# Dissipationless Phonon Hall Viscosity

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M. Barkeshli, Suk Bum Chung, Xiaoliang Qi, arXiv: 1109.5648

# Outline

- Motivation
- Hall viscosity and gravitational response
- Definition of phonon Hall viscosity
- Examples
- Physical consequences and numerical estimates

# How do we characterize gapped states?

• Theorists have many answers…

Entanglement, tensor category theory, Berry phases, etc.

• Experimentally: response to external fields

e.g. quantized Hall response: $$\mathcal{L} = \frac{\sigma_{xy}}{2}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda$$

Top. Magneto-electric Effect: for 3D TI $$\delta\mathcal{L} = \frac{\theta}{2\pi}\frac{\alpha}{2\pi}E\cdot B$$

Qi, Hughes, Zhang
Essin, Moore

But this is not enough to fully characterize different gapped states

# How do we characterize gapped states?

For example, there are infinitely many different fractional quantum Hall states with the same Hall conductance.

With rotation symmetry, there are infinite number of different **integer** quantum Hall states with same Hall conductance
(eg which Landau level is fully filled)

Some systems (superconductors) may not even have a conserved charge

This motivates us to look for new response properties

# Hall viscosity

Introduced for gapped quantum liquids by Avron et al, 1995

The stress tensor of a liquid is

$$T_{ij} = -p_{ij} - \eta_{ijkl} v_{kl} \qquad v_{kl} = \frac{1}{2}(\partial_k v_l + \partial_l v_k)$$

Anti-symmetric part of viscosity tensor = “Hall” viscosity

$$\eta^A_{ijkl} = \frac{1}{2}(\eta_{ijkl} - \eta_{klij})$$

c.f. Hall resistivity (dissipationless, T-breaking)

For 2D isotropic system, characterized by one number:

$$\eta^H \equiv \eta^A_{xxxy} = \eta^A_{xyyy}$$

$$T_{xy} = -\eta^H v_{xx}$$

# Hall viscosity

Determined by response to deformations of space: gravitational response

Avron et al, 1995

$$T_{ij} = \left\langle \frac{\partial H}{\partial g_{ij}} \right\rangle = \frac{\partial E}{\partial g_{ij}} + \eta_{ijkl}\dot{g}_{kl}$$

This gravitational response has been studied in several different contexts:

Avron, Seiler, Zograaf (1995),
Read, Rezayi (2009, 2010)
Haldane (2009)
DT Son et al, (2010)
Milovanovic (2010)
Hughes, Leigh, Fradkin (2011)
Tokatly, Vignale (2006, 2007, 2009),
Volovik (1984)

For gapped isotropic system, Hall viscosity is topological:

Read (2009),
Read, Rezayi (2010)

$$\eta^H = \frac{1}{2}\hbar n s$$

n = electron density
s = angular momentum per electron

s is topological quantum number. Defines response to curvature (Wen-Zee, 1992)

# Gravitational response

More generally, gravitational response of gapped states is very interesting.

In 2+1D, there can be gravitational Chern-Simons terms:

$$\frac{1}{4\pi}\frac{c}{24}\int d^3r\,\epsilon_{\mu\nu\lambda}\mathrm{tr}\left(\omega_\mu\partial_\nu\omega_\lambda+\frac{2}{3}\omega_\mu\omega_\nu\omega_\nu\right)$$

$\omega_\mu^{ab}$ spin connection

Non-Lorentz invariant terms:

$$\mathcal{L}=\frac{k}{4\pi}a\partial a+\frac{1}{2\pi}A\partial a+\frac{1}{2\pi}s\omega^{12}\partial a$$

Wen-Zee 1992

In 3+1D TI,

$$\int d^4x\sqrt{g}\epsilon^{cdef}R^a{}_{bcd}R^b{}_{aef}$$

Wang, Qi, Zhang 2010
Ryu, Moore, Ludwig 2010

# Problems:

In general, condensed matter systems are defined on a lattice:

no continuous translation symmetry,
no well-defined momentum current for the electrons.

**Cannot define gravitational response in general.**

Even with continuum theory, computing gravitational response may require UV regularization (Hughes, Leigh, Fradkin (2011) on Dirac model)

(Hall viscosity depends on a length scale, and gapped systems have two natural length scales, set by the gap and by the electron density)

Even when it is defined, it may be useful as a numerical diagnostic, but…

Read-Rezayi (2010)

**…we do not know how to directly measure it.**

# Phonon Hall Viscosity

Acoustic phonon effective action (insulator):

$$S_{eff} = \frac{1}{2}\int d^d x dt (\rho \partial_t u_j \partial_t u_j - \lambda_{ijkl} \partial_i u_j \partial_k u_l)$$

If time-reversal symmetry is broken, can have a correction:

$$\delta S_H = \frac{1}{2}\int d^d x dt \eta_{ijkl} \partial_i u_j \partial_k \dot{u}_l$$

Distinct effect on acoustic phonon dynamics:

- Goes like $O(k^3)$. In presence of inversion symmetry, this is the only extra term at this order.
- Anharmonic (phonon interaction) effects go like $O(k^4)$
- Disorder effects are not sensitive to T-breaking.

# Phonon Hall viscosity as adiabatic response

Assume acoustic phonon frequencies << electronic energy gap

Adiabatic approximation: electrons in their instantaneous ground state with respect to the crystal.

Effect of lattice displacements is to adiabatically change effective parameters in electron tight-binding model. $H_{t.b.}[\{\boldsymbol{u}_i\}]$

$$\boldsymbol{u}_{\boldsymbol{q}} = \frac{1}{\sqrt{N_{site}}} \sum_{\boldsymbol{n}} \boldsymbol{u}_{\boldsymbol{n}} e^{i\boldsymbol{q}\cdot\boldsymbol{n}}$$

$$\eta_{ab}(\boldsymbol{q},\omega) = \frac{1}{\omega}\frac{1}{L^d}\int dt e^{i\omega t}\left\langle\left[\frac{\partial H_{t.b.}}{\partial u_{\boldsymbol{q},a}}(t), \frac{\partial H_{t.b.}}{\partial u_{-\boldsymbol{q},b}}(0)\right]\right\rangle$$

$$\delta S = \frac{1}{2}\int d^{d+1}x d^{d+1}x' \eta_{ab}(x-x') u_a(x)\dot{u}_b(x')$$

$$\eta_{ijkl} = \frac{1}{2}\lim_{\omega\to 0}\lim_{q\to 0}\frac{\partial}{\partial q_i}\frac{\partial}{\partial q_k}\eta_{jl}(\boldsymbol{q},\omega)$$

# Phonon Hall viscosity as Berry curvature

For spatially homogeneous deformation, $w_{ij} \equiv \partial_i u_j$ is constant

Regard this as a parameter in the Hamiltonian

$$\eta_{ijkl} = \frac{1}{2} \lim_{\omega \to 0} \frac{1}{\omega} \frac{1}{L^d} \int dt e^{i\omega t} \left\langle \left[ \frac{\partial H_{t.b.}}{\partial w_{ij}}(t), \frac{\partial H_{t.b.}}{\partial w_{kl}}(0) \right] \right\rangle + (i \leftrightarrow k)$$

Determined by adiabatic Berry curvature of ground state wave function

$$i\eta_{ijkl} = \frac{1}{2} \left( \frac{\partial}{\partial w_{ij}} \langle \psi | \frac{\partial}{\partial w_{kl}} | \psi \rangle - \frac{\partial}{\partial w_{kl}} \langle \psi | \frac{\partial}{\partial w_{ij}} | \psi \rangle + (i \leftrightarrow k) \right)$$

**For finite frequency inhomogeneous deformations, continue to use DC response for freq. below energy gap.**

# Example: electrons hopping on square lattice in magnetic field

$$H = -\frac{1}{2}\sum_{\langle ij\rangle} t_{ij} e^{iA_{ij}} c_i^\dagger c_j - \frac{1}{2}\sum_{\langle\langle ij\rangle\rangle} \tilde{t}_{ij} e^{iA_{ij}} c_i^\dagger c_j + h.c.$$

For s-wave orbitals, hopping amplitudes only depend on distance between atoms

$$t_{i,i+\hat{\mathbf{x}}} \simeq t + t' u_{xx} \qquad \tilde{t}_{i,i+\hat{\mathbf{x}}\pm\hat{\mathbf{y}}} = \tilde{t} + \tilde{t}'\left(\tfrac{1}{2}(u_{xx} + u_{yy}) \pm u_{xy}\right)$$

$$\epsilon_k \simeq \frac{1}{2m^*} k_i k_j g_{ij} - (t' + \tilde{t}')(u_{xx} + u_{yy}) \qquad u_{ij} = \frac{1}{2}(\partial_i u_j + \partial_j u_i)$$

$$g_{ij} = \delta_{ij} + \delta g_{ij} \qquad \delta g = 2m^* \frac{\tilde{t}'}{2} \begin{pmatrix} (1 + \frac{t'}{\tilde{t}'})u_{xx} + u_{yy} & 2u_{xy} \\ 2u_{xy} & (1 + \frac{t'}{\tilde{t}'})u_{yy} + u_{xx} \end{pmatrix}$$

$$\frac{1}{2m^*} \equiv (t/2 + \tilde{t})$$

$$H = -\frac{1}{2m^*} g_{ij} D_i D_j - (t' + \tilde{t}')(u_{xx} + u_{yy})$$

$$\eta^H = (t/2 + \tilde{t})^{-2} \frac{\tilde{t}' t'}{4} \eta^H_{gr} \quad \sim \quad \eta^H_{gr} = N_L \hbar n / 4$$

# Physical consequences of phonon Hall viscosity

- For simplicity, consider 2D system with square lattice symmetry. Then,

$$\delta S_H = \int d^2x dt [\eta_{xxxy}(u_{xx} - u_{yy})\dot{u}_{xy} + \eta_{xxxy}(u_{xx} + u_{yy})\dot{m}_{xy}]$$

Characteristic frequency $\omega_v = \rho c^2 / \eta^H$

Dispersion shifts by $(\omega/\omega_v)^2$

Transverse and longitudinal modes get mixed: $x \equiv \omega/\omega_v$

$$e_+ \propto \begin{pmatrix} 1 \\ -ix \end{pmatrix} + \mathcal{O}(x^2) \qquad e_- \propto \begin{pmatrix} -ix \\ 1 \end{pmatrix} + \mathcal{O}(x^2)$$

Elliptical eigenmodes, phase shift of $\frac{\pi}{2}\mathrm{sgn}(\eta^H)$

## Numerical estimates

$$\omega_v = \rho c^2/\eta^H$$

$$c^2 \sim 10^{10} cm^2/s^2$$

$$\rho \sim Am_p/a^2 \sim 10^{-8} g/cm^2 \qquad \eta^H \sim \hbar n_e \qquad n_e \sim 10^{15} cm^{-2}$$

$$\omega_v \sim 10^{14} s^{-1}$$

For 1 GHz measurement, $\omega/\omega_v \sim 10^{-5}$

For 100 GHz measurement, $\omega/\omega_v \sim 10^{-3}$

Note: need $\omega \ll E_g$ (eg. 10 K ~ 2 THz) , $\omega \ll \omega_D$ ~ 10 THz

Measurable? Small strain: $|\partial u| \ll 1 \longrightarrow \omega u \ll c$

Amplitude of other mode mixed in is $(\omega/\omega_v)u$

Practically, want this to be larger than size of wave function of atom:

$$|\omega u| \gg \omega_v \times 0.1 \mathring{A}$$

$$\omega_v \ll 5 \times 10^{14} s^{-1}$$

Time-dependent x-ray diffraction has spatial resolution ~ 1 Angstrom

$\omega_v \sim 10^{14} s^{-1}$ **is borderline measurable**

# Experimental probes and possible systems

- Common probe: resonant or pulsed echo ultrasound.
  - Frequencies limited to ~ 1GHz, spatial resolution may not be good enough.

- Time dependent X-ray diffraction is used to directly image phonons, and can have spatial resolution ~ 1 Angstrom

- Need gapped systems that spontaneously break time reversal symmetry:

  e.g. quantum anomalous Hall states,

  chiral superconductors,

  ferromagnetic insulators.

  QAH states are usually 2D (3D crystals are preferable),

  Chiral superconductors usually have small gaps, and imaging phonons through x-rays in a superconductor may not be possible.

Best candidate right now: ferromagnetic insulators.

# Conclusions

- Electronic insulators can give a unique correction to phonon dynamics: phonon Hall viscosity.

- Determined by adiabatic Berry curvature of electron wave function -- gives a way to directly measure Berry curvature of electron wave function.

- Always a well-defined quantity, may be practically measurable in some systems.

- Gives a way to measure “gravitational response” in condensed matter systems