## Quantum phases of matter with Rydberg atoms

Interacting Topological Matter: Atomic, Molecular, and Optical Systems Kavli Institute for Theoretical Physics University of California, Santa Barbara
July 6,202I

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$\square \mathrm{V} \mid \mathrm{S}\{\mathrm{R}$
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Fig: https://www.caltech.edu/about/ news/quantum-innovations-achieved-using-alkaline-earth-atoms

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H_{\mathrm{Ryd}}=\sum_{i}\left[\frac{\Omega}{2}(|g\rangle\langle r|+|r\rangle\langle g|)_{i}-\Delta|r\rangle\langle r|\right]+\sum_{(i, j)} V_{|i-j|}\left(|r\rangle\left\langle\left. r\right|_{i} \otimes \mid r\right\rangle\left\langle\left. r\right|_{j}\right)\right.
$$

I. Rydberg chains

## The $Z_{3}$ chiral clock transition

2. Square lattice

Quantum Ising criticality in 2+I dimensions
3. Kagome symmetry lattices

Probing topological spin liquids
4. Theory of odd and even $Z_{2}$ spin liquids

## QPTs in a Rydberg quantum simulator



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\begin{aligned}
H_{\mathrm{Ryd}}= & \sum_{i=1}^{N} \frac{\Omega}{2}(|g\rangle\langle r|+|r\rangle\langle g|)_{i}-\Delta \sum_{i=1}^{N}|r\rangle\left\langle\left. r\right|_{i}\right. \\
& +\sum_{i<j} V_{|i-j|}\left(|r\rangle\left\langle\left. r\right|_{i} \otimes \mid r\right\rangle\left\langle\left. r\right|_{j}\right)\right.
\end{aligned}
$$

$$
V_{|i-j|} \sim \frac{1}{\left|r_{i}-r_{j}\right|^{6}}
$$

## QPTs in a Rydberg quantum simulator

Universal critical dynamics: quantum Kibble-Zurek mechanism

Tune through transition at rate $v$ :

$$
\Delta(t)=\Delta_{c}+v t
$$



Correlation length saturates!

$$
\xi \sim v^{-\nu /(1+\nu z)}
$$

Experimental probe of critical exponents

Quantum Kibble-Zurek mechanism and critical dynamics on a programmable Rydberg simulator

Alexander Keesling, Ahmed Omran, Harry Levine, Hannes Bernien, Hannes Pichler, Soonwon Choi, Rhine Samajdar, Sylvain Schwartz, Pietro Silvi, Subir Sachdev, Peter Zoller, Manuel Endres, Markus Greiner,Vladan Vuletic, and Mikhail D. Lukin, Nature 568, 207 (2019)

## Competing density-wave orders in a one-dimensional hard-boson model

## PHYSICAL REVIEW B 69, 075106 (2004) P. Fendley, K. Sengupta, S. Sachdev

$$
n_{j} \equiv b_{j}^{\dagger} b_{j}
$$

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\mathcal{H}=\sum_{j}\left[\frac{\Omega}{2}\left(b_{j}+b_{j}^{\dagger}\right)-\Delta n_{j}\right]+\sum_{i<j} V_{|i-j|} n_{i} n_{j}
$$

$$
V_{1}=\infty, \quad V_{2}=V, \quad V_{i>2}=0
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The $V=0$ case is the 'PXP' model, originally introduced in
S. Sachdev, K. Sengupta, and S.M. Girvin, PRB 66, 075128 (2002)

These models were motivated by 'tilted lattices' of bosonic atoms, and the $\mathbb{Z}_{2}$ quantum transition was observed in
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integrable
----- $\quad$ Ising first order

-     -         -             - Ising second order
chiral transition incommensurate region


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V_{1}= & \infty, \quad V_{2}=V, \quad V_{i>2}=0 \\
w= & \Omega / 2, \quad U=-\Delta
\end{aligned}
$$



Floating Phase versus Chiral Transition in a 1D Hard-Boson Model PHYSICAL REVIEW LETTERS 122, 017205 (2019) N. Chepiga and F. Mila

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\text { IC: } \quad \begin{aligned}
& \text { Haldane, Bak, Bohr, PRB 28, } \\
& \text { Schulz, PRB 28, } 2746(1983)
\end{aligned}
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\text { Von Gehlen and Rittenberg, Nucl. Phys. B, } 473 \text { (1984) }
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Quantum field theory for the chiral clock transition in one spatial dimension PHYSICAL REVIEW B 98, 205118 (2018)
S.Whitsitt, R. Samajdar, and S. Sachdev
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What is the critical field theory? First try: write the most general theory for order parameter with appropriate symmetries.

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\begin{aligned}
\Phi \rightarrow e^{2 \pi i / N} \Phi & \Phi(x, \tau) \rightarrow \Phi^{*}(-x, \tau) \\
\mathcal{S}_{\Phi}= & \int d x d \tau\left[\left|\partial_{\tau} \Phi\right|^{2}+\left|\partial_{x} \Phi\right|^{2}+i \alpha_{x} \Phi^{*} \partial_{x} \Phi\right. \\
& \left.+s_{\Phi}|\Phi|^{2}+u|\Phi|^{4}+\lambda\left(\Phi^{N}+\left(\Phi^{*}\right)^{N}\right)\right]
\end{aligned}
$$

In perturbation theory, the field condenses at nonzero momentum.

$$
\mathcal{S}_{\Phi}=\int \frac{d \omega d k}{(2 \pi)^{2}} \Phi^{*}\left[\omega^{2}+k^{2}-\alpha_{x} k+s\right] \Phi+\cdots
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In perturbation theory, the field condenses at nonzero momentum.

Can only describe transition to incommensurate phase.

$$
\langle\Phi(x) \Phi(0)\rangle \sim e^{i k_{0} x}
$$

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\qquad \begin{array}{c}
\mathbb{Z}_{N} \begin{array}{l}
\text { density wave } \\
\langle\Phi\rangle \neq 0 \\
\text { Gapped }
\end{array} \\
\hline \begin{array}{c}
\text { Incommensurate } \\
\text { density wave } \\
\langle\Phi\rangle \sim e^{i k_{I} x} \\
\text { Gapless }
\end{array} \\
\hline \begin{array}{l}
\langle\Phi\rangle=0 \\
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\hline
\end{array}
\end{gathered}
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Quantum field theory for the chiral clock transition in one spatial dimension PHYSICAL REVIEW B 98, 205118 (2018)
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Field theory for $\mathbb{Z}_{N}$ density wave ordering

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\end{aligned}
$$

is Kramers-Wannier dual to

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field theory for Bose condensation in the presence of a background $N$ boson condensate

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field theory for Bose condensation in the presence of a background $N$ boson condensate
Note: this is not a Wick rotationthere is a crucial difference in the factor of $i!!$

# Quantum field theory for the chiral clock transition in one spatial dimension 

PHYSICAL REVIEW B 98, 205118 (2018)<br>S.Whitsitt, R. Samajdar, and S. Sachdev



We performed a renormalization group analysis for the Bose gas transition in a expansion in $2-d$, with $4-N$ chosen to be order $2-d$. This led a strongly-coupled critical point with $z \neq 1$.

## Quantum field theory for the chiral clock transition in one spatial dimension

|  |  | $\mathcal{S}_{\Phi}, d>1$ |
| :---: | :---: | :---: |
| $\mathbb{Z}_{N}$ density wave <br> $\langle\Phi\rangle \neq 0$ <br> Gapped | Incommensurate density wave $\langle\Phi\rangle \sim e^{i k_{I} x}$ Gapless | $\langle\Phi\rangle=0$ <br> Gapped |

$$
\mathcal{S}_{\Psi}, d>1
$$



We performed a renormalization group analysis for the Bose gas transition in a
expansion in $2-d$, with $4-N$ chosen to be order $2-d$. This led a strongly-coupled critical point with $z \neq 1$.
I. Rydberg chains

The $Z_{3}$ chiral clock transition

## 2. Square lattice

Quantum Ising criticality in 2+1 dimensions
3. Kagome symmetry lattices

Probing topological spin liquids
4. Theory of odd and even $Z_{2}$ spin liquids


Fig: https://www.caltech.edu/about/ news/quantum-innovations-achieved-using-alkaline-earth-atoms

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$$

Rydberg atoms on the square lattice: theory


Rydberg atoms on the square lattice: experiment

$$
\tilde{\xi}=\xi\left(s / s_{0}\right)^{\mu}
$$



Star
$\tilde{\mathcal{F}}(\pi, \pi / 2)$



Quantum Phases of Matter on a 256-Atom Programmable Quantum Simulator, Sepehr Ebadi, Tout T.Wang, Harry Levine, Alexander Keesling, Giulia Semeghini, Ahmed Omran, Dolev Bluvstein, Rhine Samajdar, Hannes Pichler, Wen Wei Ho, Soonwon Choi, Subir Sachdev, Markus Greiner,Vladan Vuletic, and Mikhail D. Lukin, Nature to appear, arXiv:20I2.I228।; Pascal Scholl et al. arXiv:20I2.I2268

$$
\tilde{\Delta}=\left(\Delta-\Delta_{c}\right)\left(s / s_{0}\right)^{\kappa}
$$






First observation of Ising quantum phase transition in 2+1 dimensions
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Rydberg atoms on site-kagome lattice: theory


R. Samajdar, Wen Wei Ho, H. Pichler, M. D. Lukin, and
S. Sachdev, PNAS I I 8, e20I5785II8 (202I)
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## Probing topological spin liquids

4. Theory of odd and even $Z_{2}$ spin liquids

Mott insulator: Triangular lattice antiferromagnet

$$
H=J \sum_{\langle i j\rangle} \vec{S}_{i} \cdot \vec{S}_{j}
$$



Nearest-neighbor model has non-collinear Neel order

Mapping of booms and spins

Boon $B$

$$
B^{+} B \leq 1
$$

State $|0\rangle, B^{+}|0\rangle$ Operators


States $|\uparrow\rangle|\downarrow\rangle$ Operators

$$
\begin{aligned}
& \text { Operator } \\
& S_{+}=S_{x}+i S_{y}
\end{aligned}
$$

$$
\rightarrow S_{-}=S_{x}-i S_{y}
$$

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Nearest-neighbor model has non-collinear Neel order

## Mott insulator: Triangular lattice antiferromagnet

Spin liquid for bosons at half-filling, or a spin model with $S=1 / 2$ per unit cell
$\left.=\frac{1}{\sqrt{2}}(\uparrow \downarrow\rangle-|\downarrow \uparrow\rangle\right)$

$$
|G\rangle=\sum_{\mathcal{D}} c_{\mathcal{D}}|\mathcal{D}\rangle
$$

$\mathcal{D} \rightarrow$ dimer covering of lattice
P. Fazekas and P. W. Anderson, Philos. Mag. 30, 23 (1974).

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## Mott insulator: Triangular lattice antiferromagnet



## Excitations of the $Z_{2}$ Spin liquid

## Spinon: $\mathrm{S}_{\mathrm{z}}=1 / 2$

$e$ (boson) or $\epsilon$ (fermion) particle
$\left.=\frac{1}{\sqrt{2}}(\uparrow \downarrow\rangle-|\downarrow \uparrow\rangle\right)$


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$B_{2}^{+} \rightarrow \theta_{1}$

$$
=\frac{1}{\sqrt{2}}|\uparrow\rangle_{1}|\uparrow\rangle_{2}
$$

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- Spinons can only be created in pairs by a local operator (e.g. $B^{\dagger}$ )

A single spinon carries boson number $B^{\dagger} B=1 / 2$ : fractionalization!

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## Excitations of the $Z_{2}$ Spin liquid

## A vison

$m$ (boson) particle

$$
=\frac{1}{\sqrt{2}}(|\uparrow \downarrow\rangle-|\downarrow \uparrow\rangle)
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## Excitations of the $Z_{2}$ Spin liquid

- A spinon adiabatically transported around a vison picks up a phase factor of -1 : spinons and visons are mutual semions.
- A bound state of a spinon and a vison picks up a phase factor of -1 when exchanged with another bound state of a spinon and a vison:
- The $\epsilon$ spinon (fermion) is a bound state of the $e$ spinon (boson) and a vison $(\epsilon=e \times m)$.
- The The $e$ spinon (boson) is a bound state of the $\epsilon$ spinon (fermion) and a vison $(e=\epsilon \times m)$.

Ground state degeneracy on the torus


Ground state degeneracy on the torus

| Place |
| :---: |
| insulator |
| on a torus: |
| Obtain a |
| degenerate |
| orthogonal state |
| by modifying the |
| wavefunction on |
| a "branch-cut" |
| encircling the |
| torus. |

Ground state degeneracy on the torus
$\bigcirc=(|\uparrow \downarrow\rangle-|\downarrow \uparrow\rangle) / \sqrt{2}$


## Place

 insulator on a torus:Number of dimers crossing "branch-cut" is conserved modulo 2: there are nearly degenerate states with odd and even dimer-cuts
D.J.Thouless, PRB 36, 7187 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, Europhys. Lett. 6, 353 (I988)

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Number of dimers crossing "branch-cut" is conserved modulo 2: there are nearly degenerate states with odd and even dimer-cuts
D.J.Thouless, PRB 36, 7187 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, Europhys. Lett. 6, 353 (I988)

## Ground state degeneracy on the torus

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Topological $Z$ operator of the $Z_{2}$ Spin liquid

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\left.=\frac{1}{\sqrt{2}}(\uparrow \downarrow\rangle-|\downarrow \uparrow\rangle\right)
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## Simplest example with time-reversal symmetry: " $Z_{2}$ spin liquid" or "toric code"

- Anyons: $\mathbb{1}, e, m, \epsilon$. The $e, m, \epsilon$ anyons cannot be created from the ground state ( $\mathbb{1}$ ) by any local operator.
- The $e$ and $\epsilon$ are spinons, the $m$ is the 'vison'.
- Self statistics: $e$ and $m$ are bosons, while $\epsilon$ is a fermion.
- Mutual statistics: Any pair of $e, m, \epsilon$ are mutual semions i.e. one anyon picks up a ( -1 ) upon encircling any other type of anyon.
- Fusion rules: $e \times m=\epsilon, e \times \epsilon=m, m \times \epsilon=e, e \times e=\epsilon \times \epsilon=m \times m=\mathbb{1}$.
- 4-fold ground state degeneracy on a torus.
- Emergent, deconfined $\mathbb{Z}_{2}$ gauge field.
- No protected edge states in general, but could appear with special symmetries.
- Topological entanglement entropy $=\ln 2$.

The $\mathbb{Z}_{2}$ spin liquid was obtained in N. Read and S. Sachdev, Phys. Rev. Lett. 66, 1773 (1991) and X.-G. Wen, Phys. Rev. B 44, 2664 (1991). A. Kitaev, arXiv:quant-ph/9707021 described the toric code.

## Odd and even $Z_{2}$ spin liquids

## Berry phase of vison motion



(d)

To return to the initial state, we need a gauge transformation factor of -1 for each dimer ending on the red circle: this yields a factor $e^{i \pi S}$, because there are $2 S$ dimers on each site.

[^0]
## Odd and even $Z_{2}$ spin liquids

- The spinons carry spin $S_{z}=1 / 2$ boson number $B^{\dagger} B=1 / 2$.
- $\mathbb{Z}_{2}$ spin liquids of bosons (more generally, in systems with a global $U(1)$ symmetry) must obey contraints associated with a 'tHooft anomaly' which is determined by the boson filling $n$.
- On a square lattice, the single vison state exhibit 'translational symmetry fractionalization' with

$$
T_{x} T_{y}=T_{y} T_{x} e^{2 \pi i n}
$$

with $n$ integer or half-integer.

- For antiferromagnets of spin $S$, the translational symmetry fractionalization is

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- More generally, any $\mathbb{Z}_{2}$ spin liquid, even without a conserved $\mathrm{U}(1)$, can exhibit symmetry fractionalization, with $T_{x} T_{y}=T_{y} T_{x}$ for an even $\mathbb{Z}_{2}$ spin liquid, and $T_{x} T_{y}=-T_{y} T_{x}$ for an odd $\mathbb{Z}_{2}$ spin liquid on the square lattice (generalizes to other lattices)
I. Rydberg chains

The $Z_{3}$ chiral clock transition
2. Square lattice

Quantum Ising criticality in 2+1 dimensions
3. Kagome symmetry lattices

Probing topological spin liquids
4. Theory of odd and even $Z_{2}$ spin liquids

R. Samajdar, Wen Wei Ho, H. Pichler, M. D. Lukin, and
S. Sachdev, PNAS I I 8, e20I5785II8 (202I)

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Rydberg atoms on site-kagome lattice: theory



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The number of dimers is not conserved, and so the $\mathbb{Z}_{2}$ gauge theory has finite gap matter fields.

Rydberg atoms on site-kagome lattice: theory
R. Moessner, S. L. Sondhi, Phys. Rev.Lett. 86, 1881

K. Roychowdhury,
S. Bhattacharjee, F. Pollmann, Phys. Rev.B 92, 075141 (2015).

$$
[\delta / \Omega=3.30]
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## Solid phase of the odd quantum dimer model on the triangular lattice


‘Hard boson’ of Fendley, Sengupta, Sachdev $\Rightarrow$ 'Dimer' on triangular lattice!
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Rydberg atoms on link-kagome lattice: theory

$$
H=\frac{\Omega}{2} \sum_{i} P \sigma_{i}^{x} P-\delta \sum_{i} n_{i}
$$

we put the model on an infinitely-long cylinder

$\rightarrow$ use density matrix renormalization group (DMRG)
'Hard boson' of
(White '92, Stoudenmire '13, Hauschild `18) Fendley, Sengupta, Sachdev $\Rightarrow$ 'Dimer’ on kagome lattice!


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Quantum liquid phase of the odd quantum dimer model on the
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## Probing Topological Spin Liquids on a Programmable Quantum Simulator

G. Semeghini, H. Levine, A. Keesling, S. Ebadi, T.T.Wang, D. Bluvstein, R. Verresen, H. Pichler, M. Kalinowski, R. Samajdar, A. Omran, S. Sachdev, A.Vishwanath, M. Greiner,V.Vuletic, M. D. Lukin, arXiv:2I04.04II9

Rydberg atoms on the link-kagome lattice: experiment


C


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Rydberg atoms on the link-kagome lattice: experiment

## Measurement of the topological $Z$ operator

A

$$
Z=-A \cdot:\left\{\begin{array}{l}
\Delta \rightarrow \Delta \\
\Delta \rightarrow \Delta \\
\triangle \rightarrow-\mathbb{Q} \\
\Delta \rightarrow-\Delta
\end{array}\right.
$$

B


$$
\langle Z\rangle=-1
$$

$\langle Z\rangle=(-1)^{\# x}$



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Rydberg atoms on the
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Can also define a topological $X$ operator which resonates between different dimer configurations of the kagome dimer model.
$X Z=(-1)^{\text {number of intersections }} Z X$
N. Schuch, D. Poilblanc, J.I. Cirac,
D. Perez-Garcia, PRB 86, 115108 (2012).
R. Verresen, M. D. Lukin,
A. Vishwanath, arXiv: 2011.12310


Rydberg atoms on site-kagome lattice: theory


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Rydberg atoms on site-kagome lattice: theory

Work in progress:
Zheng Yan Zi Yang Meng Rhine Samajdar


$$
\begin{aligned}
& -h \sum_{\ell}(|\bullet \bullet\rangle\langle\multimap|+|\Longleftrightarrow\rangle\langle\bullet \bullet|)-\mu \sum_{\ell}|\bullet\rangle\langle\multimap|
\end{aligned}
$$

Rydberg atoms on site-kagome lattice: theory

Work in progress:
Zheng Yan Zi Yang Meng Rhine Samajdar


Related to Ising gauge theory $\left(\sigma_{i j}\right)$ with matter fields $\left(\tau_{i}\right)$ :

$$
H=-K \prod_{\square} \sigma_{i j}^{z}-g \sum_{\langle i j\rangle} \sigma_{i j}^{x}-J \sum_{\langle i j\rangle} \tau_{i}^{z} \sigma_{i j}^{z} \tau_{j}^{z}-h \sum_{i} \tau_{i}^{x}
$$

$$
\begin{aligned}
H= & -J \sum_{r}(|\boldsymbol{N}\rangle\langle\boldsymbol{\nabla}|+\text { H.c. })+V_{2} \sum_{r}(|\boldsymbol{N}\rangle\langle\boldsymbol{N}|+|\boldsymbol{\rightharpoonup}\rangle\langle\boldsymbol{\nabla}|) \\
& -h \sum_{\ell}(|\boldsymbol{\bullet}\rangle\langle\boldsymbol{\bullet}|+|\boldsymbol{\bullet}\rangle\langle\boldsymbol{\bullet}|)-\mu \sum_{\ell}|\boldsymbol{\bullet}\rangle\langle\boldsymbol{\bullet}|
\end{aligned}
$$



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