Quantum transport and absence of Anderson localization in strong and weak topological insulators

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LBL & UC Berkeley

KITP, Wed. ,Oct. 5 2011
- discussion -

Refs: PRL, **99**, 106801, (2007)
PRL, **105**, 156803, (2011)
arXiv:1109.3201

Collaborators

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Classification scheme

Cartan nomenclature	TRS	PHS	SLS	Hamiltonian	NLSM (ferm. replicas)	$d=1$	$d=2$	$d=3$
A (unitary)	0	0	0	$U(N)$	$U(2n)/U(n)\times U(n)$	-	$\mathbb{Z}$	-
AI (orthogonal)	$+1$	0	0	$U(N)/O(N)$	$Sp(2n)/Sp(n)\times Sp(n)$	-	-	-
AII (symplectic)	-1	0	0	$U(2N)/Sp(2N)$	$O(2n)/O(n)\times O(n)$	-	$\mathbb{Z}_2$	$\mathbb{Z}_2$
AIII (chiral unit.)	0	0	1	$U(N+M)/U(N)\times U(M)$	$U(n)$	$\mathbb{Z}$	-	$\mathbb{Z}$
BDI (chiral orthog.)	$+1$	$+1$	1	$SO(N+M)/SO(N)\times SO(M)$	$U(2n)/Sp(n)$	$\mathbb{Z}$	-	-
CII (chiral sympl.)	-1	-1	1	$Sp(2N+2M)/Sp(2N)\times Sp(2M)$	$U(2n)/O(2n)$	$\mathbb{Z}$	-	$\mathbb{Z}_2$
D	0	$+1$	0	$SO(2N)$	$O(2n)/U(n)$	$\mathbb{Z}_2$	$\mathbb{Z}$	-
C	0	-1	0	$Sp(2N)$	$Sp(n)/U(n)$	-	$\mathbb{Z}$	-
DIII	-1	$+1$	1	$SO(2N)/U(N)$	$O(2n)$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
CI	$+1$	-1	1	$Sp(2N)/U(N)$	$Sp(n)$	-	-	$\mathbb{Z}$

What is the effect of topological term on the Anderson localization?

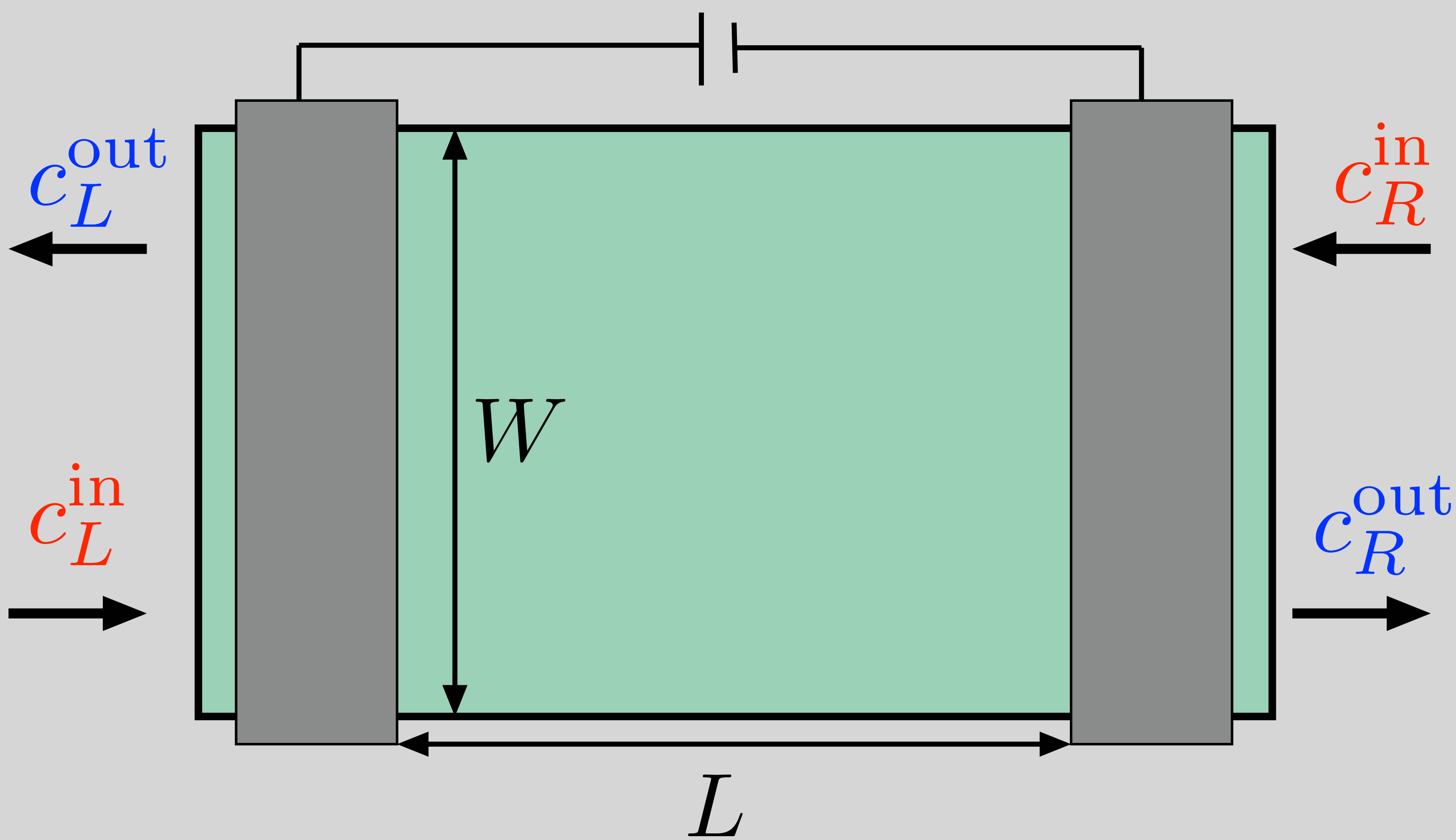
Symplectic class in 3D: Strong vs. weak topological insulators

Schnyder, Ryu, Furusaki and Ludwig, (Landau Memorial Conf.;arXiv:0905.2029), AIP Conf. Proc. 1134, 10 (2009).

Symplectic class (AII) -- Consequences of time reversal

Time reversal: $T^2 = -1$ $\quad THT^{-1} = H$

Schematic setup and basic definitions



$$\begin{pmatrix} c_L^{\text{out}} \\ c_R^{\text{out}} \end{pmatrix} = S \begin{pmatrix} c_L^{\text{in}} \\ c_R^{\text{in}} \end{pmatrix}$$

$$S = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix}$$

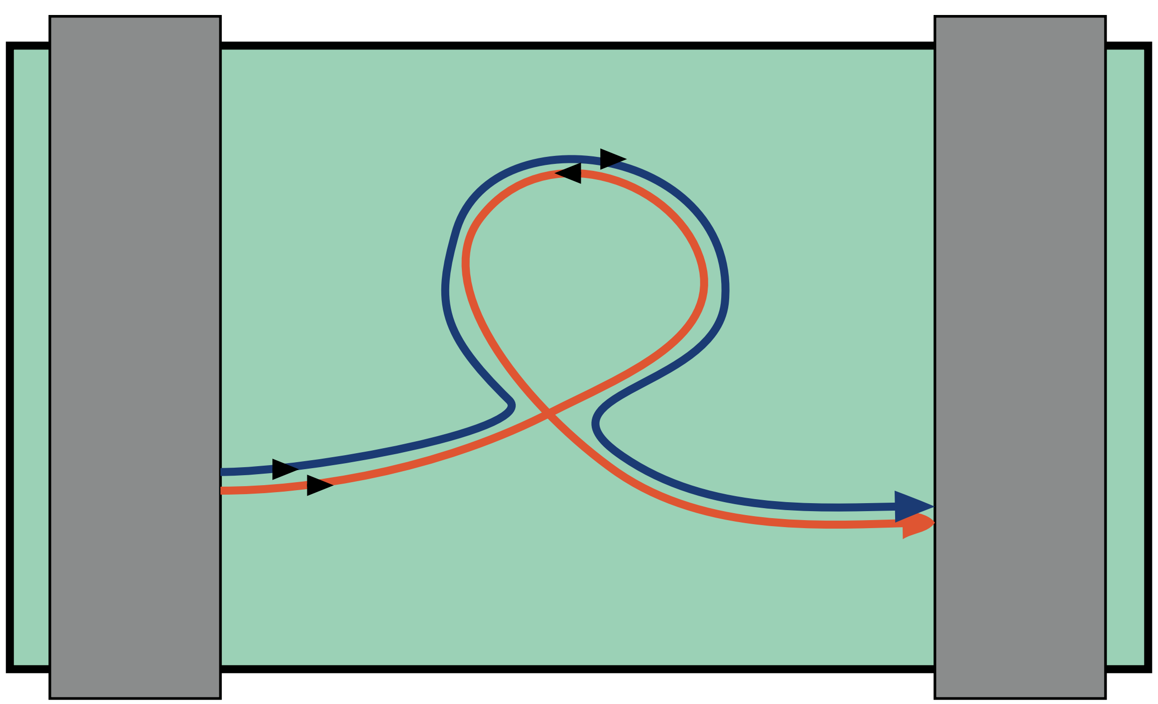
Landauer conductance: $G = \text{tr}\, t^\dagger t = \text{tr}\,(1 - r^\dagger r) = \sigma W/L$

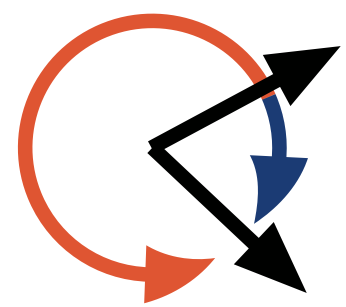
Symplectic class (AII) -- Consequences of time reversal

Time reversal: $T^2 = -1$ $THT^{-1} = H$

i) $S^T = -S \Rightarrow$
- Absence of backscattering (also for multiple scattering)
- Kramers' degeneracy of transmission eigenvalues
- Perfectly transmitted mode if odd number of modes (minimum conductance of e^2/h)

ii) Weak anti-localization $\sigma = \sigma_0 + \frac{1}{\pi} \ln L$





Single parameter scaling and non-linear sigma model

Effective low energy field theory of a strong TI:

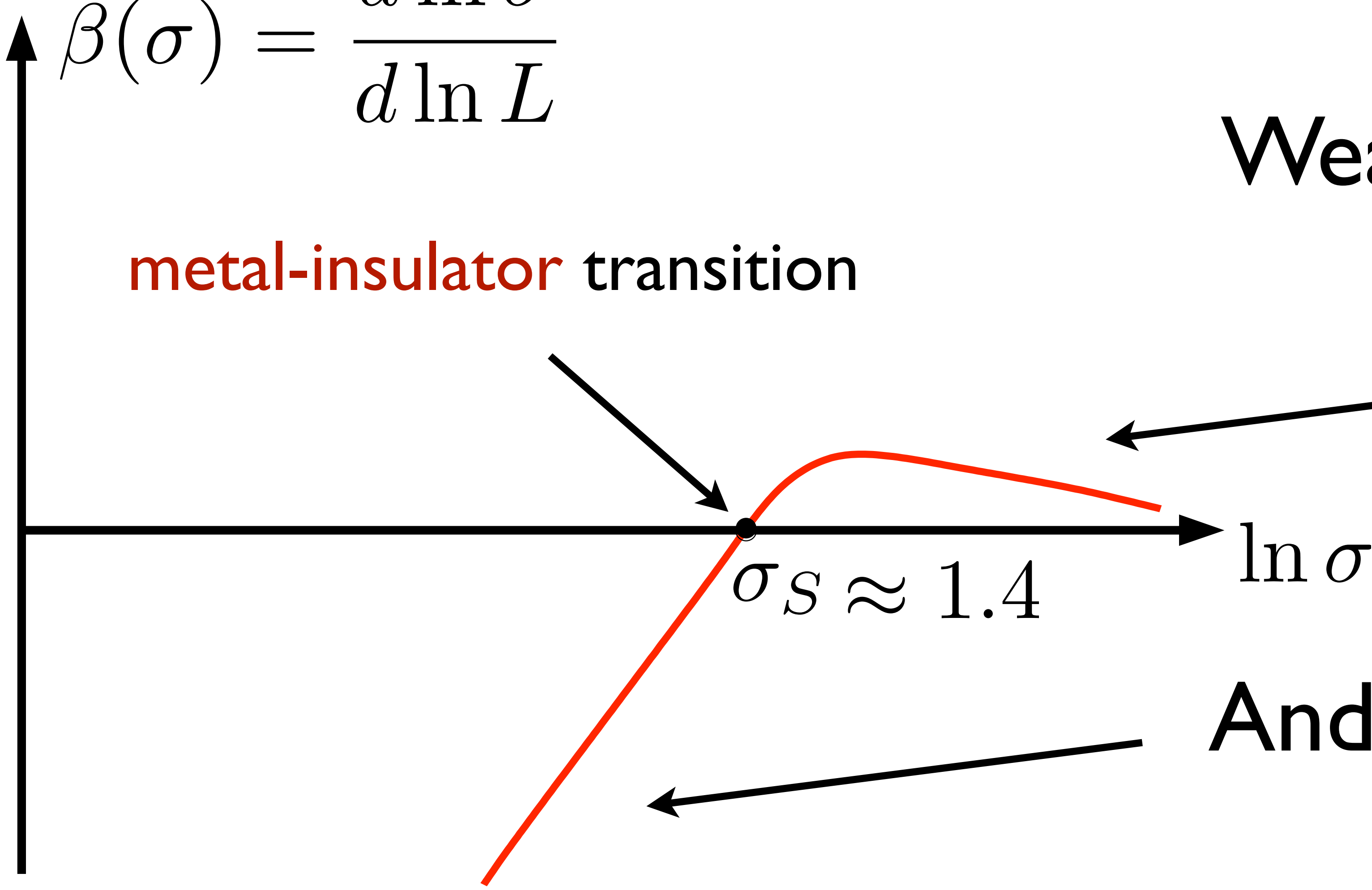
S. Ryu, C. Mudry, H. Obuse, and A. Furusaki, Phys. Rev. Lett. **99**,116601 (2007).
P. M. Ostrovsky, I. V. Gornyi, and A. D. Mirlin, Phys. Rev. Lett. **98**, 256801 (2007).

$$S = S^{\mathrm{NL}\sigma\mathrm{M}}_{\mathrm{symp}} + S_{\mathrm{topological}}$$

Without topological term:

S. Hikami, A. I. Larkin and Y. Nagaoka (1980)
P. Markoš and L. Schweitzer (2006)

$$\beta(\sigma) = \frac{d\ln\sigma}{d\ln L}$$



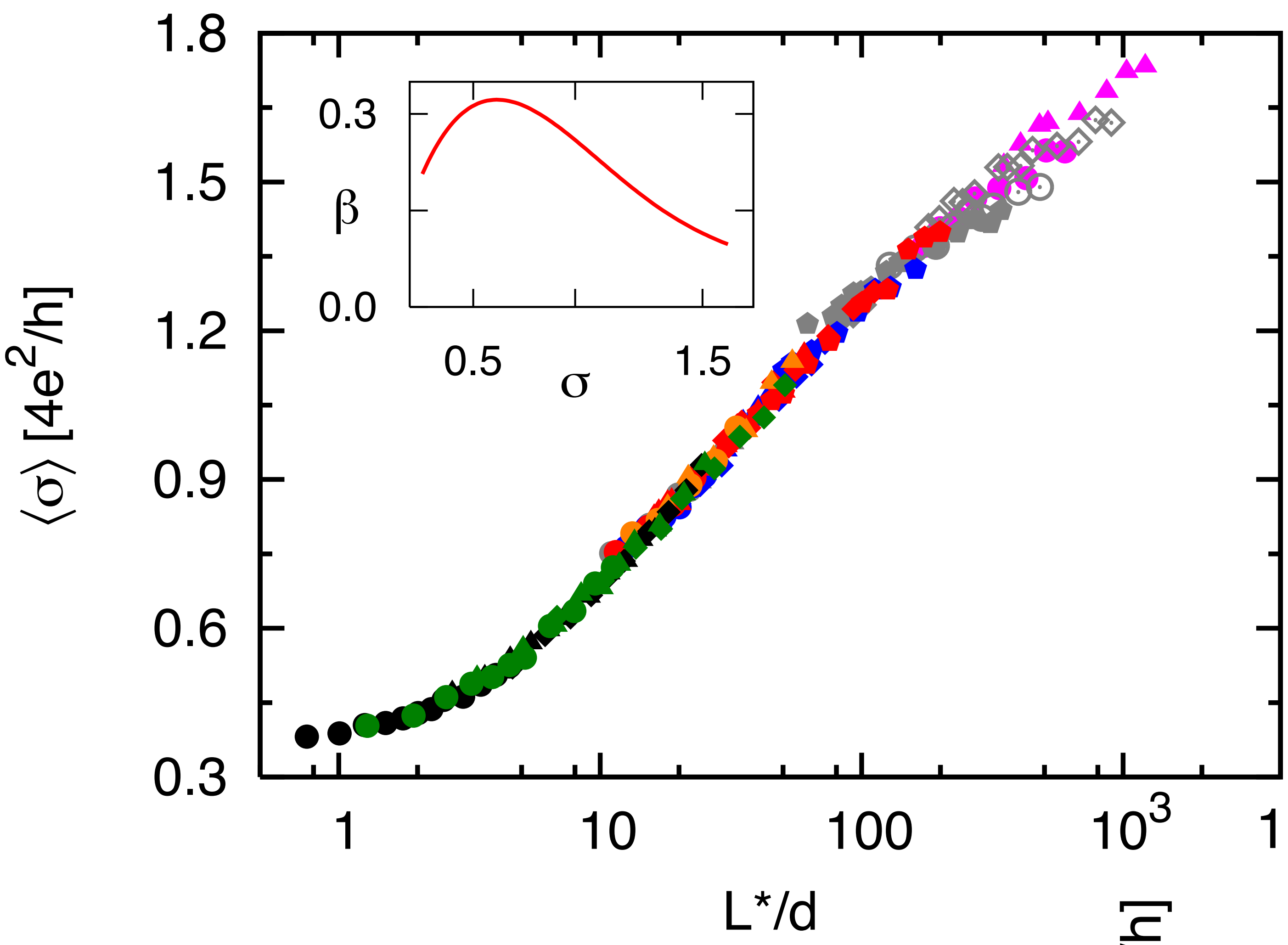
Weak anti-localization:

$$\beta(\sigma) \sim \frac{1}{\pi\sigma}$$

Anderson-localization

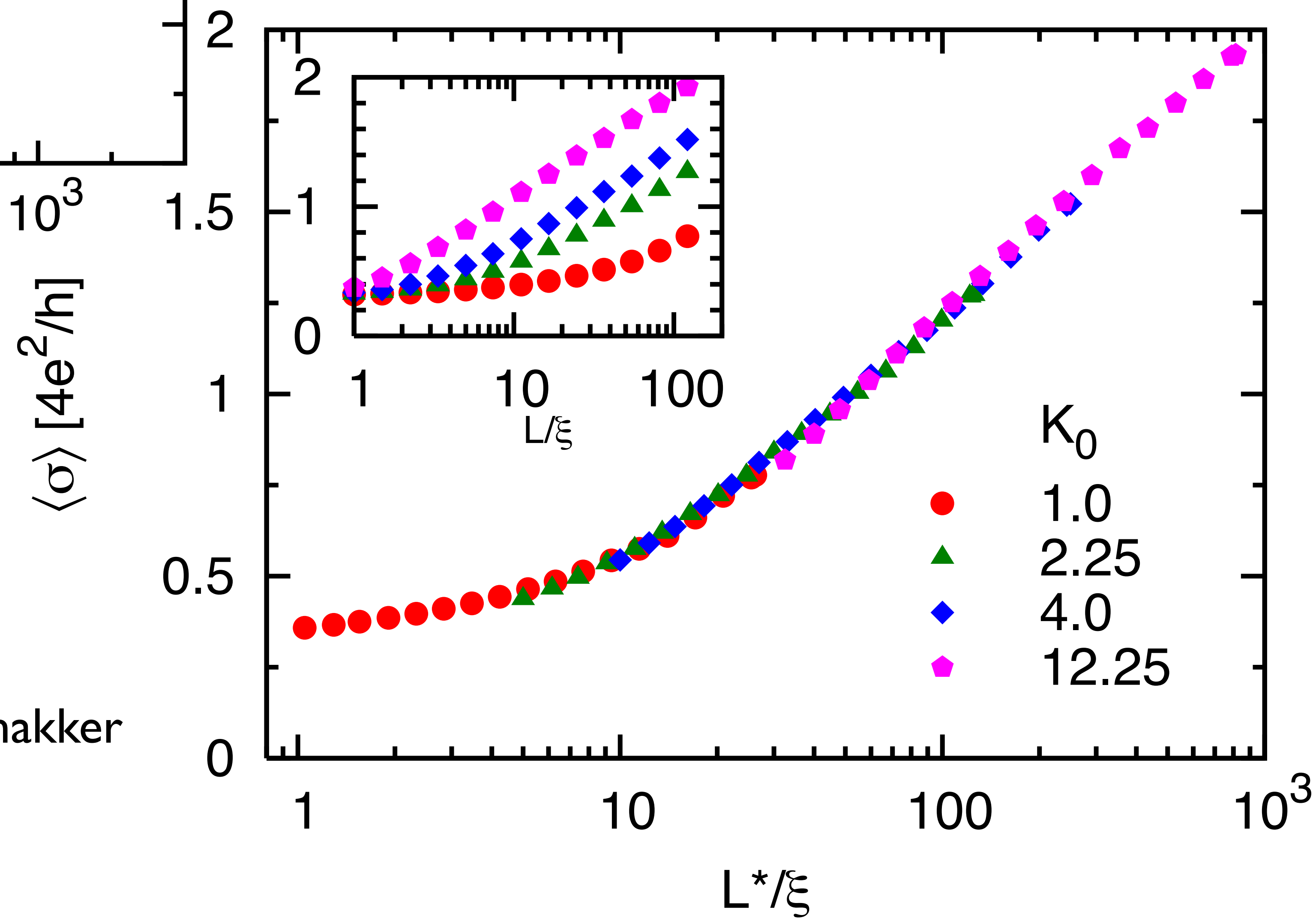
$$\beta(\sigma) \sim -\ln\sigma$$

Landauer conductivity, single parameter scaling and metallic fixed point



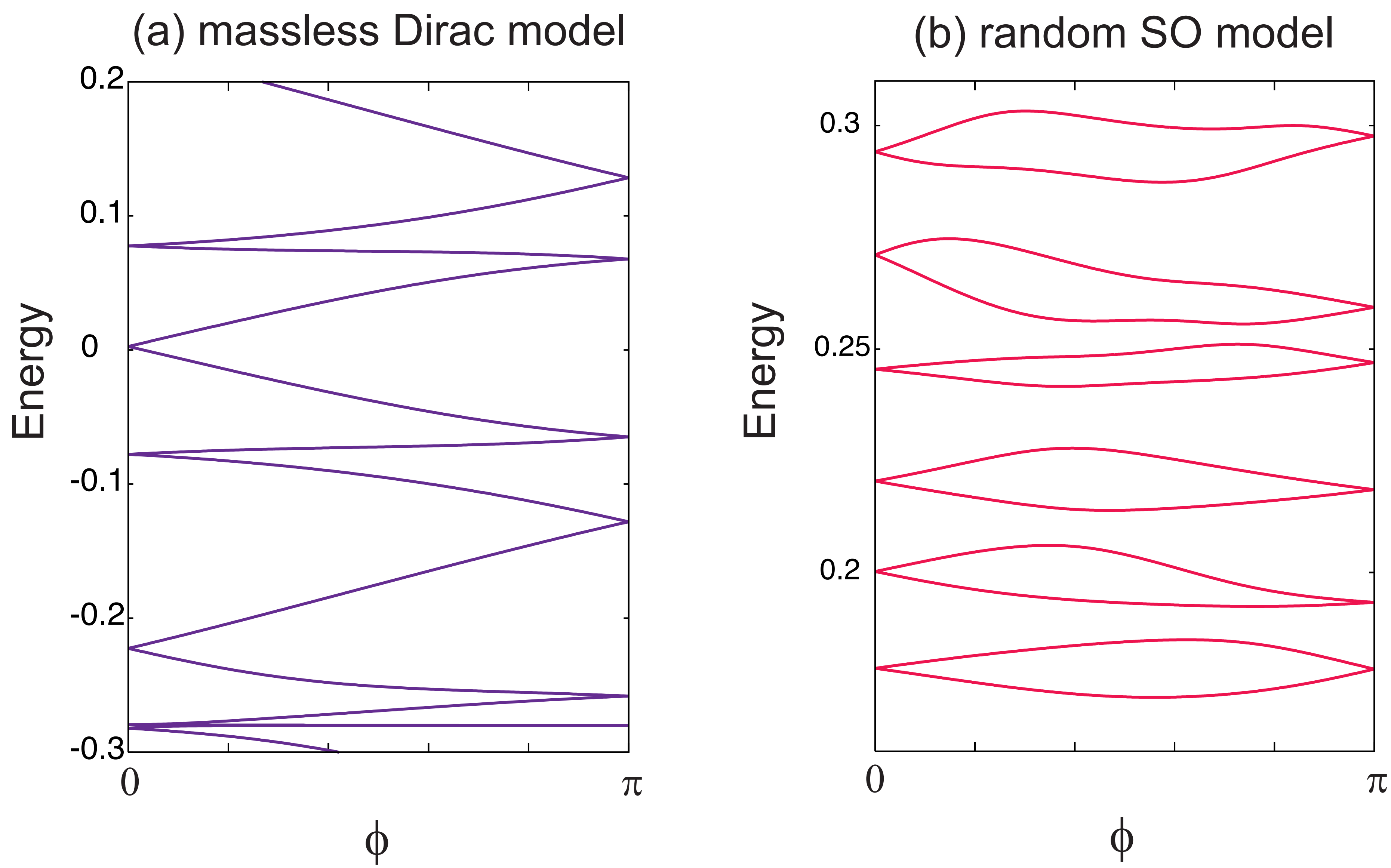
$$H = v\mathbf{p}\cdot\sigma + V(\mathbf{r})$$

$$\langle V(\mathbf{r})V(\mathbf{r}')\rangle = K_0 K(\mathbf{r}-\mathbf{r}')$$



J. H. B,, J. Tworzydlo, P.W. Brouwer, C.W.J. Beenakker
PRL **99**, 106801, (2007)

Effect of topological term: band topology
-- Twisted boundary conditions --



Thouless conductance: $g \sim \left(\frac{\Delta E}{\delta E}\right)^{\alpha}$

K. Nomura, M. Koshino, S. Ryu, PRL **99**, 146806 (2007)

Single parameter scaling and non-linear sigma model

Effective low energy field theory of a strong TI:

S. Ryu, C. Mudry, H. Obuse, and A. Furusaki, Phys. Rev. Lett. **99**,116601 (2007).
P. M. Ostrovsky, I. V. Gornyi, and A. D. Mirlin, Phys. Rev. Lett. **98**, 256801 (2007).

$$S = S^{\mathrm{NL}\sigma\mathrm{M}}_{\mathrm{symp}} + S_{\mathrm{topological}}$$

With topological term:

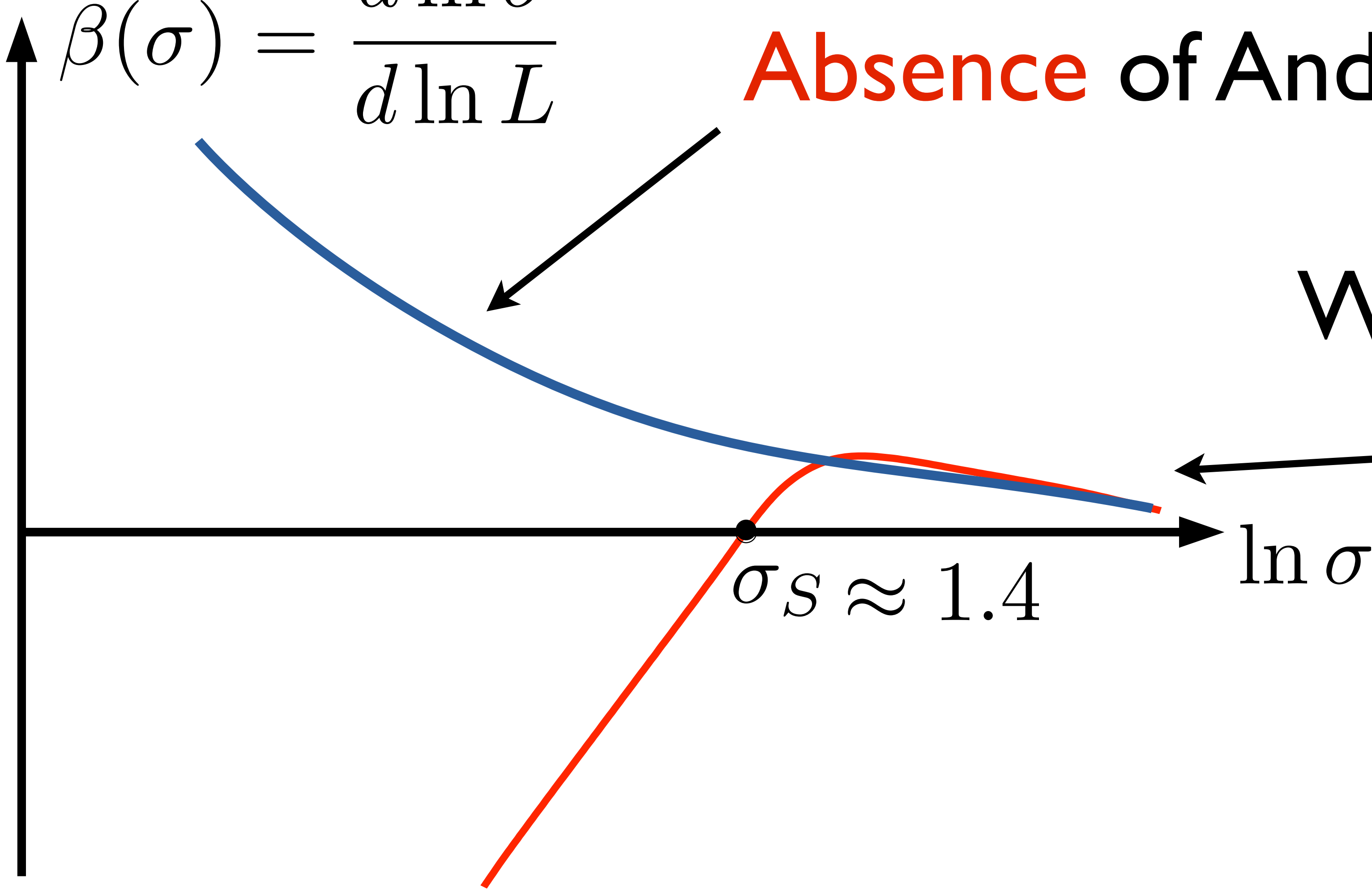
J. H. B,, J. Tworzydlo, P. W. Brouwer, C. W. J. Beenakker
PRL **99**, 106801, (2007)

$$\beta(\sigma) = \frac{d\ln\sigma}{d\ln L}$$

Absence of Anderson localization

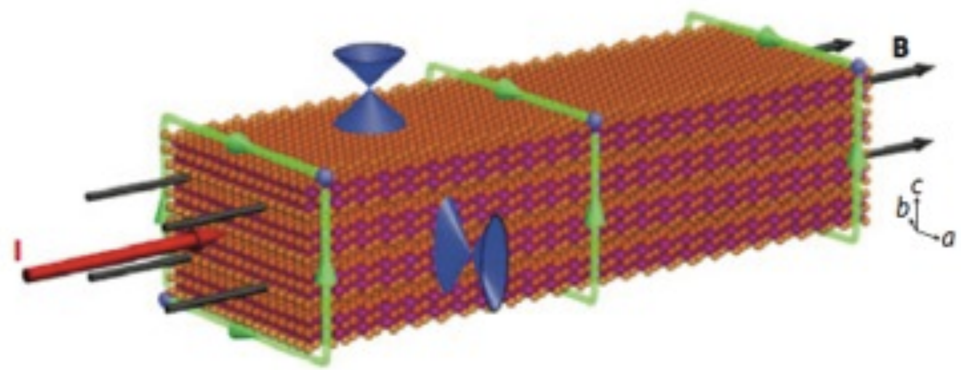
Weak anti-localization:

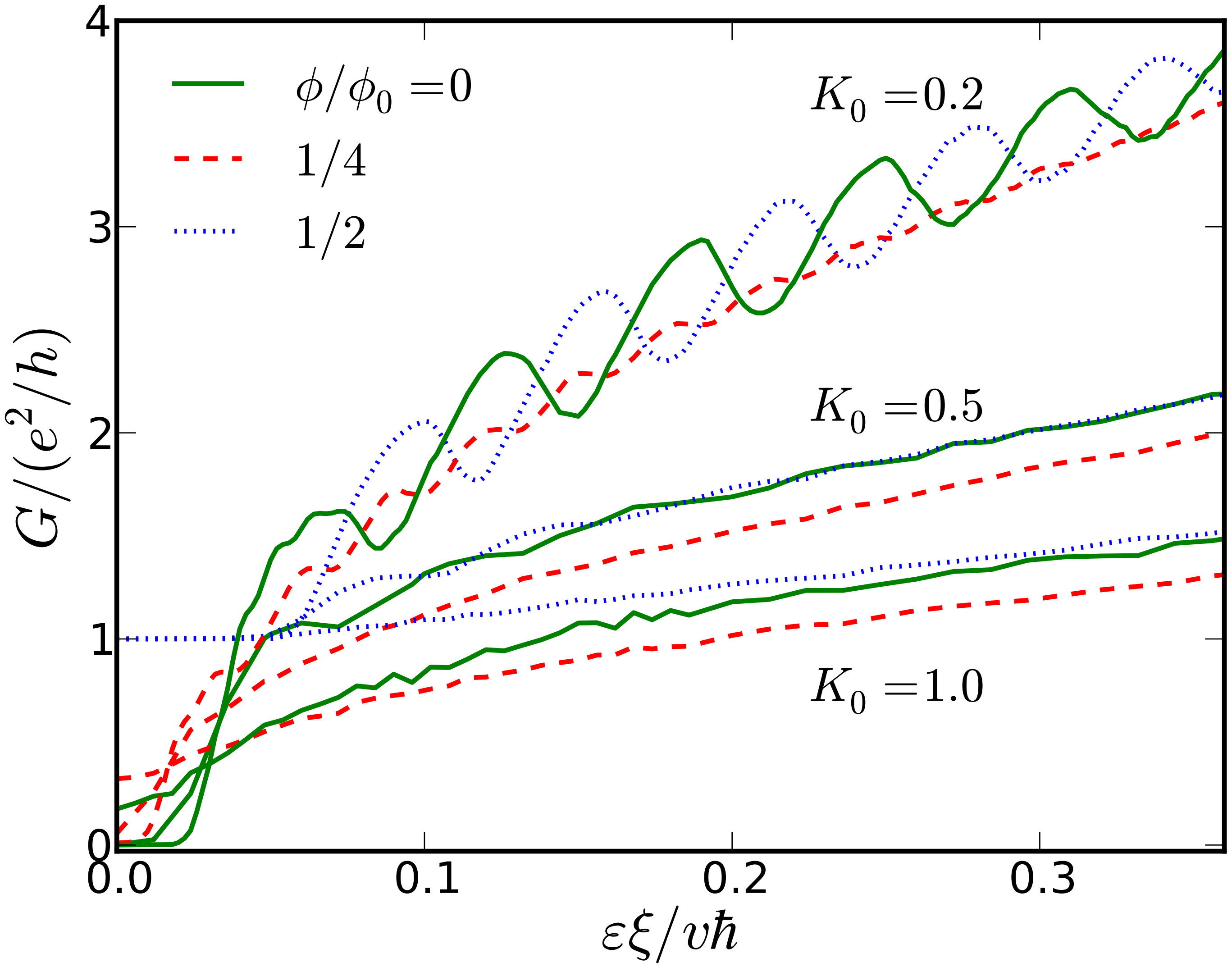
$$\beta(\sigma) \sim \frac{1}{\pi\sigma}$$

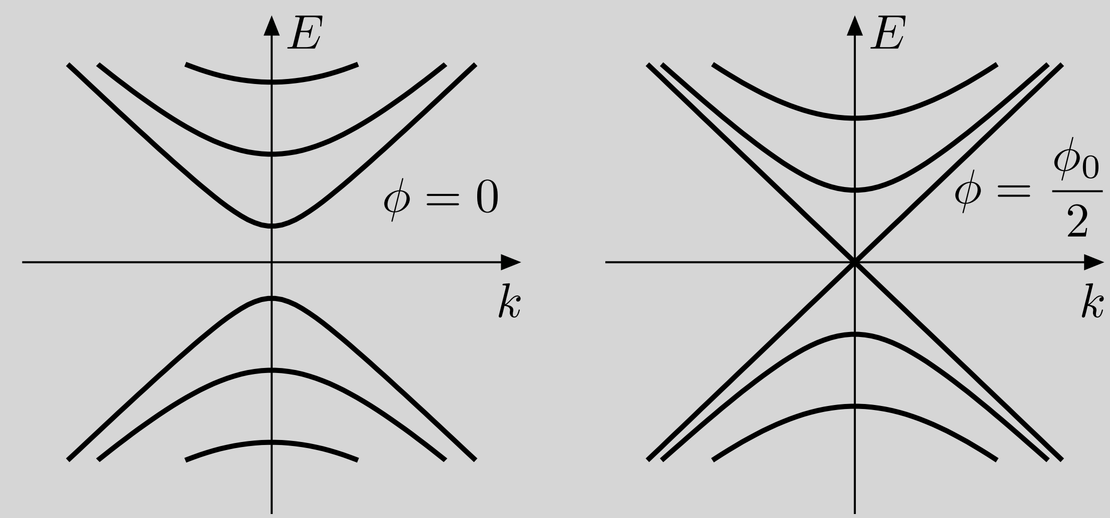


Aharonov-Bhom oscillations in nanowires

J.H.B., P.W. Brouwer, and J.E. Moore, PRL (2010)







Perfectly transmitted mode at nonzero flux due to Berry phase

Three regimes:

i) Dirac point. period h/e, min G
ii) Weak disorder, away from Dirac point, period h/e, min or max G
iii) Strong disorder, away from Dirac point, period h/2e (WAL)

nature materials

LETTERS

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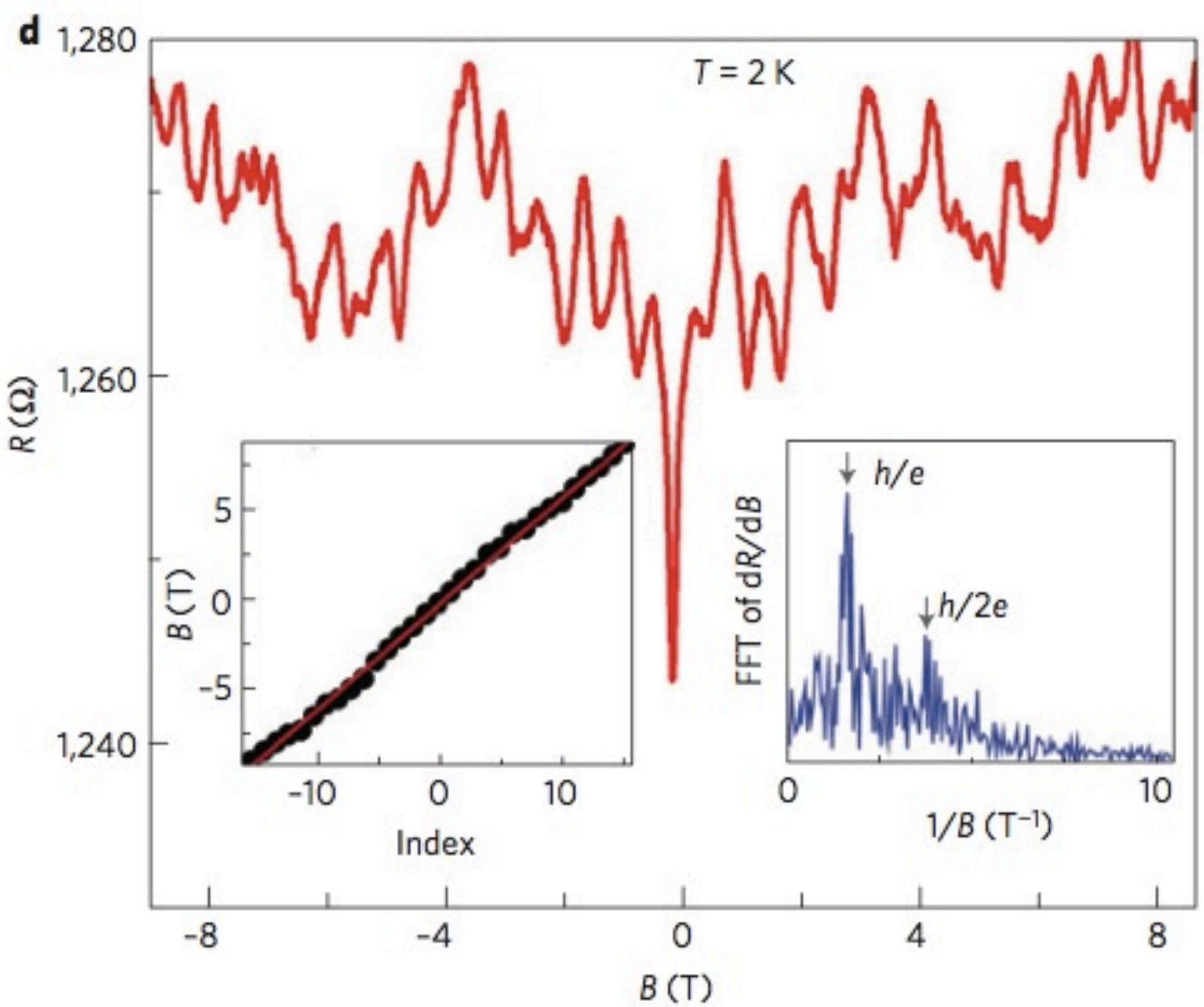
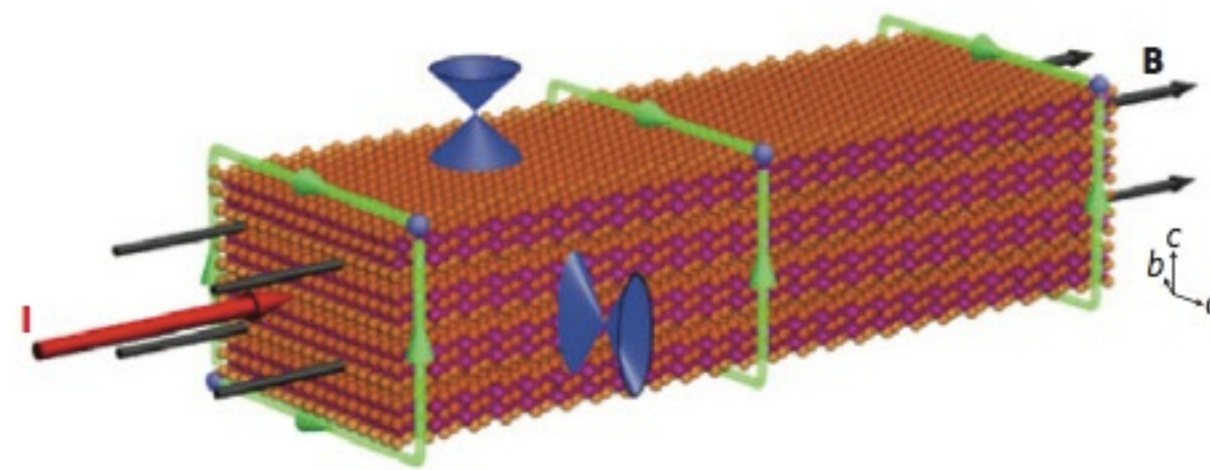
Aharonov–Bohm interference in topological insulator nanoribbons

Hailin Peng[1,2]*, Keji Lai[3,4]*, Desheng Kong[1], Stefan Meister[1], Yulin Chen[3,4,5], Xiao-Liang Qi[4,5], Shou-Cheng Zhang[4,5], Zhi-Xun Shen[3,4,5] and Yi Cui[1]†





- Oscillations of the magnetocondutance with period h/e (one flux quantum)
- Maximum conductance at zero flux
- No periodic oscillations in wide ribbons

Quantum transport in weak TI's

R. S. K. Mong, J.H.B, J.E. Moore, arXiv:1109.3201

$$H = \hbar v_D \tau^0 (\sigma^x k_x + \sigma^y k_y) + V(\mathbf{r})$$

$$V(\mathbf{r}) = \sum_{\alpha\beta} V_{\alpha\beta}(\mathbf{r})\, \tau^\alpha \otimes \sigma^\beta$$

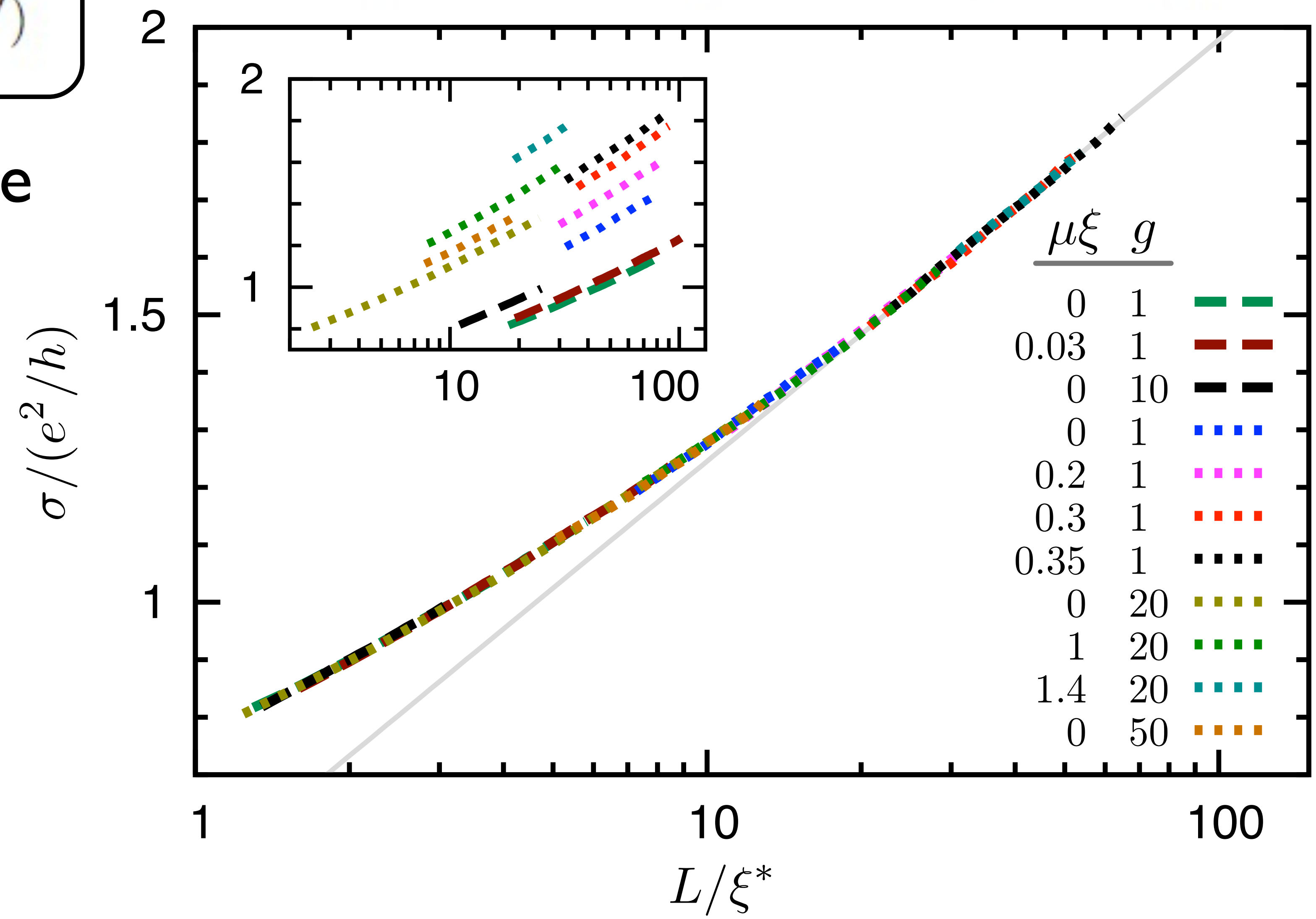
$$\langle \delta V_{\alpha\beta}(\mathbf{r})\, \delta V_{\alpha\beta}(\mathbf{r}') \rangle = g_{\alpha\beta} K(\mathbf{r} - \mathbf{r}')$$

Disorder structure	Disorder type	Notation
$V_{x0} \cdot \tau^x$	scalar potential (2×AII)	
$V_{yx} \cdot \tau^y \sigma^x$	gauge potential (2×AIII)	
$V_{yy} \cdot \tau^y \sigma^y$	gauge potential (2×AIII)	
$V_{yz} \cdot \tau^y \sigma^z$	mass (2×D)	$m = \langle V_{yz} \rangle$
$V_{z0} \cdot \tau^z$	scalar potential (2×AII)	
$V_{00} \cdot \mathbb{1}$	scalar potential (2×AII)	$\mu = -\langle V_{00} \rangle$

No topological term in the NLσM!

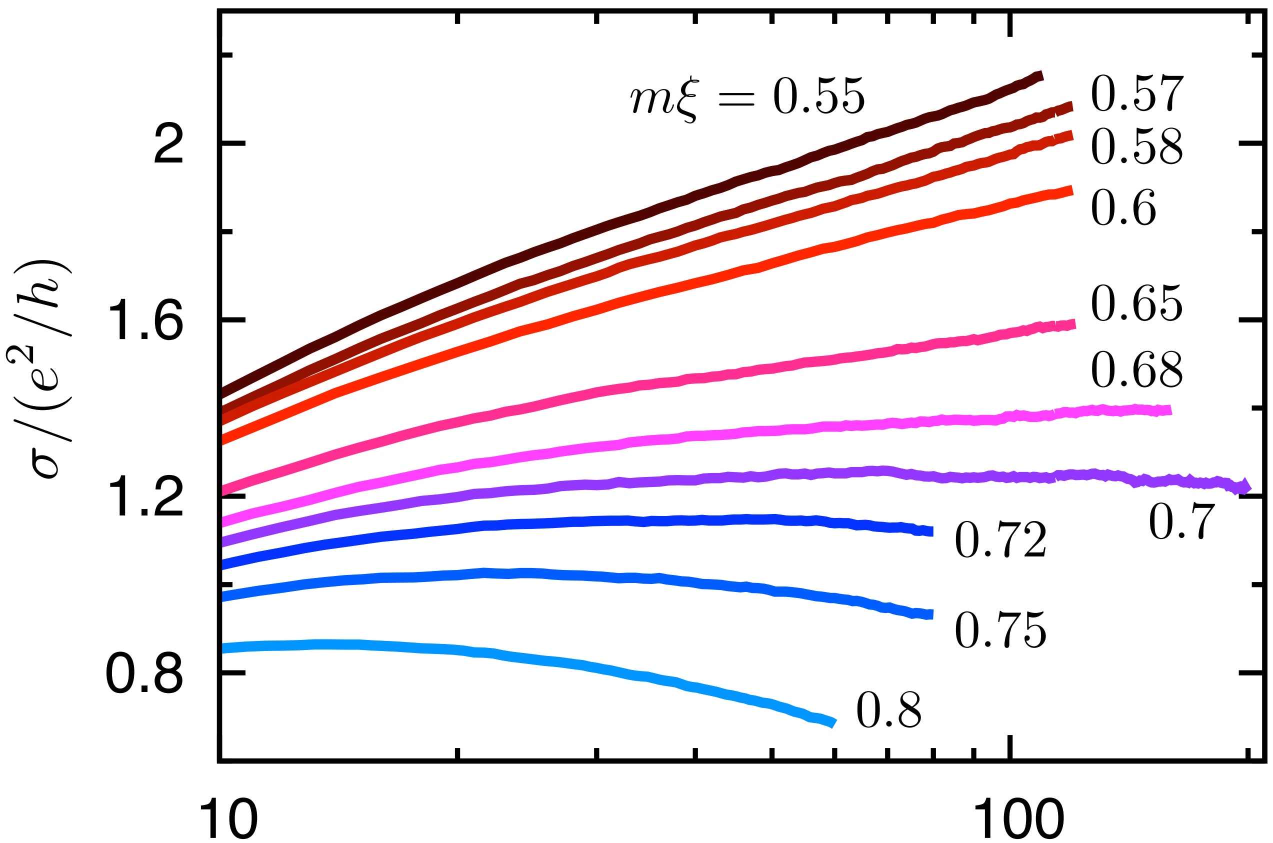
but

for $m = 0$
Single parameter scaling.
Flow to symplectic metal.

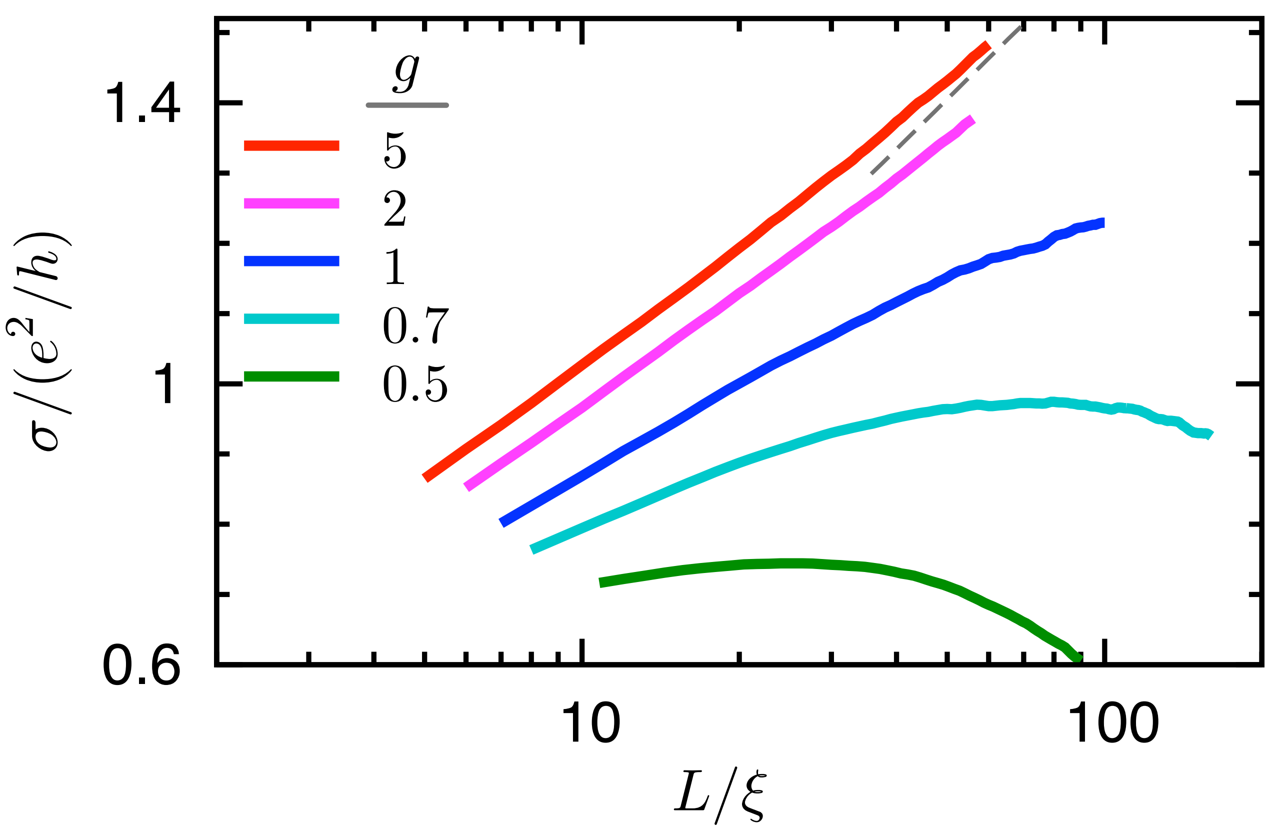


See also: Ringel, Kraus and Stern, arXiv:1105.4351

Effect of nonzero mass m

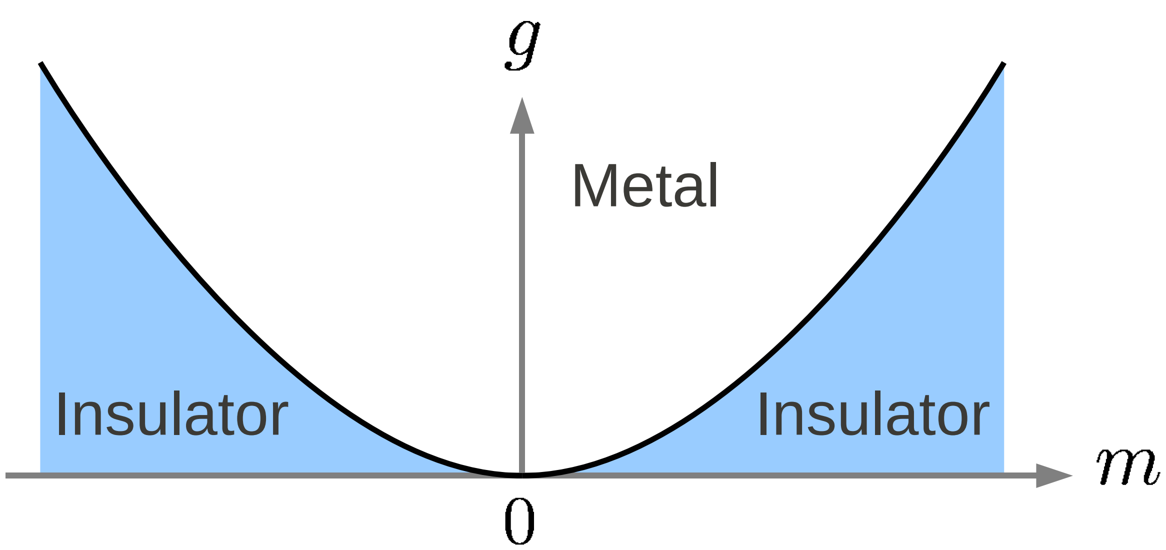


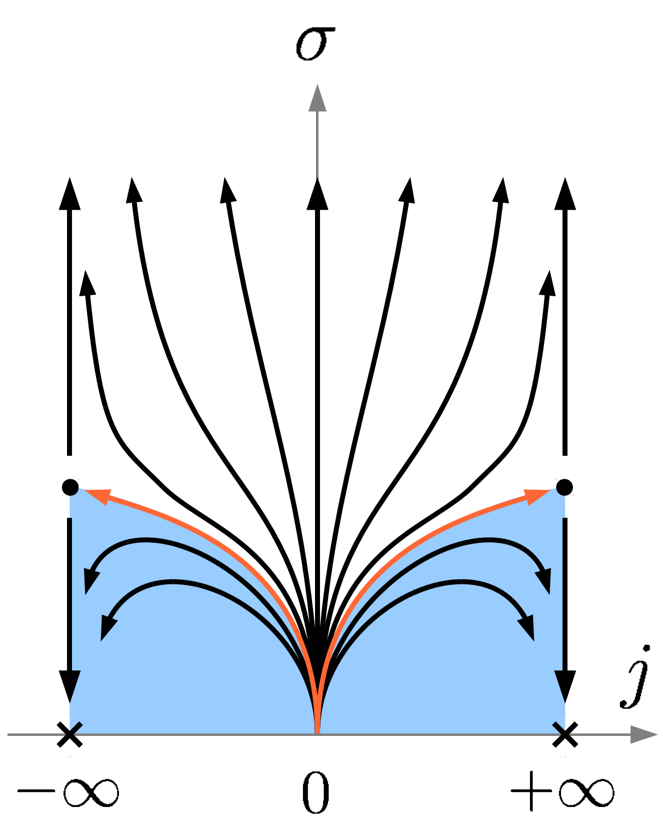
Increasing mass drives the system insulating

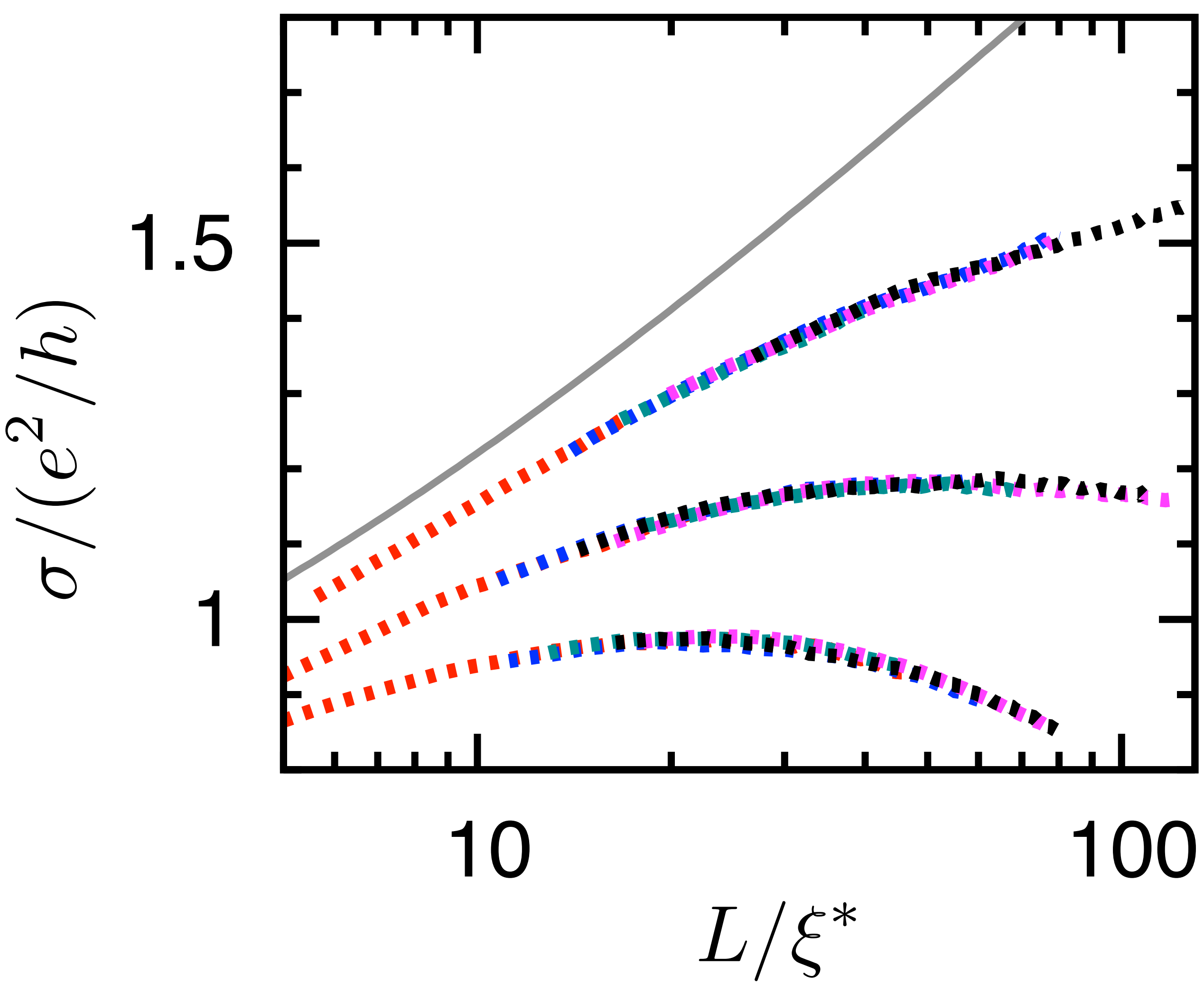


Increasing disorder drives the system metallic

Phase diagram and possible two parameter scaling



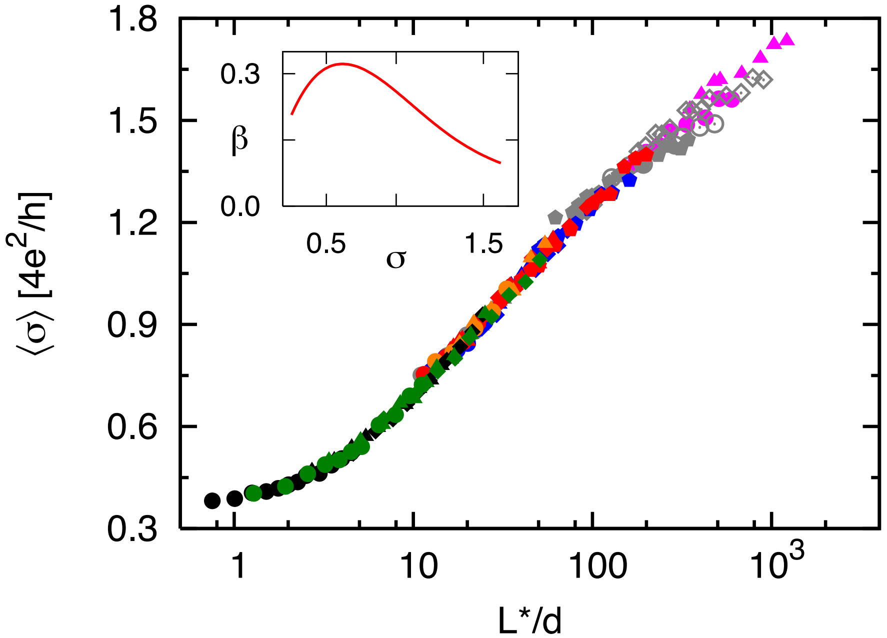




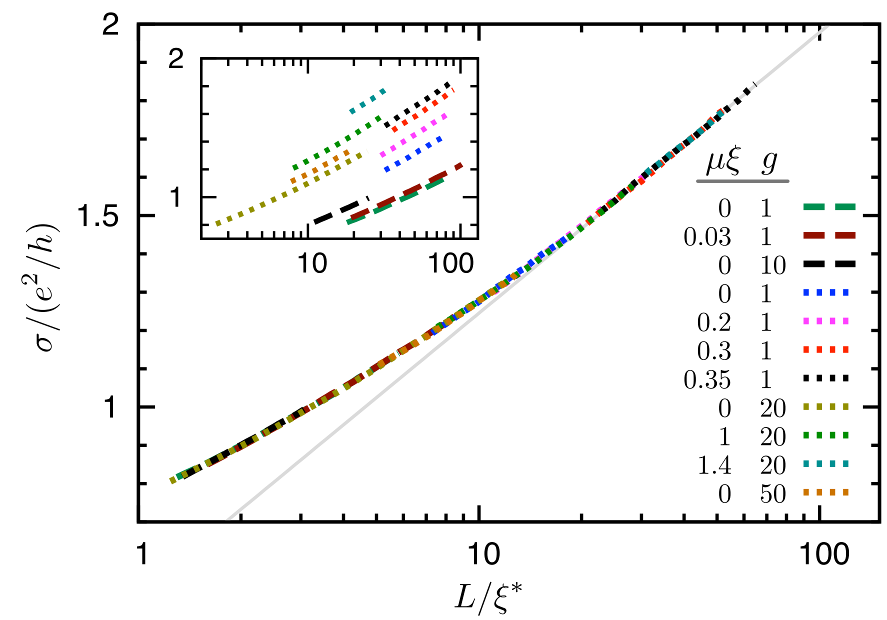
See also: Obuse, Furusaki, Ryu and Mudri PRB, 2007
Essin and Moore, RRB 2007, Shindou and Murakami PRB 2009

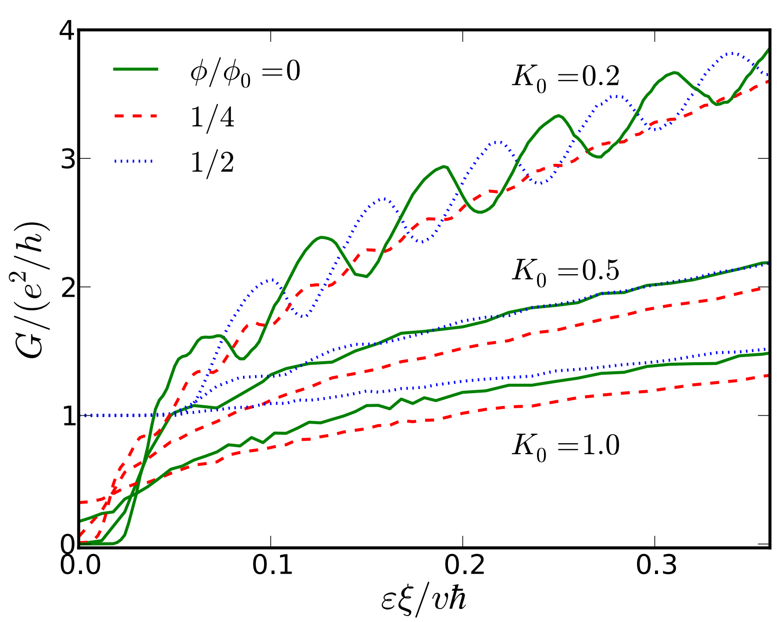
Conclusions

Disorder always drives the strong insulator surface into the symplectic metal phase, characterized by weak anti-localization



In weak topological insulators, similar behavior is observed in the absence of mass. Including mass gives arise to two parameter scaling





WAL also obtained in wires, expect close to the Dirac point (dominated by perfectly transmitted mode) or at very weak disorder