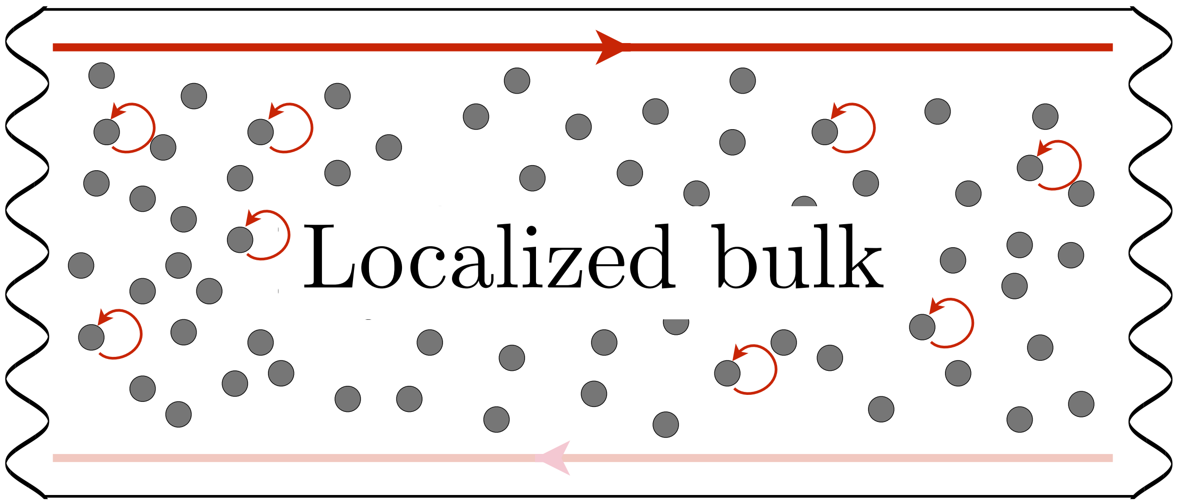
See, for example:

Abanin, De Roeck, and Huveneers, PRL (2015).

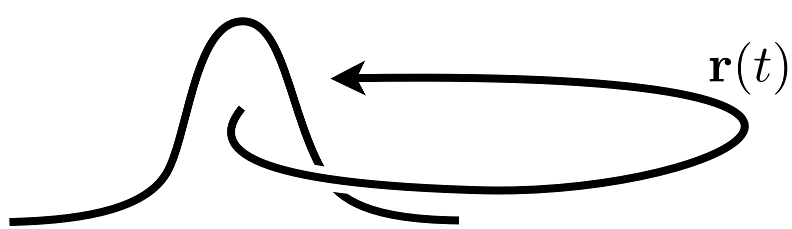
Bukov, D'Alessio, Polkovnikov, Adv. in Phys. (2015).

Abanin, De Roeck, Ho, and Huveneers, PRB (2017).

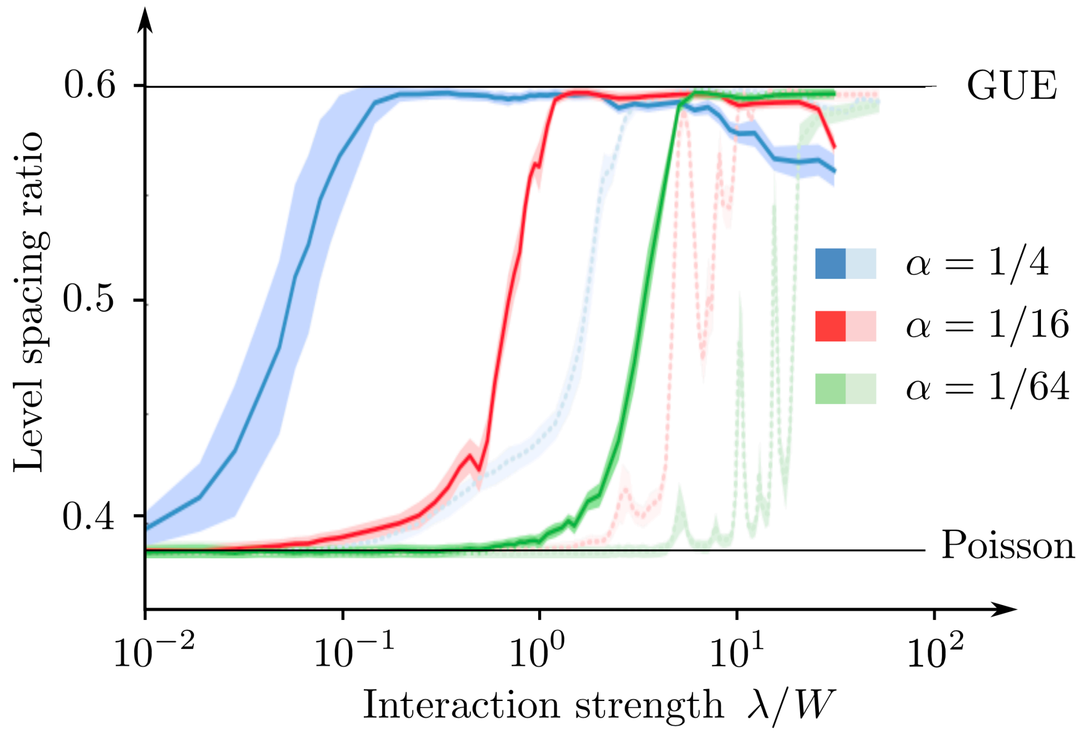
AFI occurs for strong, intermediate-frequency driving



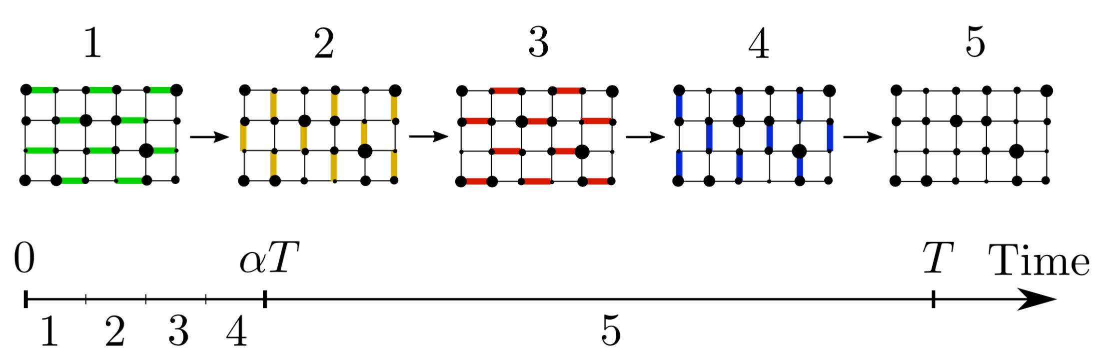
Particles move $|\Delta \mathbf{r}| \sim a$ in time T



AFI consistent with MBL (if it exists in 2D)

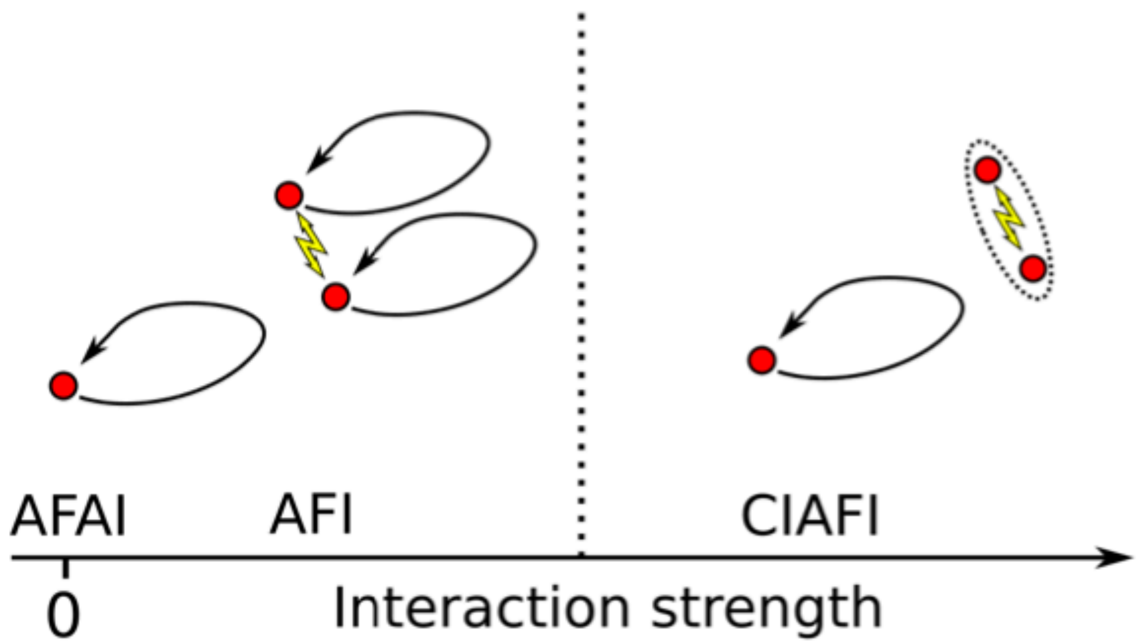


Additional parameter α tunes localization length



Nathan, Abanin, Berg, Lindner, and MR, PRB (2019).

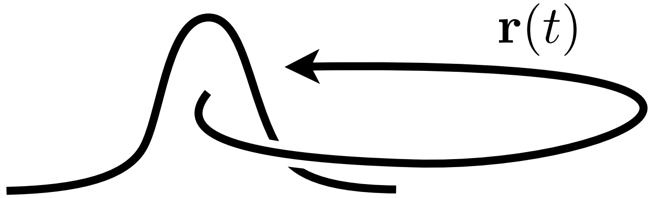
Strong interactions induce transitions to topologically distinct, “correlation-induced anomalous Floquet insulator” phases



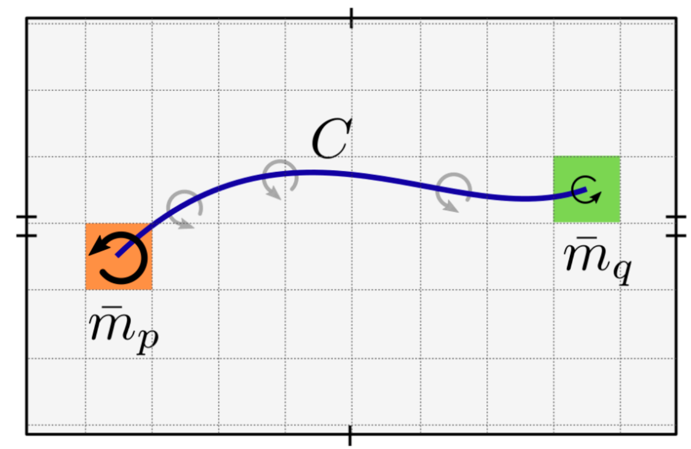
Nathan, Abanin, Lindner, Berg, and MR, SciPost (to appear); arXiv:1907.12228.

Micromotion (circulating currents) in many-body system captured by magnetization

Single particle



Many-body



Orbital moment:

$$M(t) = \frac{1}{2}\mathbf{r}(t) \times \partial_t \mathbf{r}(t)$$

Time-averaged magnetization density at plaquette *p*:

$$\bar{m}_p \equiv \lim_{\tau\to\infty} \frac{1}{\tau}\int_0^\tau dt\, U^\dagger(t)\hat{m}_p(t)U(t)$$

Total time-averaged magnetic moment:

$$\bar{M} = \sum_p \bar{m}_p a^2$$

Novel phases protected by “k-particle localization”

Long time averaged observables admit (truncated) LIOM-like description

$$\bar{m}_p = \sum_{\alpha_1} m^p_{\alpha_1} \hat{n}_{\alpha_1} + \sum_{\alpha_1 \alpha_2} m^p_{\alpha_1 \alpha_2} \hat{n}_{\alpha_1} \hat{n}_{\alpha_2} + \cdots \ (k \text{ terms})$$

Proof of existence of *k*-particle localization for any finite *k:*

Aizenman and Warzel, Comm. Math. Phys. (2009).

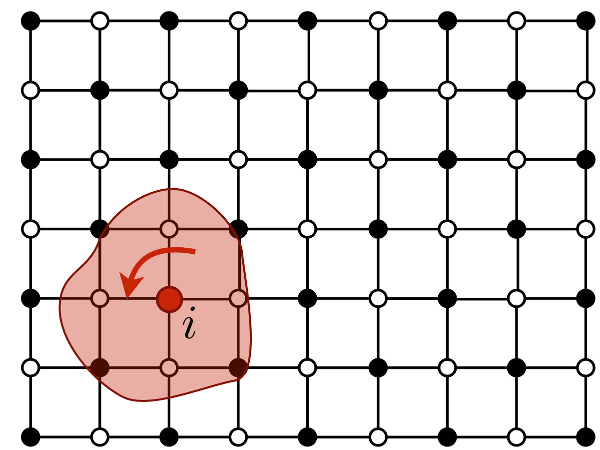
$\mathrm{Tr}_\ell[\bar{M}]$ in ℓ particle subspace is quantized for each $\ell \leq k$

Single particle:

particle on site *i* → system area (Na^2)

$$\sum_i \underbrace{\langle\, i \,|\, \bar{M} \,|\, i \,\rangle}_{\bar{M}_i} = \mu_1 L^2/T$$

integer invariant



Nathan, Abanin, Lindner, Berg, and MR, SciPost (to appear); arXiv:1907.12228.

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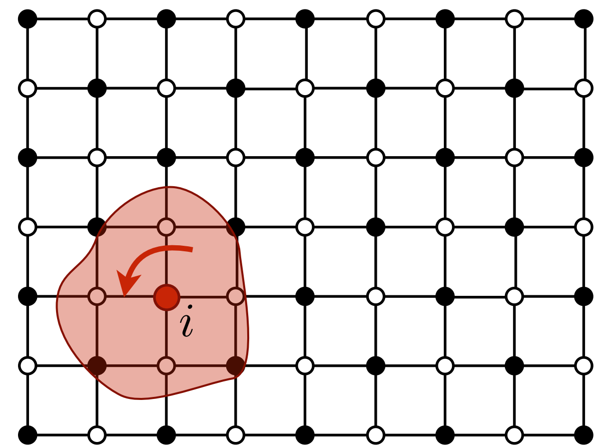


Two particles:

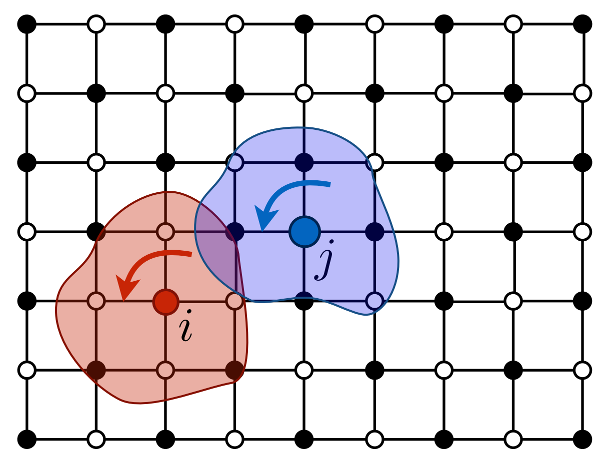
magnetization for particles initialized on sites *i, j*

$$\bar{M}_{ij} = \bar{M}_i + \bar{M}_j + C_{ij}$$

"magnetization cumulant"

$$S_2 \equiv \sum_{i<j} C_{ij} = \mu_2 L^2/T$$

integer invariant

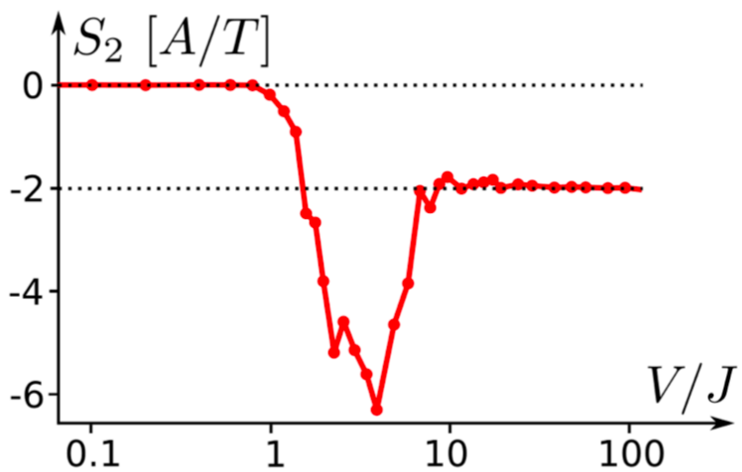


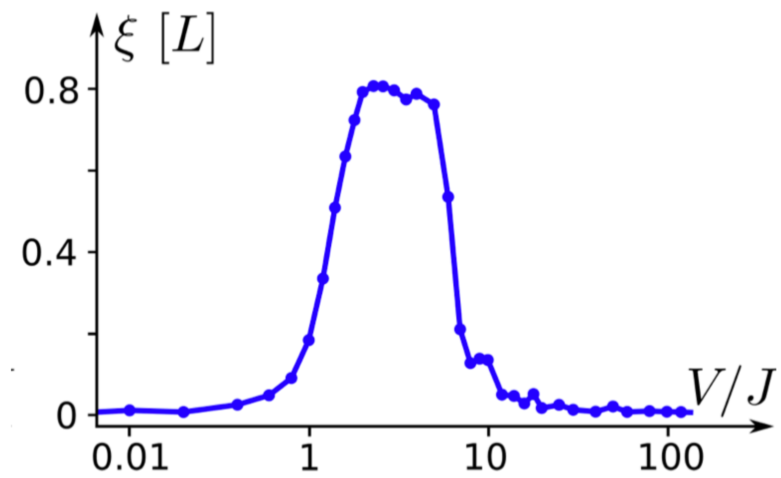
$S_2 \neq 0$: new topological phase without non-interacting limit

Nathan, Abanin, Lindner, Berg, and MR, SciPost (to appear); arXiv:1907.12228.

Transition to correlation-induced AFI for increasing strength of Hubbard interaction

Numerics: average magnetic moments of 4 spin-1/2 particles on 6 x 6 lattice





Nathan, Abanin, Lindner, Berg, and MR, SciPost (to appear); arXiv:1907.12228.

Summary and open questions

Periodic driving brings new features in topology, many-body dynamics

Micromotion is crucial in the "new" phenomena of Floquet systems

Nontrivial topology of AFI manifested in quantized current at large bias

AFI compatible with MBL; interactions bring new correlation-induced AFI phases

For a recent review and technical guide, see:

MR and N. H. Lindner, arXiv:1909.02008; *to appear in Nature Reviews Physics* (2020).

MR and N. H. Lindner, arXiv:2003.08252; *"The Floquet engineer's handbook."*

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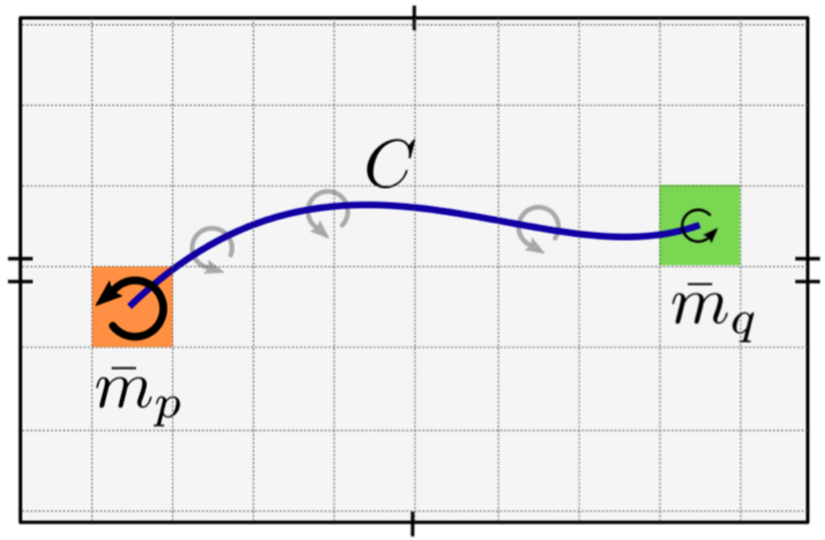
Support for this work provided by:

Trace of the magnetization density is a topological invariant



$$\bar{m}_p \equiv \lim_{\tau\to\infty} \frac{1}{\tau}\int_0^\tau dt\, U^\dagger(t)\hat{m}_p(t)U(t)$$

$$\bar{I}_C = \bar{m}_p - \bar{m}_q$$

current across cut C

$$\mathrm{Tr}_\ell[\bar{I}_C] = \mathrm{Tr}_\ell[\bar{m}_p] - \mathrm{Tr}_\ell[\bar{m}_q]$$

$$\Rightarrow \mathrm{Tr}_\ell[\bar{m}_p] = \mathrm{Tr}_\ell[\bar{m}_q]$$

***All plaquettes must have the same magnetization density, for any disorder realization that preserves localization

Long-time average of operator is diagonal in terms of LIOMs

$$U(T) = e^{-iT\left(\sum_\alpha \varepsilon^{(1)}_\alpha n_\alpha + \frac{1}{2}\sum_{\alpha,\beta}\varepsilon^{(2)}_{\alpha\beta} n_\alpha n_\beta + \cdots\right)}$$

$$\bar{m}_p = \lim_{\tau\to\infty}\frac{1}{\tau}\int_0^\tau dt\; U^\dagger(t) m_p(t) U(t)$$

$$\bar{m}_p = \sum_\alpha \hat{n}_{\alpha_1}\frac{\partial\varepsilon_{\alpha_1}}{\partial\phi_p} + \frac{1}{2}\sum_{\alpha_1\alpha_2}\hat{n}_{\alpha_1}\hat{n}_{\alpha_2}\frac{\partial\varepsilon_{\alpha_1\alpha_2}}{\partial\phi_p} + \ldots$$

quantized:
see single particle sector

also quantized:
additional "many-body"
invariants

ϕ_p : flux through plaquette p