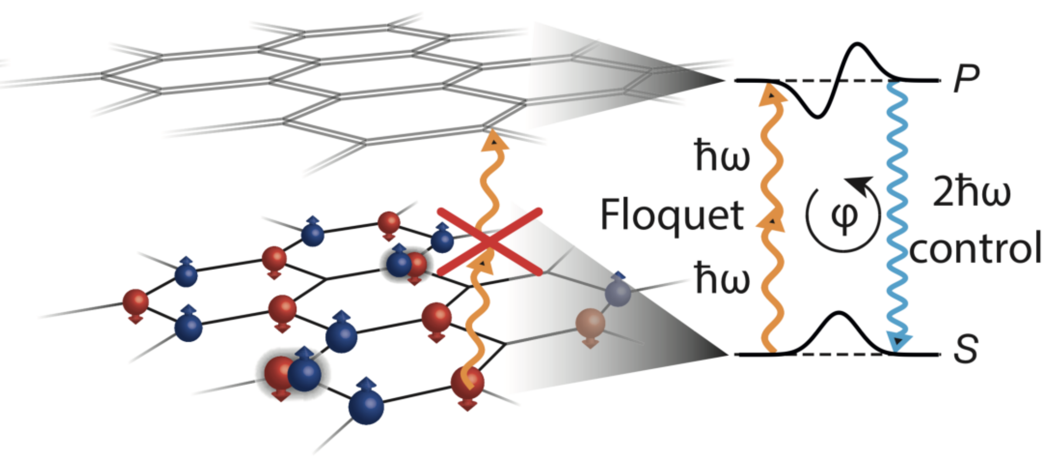
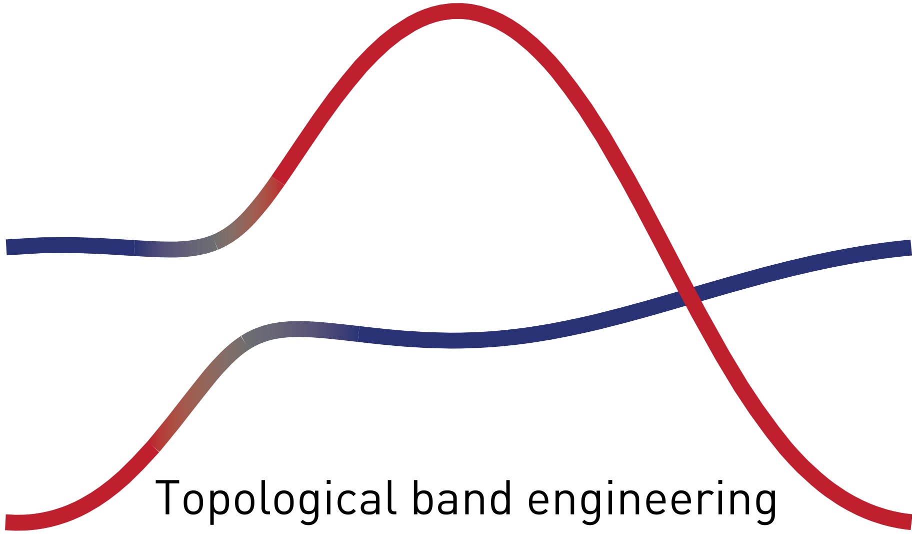
Multi-frequency floquet engineering



Suppress inter-band heating

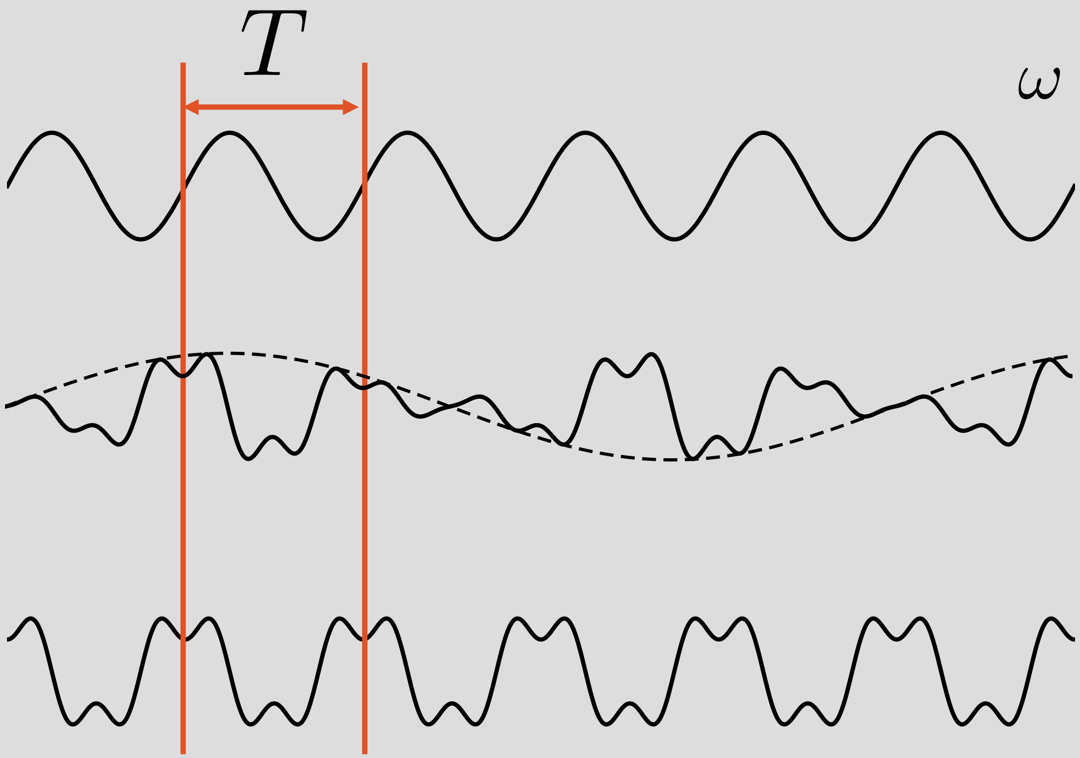
Viebahn et al. PRX 2021



Topological band engineering
Floquet Thouless pumping

Sandholzer et al. (in prep), Minguzzi et al. (in prep)

$$\omega = \frac{2\pi}{T}$$



$$\mathcal{H}(\tau) = \mathcal{H}(\tau + T)$$

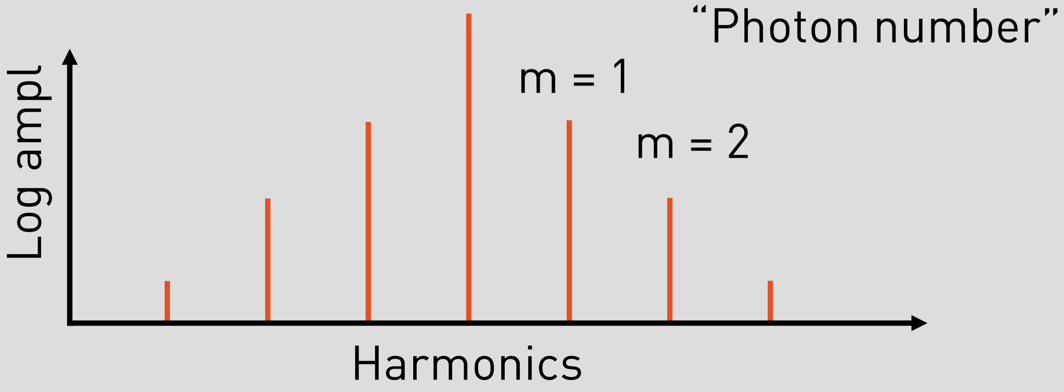
$$|\psi_n(\tau)\rangle = e^{-i\epsilon_n \tau/\hbar} |u_n(\tau)\rangle$$

Periodic Floquet mode

$$|u_n(\tau)\rangle = |u_n(\tau + T)\rangle$$

Fourier decomposition

$$|\psi_n(\tau)\rangle = e^{-i\epsilon_n \tau/\hbar} \sum_{m=-\infty}^{+\infty} e^{-im\omega\tau} |n, m\rangle$$



$$|\psi_n(\tau)\rangle = e^{-i\epsilon_n \tau/\hbar} \sum_{m=-\infty}^{+\infty} e^{-im\omega\tau} \, |n, m\rangle$$

$$\xRightarrow{\text{Schrödinger}} \quad (\epsilon_n + m\hbar\omega) \, |n, m\rangle = \sum_{m'=-\infty}^{+\infty} \mathcal{H}_{m-m'} \, |n, m'\rangle$$

$$|\psi_n(\tau)\rangle = e^{-i\epsilon_n \tau/\hbar} \sum_{m=-\infty}^{+\infty} e^{-im\omega\tau} |n, m\rangle$$

Schrödinger

$$\Longrightarrow \quad (\epsilon_n + m\hbar\omega) |n, m\rangle = \sum_{m'=-\infty}^{+\infty} \mathcal{H}_{m-m'} |n, m'\rangle$$

Single-frequency drive

$$\mathcal{H}(\tau) = \mathcal{H}_0 + V e^{i\omega\tau} + V^\dagger e^{-i\omega\tau}$$

Extended space Hamiltonian $\Longrightarrow$

$$\begin{pmatrix} \ddots & V & 0 & & \\ V^\dagger & \mathcal{H}_0 + \hbar\omega & V & 0 & \\ 0 & V^\dagger & \mathcal{H}_0 & V & 0 \\ & 0 & V^\dagger & \mathcal{H}_0 - \hbar\omega & V \\ & & 0 & V^\dagger & \ddots \end{pmatrix}$$

Example: 3-photon resonance

$$\mathcal{H}(\tau) = \frac{\hbar\omega_0}{2}\sigma_z + \mu B \ \sigma_x \ \cos\omega\tau$$

$$3\omega \simeq \omega_0 \ \Rightarrow \ \omega_0 - 3\omega \simeq 0$$

Example: 3-photon resonance

$$\mathcal{H}(\tau) = \frac{\hbar\omega_0}{2}\sigma_z + \mu B\ \sigma_x\ \cos\omega\tau$$

$$3\omega \simeq \omega_0 \ \Rightarrow\ \omega_0 - 3\omega \simeq 0$$

$$\begin{pmatrix}
\ddots & & & & & & & \\
& \begin{pmatrix} 0 & \\ & \omega_0 \end{pmatrix} & \begin{pmatrix} 0 & \mu B \\ \mu B & 0 \end{pmatrix} & & & & & \\
& & \begin{pmatrix} -\omega & \\ & \omega_0 - \omega \end{pmatrix} & \begin{pmatrix} 0 & \mu B \\ \mu B & 0 \end{pmatrix} & & & & \\
& & & \begin{pmatrix} -2\omega & \\ & \omega_0 - 2\omega \end{pmatrix} & \begin{pmatrix} 0 & \mu B \\ \mu B & 0 \end{pmatrix} & & & \\
& & & & \begin{pmatrix} -3\omega & \\ & \omega_0 - 3\omega \end{pmatrix} & & & \\
& & & & & & & \ddots
\end{pmatrix}$$

$$\Omega_{3\text{-photon}} \simeq \frac{(\mu B)^3}{\omega^2}$$

Example: interference

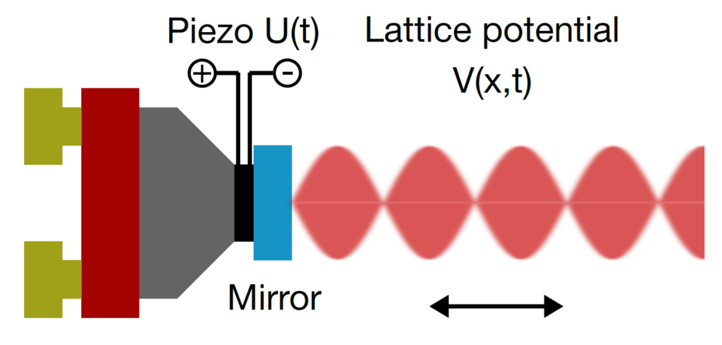
$$\mathcal{H}(\tau) = \frac{\hbar\omega_0}{2}\sigma_z + \mu B\ \sigma_x\ \cos\omega\tau + \mu B_3\ \sigma_x\ \cos(3\omega\tau + \varphi)$$

$$\omega_0 - 3\omega \simeq 0$$

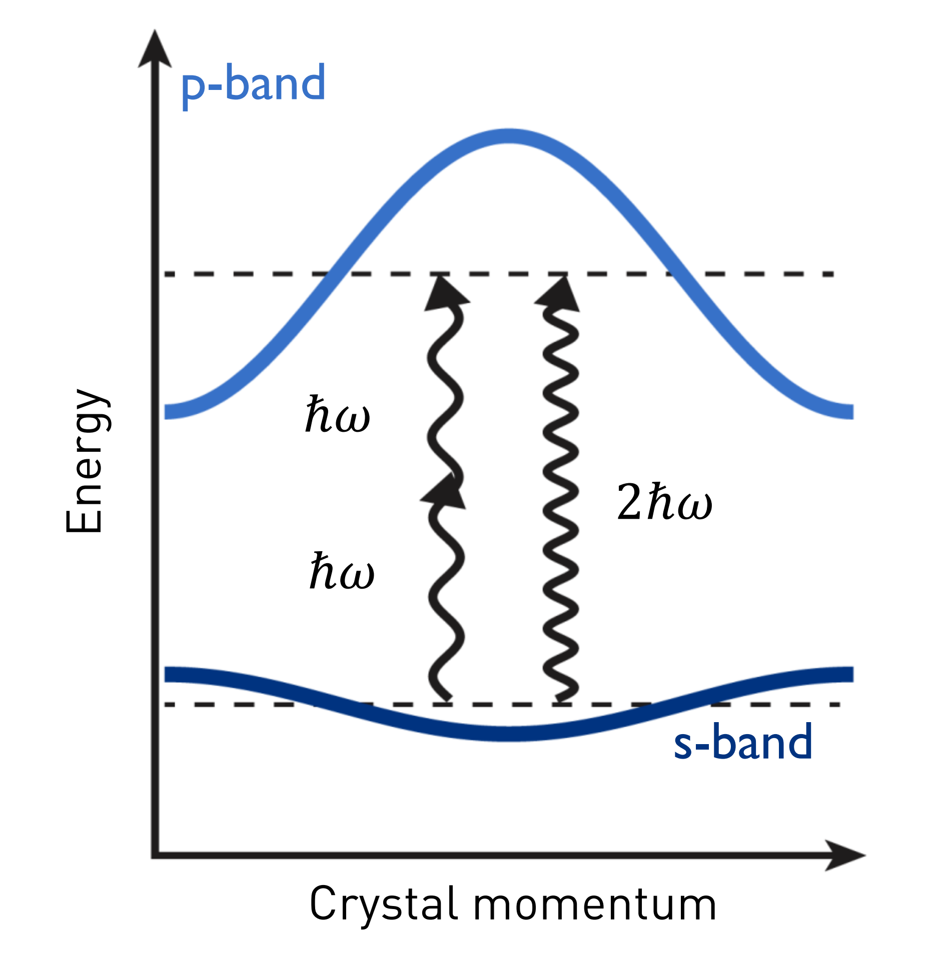
$$\begin{pmatrix}
\ddots & & & & & \\
& \begin{pmatrix}0 & \\ & \omega_0\end{pmatrix} & \begin{pmatrix}0 & \mu B\\ \mu B & 0\end{pmatrix} & & \begin{pmatrix}0 & \mu B_3 e^{i\varphi}\\ \mu B_3 e^{-i\varphi} & 0\end{pmatrix} & \\
& & \begin{pmatrix}-\omega & \\ & \omega_0 - \omega\end{pmatrix} & \begin{pmatrix}0 & \mu B\\ \mu B & 0\end{pmatrix} & & \\
& & & \begin{pmatrix}-2\omega & \\ & \omega_0 - 2\omega\end{pmatrix} & \begin{pmatrix}0 & \mu B\\ \mu B & 0\end{pmatrix} & \\
& & & & \begin{pmatrix}-3\omega & \\ & \omega_0 - 3\omega\end{pmatrix} & \\
& & & & & \ddots
\end{pmatrix}$$

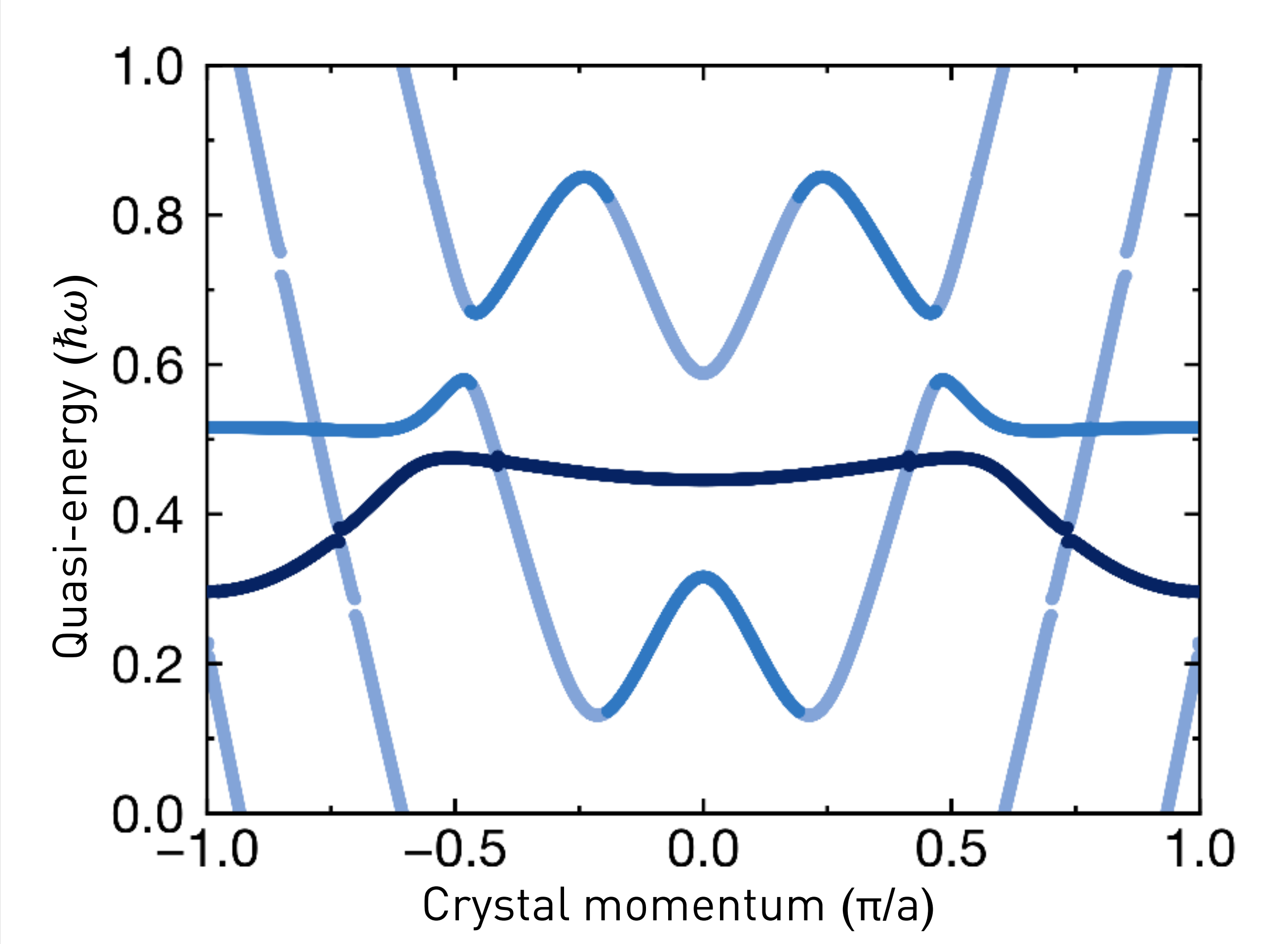
$$\Omega_{3\text{-photon}} \simeq \frac{(\mu B)^3}{\omega^2} + \mu B_3 e^{i\varphi}$$

Extended systems: Floquet-Bloch bands

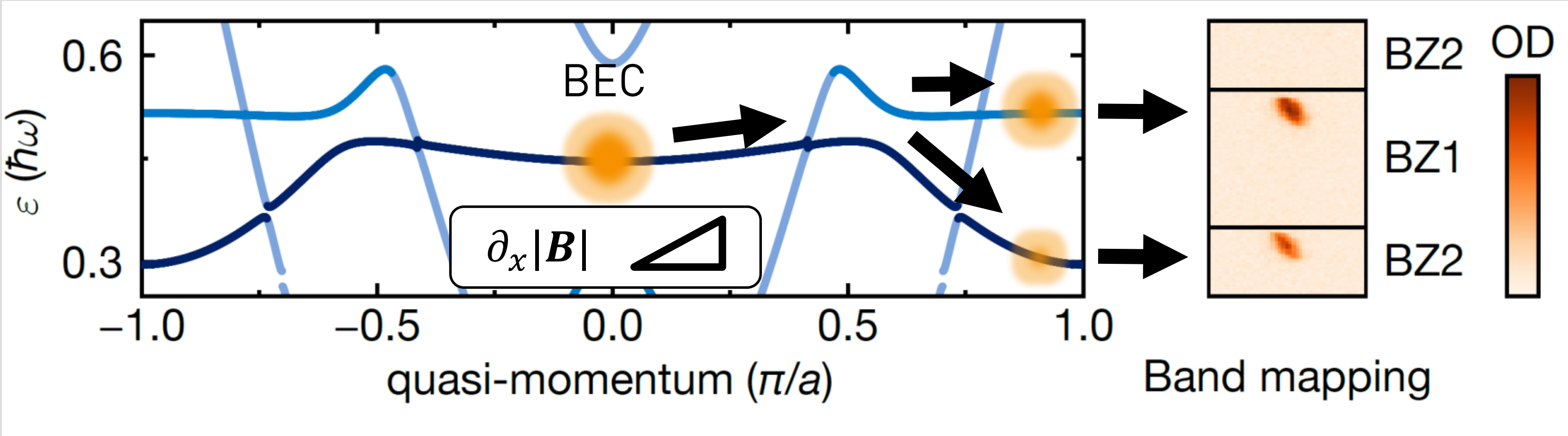


$$H = \frac{\hat{p}^2}{2m} + \frac{V_0}{2}\cos(2k_L\hat{x}) + F(t)\,\hat{x}$$

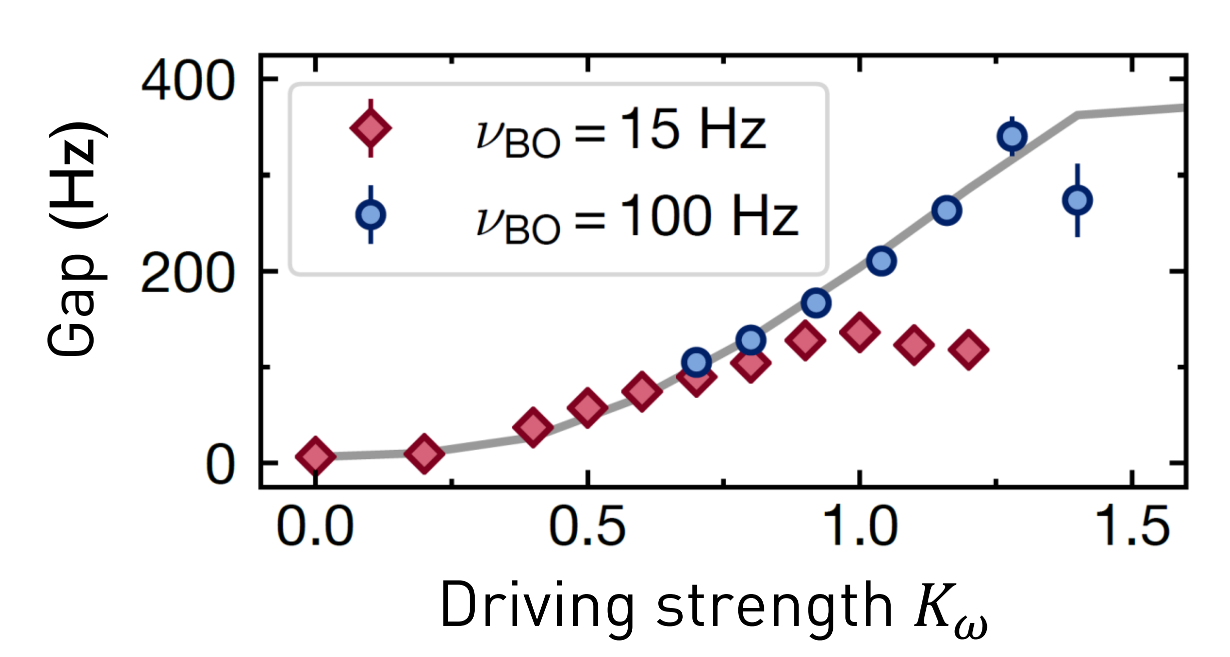




1ω shaking



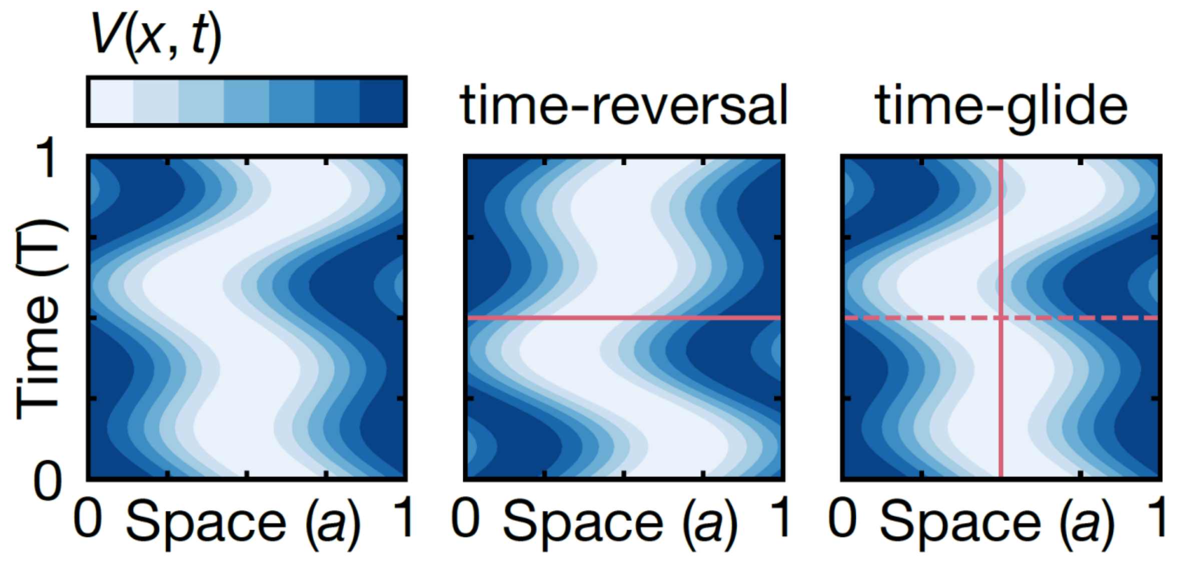
$$\frac{aF(t)}{\hbar\omega} = K_\omega \cos(\omega t)$$

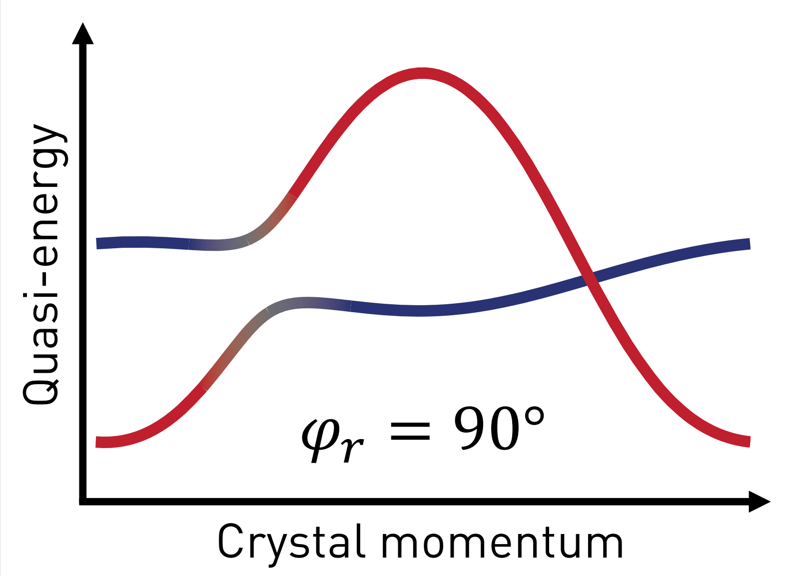


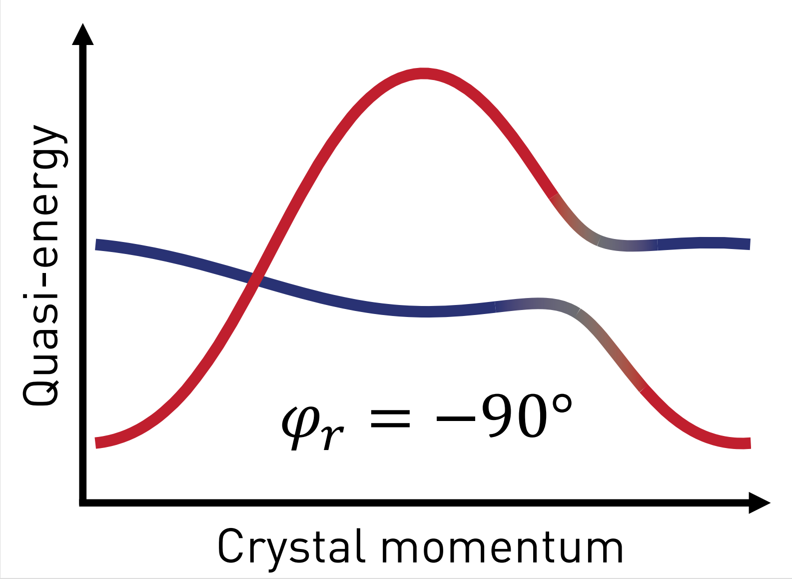
Sandholzer et al. (in prep)

1ω -2ω shaking

$K_\omega = 1.0 \quad K_{2\omega} = 0.3$







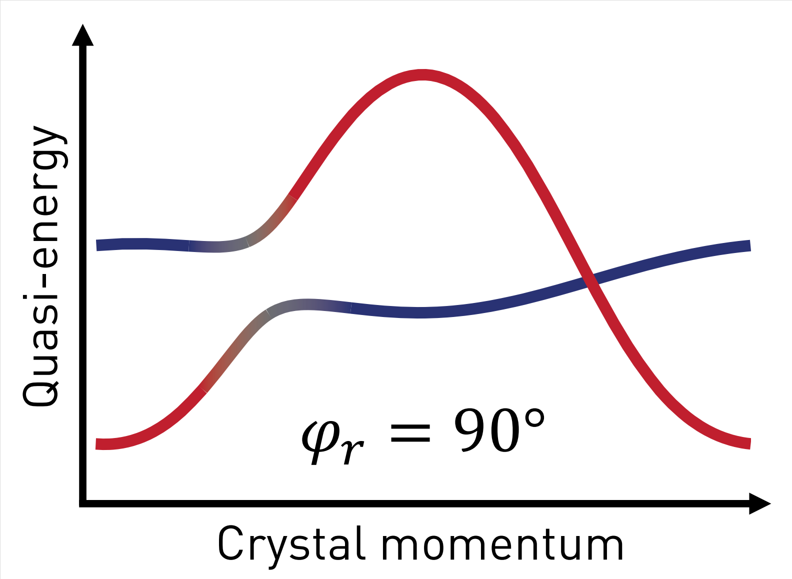
time-reversal: $V(x,t) \rightarrow V(x,-t)$

time-glide: $V(x,t) \rightarrow V(-x,-t-T/2)$

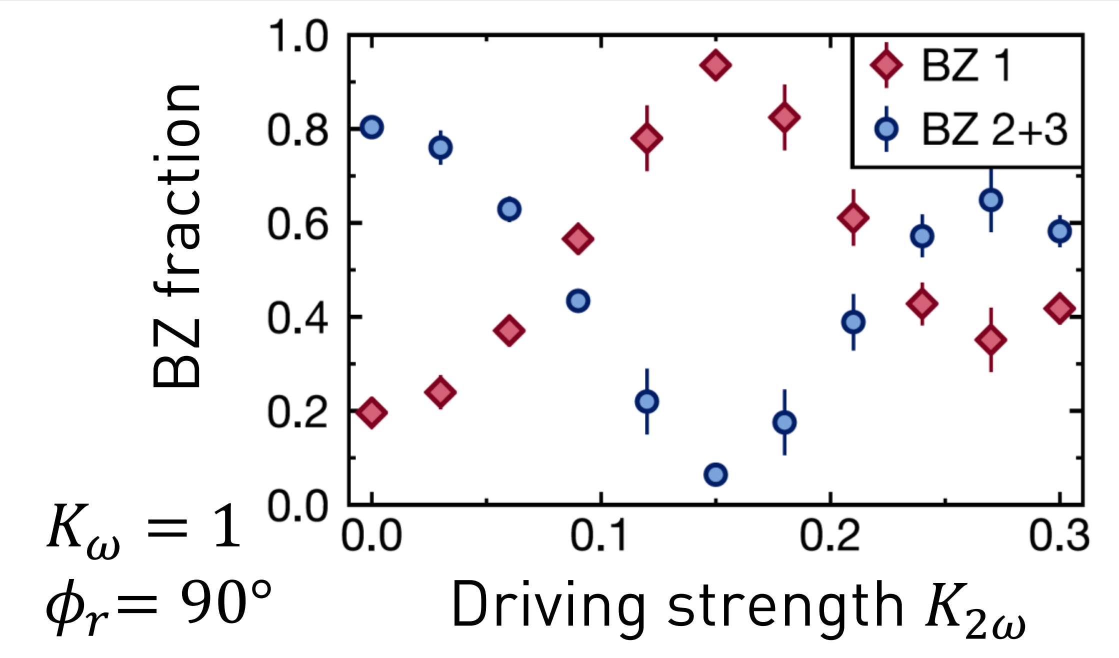
$$\frac{aF(t)}{\hbar\omega} = K_\omega \cos(\omega t) + 2K_{2\omega} \cos(2\omega t + \phi_r)$$

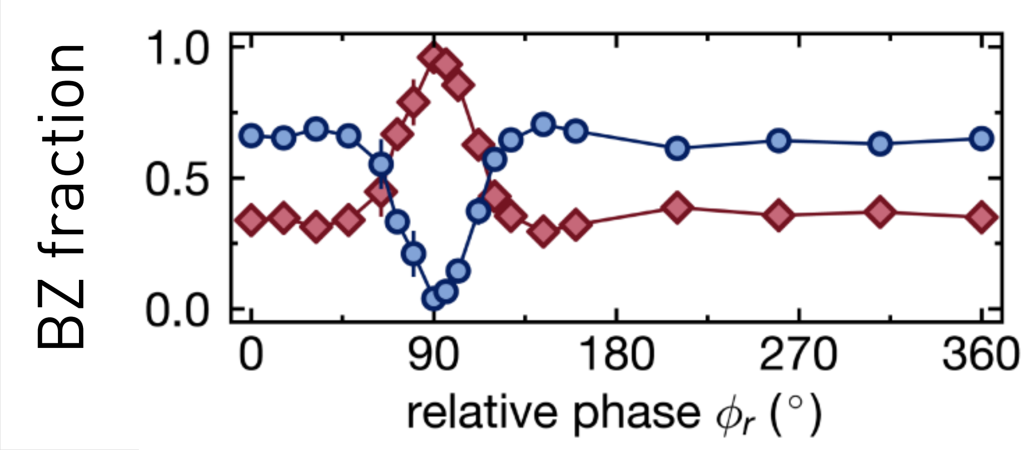
Struck et al. PRL 2012 (Peierls phase)
Aidelsburger et al. PRL 2011 (magnetic fluxes)
Xu & Wu PRL 2018 (time-glide symmetry)

1ω -2ω shaking



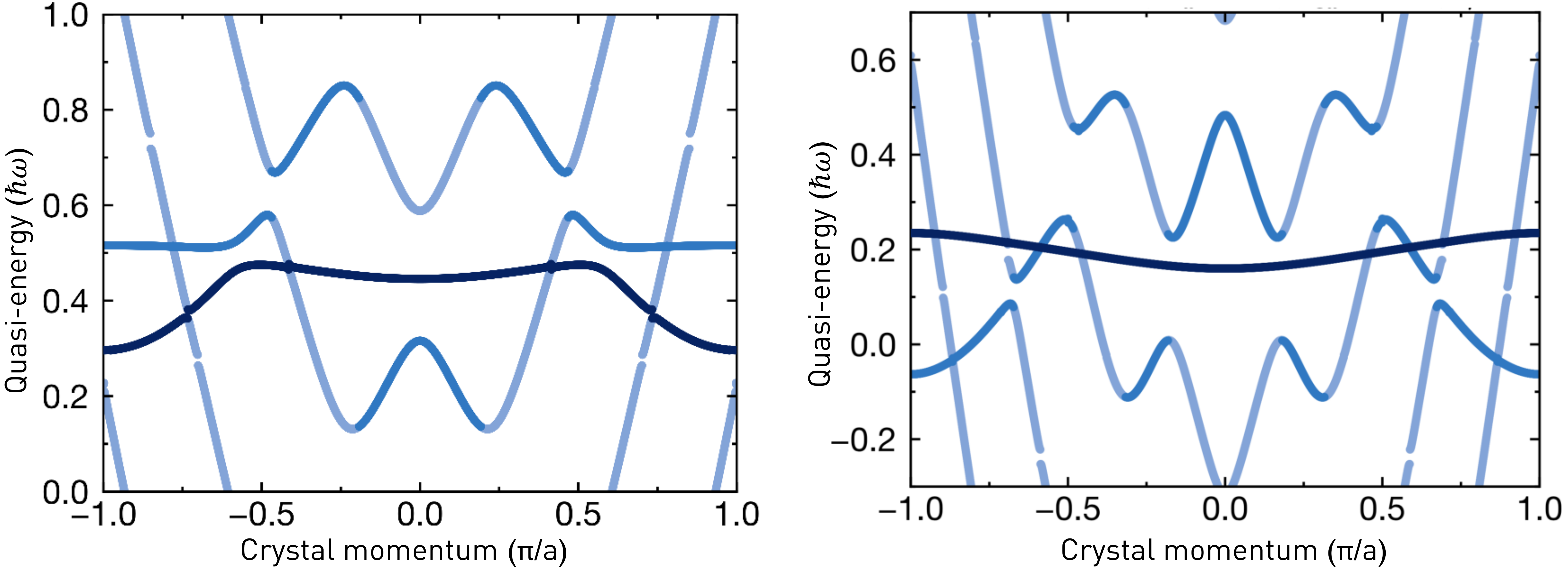
$$\frac{aF(t)}{\hbar\omega} = K_\omega \cos(\omega t) + 2K_{2\omega} \cos(2\omega t + \phi_r)$$





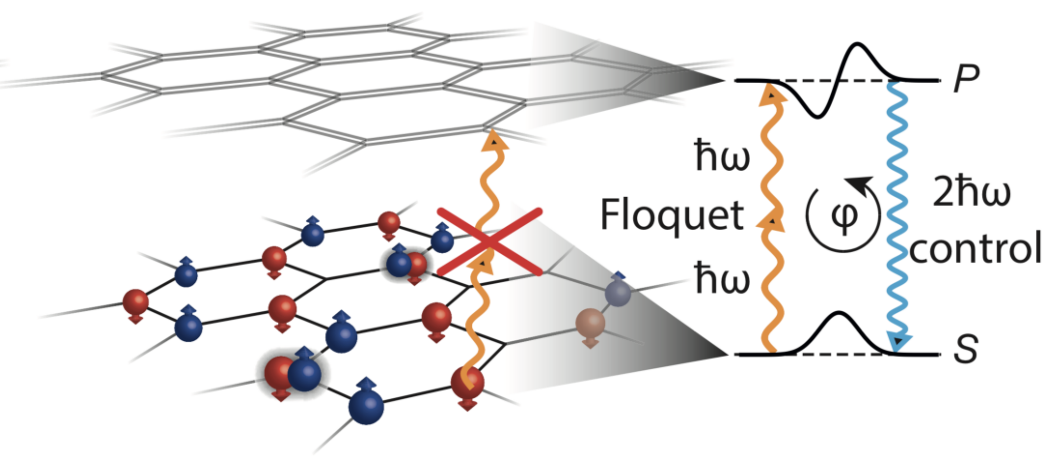
Sandholzer et al. (in prep)

1ω -3ω shaking



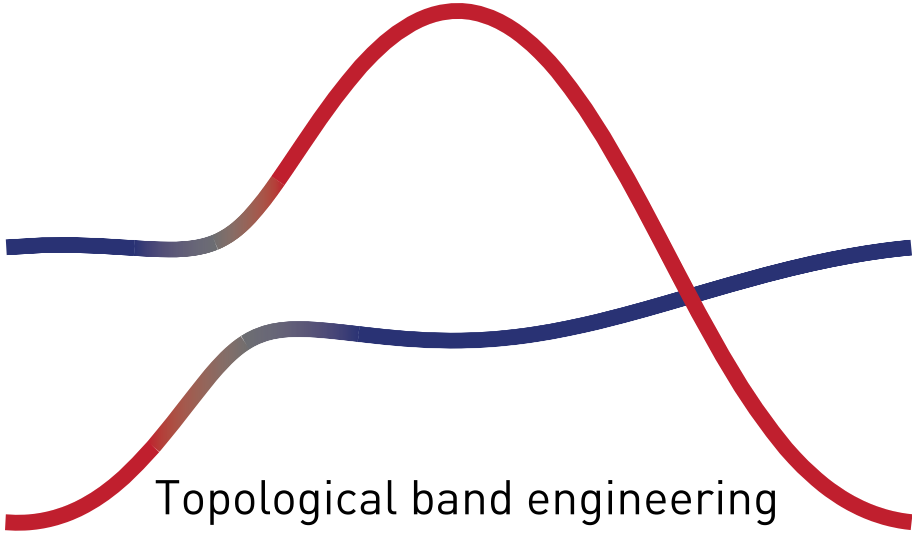
Schiavoni et al. PRL 2003
Zhuang et al. PRL 2013
Niu et al. Opt. Express 2015

Multi-frequency floquet engineering



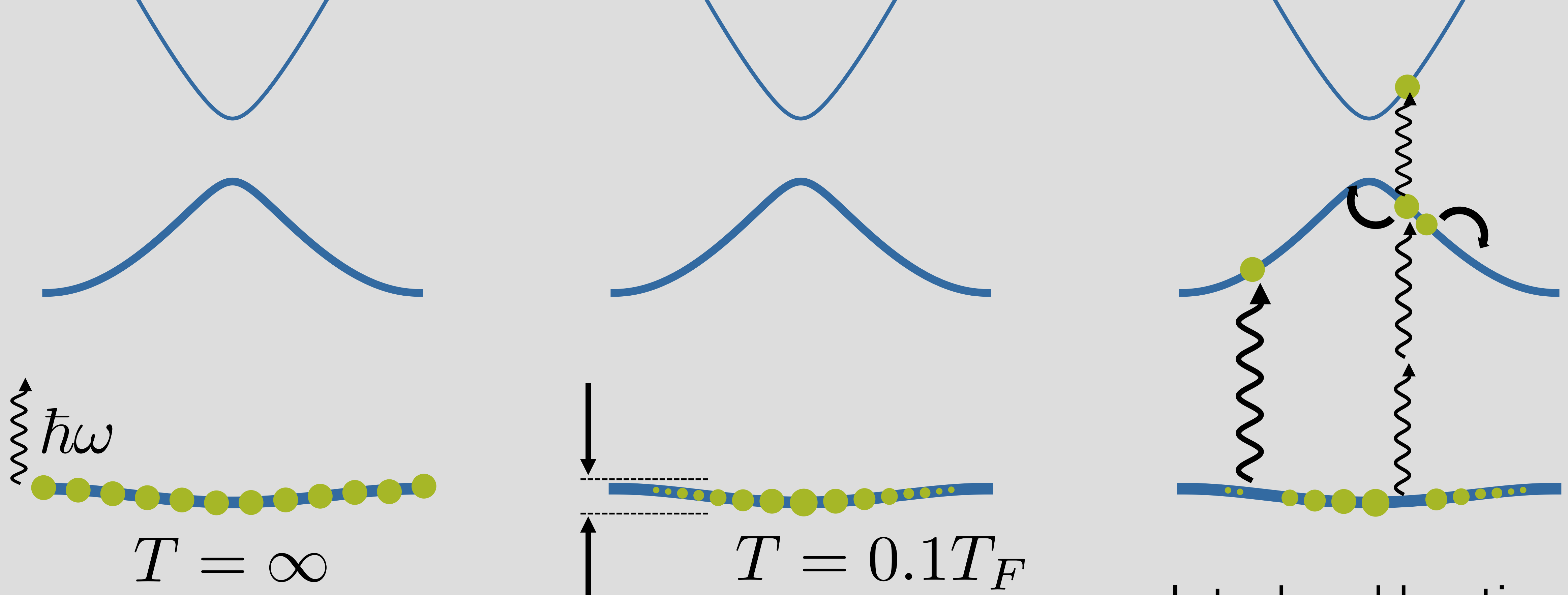
Suppress inter-band heating

Viebahn et al. PRX 2021



Topological band engineering
Floquet Thouless pumping

Sandholzer et al. (in prep), Minguzzi et al. (in prep)



Intraband "Floquet" heating

bandwidth W

Interband heating

Rate $\propto e^{-\frac{\hbar\omega}{W}}$

Static

Multi-photon resonances

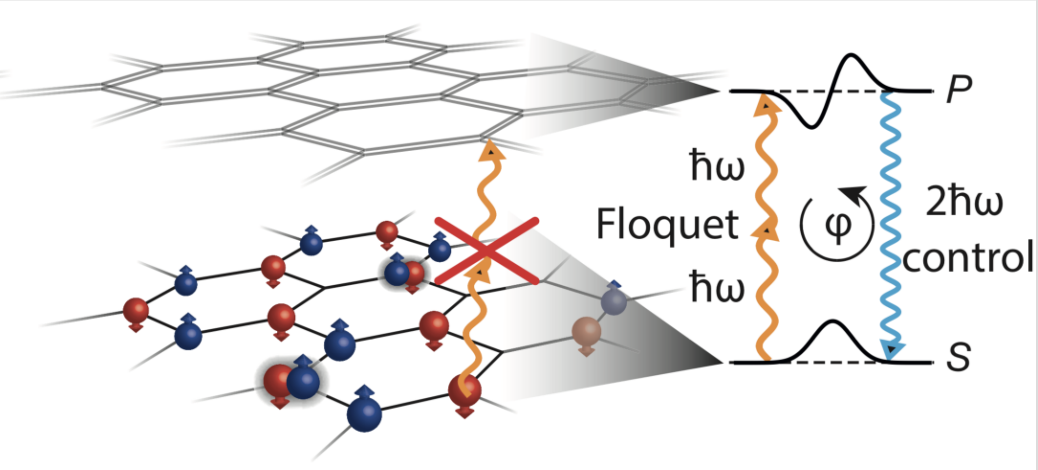
Messer et al. PRL 2018
Rubio-Abadal et al. PRX 2020
Singh et al. PRX 2019

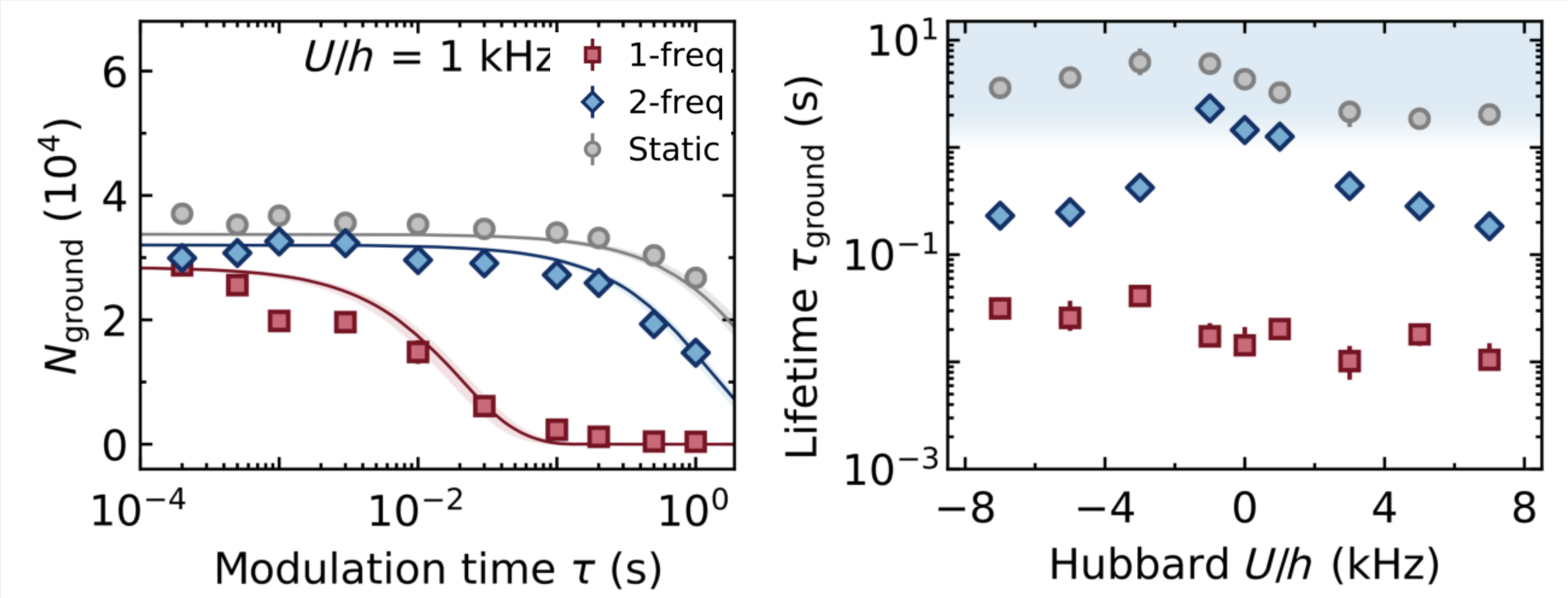
Compromise: Sun & Eckardt PRR 2020

Weinberg et al. PRA 2015
Reitter et al. PRL 2017

1ω -2ω amplitude modulation

Driven Fermi-Hubbard model



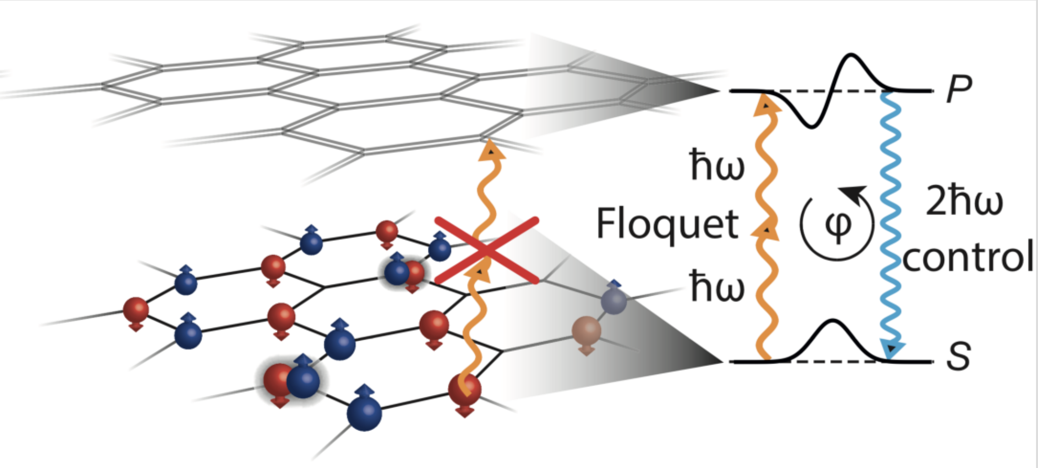


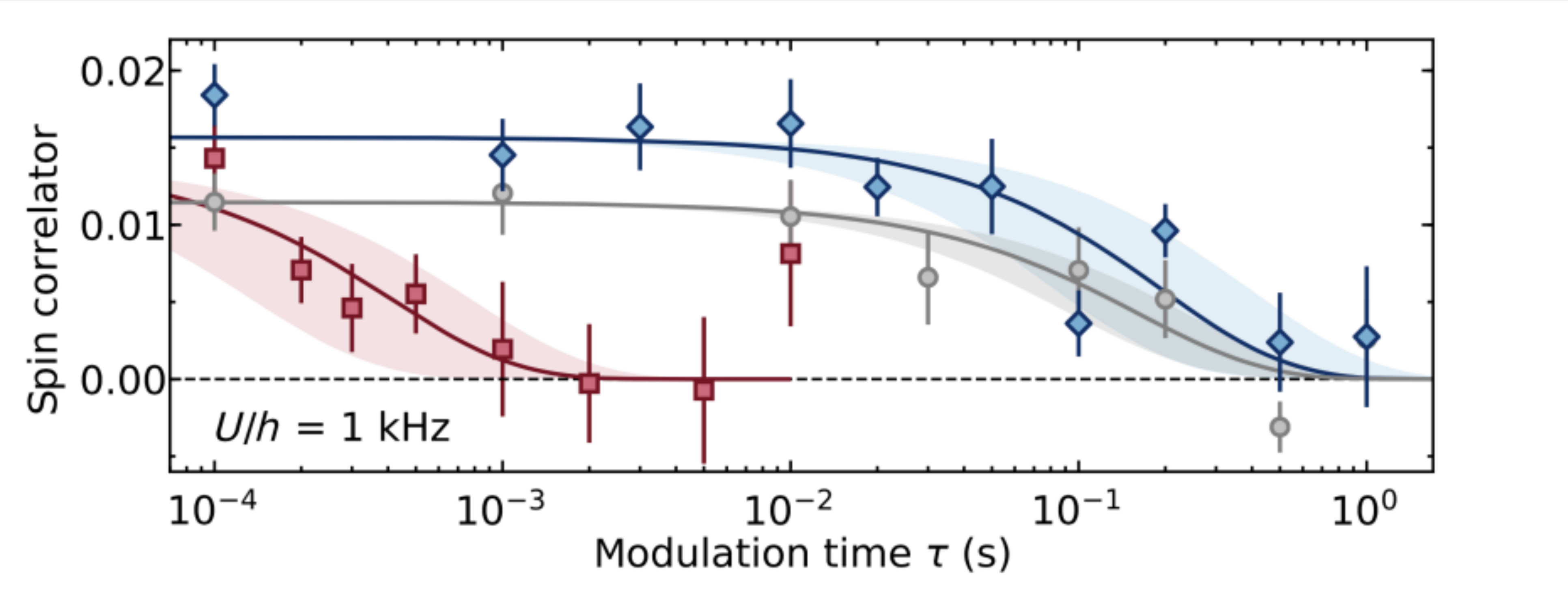
Viebahn et al. PRX 2021

$$(t_x, t_y, t_z)/h \;=\; (340, 90, 106)\ \mathrm{Hz}$$

1ω -2ω amplitude modulation

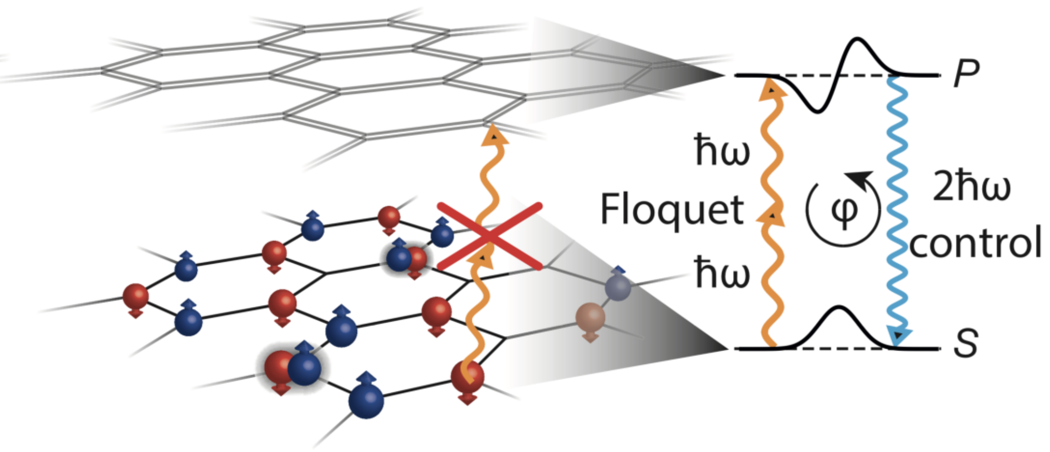
Driven Fermi-Hubbard model





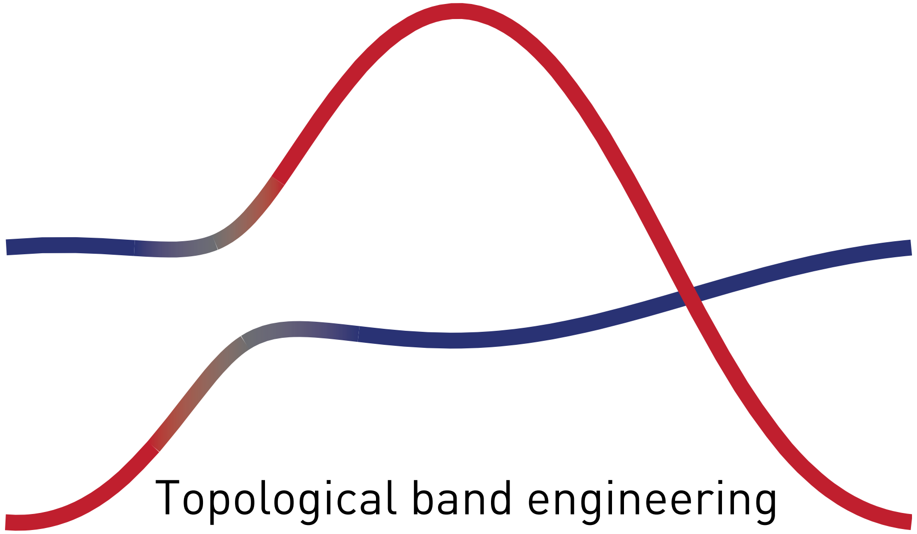
Viebahn et al. PRX 2021

Multi-frequency floquet engineering



Suppress inter-band heating

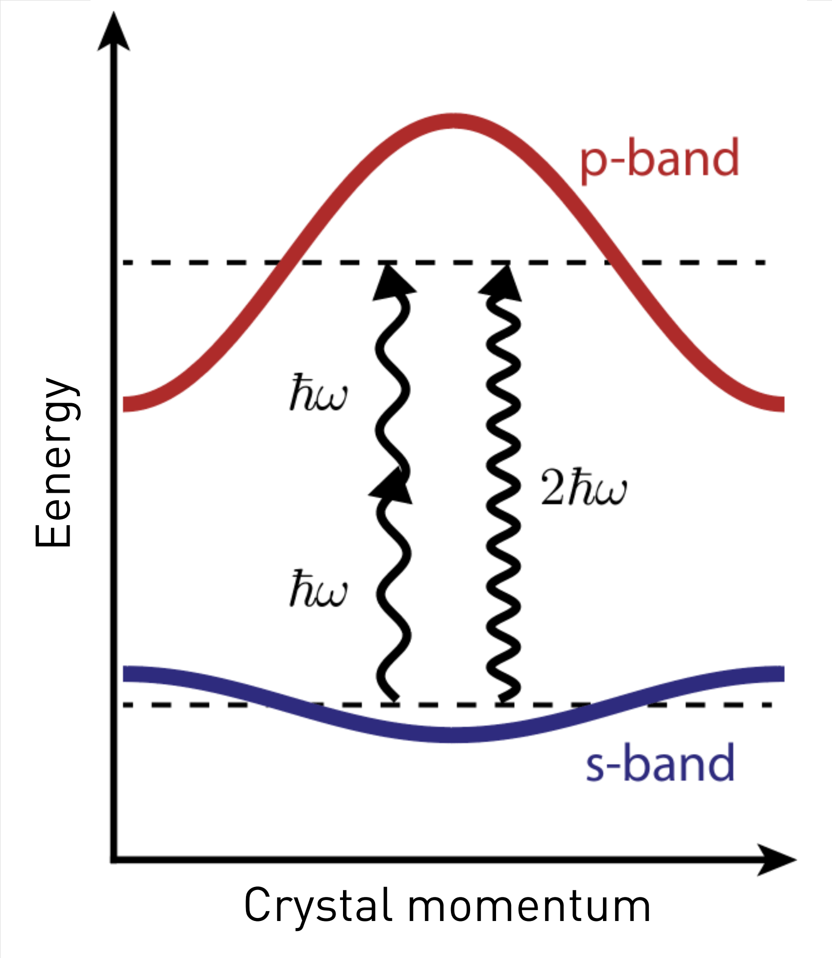
Viebahn et al. PRX 2021

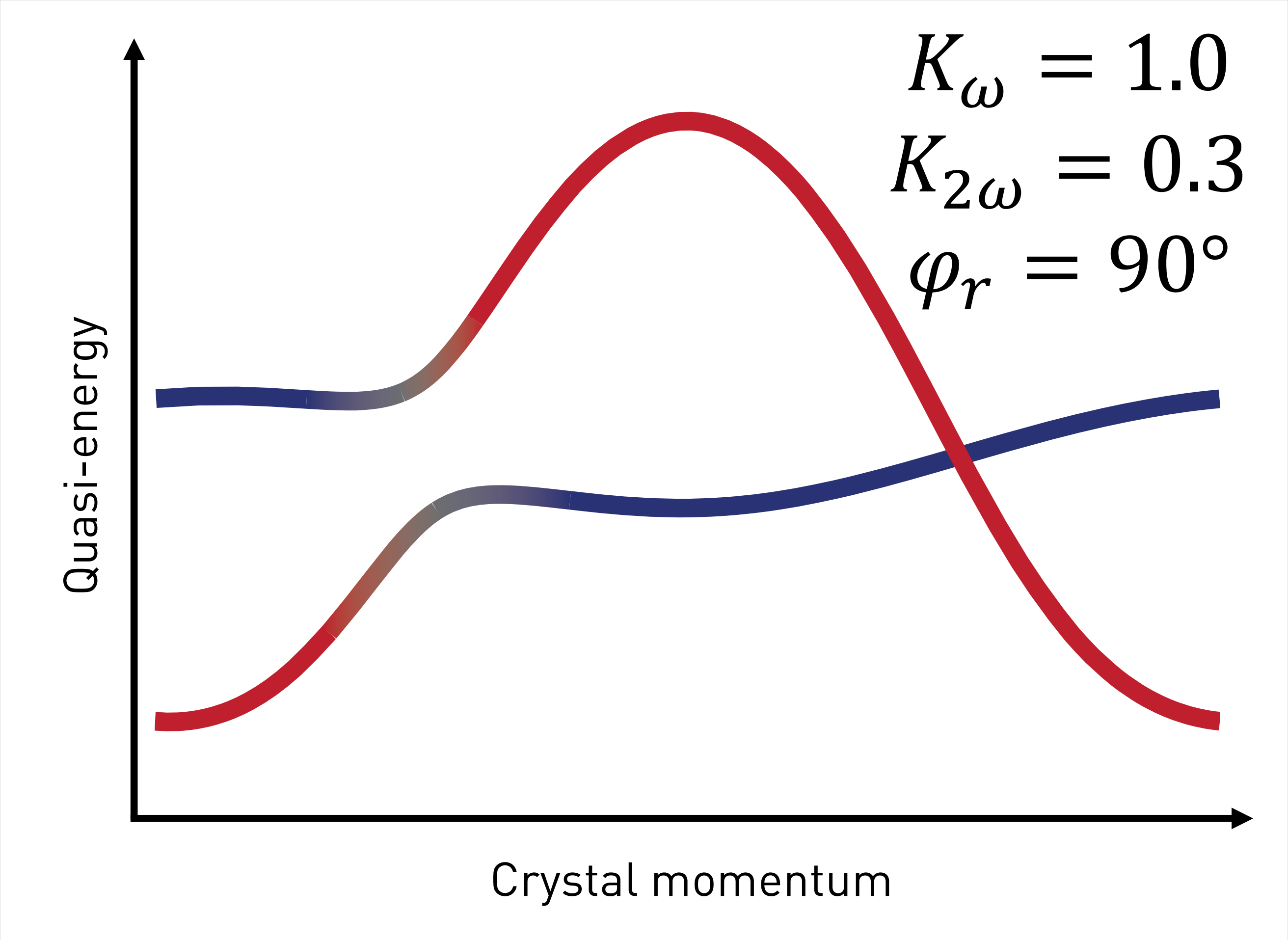


Topological band engineering
Floquet Thouless pumping

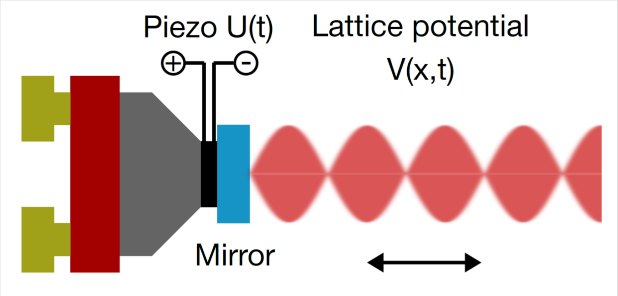
Sandholzer et al. (in prep), Minguzzi et al. (in prep)

1ω -2ω shaking



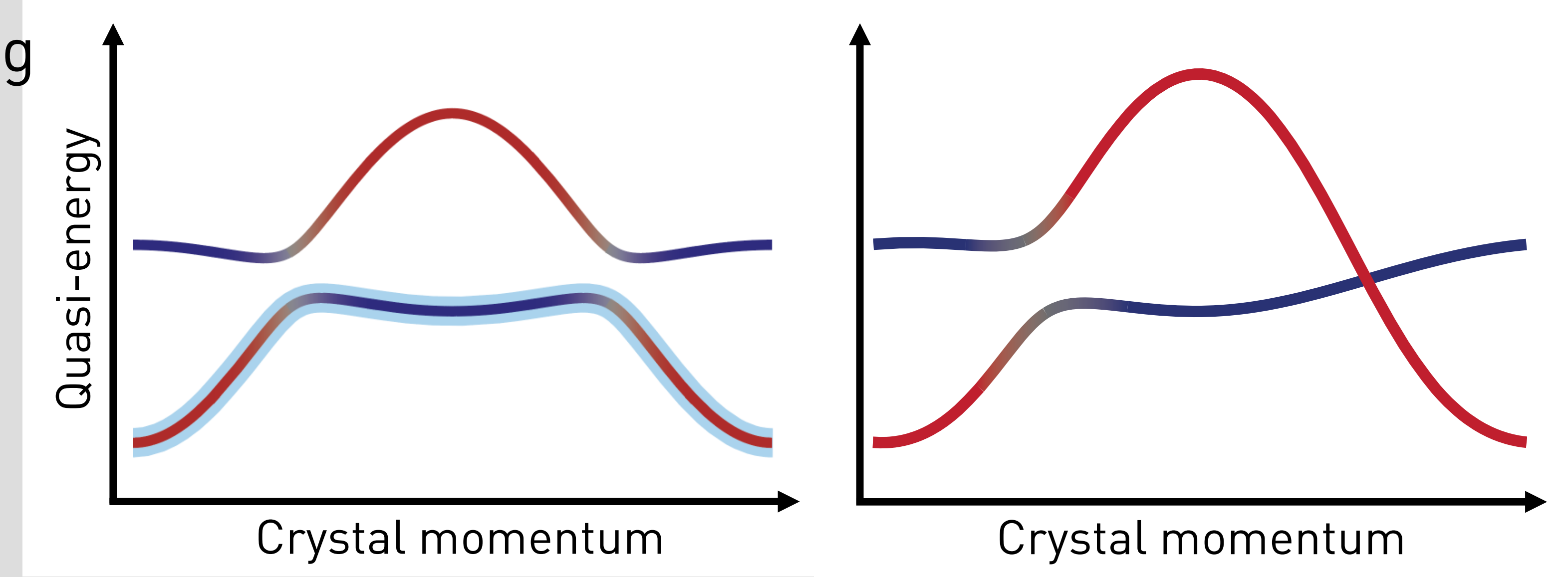


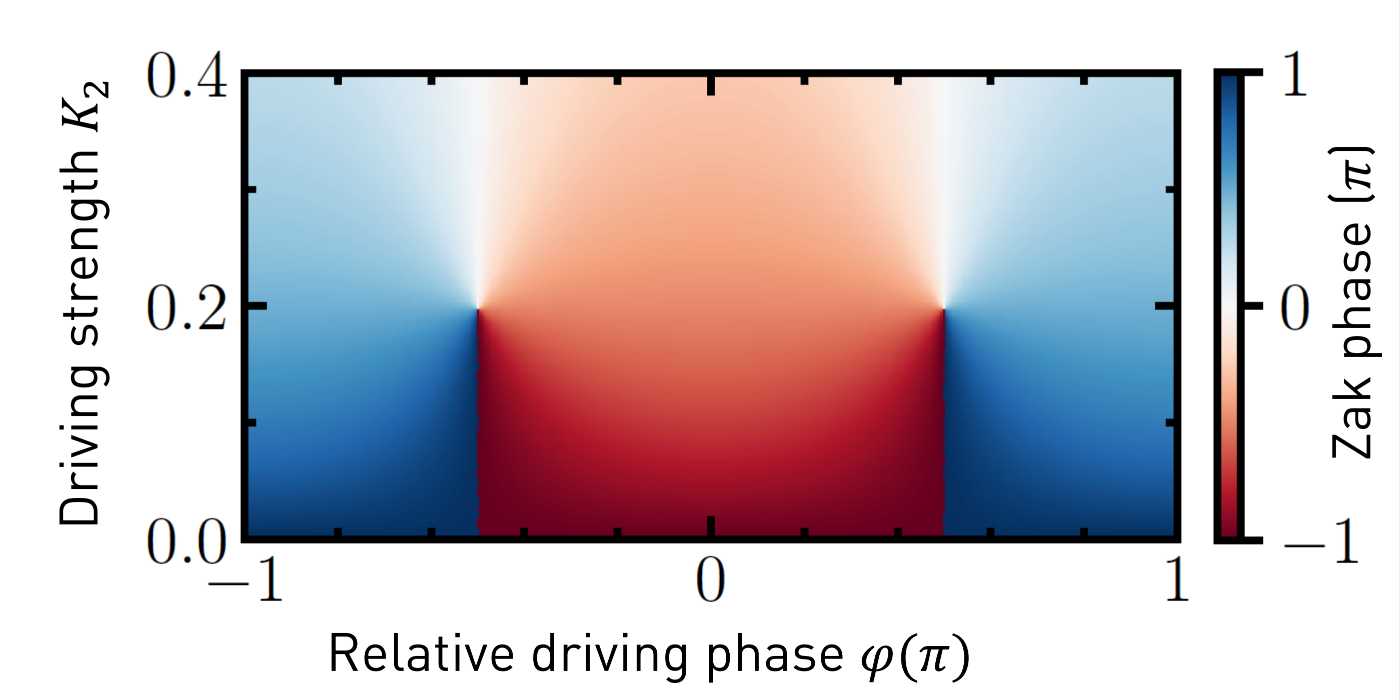
$$\frac{aF(t)}{\hbar\omega} = K_\omega \cos(\omega t) + 2K_{2\omega} \cos(2\omega t + \varphi_r)$$



Sun & Lim PRB (2017)
Kang & Shin PRA (2020)

1ω -2ω shaking

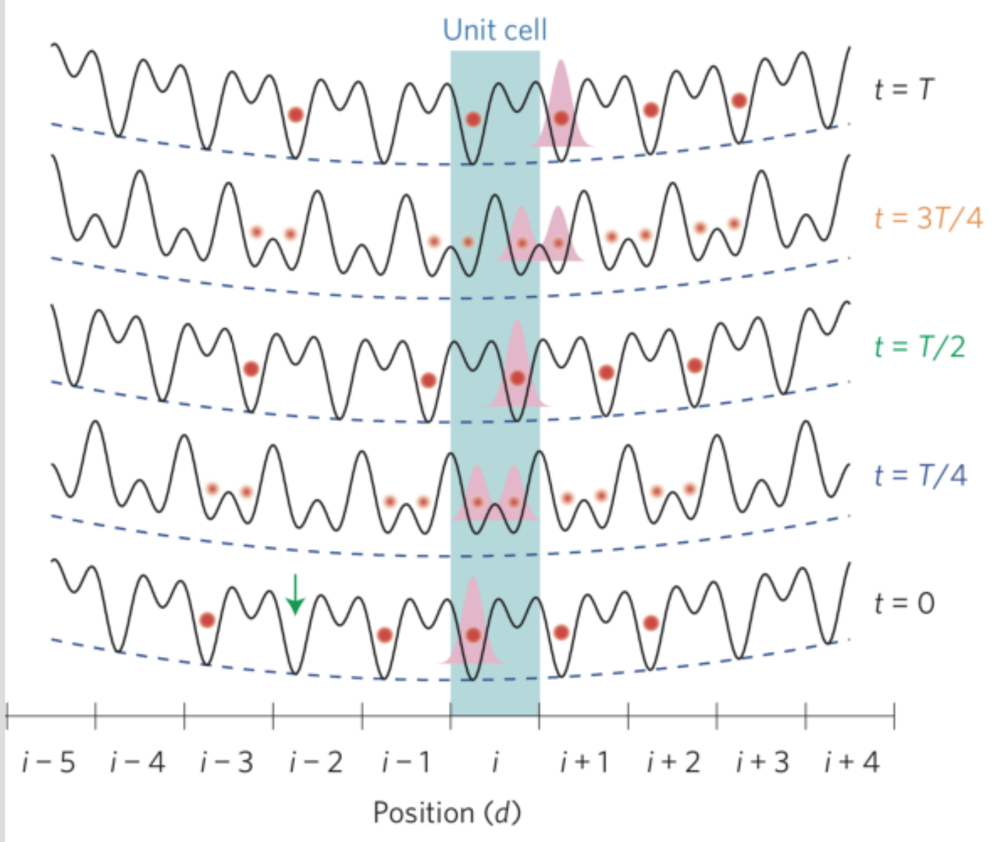




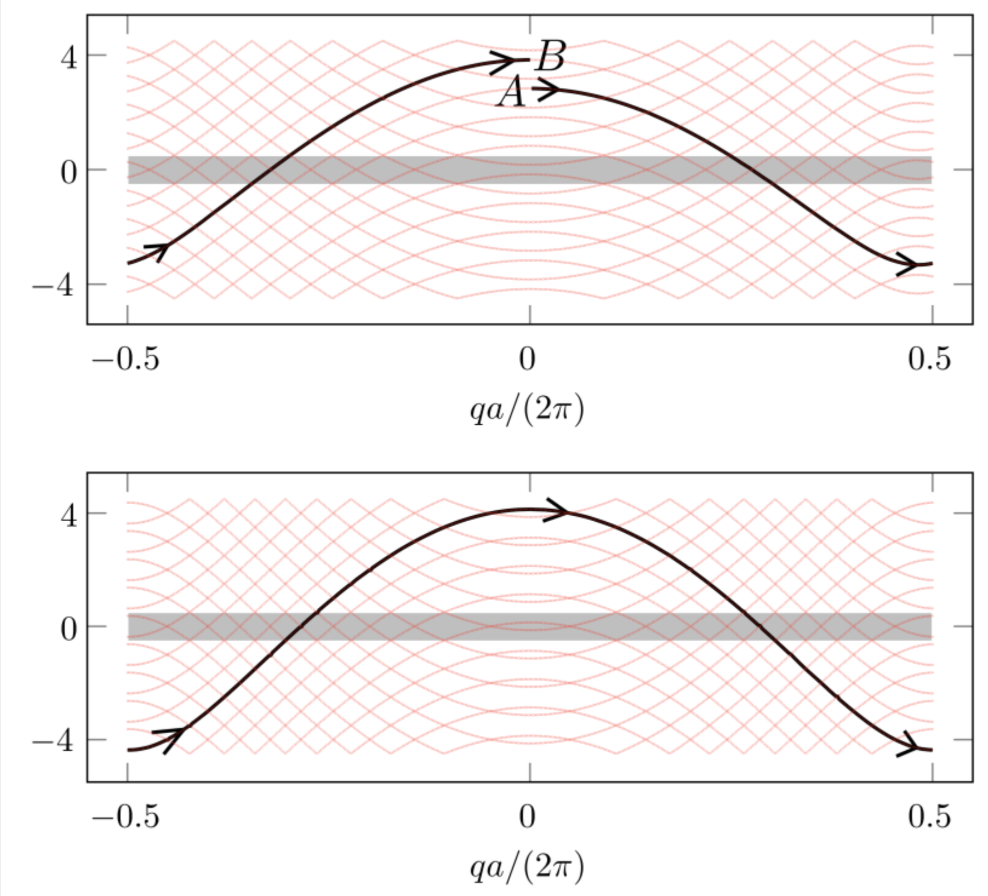
Sun & Lim PRB (2017)
Kang & Shin PRA (2020)

$$\text{Zak phase} = i\int_{\text{BZ}} \langle u_n(q)|\partial_q u_n(q)\rangle\, dq$$

Sliding potentials

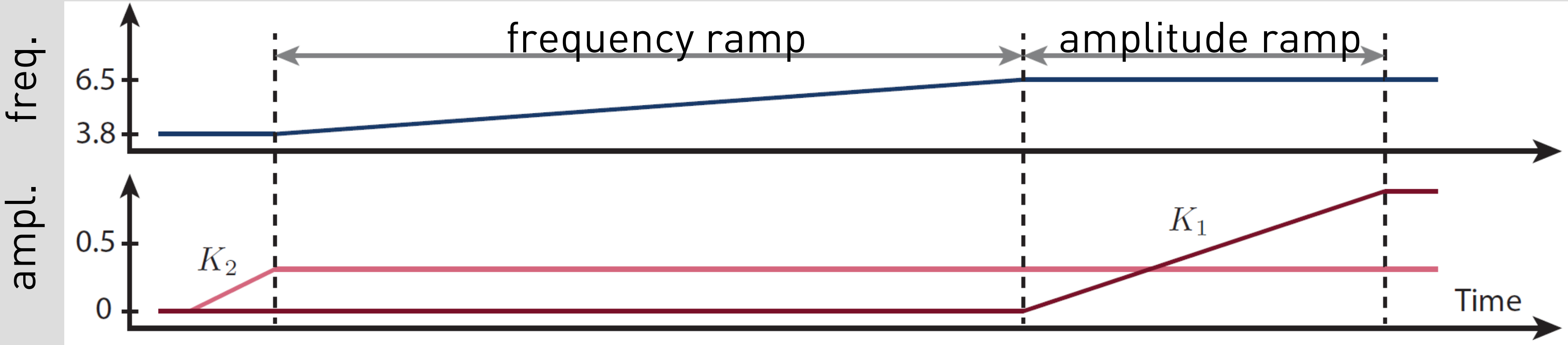


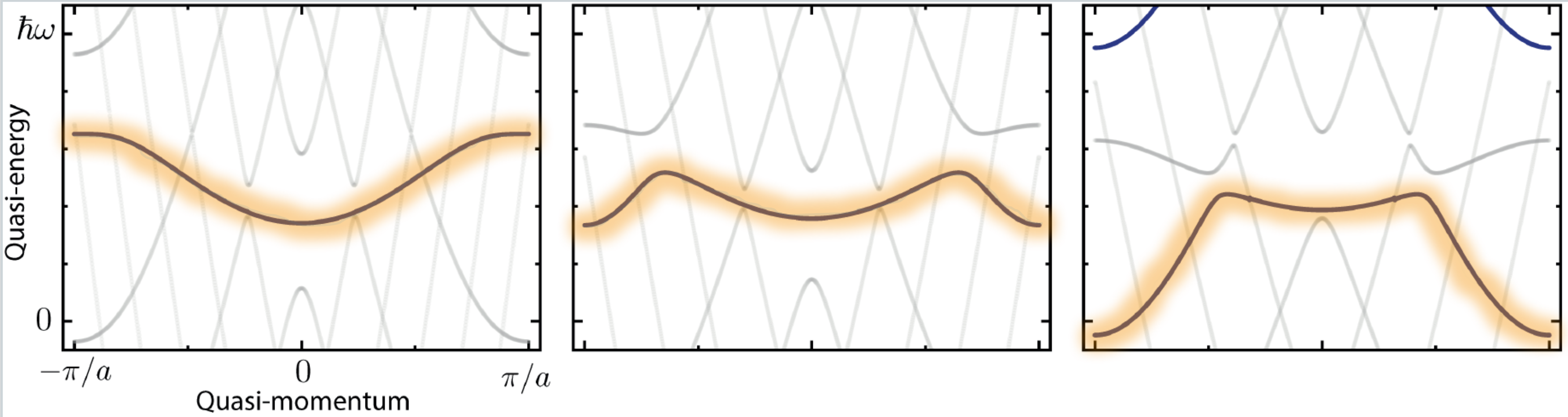
Slow Floquet description

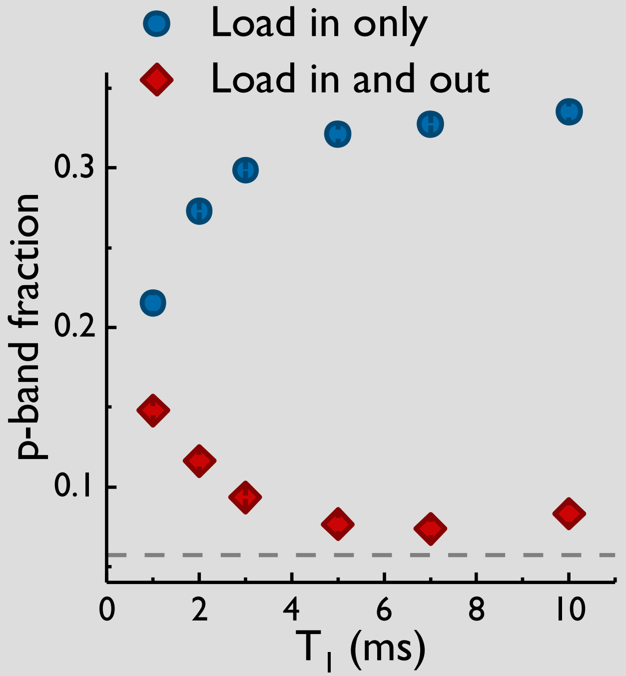


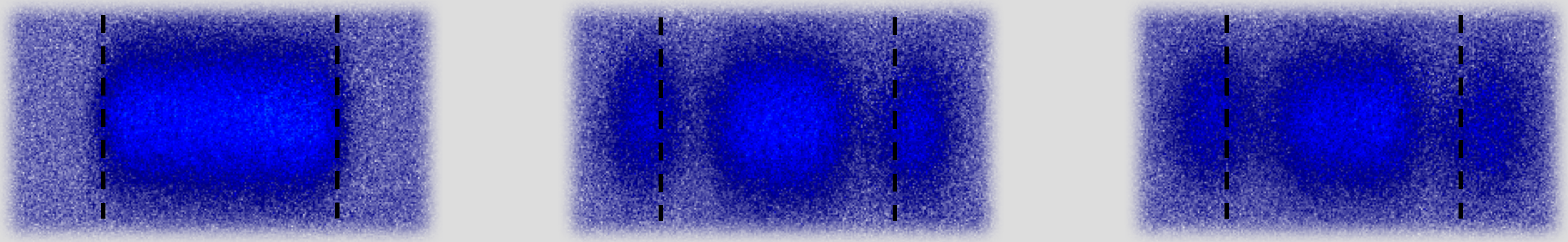
Lohse et al. Nat. Phys. 2015
Nakajima et al. Nat. Phys. 2016
Nakajima et al. Nat. Phys. 2021
Cerjan et al. Light Sci. Appl. 2020
Lu et al. PRL 2016

Kitagawa et al. PRB 2010
Cooper, Dalibard, & Spielman RMP 2019

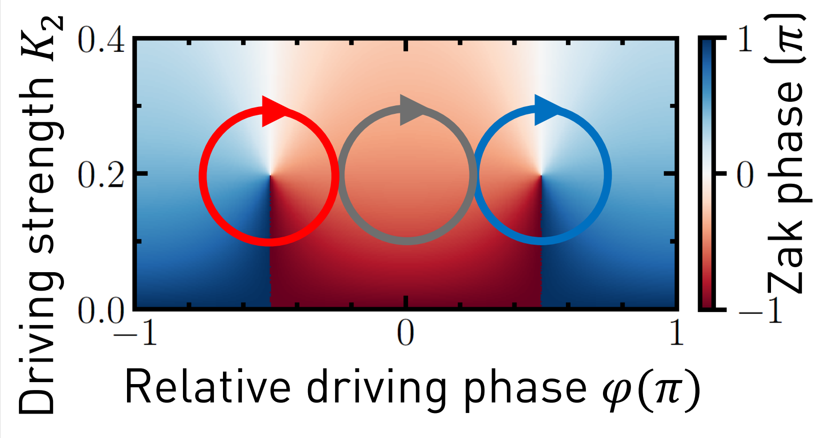


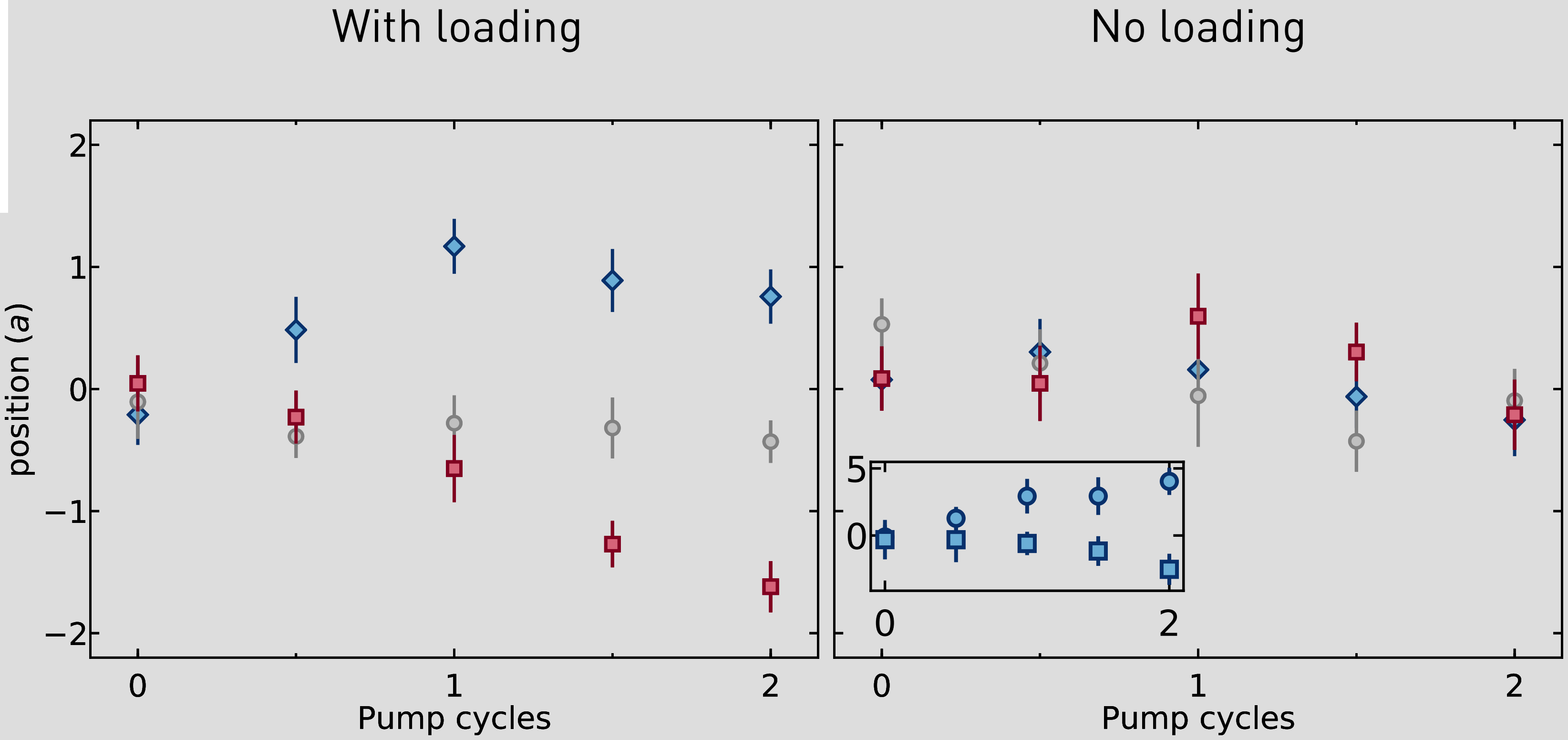


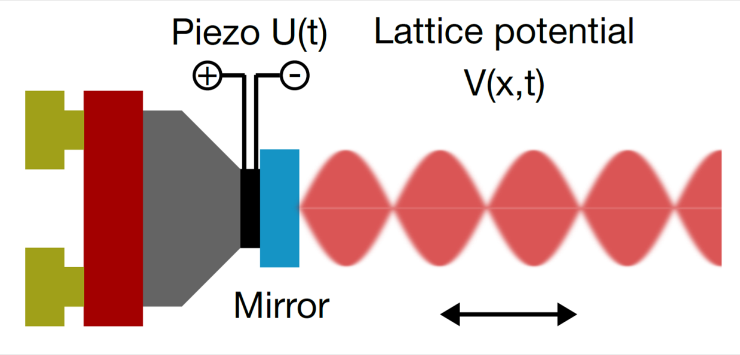




Minguzzi et al. (in prep)







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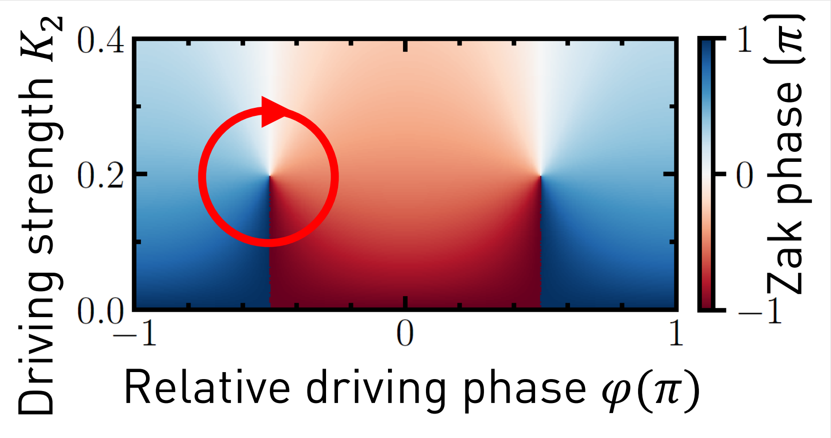
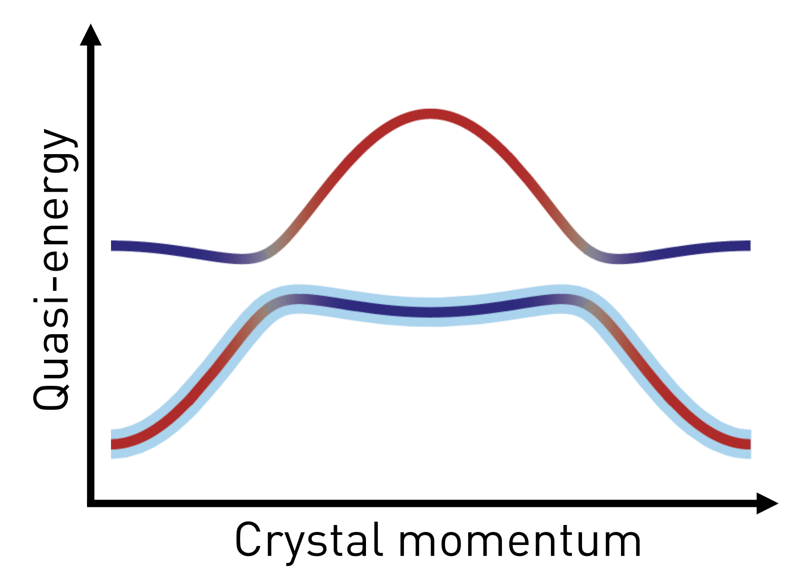
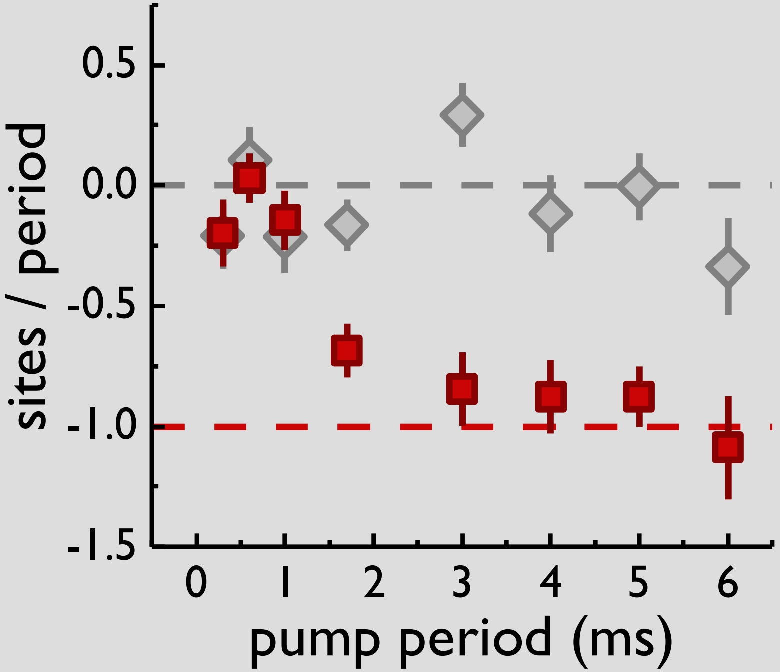
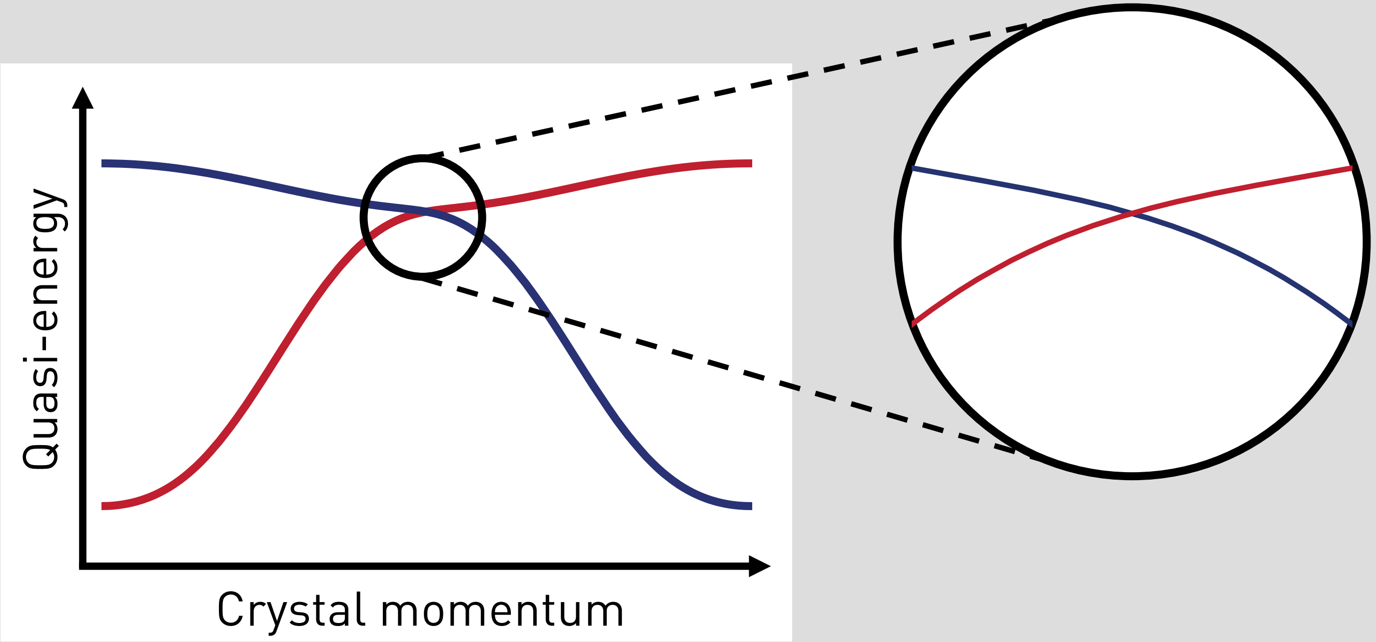


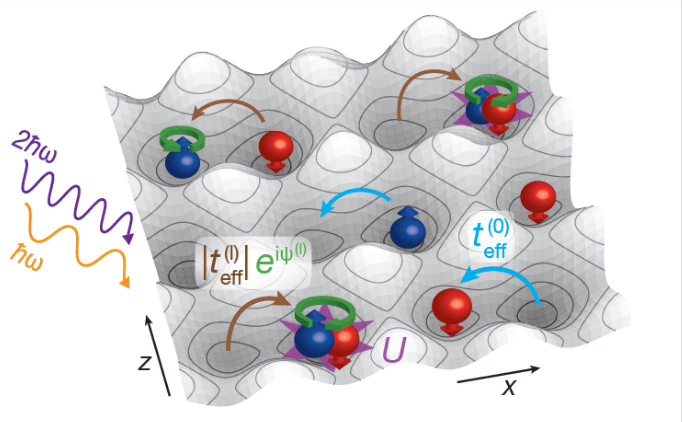




Minguzzi et al. (in prep)

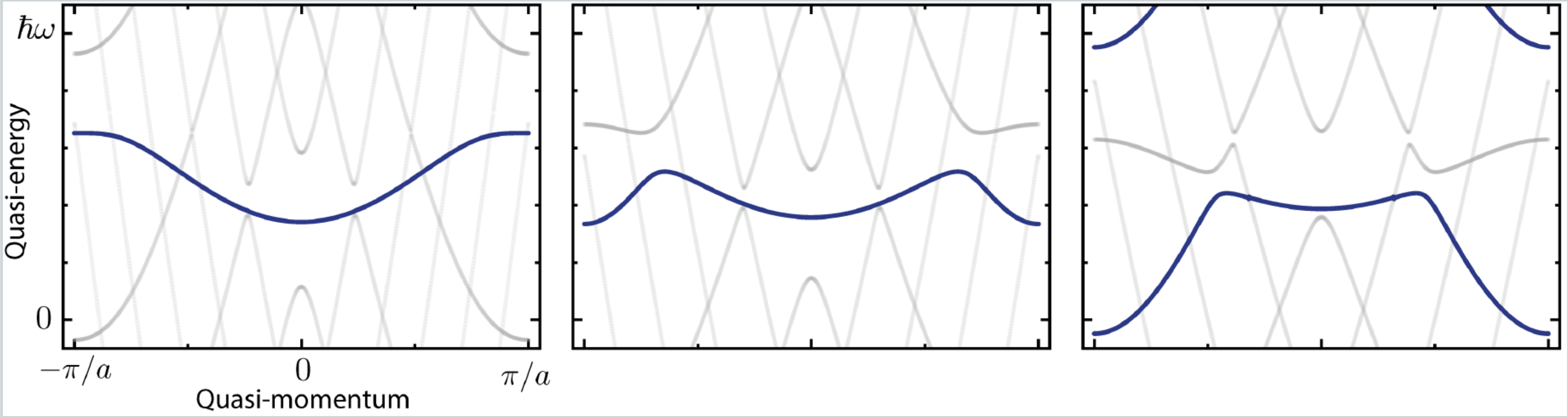


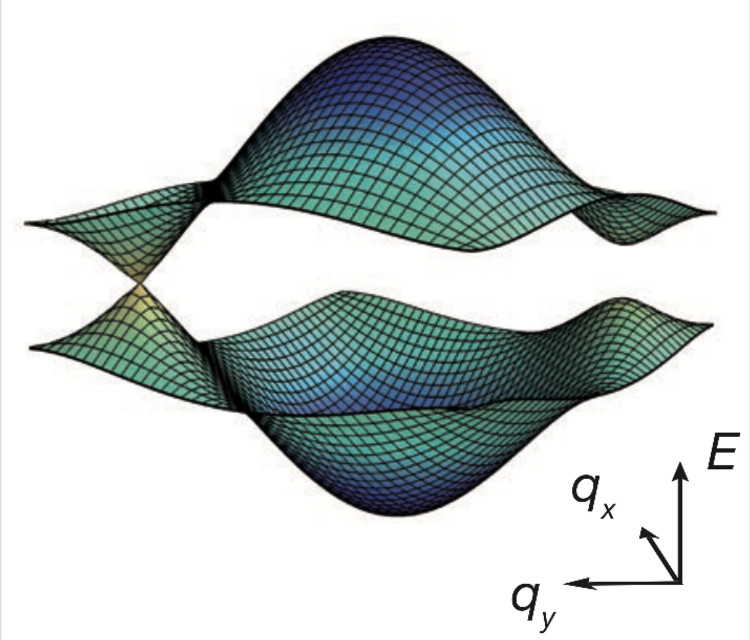
"Helical Floquet bands" Budich, Hu, & Zoller 2017



Density-dependent
Peierls phases

Görg et al. Nat. Phys. 2019





Adiabatic preparation of Chern insulator
Dauphin et al. 2D Mater. 2017 and others


```
>>K9_OL-6-0-0_Dephase_ShakeX-K1-0.8-K2-0.15-Ak-m0.09-Aphi-45_T4-10ms-Tp-5ms_vs-AtNb-phi-loading
>>K9_OL-6-0-0_Dephase_ShakeX-K1-0.8-K2-0.15-Ak-m0.09-Aphi-m45_T4-10ms-Tp-5ms_vs-AtNb-phi-loading
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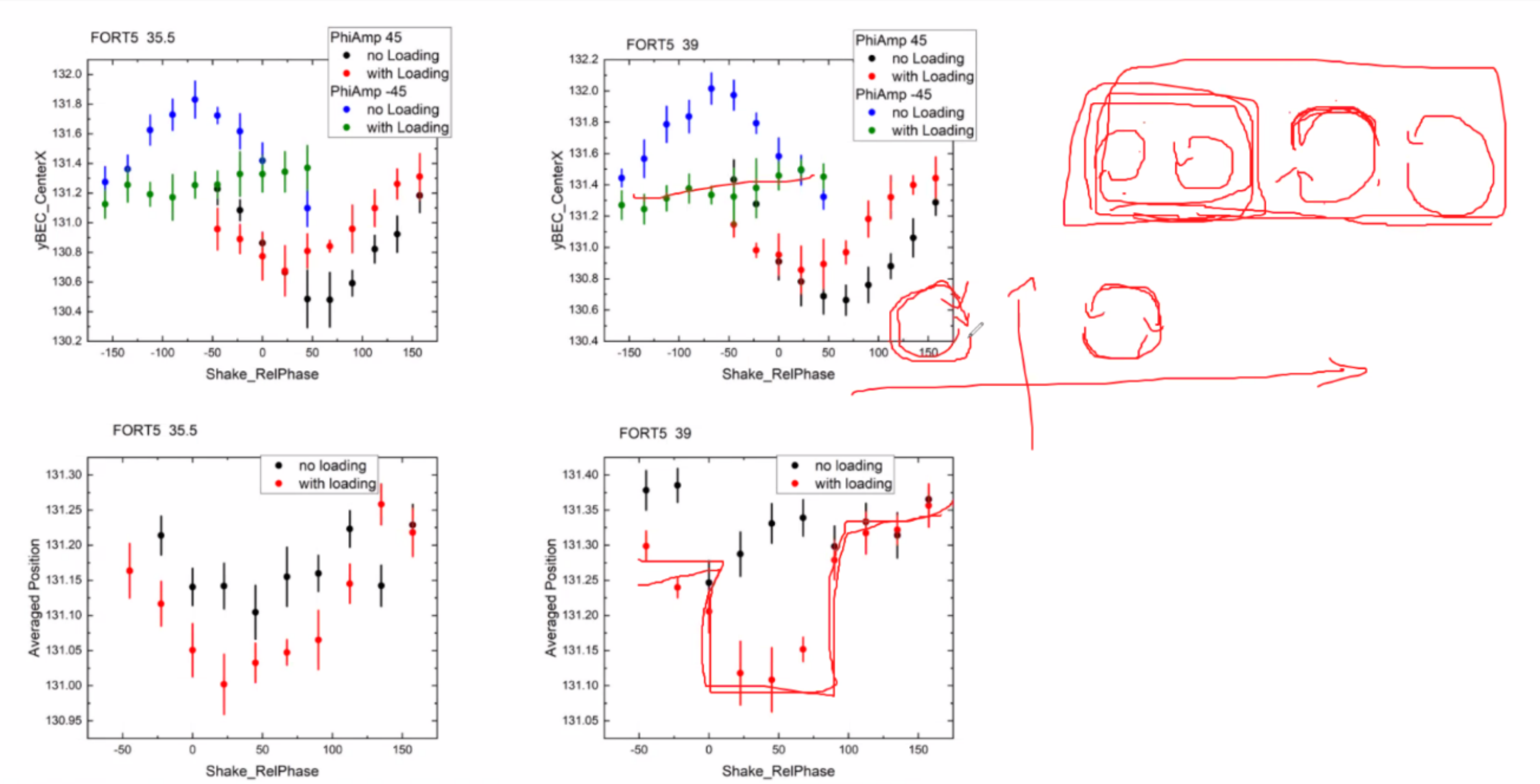

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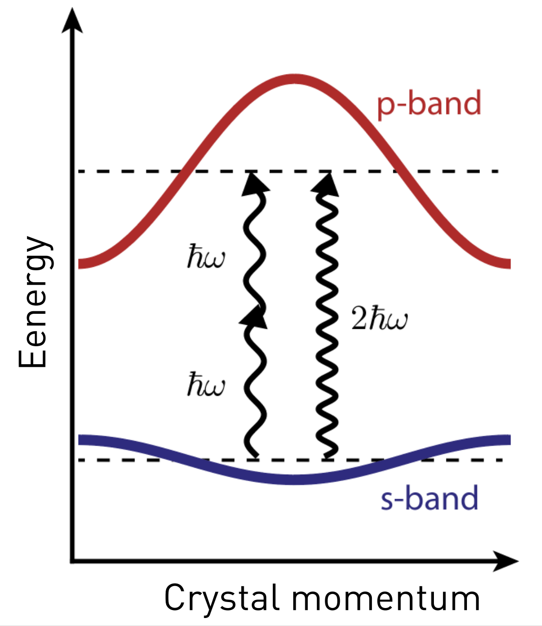
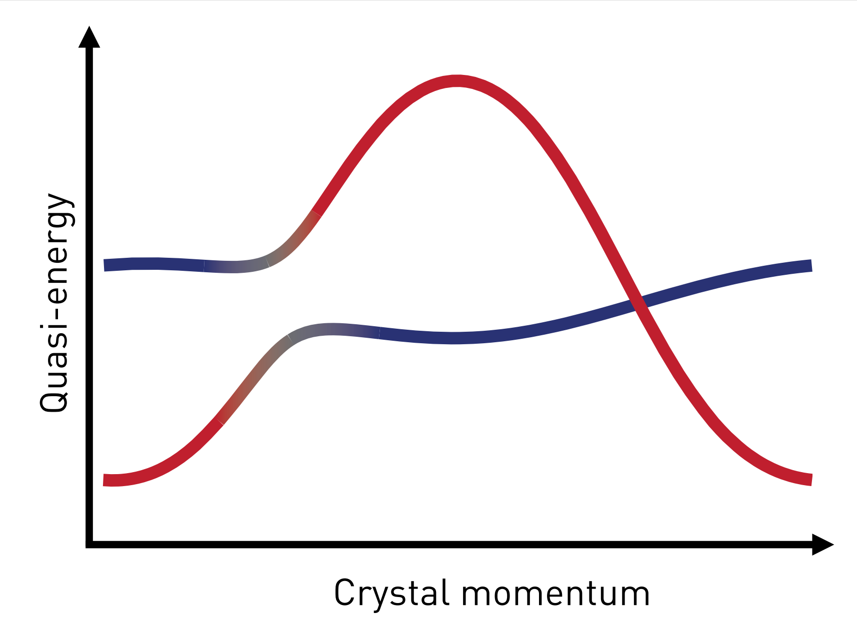
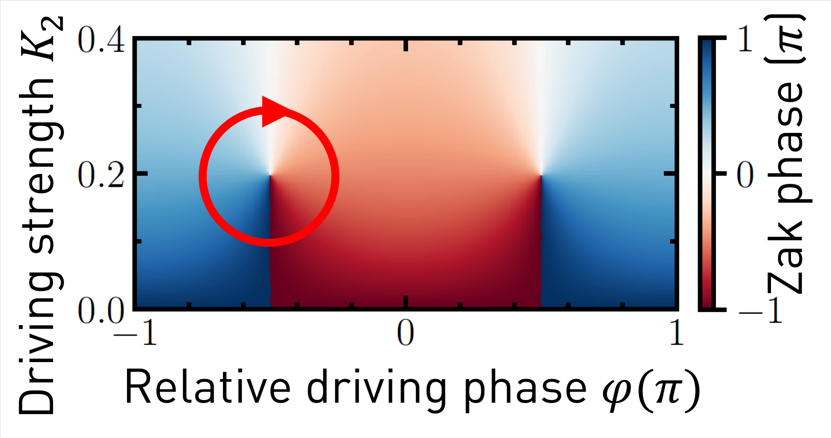
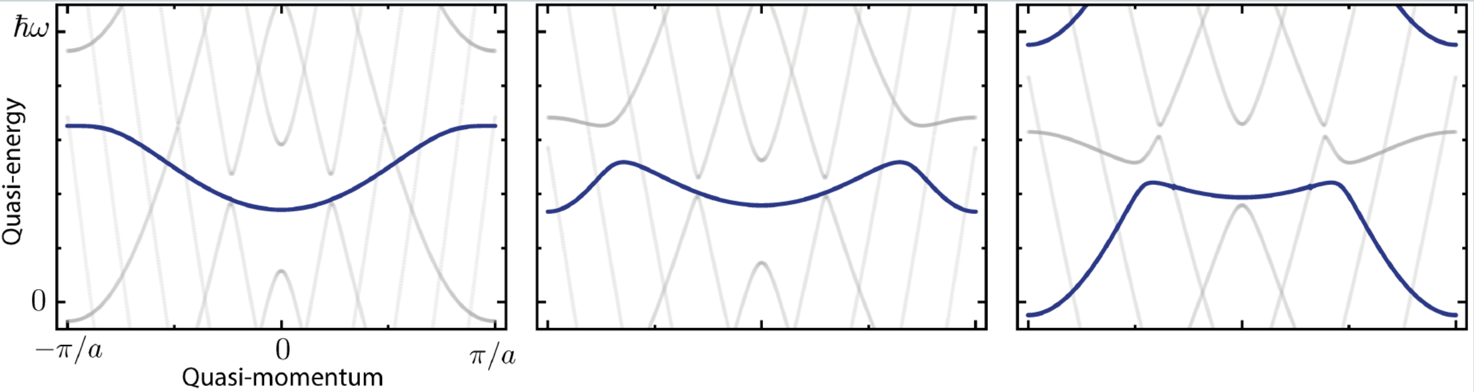
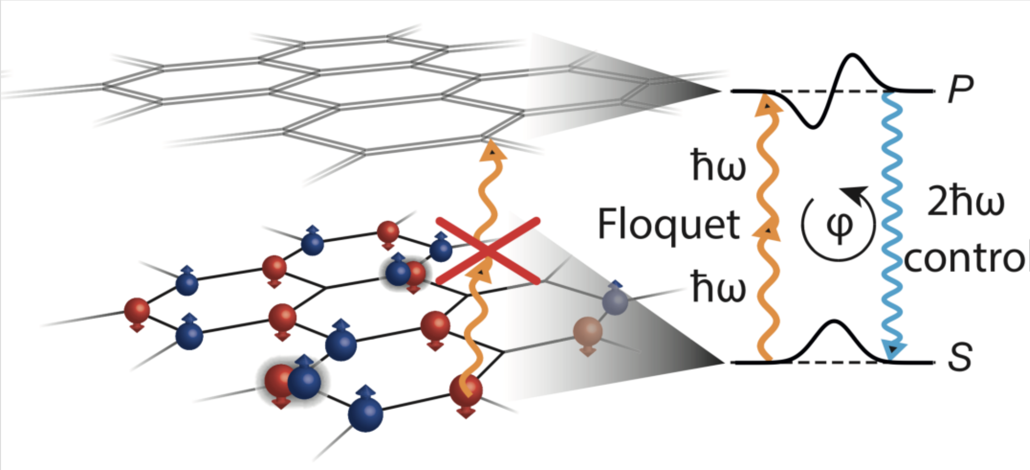


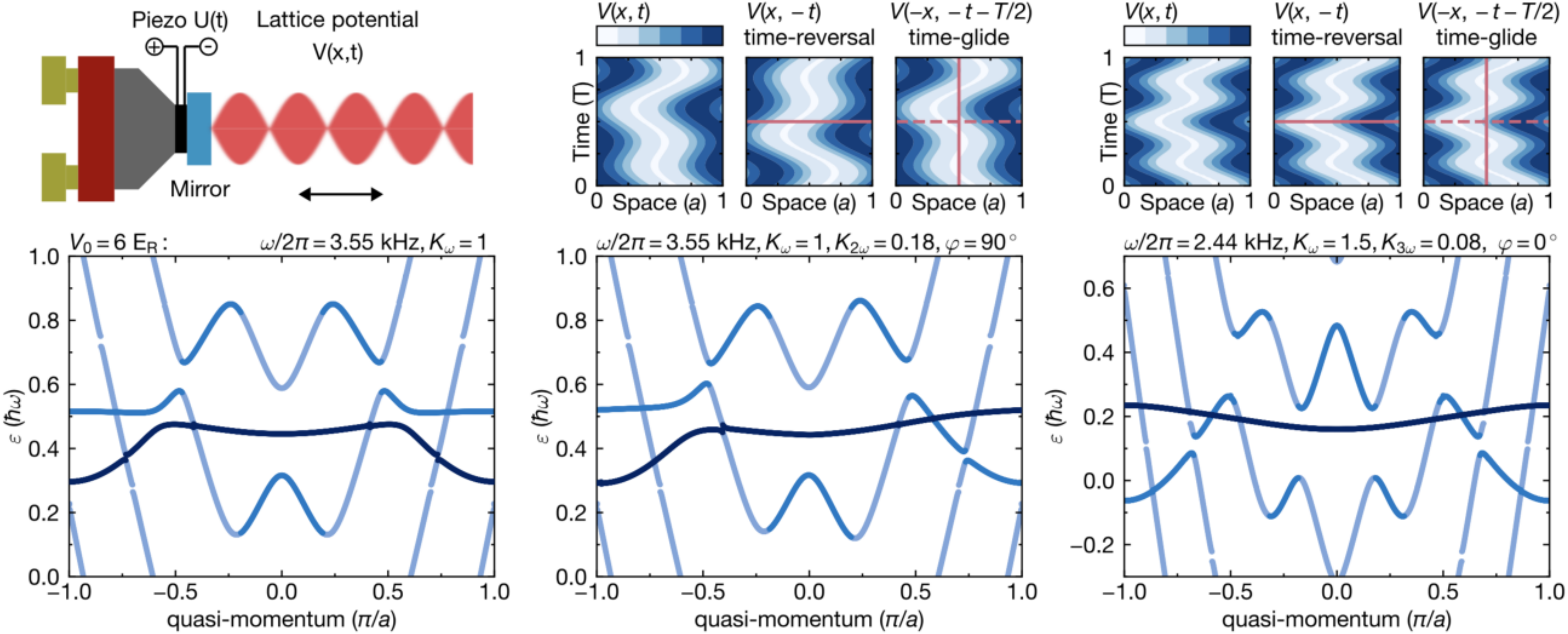




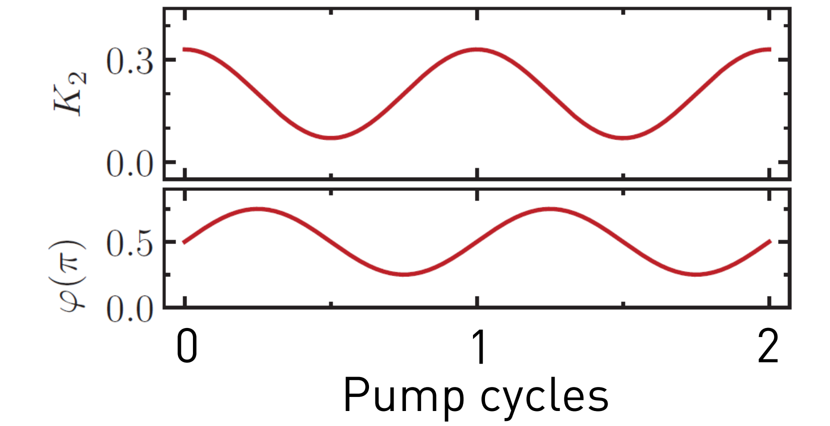




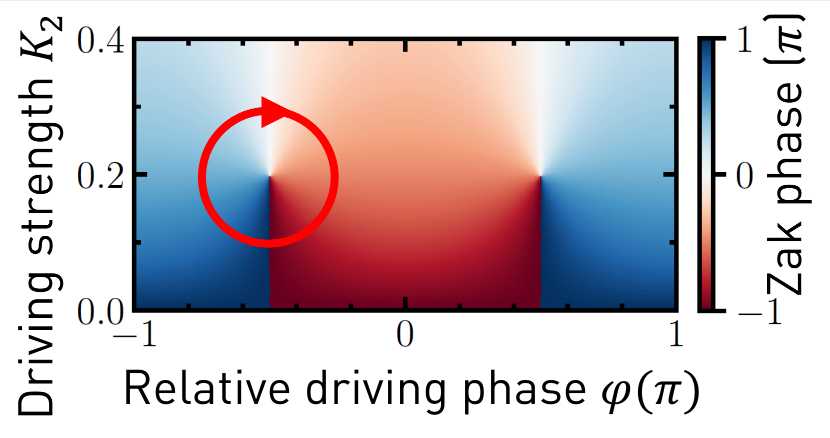
$$\begin{pmatrix} \ddots & & & & \\ & \begin{pmatrix}0 & \\ & \omega_0\end{pmatrix} & \begin{pmatrix}0 & \mu B\\ \mu B & 0\end{pmatrix} & & \begin{pmatrix}0 & \mu B_3 e^{i\varphi}\\ \mu B_3 e^{-i\varphi} & 0\end{pmatrix} \\ & & \begin{pmatrix}-\omega & \\ & \omega_0-\omega\end{pmatrix} & \begin{pmatrix}0 & \mu B\\ \mu B & 0\end{pmatrix} & \\ & & & \begin{pmatrix}-2\omega & \\ & \omega_0-2\omega\end{pmatrix} & \begin{pmatrix}0 & \mu B\\ \mu B & 0\end{pmatrix} \\ & & & & \begin{pmatrix}-3\omega & \\ & \omega_0-3\omega\end{pmatrix} \\ & & & & & \ddots \end{pmatrix}$$



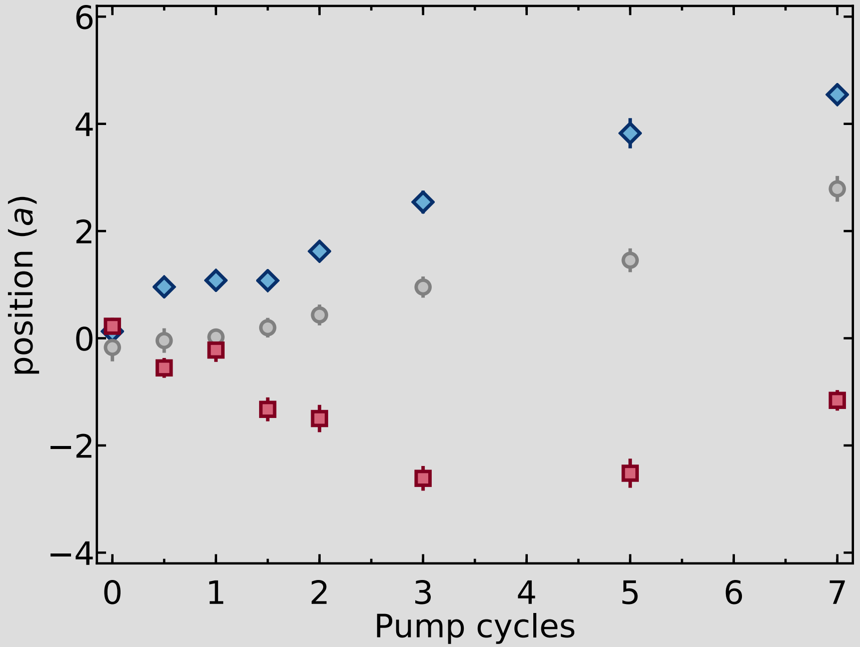
Piezo U(t)
Lattice potential
V(x,t)
Mirror
V(x, t)
V(x, −t)
time-reversal
V(−x, −t − T/2)
time-glide
Time (T)
Space (a)
$V_0 = 6\ E_R$:
$\omega/2\pi = 3.55$ kHz, $K_\omega = 1$
$\omega/2\pi = 3.55$ kHz, $K_\omega = 1$, $K_{2\omega} = 0.18$, $\varphi = 90^\circ$
$\omega/2\pi = 2.44$ kHz, $K_\omega = 1.5$, $K_{3\omega} = 0.08$, $\varphi = 0^\circ$
ε ($\hbar\omega$)
quasi-momentum (π/a)



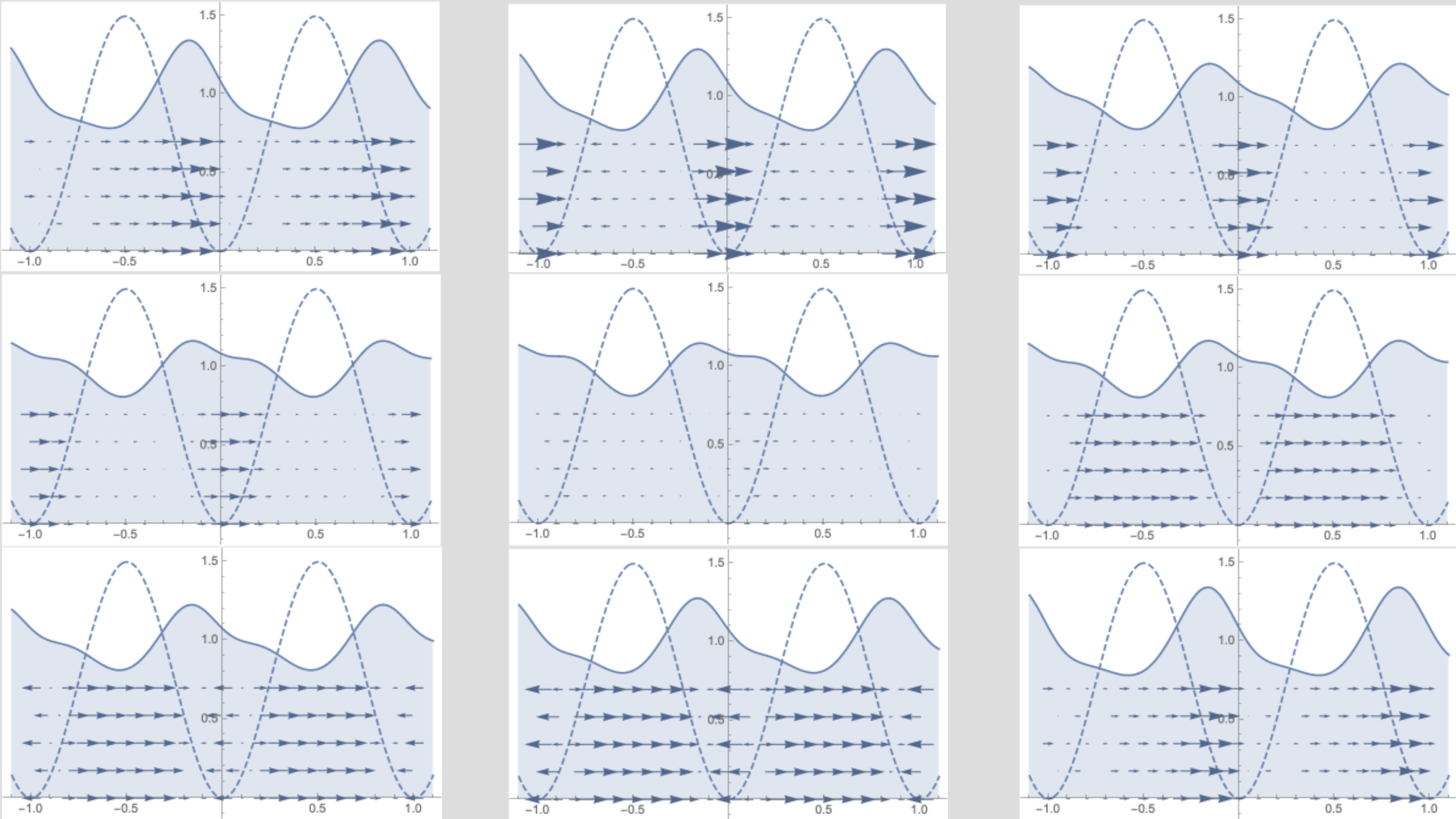
K_2
0.3
0.0
$\varphi(\pi)$
0.5
0.0
0
1
2
Pump cycles

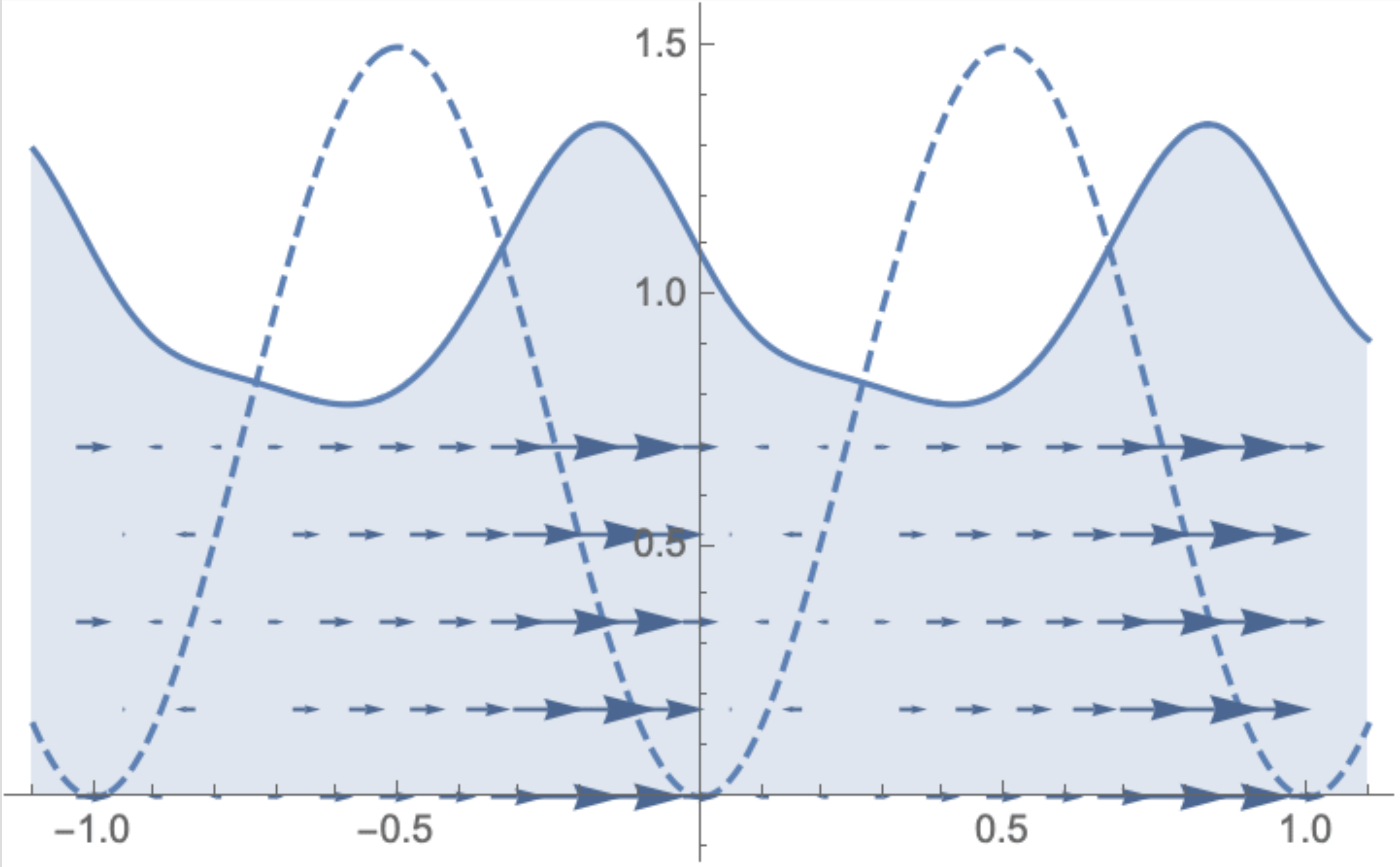


Driving strength K_2
0.4
0.2
0.0
−1
0
1
Relative driving phase $\varphi(\pi)$
Zak phase [π]
1
0
−1



position (a)
6
4
2
0
−2
−4
0
1
2
3
4
5
6
7
Pump cycles


1.5
1.0
0.5
−1.0
−0.5
0.5
1.0


1.5
1.0
0.5
−1.0
−0.5
0.5
1.0