Quantum phases of matter with Rydberg atoms





- Interacting Topological Matter: Atomic, Molecular, and Optical Systems Kavli Institute for Theoretical Physics University of California, Santa Barbara **July 6, 2021**
 - Subir Sachdev
 - Talk online: sachdev.physics.harvard.edu





Rhine Samajdar



Seth Whitsitt





Hannes Pichler

Mikhail Lukin





excited state (large principle quantum number)

optical tweezer (traps atom) $H_{\text{Ryd}} = \sum_{i} \left[\frac{\Omega}{2} \left(|g\rangle \langle r| + |r\rangle \langle g| \right)_{i} - \Delta |r\rangle \langle r| \right] + \sum_{(i,j)} V_{|i-j|} \left(|r\rangle \langle r|_{i} \otimes |r\rangle \langle r|_{j} \right)$

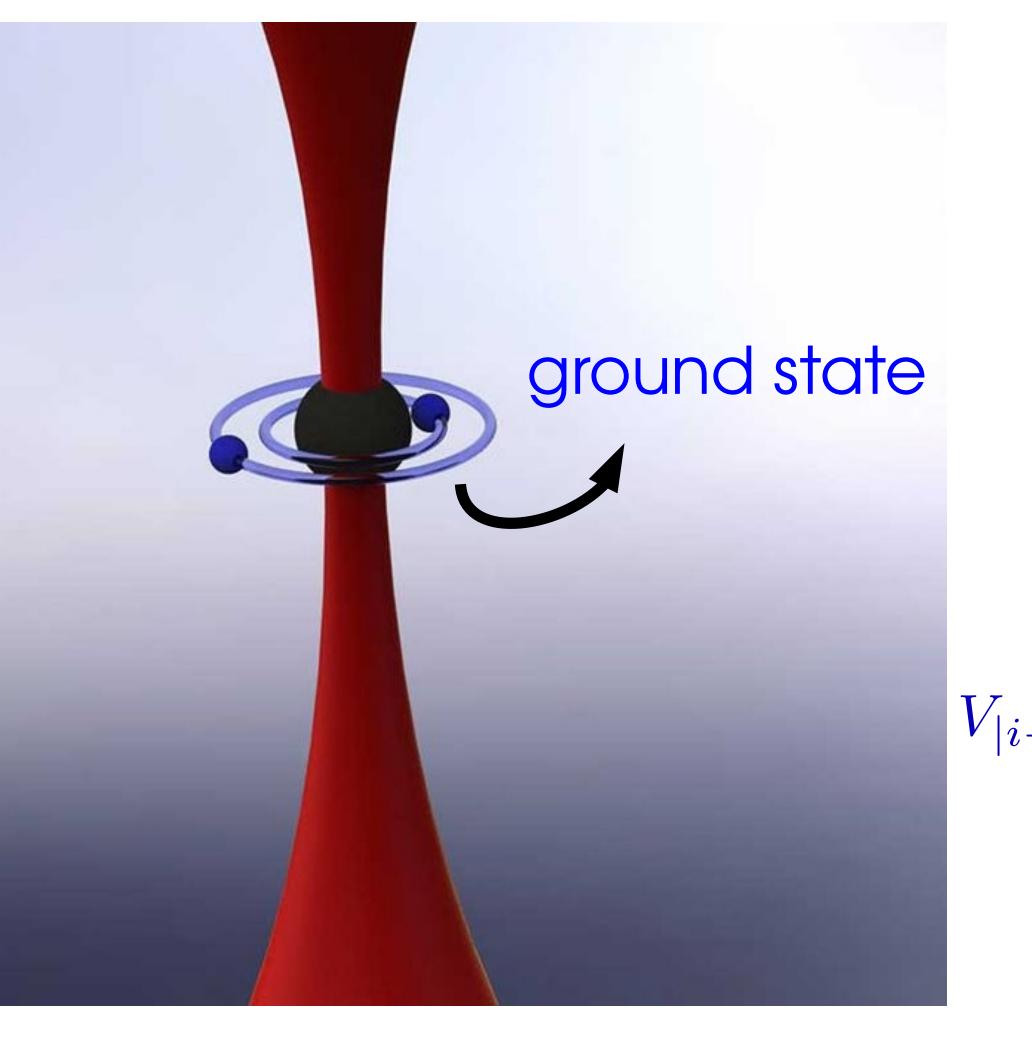


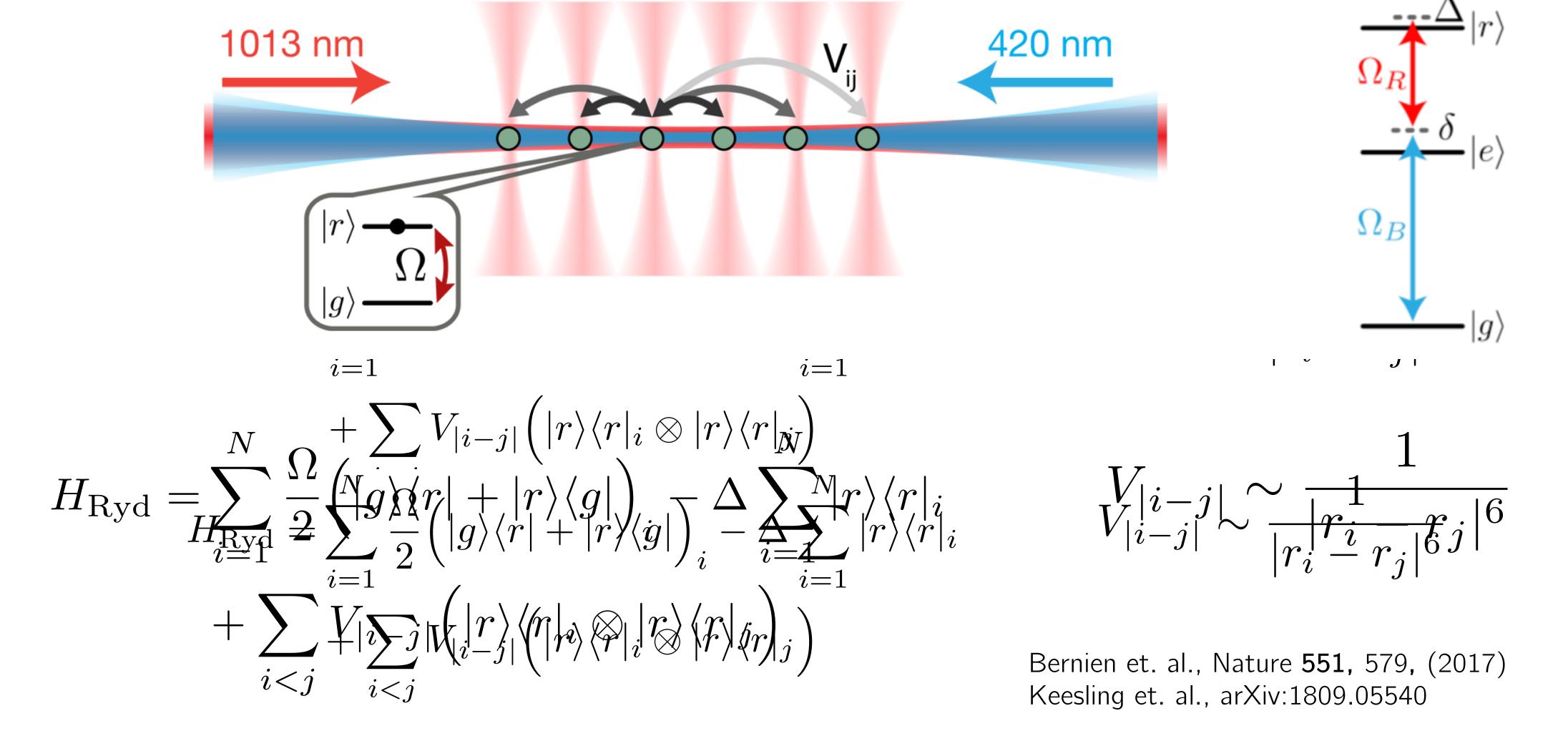
Fig: https://www.caltech.edu/about/ news/quantum-innovations-achievedusing-alkaline-earth-atoms



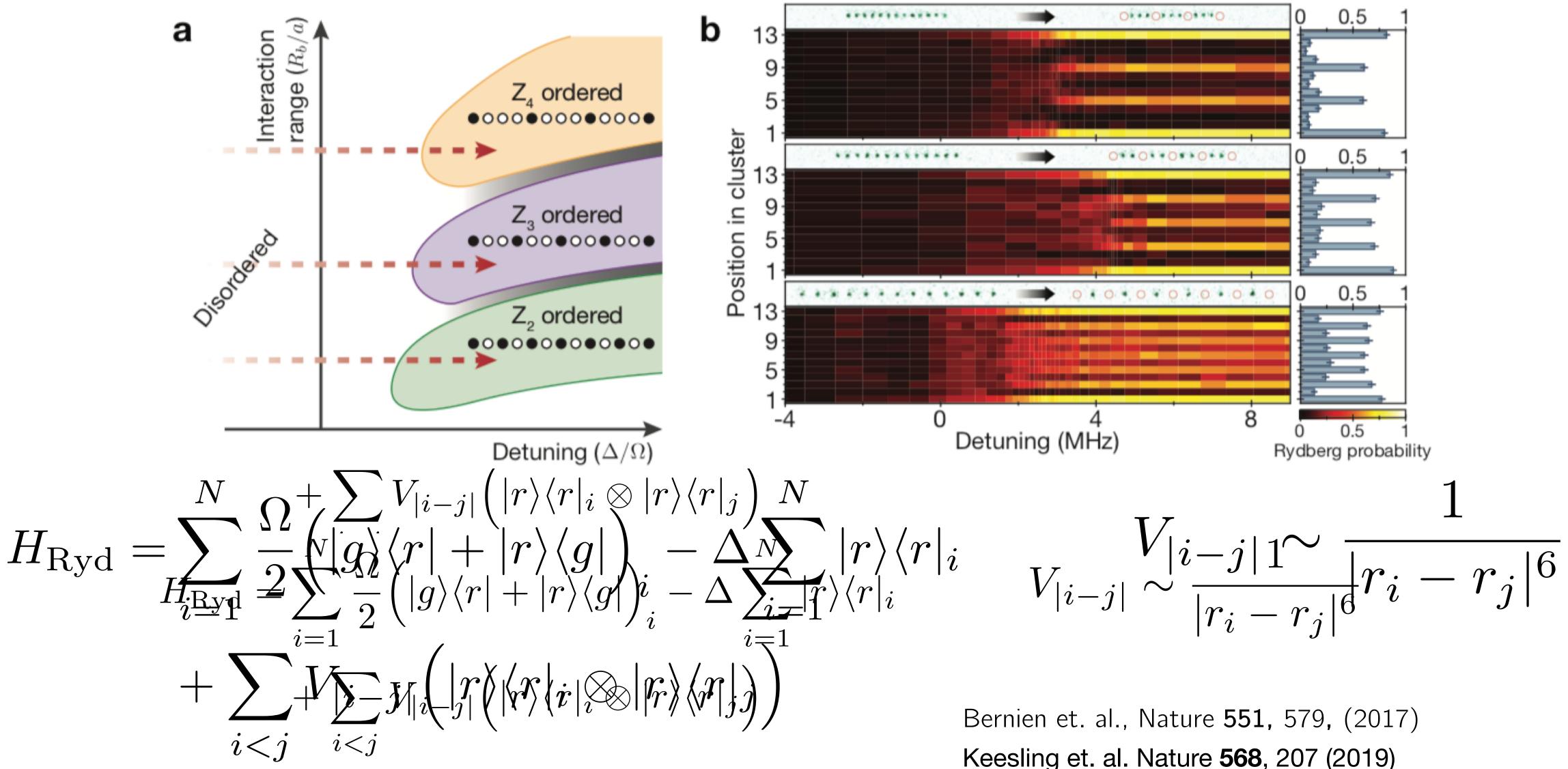
I. Rydberg chains The Z₃ chiral clock transition 2. Square lattice Quantum Ising criticality in 2+1 dimensions 3. Kagome symmetry lattices Probing topological spin liquids 4. Theory of odd and even Z₂ spin liquids



QPTs in a Rydberg quantum simulator



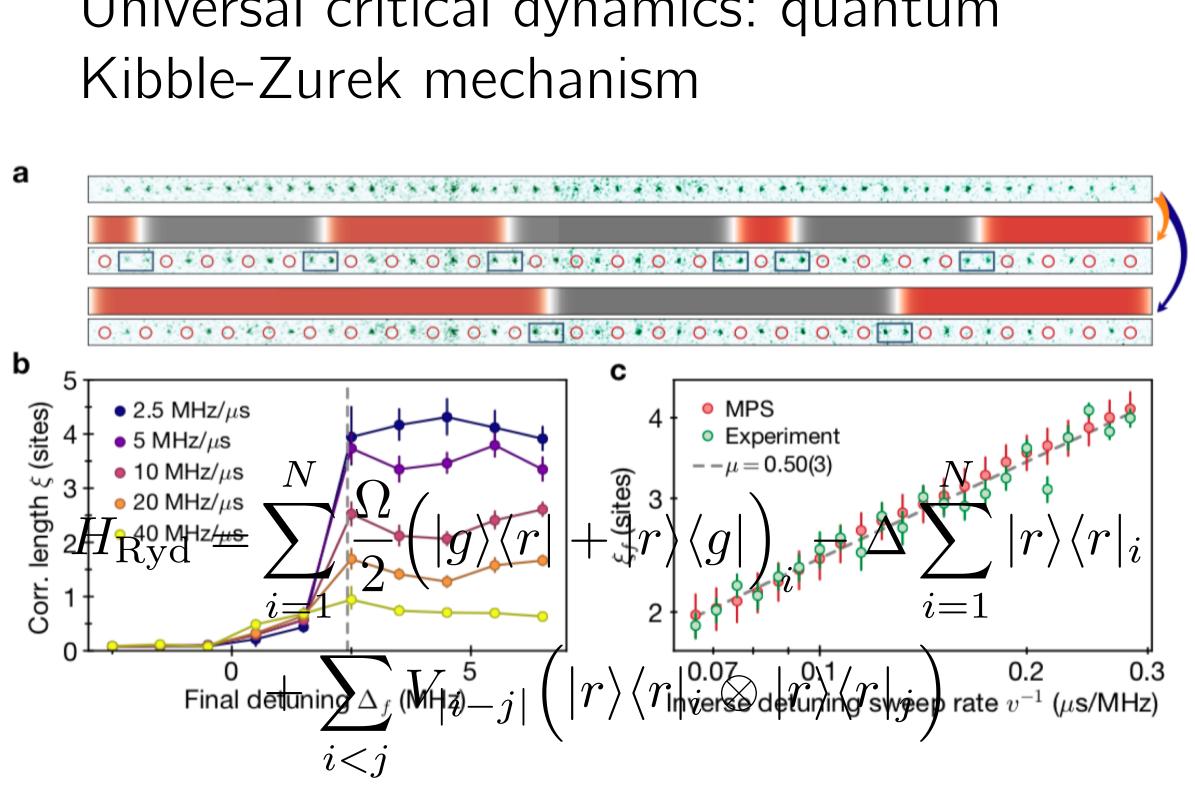
QPTs in a Rydberg quantum simulator



Keesling et. al. Nature **568**, 207 (2019)

QPTs in a Rydberg quantum simulator

Universal critical dynamics: quantum

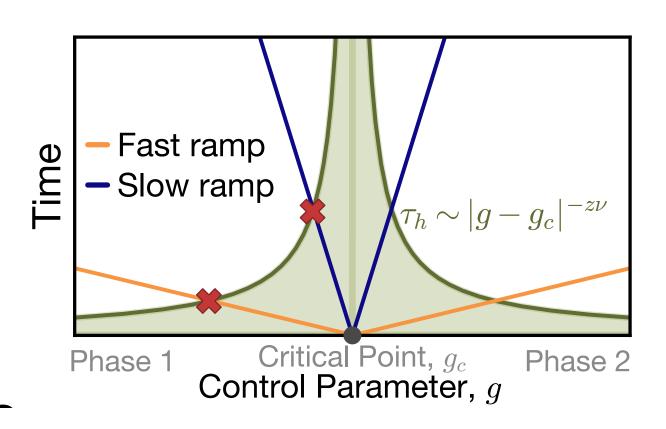


Quantum Kibble-Zurek mechanism and critical dynamics on a programmable Rydberg simulator

Alexander Keesling, Ahmed Omran, Harry Levine, Hannes Bernien, Hannes Pichler, Soonwon Choi, Rhine Samajdar, Sylvain Schwartz, Pietro Silvi, Subir Sachdev, Peter Zoller, Manuel Endres, Markus Greiner, Vladan Vuletic, and Mikhail D. Lukin, Nature 568, 207 (2019)

Tune through transition at rate v: $\Delta(t) = \Delta_c + vt$ Correlation length saturates! $\xi \sim v^{-\nu/(1+\nu z)}$ Experimental probe of critical

 $expansion entry \sim$



 $\frac{|r_i - r_j|^6}{|r_i - r_j|^6}$

Competing density-wave orders in a one-dimensional hard-boson model PHYSICAL REVIEW B 69, 075106 (2004) P. Fendley, K. Sengupta, S. Sachdev

$$n_{j} \equiv b_{j}^{\dagger}b_{j}$$
$$\mathcal{H} = \sum_{j} \left[\frac{\Omega}{2} \left(b_{j} + b_{j}^{\dagger}\right) - \Delta n_{j}\right] + \sum_{i < j} V_{|i-j|}n$$
$$V_{1} = \infty, \quad V_{2} = V, \quad V_{i>2} = 0$$

The V = 0 case is the 'PXP' model, originally introduced in S. Sachdev, K. Sengupta, and S.M. Girvin, PRB 66, 075128 (2002)

These models were motivated by 'tilted lattices' of bosonic atoms, and the \mathbb{Z}_2 quantum transition was observed in J. Simon, W. S. Bakr, Ruichao Ma, M. Eric Tai, P. M. Preiss, M. Greiner, Nature **472**, 307 (2011).

 $n_i n_j$

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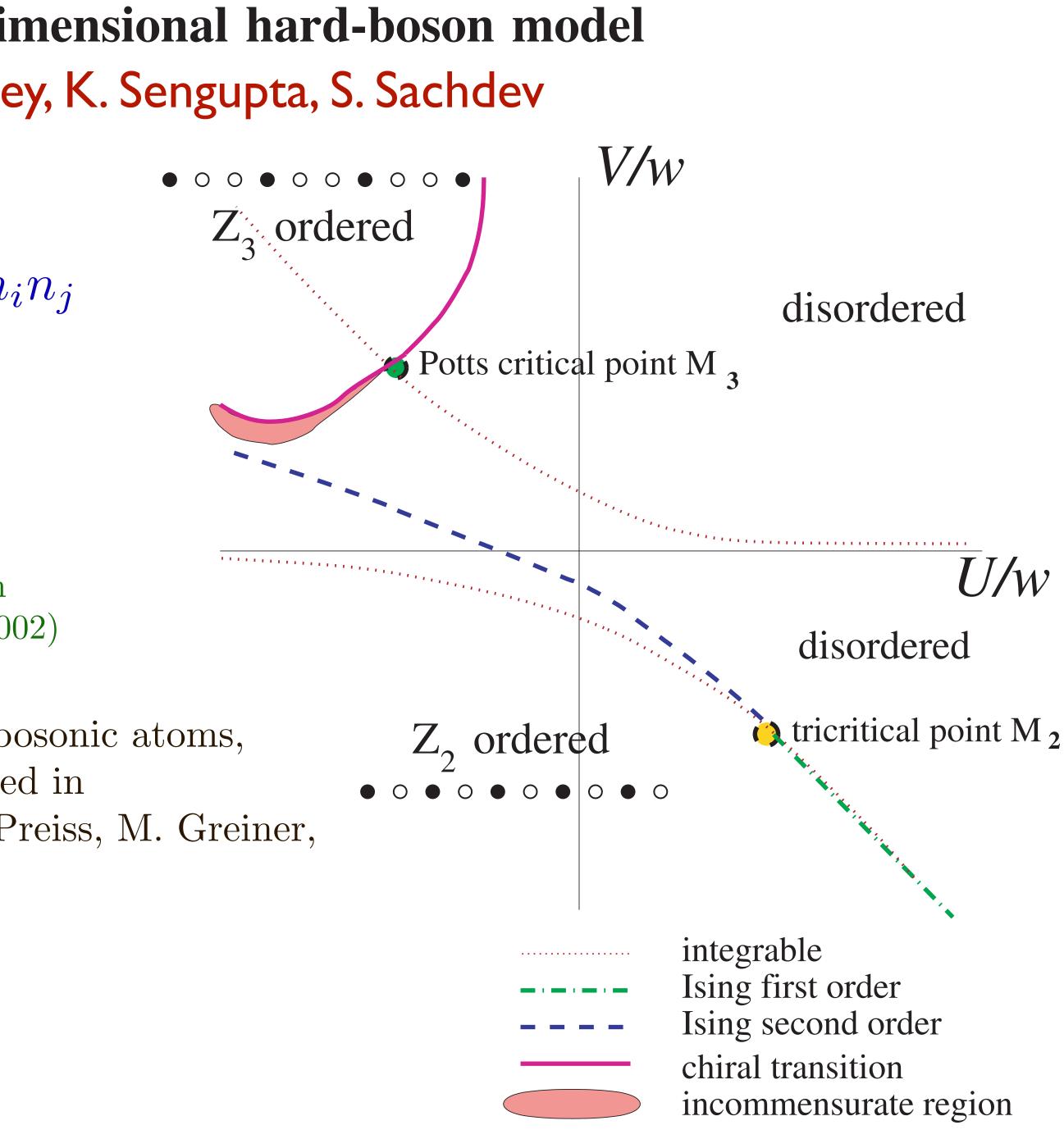
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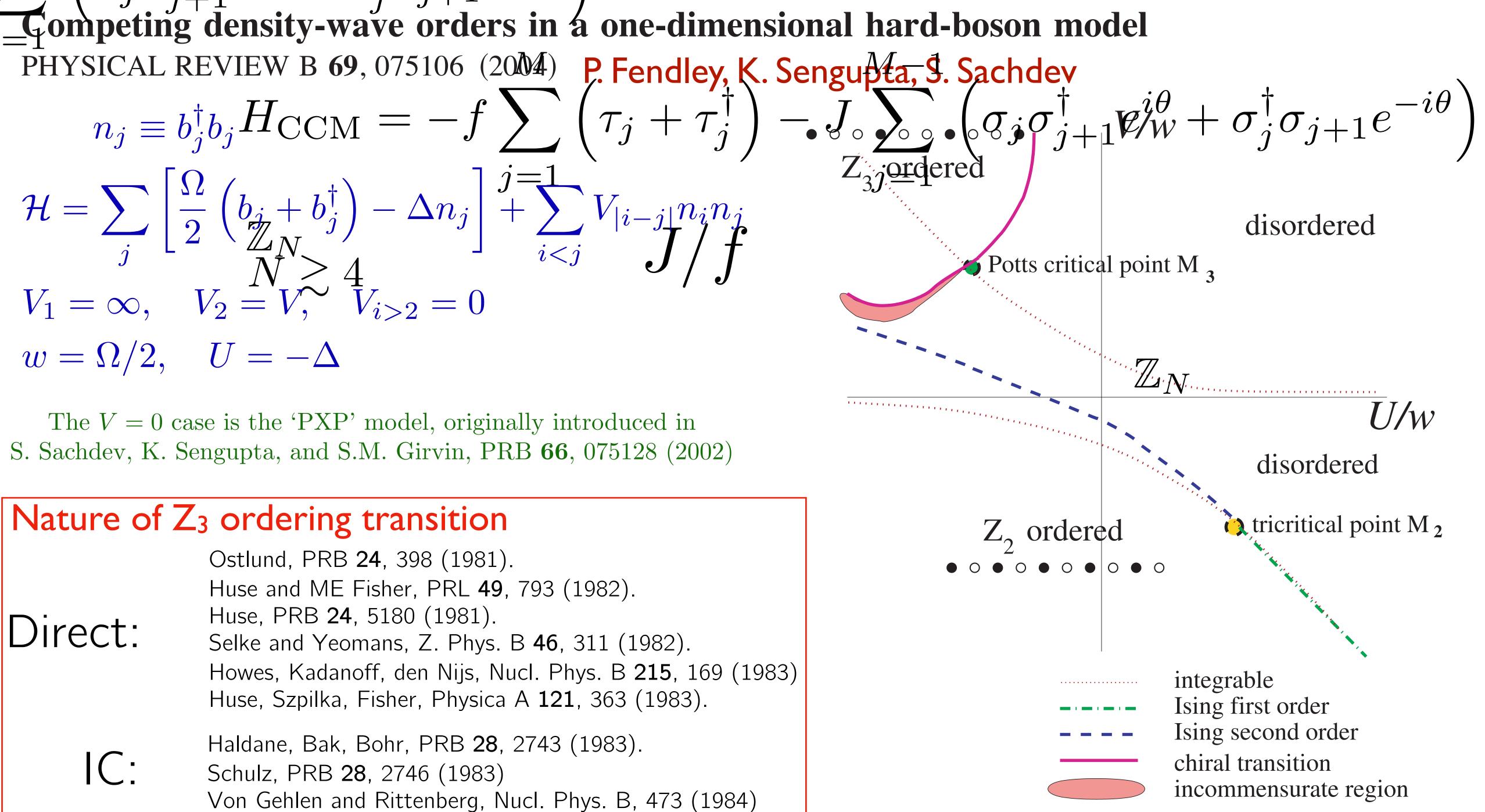
$$V_{1} = \infty, \quad V_{2} = V, \quad V_{i>2} = 0$$

$$w = \Omega/2, \quad U = -\Delta$$

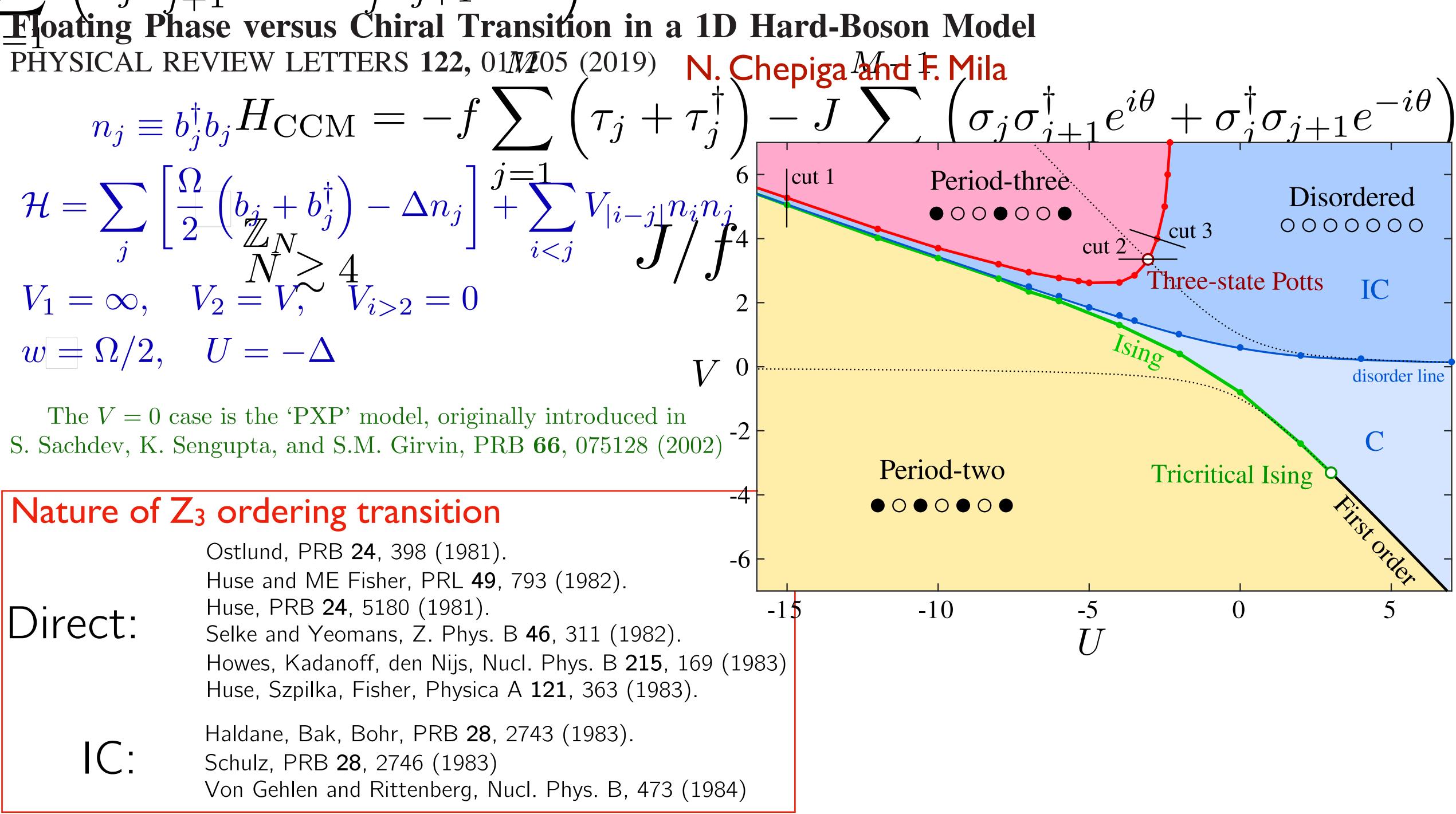
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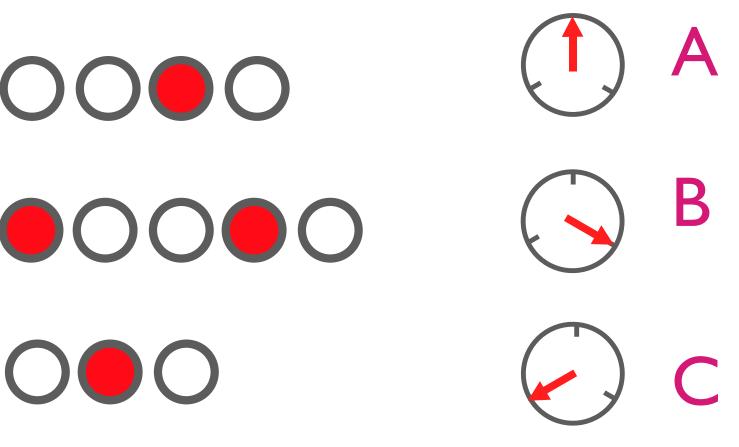
Direct:	Ostlund, PRB 24 , 398 (1981). Huse and ME Fisher, PRL 49 , 793 (1982). Huse, PRB 24 , 5180 (1981). Selke and Yeomans, Z. Phys. B 46 , 311 (1982). Howes, Kadanoff, den Nijs, Nucl. Phys. B 215 , 16 Huse, Szpilka, Fisher, Physica A 121 , 363 (1983)
IC:	Haldane, Bak, Bohr, PRB 28 , 2743 (1983). Schulz, PRB 28 , 2746 (1983) Von Gehlen and Rittenberg, Nucl. Phys. B, 473 (



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$\mathbf{OOOOOOOOOOOOOOOOO}$

A A type II DW type I DW







What is the critical field theory? First try: write the most general theory for order parameter with appropriate symmetries. $\Phi \rightarrow e^{2\pi i/N} \Phi \qquad \Phi(x, \tau) \rightarrow \Phi^*(-)$

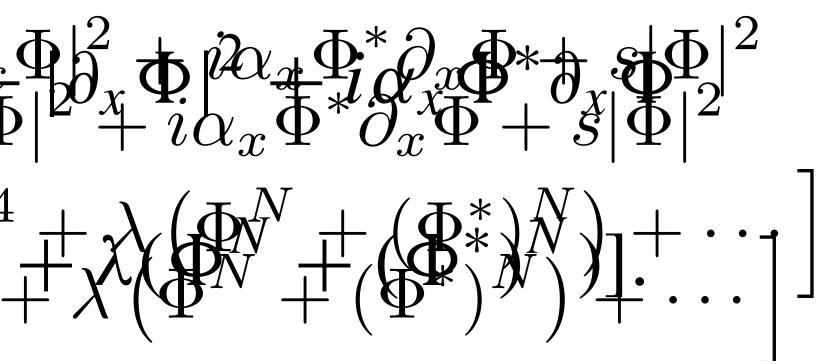
$$\begin{split} \Phi \to e^{2\pi i/N} \Phi \\ \mathcal{S}_{\Phi} &= \int dx dx dx \left[\partial_{\tau} \Phi \right]_{\tau}^{2} \Phi \left[\partial_{\tau} \Phi \right]_{\tau}^{2} \Phi \\ + s_{\Phi} \left[\Phi \right]_{\tau}^{2} \Phi \left[\partial_{\tau} \Phi \right]_{\tau}^{2} + \left[\partial_{x} \Phi \right]_{\tau}^{2} \Phi \\ + s_{\Phi} \left[\Phi \right]_{\tau}^{2} \Phi \left[\partial_{\tau} \Phi \right]_{\tau}^{2} + \left[\partial_{x} \Phi \right]_{\tau}^{4} \Phi \\ \end{split}$$

In perturbation theory, the field condenses at nonzero momentum.

$$\mathcal{S}_{\Phi} = \int \frac{d\omega \, dk}{(2\pi)^2} \, \Phi^* \left[\omega^2 + k^2 - \alpha_x k + s \right] \Phi + \cdots$$

$$\Phi(x,\tau) \to \Phi^*(-x,\tau)$$

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 $\rightarrow e^{2\pi i/N} \Phi \text{ with appropriate } (at e \pi) \text{ mine} \Phi^* e(x, \tau) \rightarrow \Phi^*(-x, \tau) \\ \Phi \rightarrow e^{2\pi i/N} \Phi \qquad \Phi(x, \tau) \rightarrow \Phi^*(-x, \tau) \\ \Phi \rightarrow e^{2\pi i/N} \Phi \qquad \Phi(x, \tau) \rightarrow \Phi^*(-x, \tau)$ $d\tau \left[|\partial_{\tau} \Phi|^2 \mathcal{S}_{\Phi} \right] \overset{i}{\to} \overset{i}{\to$ $+ u |\Phi|^4 + \lambda \left(\Phi |^2 + \mu \Phi |^4 + \lambda \Phi |^4 + \mu \Phi |^4 +$

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Carponty describe transition to known strate that is the second stra $\sim (k - k_0)^{\sigma} = \int \frac{1}{(2\pi)^2} \langle \Phi(x)\Phi(0) \rangle \sim e^{ik_0x}$

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What is the critical field theory? First try: write the most general theory for order parameter with appropriate symmetries.

$$\begin{aligned} \Phi \to e^{2\pi i/N} \Phi & \Phi(x,\tau) \to \Phi^*(-x,\tau) \\
S_{\Phi} \stackrel{S_{\Phi}}{=} \int dy d\tau \begin{bmatrix} d \tau & d \tau \\ d \tau & d \tau \end{bmatrix} \begin{vmatrix} \partial_{\tau} & \Phi \end{vmatrix}^2 + \begin{bmatrix} \partial_{x} & \Phi \end{vmatrix}^2 \\ + \begin{bmatrix} \partial_{x} & \Phi \end{bmatrix}^2 & \pm \begin{bmatrix} \partial_{x} & \Phi \end{bmatrix}^2 \\ + \begin{bmatrix} \partial_{x} & \Phi \end{bmatrix}^2 & \pm \begin{bmatrix} \partial_{x} & \Phi \end{bmatrix}^2 \\ + \begin{bmatrix} \partial_{x} & \Phi \end{bmatrix}^2 & \pm \begin{bmatrix} \partial_{x} & \Phi \end{bmatrix}^2 \\ + \begin{bmatrix} \partial_{x} & \Phi \end{bmatrix}^2 & \pm \begin{bmatrix} \partial_{x} & \Phi \end{bmatrix}^2 \\ + \begin{bmatrix} \partial_{x} & \Phi \end{bmatrix}^2 & \pm \begin{bmatrix} \partial_{x} & \Phi \end{bmatrix}^2 \\ + \begin{bmatrix} \partial_{x} & \Phi$$

$$\mathbb{Z}_N$$
 density wave
 $\langle \Phi \rangle \neq 0$
Gapped



PHYSICAL REVIEW B 98, 205118 (2018) Field theory for \mathbb{Z}_N density wave ordering is Kramers-Wannier dual to

of a background N boson condensate

- Quantum field theory for the chiral clock transition in one spatial dimension S.Whitsitt, R. Samajdar, and S. Sachdev
 - $S_{\Phi} = \int dx \, d\tau [|\partial_{\tau} \Phi|^2 + |\partial_x \Phi|^2 + i\alpha_x \Phi^* \partial_x \Phi$
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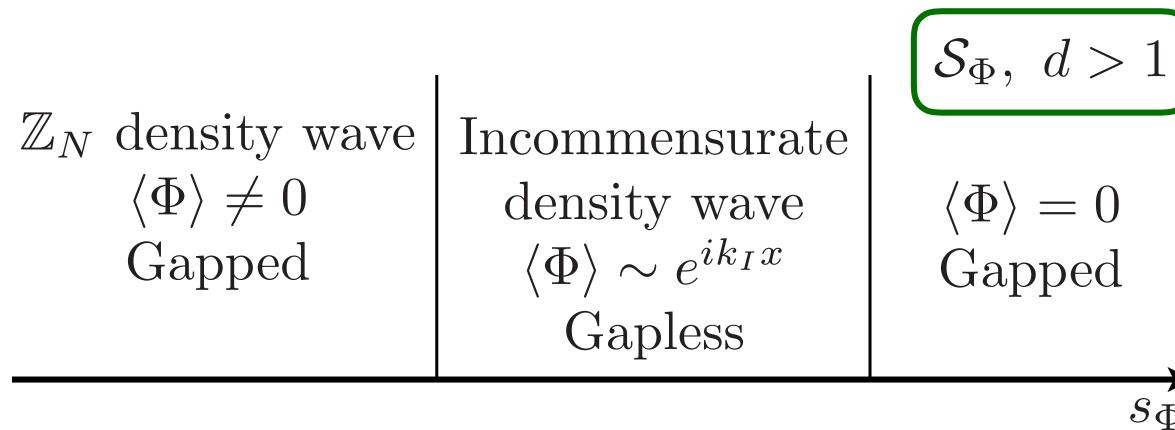


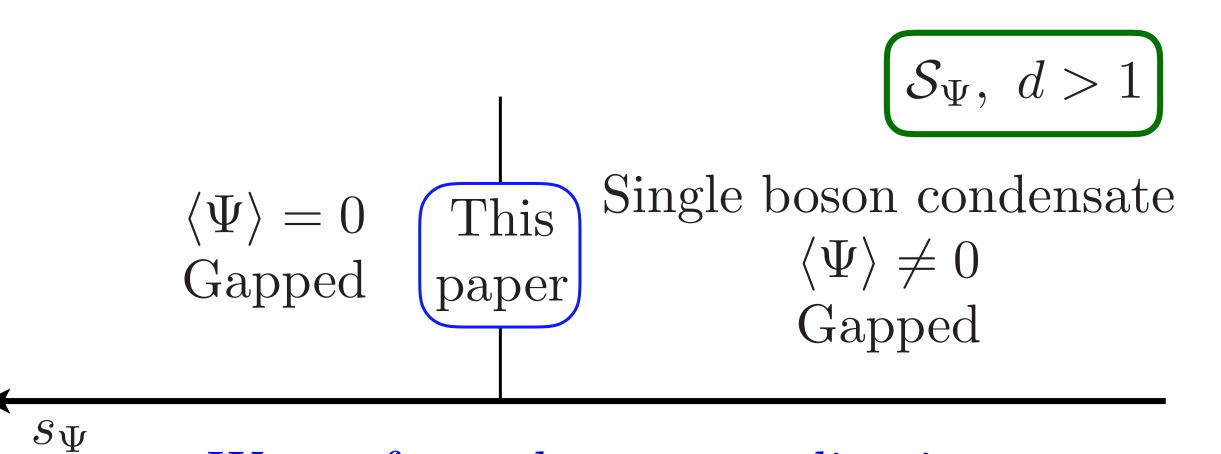
PHYSICAL REVIEW B 98, 205118 (2018) Field theory for \mathbb{Z}_N density wave ordering is Kramers-Wannier dual to

of a background N boson condensate Note: this is not a Wick rotation—

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 - field theory for Bose condensation in the presence
 - there is a crucial difference in the factor of i !!



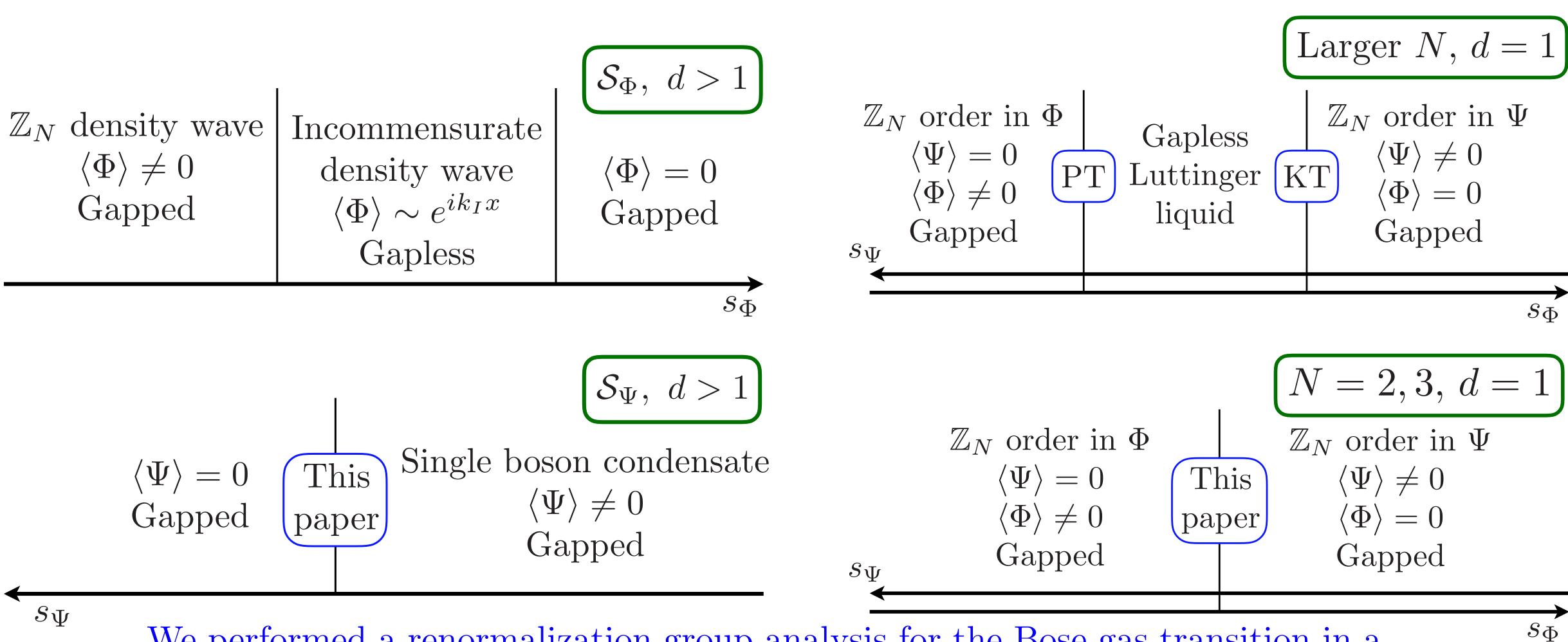


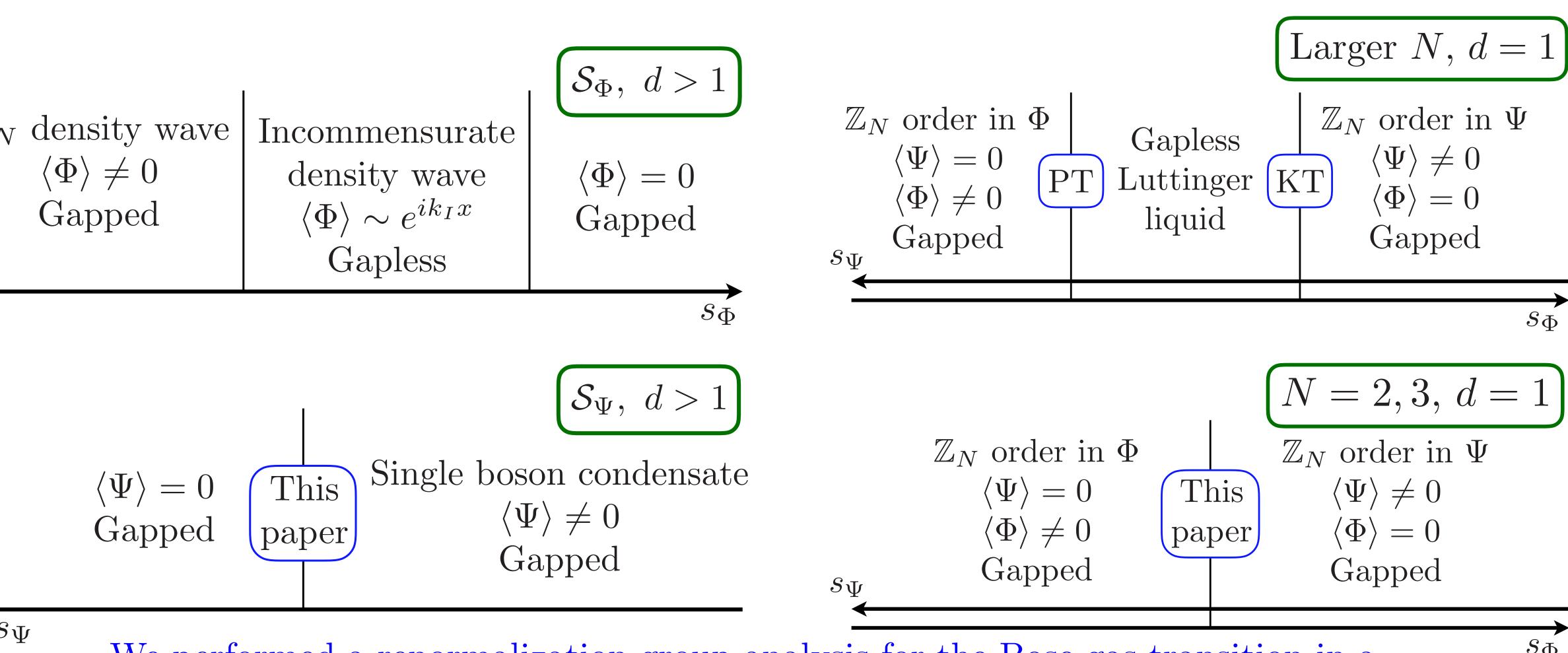


 $S\Phi$

We performed a renormalization group analysis for the Bose gas transition in a expansion in 2 - d, with 4 - N chosen to be order 2 - d. This led a strongly-coupled critical point with $z \neq 1$.

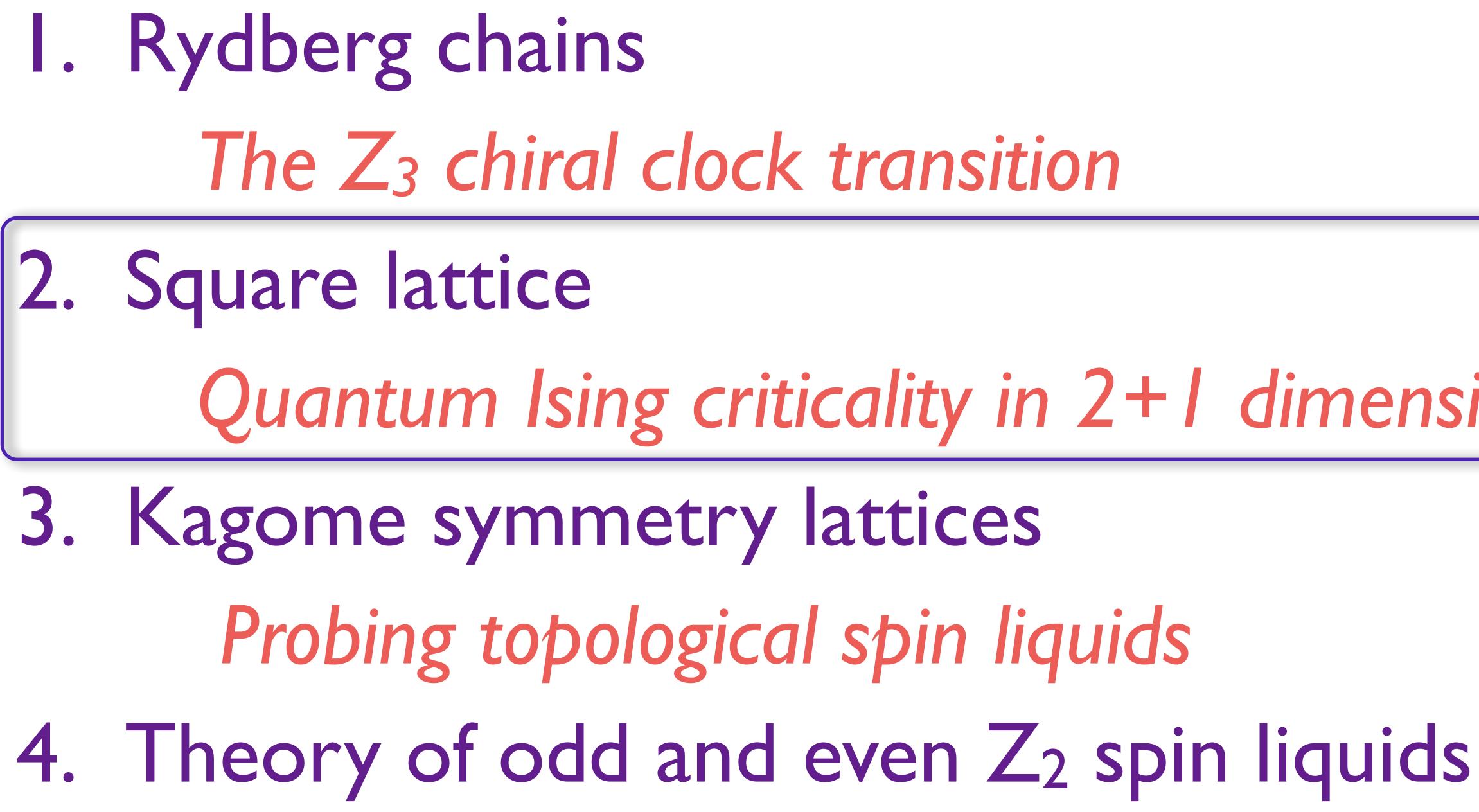






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Quantum Ising criticality in 2+1 dimensions



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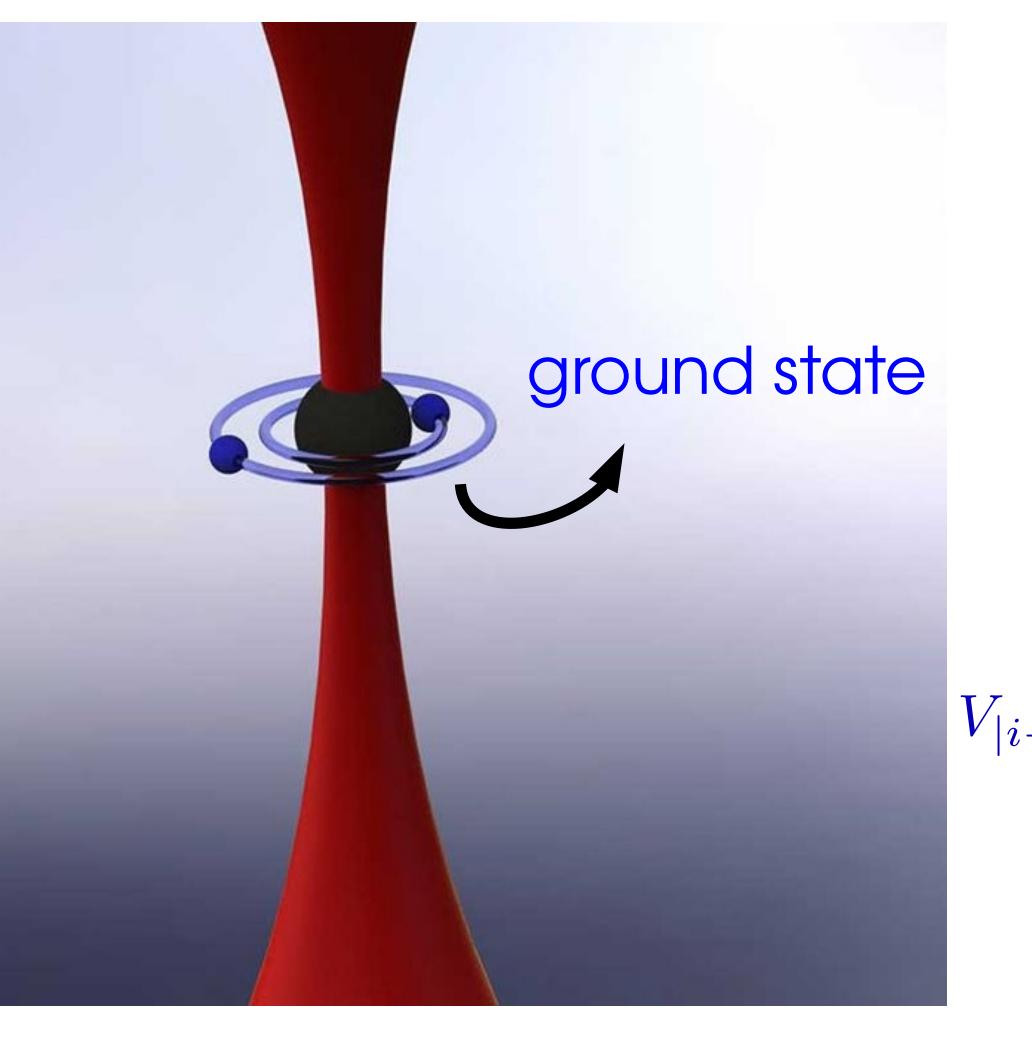
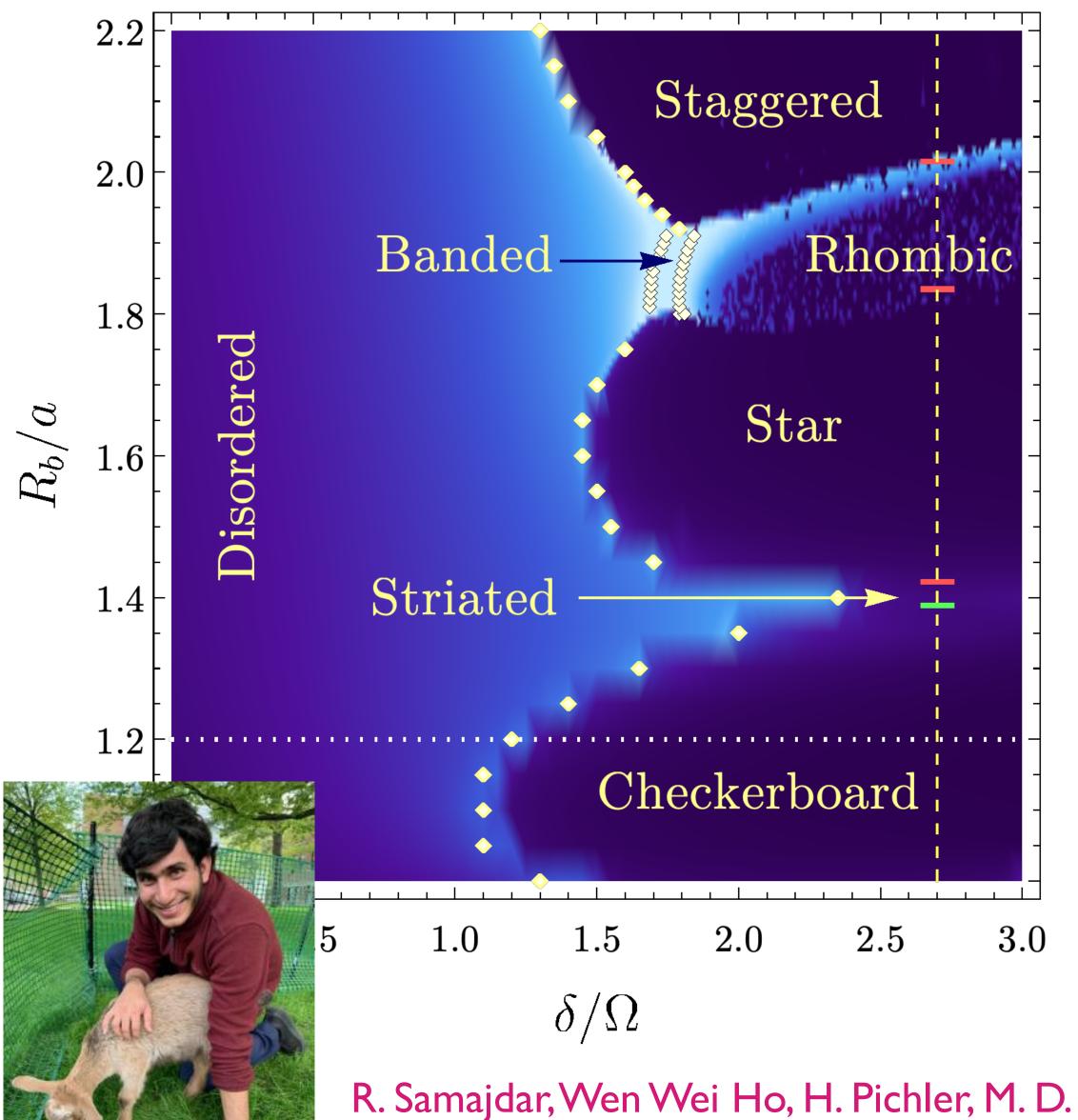


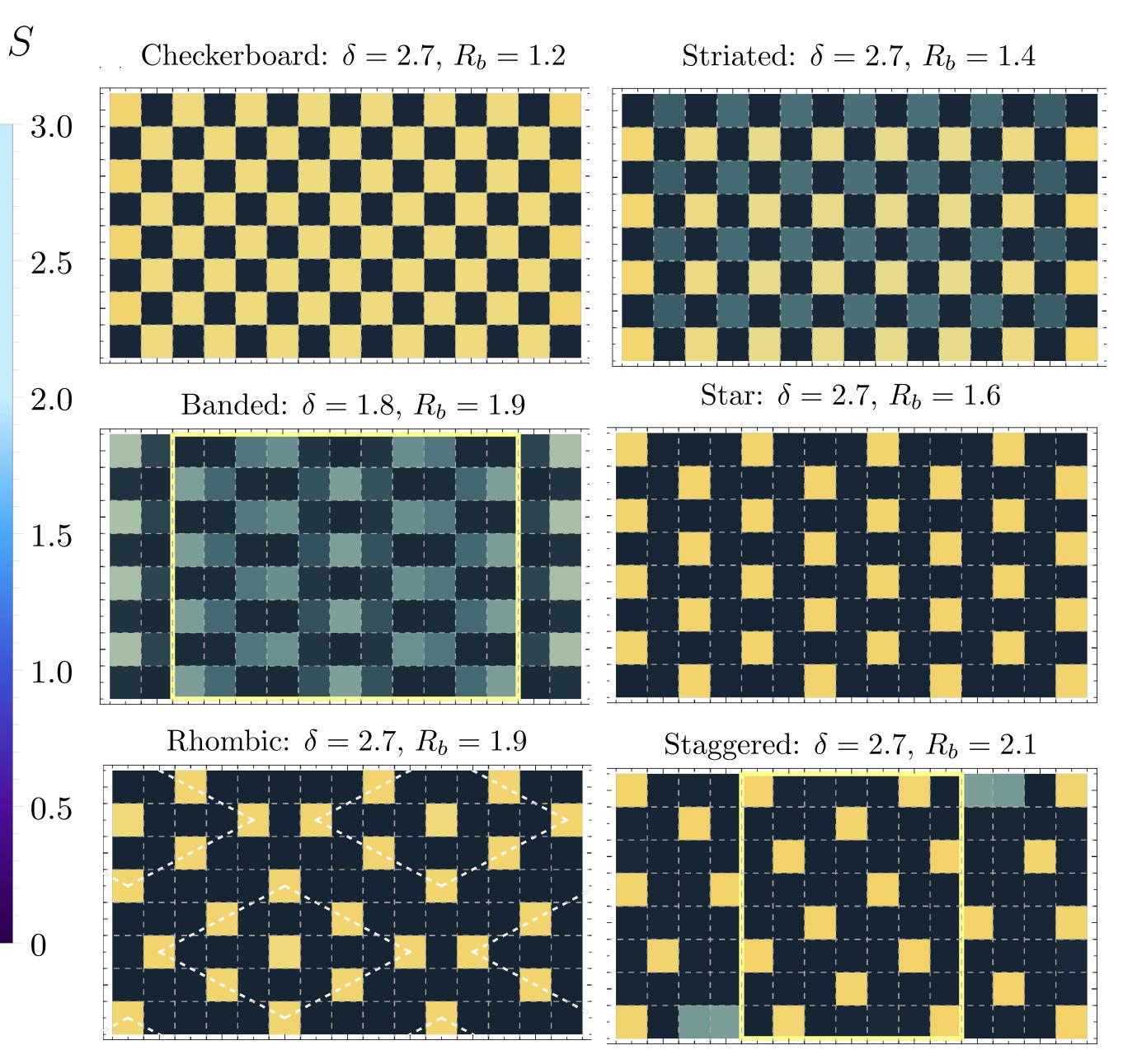
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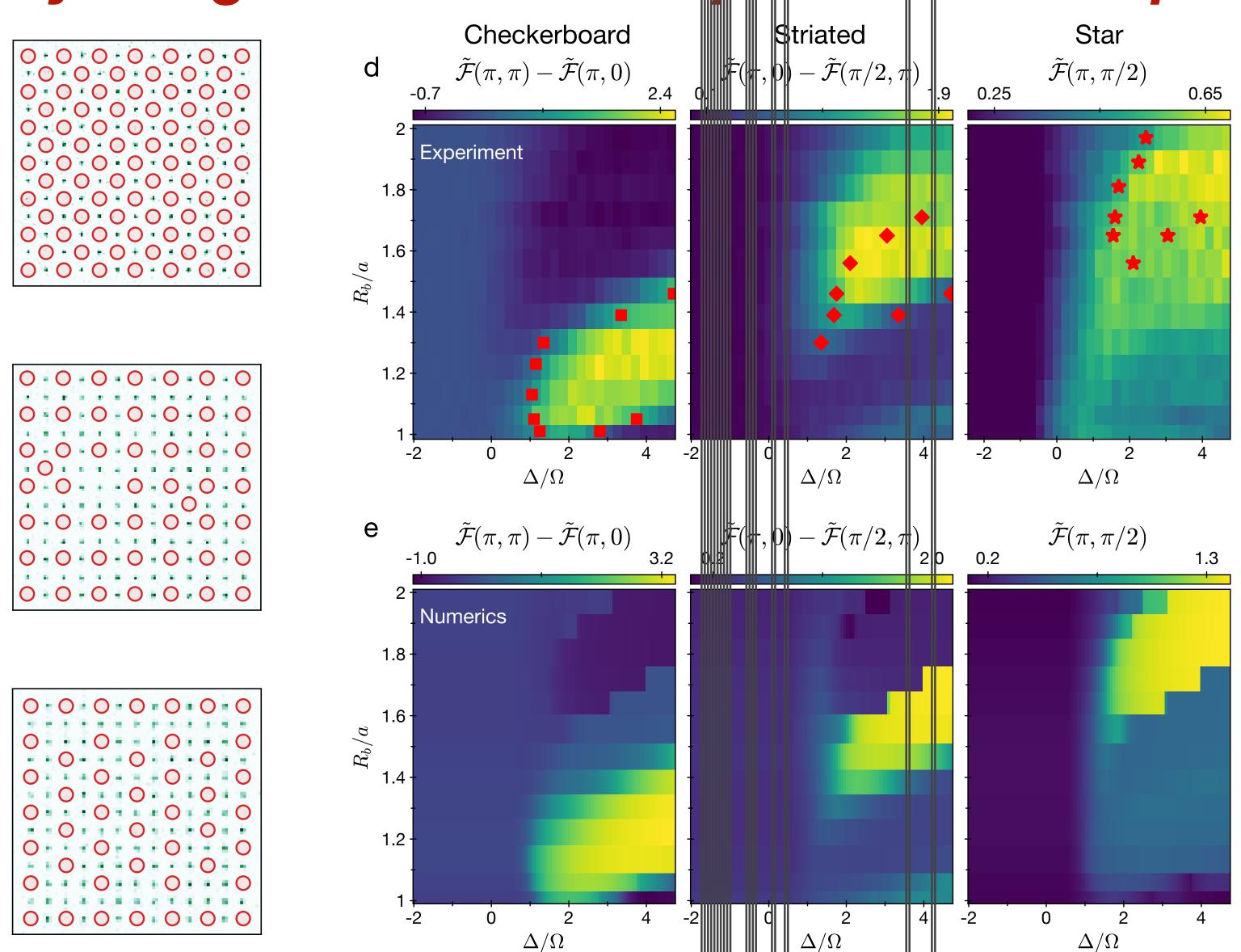
Rydberg atoms on the square lattice: theory



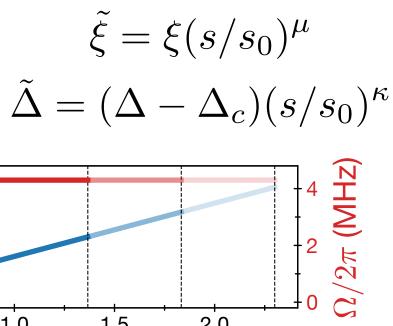
Lukin, S. Sachdev, PRL **124**, 103601 (2020)

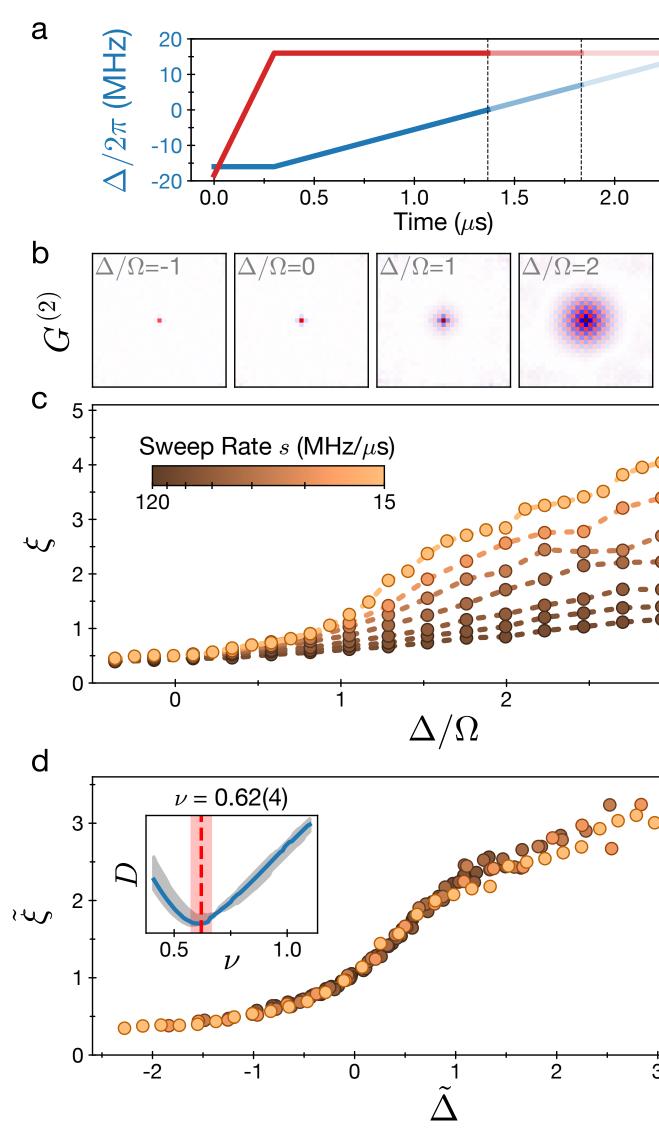


Rydberg atoms on the square lattice: experiment

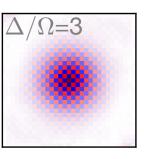


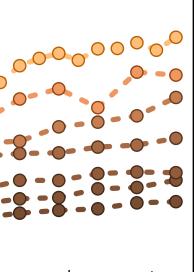
Quantum Phases of Matter on a 256-Atom Programmable Quantum Simulator, Sepel r Ebadi, Tout T. Wang, Harry Levine, Alexander Keesling, Giulia Semeghini, Ahmed Omran, Dolev Bluvstein, Rhine Sama dar, Hannes Pichler, Wen Wei Ho, Soonwon Choi, Subir Sachdev, Markus Greiner, Vladan Vuletic, and Mikhail D. Lukin, Nature to appear, arXiv:2012.12281; Pascal Scholl et al. arXiv:2012.12268

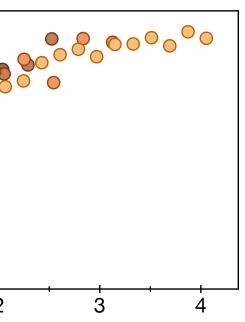




First observation of Ising quantum phase transition in 2+1 dimensions









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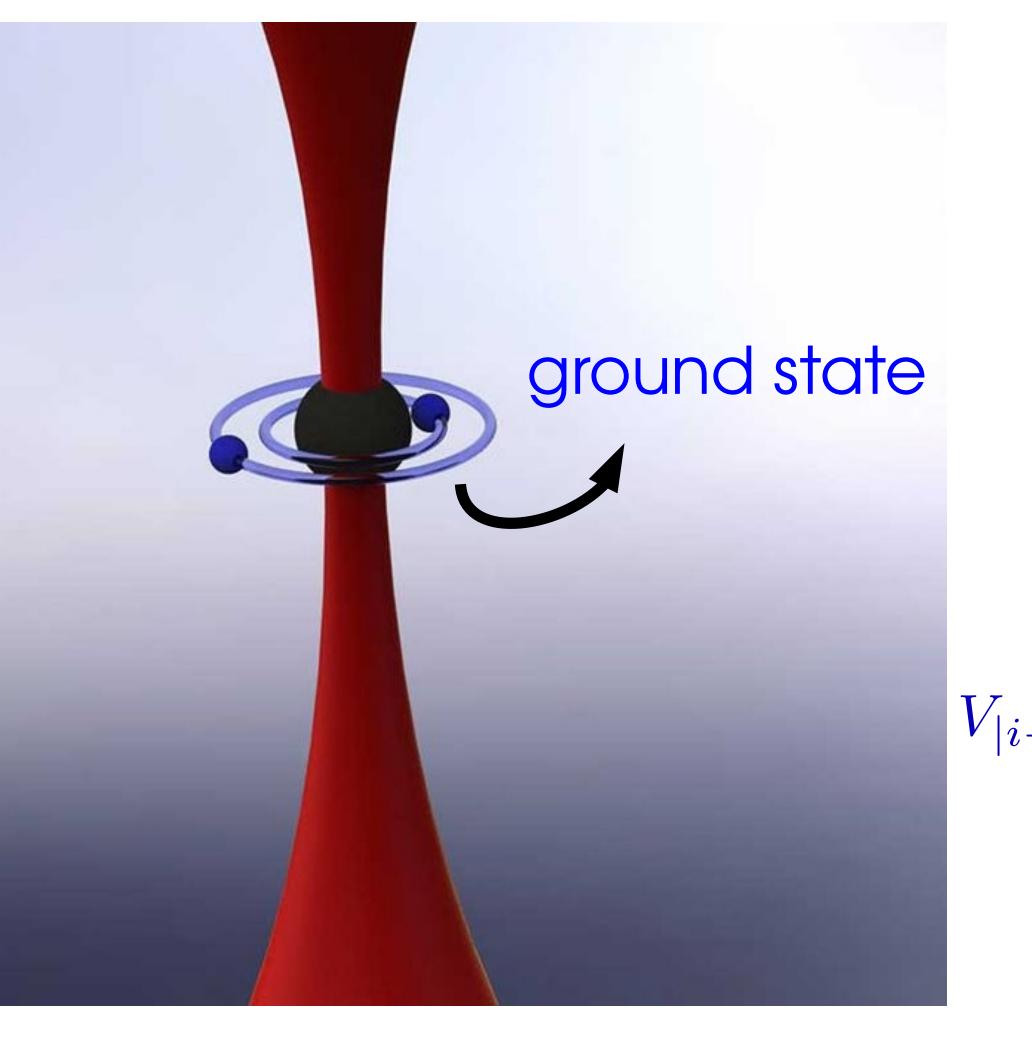
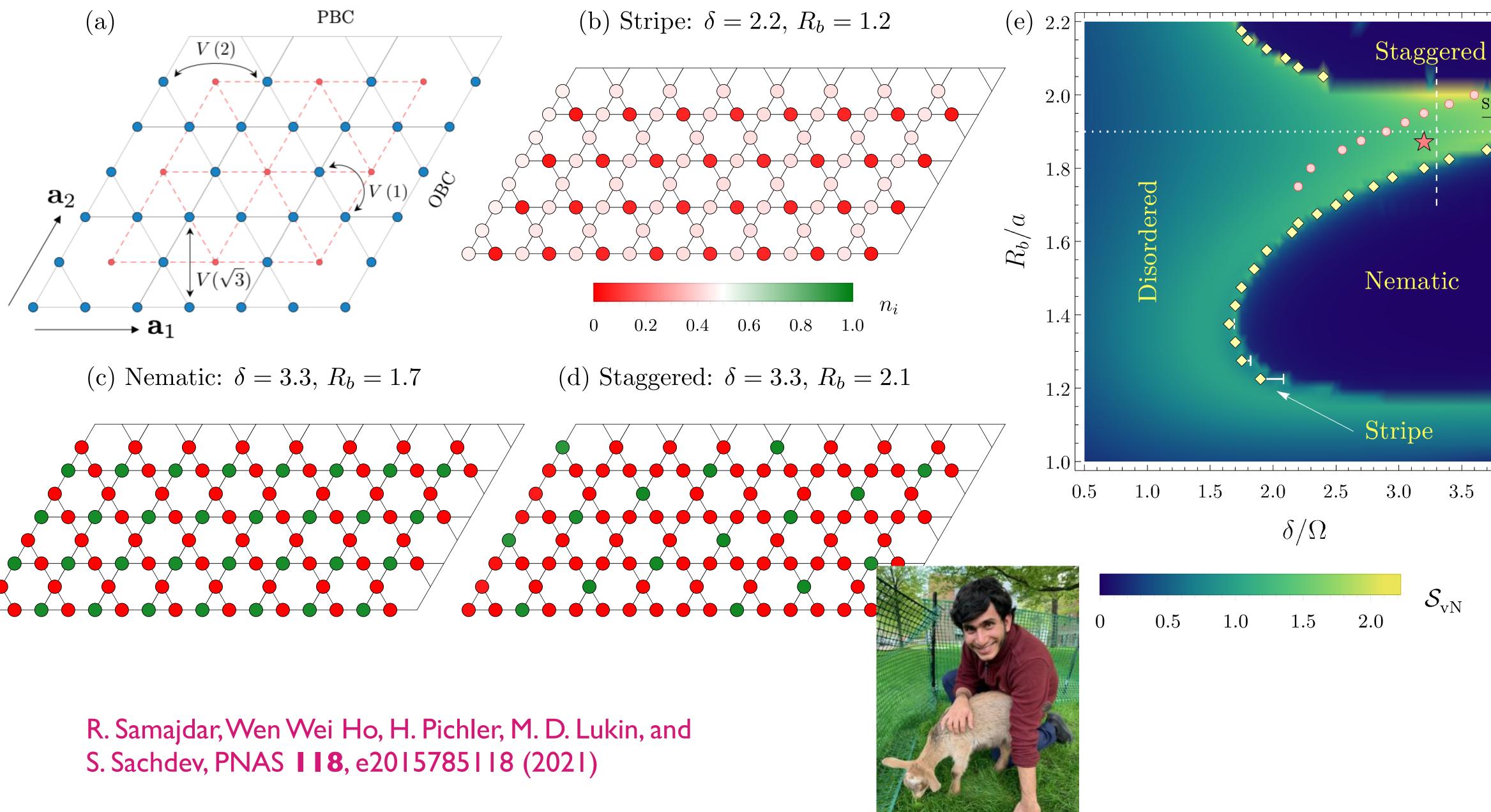
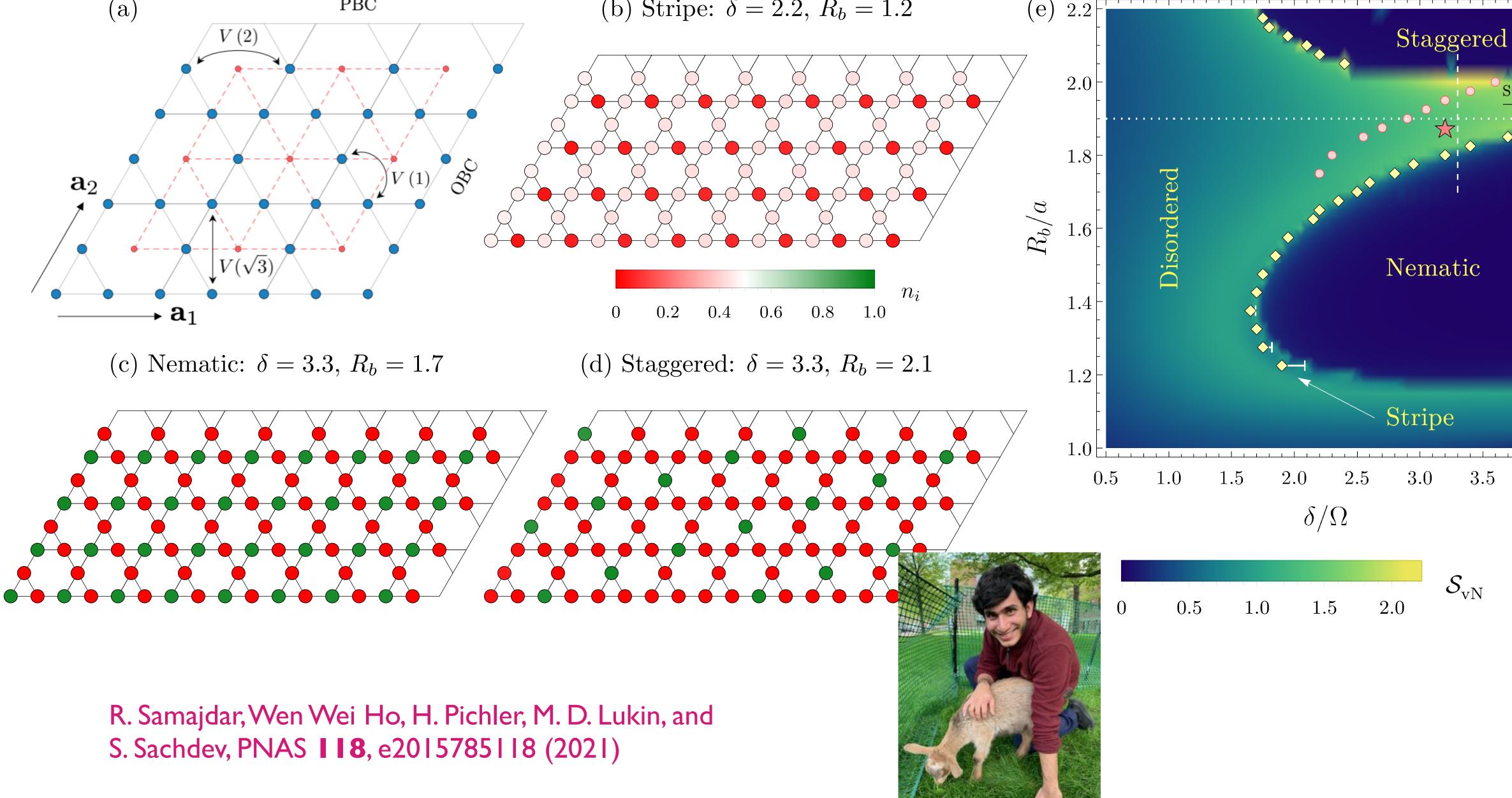


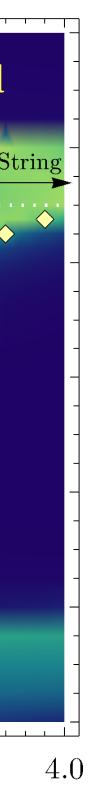
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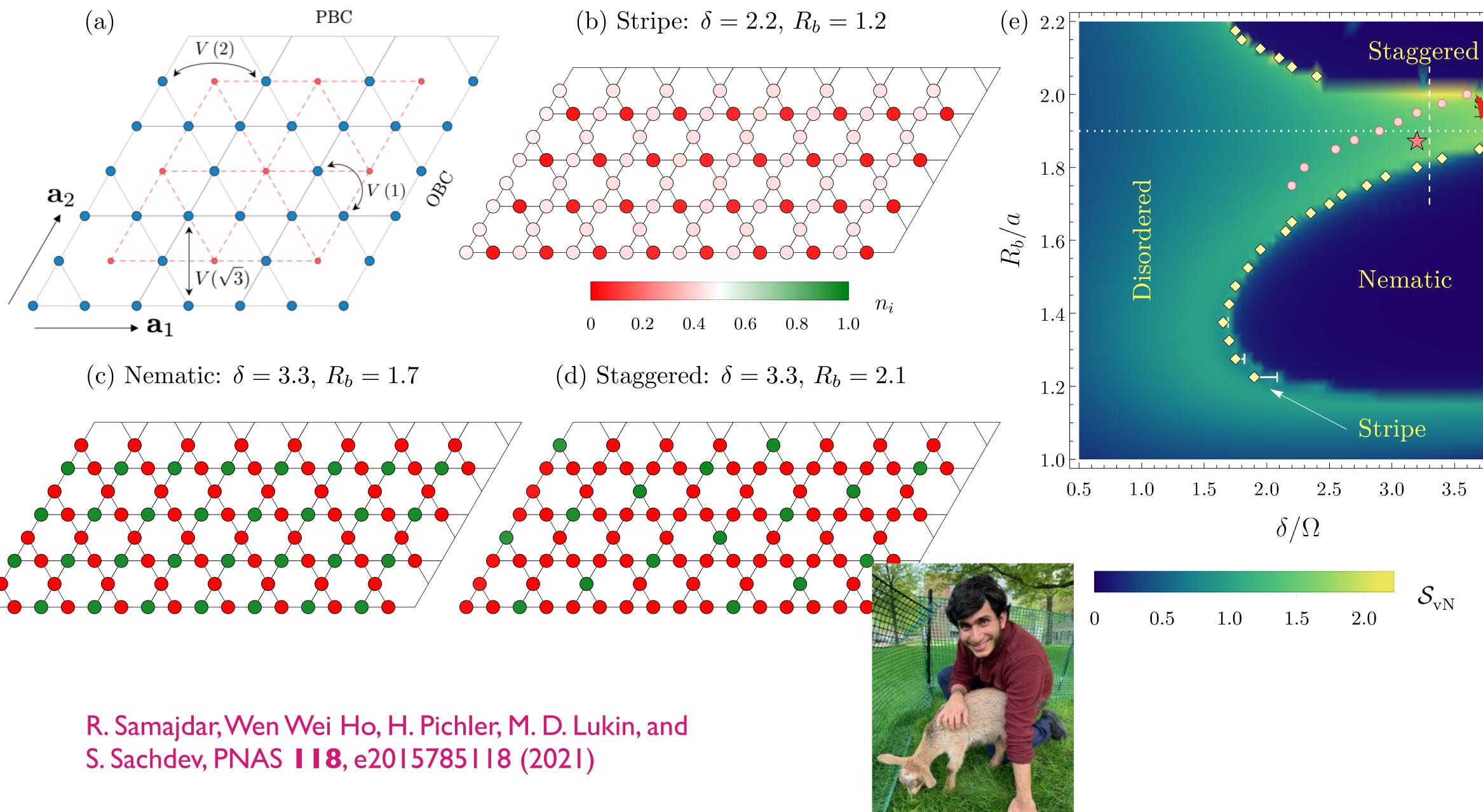
Rydberg atoms on site-kagome lattice: theory

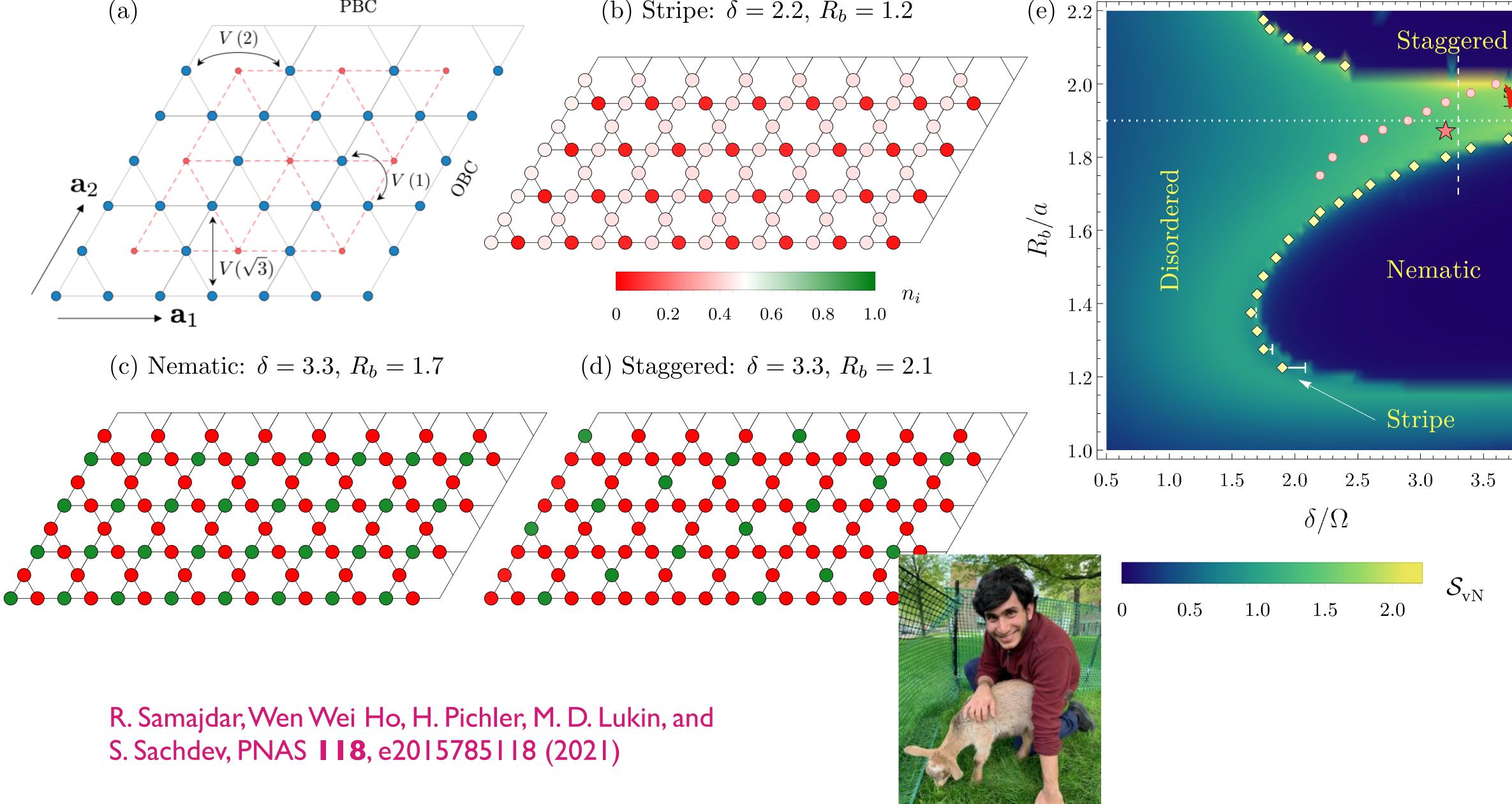


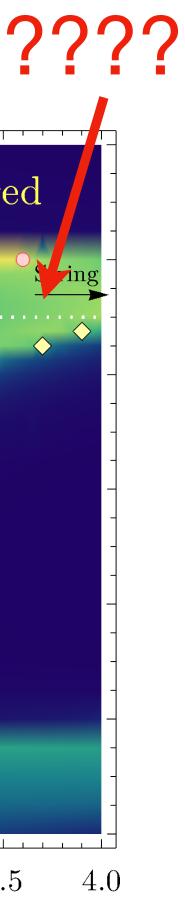




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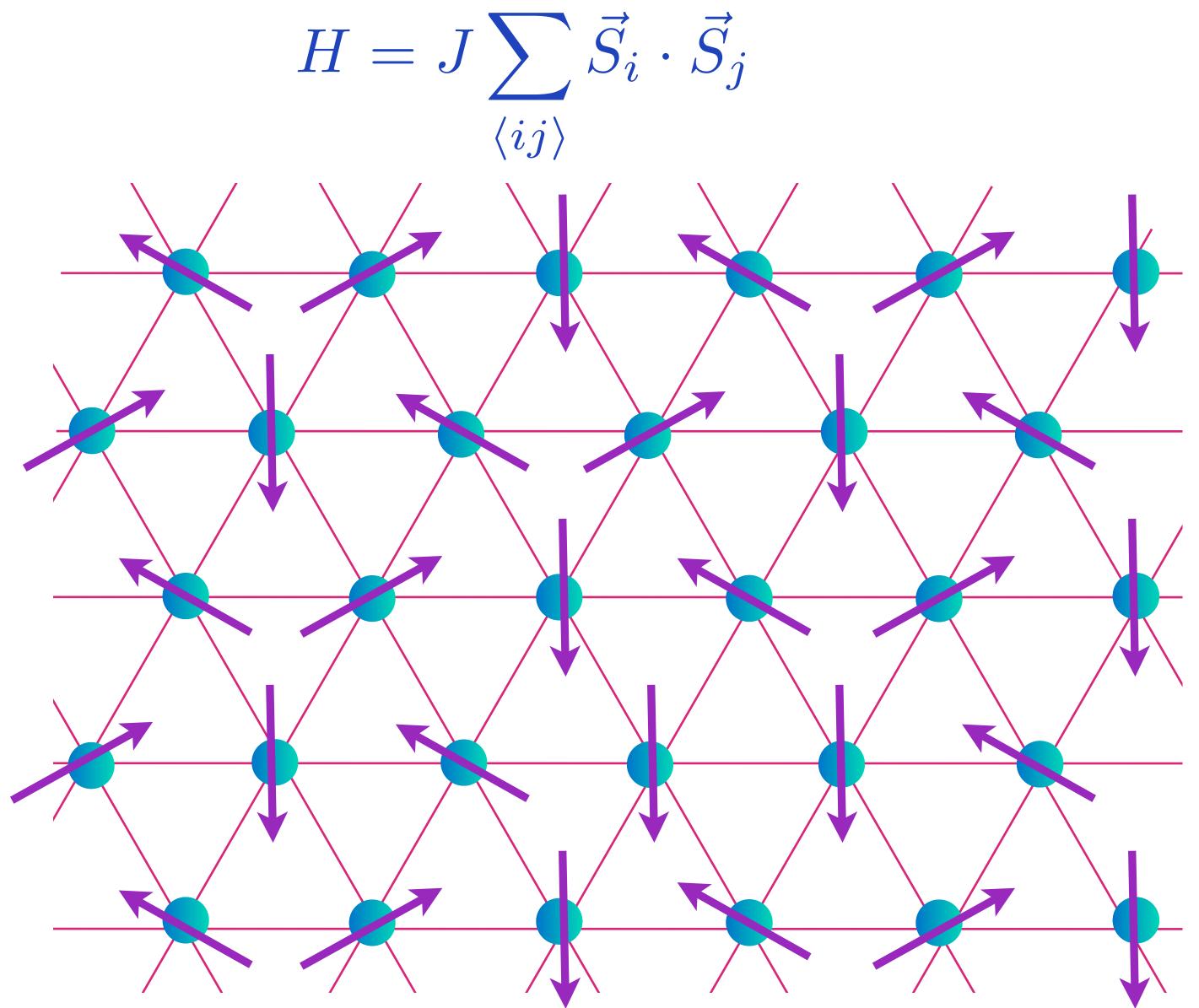


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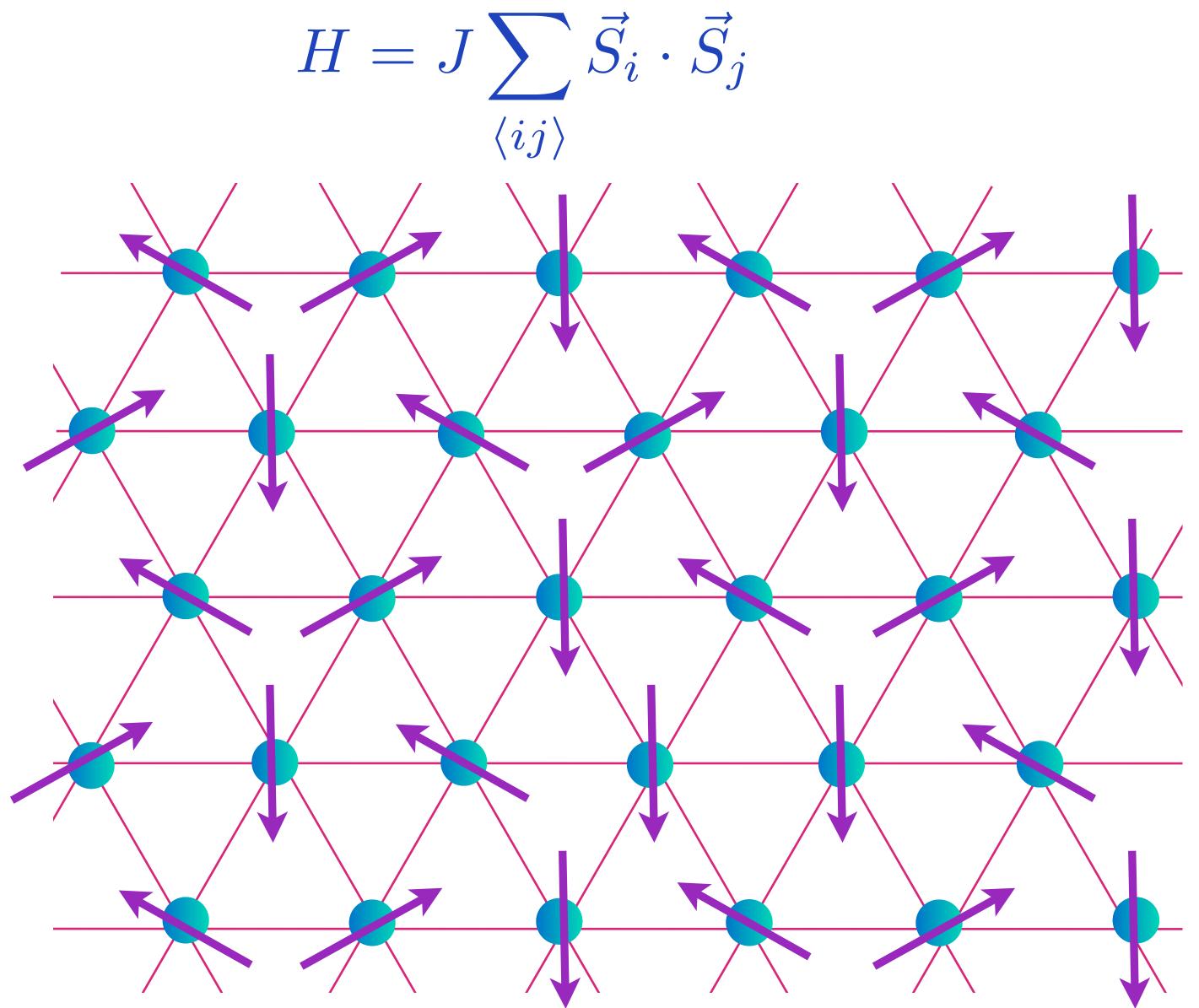
Mott insulator: Triangular lattice antiferromagnet



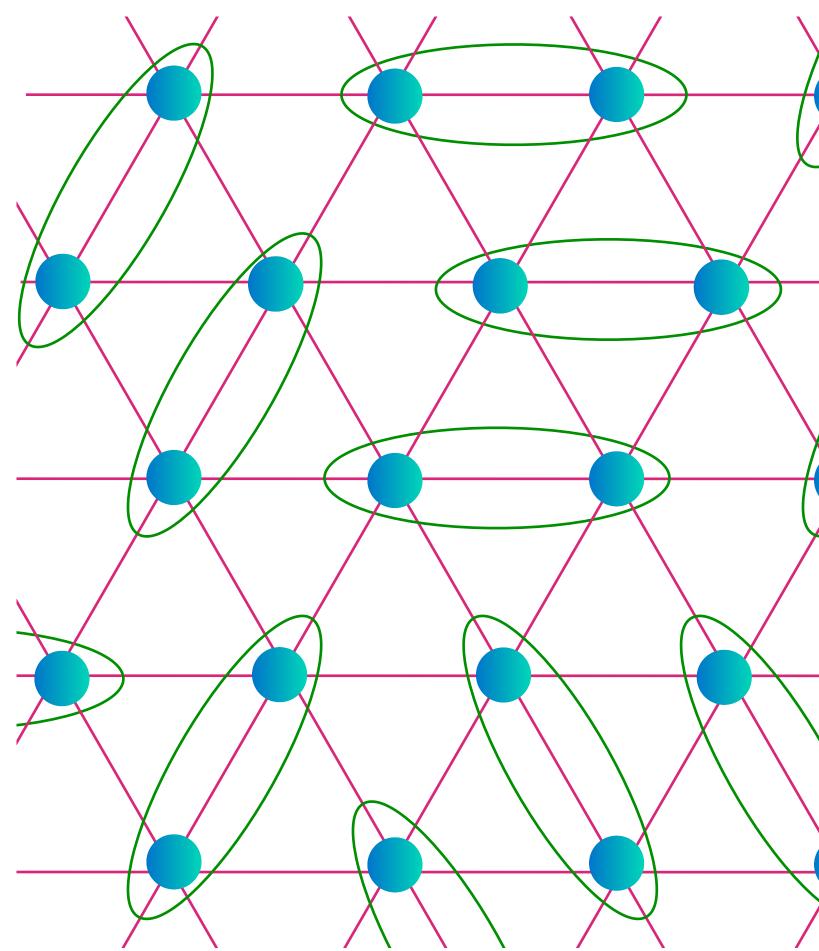
Nearest-neighbor model has non-collinear Neel order

Mapping of bosons and spins Spin Š Boron B $\tilde{\xi}^{z} = \frac{5}{4}$ $B^{+}B \leq I$ states 177 127 State $|0\rangle$, $B^{+}|0\rangle$ Operators Operators $\neq S_{+} = S_{\times} + iS_{\vee}$ $\Rightarrow S_{-} = S_{\times} - iS_{\vee}$ $B^{\dagger}B^{}-\frac{1}{2}\in$

Mott insulator: Triangular lattice antiferromagnet

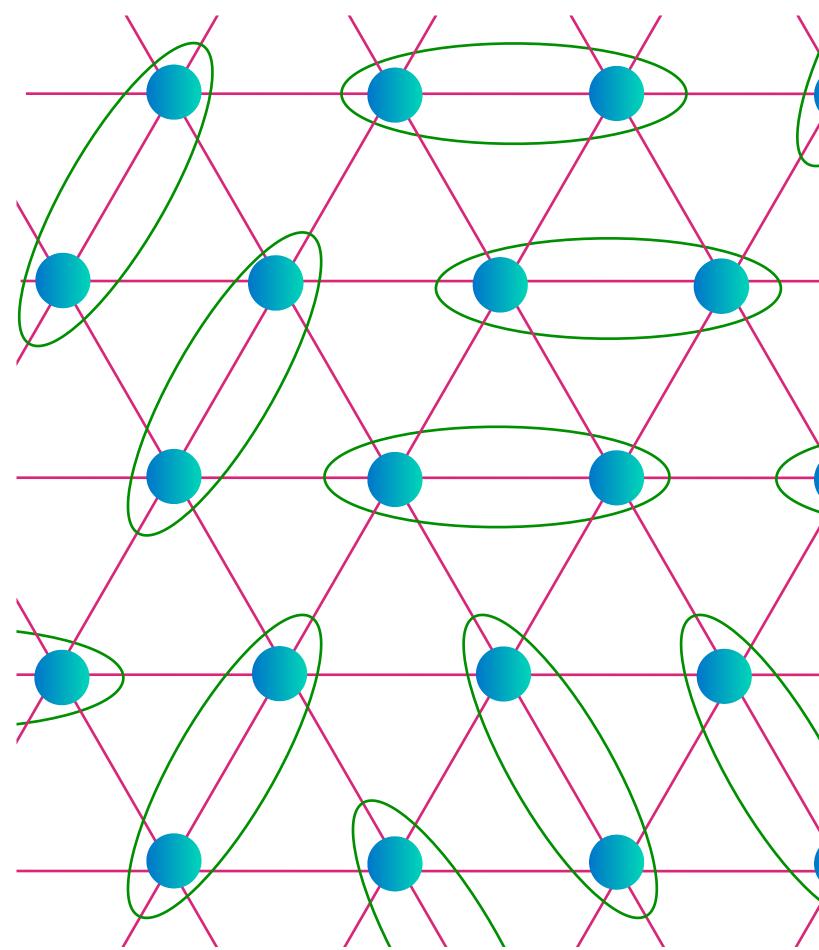


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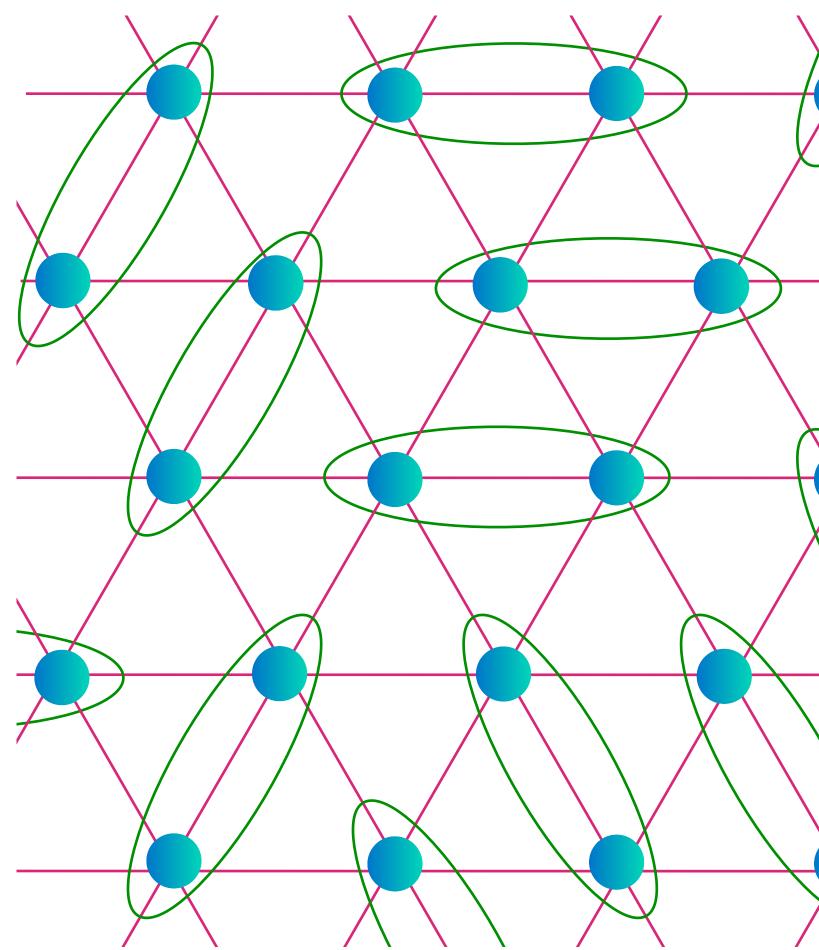
P. Fazekas and P. W. Anderson, *Philos. Mag.* **30**, 23 (1974).





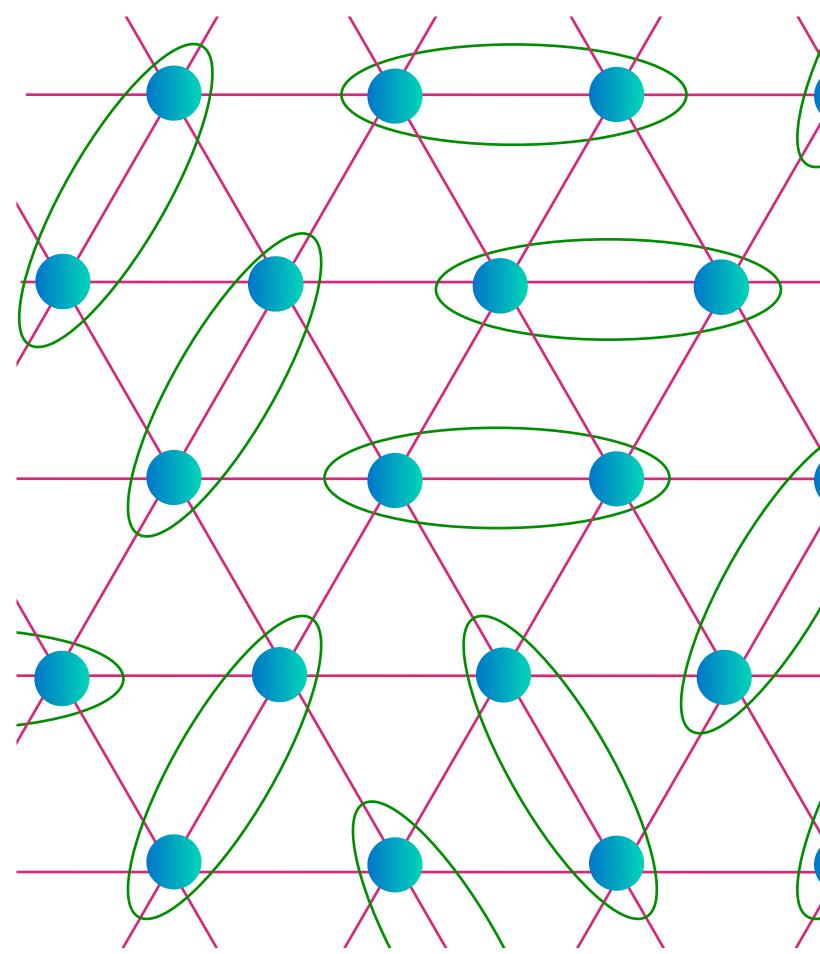
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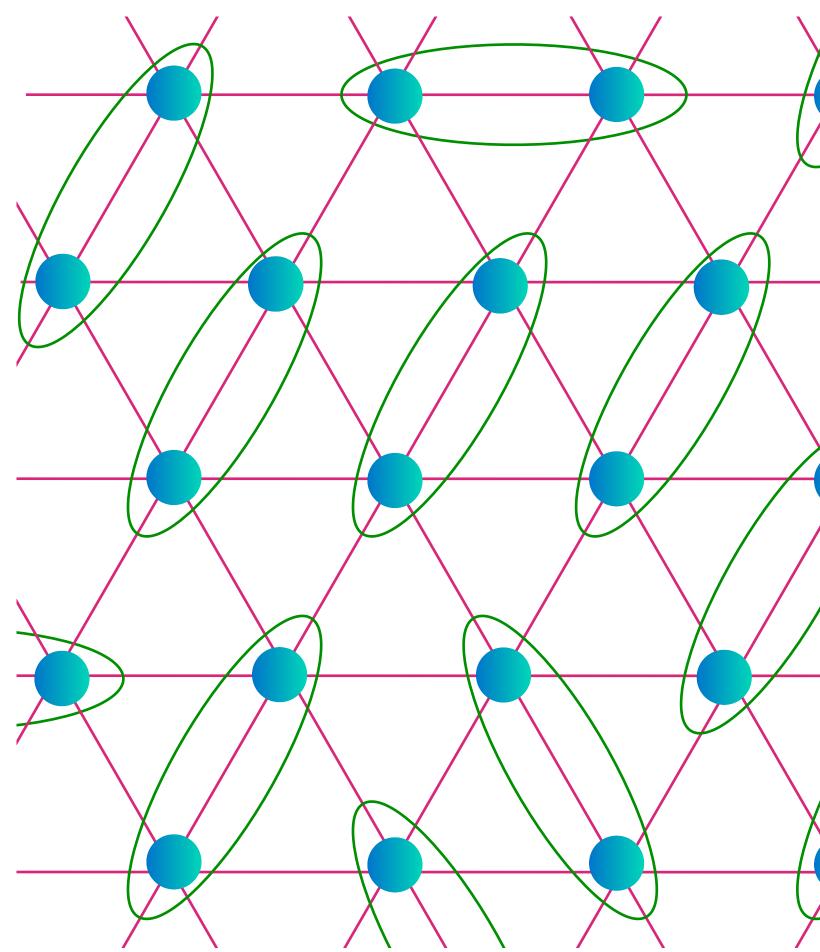
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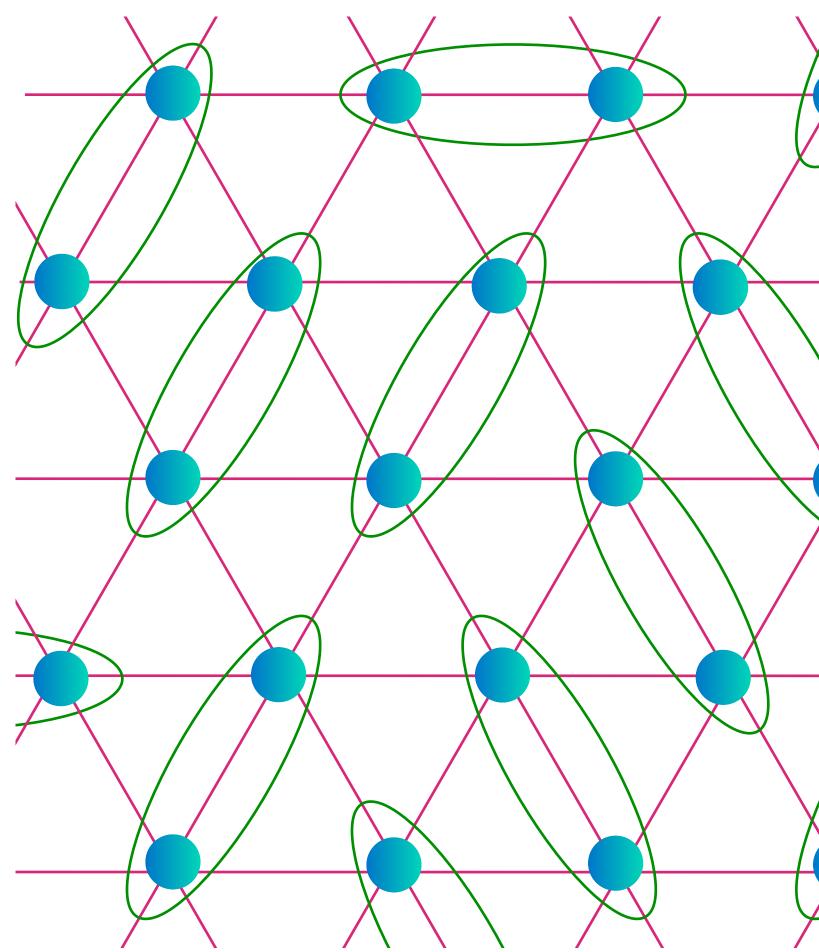




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Spin liquid for bosons at half-filling, or a spin model with S=1/2 per unit cell $=\frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle-\left|\downarrow\uparrow\right\rangle\right)$ $|G\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}} |\mathcal{D}\rangle$ $\mathcal{D} \to \text{dimer covering}$ of lattice

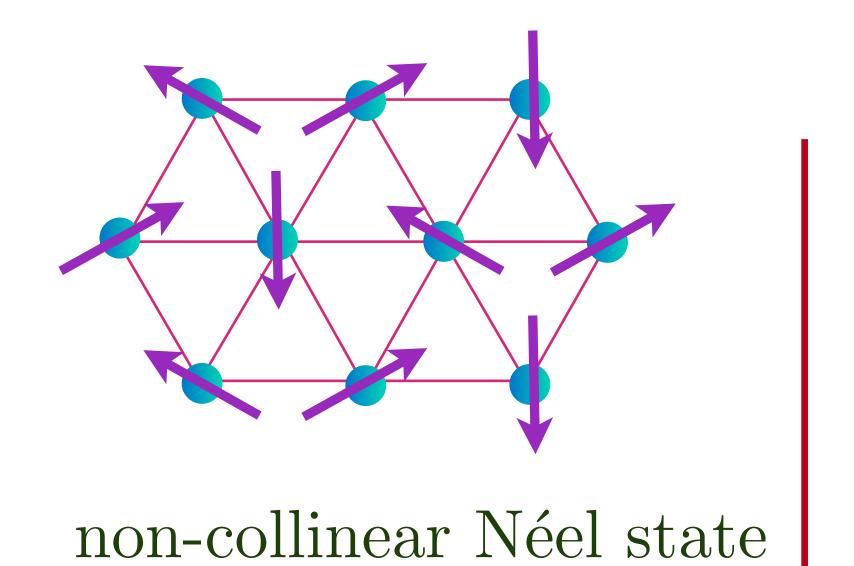


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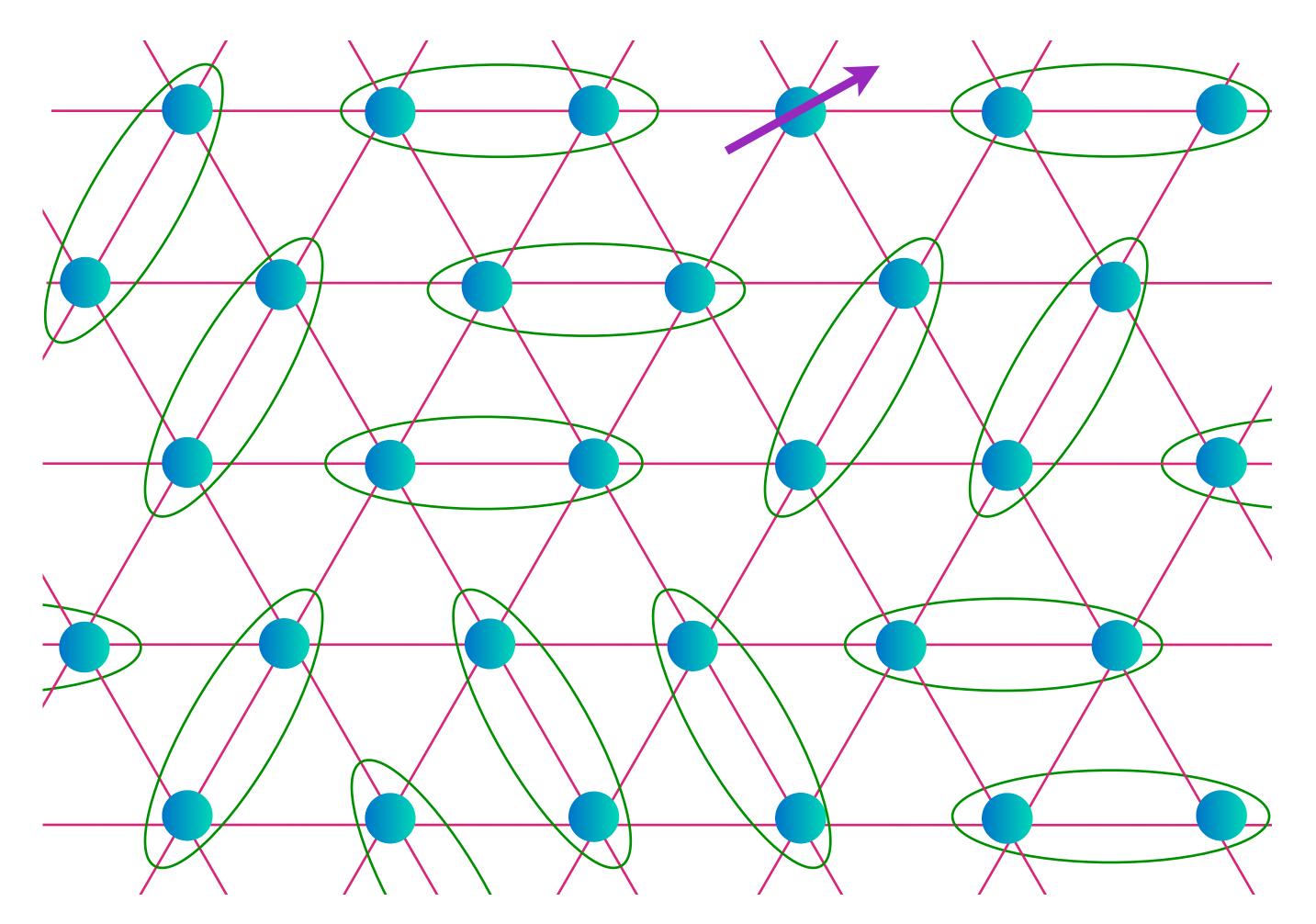
 S_{C}

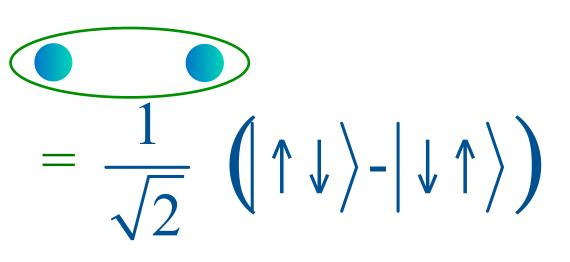
Z_2 spin liquid with neutral S = 1/2 spinons and **vison** excitations

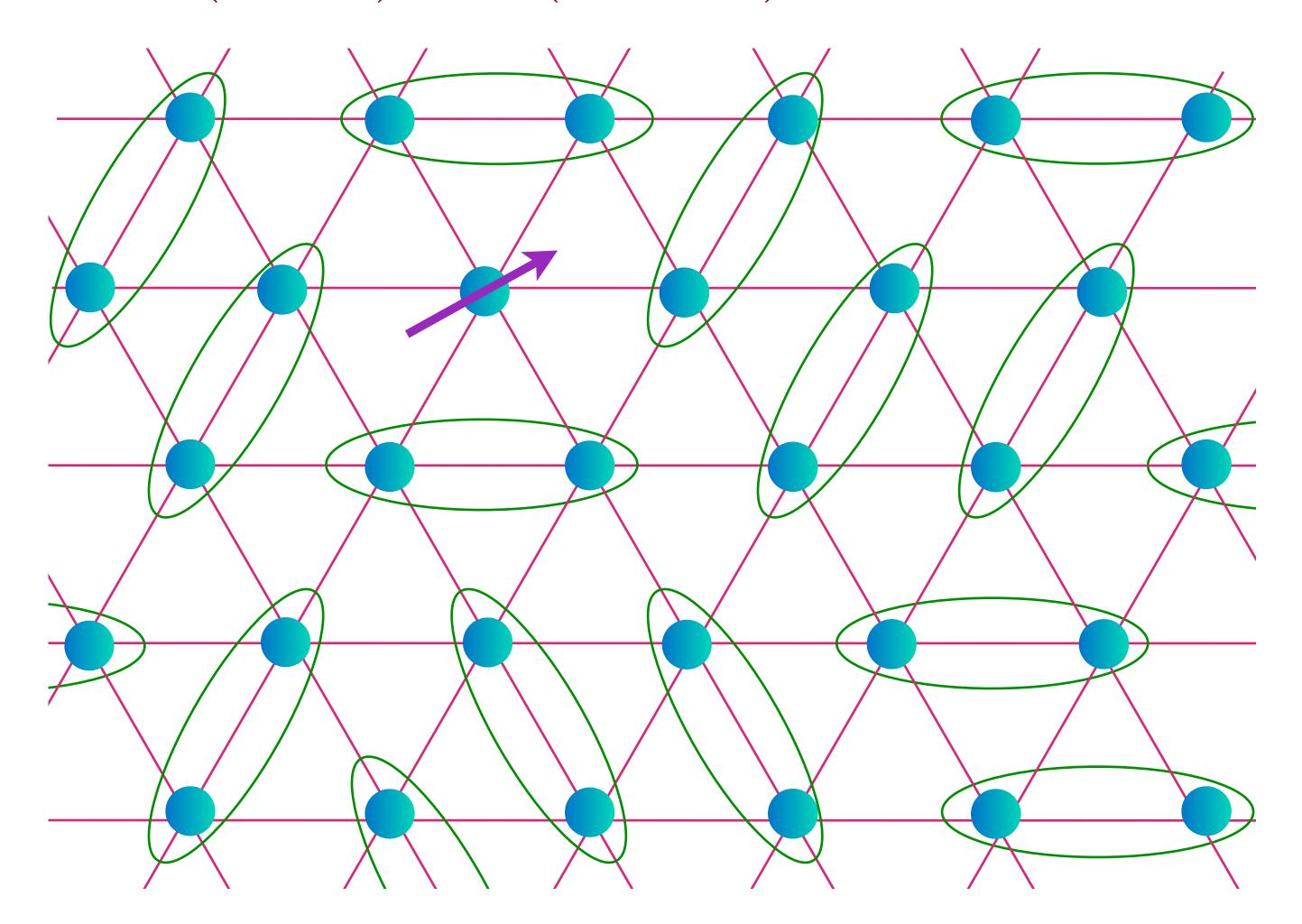


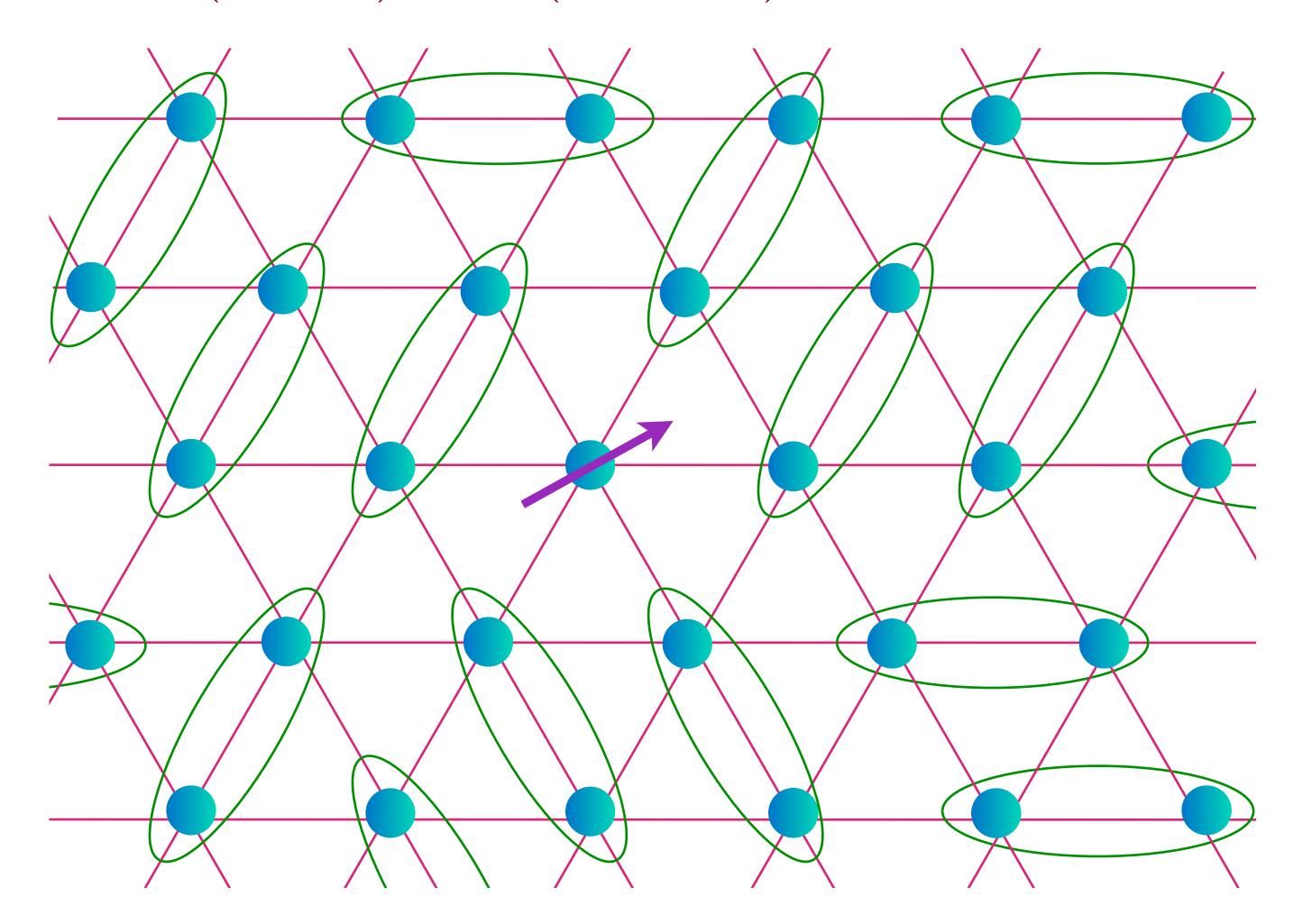
N. Read and S. Sachdev, Phys. Rev. Lett. 66, 1773 (1991) X.-G. Wen, *Phys. Rev. B* 44, 2664 (1991)

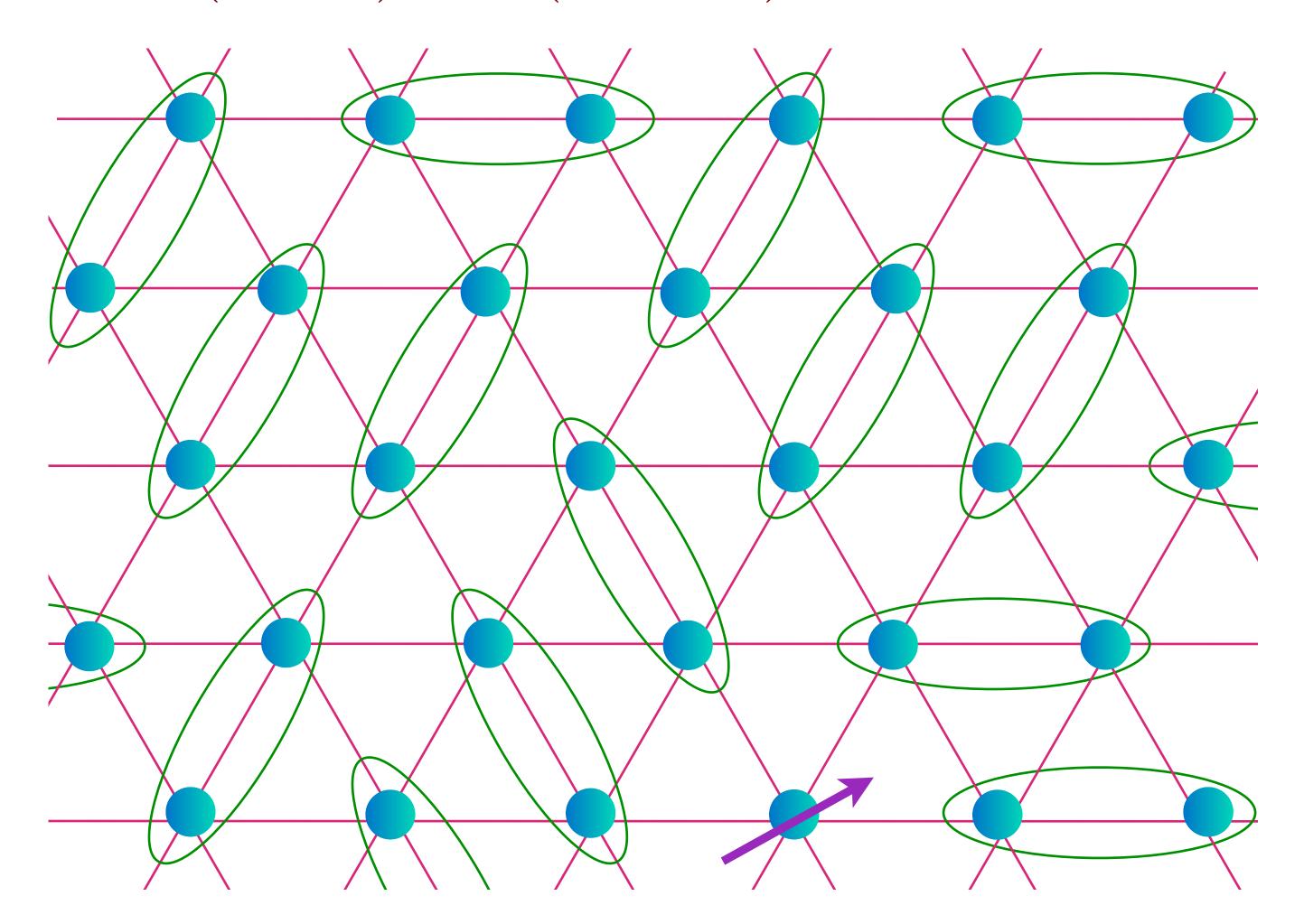


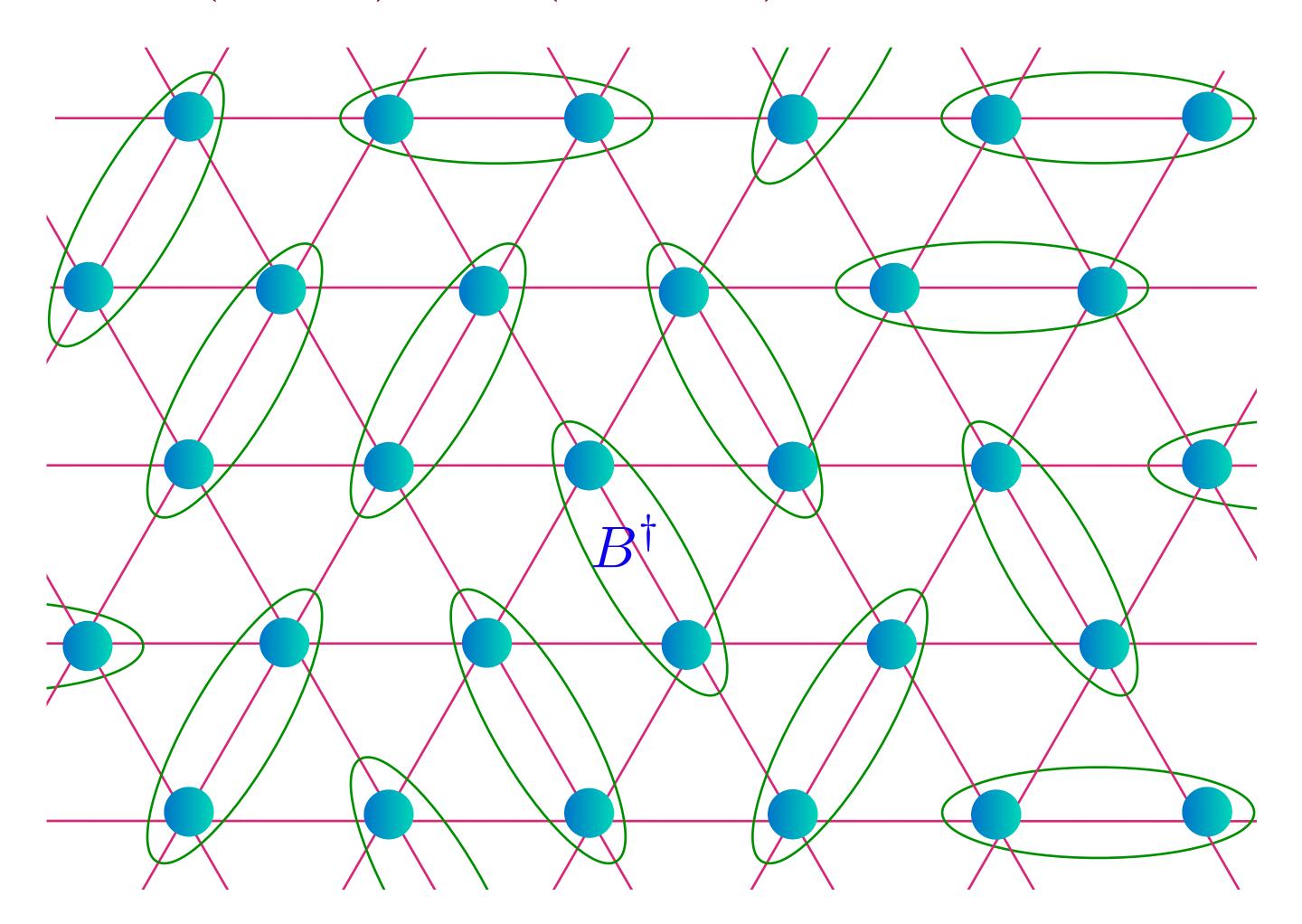




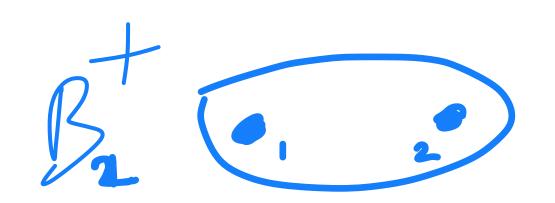






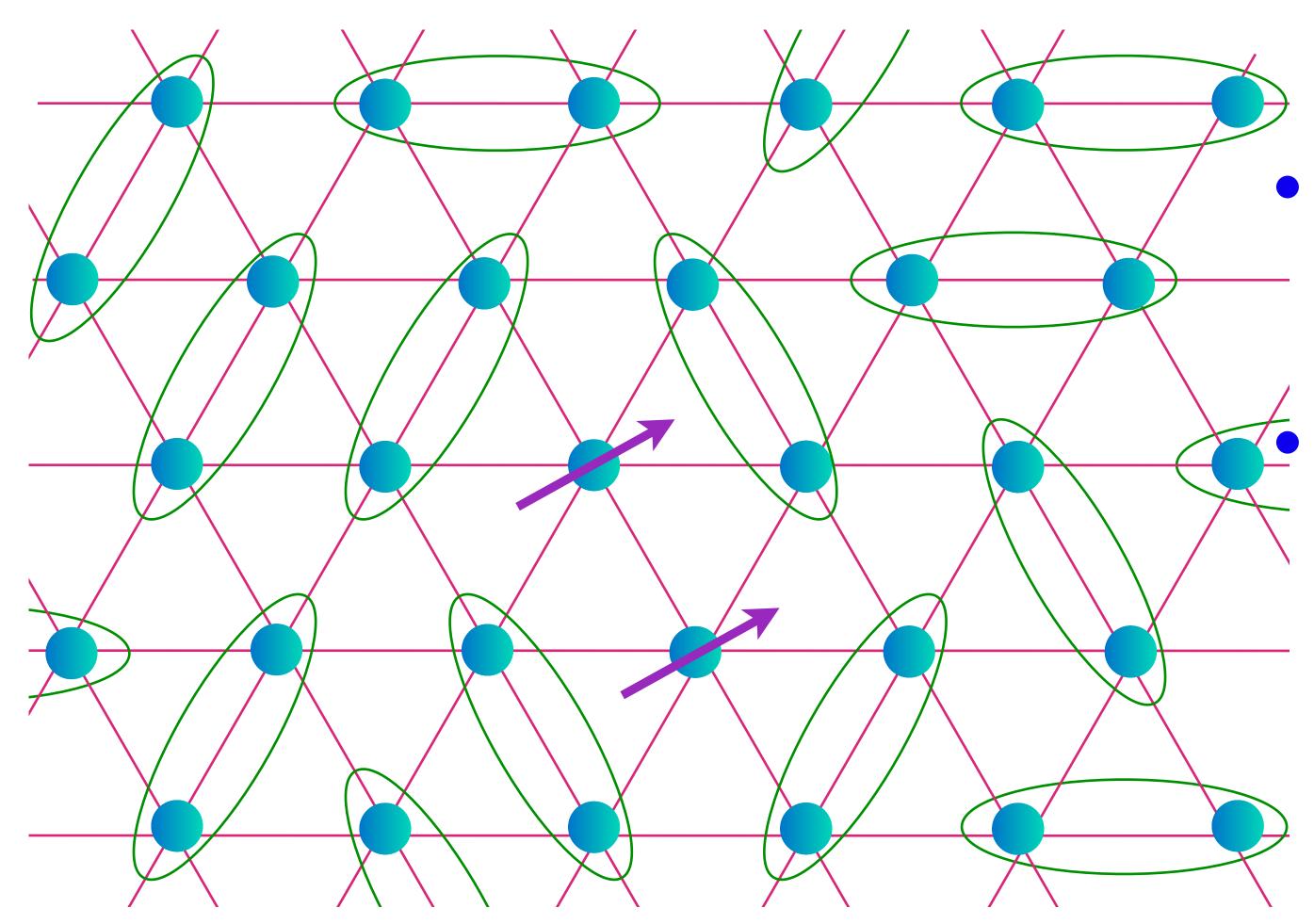


 $=\frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle-\left|\downarrow\uparrow\right\rangle\right)$



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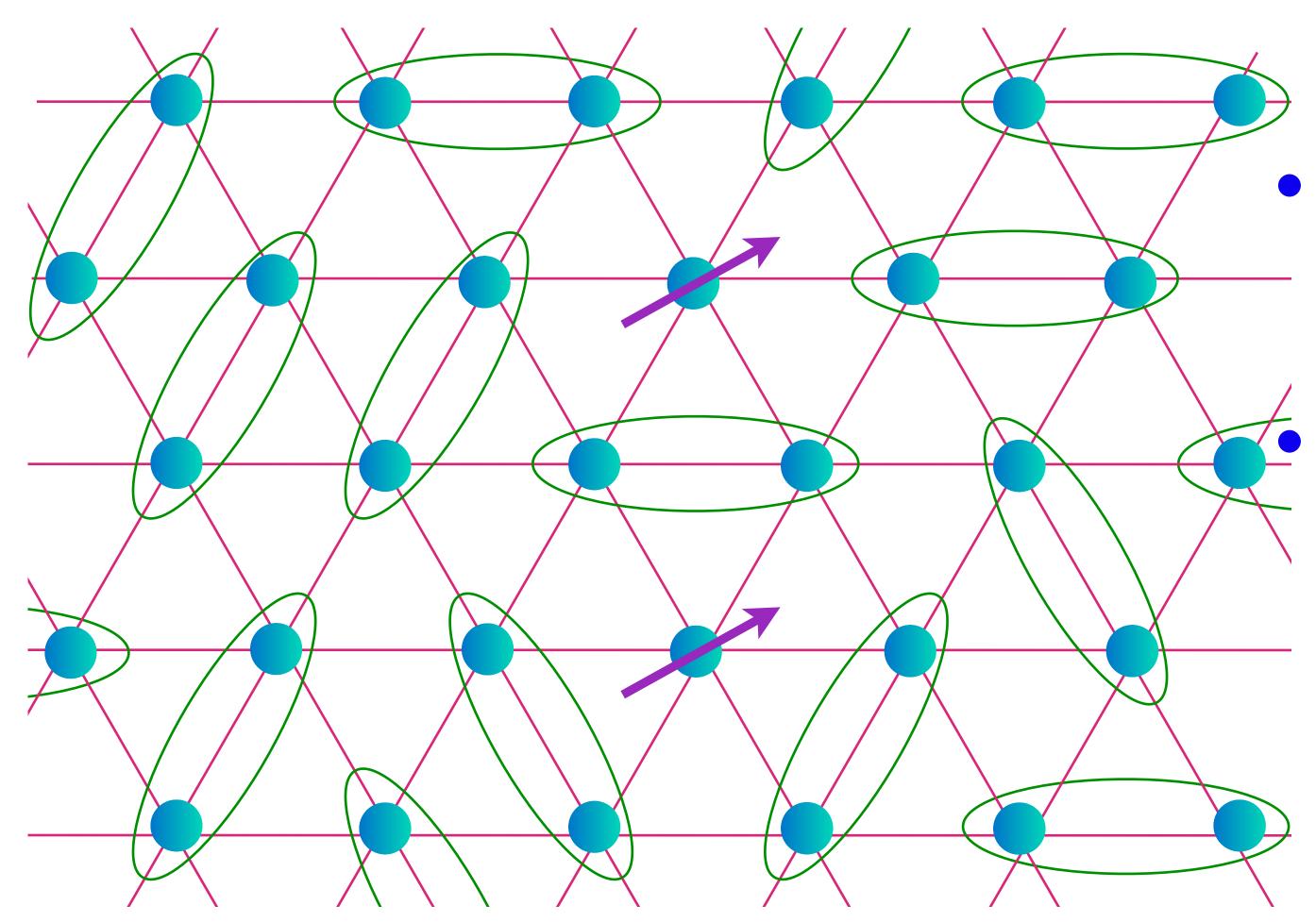


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• Spinons can only be created in pairs by a local operator $(e.g. B^{\dagger})$





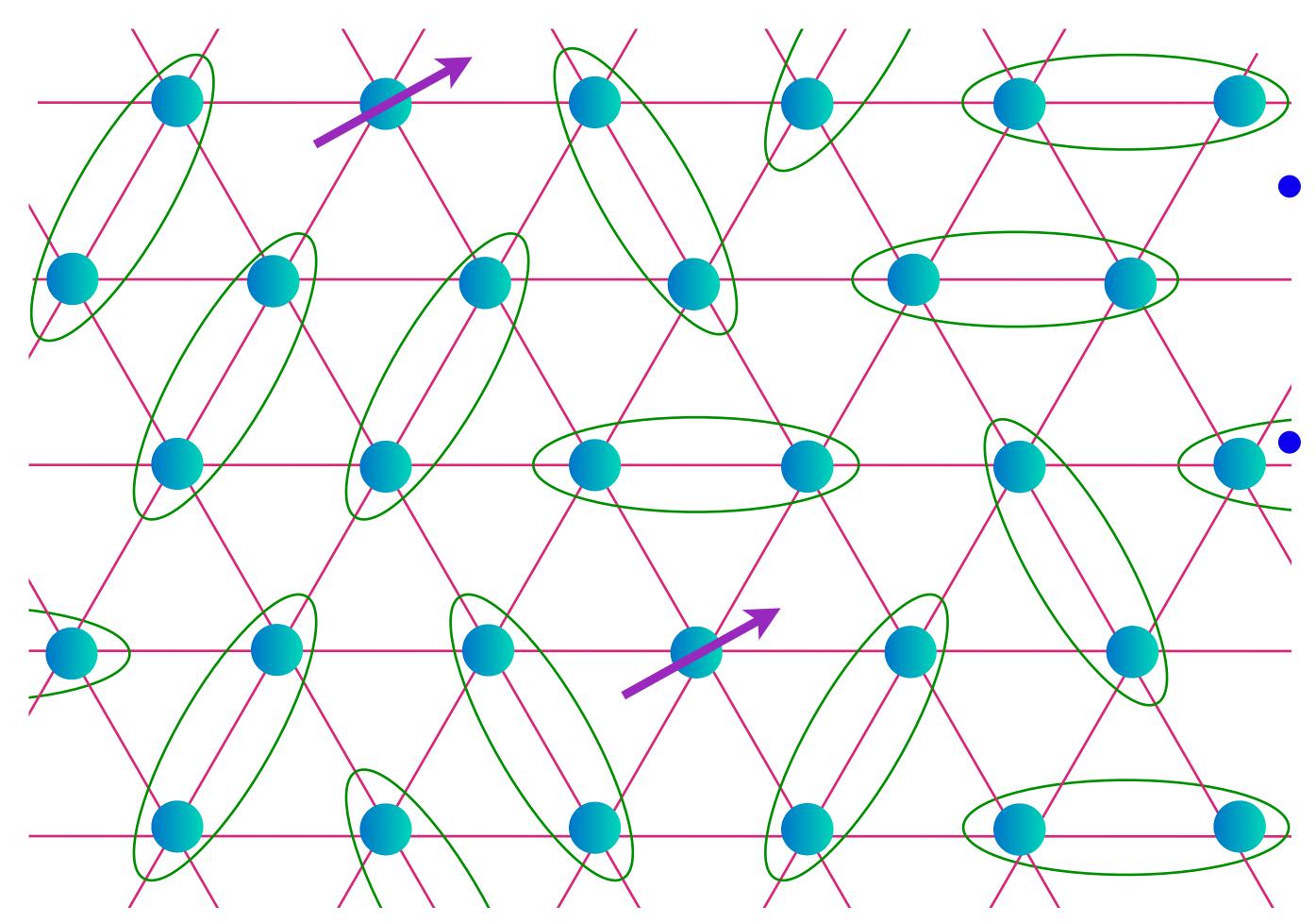


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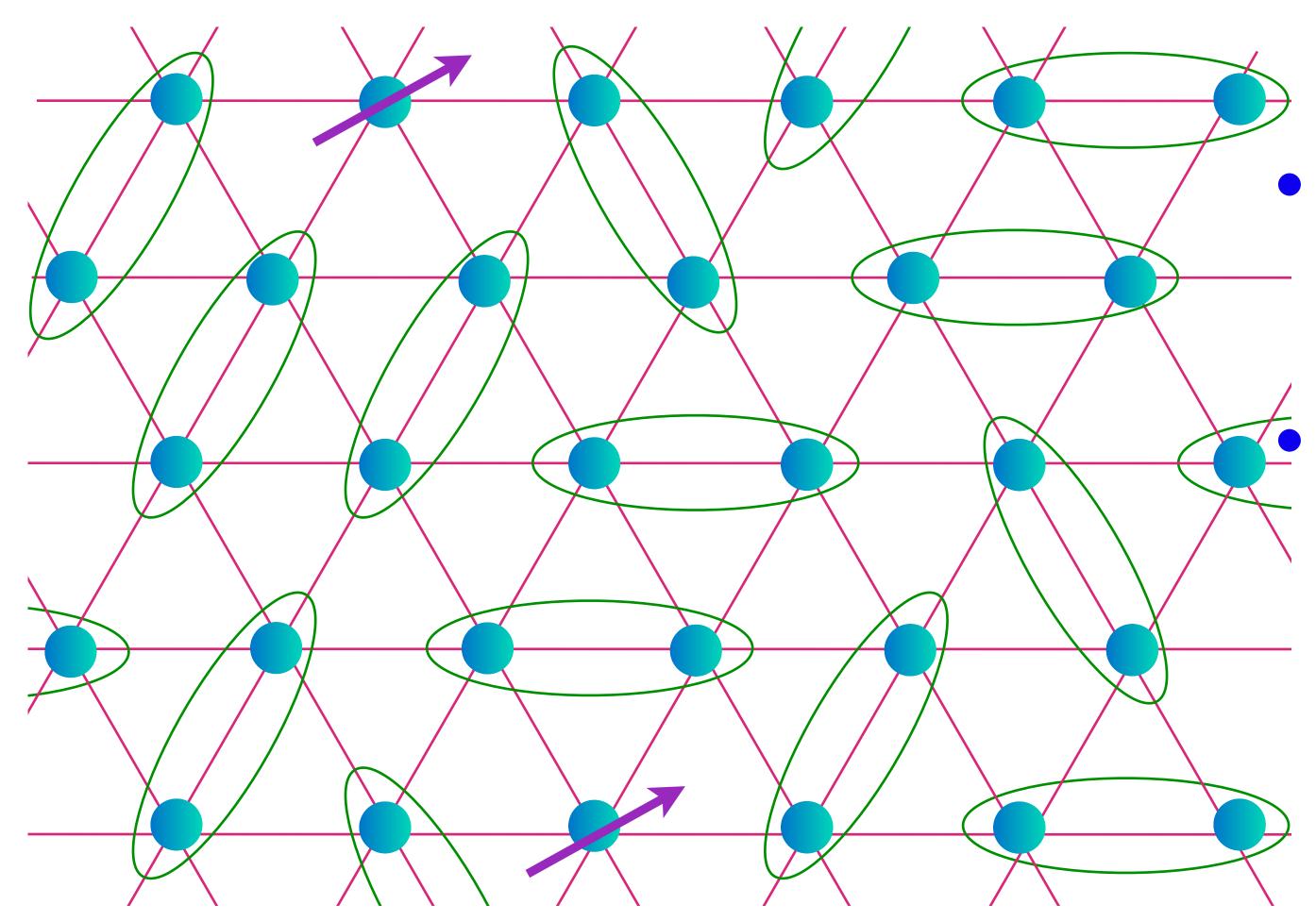


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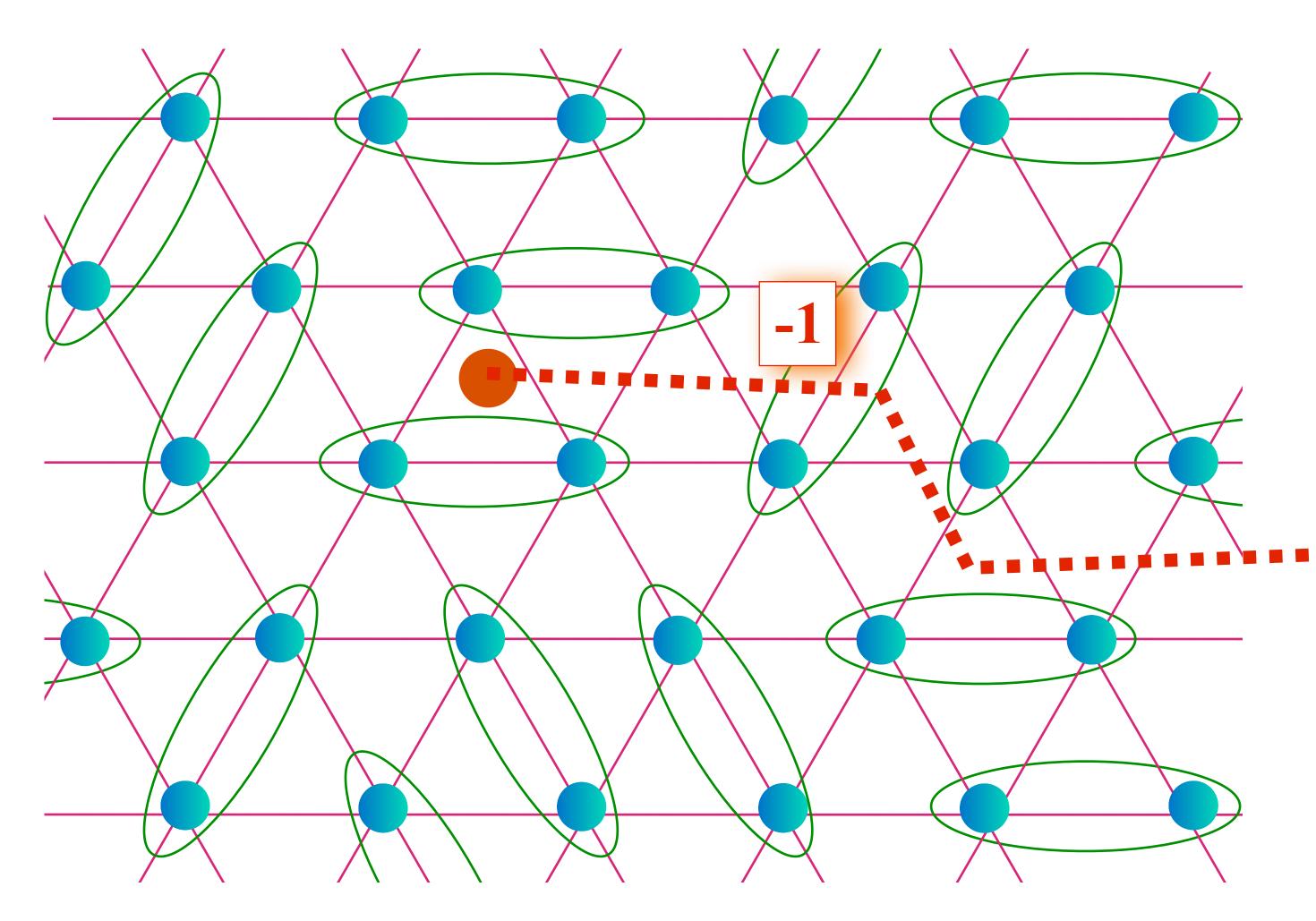
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<u>A vison</u> m (boson) particle





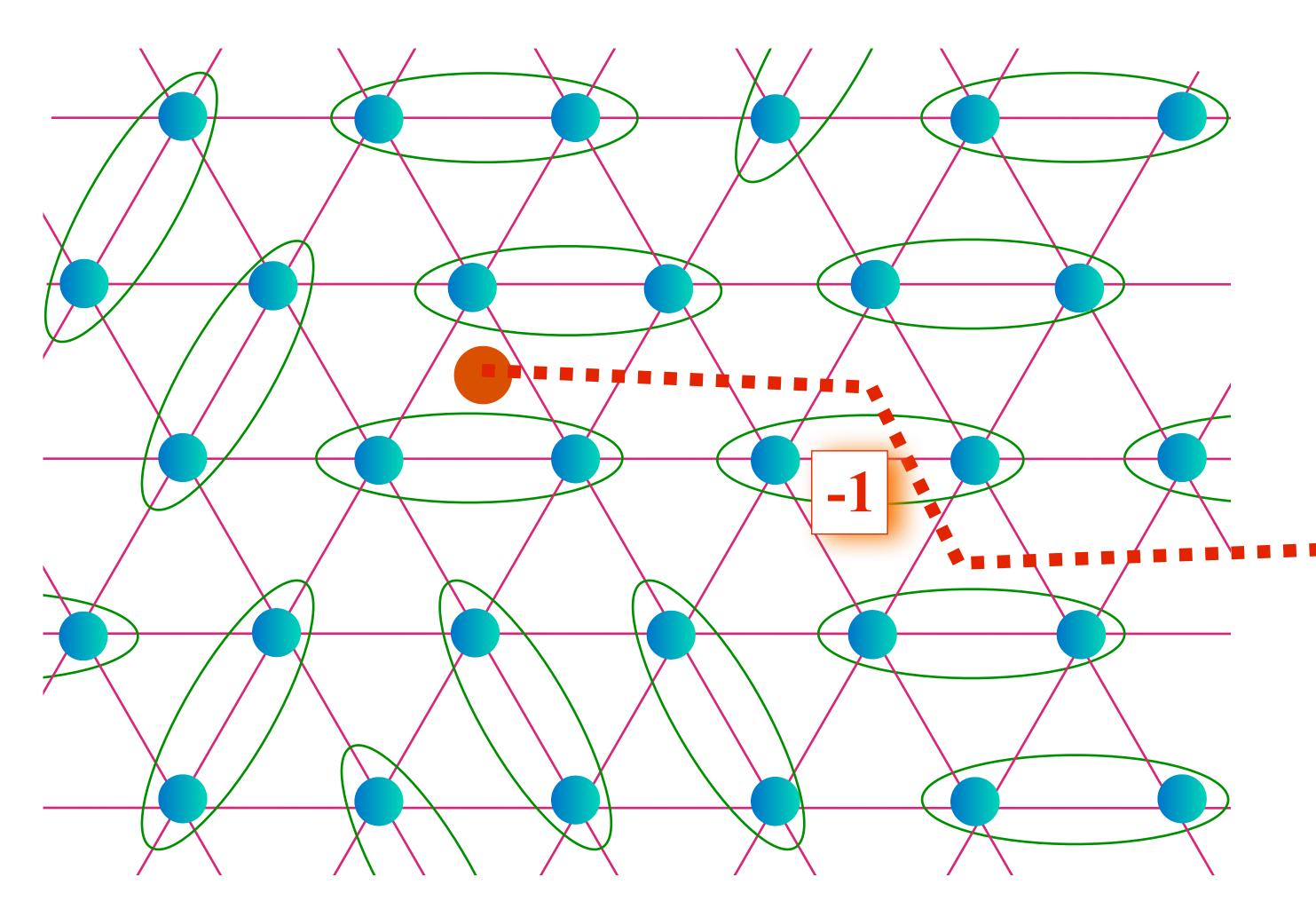
 $|v\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}} (-1)^{n_{\mathcal{D}}} |\mathcal{D}\rangle$

- $\mathcal{D} \to \text{dimer covering}$
 - of lattice
- $n_{\mathcal{D}} \to \text{number of dimens}$
 - crossing red line





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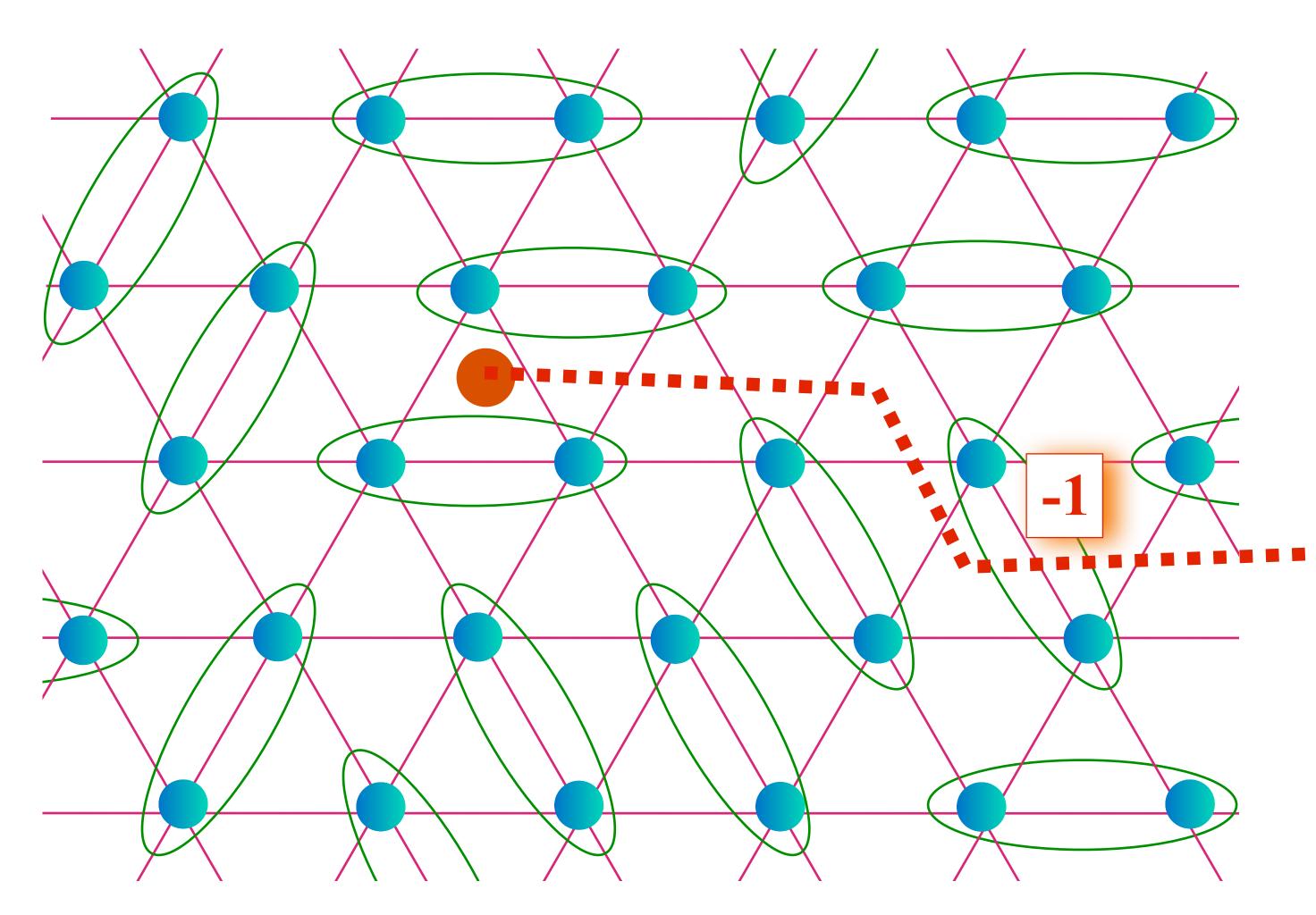
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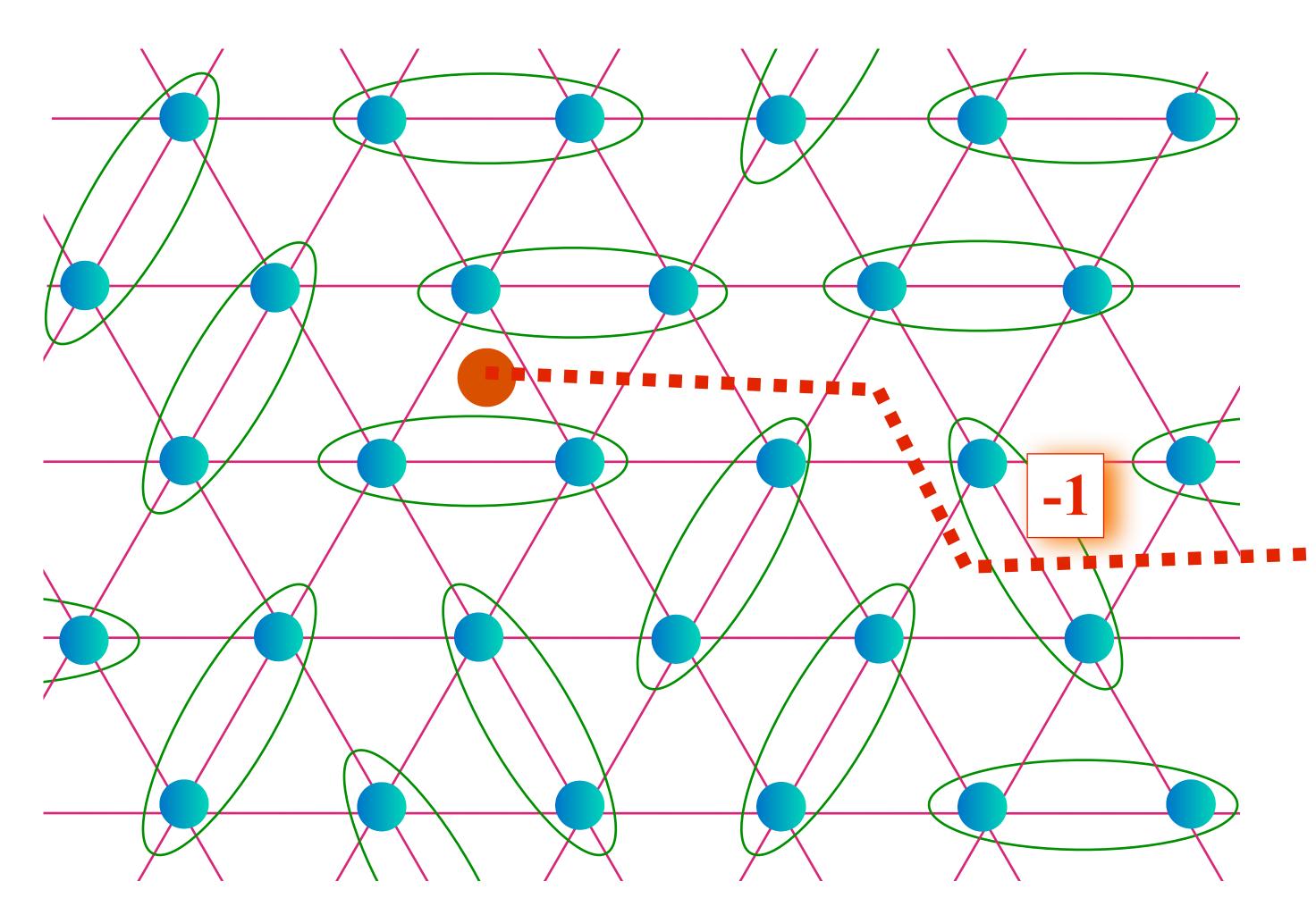
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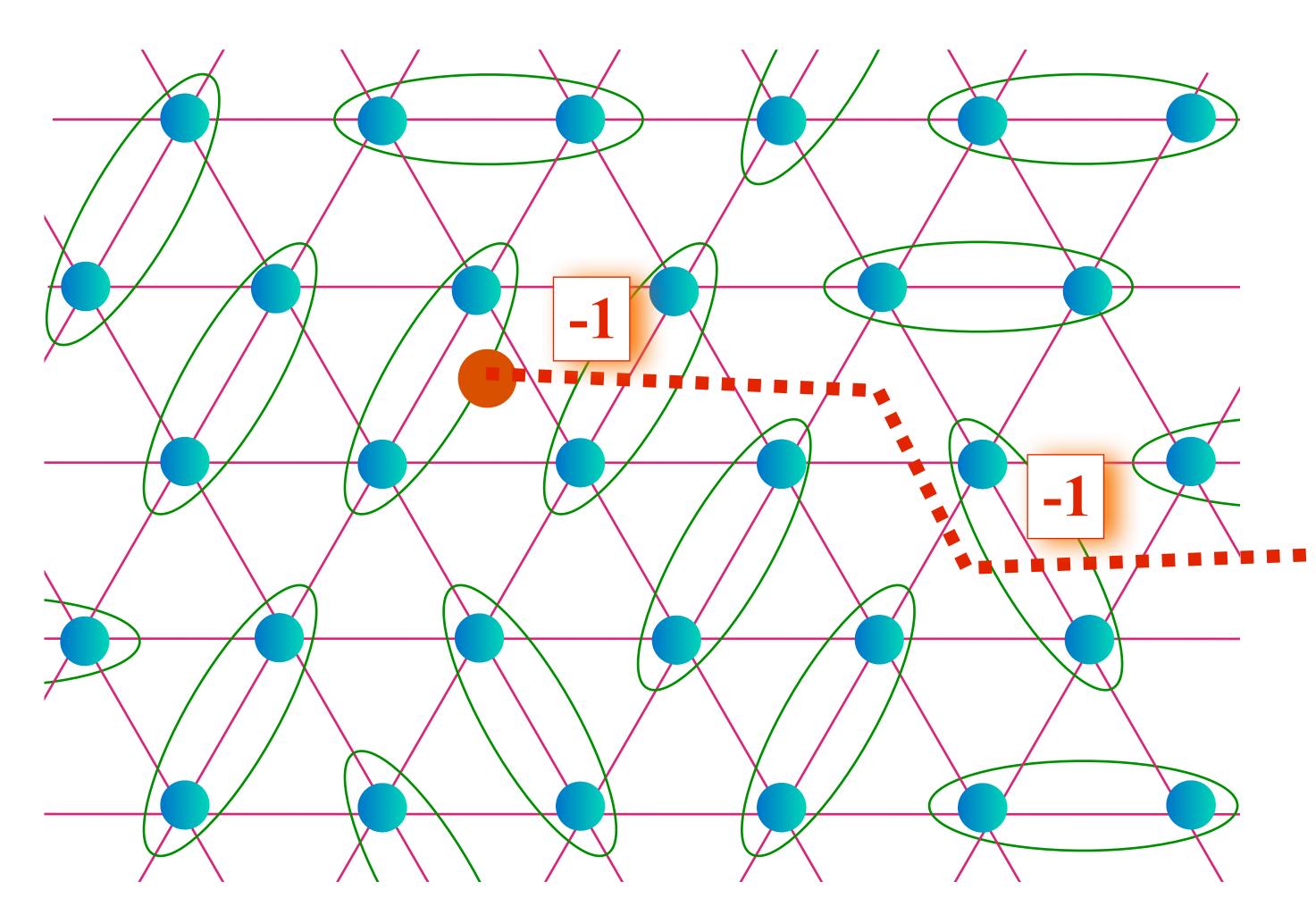
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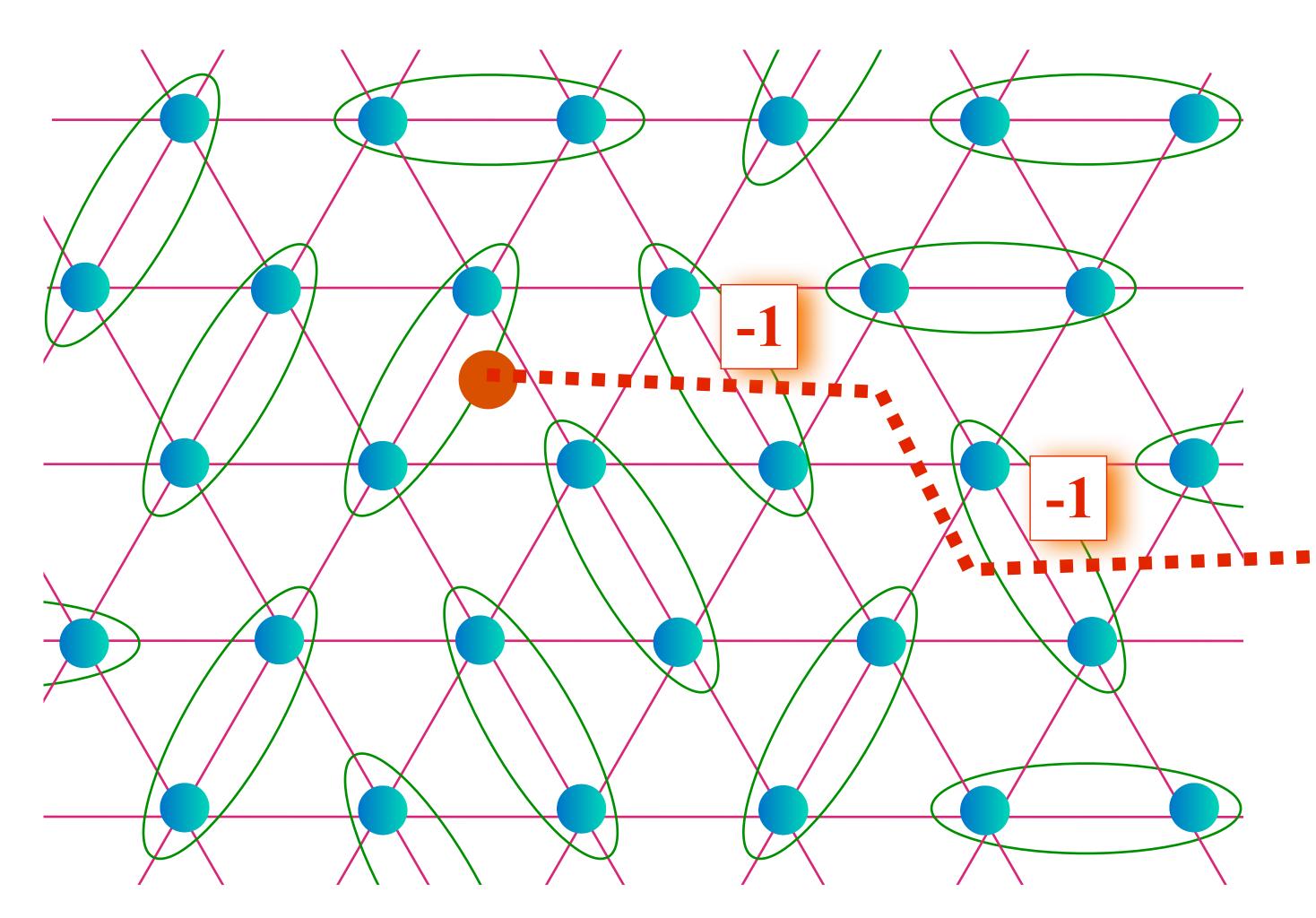
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 - crossing red line





Excitations of the Z₂ Spin liquid

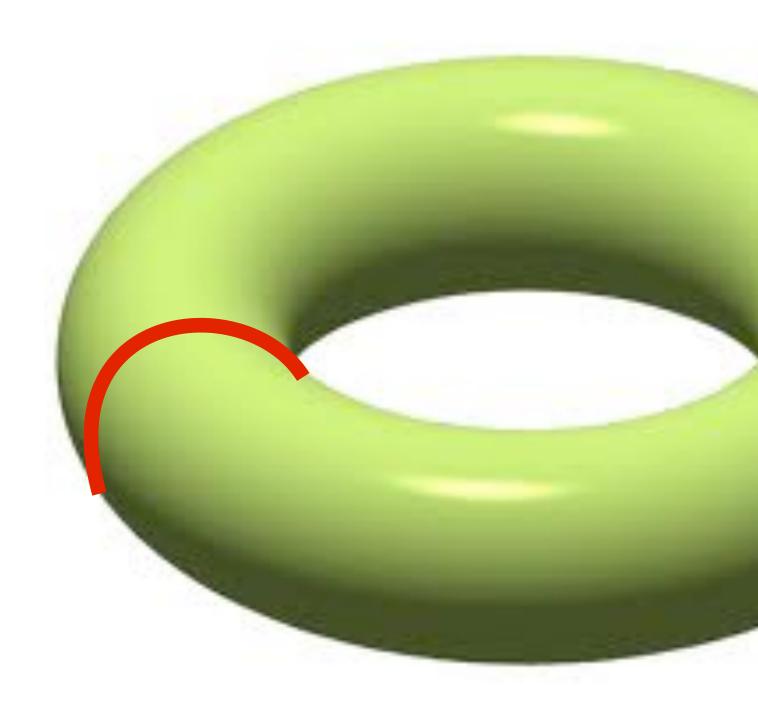
- A spinon adiabatically transported around a vison picks up a phase factor of -1: spinons and visons are mutual semions.
- A bound state of a spinon and a vison picks up a phase factor of −1 when exchanged with another bound state of a spinon and a vison:
 - The ϵ spinon (fermion) is a bound state of the *e* spinon (boson) and a vison ($\epsilon = e \times m$).
 - The The *e* spinon (boson) is a bound state of the ϵ spinon (fermion) and a vison ($e = \epsilon \times m$).

Ground state degeneracy on the torus



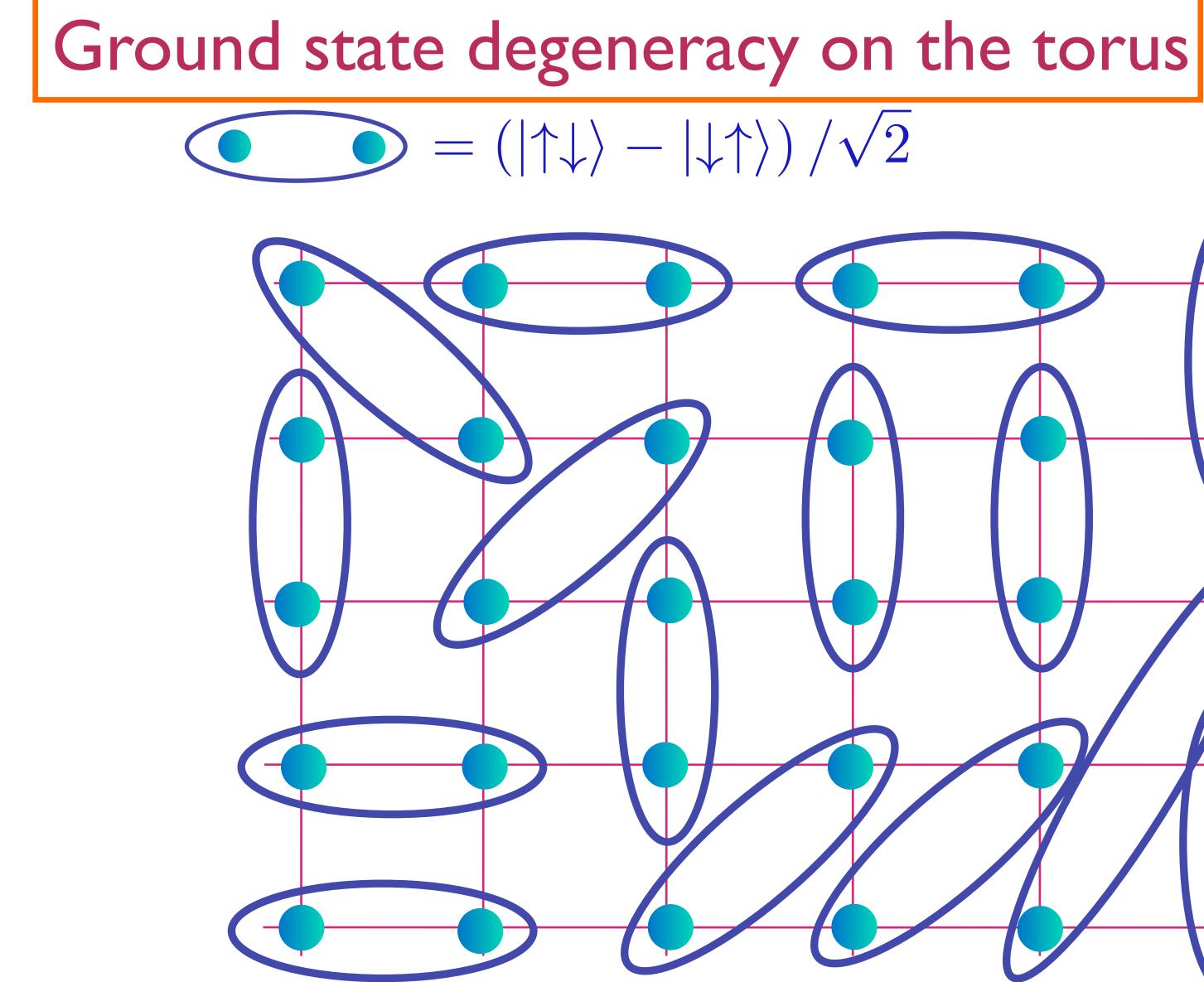
Place insulator on a torus:

Ground state degeneracy on the torus

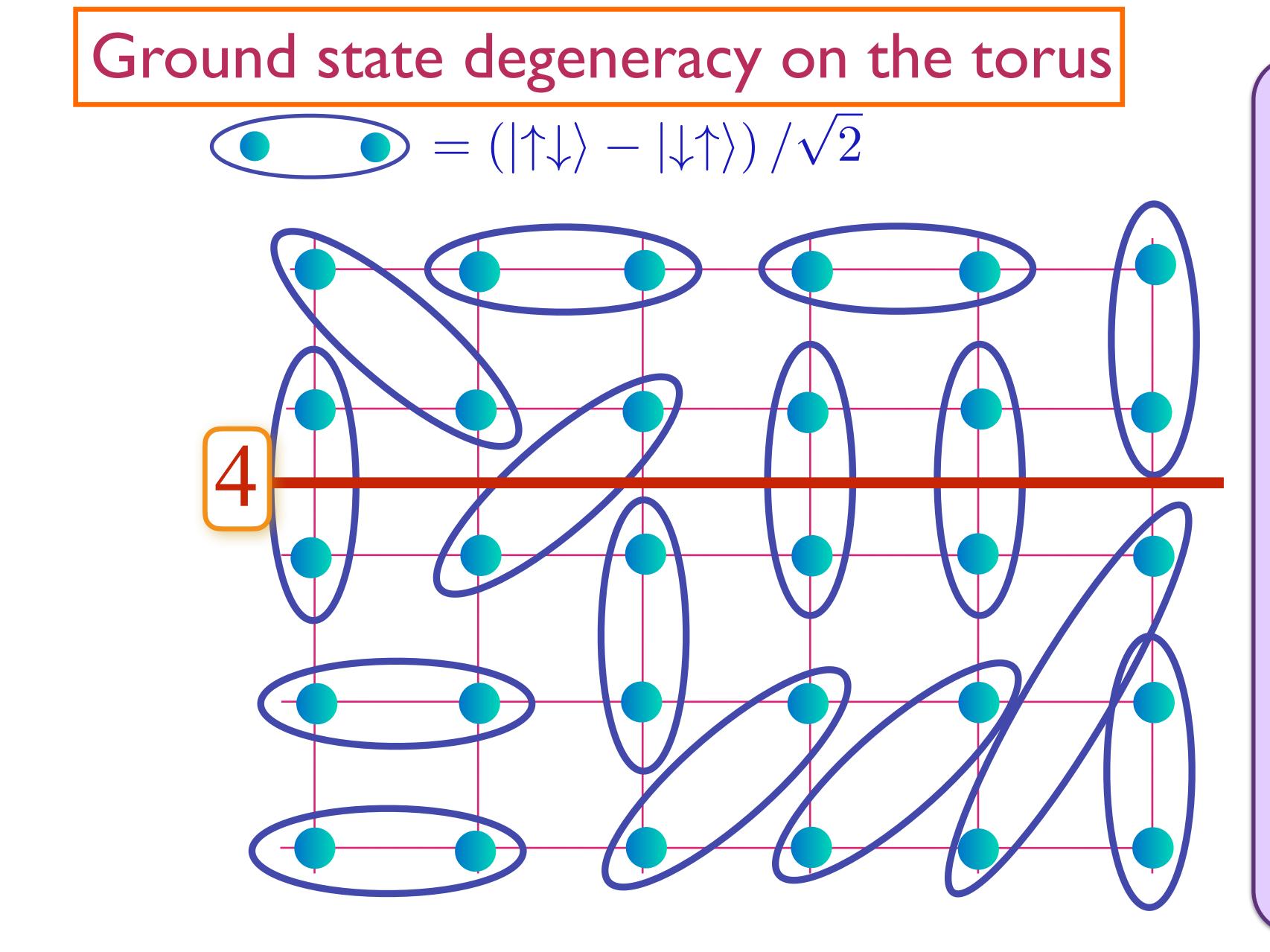


Place insulator on a torus:

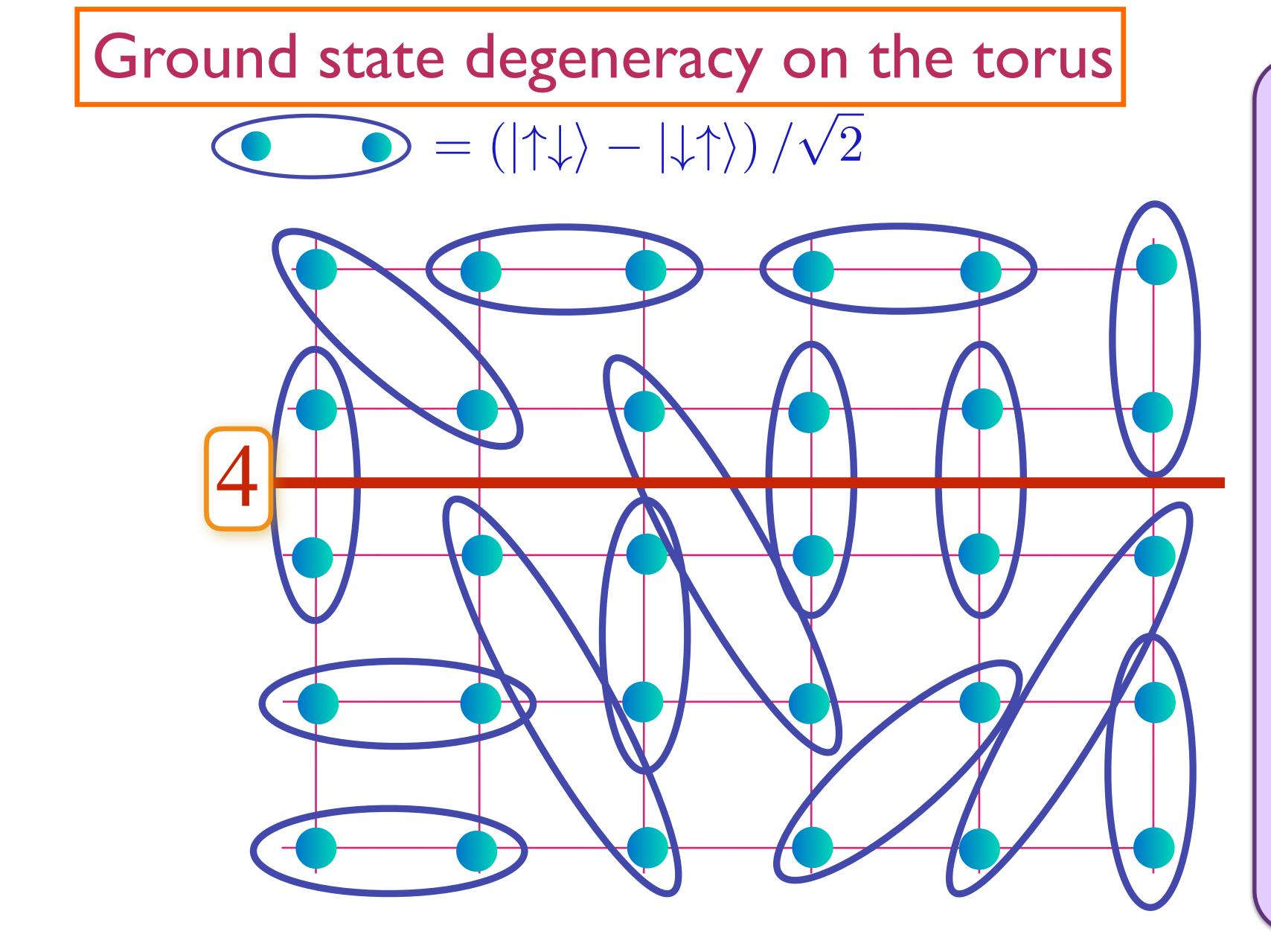
Obtain a degenerate orthogonal state by modifying the wavefunction on a "branch-cut" encircling the torus.



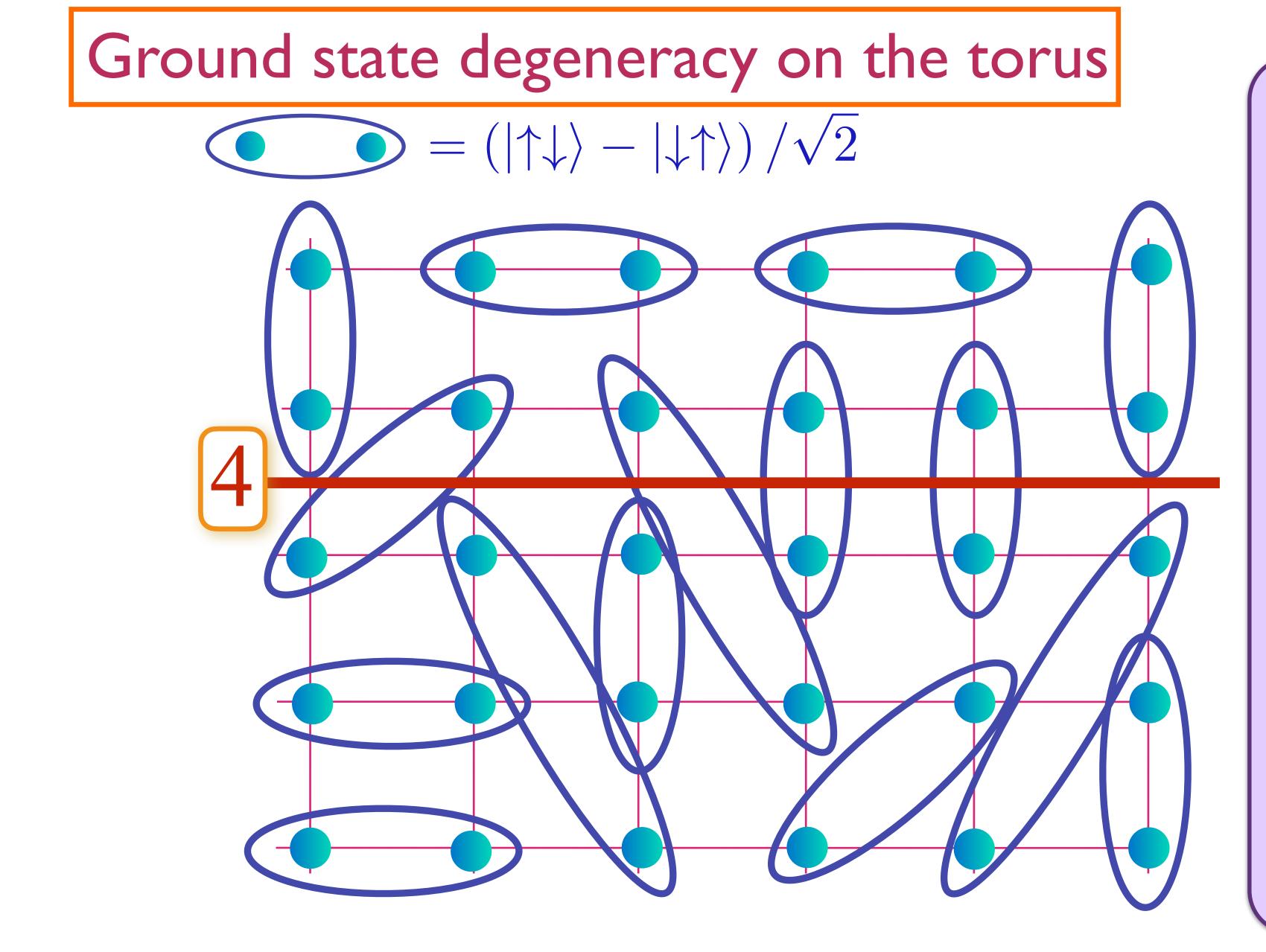
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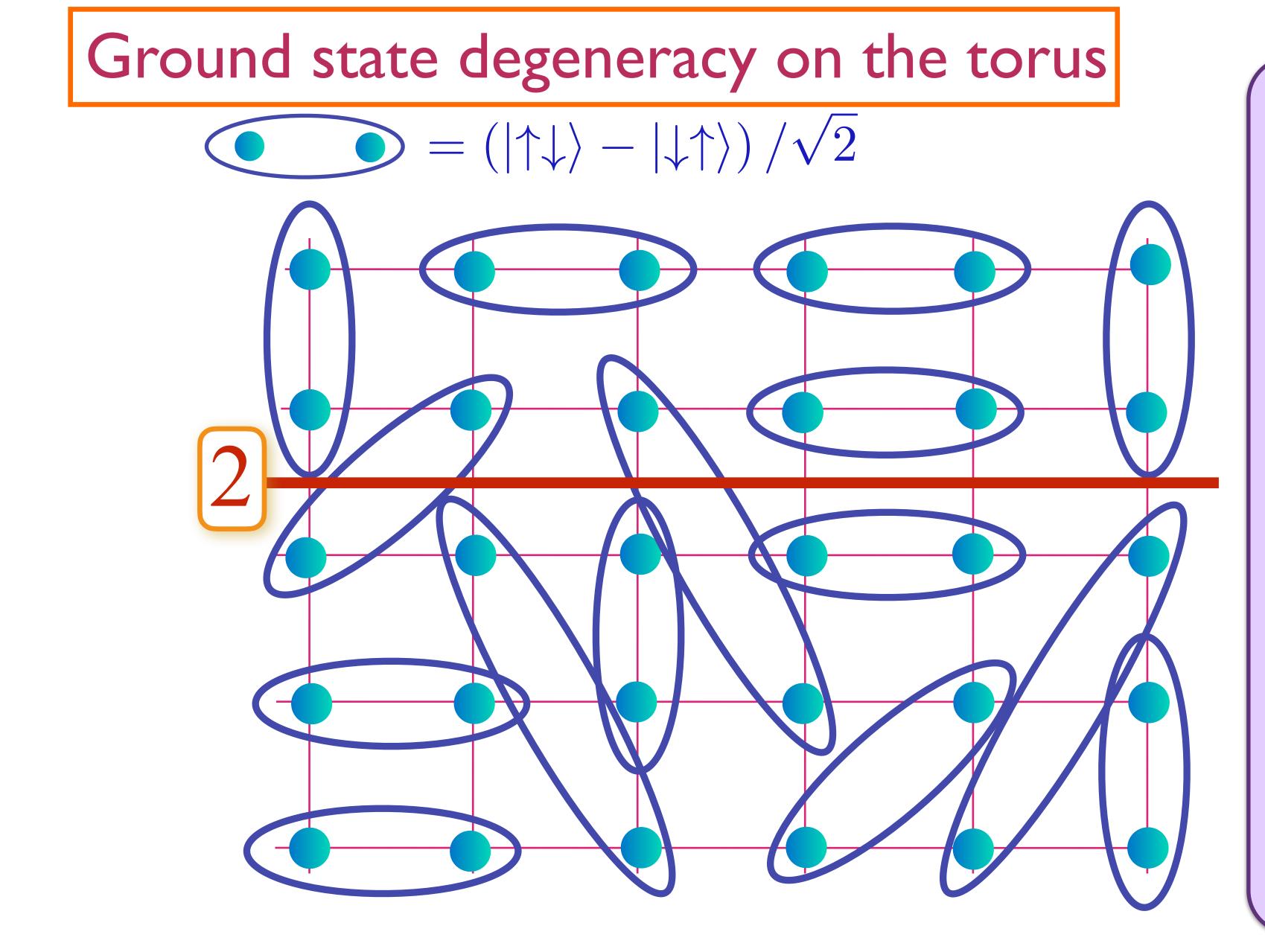
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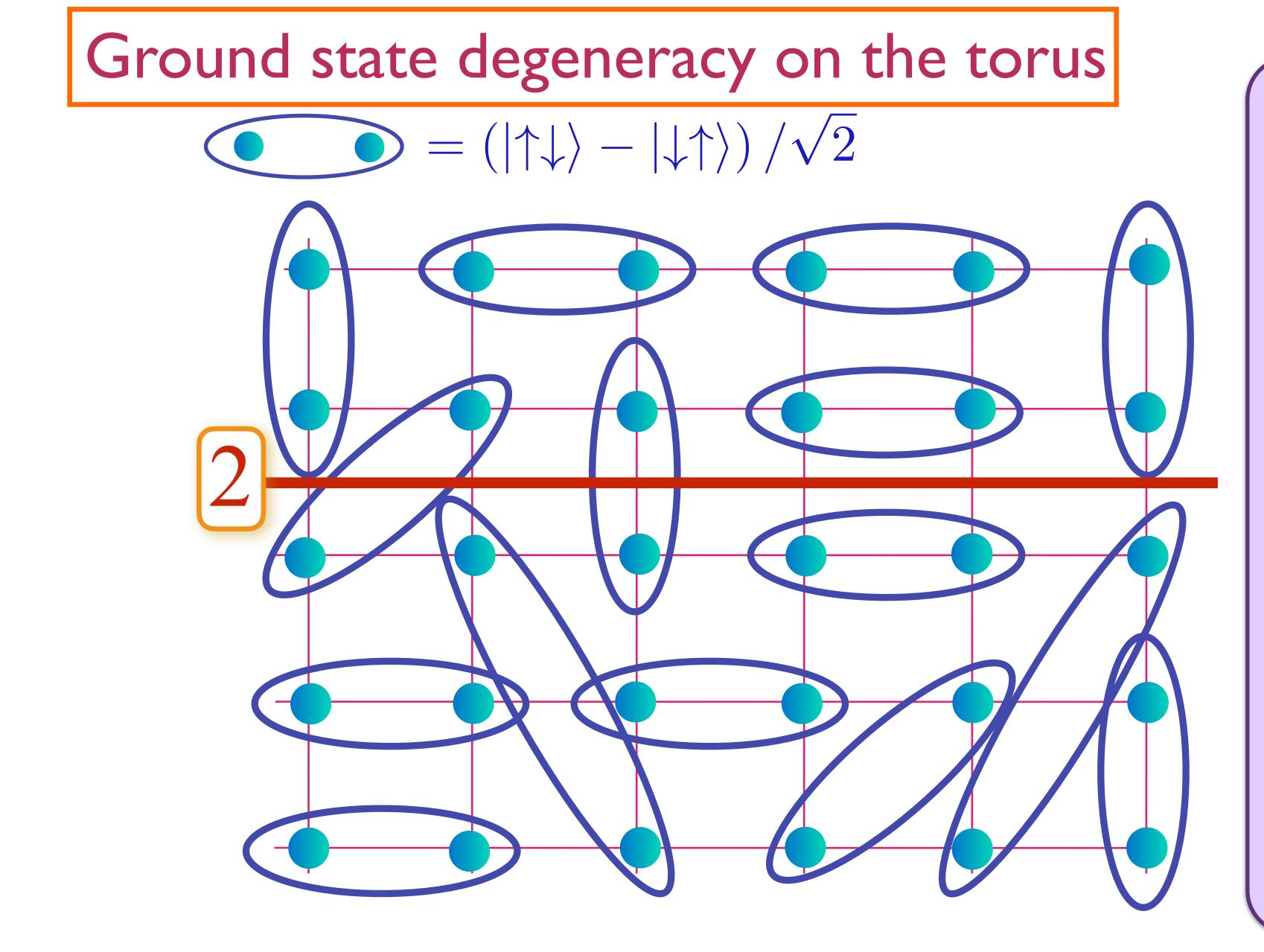
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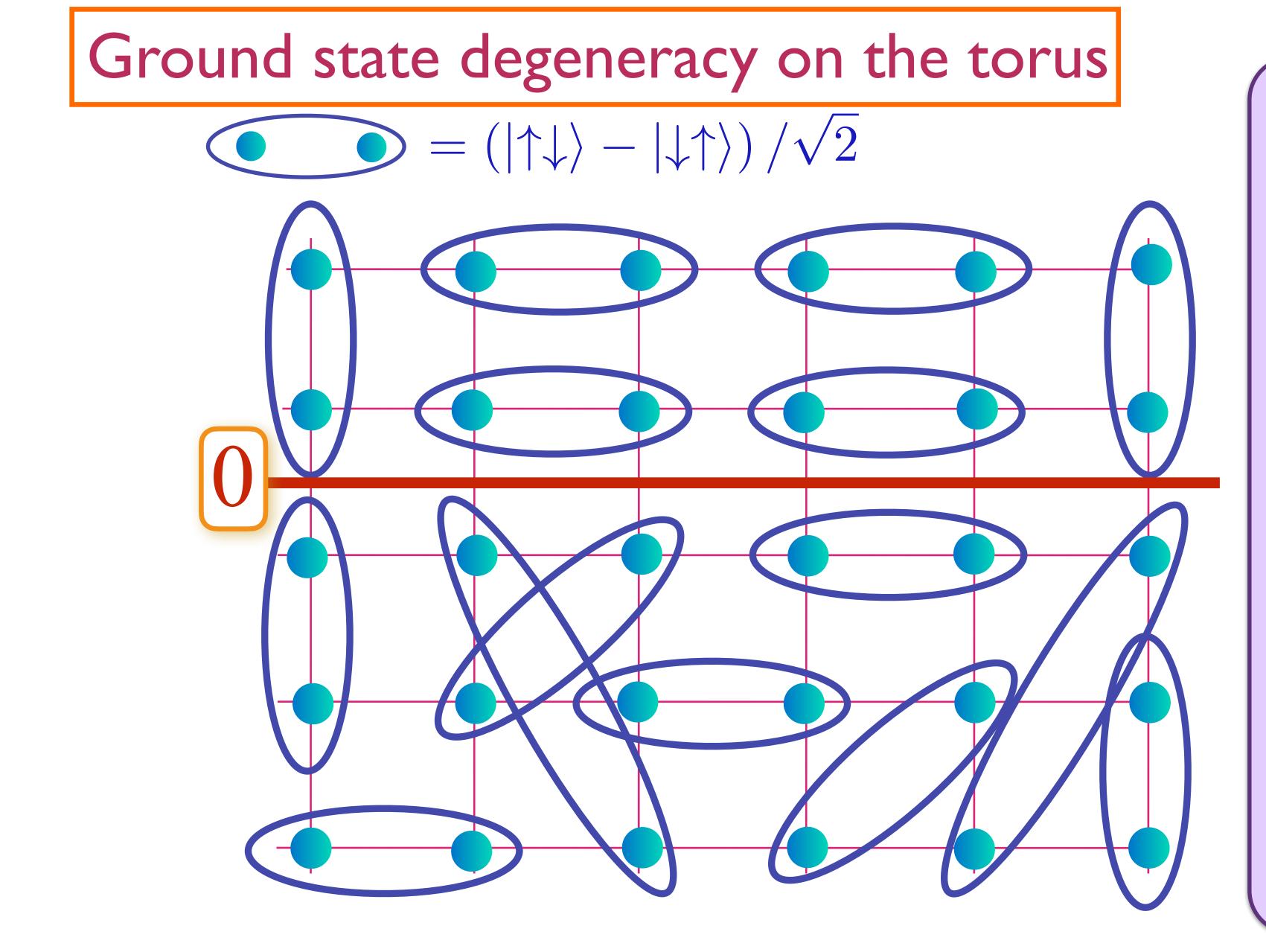
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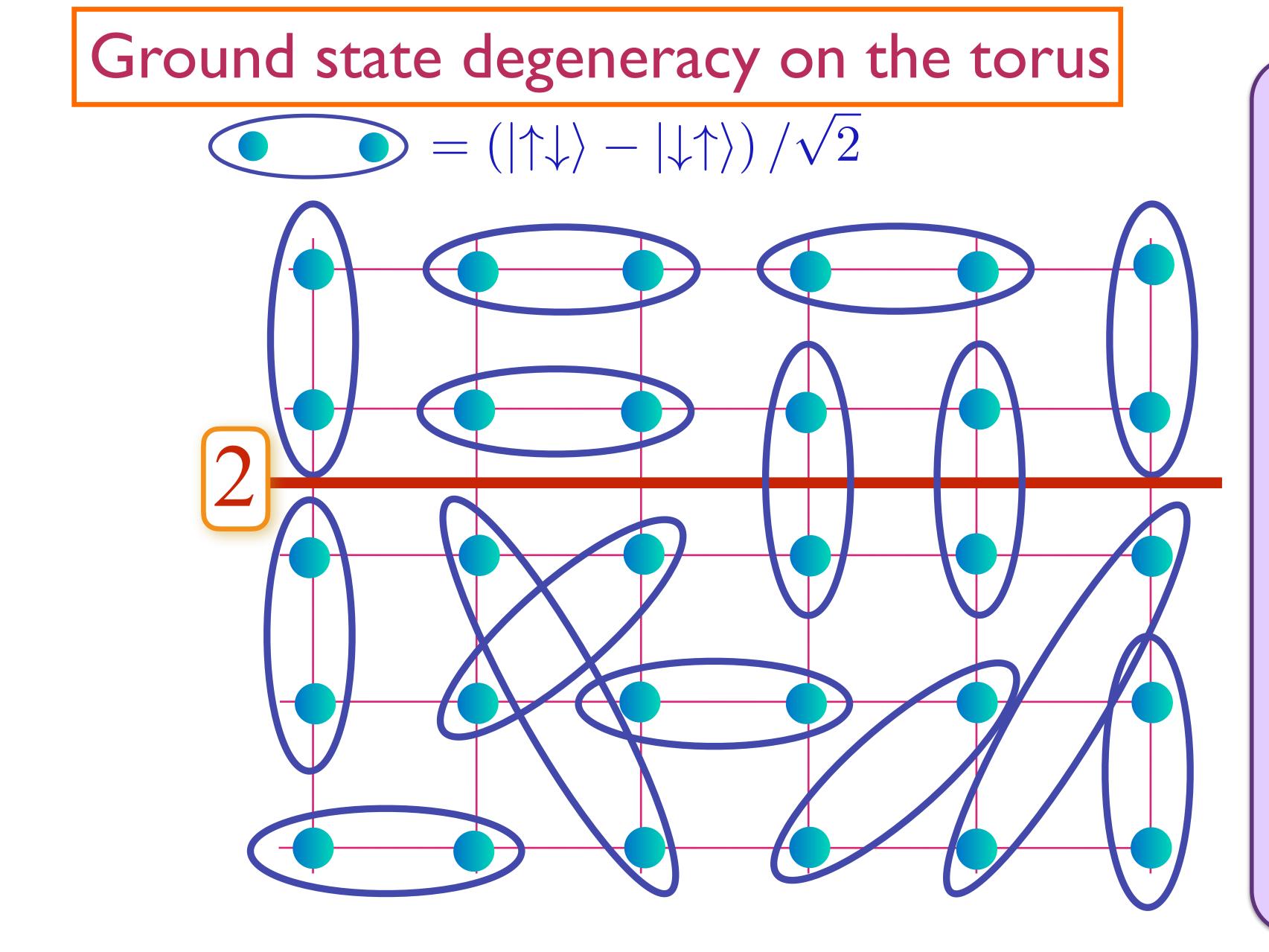
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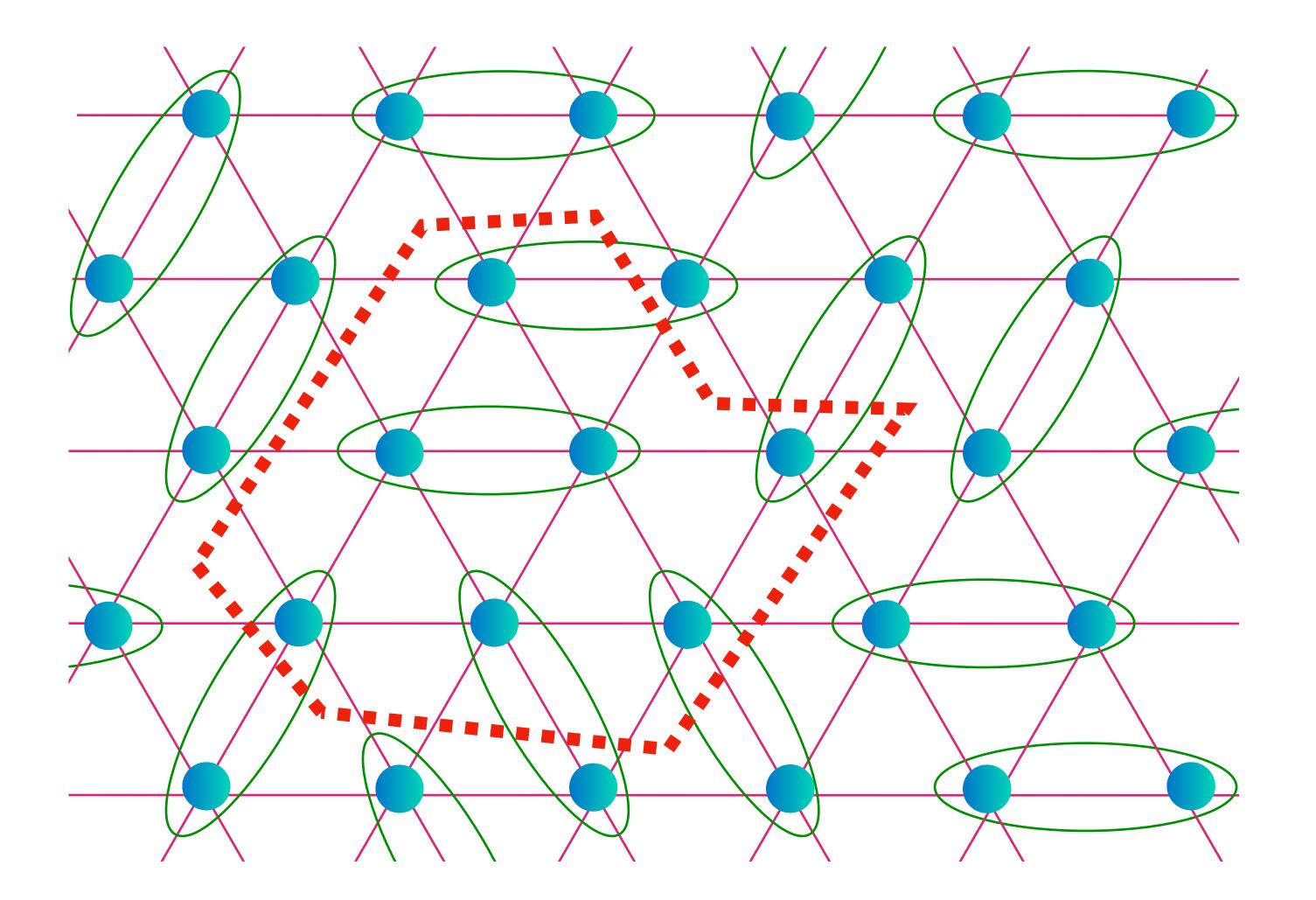
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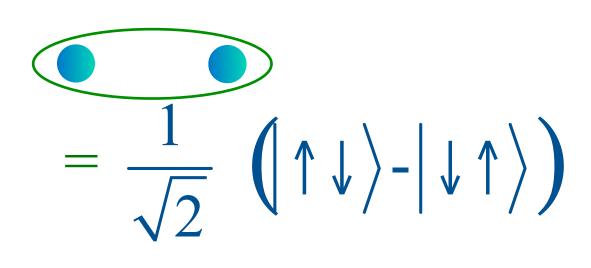
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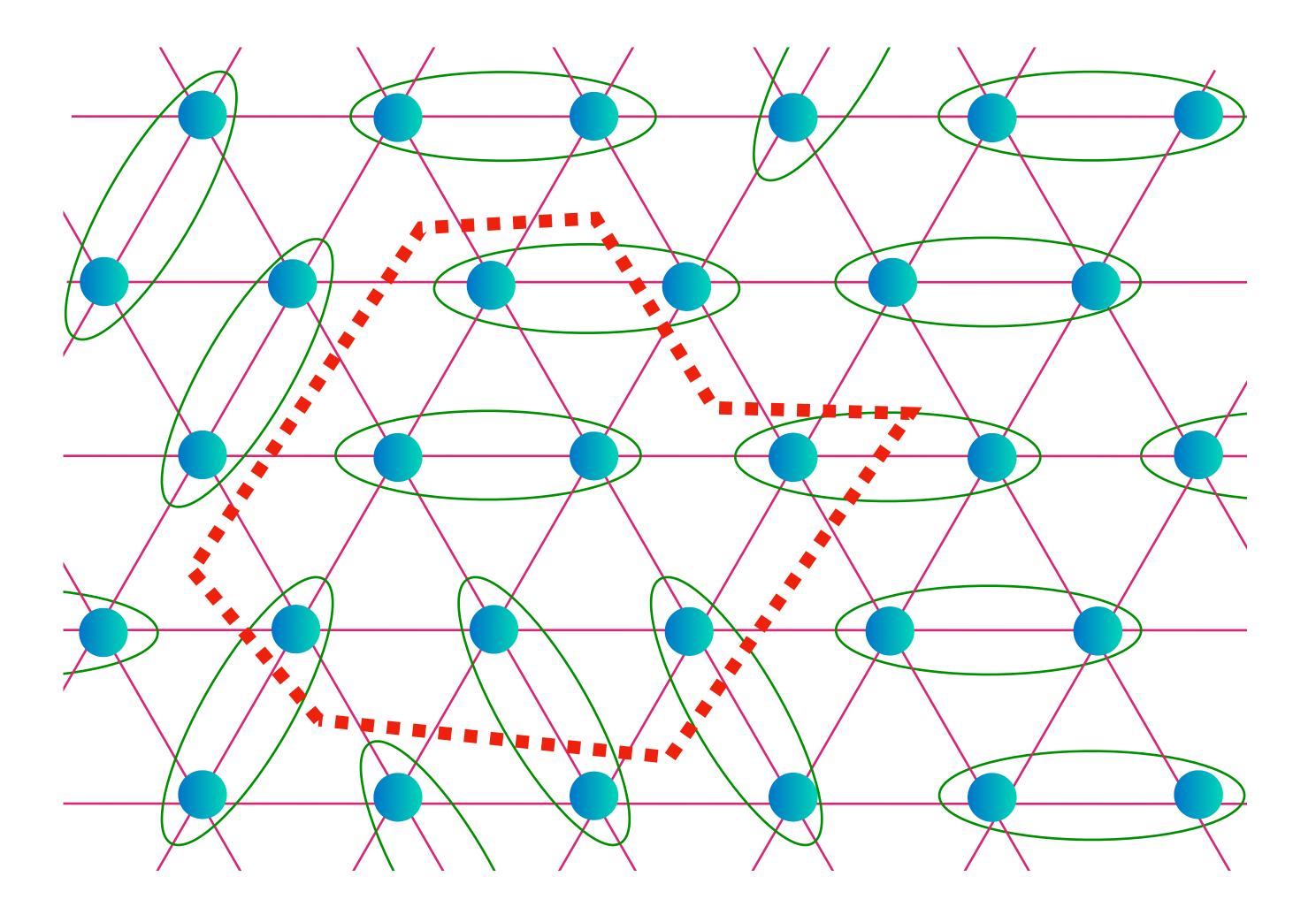




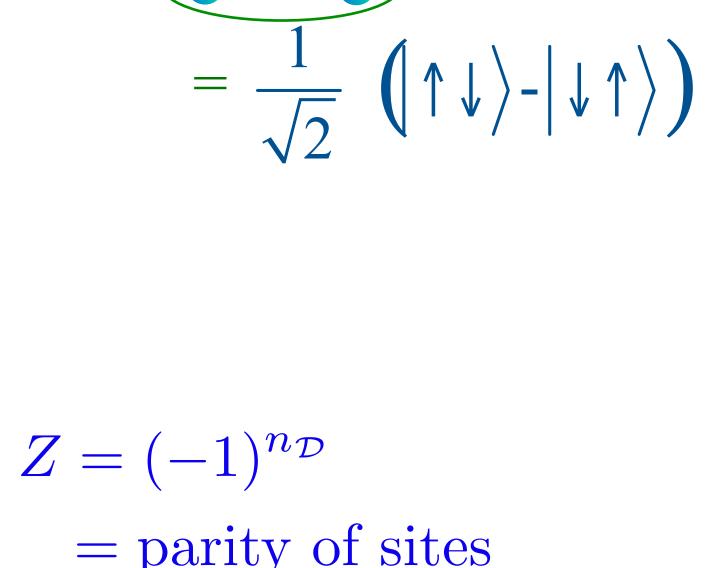


 $Z = (-1)^{n_{\mathcal{D}}}$ = parity of sites enclosed by red line $n_{\mathcal{D}} \to \text{number of dimens}$ crossing red line



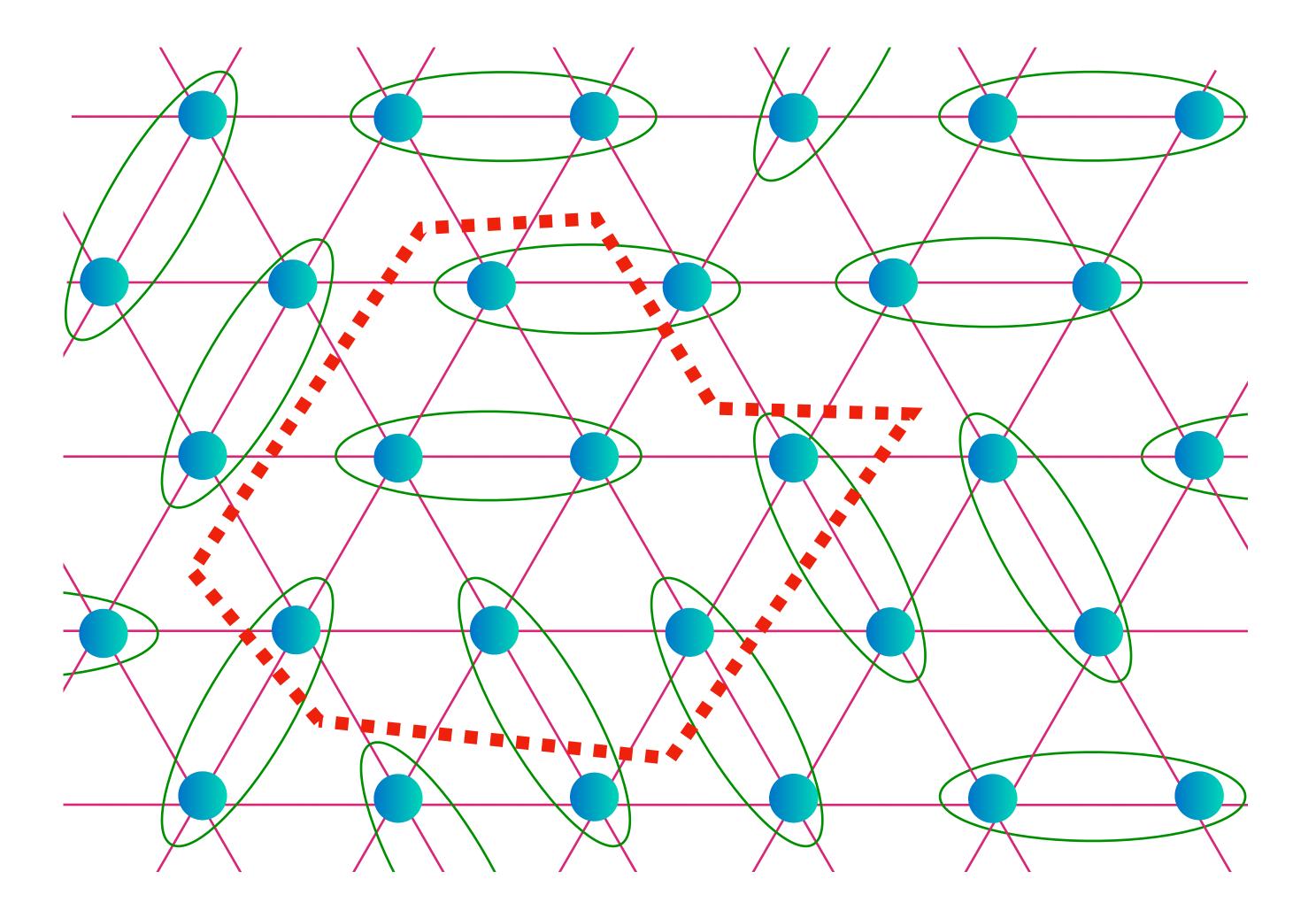




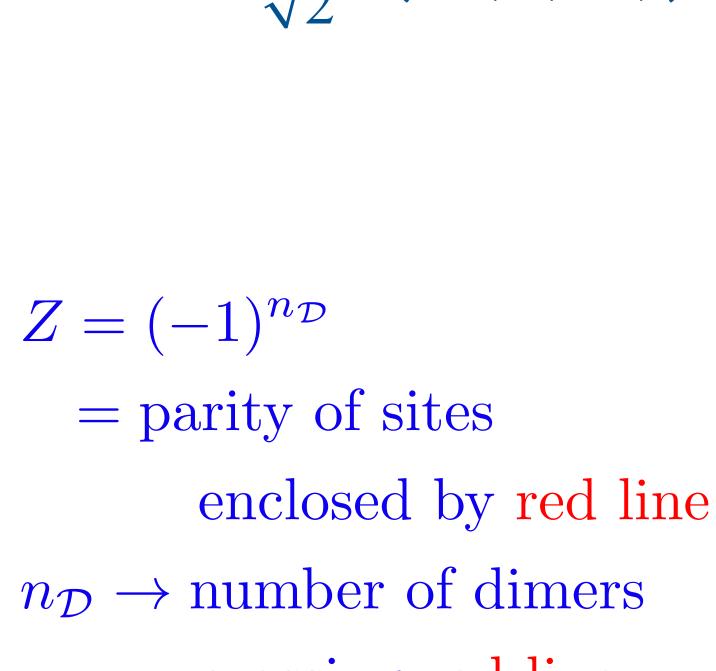


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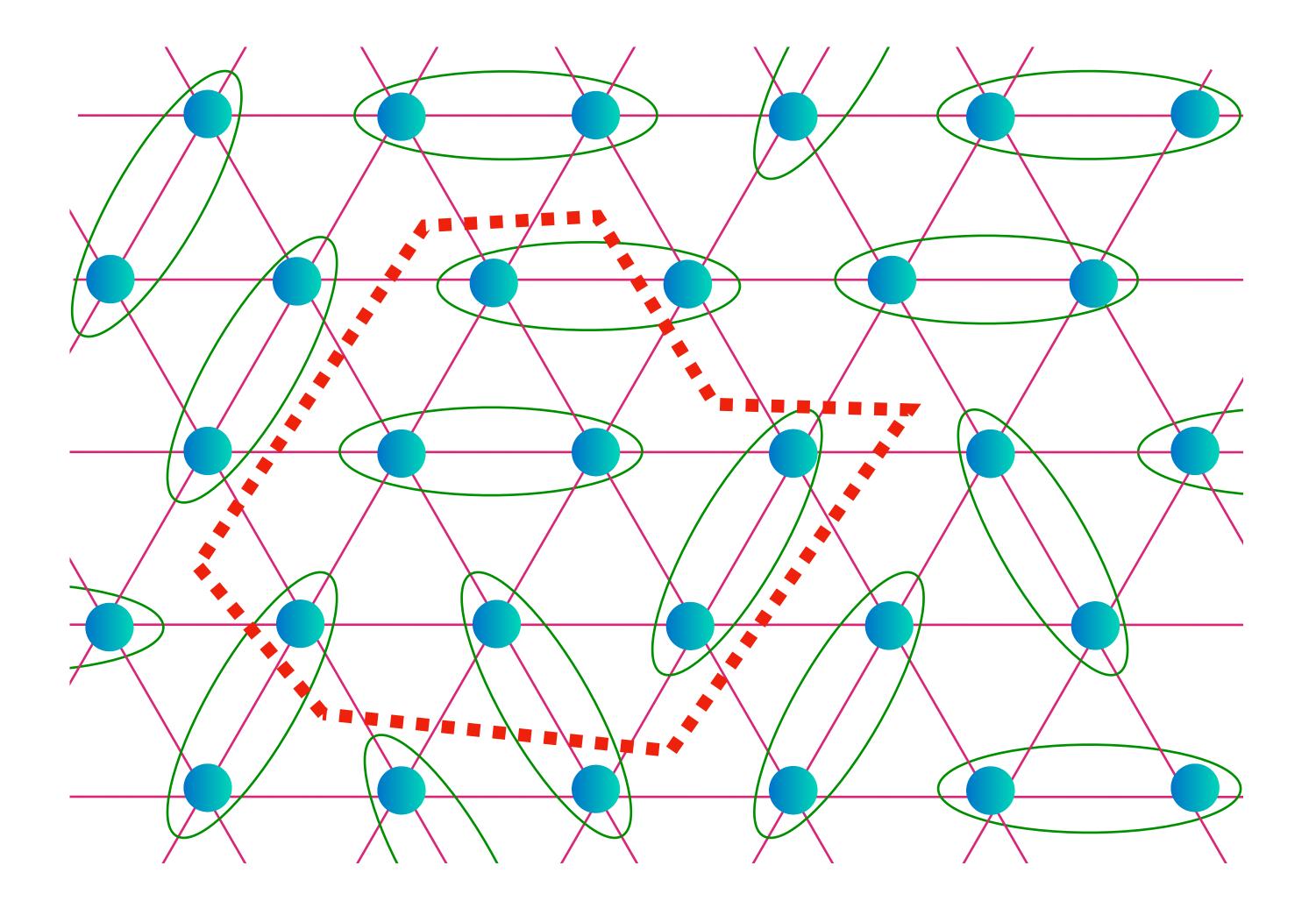




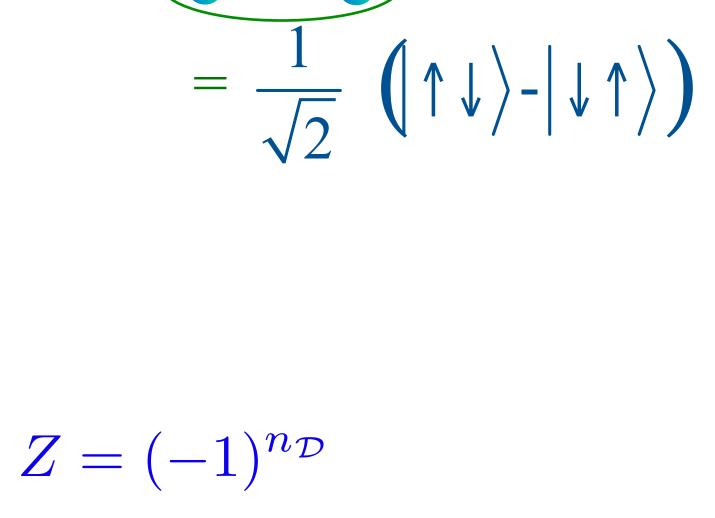


crossing red line



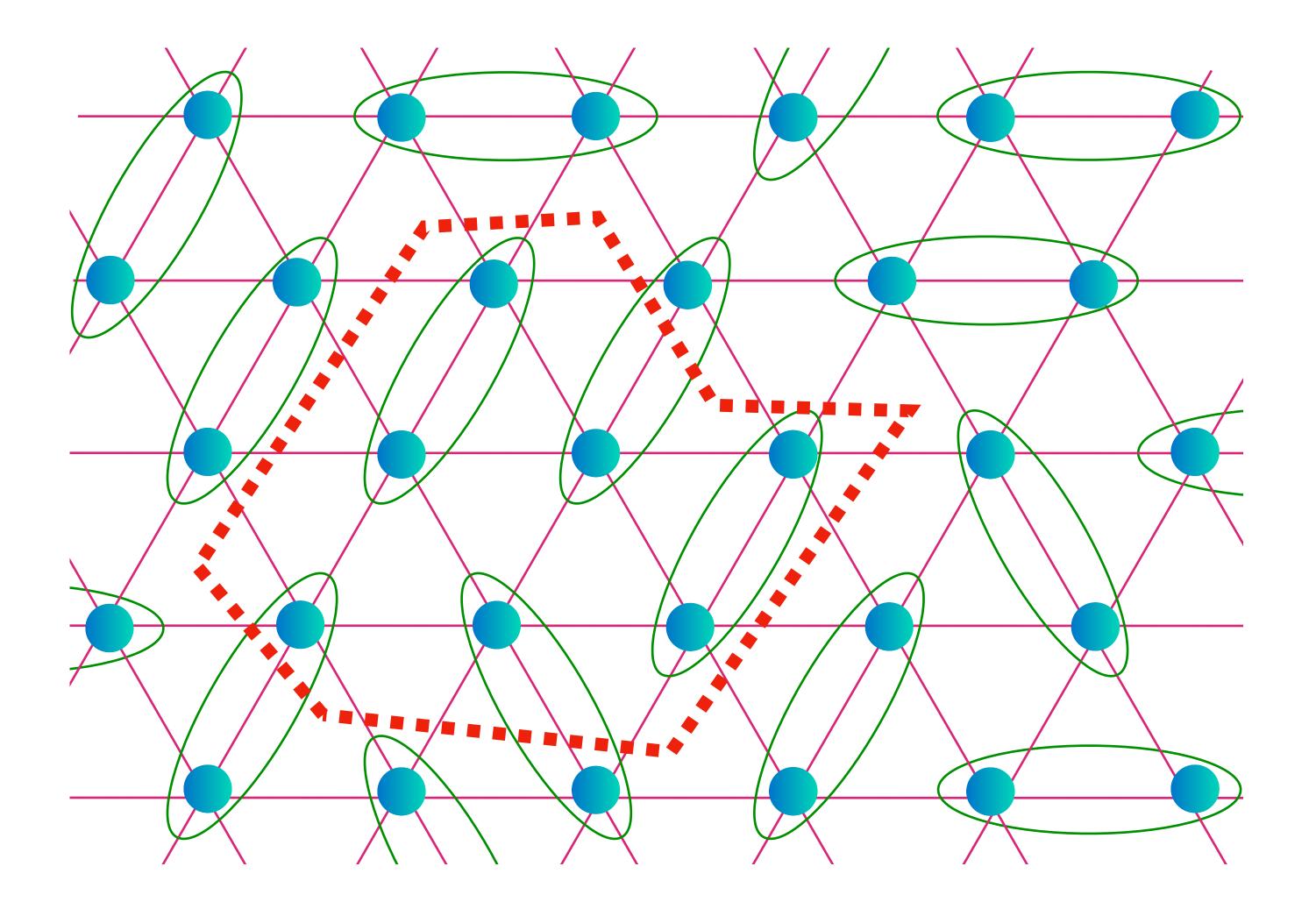




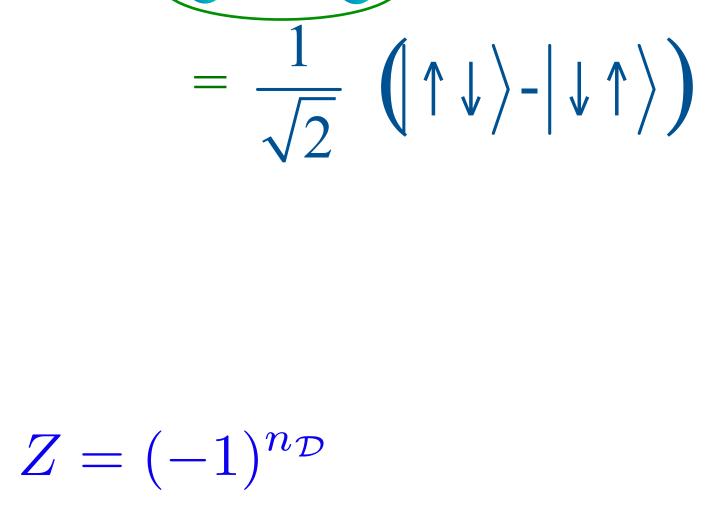


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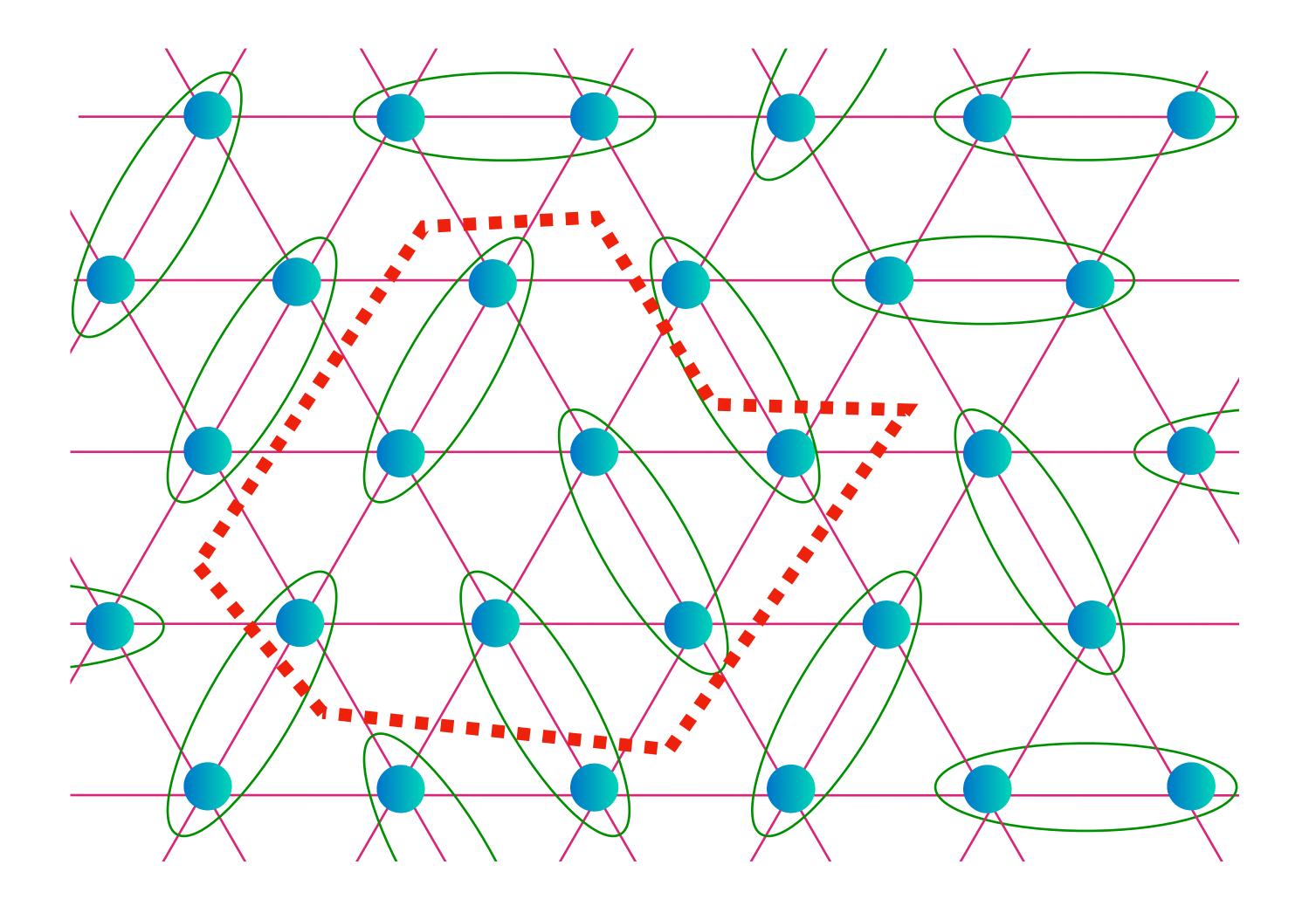




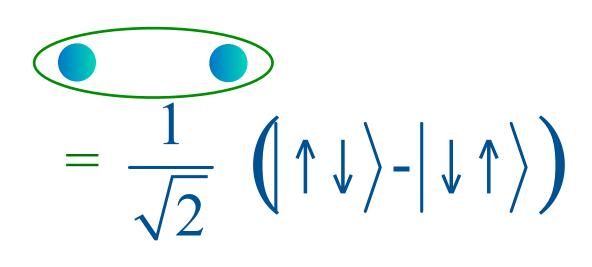


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Simplest example with time-reversal symmetry: "*Z*₂ spin liquid" or "toric code"

- by any local operator.
- The e and ϵ are spinons, the m is the 'vison'.
- Self statistics: e and m are bosons, while ϵ is a fermion.
- (-1) upon encircling any other type of anyon.
- Fusion rules: $e \times m = \epsilon, e \times \epsilon = m, m \times \epsilon = e, e \times e = \epsilon \times \epsilon = m \times m = 1.$
- 4-fold ground state degeneracy on a torus.
- Emergent, deconfined \mathbb{Z}_2 gauge field.
- Topological entanglement entropy $= \ln 2$.

• Anyons: 1, e, m, ϵ . The e, m, ϵ anyons cannot be created from the ground state (1)

• Mutual statistics: Any pair of e, m, ϵ are mutual semions *i.e.* one anyon picks up a

• No protected edge states in general, but could appear with special symmetries.

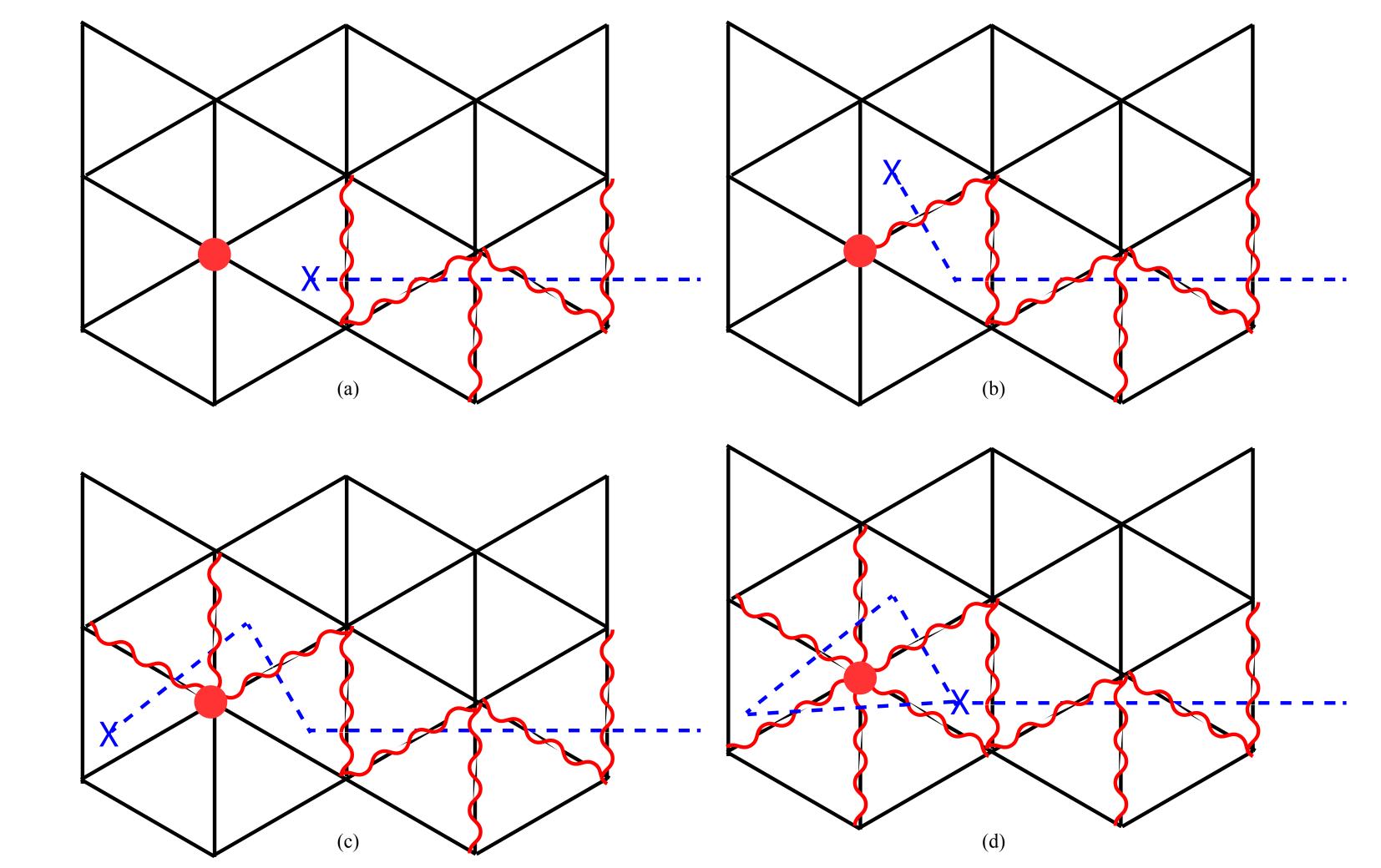
The \mathbb{Z}_2 spin liquid was obtained in N. Read and S. Sachdev, Phys. Rev. Lett. 66, 1773 (1991) and X.-G. Wen, Phys. Rev. B 44, 2664 (1991). A. Kitaev, arXiv:quant-ph/9707021 described the toric code.

Odd and even Z_2 spin liquids

Berry phase

of vison

motion



To return to the initial state, we need a gauge transformation factor of -1 for each dimer ending on the red circle: this yields a factor $e^{i\pi S}$, because there are 2S dimers on each site.

> R. A. Jalabert and S. Sachdev, PRB 44, 686 (1991); S. Sachdev and M Vojta, J. Phys. Soc. Jpn. Suppl. B **69**, (2000); T. Senthil and M.P.A. Fisher, PRB **62**, 7850 (2000).



Odd and even Z_2 spin liquids

- The spinons carry spin $S_z = 1/2$ boson number $B^{\dagger}B = 1/2$.

tion' with

with *n* integer or half-integer.

 $T_x T_y = T_y T_x e^{2\pi i S}.$

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• \mathbb{Z}_2 spin liquids of bosons (more generally, in systems with a global U(1) symmetry) must obey contraints associated with a 'tHooft anomaly' which is determined by the boson filling n.

- On a square lattice, the single vison state exhibit 'translational symmetry fractionaliza-

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 \mathbb{Z}_2 spin liquid on the square lattice (generalizes to other lattices)

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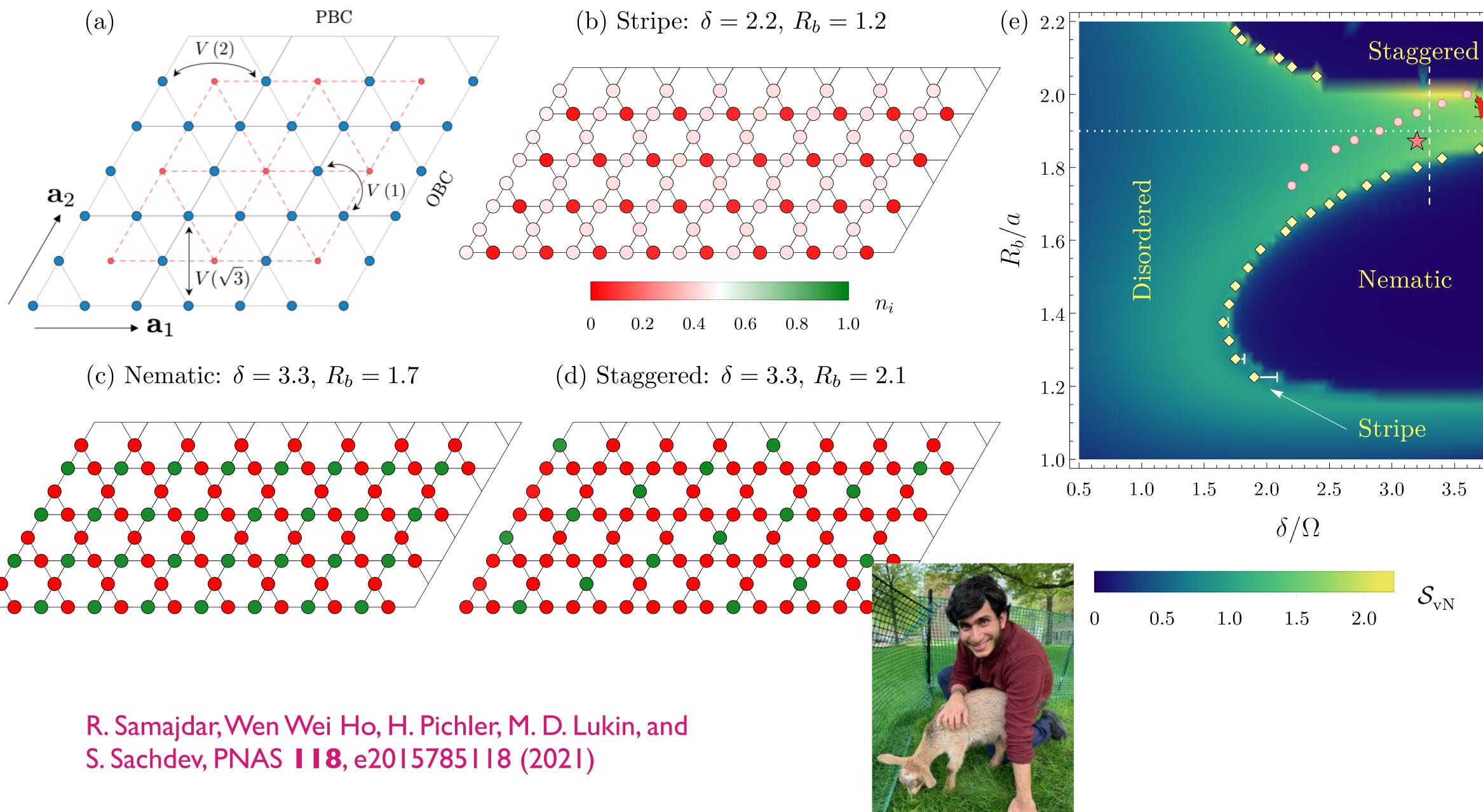
• More generally, any \mathbb{Z}_2 spin liquid, even without a conserved U(1), can exhibit symmetry *fractionalization*, with $T_x T_y = T_y T_x$ for an even \mathbb{Z}_2 spin liquid, and $T_x T_y = -T_y T_x$ for an odd

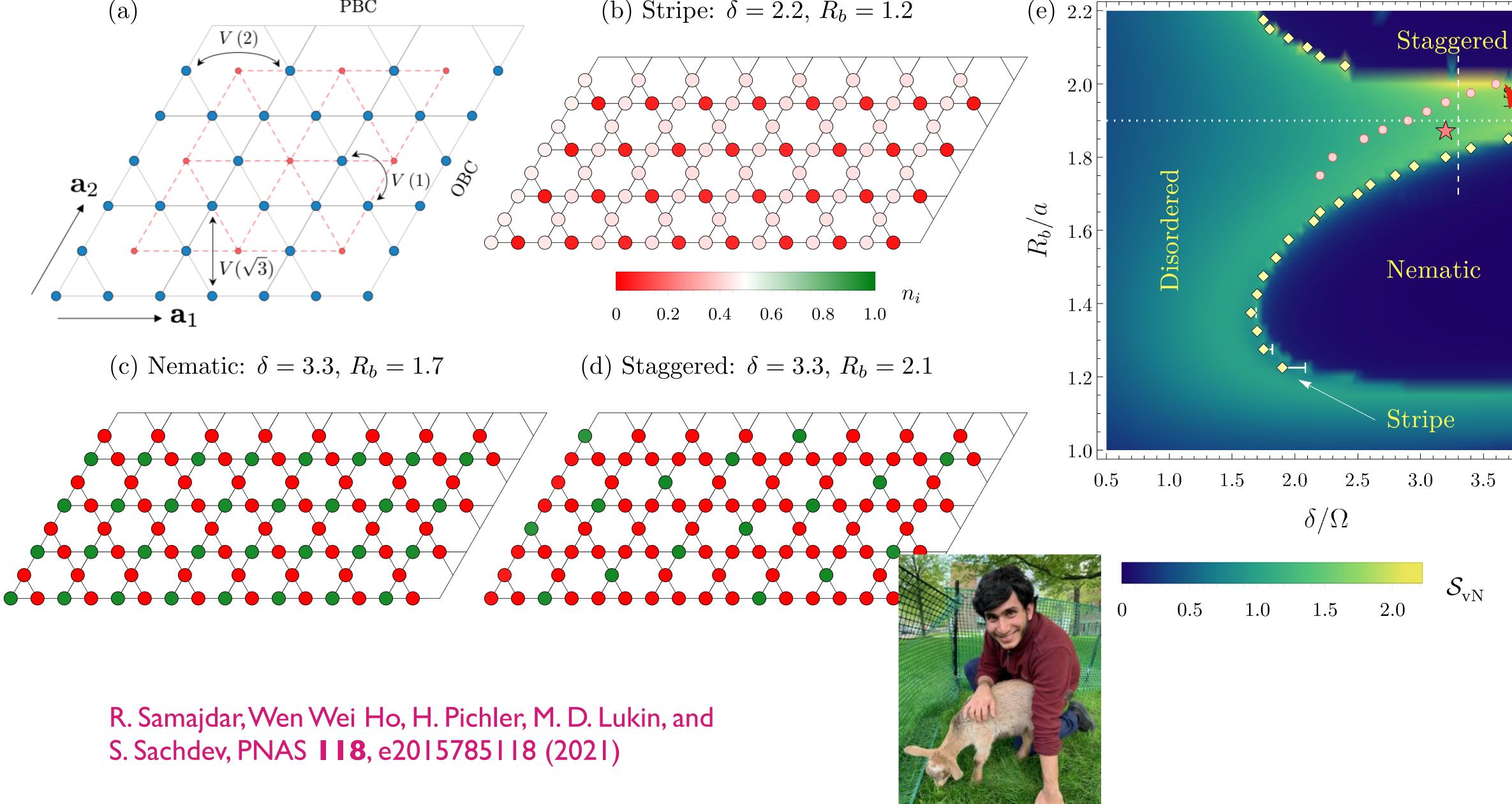


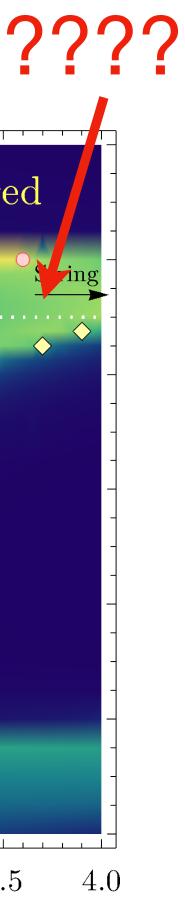


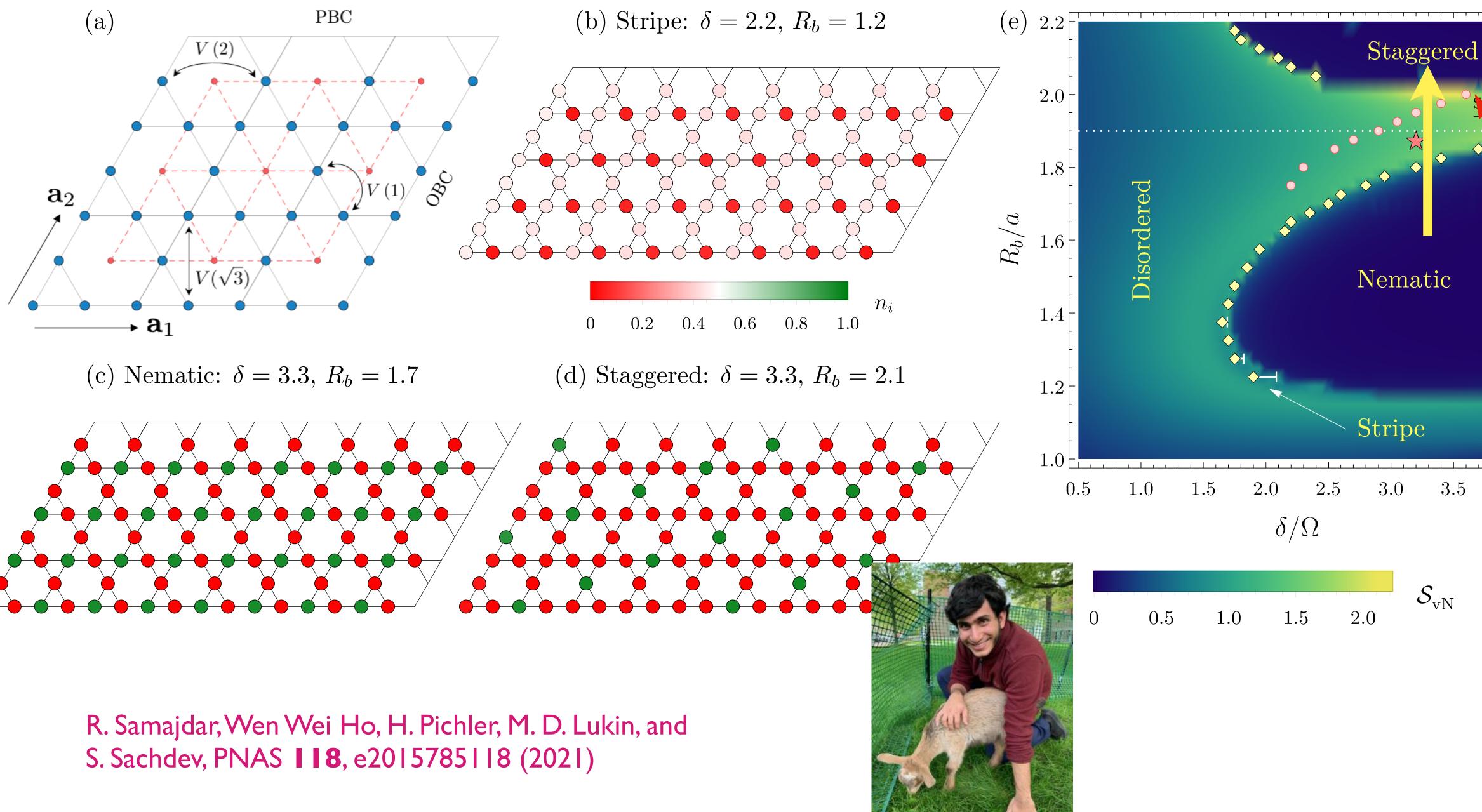
I. Rydberg chains The Z₃ chiral clock transition 2. Square lattice Quantum Ising criticality in 2+1 dimensions 3. Kagome symmetry lattices Probing topological spin liquids 4. Theory of odd and even Z₂ spin liquids

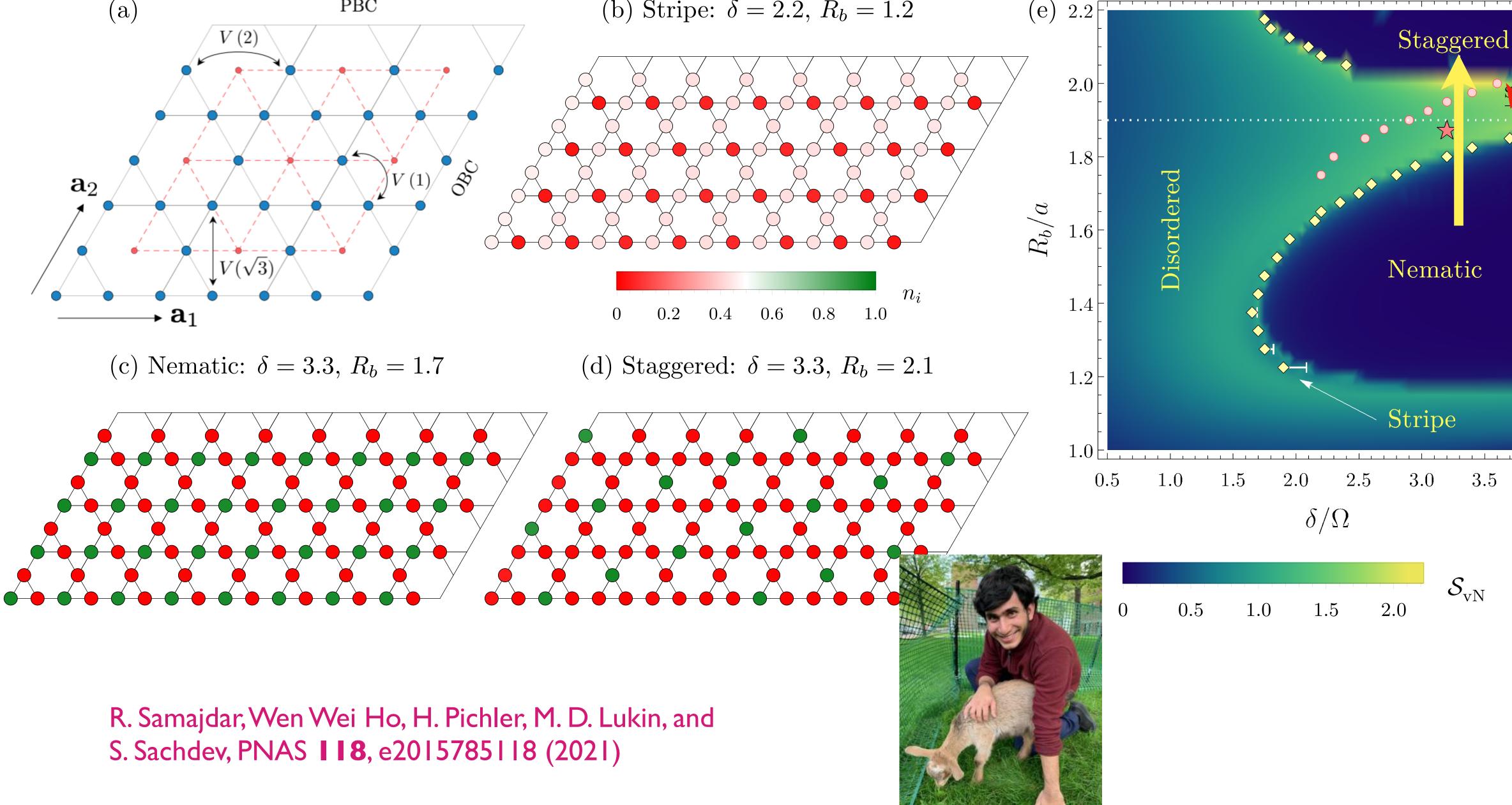




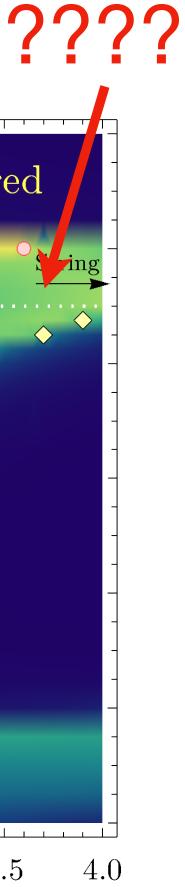


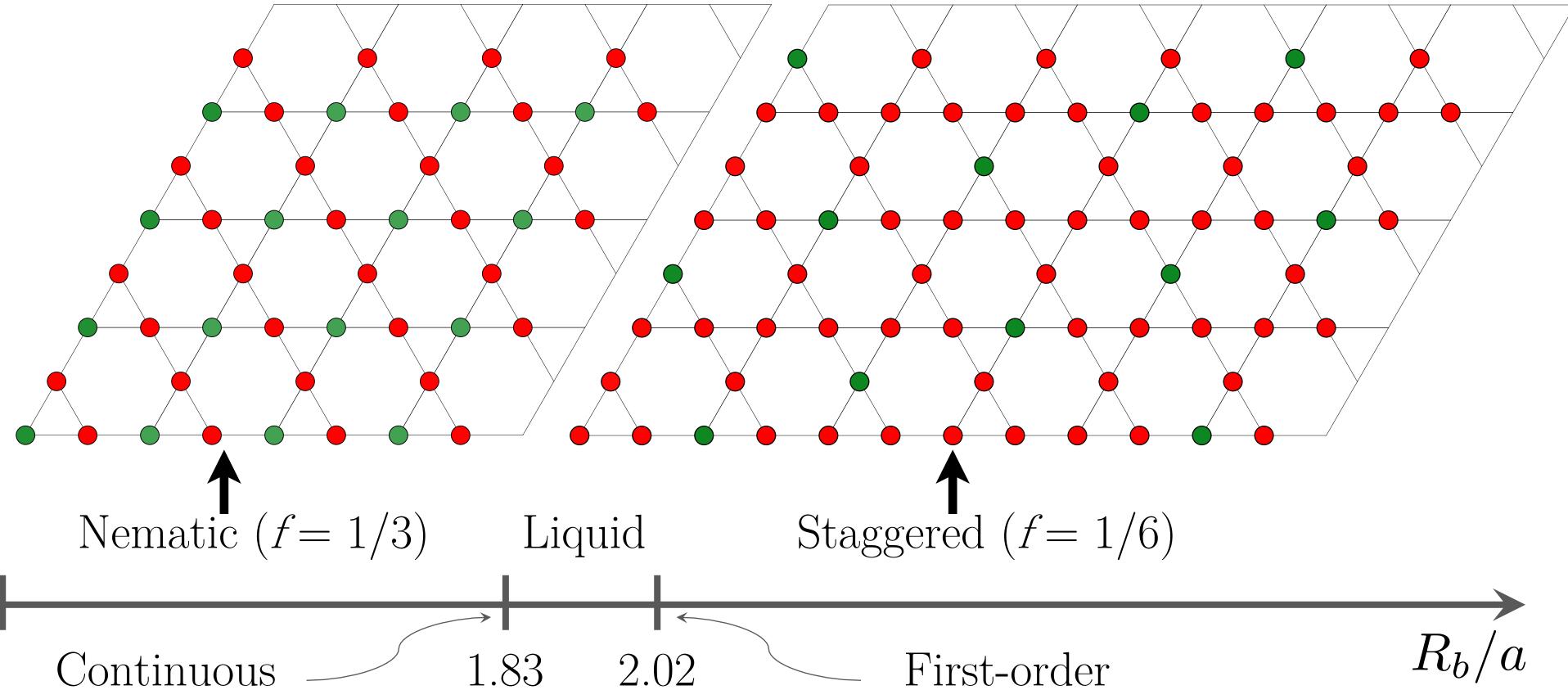










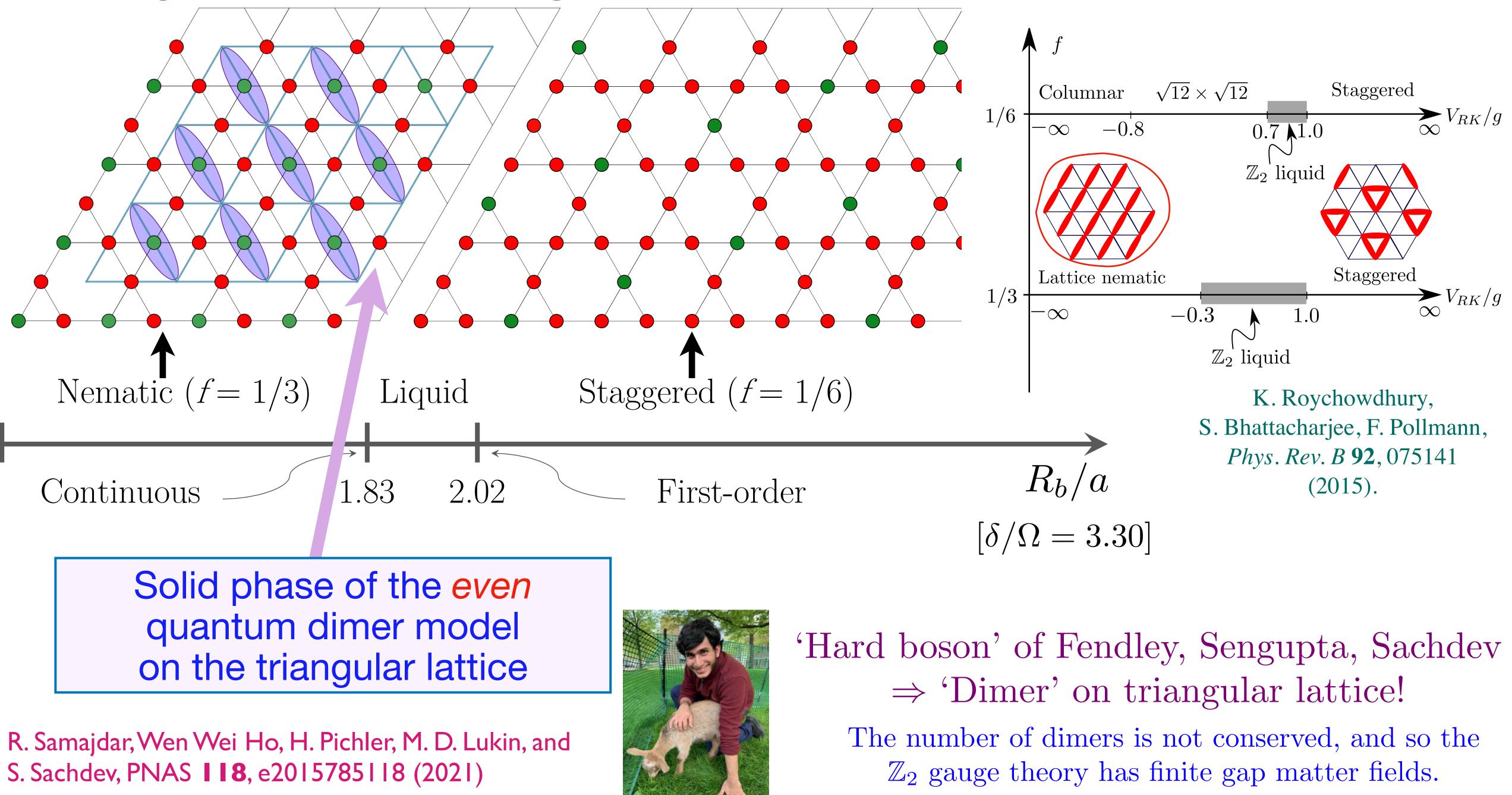


R. Samajdar, Wen Wei Ho, H. Pichler, M. D. Lukin, and S. Sachdev, PNAS **II8**, e2015785118 (2021)

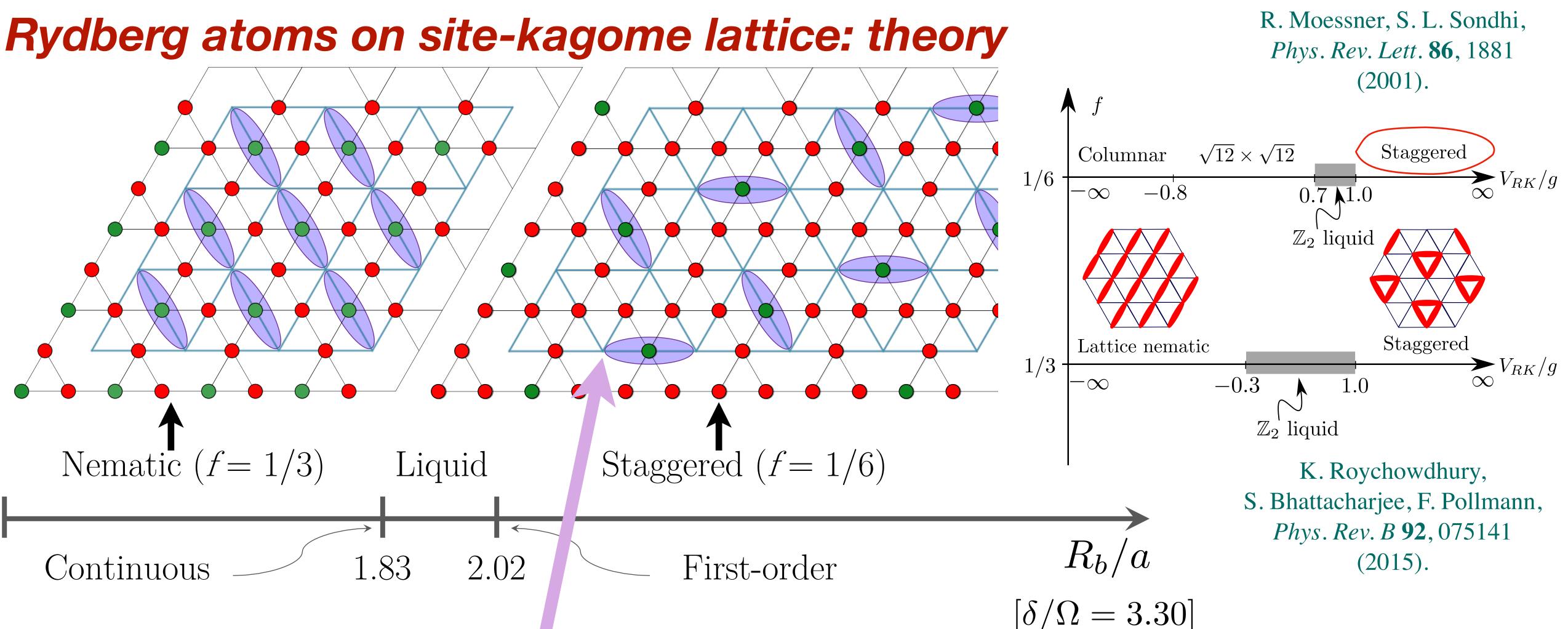


 $[\delta/\Omega = 3.30]$







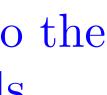


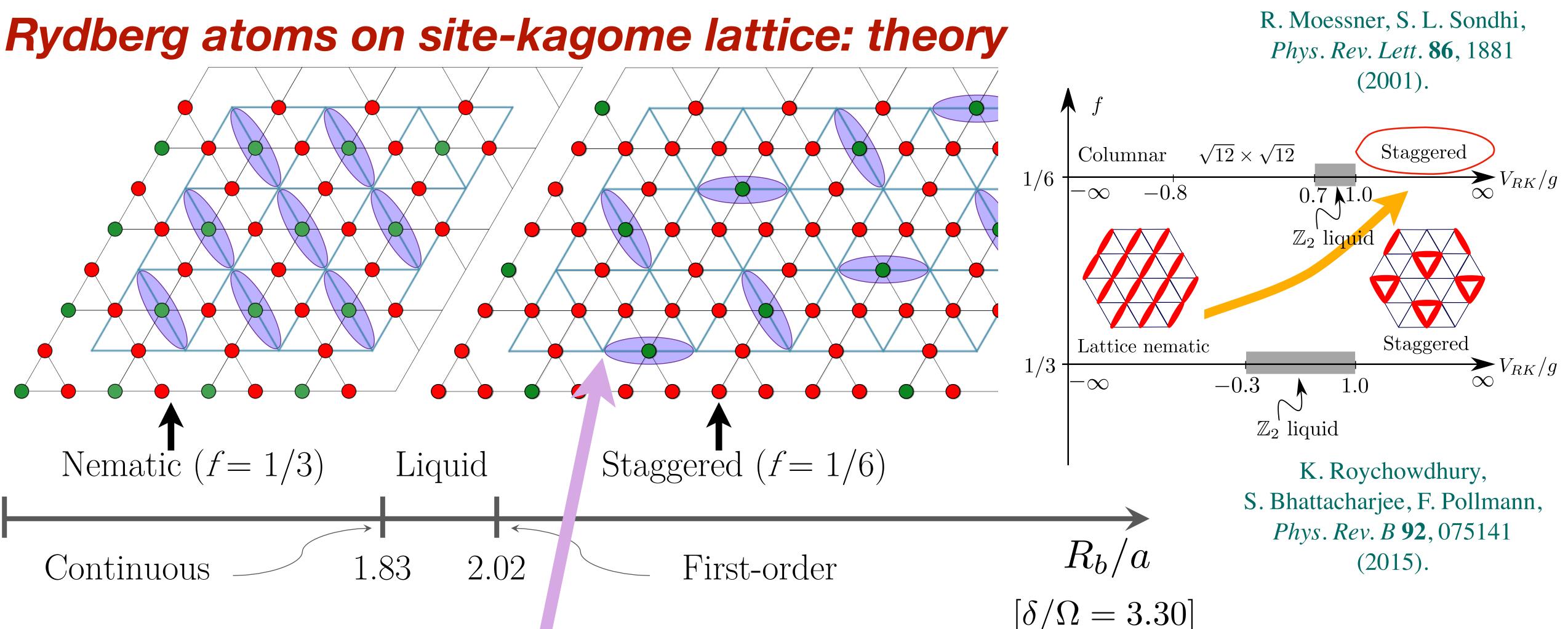
Solid phase of the odd quantum dimer model on the triangular lattice

R. Samajdar, Wen Wei Ho, H. Pichler, M. D. Lukin, and S. Sachdev, PNAS **18**, e2015785118 (2021)







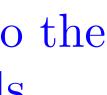


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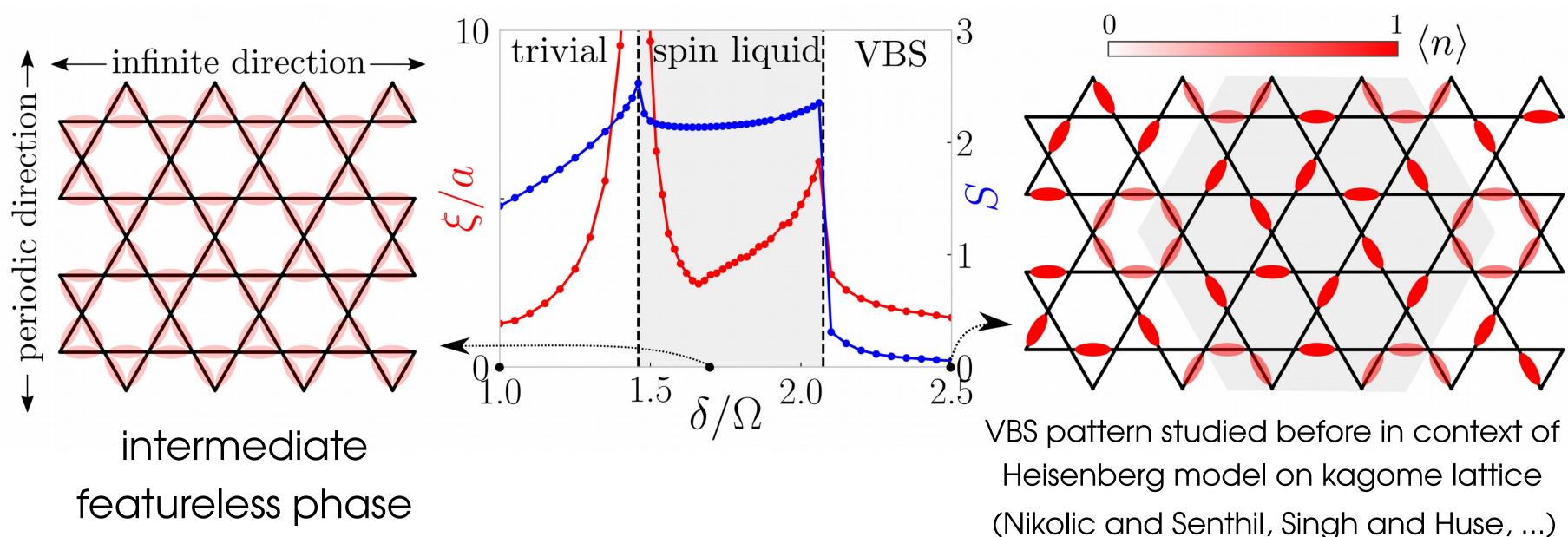


$$H = \frac{\Omega}{2} \sum_{i} P \sigma_i^x P - \delta \sum_{i} n_i$$

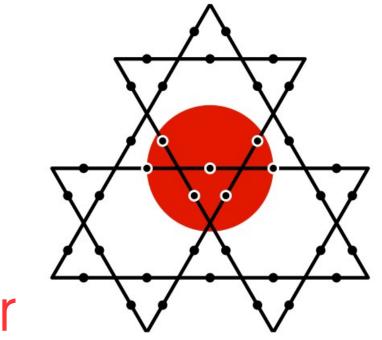
we put the model on an infinitely-long cylinder

 \rightarrow use density matrix renormalization group (DMRG)

(White `92, Stoudenmire `13, Hauschild `18)



Ruben Verresen, Mikhail D. Lukin, Ashvin Vishwanath, arXiv: 2011.12310



'Hard boson' of Fendley, Sengupta, Sachdev \Rightarrow 'Dimer' on kagome lattice!

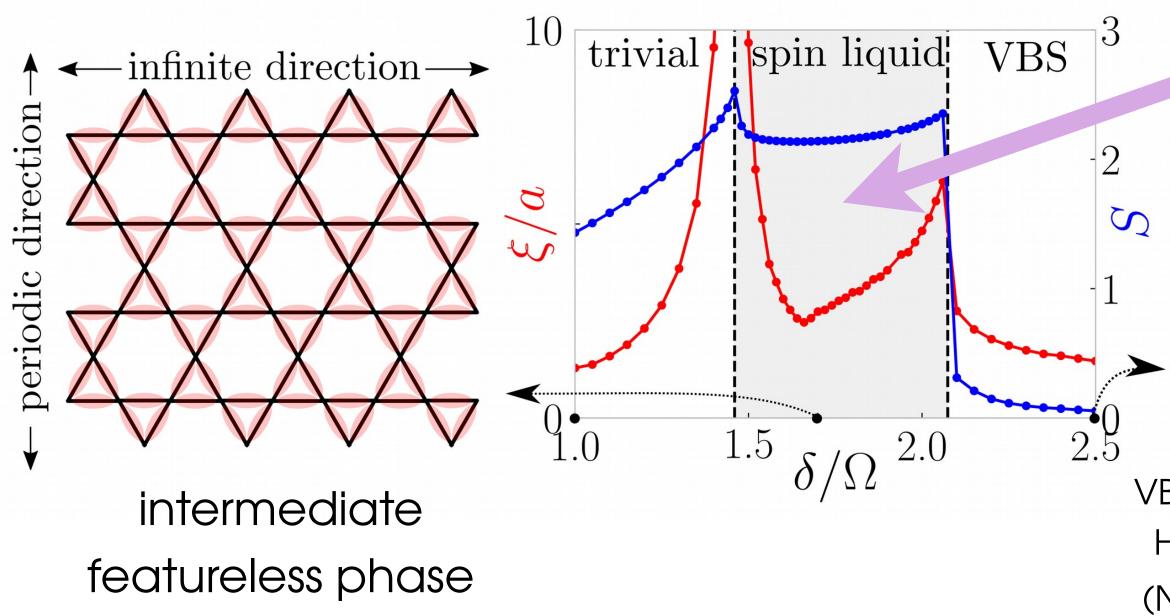
(Nikolic and Senthil, Singh and Huse, ...)

The number of dimers is not conserved, and so the \mathbb{Z}_2 gauge theory has finite gap matter fields.



$$H = \frac{\Omega}{2} \sum_{i} P \sigma_i^x P - \delta \sum_{i} n_i$$

we put the model on an infinitely-long cylinder \rightarrow use density matrix renormalization group (DMRG) (White `92, Stoudenmire `13, Hauschild `18) 10 $\langle n \rangle$ trivial spin liquid VBS 2 \mathcal{O} S u



Ruben Verresen, Mikhail D. Lukin, Ashvin Vishwanath, arXiv: 2011.12310

Quantum liquid phase of the odd quantum dimer model on the kagome lattice

VBS pattern studied before in context of Heisenberg model on kagome lattice (Nikolic and Senthil, Singh and Huse, ...)

> The number of dimers is not conserved, and so the \mathbb{Z}_2 gauge theory has finite gap matter fields.

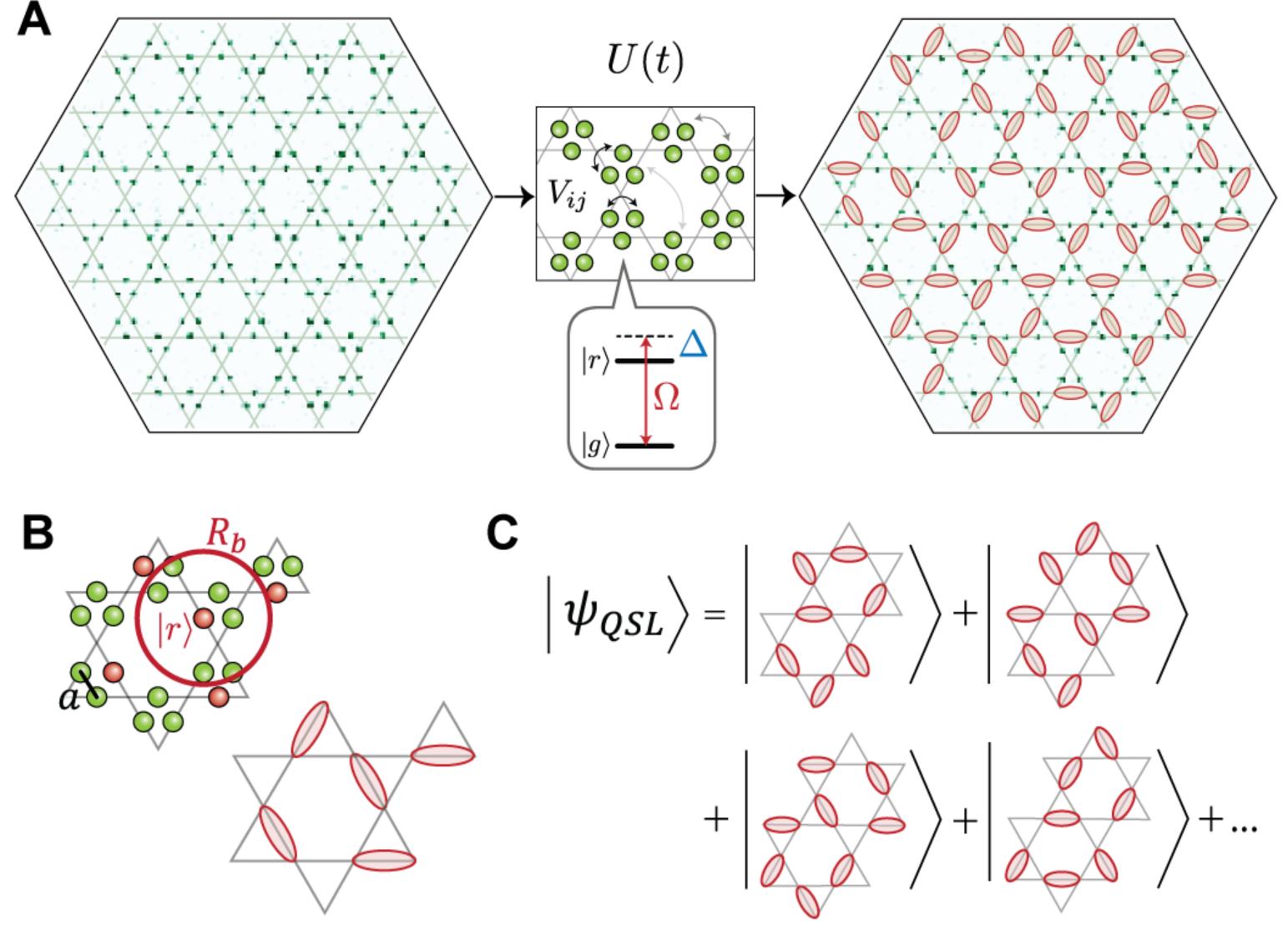


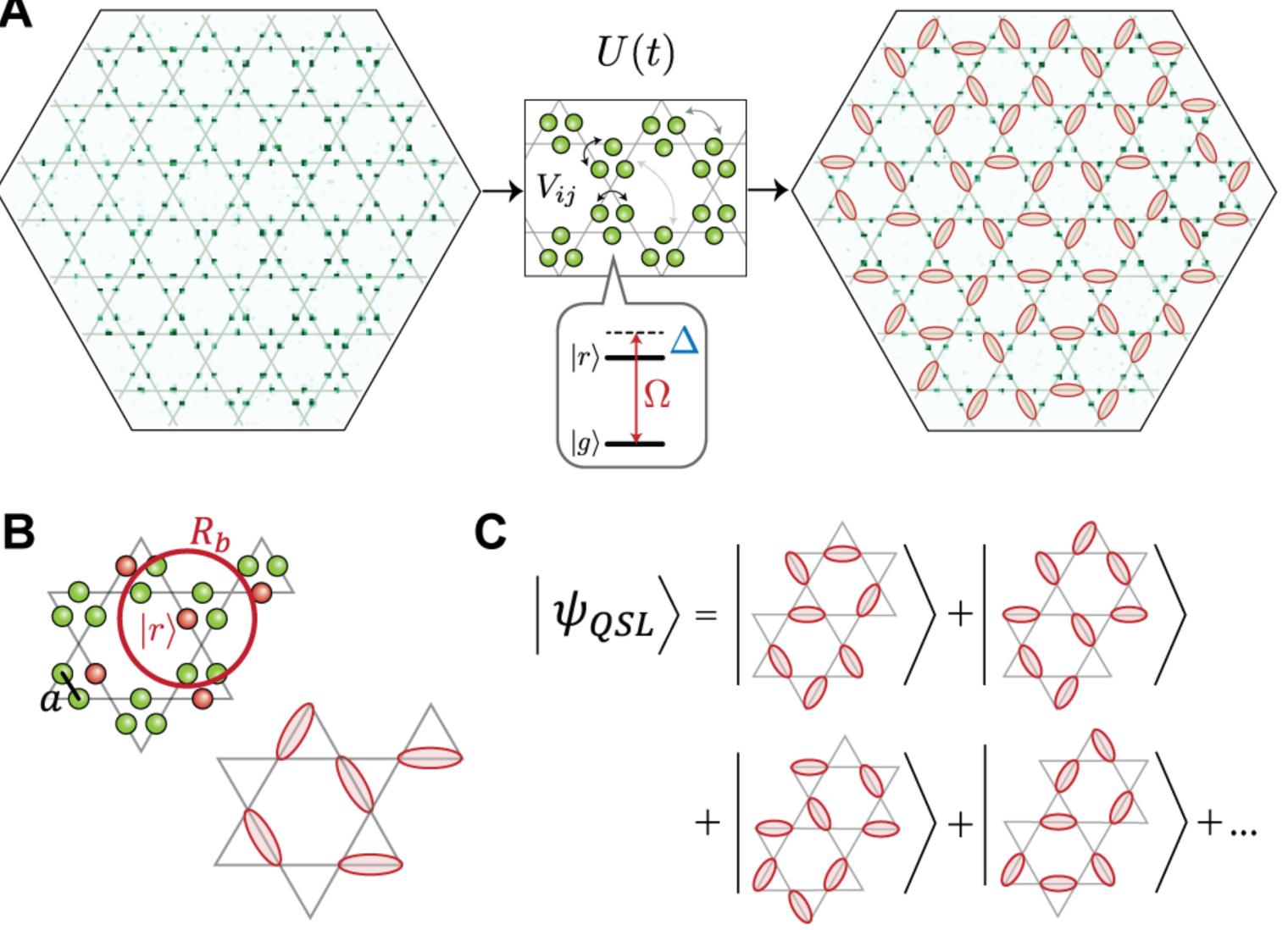


Probing Topological Spin Liquids on a Programmable Quantum Simulator

G. Semeghini, H. Levine, A. Keesling, S. Ebadi, T.T. Wang, D. Bluvstein, R. Verresen, H. Pichler, M. Kalinowski, R. Samajdar, A. Omran, S. Sachdev, A. Vishwanath, M. Greiner, V. Vuletic, M. D. Lukin, arXiv:2104.04119

Rydberg atoms on the link-kagome lattice: experiment

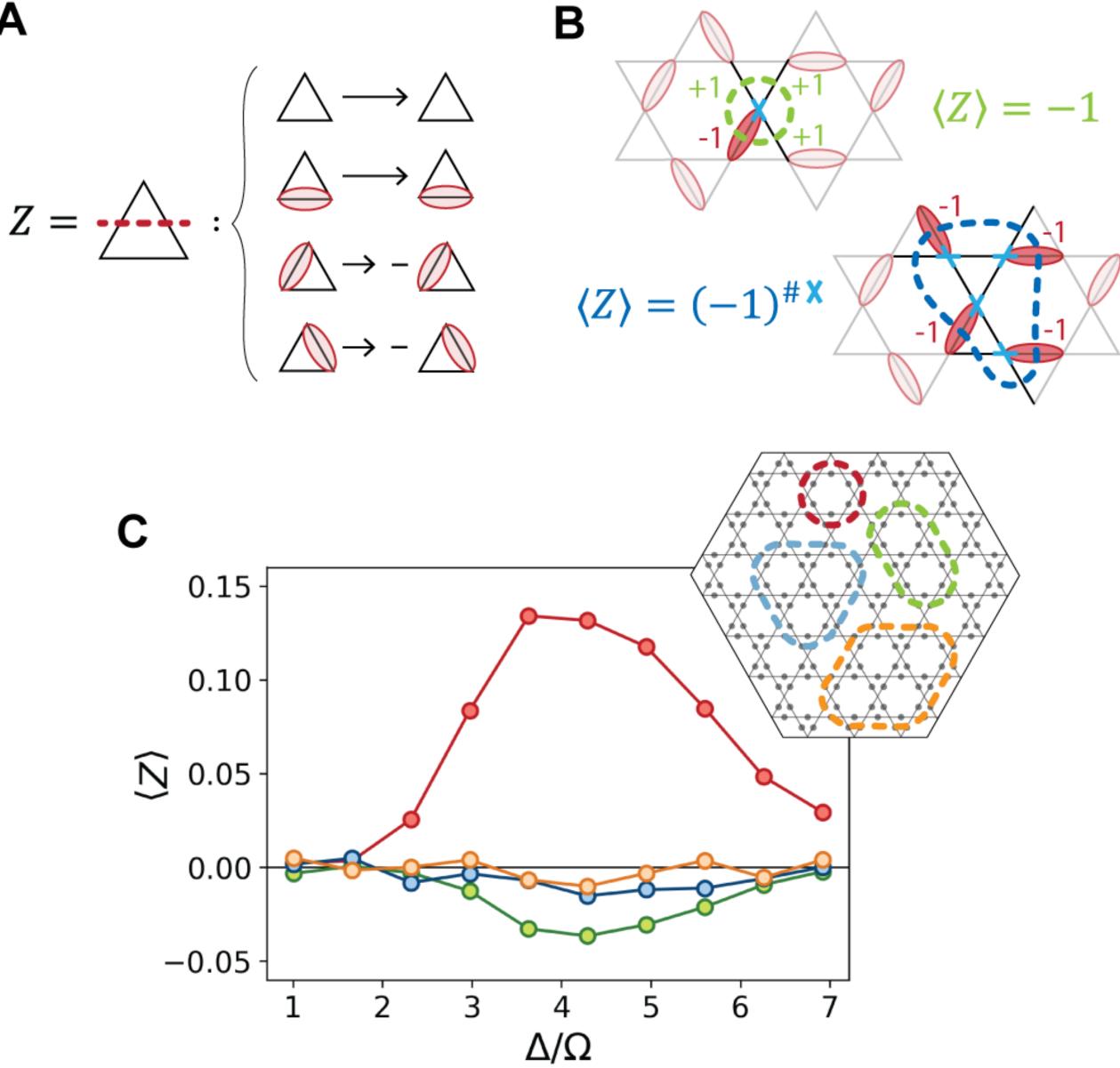






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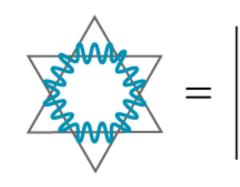


Measurement of the topological Z operator



Probing Topological Spin Liquids on a Programmable Quantum Simulator G. Semeghini, H. Levine, A. Keesling, S. Ebadi, T.T. Wang, D. Bluvstein, R. Verresen, H. Pichler, M. Kalinowski, R. Samajdar, A. Omran, S. Sachdev, A. Vishwanath, M. Greiner, V. Vuletic, M. D. Lukin, arXiv:2104.04119

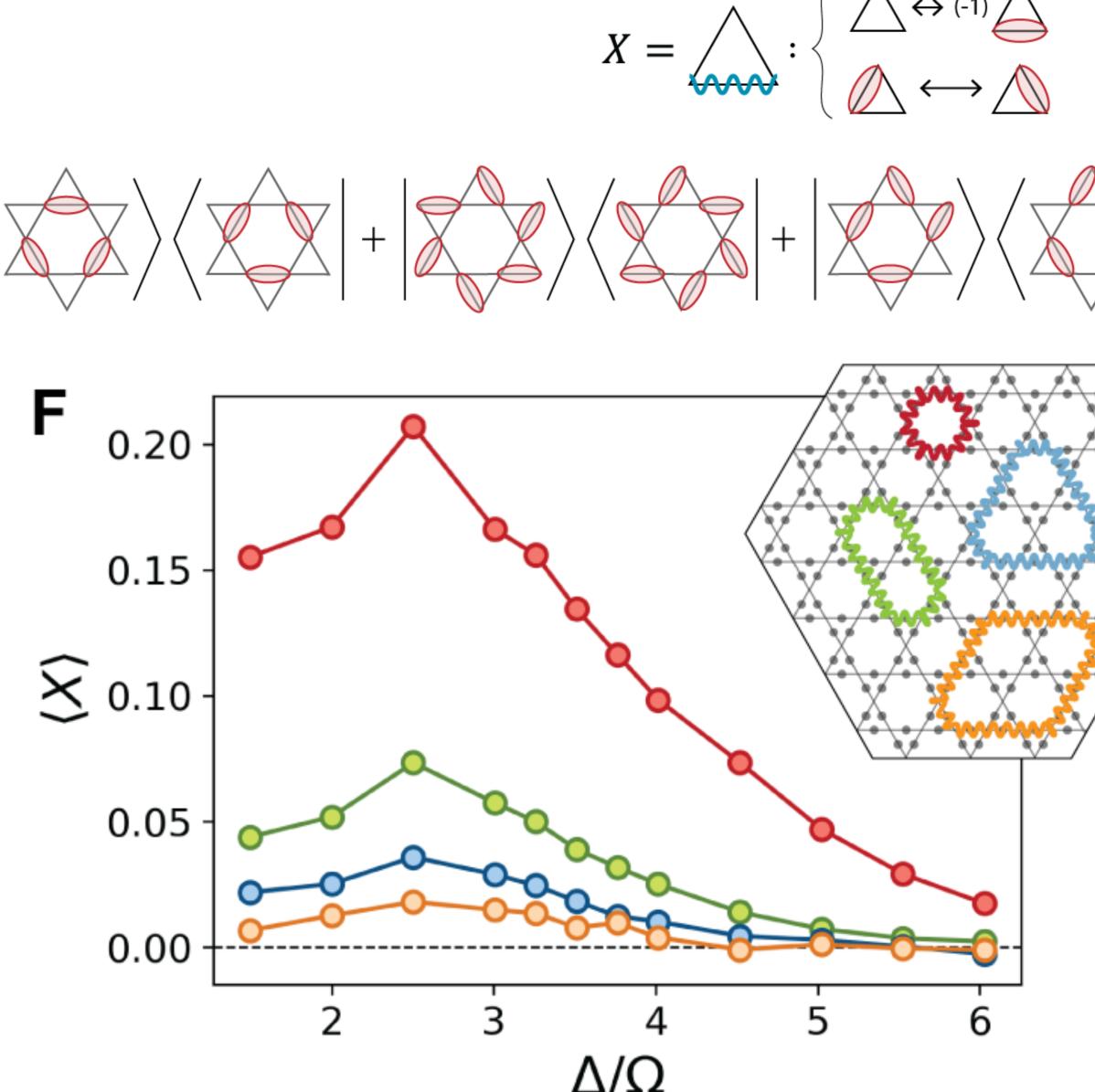
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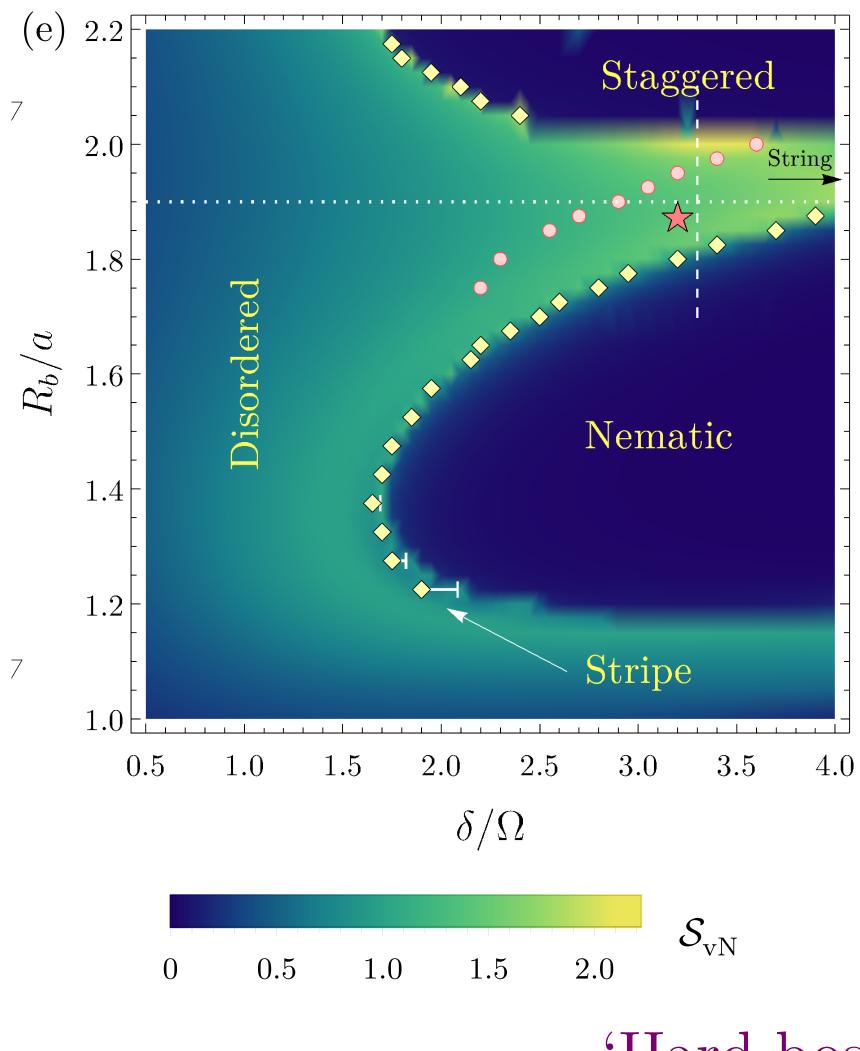
Can also define a topological X operator which resonates between different dimer configurations of the kagome dimer model.

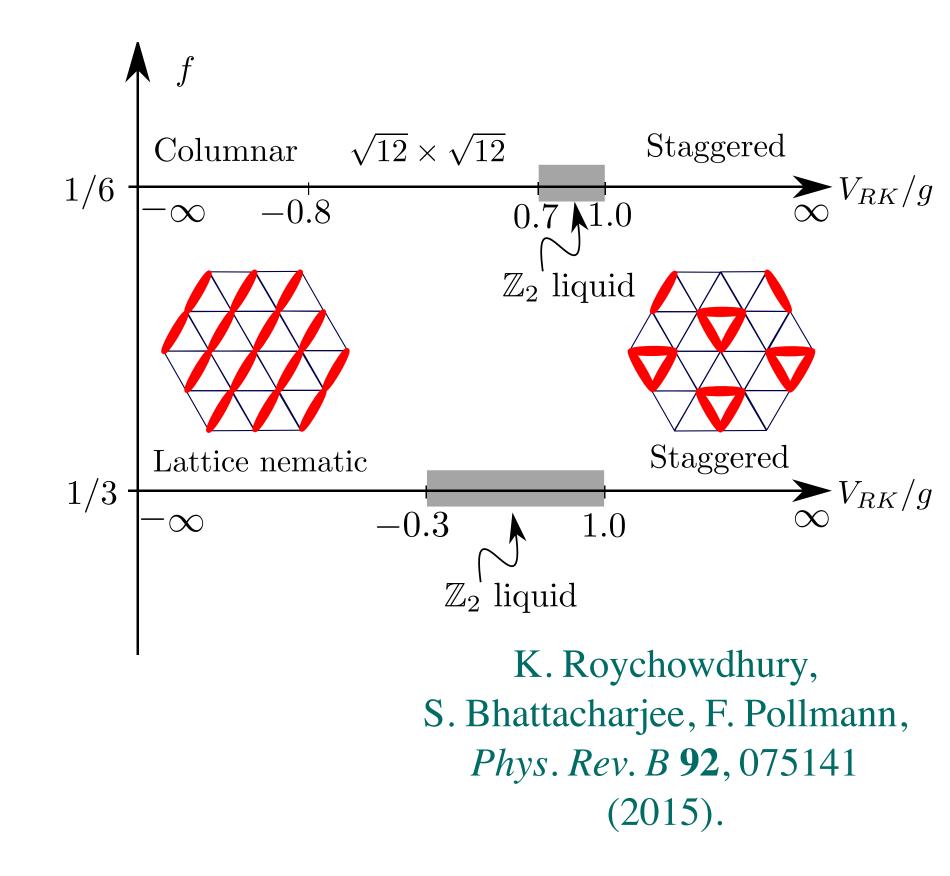
 $XZ = (-1)^{\text{number of intersections}} ZX$

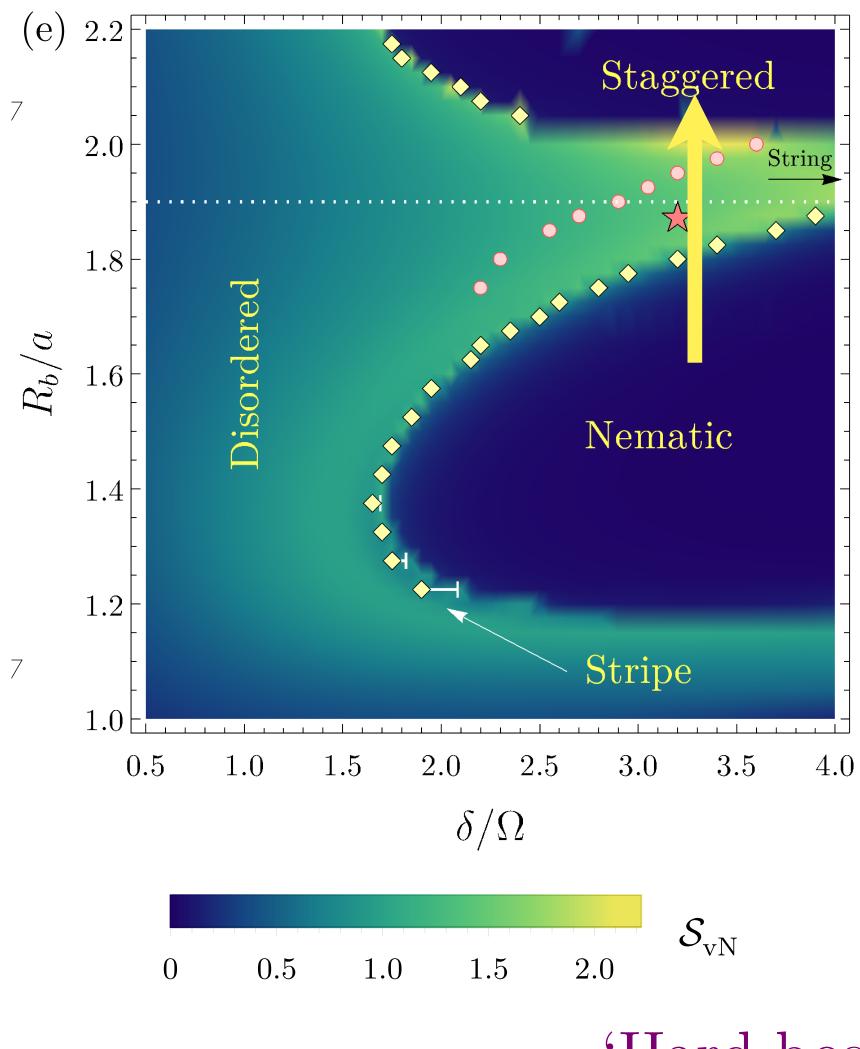
N. Schuch, D. Poilblanc, J.I. Cirac, D. Perez-Garcia, PRB 86, 115108 (2012). R. Verresen, M. D. Lukin, A. Vishwanath, arXiv: 2011.12310

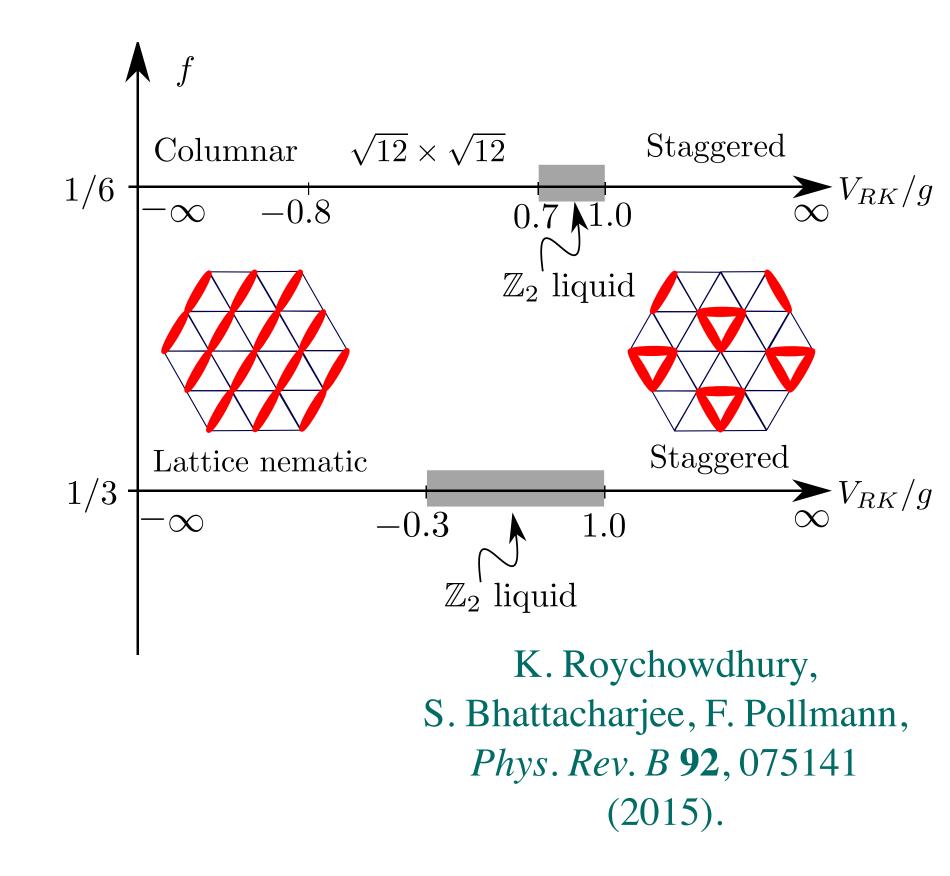


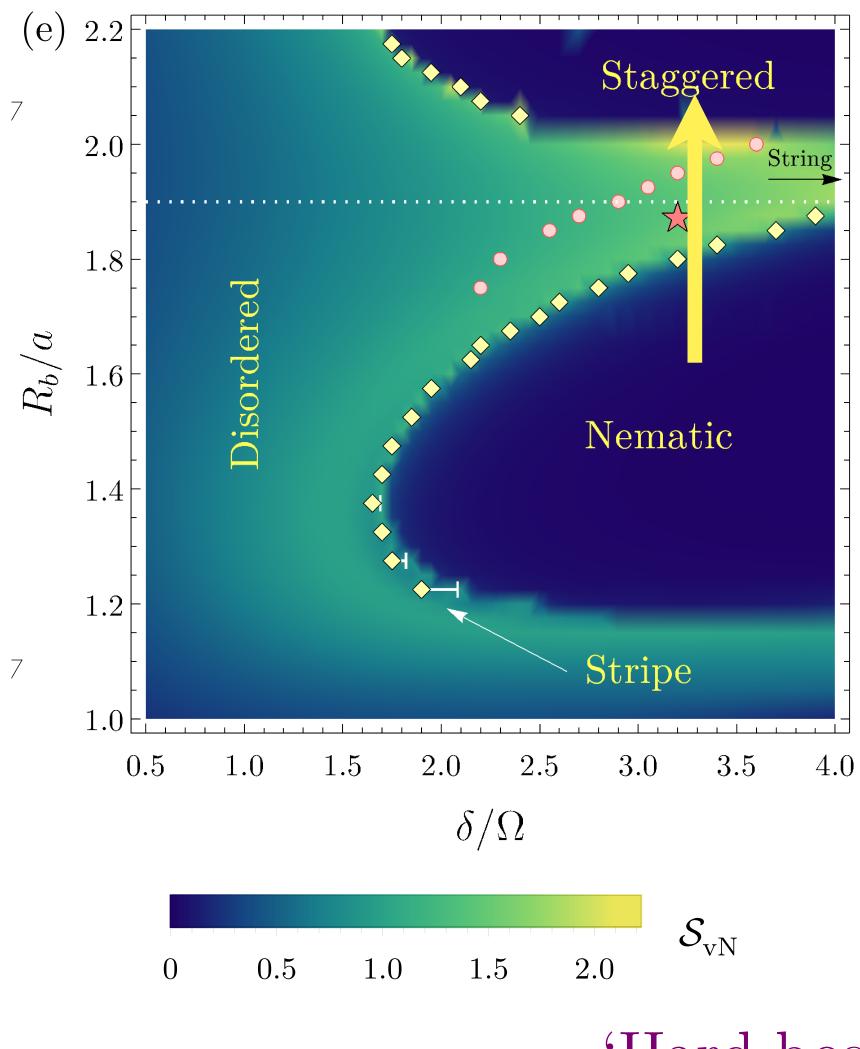


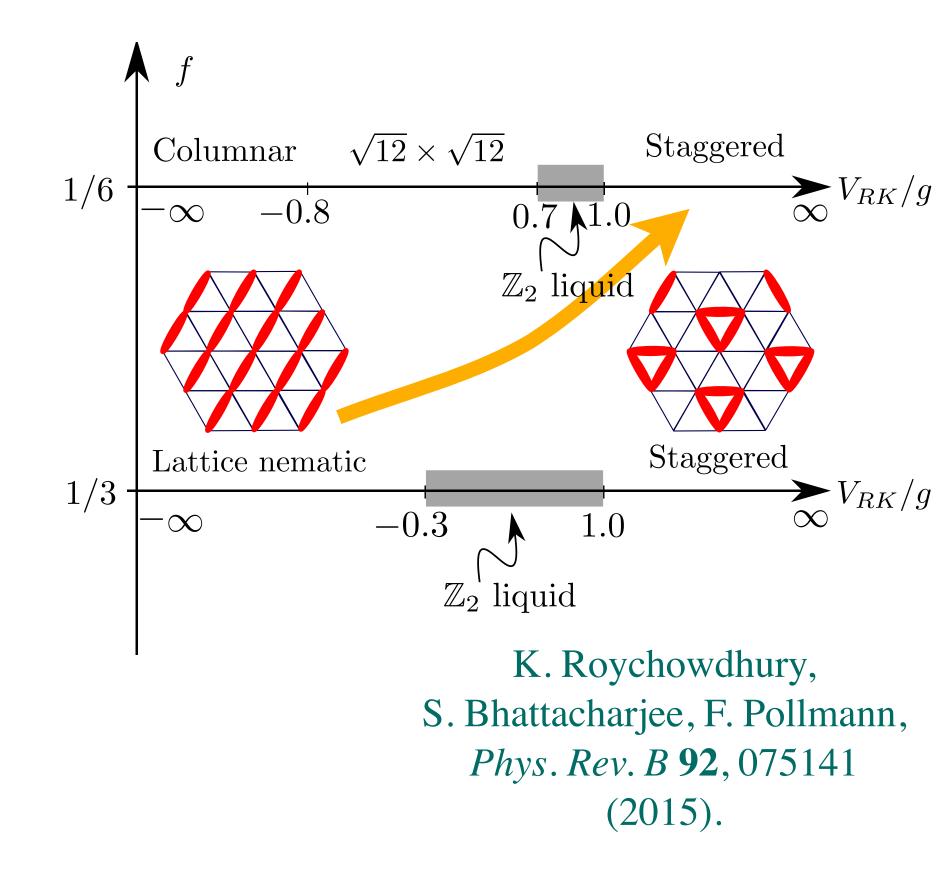




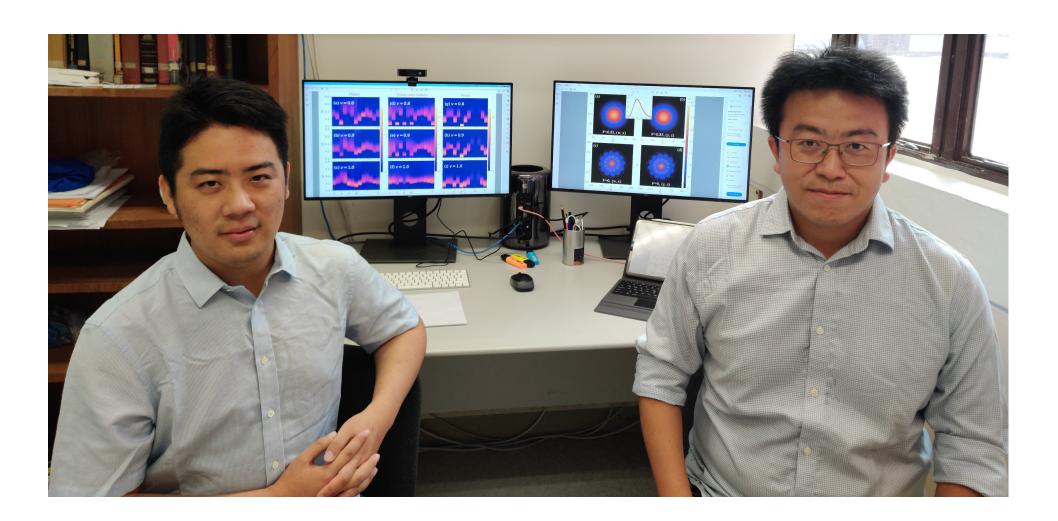


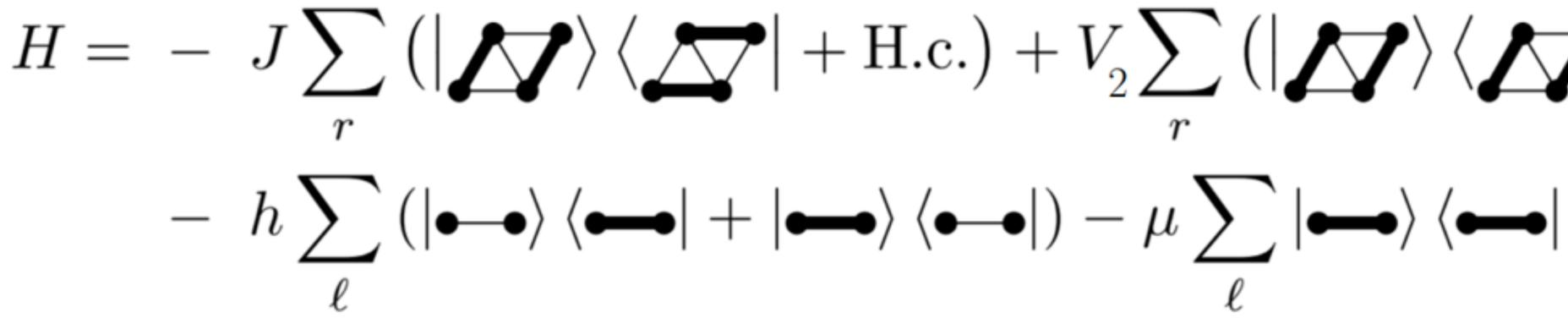






Work in progress: Zheng Yan Zi Yang Meng **Rhine Samajdar**

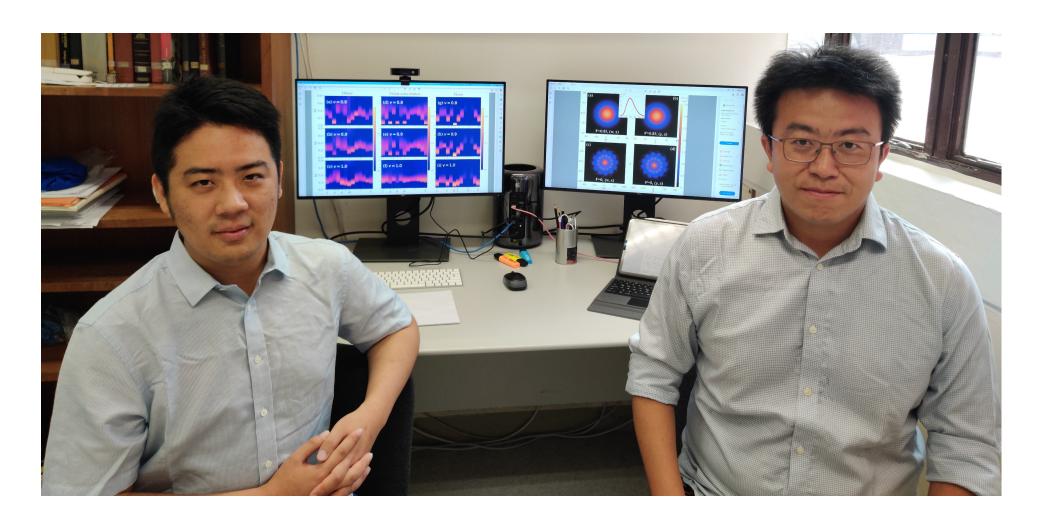




 $H = -J\sum \left(\left| \swarrow \right\rangle \left\langle \bigtriangleup \right| + \text{H.c.} \right) + V_2 \sum \left(\left| \bigtriangleup \right\rangle \left\langle \bigtriangleup \right| + \left| \bigtriangleup \right\rangle \left\langle \bigtriangleup \right| \right)$



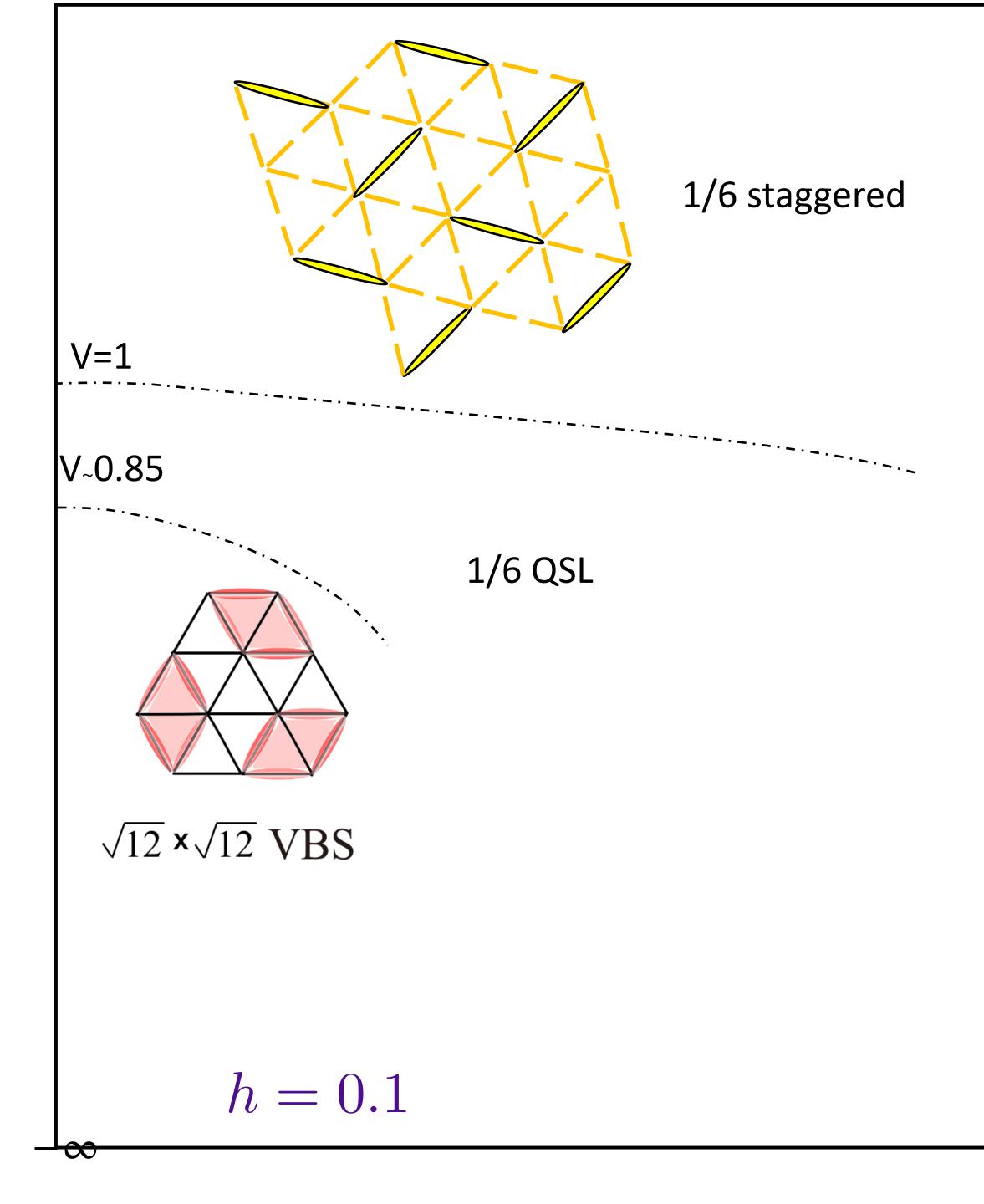
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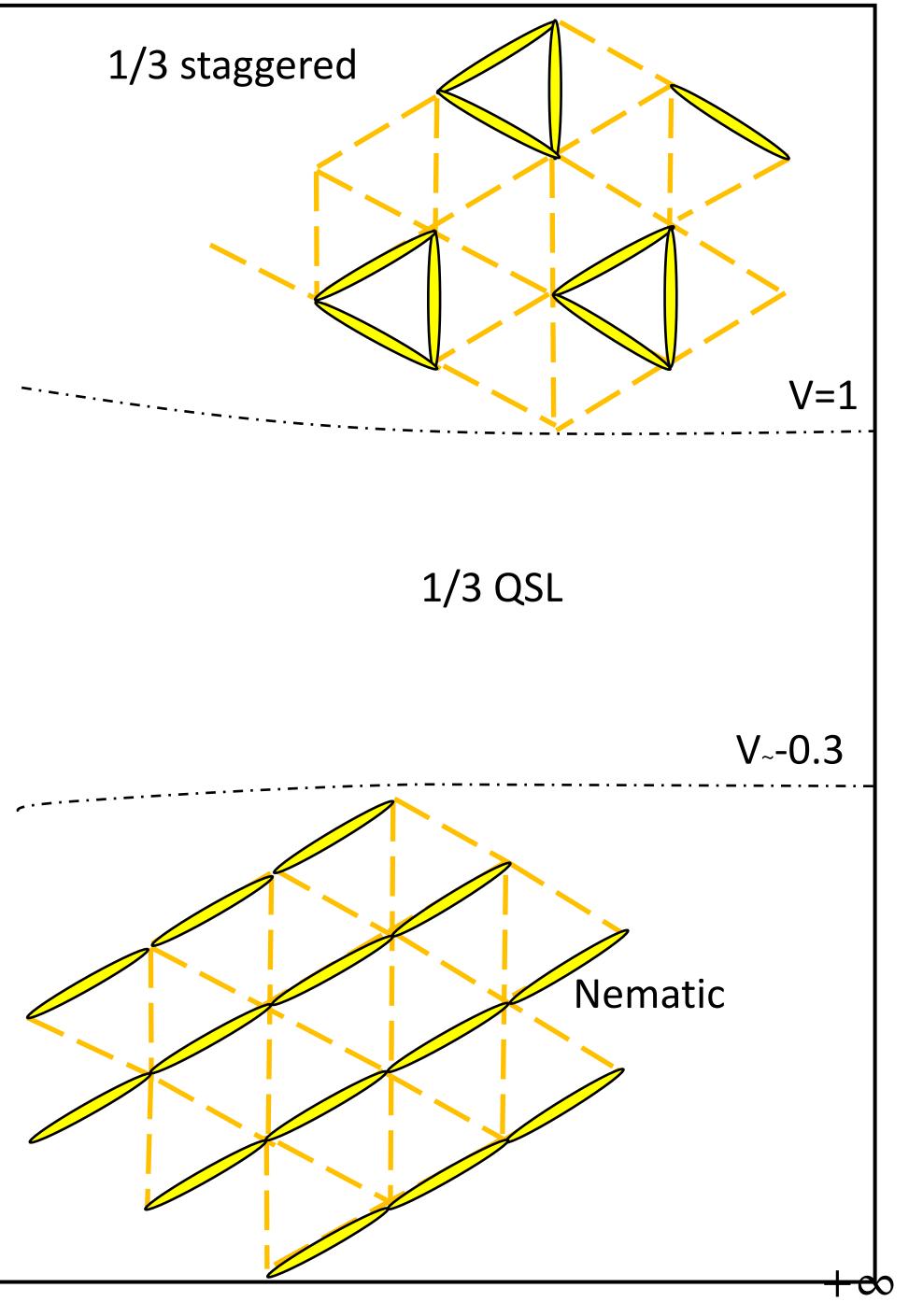


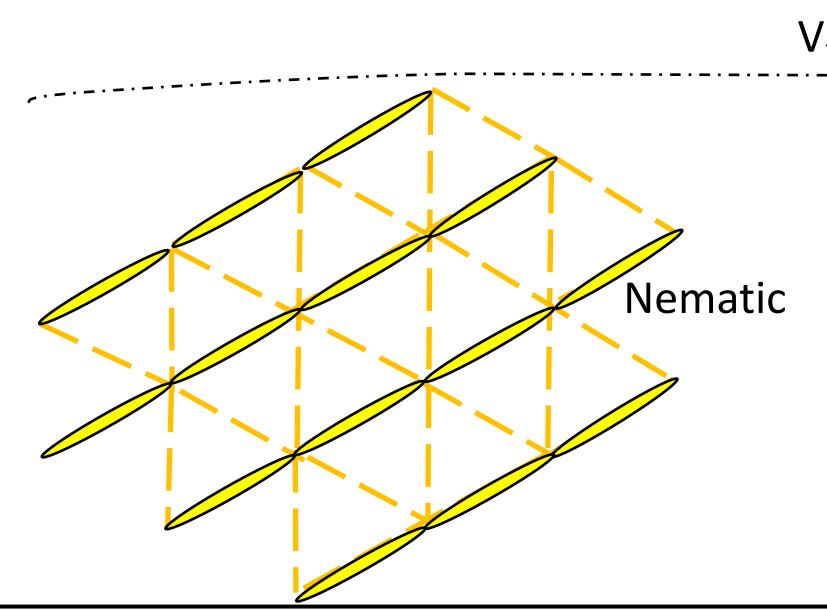
Related to Ising gauge theory (σ_{ij}) with matter fields (τ_i) : $H = -K \prod_{\Box} \sigma_{ij}^z - g \sum_{\langle ij \rangle} \sigma_{ij}^x - J \sum_{\langle ij \rangle} \tau_i^z \sigma_{ij}^z \tau_j^z - h \sum_i \tau_i^x$

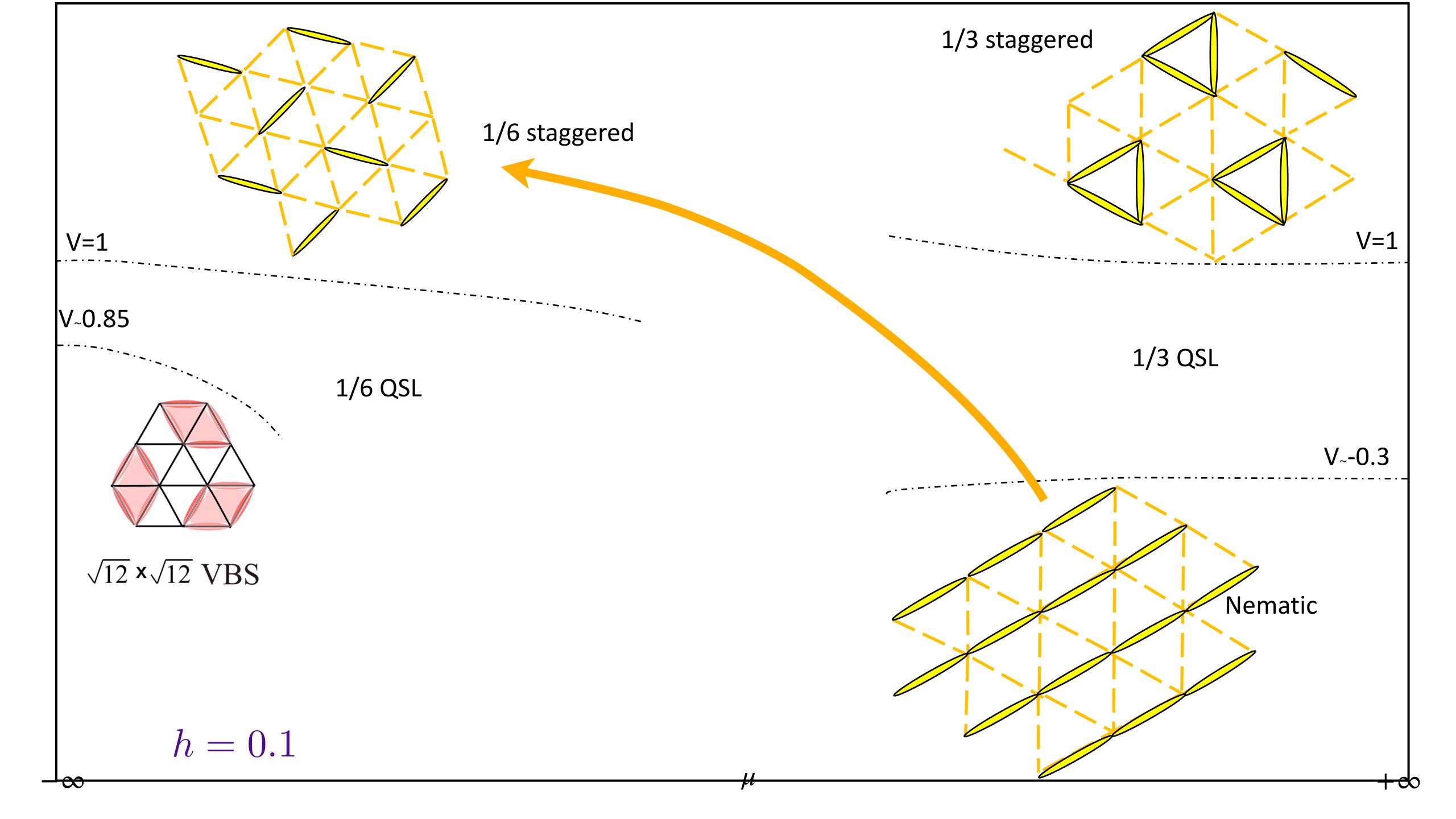
$$H = -J\sum_{r} \left(\left| \bigwedge \right\rangle \left\langle \bigwedge \right| + \text{H.c.} \right) + V_{2}\sum_{r} \left(\left| \bigwedge \right\rangle \left\langle \bigwedge \right| + \left| \bigwedge \right\rangle \right\rangle \left\langle \bigwedge \right| + \left| \bigwedge \right\rangle \left\langle \bigwedge \right| + \left| \bigwedge \right\rangle \left\langle \bigwedge \right| + \left| \bigwedge \right\rangle \left\langle \bigwedge \right| \right\rangle + V_{2}\sum_{r} \left(\left| \bigwedge \right\rangle \left\langle \bigwedge \right| + \left| \bigwedge \right\rangle \left\langle \bigwedge \right| \right\rangle + V_{2}\sum_{r} \left(\left| \bigwedge \right\rangle \left\langle \bigwedge \right| + \left| \bigwedge \right\rangle \left\langle \bigwedge \right| \right\rangle + V_{2}\sum_{r} \left(\left| \bigwedge \right\rangle \left\langle \bigwedge \right| + \left| \bigwedge \right\rangle \left\langle \bigwedge \right\rangle + V_{2}\sum_{r} \left(\left| \bigwedge \right\rangle \left\langle \bigwedge \right\rangle \left\langle \bigwedge \right\rangle + V_{2}\sum_{r} \left(\left| \bigwedge \right\rangle \left\langle \bigwedge \right\rangle \right\rangle + V_{2}\sum_{r} \left(\left| \bigwedge \right\rangle \left\langle \bigwedge \right\rangle \left\langle \bigwedge \right\rangle + V_{2}\sum_{r} \left(\left| \bigwedge \right\rangle \left\langle \bigwedge \right\rangle \right\rangle + V_{2}\sum_{r} \left(\left| \bigwedge \right\rangle \left\langle \bigwedge \right\rangle \left\langle \bigwedge \right\rangle + V_{2}\sum_{r} \left(\left| \bigwedge \right\rangle \left\langle \bigwedge \right\rangle \left\langle \bigwedge \right\rangle + V_{2}\sum_{r} \left(\left| \bigwedge \right\rangle \left\langle \bigwedge \right\rangle \left\langle \bigwedge \right\rangle + V_{2}\sum_{r} \left(\left| \bigwedge \right\rangle \left\langle \bigwedge \right\rangle \left\langle \bigwedge \right\rangle + V_{2}\sum_{r} \left(\left| \bigwedge \right\rangle \left\langle \bigwedge \right\rangle \left\langle \bigwedge \right\rangle + V_{2}\sum_{r} \left(\left| \bigwedge \right\rangle \left\langle \bigwedge \right\rangle \left\langle \bigwedge \right\rangle + V_{2}\sum_{r} \left(\left| \bigwedge \right\rangle \left\langle \bigwedge \right\rangle \left\langle \bigwedge \right\rangle + V_{2}\sum_{r} \left(\left| \bigwedge \right\rangle \left\langle \bigwedge \right\rangle \left\langle \bigwedge \right\rangle + V_{2}\sum_{r} \left(\left| \bigwedge \right\rangle \left\langle \bigwedge \right\rangle \left\langle \bigwedge \right\rangle + V_{2}\sum_{r} \left(\left| \bigwedge \right\rangle \left\langle \bigwedge \right\rangle \left\langle \bigwedge \right\rangle + V_{2}\sum_{r} \left(\left| \bigwedge \right\rangle \left\langle \bigwedge \right\rangle \left\langle \bigwedge \right\rangle + V_{2}\sum_{r} \left(\left| \bigwedge \right\rangle \left\langle \bigwedge \right\rangle + V_{2}\sum_{r} \left(\left| \bigwedge \right\rangle \left\langle \bigwedge \right\rangle \left\langle \bigwedge \right\rangle + V_{2}\sum_{r} \left(\left| \bigwedge \right\rangle \left\langle \bigwedge \right\rangle + V_{2}\sum_{r} \left(\left| \bigwedge \right\rangle \left\langle \bigwedge \right\rangle + V_{2}\sum_{r} \left(\left| \bigwedge \right\rangle \left\langle \bigwedge \right\rangle + V_{2}\sum_{r} \left(\left| \bigwedge \right\rangle \left\langle \bigwedge \right\rangle + V_{2}\sum_{r} \left(\left| \bigcap \right\rangle + V_{2}\sum$$











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