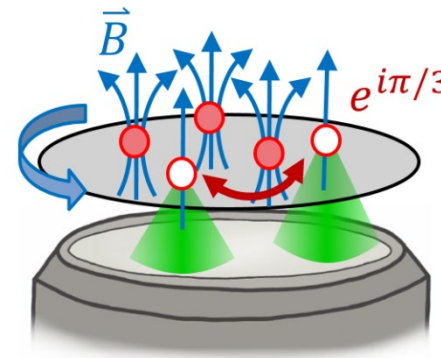
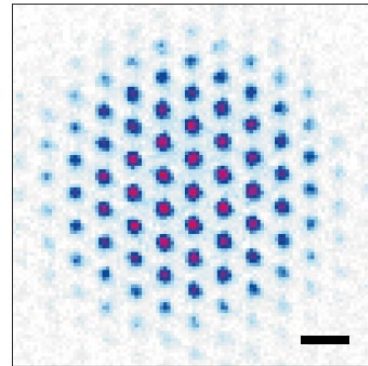


Experimental Floquet techniques for realizing topological models with ultracold atoms



Christof Weitenberg, University of Hamburg

KITP Workshop TOPOLOGY21

June 3rd 2021

Outline

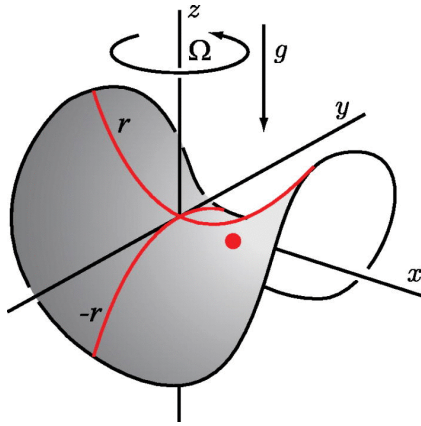
1. Floquet systems, how to engineer topology? (general introduction)
2. How to probe topology? (focus on experiments in Hamburg)
3. Interacting topological matter under the microscope (Outlook)

Floquet Engineering – tailoring a system by a periodic drive

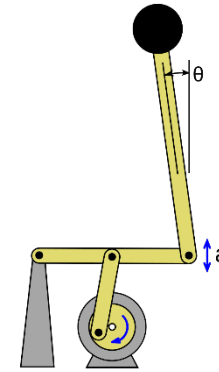
Floquet theory

Periodically driven system = effective Hamiltonian + micromotion

Oscillating saddle point potential (e.g. ion traps):
stable trapping



Kapitza's pendulum (fast drive of pivot point):
upright state as new stable equilibrium position

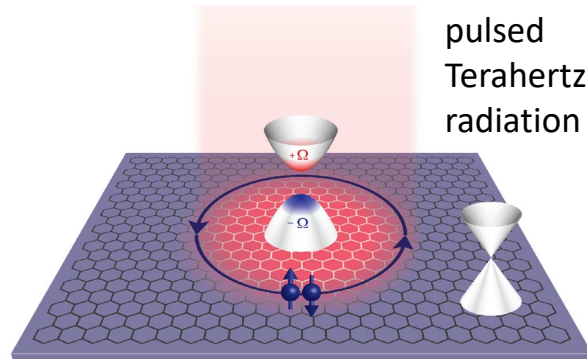


Motivation: effective system has new properties

- 1) otherwise not accessible in the specific system (trapping with saddle point, artificial gauge fields)
- 2) fundamentally impossible in a static system (upright pendulum, anomalous phases).

Why ultracold atoms?

Solid state crystal



Complicated environment

- Terahertz radiation,
- pulsed lasers for high intensity,
- ultrafast measurement schemes

[McIver et al., Nat. Phys. 16, 38 \(2020\).](#)

Toolbox of techniques: lattice shaking, amplitude modulation, additional Raman beams, modulation of the interaction strength, modulation of external field gradients, combination of the above,...

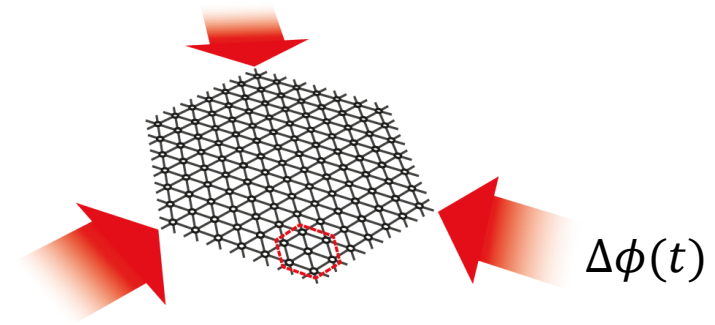
Toolbox of concepts: tunnel renormalization, band hybridization, laser-assisted tunneling,...

[Oka & Kitamura, Annual Rev. Cond. Mat. Phys. 10, 387 \(2019\).](#)

[Rudner & Lindner, Nat. Rev. Phys. 2, 229 \(2020\).](#)

[Photonics: Ozawa et al., Rev. Mod. Phys. 91, 015006 \(2019\).](#)

Cold atoms in optical lattices



Quantum simulation platform

- high control (versatile driving schemes)
- easily accessible time scales (kHz driving).
- Large driving amplitudes possible
- AND: Floquet required for topology

[Bukov, D'Alessio, & Polkovnikov, Adv. Phys. 64, 139 \(2015\).](#)

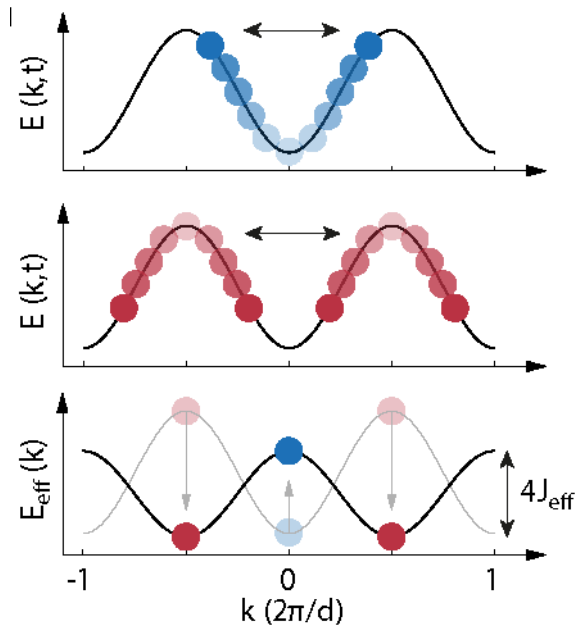
[Eckardt, Rev. Mod. Phys. 89, 011004 \(2017\).](#)

[Goldman & Dalibard, Phys. Rev. X 4, 031027 \(2014\).](#)

[Weitenberg & Simonet, arXiv:2102.07009 \(2021\).](#)

Tunnel renormalization in driven lattices

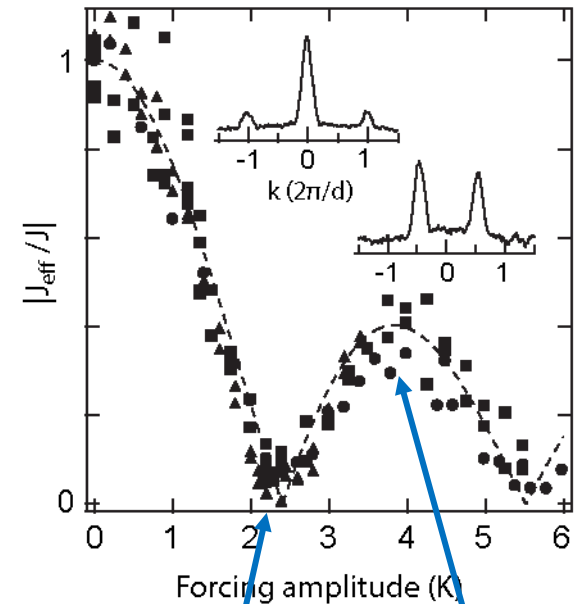
Simple driven 1d lattice



Tight binding dispersion $E(k) = -2J\cos(kd)$
 Oscillating inertial force $F(t) = F_0\cos(\Omega t)$
 with driving strength $K_0 = dF_0/\hbar\Omega$

Lowest order of $1/\Omega$ expansion: $\hat{H}_{\text{eff}} = \langle \hat{H}(t) \rangle_T$
 Renormalized dispersion with $J_{\text{eff}} = J \mathcal{J}_0(K_0)$
 (Bessel function of the first kind of order zero)

Lignier et al., PRL 99, 220403 (2007).



Dynamical
localization

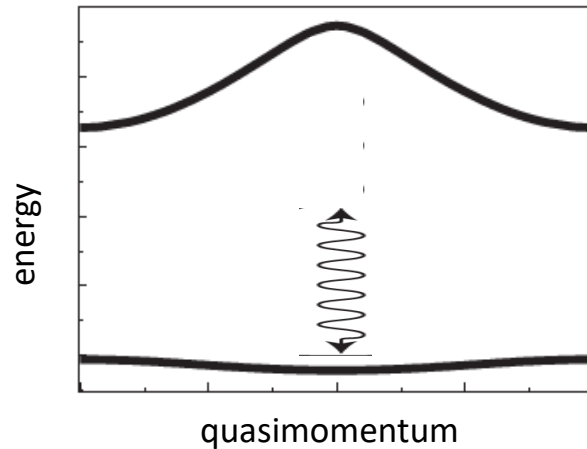
Band inversion
 $\mathcal{J}_0(K_0) < 0$
BEC at $k = \pi/d$

Remark 1: In driven triangular lattice, $J < 0$ leads to frustration and intriguing magnetic phases of the classical XY model
 Struck, et al., Science 333, 996 (2011). Struck, et al., Nat. Phys. 9, 738 (2013).

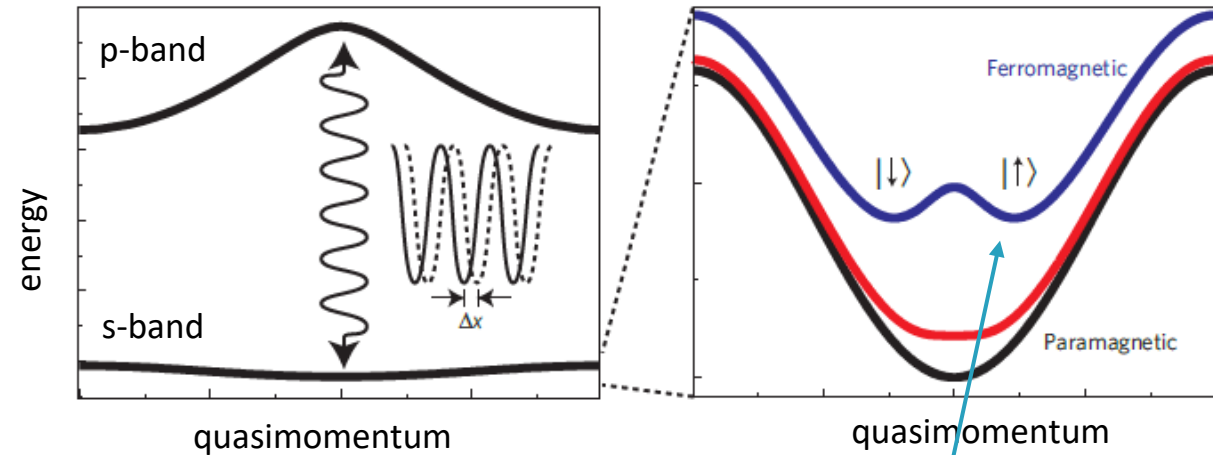
Remark 2: Same physics in static lattice with external force (e.g. oscillating magnetic field gradient)
 Jotzu et al., PRL 115, 073002 (2015).

Remark 3: In Hubbard model, renormalize spin exchange interactions
 Coulthard/Jaksch et al., PRB 96, 085104 (2017). Görg/Esslinger et al., Nature 553, 481 (2018).

Hybridization of bands



Off-resonant drive: tunnel renormalization
(single band picture)



Resonant drive: hybridization of bands, dressing of Bloch states
(compare dressing of atomic states by light)

Drastically modified dispersion (beyond the cosine-shape of an s-band)
e.g. double-well dispersion (map to spin model)

[Parker, Ha & Chin, Nat. Phys. 9, 769 \(2013\).](#)

[Clark, Feng & Chin, Science 354, 606 \(2016\).](#)

[Fujiwara et al., PRL 122, 10402 \(2019\).](#)

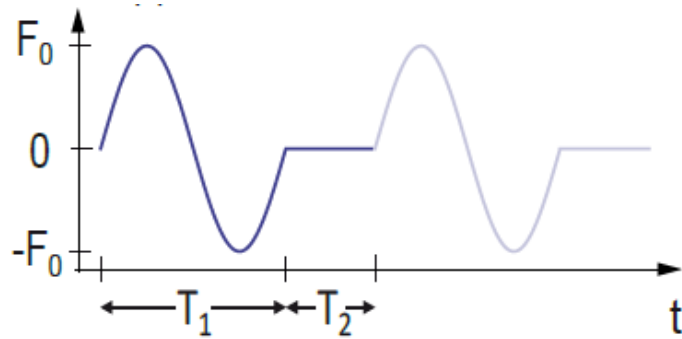
Remark 1: Change of perspective: Spectroscopy versus Floquet states, adiabatic preparation of Floquet states

Remark 2: Resonant coupling also between two s-bands in a non-Bravais lattice with sublattice offset (see later)

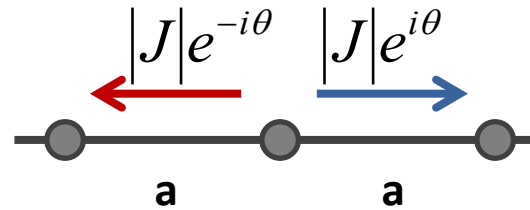
[Fläschner et al., Science 352, 1091 \(2016\).](#)

Tunable Peierls phase via lattice shaking

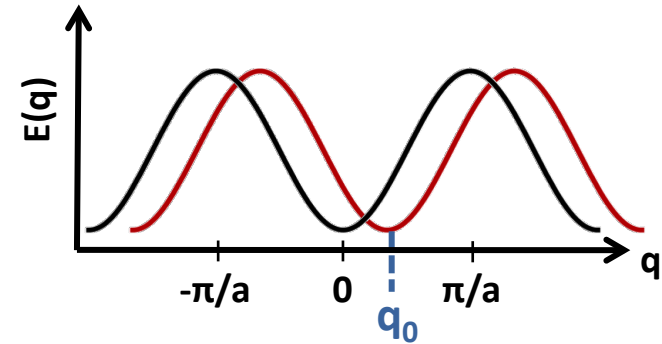
Explicitly break time-reversal symmetry (TRS) in the shaking protocol



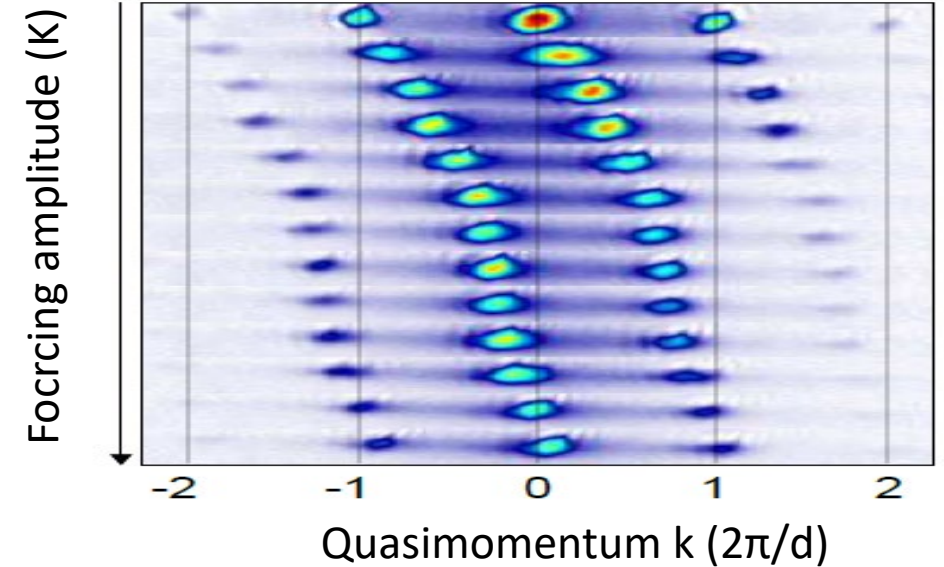
Obtain Peierls phase θ on the tunneling element



Corresponds to shifted dispersion with minimum at $q_0 = \theta/a$

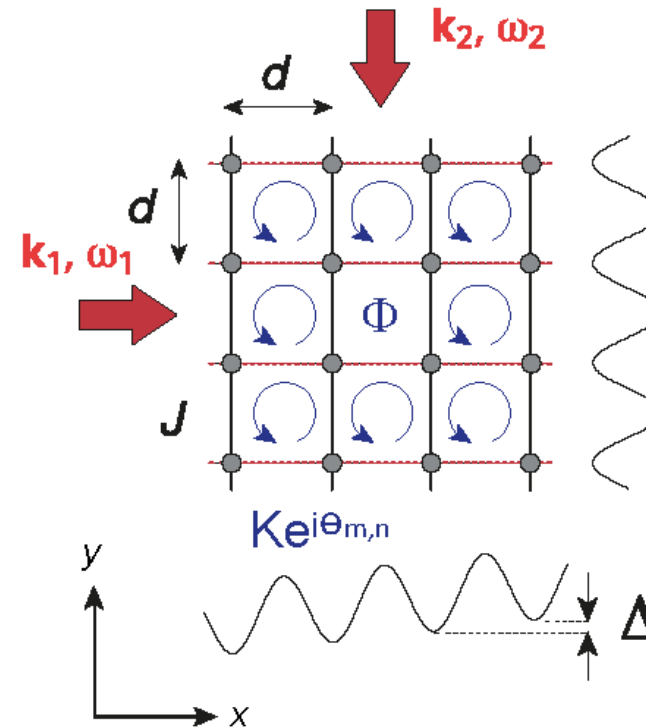
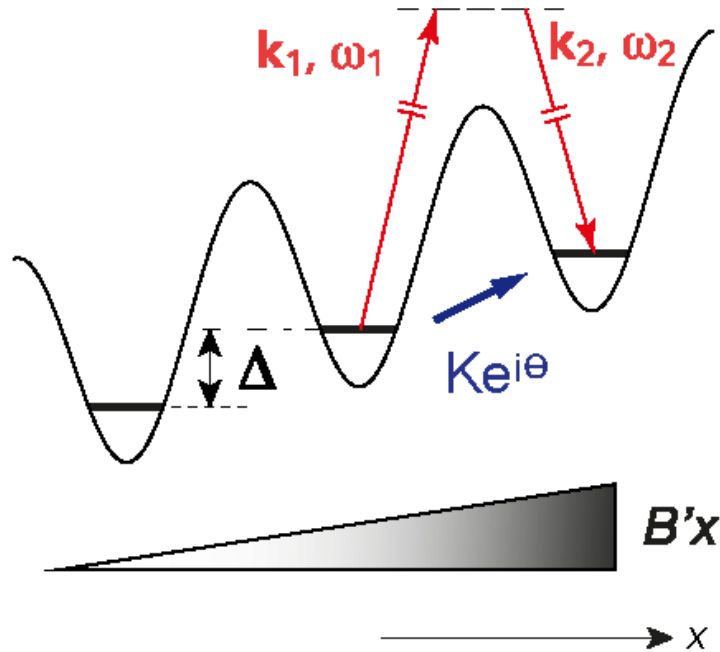


Directly reveal via BEC at dispersion minimum



More generally: break TRS via relative phase of two modulations (e.g. shaking along x and y in 2d system)

Laser-assisted tunneling



- Suppress and restore tunneling via two-photon coupling
- Restored tunneling $K_{\text{pert}} = K e^{-i \delta \mathbf{k} \cdot \mathbf{R}_{m,n}}$ (acquire local laser phase)
- Compare to homogeneous phase for lattice shaking

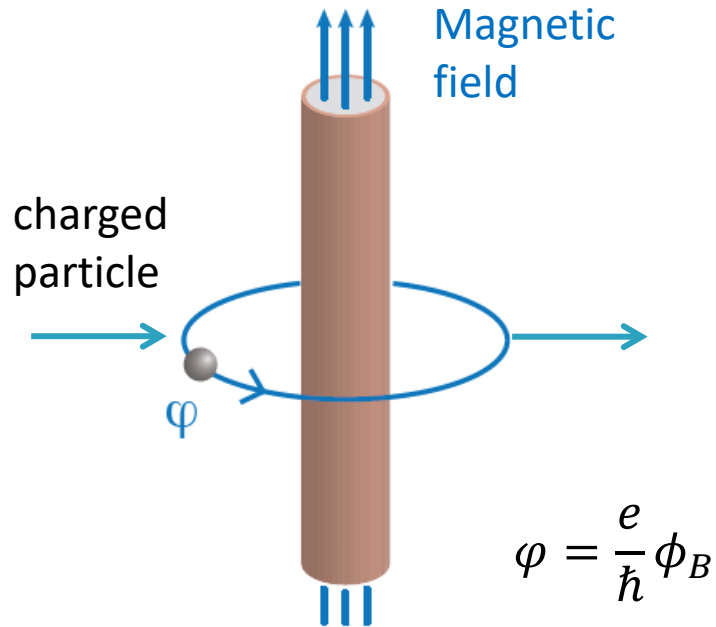
- Secondary moving lattice with $\delta \mathbf{k} = \mathbf{k}_1 - \mathbf{k}_2$,
- Amplitude modulation with locally varying phase

Remark: with Hubbard U , make resonant tunneling occupation dependent

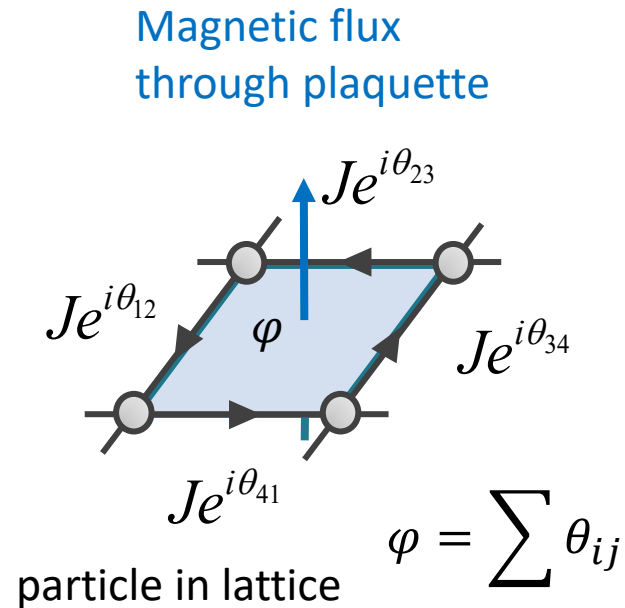
Proposal: Jaksch & Zoller, *New J. Phys.* 5, 56 (2003). Gerbier & Dalibard, *New J. Phys.* 12, 033007 (2010)
 Miyake, Ketterle et al. *PRL* 111, 185302 (2013). Aidelsburger, Bloch et al. *PRL* 111, 1185301 (2013).

Artificial gauge fields

Aharonov Bohm phase



Peierls substitution



Artificial gauge field:

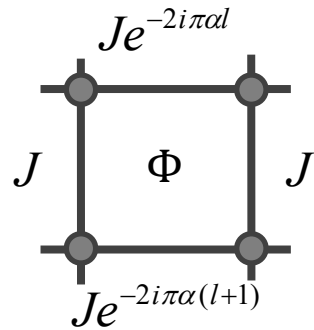
- Implement Peierls phases directly (via Floquet)
- Very large effective magnetic fields accessible

Remark 1: One implements a specific gauge by choosing the Peierls phases (therefore the name)

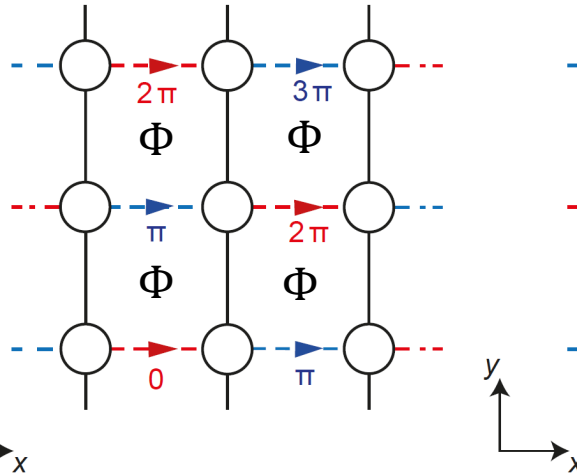
Remark 2: Gauge choice can make subtle differences, e.g. in adiabatic preparation, [Wang et al., arXiv:2009.00560 \(2020\)](#).

Floquet realization of the Harper-Hofstadter model

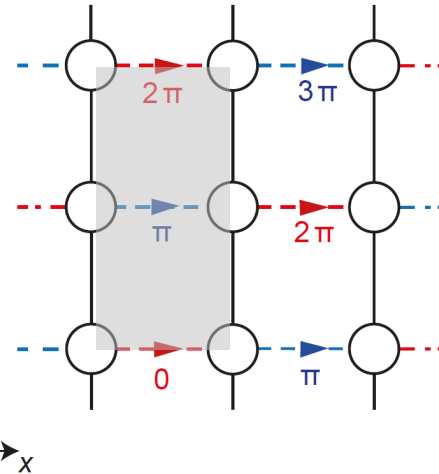
General flux α



Flux $\alpha = 1/2, \Phi = \pi$

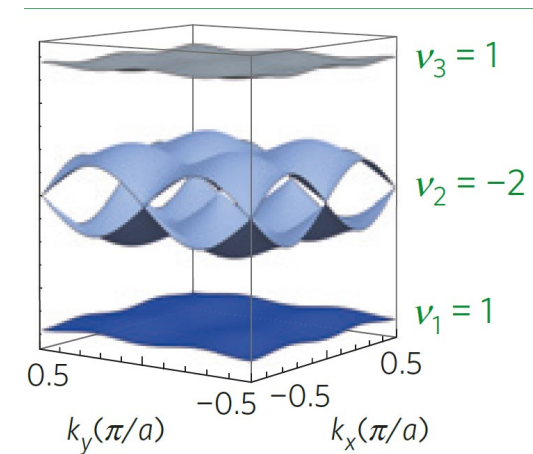
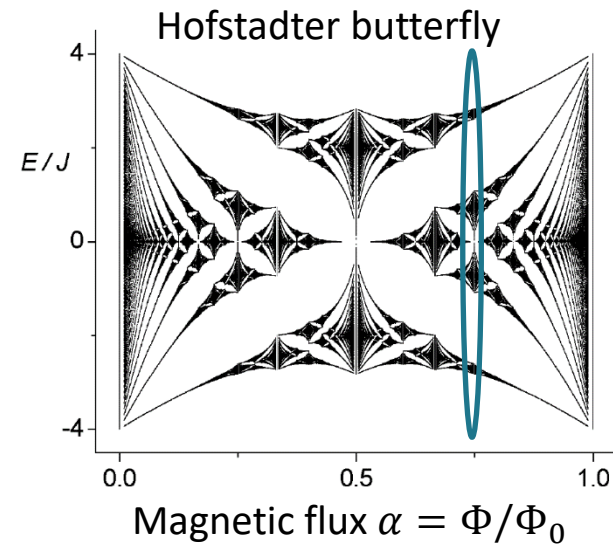


Magnetic unit cell



Harper-Hofstadter model

- Square lattice with homogeneous magnetic flux Φ per plaquette (lattice version of quantum Hall effect).
- Rational values of $\alpha = \Phi/\Phi_0 = q/p$: splitting into p subbands (Hofstadter butterfly)
- Increased spatial period pa : magnetic unit cell, smallest unit cell with an integer number of flux quanta.
- Realize via laser-assisted tunneling (e.g. in the Landau gauge $\mathbf{A} = -By \mathbf{e}_x$)



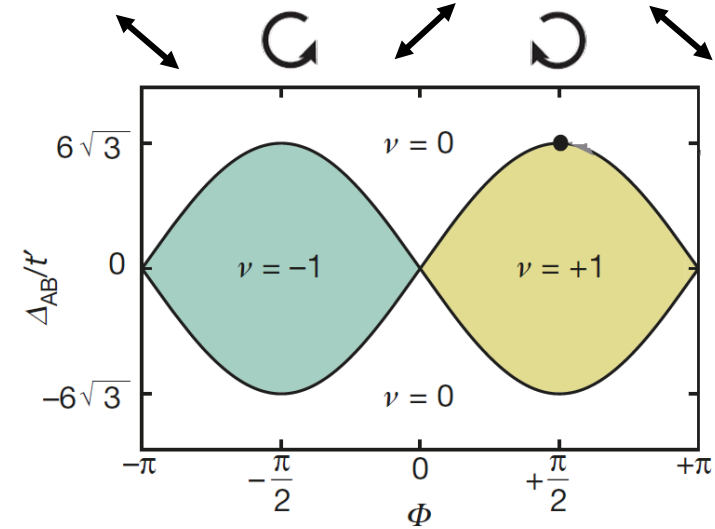
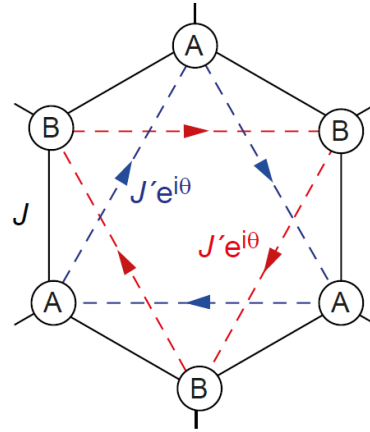
Proposal: Jaksch & Zoller, *New J. Phys.* 5, 56 (2003). Gerbier & Dalibard, *New J. Phys.* 12, 033007 (2010) Miyake, Ketterle et al. *PRL* 111, 185302 (2013). Aidelsburger, Bloch et al. *PRL* 111, 1185301 (2013).

Floquet realization of the Haldane model

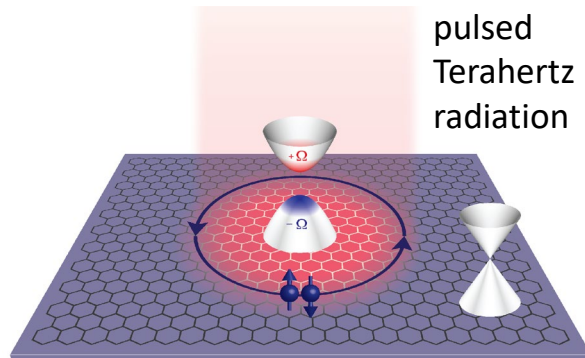
Haldane model

- Honeycomb lattice with Peierls phases θ on the NNN tunneling
- Staggered flux in subplaquettes, no net magnetic flux (shaking sufficient)
- Realize by elliptical shaking with phase ϕ , which maps onto the Peierls phase θ .

Haldane, PRL 61, 2015 (1988)

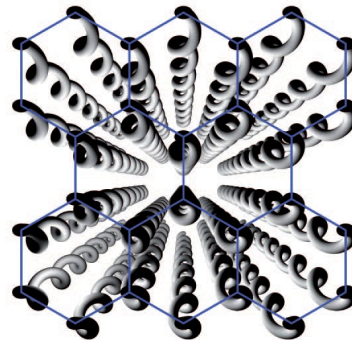


Floquet engineering of the Haldane model



Terahertz-driven graphene

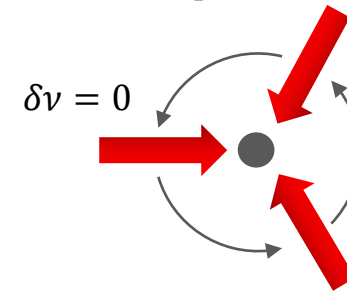
Oka & Aoki, PRB 79, 081406(R) (2009)
McIver et al. Nat. Phys. 16, 38 (2020)



Photonic wave guides

Rechtsman et al.
Nature 496, 196 (2013)

$$\delta\nu = 2\Delta\nu[\cos(\omega t) + \sqrt{3}\sin(\omega t)]$$



$$\delta\nu = 2\Delta\nu[-\cos(\omega t) + \sqrt{3}\sin(\omega t)]$$

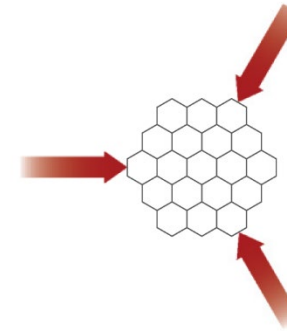
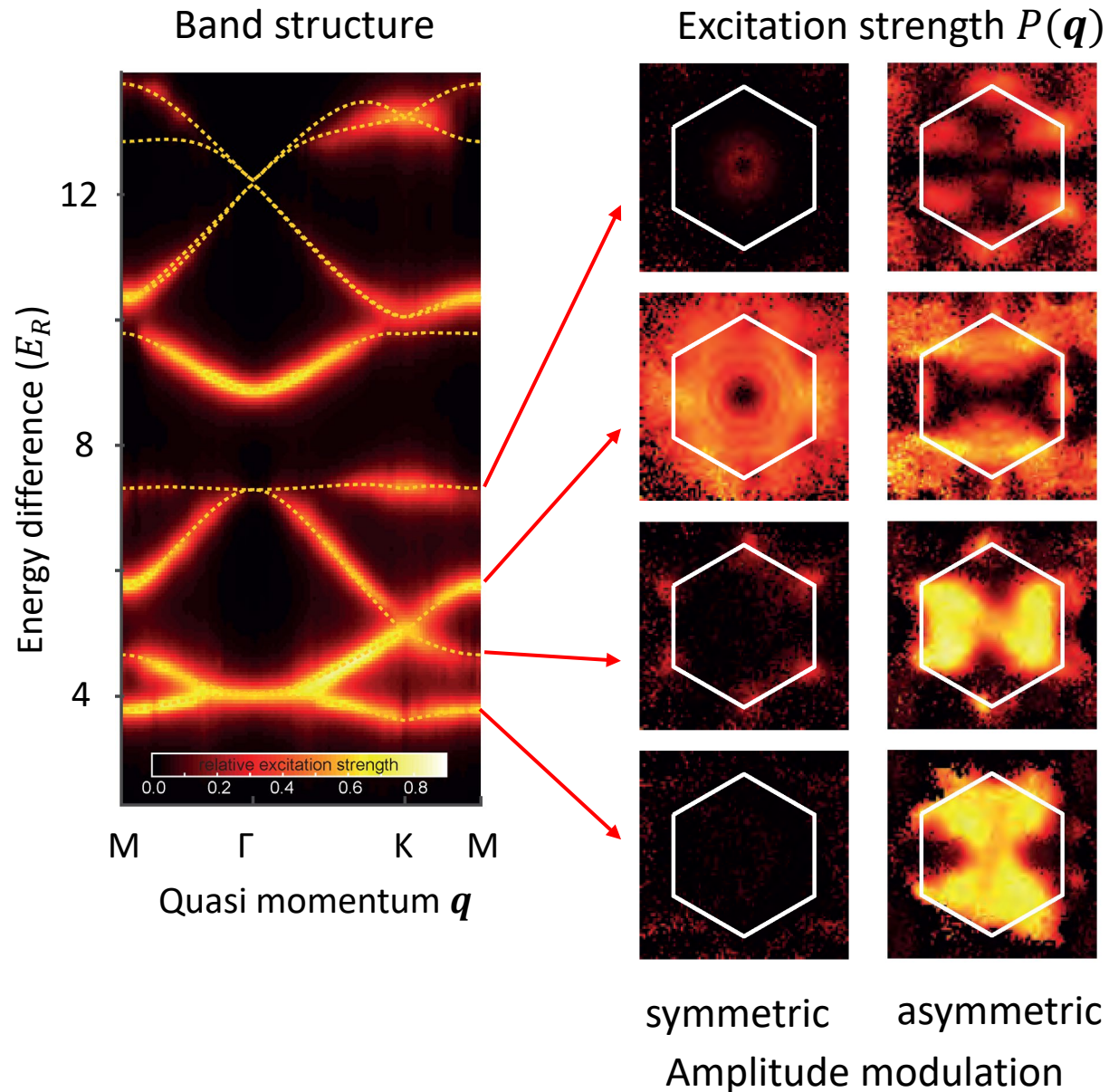
Lattice shaking

Jotzu et al. Nature 515, 237 (2014)
Fläschner et al.,
Science 352, 1091 (2016)

Outline

1. Floquet systems, how to engineer topology? (general introduction)
2. How to probe topology? (focus on experiments in Hamburg)
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Multiband Spectroscopy of the Honeycomb Lattice



- Excitation strength depends on the perturbation operator
- Access information on the eigenstates

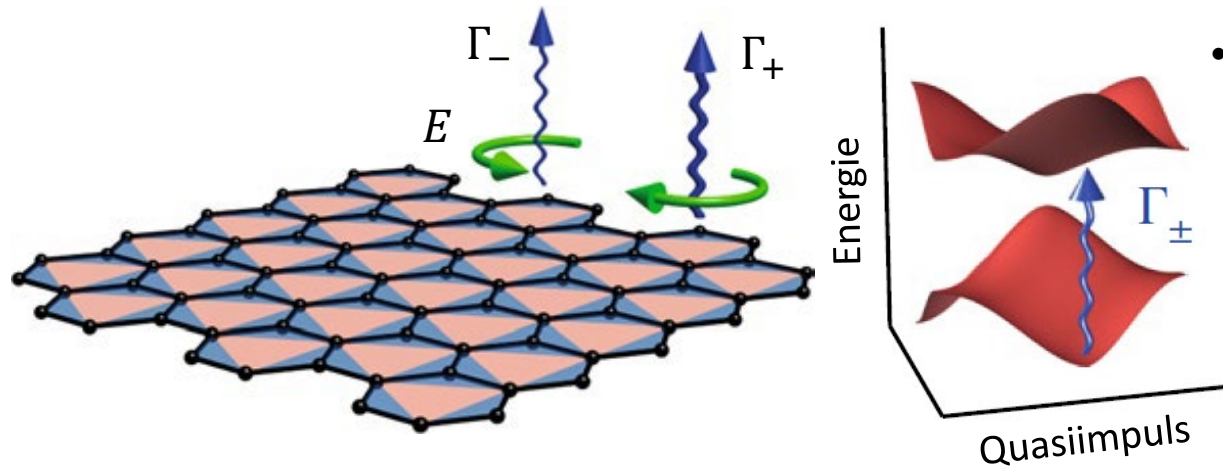
Can one also obtain the topology from the excitation strength?

Quantized circular Dichroism

Topology from spectroscopy

- Choose suitable perturbation: $\hat{H}_{\pm}(t) = \hat{H}_0 + 2E\{\cos(\omega t) \hat{x} \pm \sin(\omega t) \hat{y}\}$
- Excitation rates $\Gamma_+(\omega) \neq \Gamma_-(\omega)$
- Differential integrated rate: $\Delta\Gamma_{\pm}^{\text{int}} \equiv \int d\omega[\Gamma_+(\omega) - \Gamma_-(\omega)]/2$

$$\text{Quantized response: } \Delta\Gamma_{\pm}^{\text{int}}/A_{\text{cell}} = (1/\hbar^2)C \cdot E^2$$



- Dissipative counterpart to the Hall conductivity
- Measurement without Transport

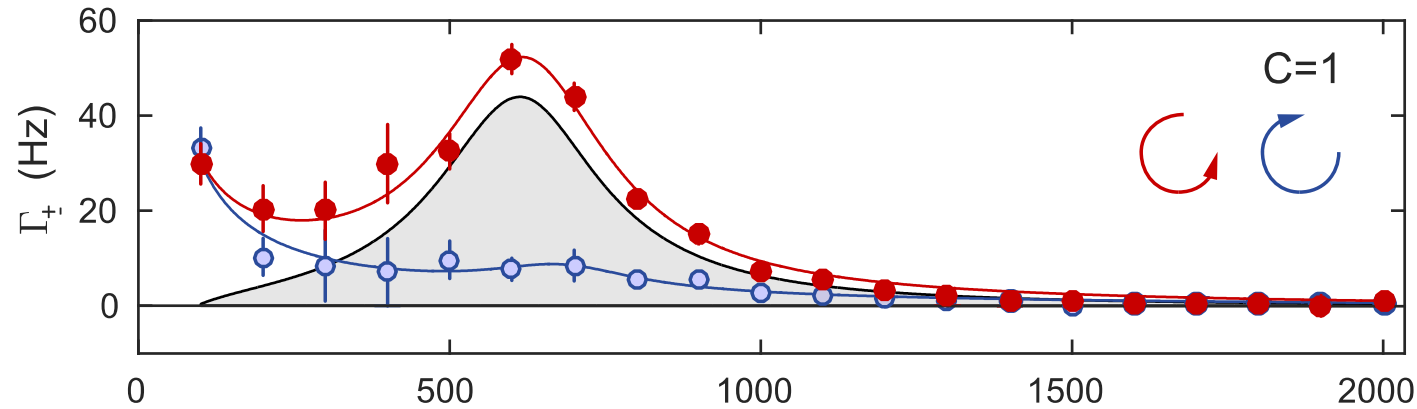
Separation of time scales

- In our case, the system \hat{H}_0 is already shaken ($\omega_{\text{FI}}/2\pi \sim 6$ kHz)
- Spectroscopy is an additional shaking with smaller frequency ($\omega/2\pi \sim 1$ kHz)

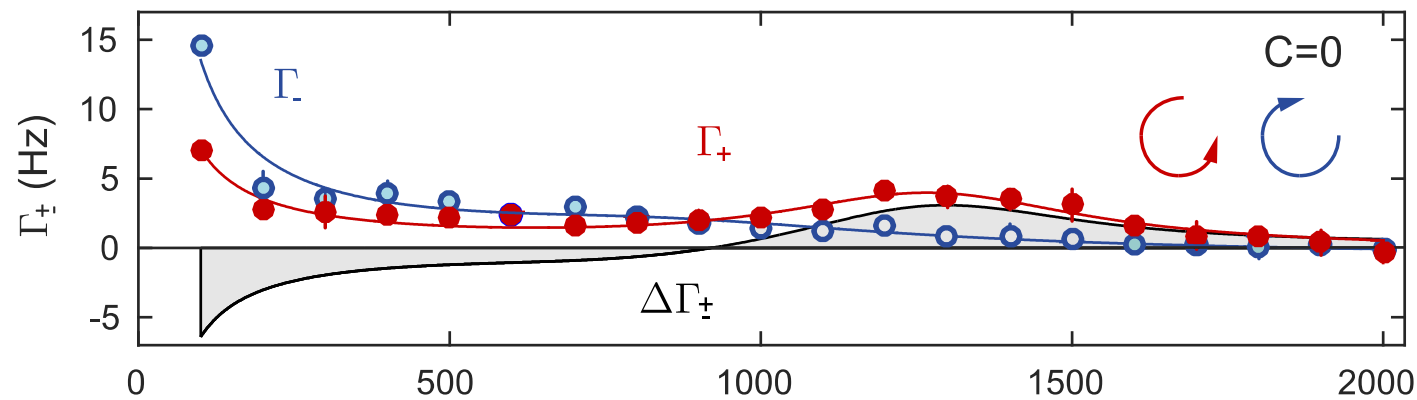
Proposal: D. T. Tran/Goldman et al., *Science Adv.* 3, e1701207 (2017).

Hall drift measurement: M. Aidelsburger et al., *Nature Phys.* 11, 162 (2015)

Chiral Spectra



$$C_{\text{exp}} = 0.92(12)$$



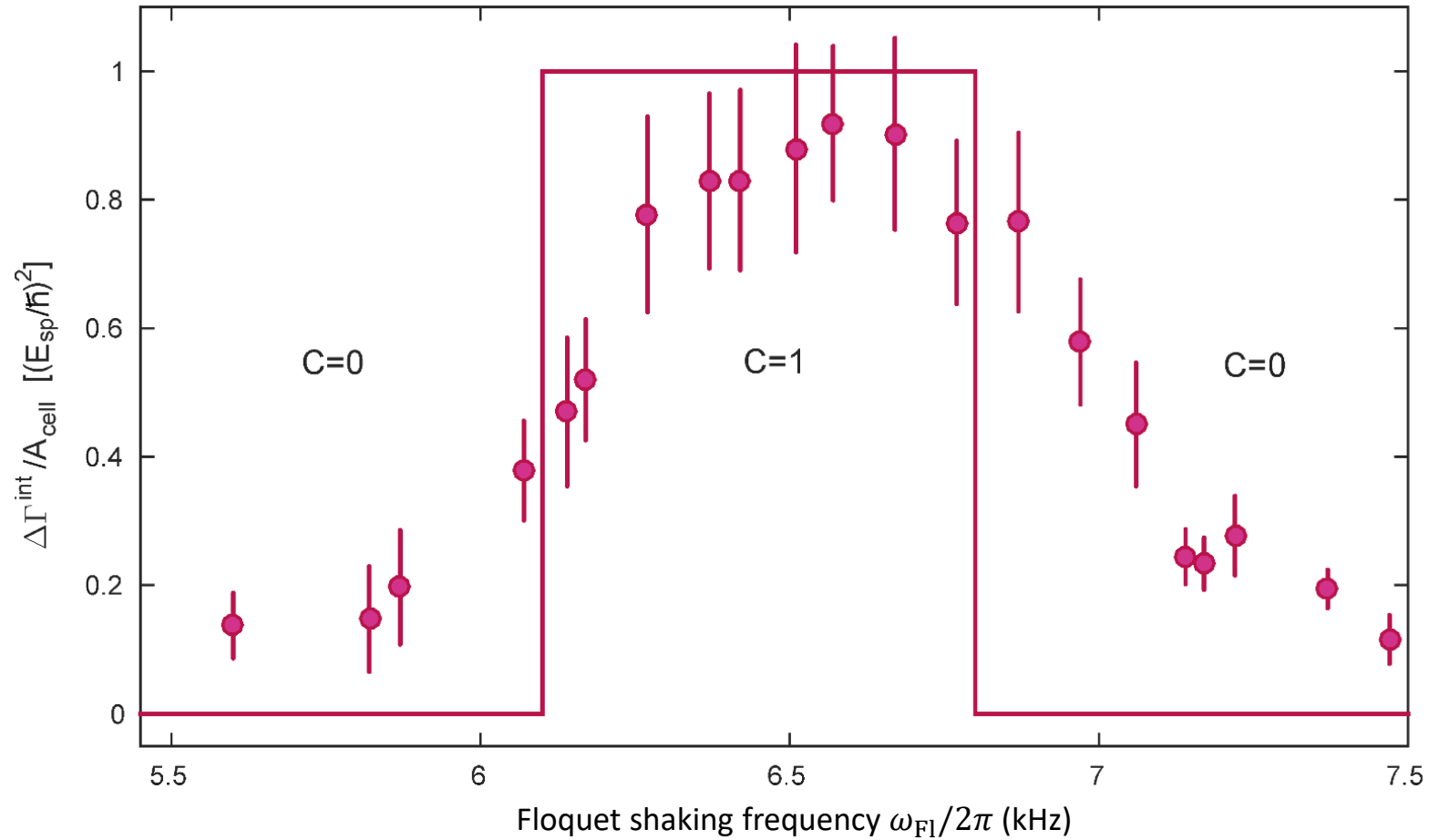
$$C_{\text{exp}} = 0.12(4)$$

Spectroscopy frequency (Hz)

Gray area $\Delta\Gamma_{\pm}^{\text{int}} = \int d\omega [\Gamma_{+}(\omega) - \Gamma_{-}(\omega)]/2$

$$\Delta\Gamma_{\pm}^{\text{int}}/A_{\text{cell}} = C_{\text{exp}} (E_{\text{sp}}/\hbar)^2$$

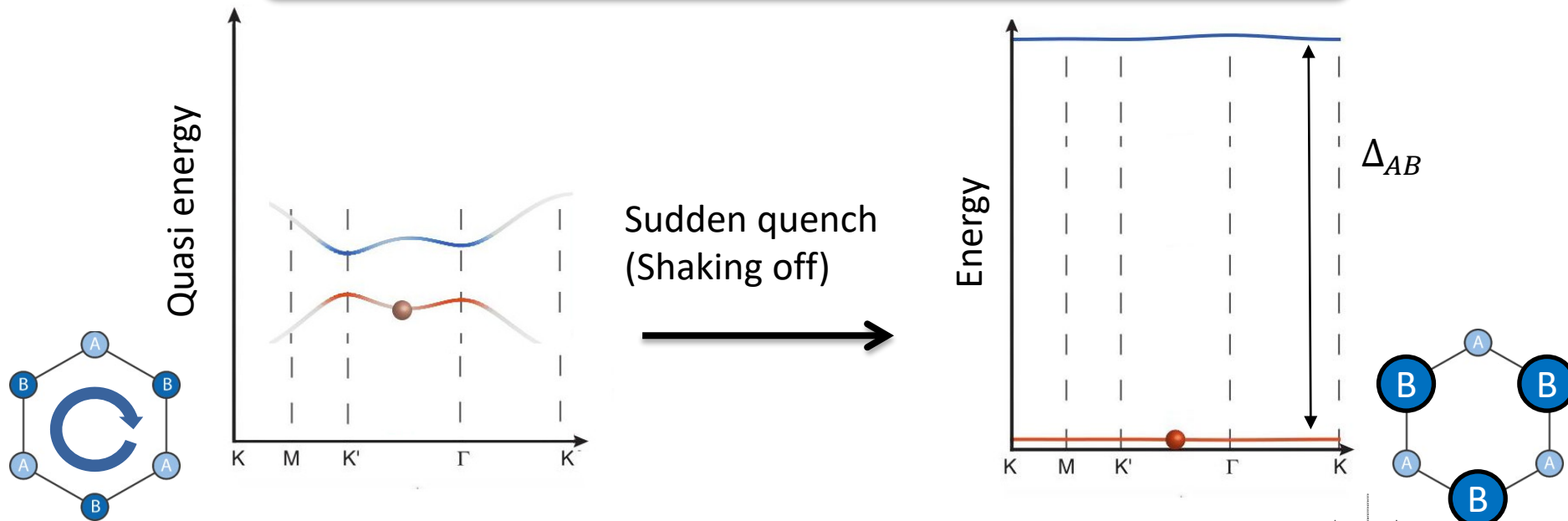
Dichroism Signal over the topological Phase Transition



- Experimental demonstration of a new topological effect
- Promising approach to detect fractional quantum Hall states
[Repellin & Goldman, PRL 122, 166801 \(2019\).](#)

Bloch State Tomography of Floquet Systems

Can one detect the geometry of the eigenstates directly?

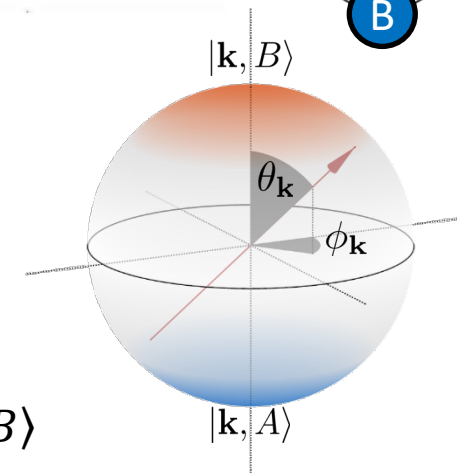


- Initial eigenstates of the Floquet system:

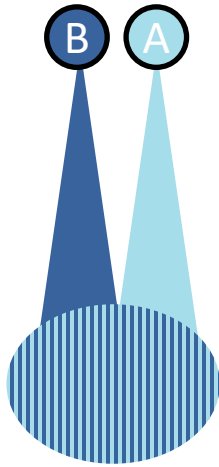
$$|k\rangle = -\sin\left(\frac{\theta_k}{2}\right) \exp(-i\phi_k) |k, A\rangle + \cos\left(\frac{\theta_k}{2}\right) |k, B\rangle$$

- Time evolution after the quench (projection onto Basis states):

$$|k\rangle = -\sin\left(\frac{\theta_k}{2}\right) \exp(-i\phi_k) |k, A\rangle + \underline{e^{i\frac{\Delta_{AB}t}{\hbar}}} \cos\left(\frac{\theta_k}{2}\right) |k, B\rangle$$

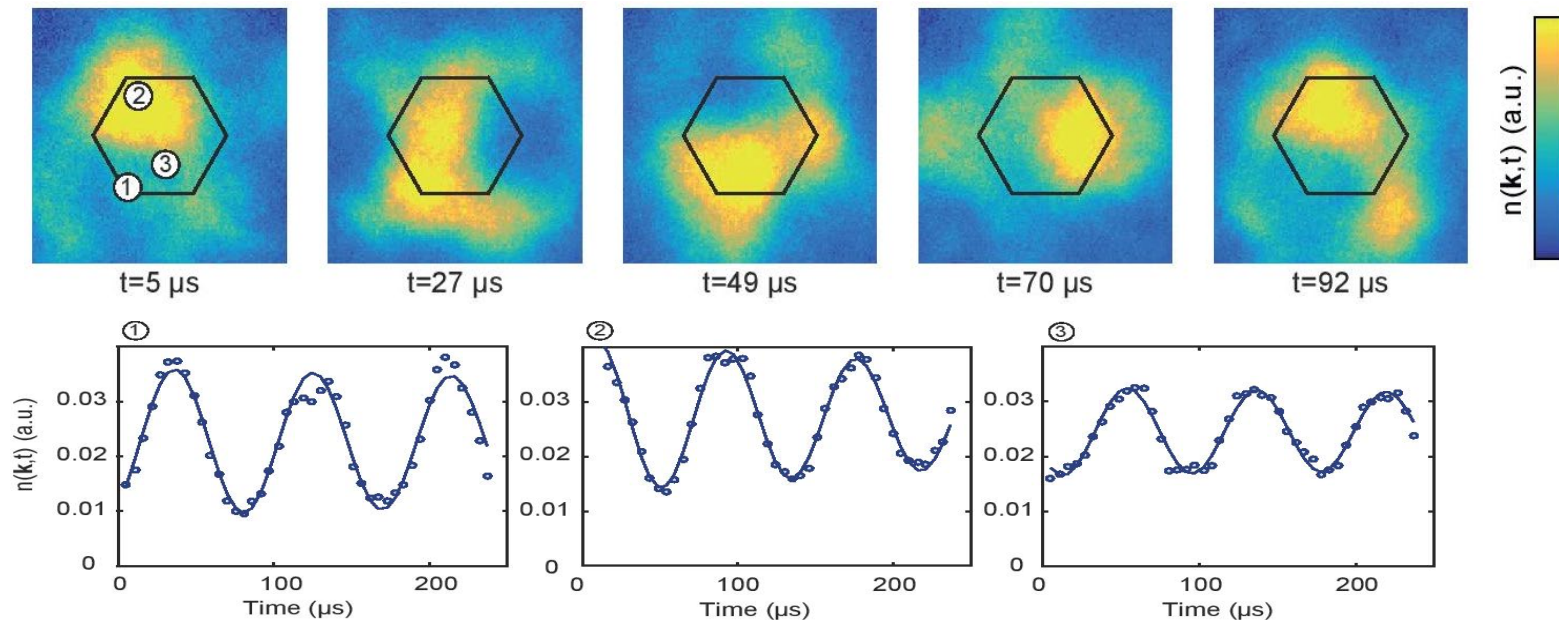


Bloch State Tomography of Floquet Systems



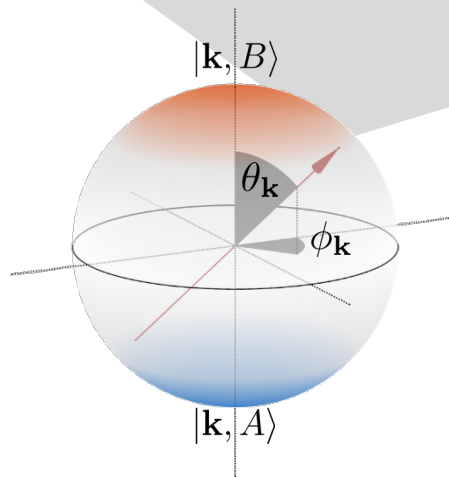
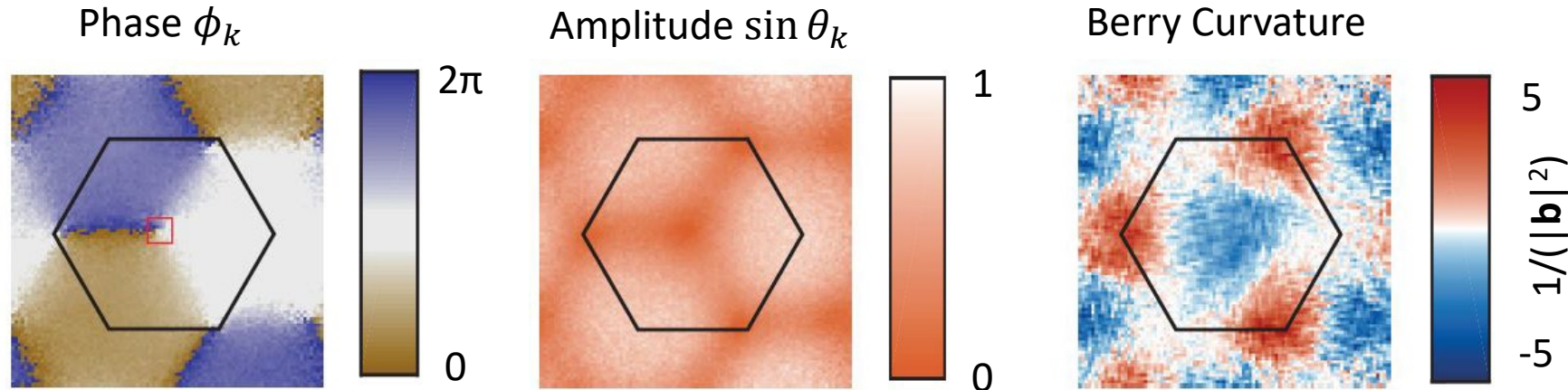
$$|k\rangle = -\sin\left(\frac{\theta_k}{2}\right)\exp(-i\phi_k)|k,A\rangle + \underline{e^{i\frac{\Delta_{AB}t}{\hbar}}}\cos\left(\frac{\theta_k}{2}\right)|k,B\rangle$$

$$n(k,t) = |c|^2(1 - \sin\theta_k \cos(t\Delta_{AB}/\hbar + \phi_k))$$



Proposal: P. Hauke, M. Lewenstein, A. Eckardt, PRL 113, 045303 (2014).
Experiment: N. Fläschner et al., Science 352, 1091 (2016).

Measurement of the Berry Curvature



Sublattice pseudo-spin

Berry curvature from derivatives of the data:

$$\mathbf{\Omega}_-(\mathbf{k}) = \nabla_{\mathbf{k}} \times i\hbar \langle u_{\bar{k}} | \frac{\partial}{\partial \mathbf{k}} | u_{\bar{k}} \rangle = -\frac{1}{2} \sin \theta (\partial_{k_x} \theta \partial_{k_y} \phi - \partial_{k_y} \theta \partial_{k_x} \phi) \hat{e}_z$$

Chern number from integration:

$$C_- = \frac{1}{2\pi} \iint_{\text{BZ}} \mathbf{\Omega}_-(\mathbf{k}) \cdot d\mathbf{k} = 0.005 \pm 0.003 \quad (\text{integer, but trivial})$$

How can we study non-trivial bands despite the heating?

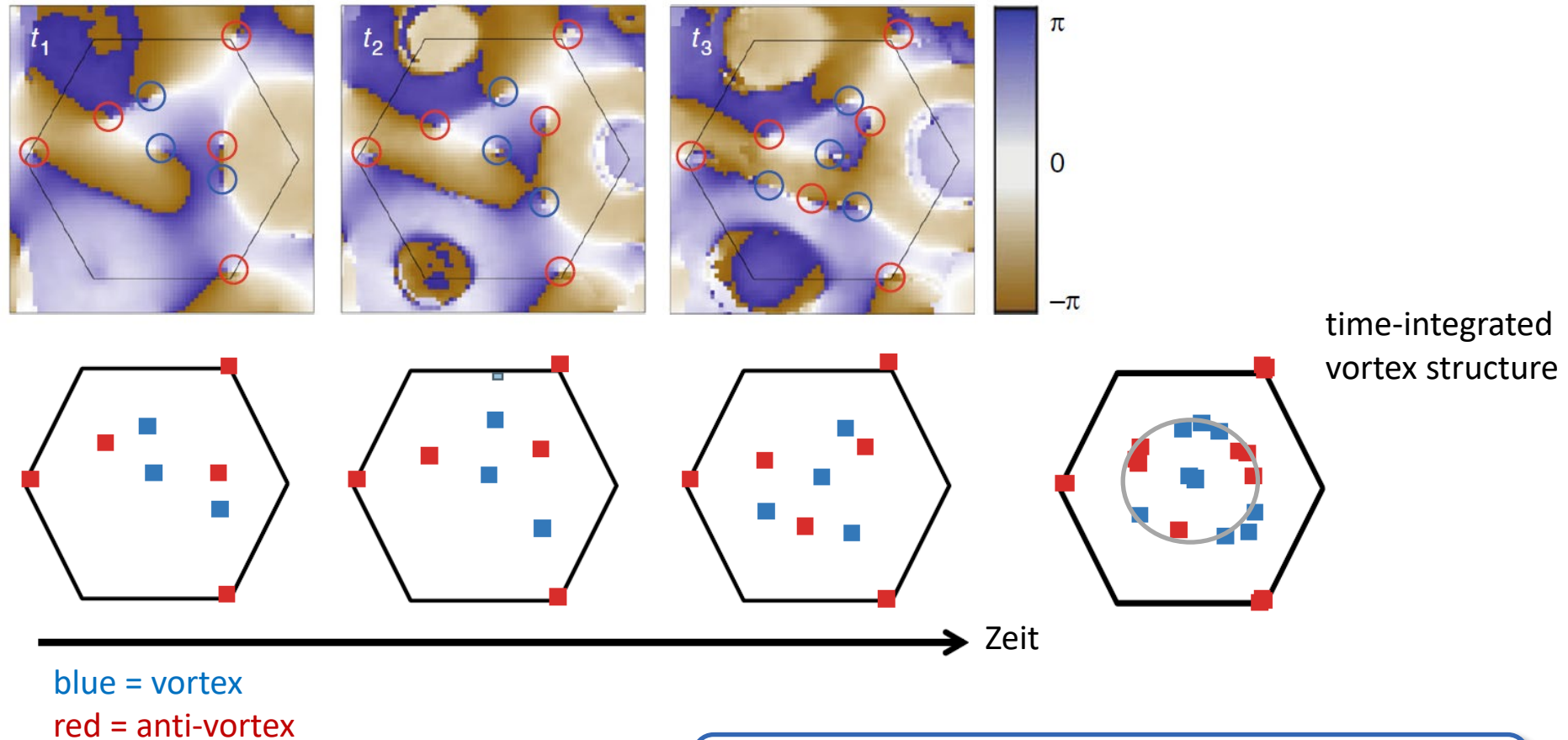
Tomography of the dynamics after quench into Floquet system

N. Fläschner et al., Science 352, 1091 (2016).

Proposal: P. Hauke, M. Lewenstein, A. Eckardt, PRL 113, 045303 (2014).

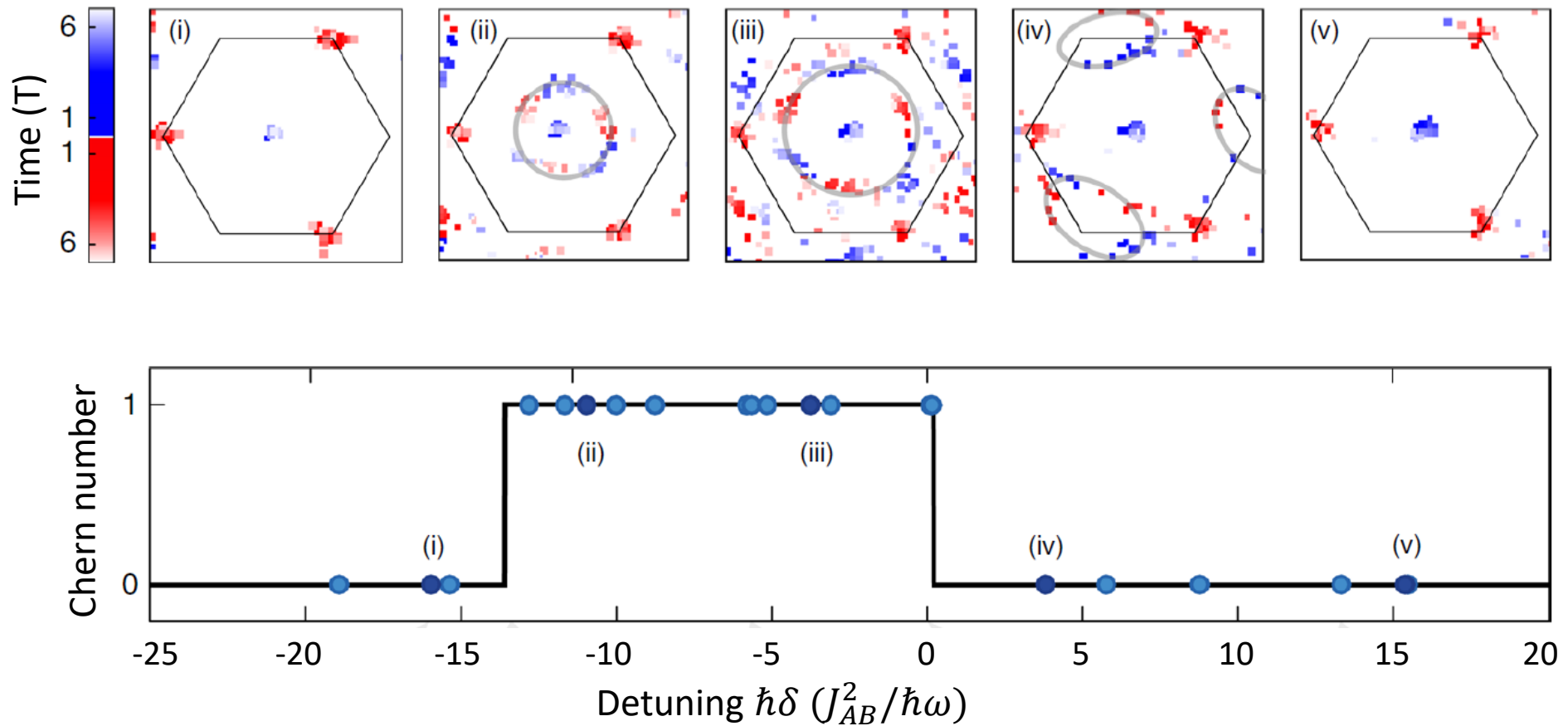
Appearance of dynamical vortices

Quench into the Floquet system: Phase profiles for different evolution times



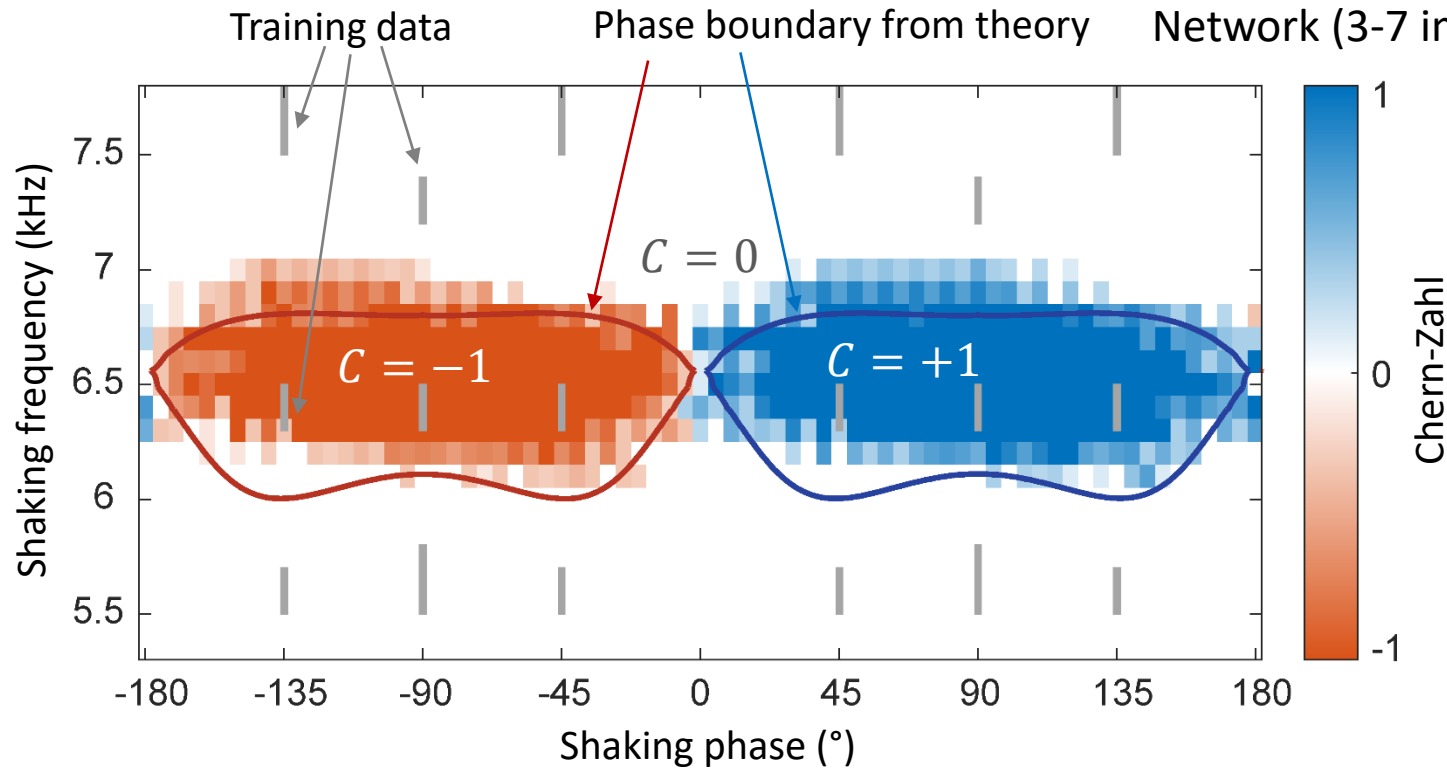
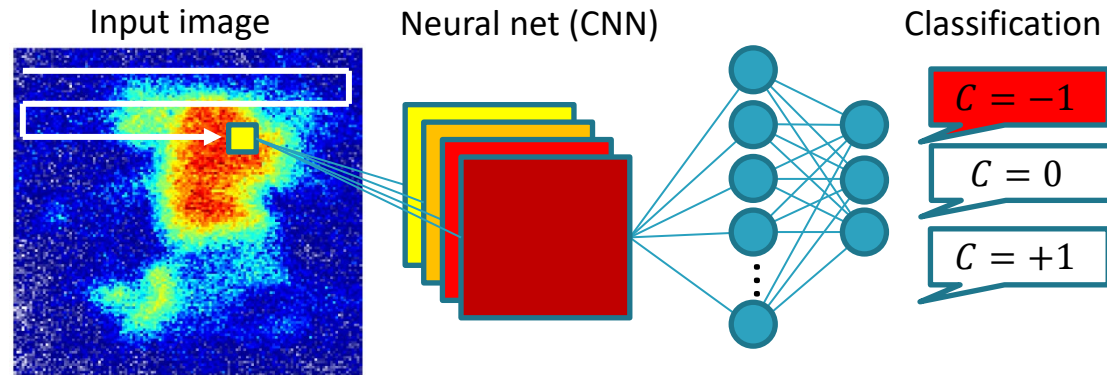
Linking number (dynamics)
= Chern number of final system (static)

Linking number: Topology from dynamics

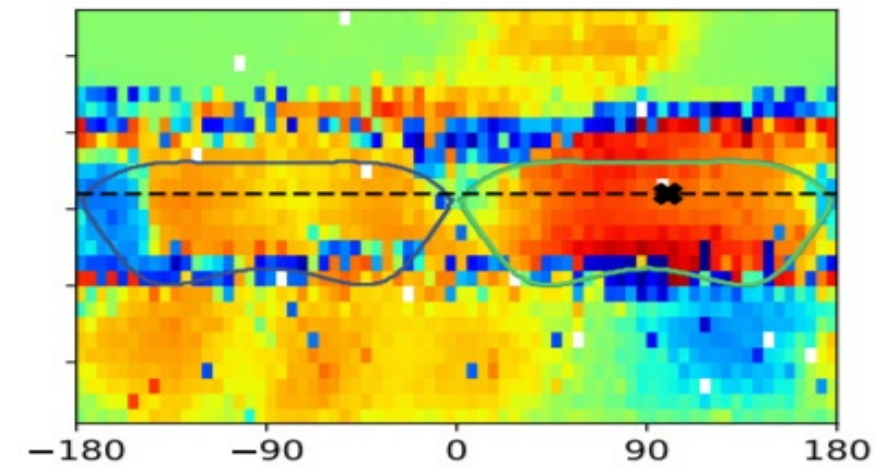


Connect topology and dynamics

Topological phase diagram via machine learning



Unsupervised machine learning
(Influence Function)



Outline

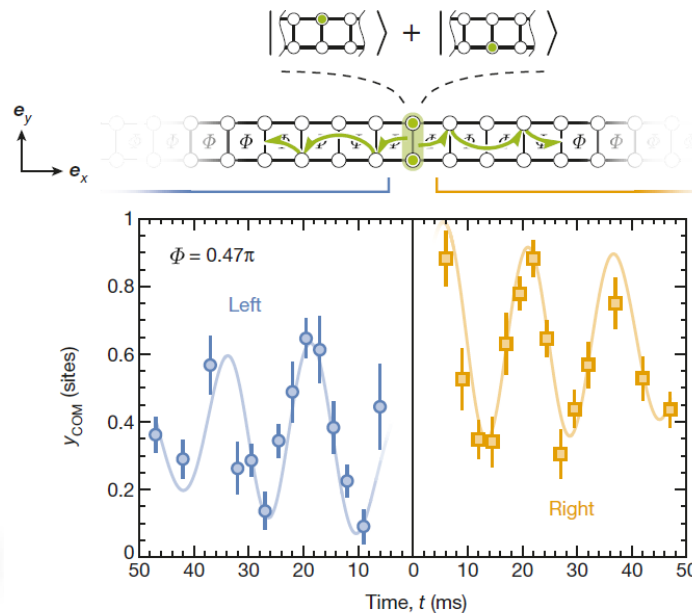
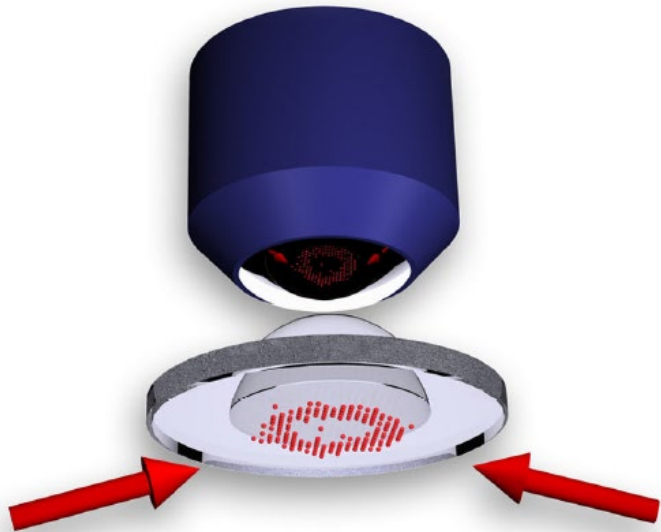
1. Floquet systems, how to engineer topology? (general introduction)
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Towards interacting topological matter

Ultracold topological matter under the microscope

- Real-space approach, tailored potentials
- Prepare and detect low filling factor

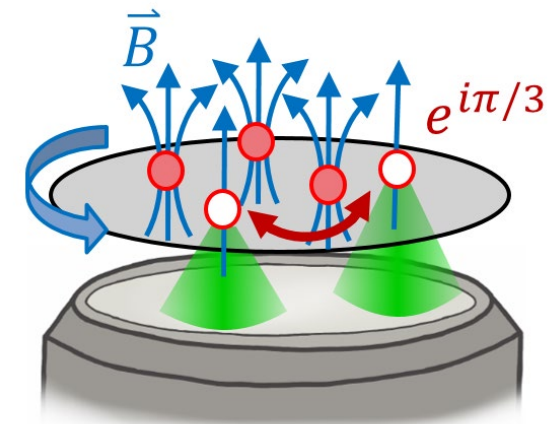
Two bosons in a flux ladder



Tai, Greiner et al. Nature 546, 519 (2017)

Quantum Hall droplets

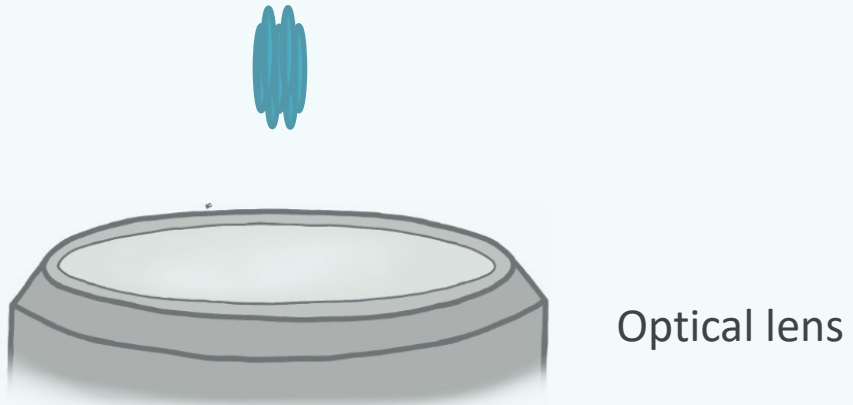
Rotating microtraps, bulk systems



Optimized adiabatic preparation
Andrade et al., arXiv:2009.08943 (2020)

New approach: Quantum gas magnifier

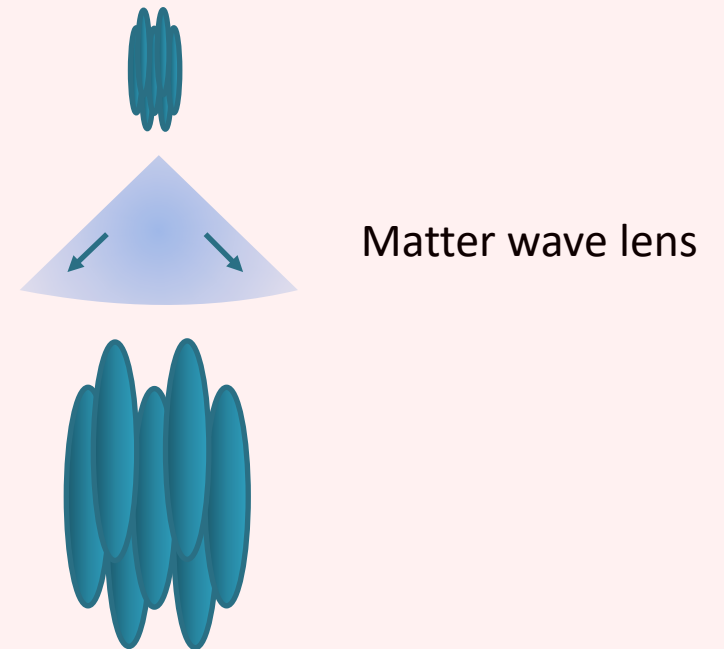
High resolution imaging



Getting closer to the quantum system....

...or bring it closer to us!

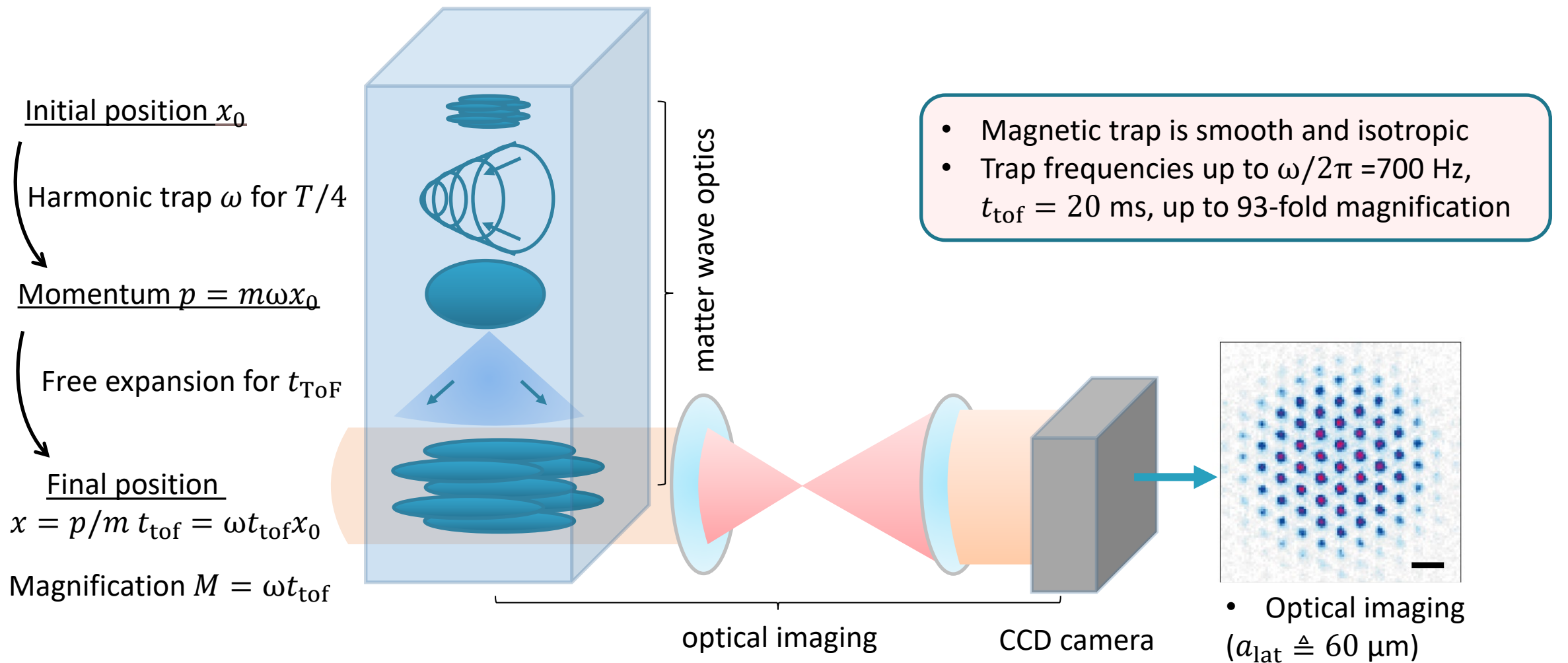
Quantum Gas Magnifier



single-shot, sub-lattice resolved
images of the 3D system!

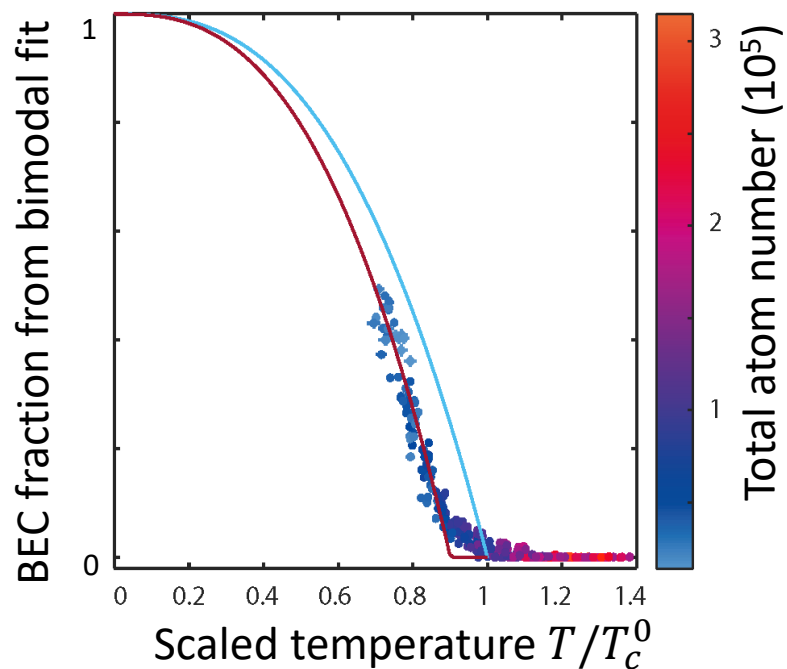
Working principle of the Quantum Gas Magnifier

Array of „tubes“, triangular lattice, $\sim 10^5$ ^{87}Rb atoms



Precision measurements with the quantum gas magnifier

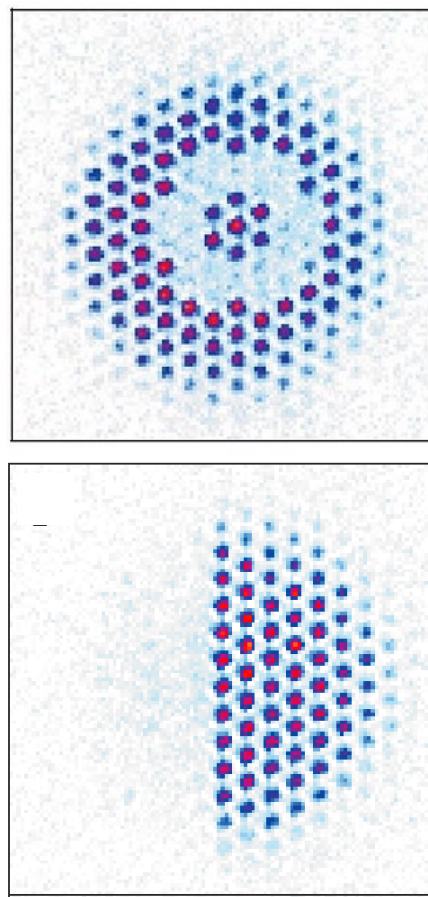
Precision thermodynamics



- Cross over regime with power law $\alpha = 2.69(1)$
- Strong interaction shift $T_c = 0.89(1) T_c^0$

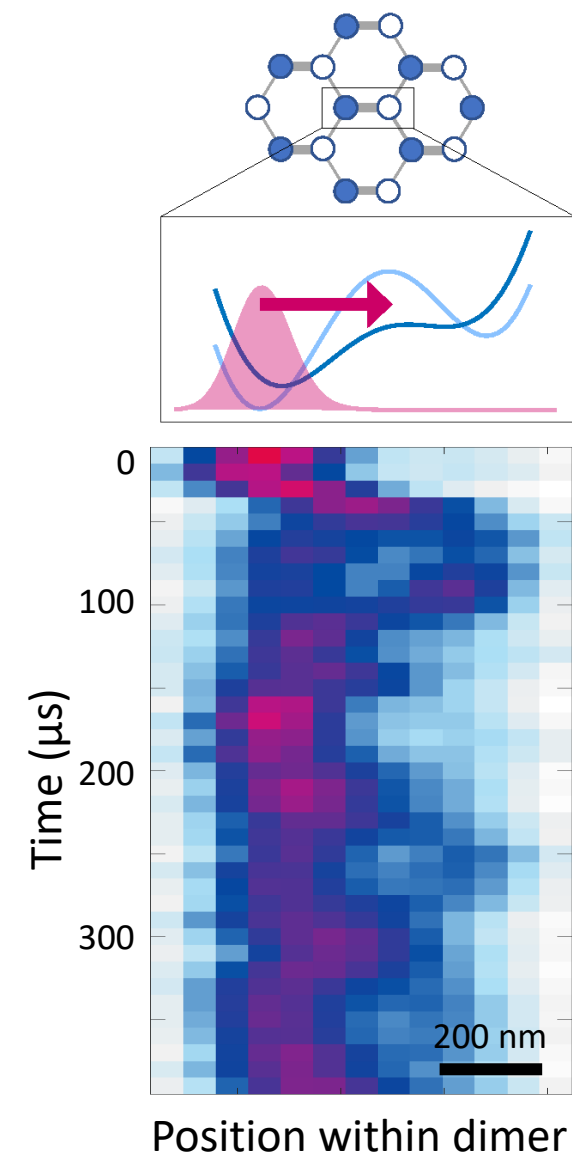
Theory?

Local RF addressing



- Prepare arbitrary density patterns
- Full functionality of a quantum gas microscope

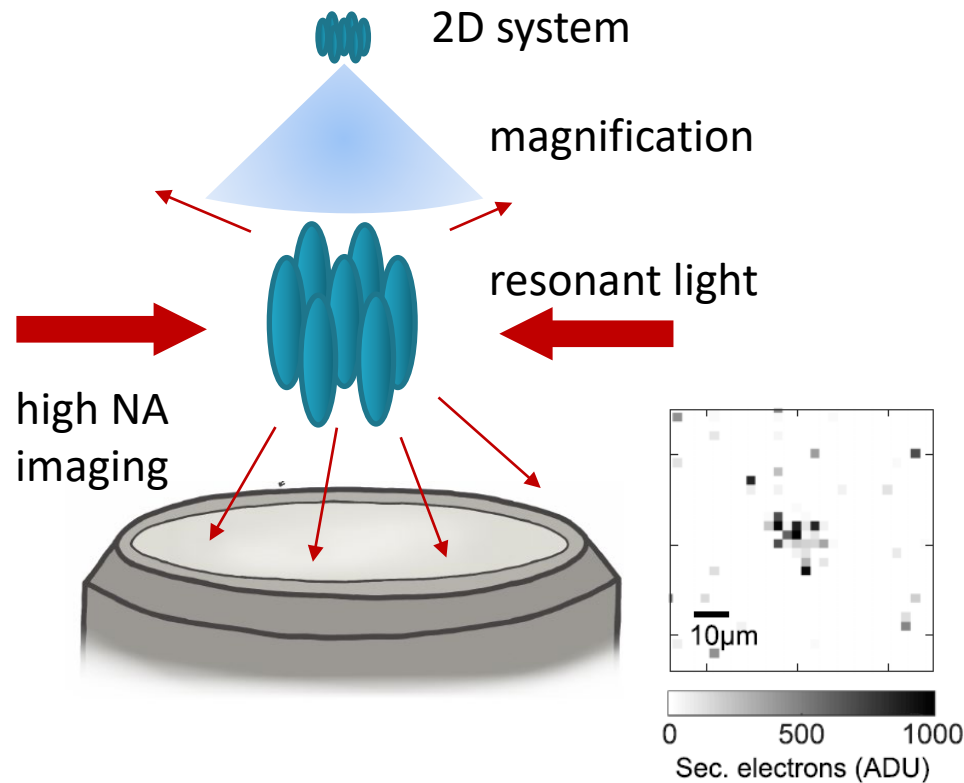
Resolving nanoscale dynamics



Towards the single-atom resolved regime

Quantum gas microscope 2.0

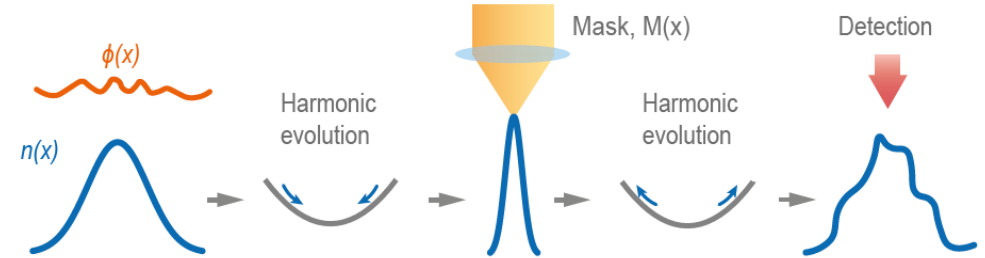
- Single-atom regime via free-space fluorescence imaging
- Magnified lattice spacing is larger than diffusive expansion



Bergschneider/Jochim et al., PRA 97, 063613 (2018).
Bücker/Schmiedmayer, New J. Phys. 11, 103039 (2009).

Access coherence properties with high spatial resolution

- Manipulation in Fourier space
- Image Talbot revivals



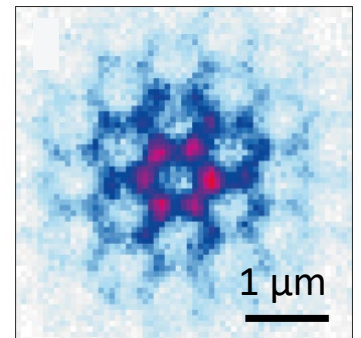
Murthy & Jochim, arXiv:1911.10824 (2019).

Santra/Ott et al., Nature Commun. 8, 15601 (2017).

New possibilities

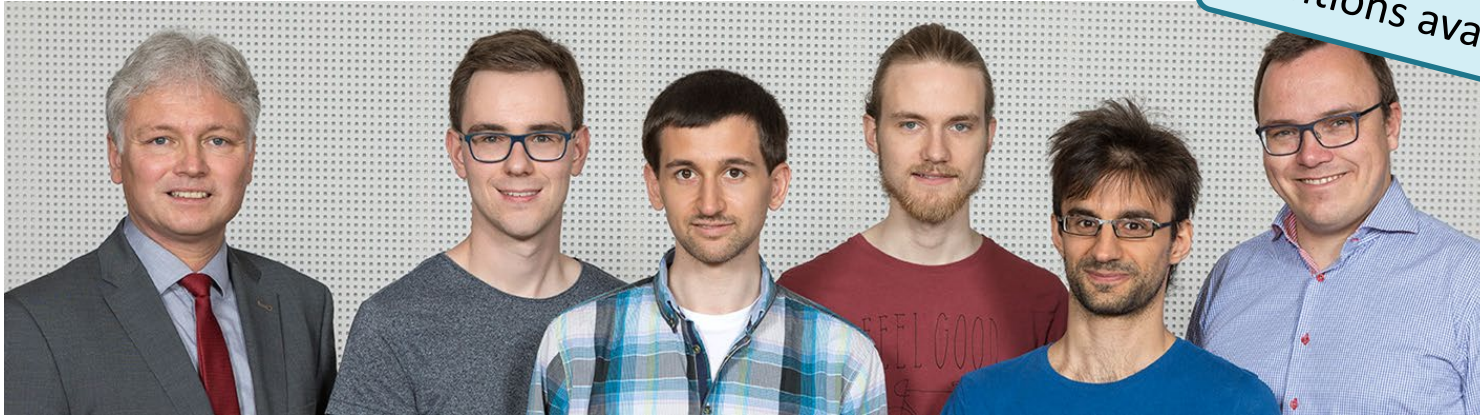
- Applicable to species, where laser cooling or deep optical lattices are not available
- Extend microscopes to exotic atoms
- Also new regimes, lattice geometries
- No parity projection

New real-space access to
quantum many-body systems



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Hamburg experimental team



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Collaborators

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Lithium team:

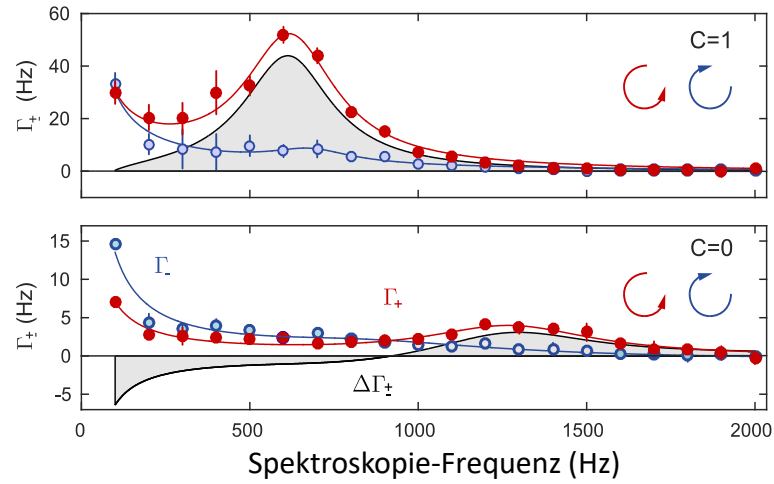
Mathis Fischer, Justus Brüggenjürgen

Funding

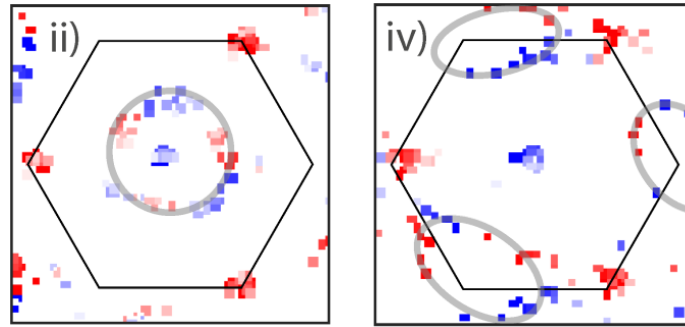


Summary

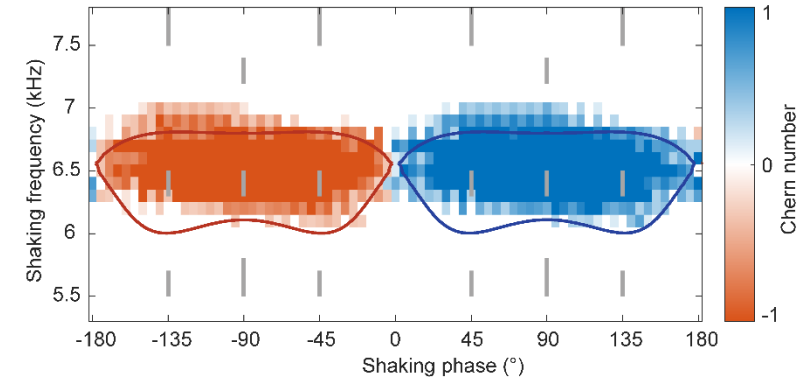
Quantized circular dichroism



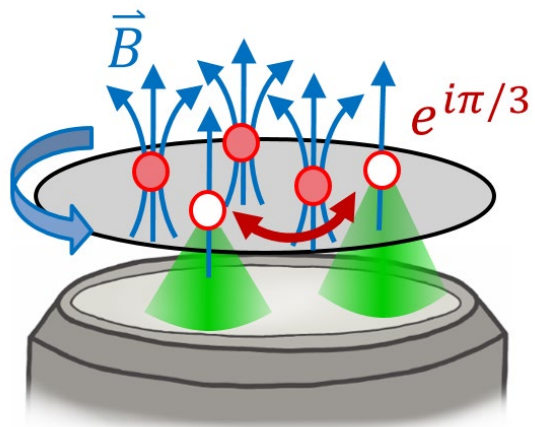
State tomography and linking number



Machine learning of topol. phases



Quantum Hall droplets



Quantum gas magnifier

