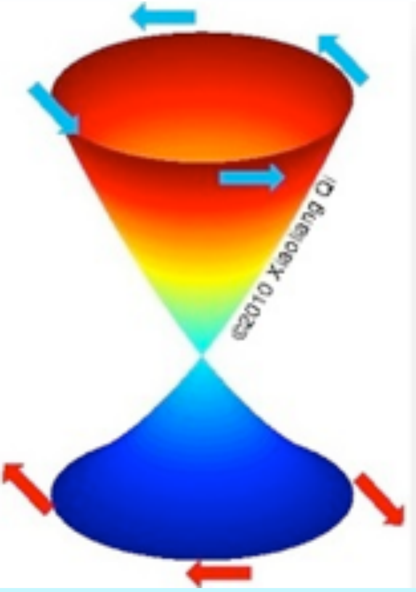
Topomat11:Topological Insulators & Superconductors, Nov.3, 2011



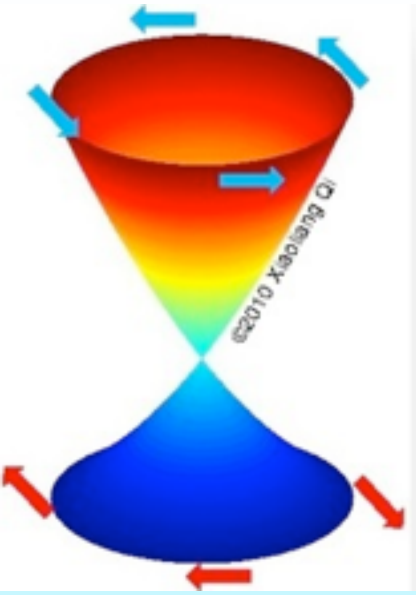
Quantized Berry Phases for Characterization of Short-Range Entangled States in d-Dimensions

Institute of Physics & TIMS, Univ. Tsukuba
Kavli Institute for Theoretical Physics, UCSB
Yasuhiro Hatsugai

YH & I. Maruyama, EPL 95, 20003 (2011), arXiv:1009.3792
I. Maruyama, S. Tanaya, M.Arikawa & YH. , arXiv:1103.1226

The Kavli Institute for Theoretical Physics
University of California, Santa Barbara

University of Tsukuba



Collaborators

I. Maruyama, Osaka Univ.

H. Katsura, Gakushuin Univ. (Univ. Tokyo)

T. Hirano, (Univ.Tokyo)

M. Arikawa, (Univ. of Tsukuba)

S. Tanaya, Univ. of Tsukuba

The Kavli Institute for Theoretical Physics
University of California, Santa Barbara

University of Tsukuba

Plan

- ☆ *Short range entanglement, symmetry & quantization*
  - ☆ *Adiabatic principle with symmetry*
  - ☆ *Gauge freedom for entangled state*
- ☆ *Two types of topological invariants for "order parameters"*
  - ☆ *Chern numbers in even dimensions*
  - ☆ *Quantized Berry phases in odd dimensions*
- ☆ *Examples in 1D, 2D, 3D and ...*
  - ☆ *Integer spin chains with dimerization*
  - ☆ *Random hopping models*
  - ☆ *Orthogonal dimers in 2D*
  - ☆ *Generalized dimers in Kagome, Pyrochlore ...*
    : d-Dim. fermions with frustration

- Short range entanglement, symmetry & quantization
  - Adiabatic principle with symmetry
  - Gauge freedom for entangled state
- Two types of topological invariants
  - Chern numbers in even dimensions
  - Quantized Berry phases in odd dimensions
- Examples in 1D, 2D, 3D and ...
  - Integer spin chains with dimerization
  - Random hopping models
  - Orthogonal dimers in 2D
  - Generalized dimers in Kagome, Pyrochlore ...
    : d-Dim. fermions with frustration

Adiabatic principle for gapped systems

- Gapped quantum (spin) liquids
  - No symmetry breaking
  - No low energy excitations (Nambu-Goldstone)

Adiabatic principle for gapped systems

- Gapped quantum (spin) liquids
  - No symmetry breaking
  - No low energy excitations (Nambu-Goldstone)

Topological order !?

Adiabatic principle for gapped systems

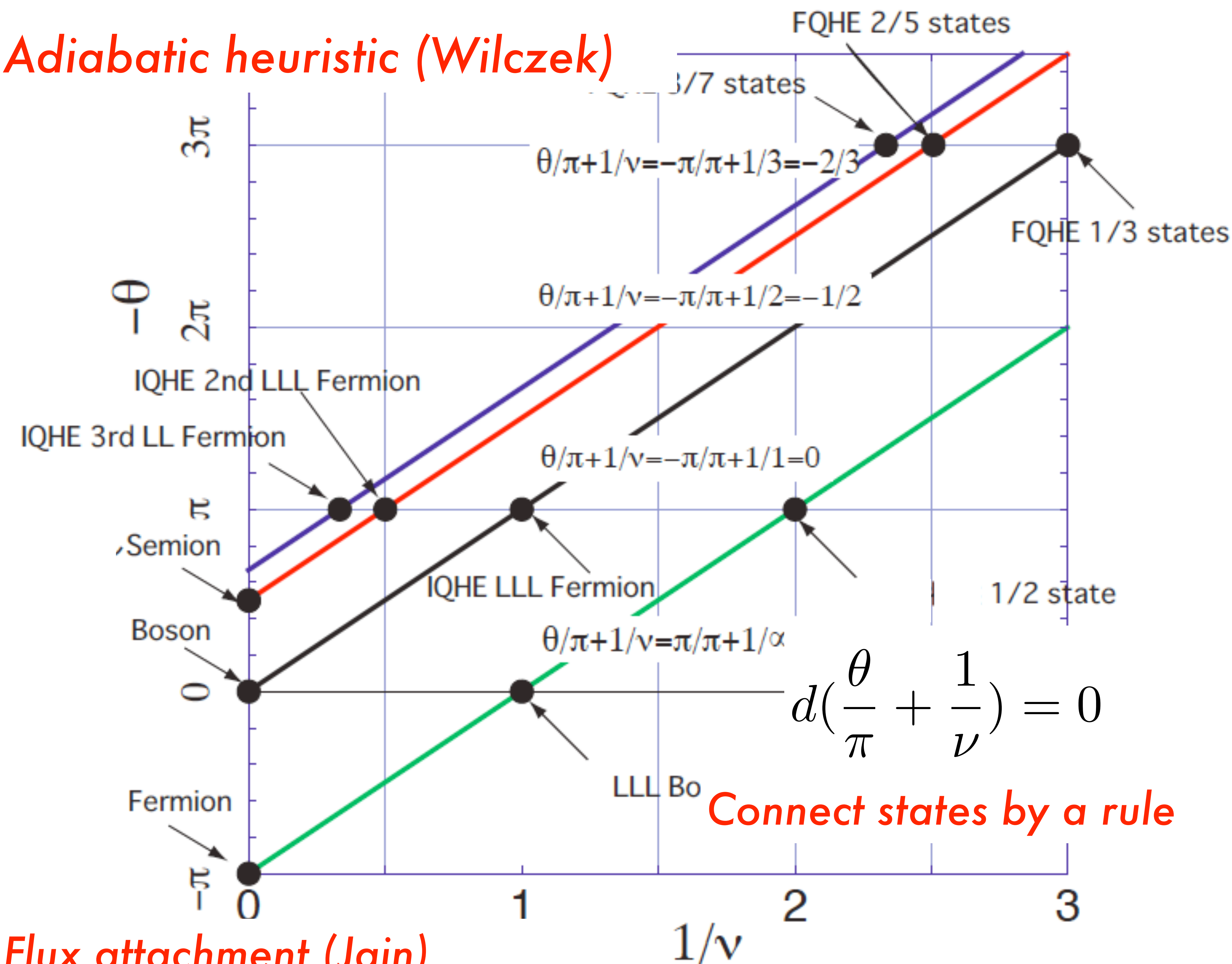
- Gapped quantum (spin) liquids
  - No symmetry breaking
  - No low energy excitations (Nambu-Goldstone)

Topological order !?

- Topological characterization for gapped system
  - Example: Adiabatic principle: a lesson from the QHE
    - flux attachment (Jain)
    - Adiabatic heuristic argument (Wilczek)

Adiabatic principle for gapped systems

Adiabatic heuristic (Wilczek)



$$d(\frac{\theta}{\pi}+\frac{1}{\nu})=0$$

Connect states by a rule

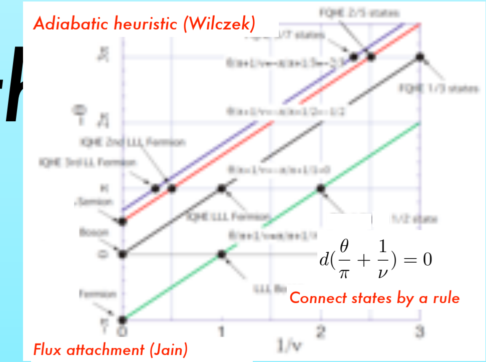
Flux attachment (Jain)

Adiabatic principle for gapped systems

- Gapped quantum (spin) liquids
  - No symmetry breaking
  - No low energy excitations (Nambu-Goldstone)

Topological order !?

- Topological characterization for gapped system
  - Example: Adiabatic principle: a lesson from th
    - flux attachment (Jain)
    - Adiabatic heuristic argument (Wilczek)



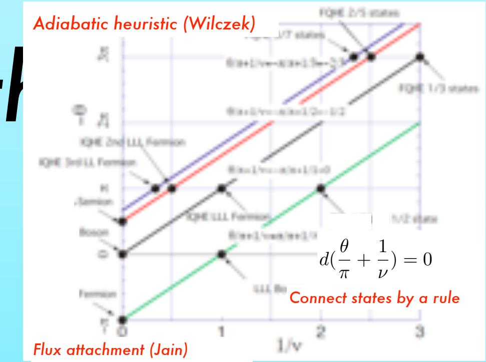
- Collect gapped phases and classify into several classes by adiabatic continuation

Adiabatic principle for gapped systems

- Gapped quantum (spin) liquids
  - No symmetry breaking
  - No low energy excitations (Nambu-Goldstone)

Topological order !?

- Topological characterization for gapped system
  - Example: Adiabatic principle: a lesson from th
    - flux attachment (Jain)
    - Adiabatic heuristic argument (Wilczek)



- Collect gapped phases and classify into several classes by adiabatic continuation

- Label of the Class : Adiabatic invariant (topological number)

Characterization of short range entangled states

Generic short range entangled states

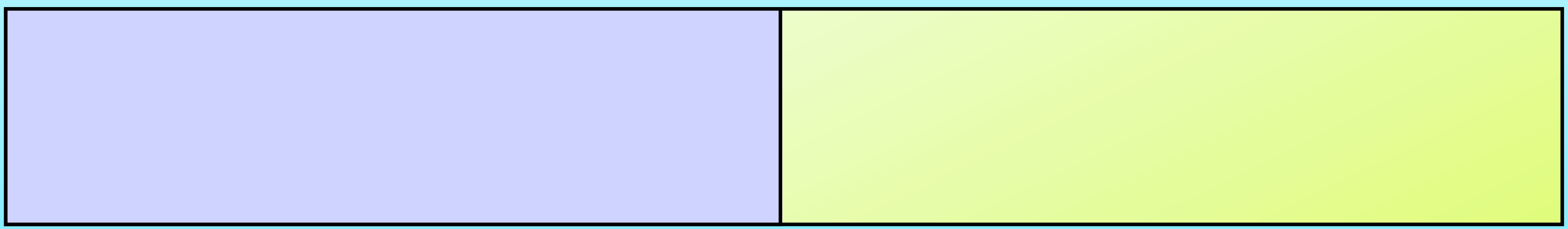
topologically single phase (too simple ?)

Characterization of short range entangled states

Generic short range entangled states

topologically single phase (too simple ?)

With some symmetry A



YH, '06

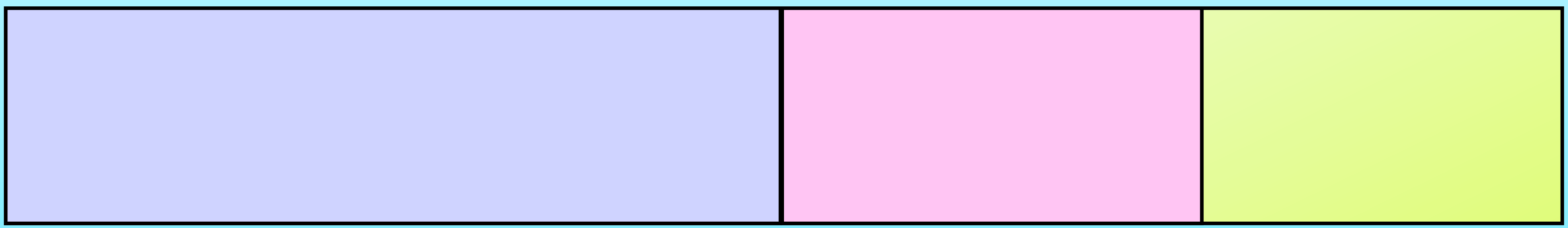
Chen-Gu-Wen, '10
Pollmann et al., '10

Characterization of short range entangled states

Generic short range entangled states

topologically single phase (too simple ?)

With some symmetry A, B



YH, '06

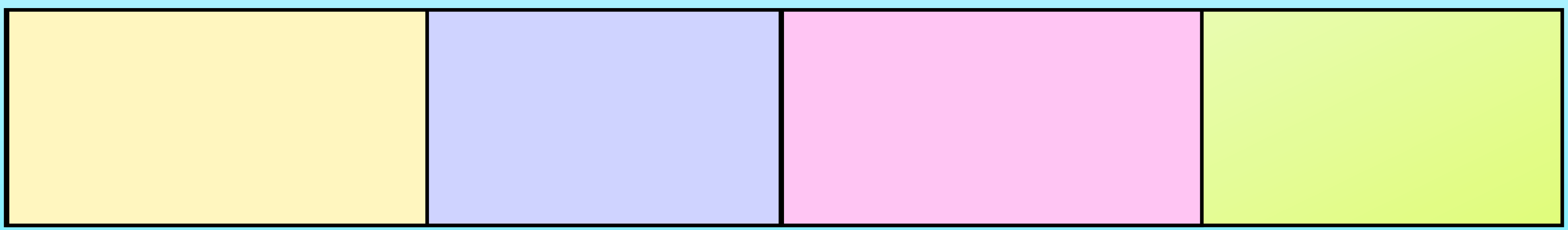
Chen-Gu-Wen, '10
Pollmann et al., '10

Characterization of short range entangled states

Generic short range entangled states

topologically single phase (too simple ?)

With some symmetry A, B, C



YH, '06

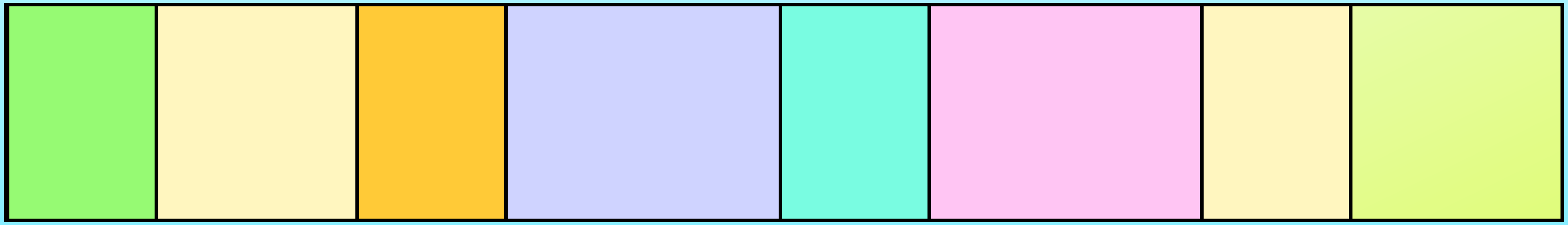
Chen-Gu-Wen, '10
Pollmann et al., '10

Characterization of short range entangled states

Generic short range entangled states

topologically single phase (too simple ?)

With some symmetry A, B, C



Multiple phases with SYMMETRIES

YH, '06

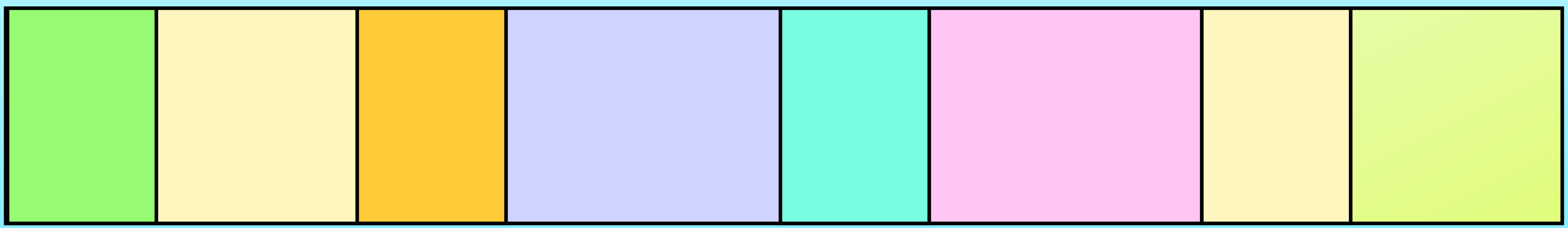
Chen-Gu-Wen, '10
Pollmann et al., '10

Characterization of short range entangled states

Generic short range entangled states

topologically single phase (too simple ?)

With some symmetry A, B, C



YH, '06 Chen-Gu-Wen, '10 Pollmann et al., '10

Multiple phases with SYMMETRIES

Time-reversal*

"Many body" Particle-hole* (Chiral symmetry)

Inversion

*Anti Unitary

$Z_Q \quad : \; 1 \to 2, 2 \to 3, \cdots, Q \to 1$

Symmetry in physics

Text book

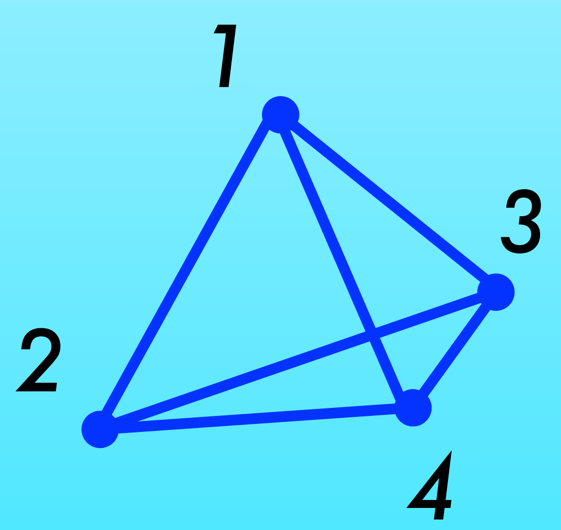
Labeling of quantum states

- ☆Conservation law $[H,G]=0$ $\quad t_{2g}\ e_g \cdots$

We are now using it as

Symmetry protection of adiabatic process

- ☆Chiral symmetry
- ☆Particle-Hole symmetry
- ☆Time-reversal symmetry
- ☆Inversion symmetry
- ☆Z_Q symmetry: S_Q reduced into Z_Q with gauge twists



S_4

Symmetry in physics

Text book

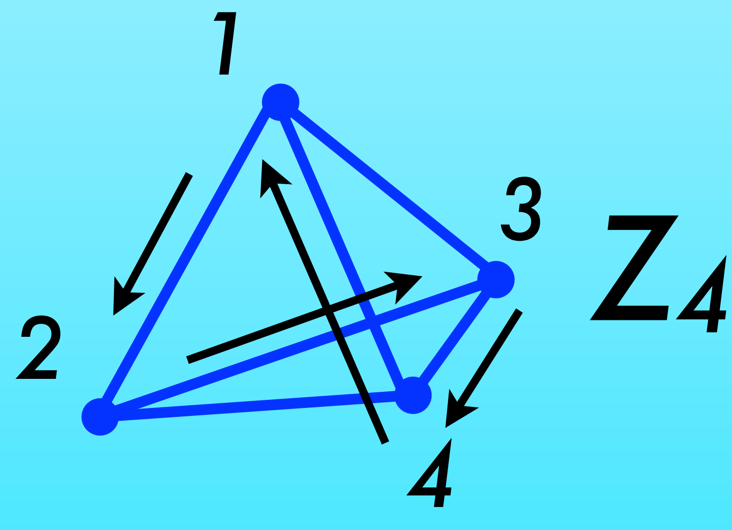
Labeling of quantum states

- ☆Conservation law $[H, G] = 0$ t_{2g} $e_g \cdots$

We are now using it as

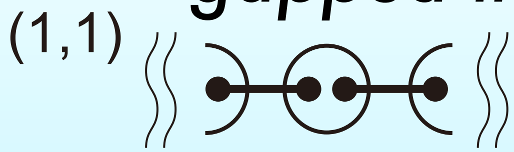
Symmetry protection of adiabatic process

- ☆Chiral symmetry
- ☆Particle-Hole symmetry
- ☆Time-reversal symmetry
- ☆Inversion symmetry
- ☆Z_Q symmetry: S_Q reduced into Z_Q with gauge twists



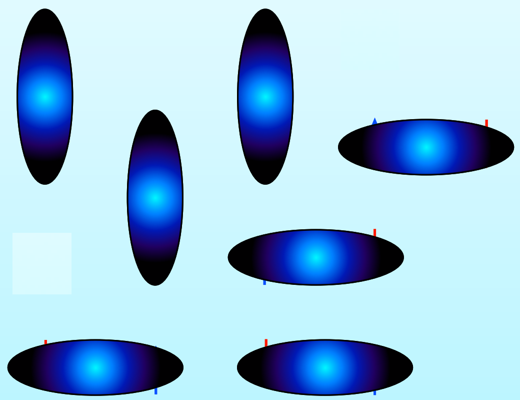
Short range entangled states

Ex.1) AKLT state

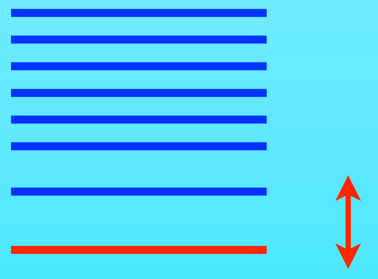


gapped integer spin chain

Ex.2) Collection of singlets



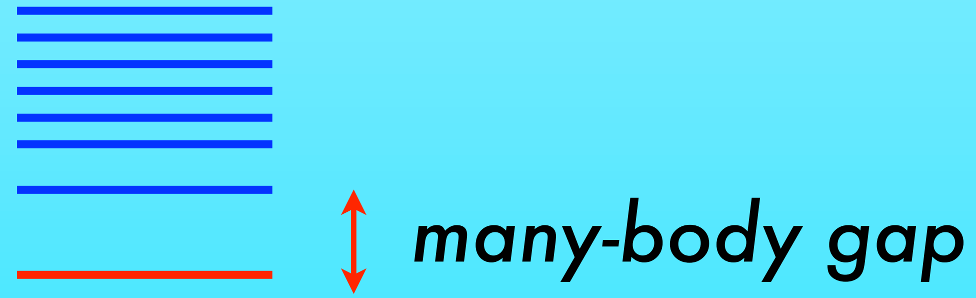
Something complicated but gapped



Short range entangled states

Adiabatic deformation !
gap remains open

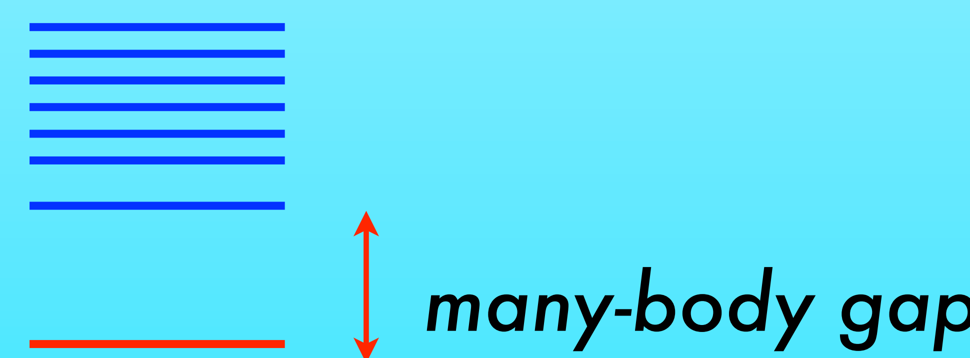
Something complicated
but gapped



Short range entangled states

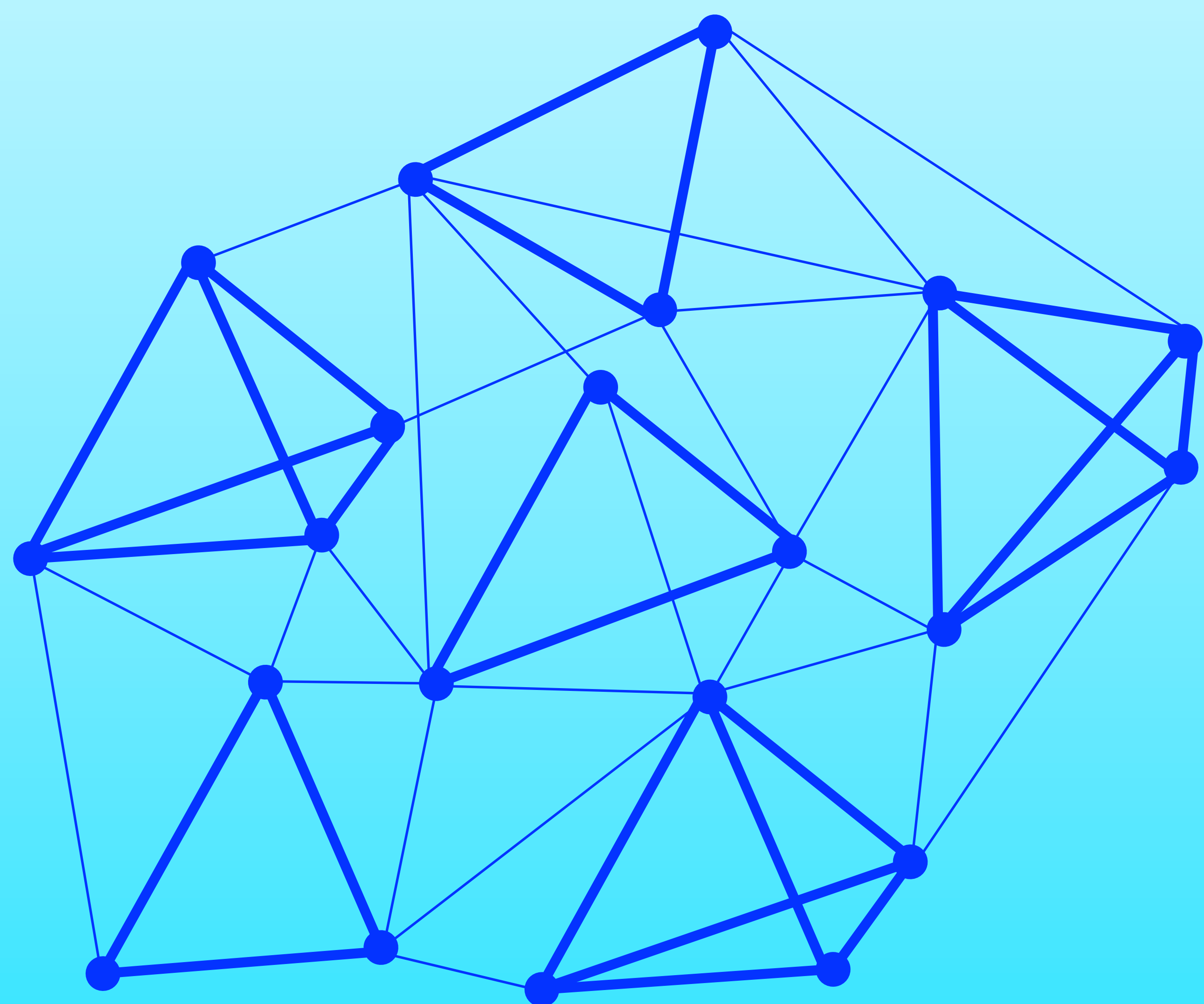
Adiabatic deformation !
gap remains open

Something complicated
but gapped

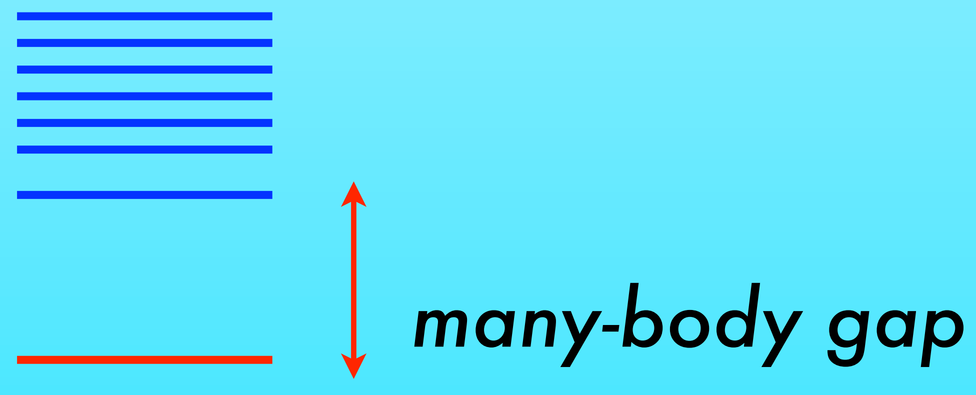


Short range entangled states

Adiabatic deformation !
gap remains open

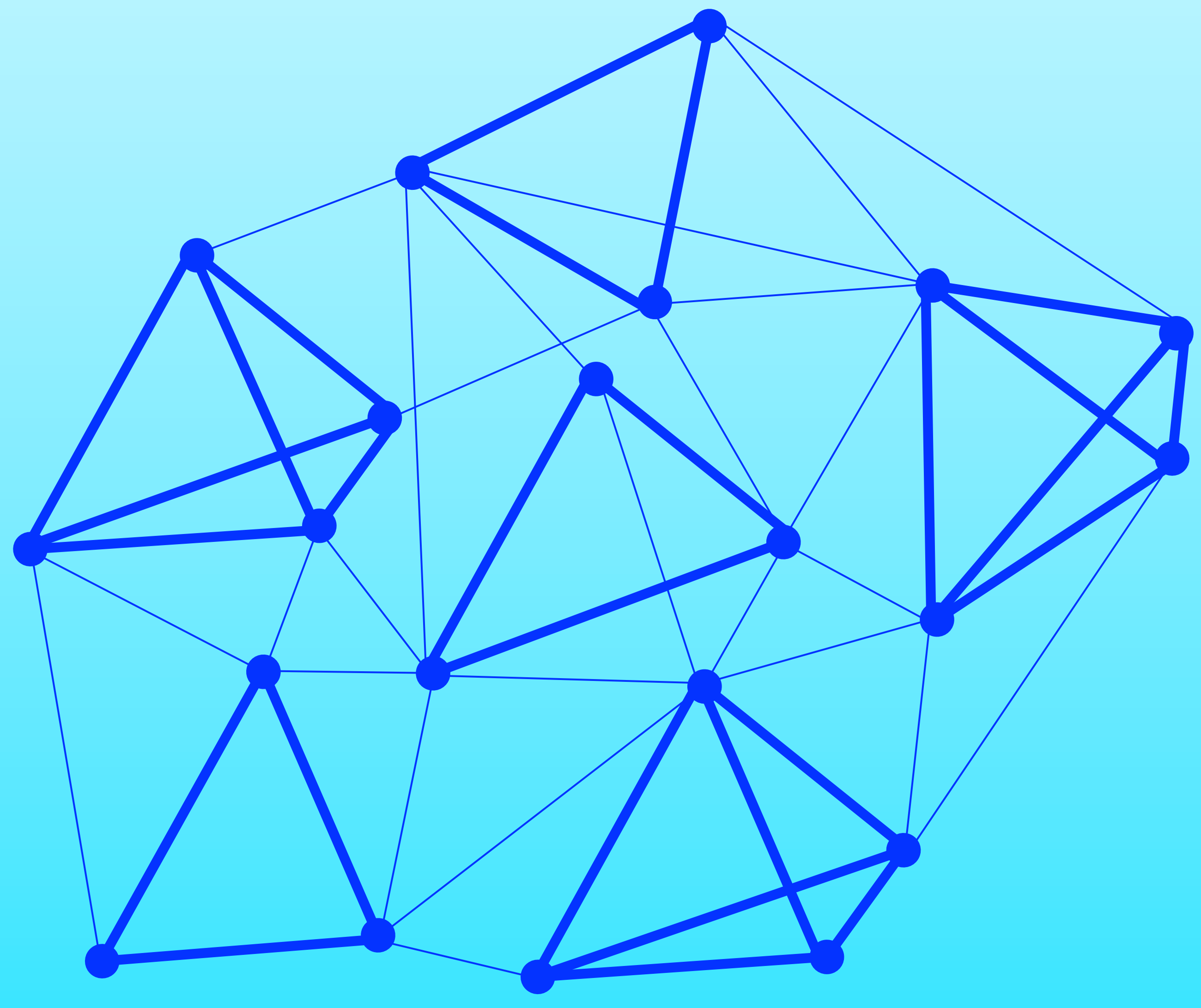


Something complicated
but gapped

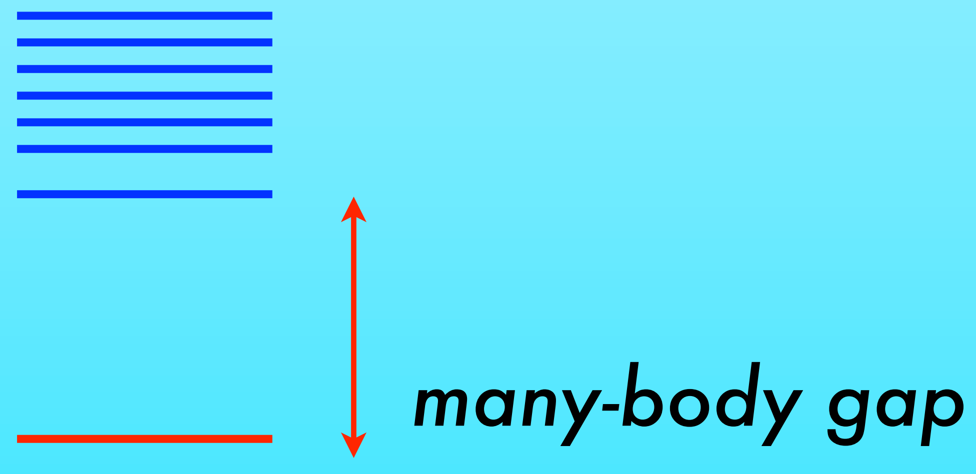


Short range entangled states

Adiabatic deformation !
gap remains open



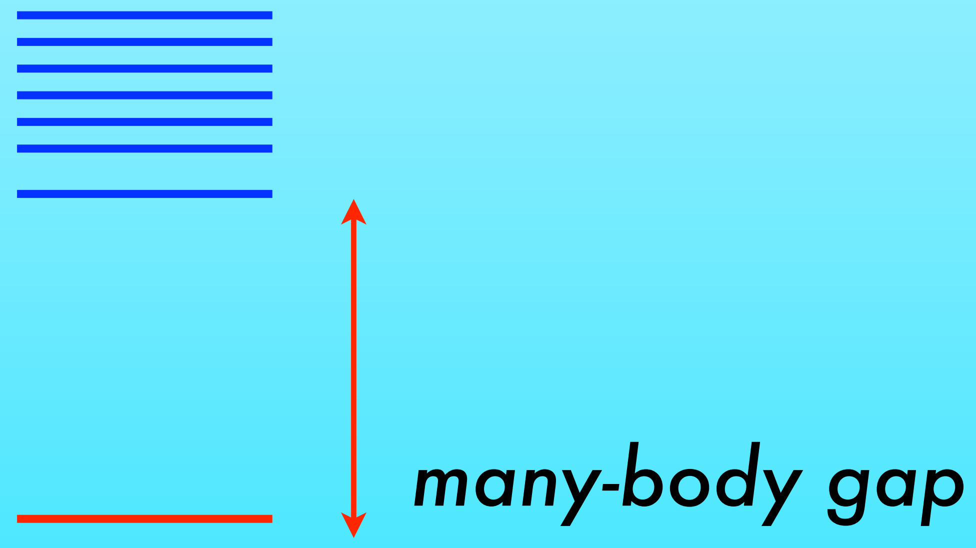
Something complicated
but gapped



Short range entangled states

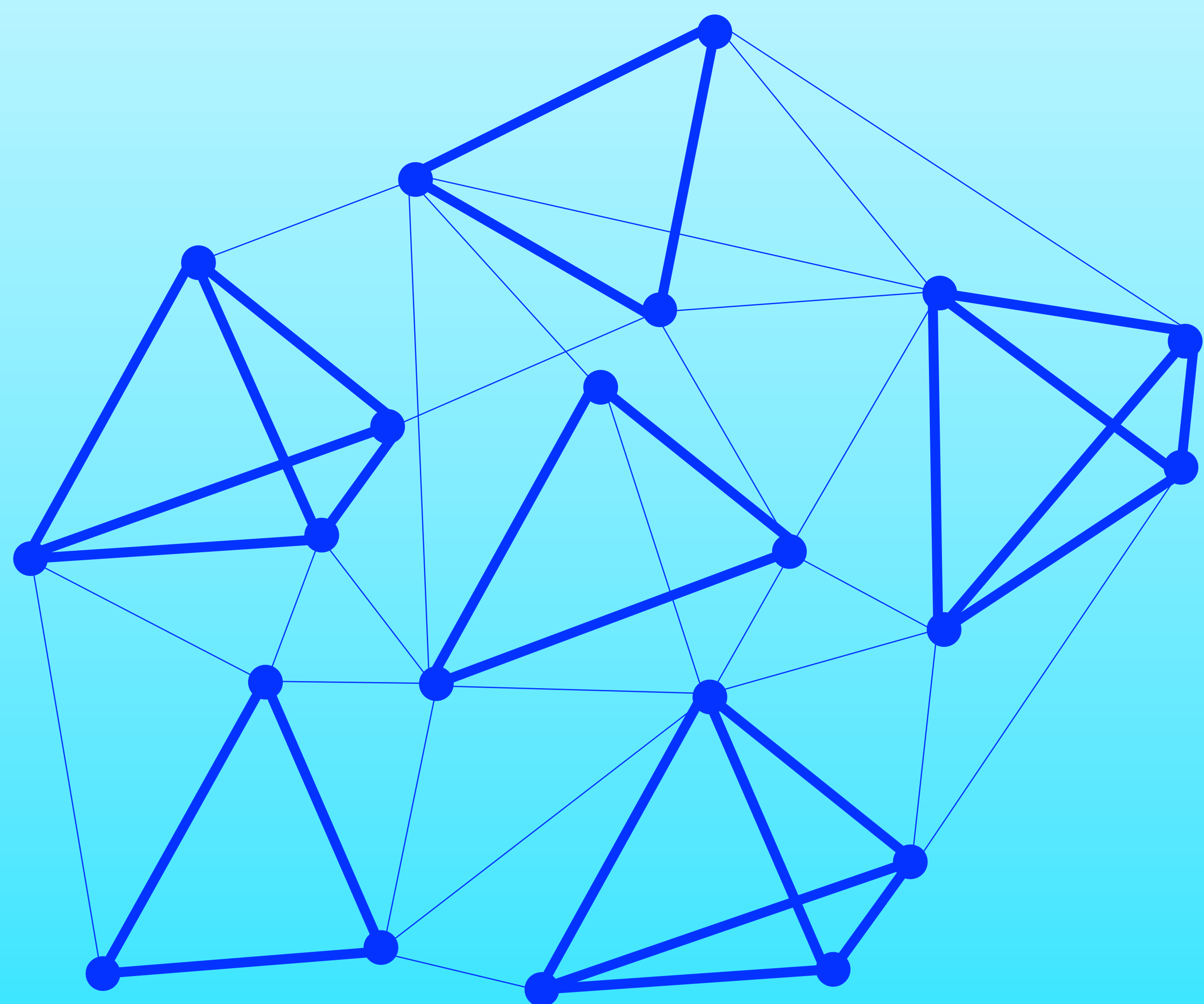
Adiabatic deformation !
gap remains open

Something complicated
but gapped

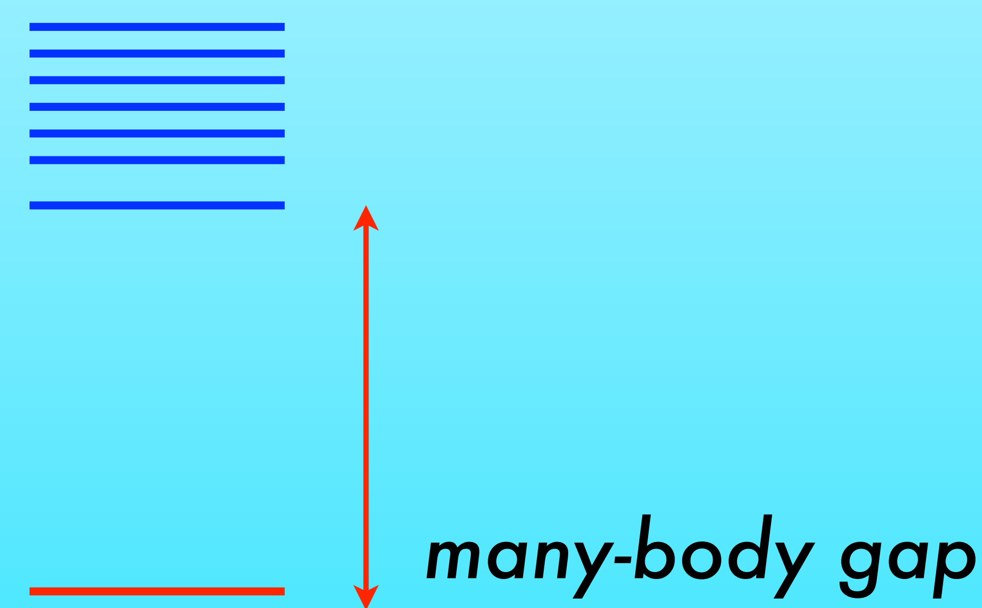


Short range entangled states

Adiabatic deformation !
gap remains open

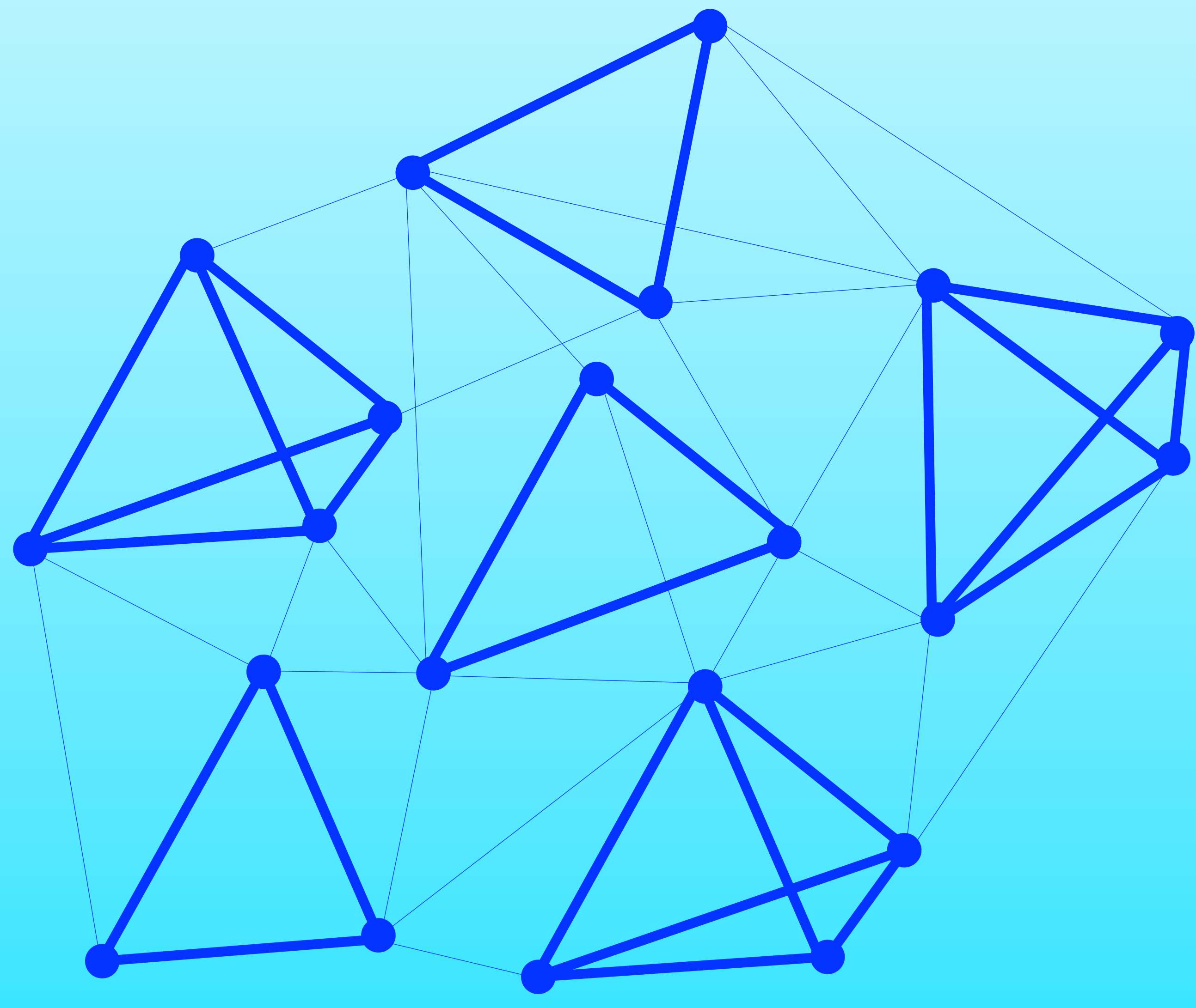


Something complicated
but gapped

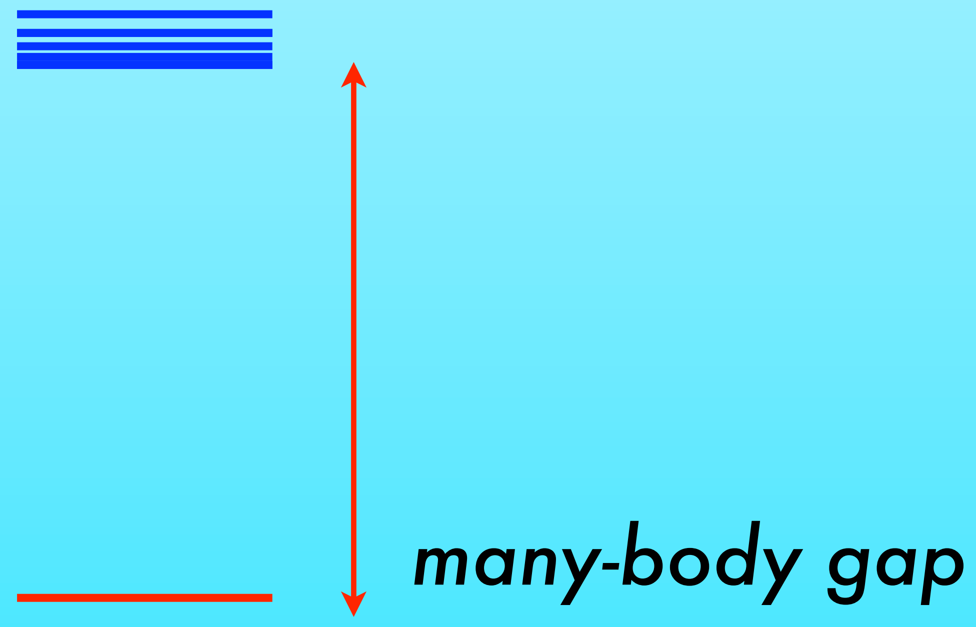


Short range entangled states

Adiabatic deformation !
gap remains open



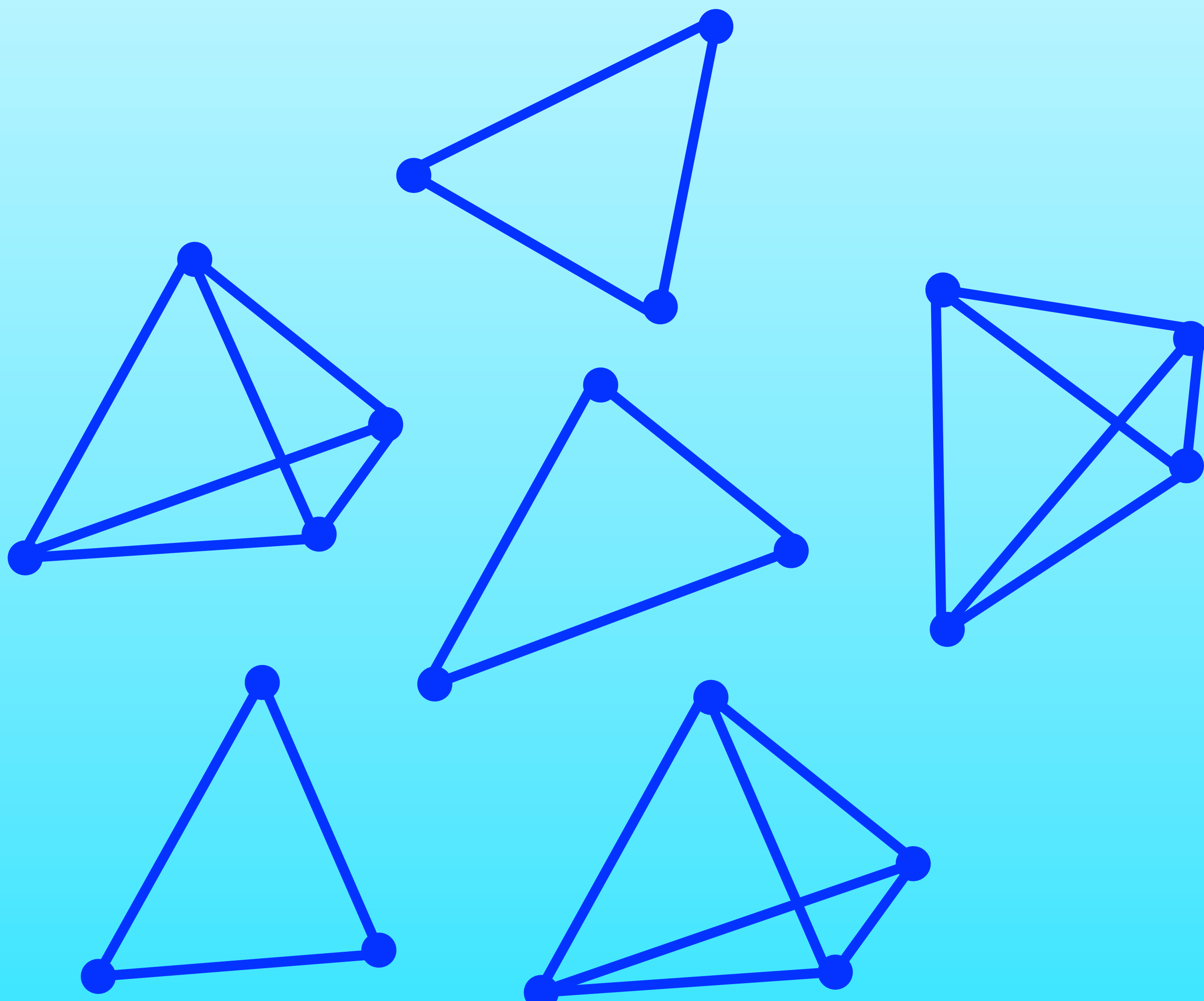
Something complicated
but gapped



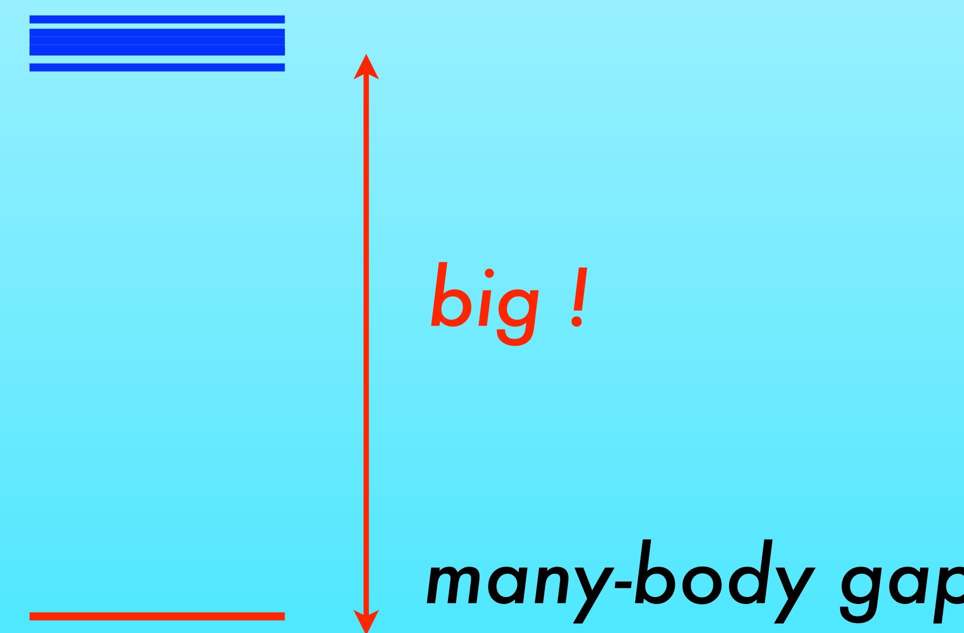
Short range entangled states

Adiabatic deformation !
gap remains open

Decoupled !

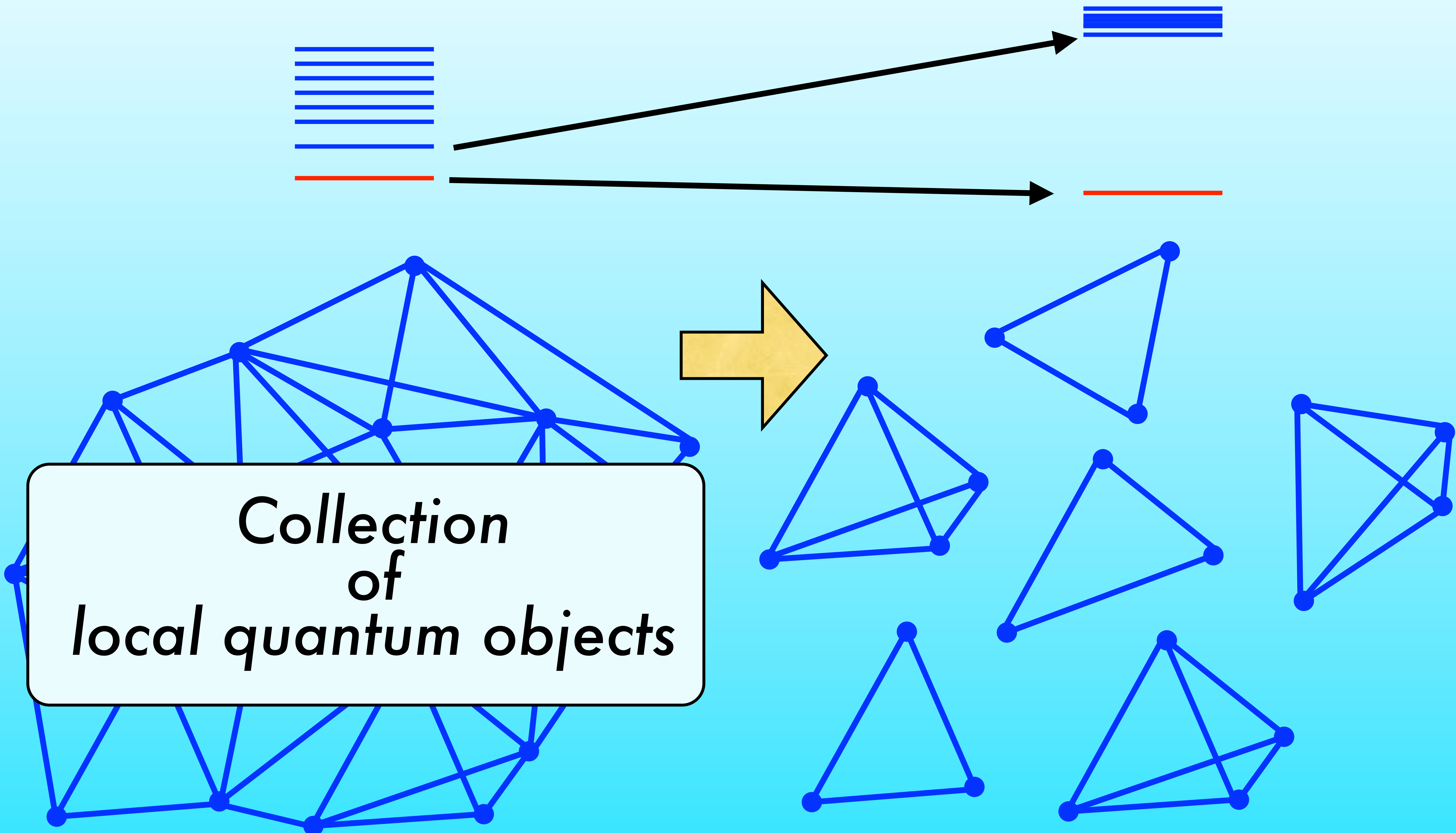


Something very simple
& gapped



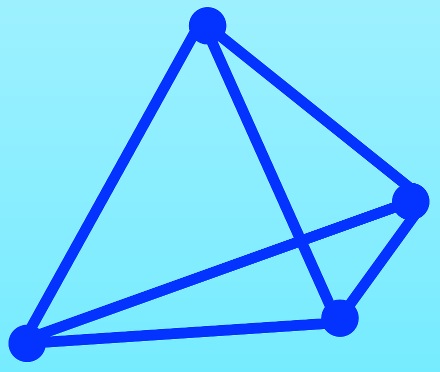
Short range entangled states

Adiabatic process to be decoupled: gap remains open

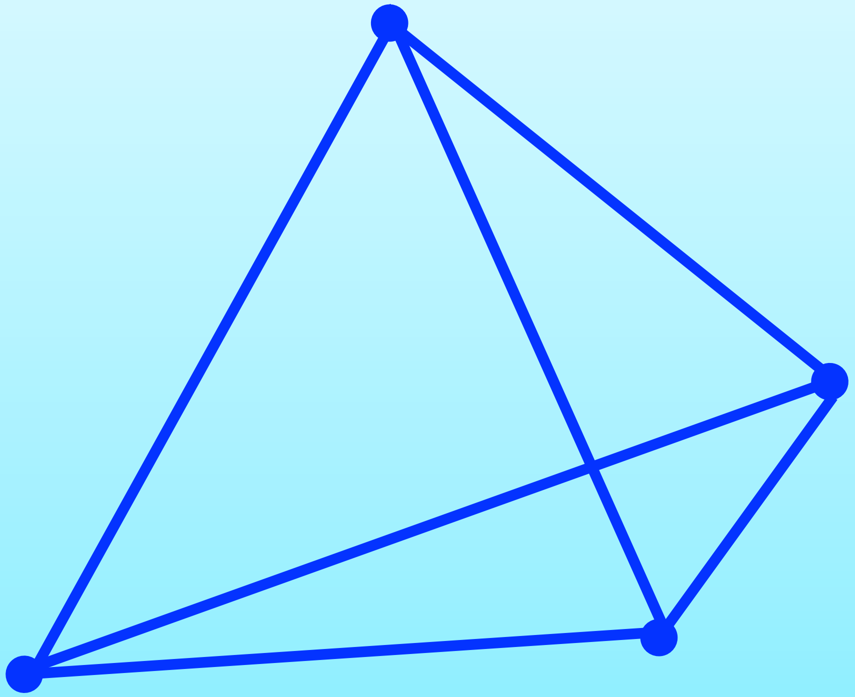


(My) def. of short range entangled state

How to characterize local object ?

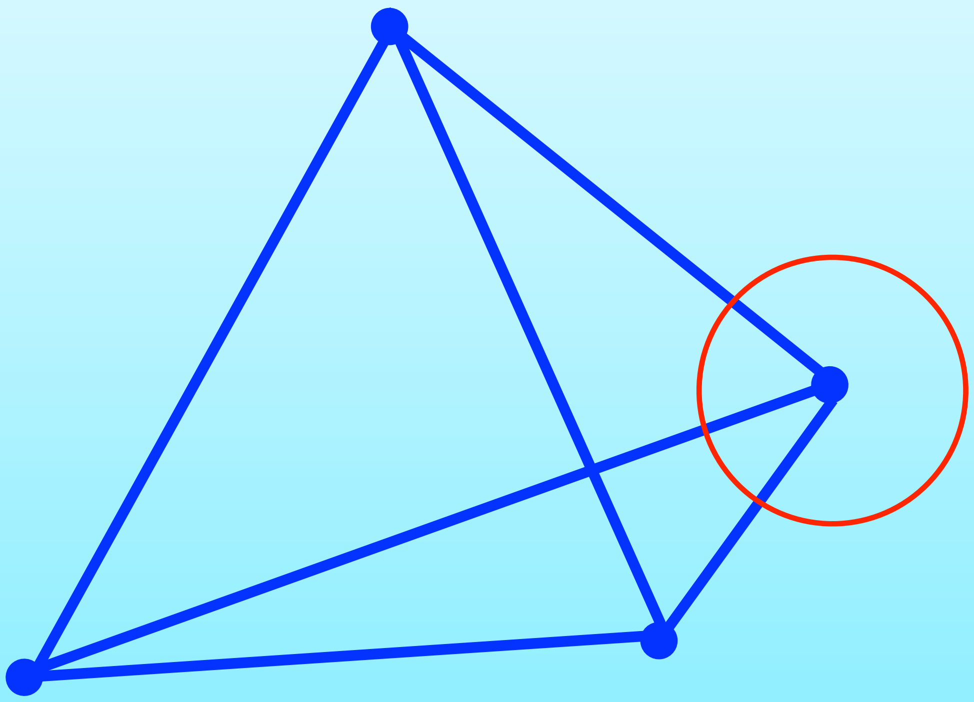


How to characterize local object ?



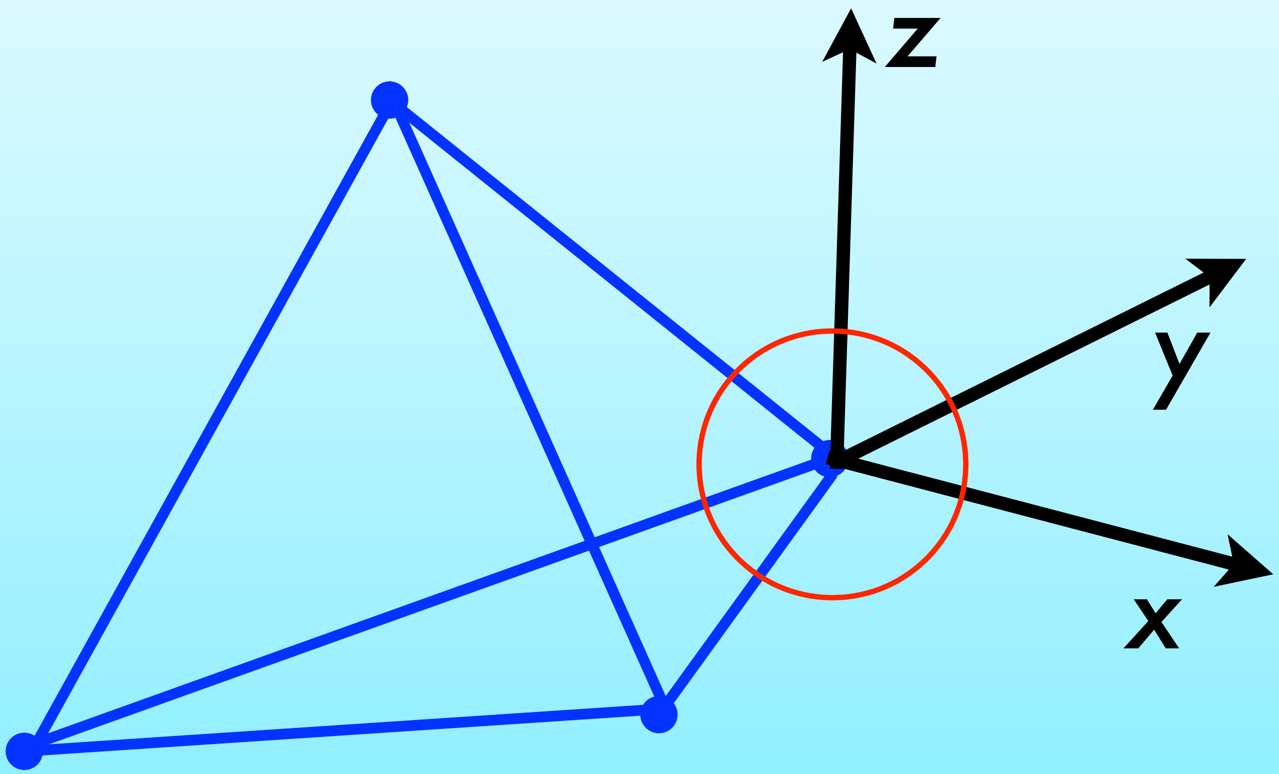
How to characterize local object ?

Consider a gauge transform at some site



How to characterize local object ?

Consider a gauge transform at some site

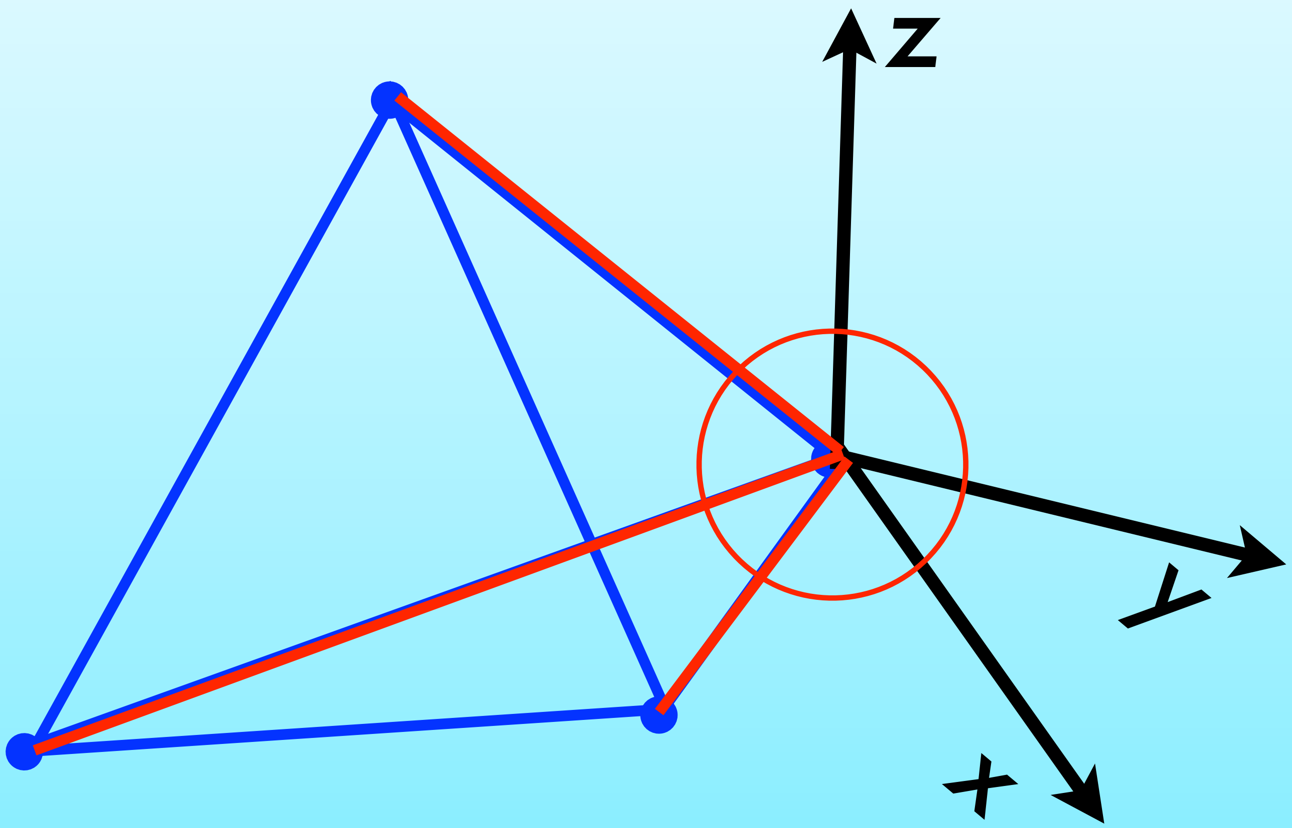


$$|\psi(\theta)\rangle = U(\theta)|\psi(0)\rangle$$

$$U(\theta) = e^{i(S-S_z)\theta}$$

How to characterize local object ?

Consider a gauge transform at some site

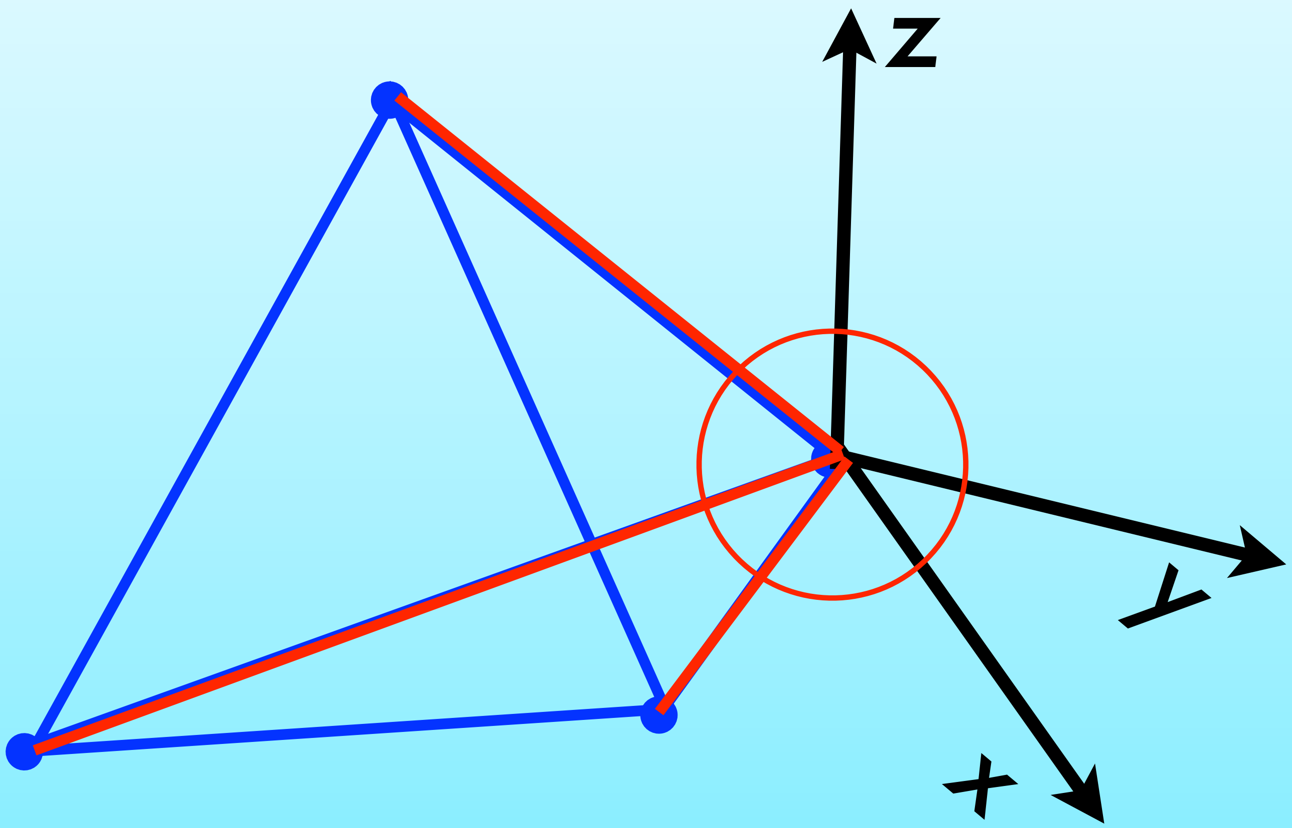


$$|\psi(\theta)\rangle = U(\theta)|\psi(0)\rangle$$

$$U(\theta) = e^{i(S-S_z)\theta}$$

How to characterize local object ?

If decoupled, the twist by the transformation is gauged away !

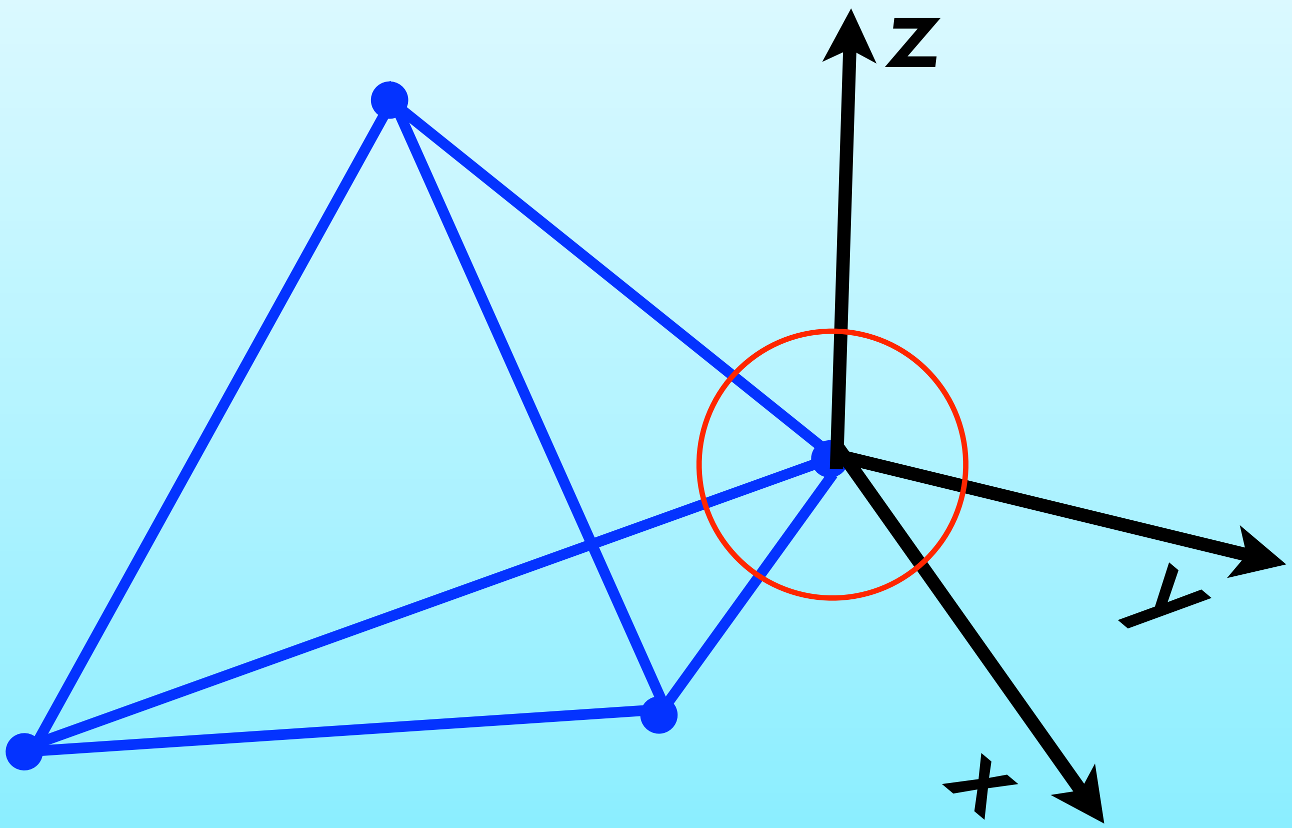


$$|\psi(\theta)\rangle = U(\theta)|\psi(0)\rangle$$

$$U(\theta) = e^{i(S-S_z)\theta}$$

How to characterize local object ?

If decoupled, the twist by the transformation is gauged away !

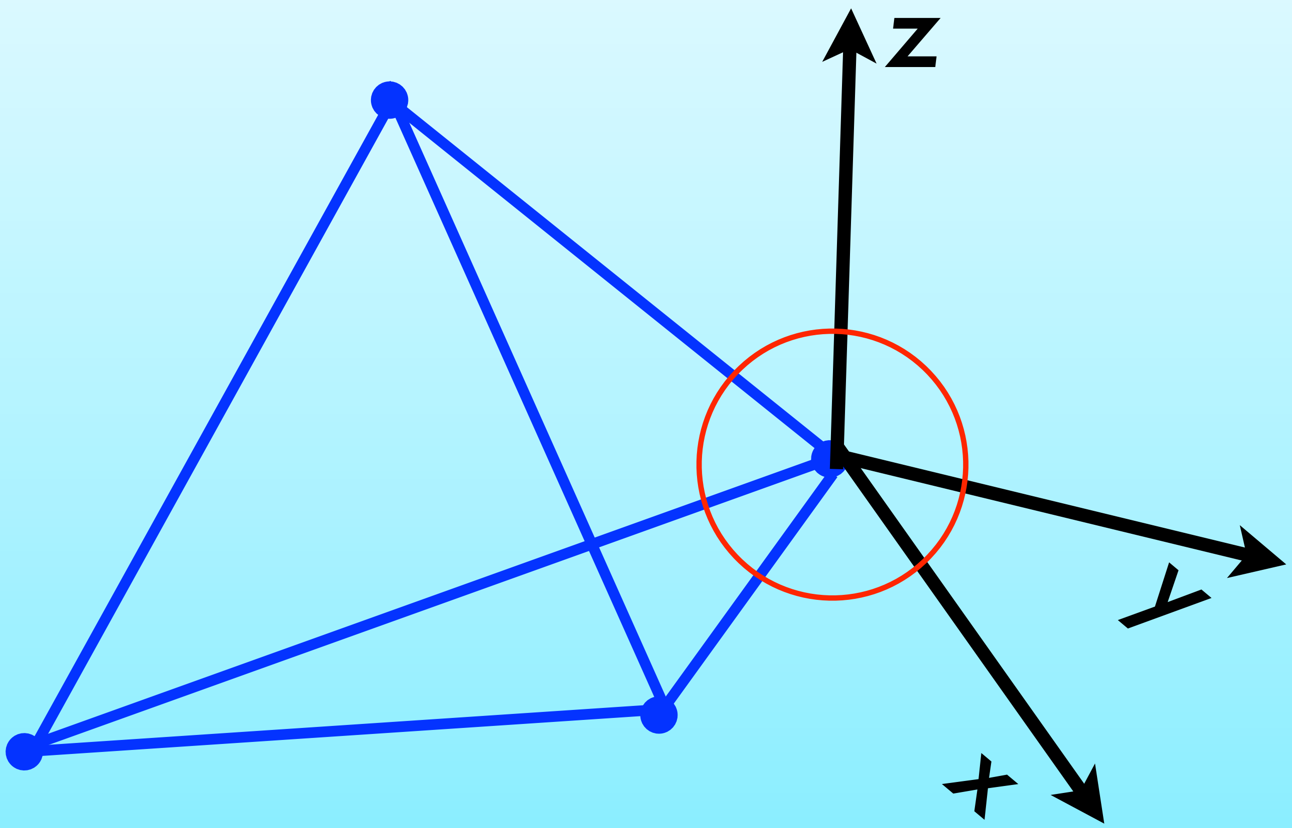


$$|\psi(\theta)\rangle = U(\theta)|\psi(0)\rangle$$

$$U(\theta) = e^{i(S-S_z)\theta}$$

How to characterize local object ?

If decoupled, the twist by the transformation is gauged away !



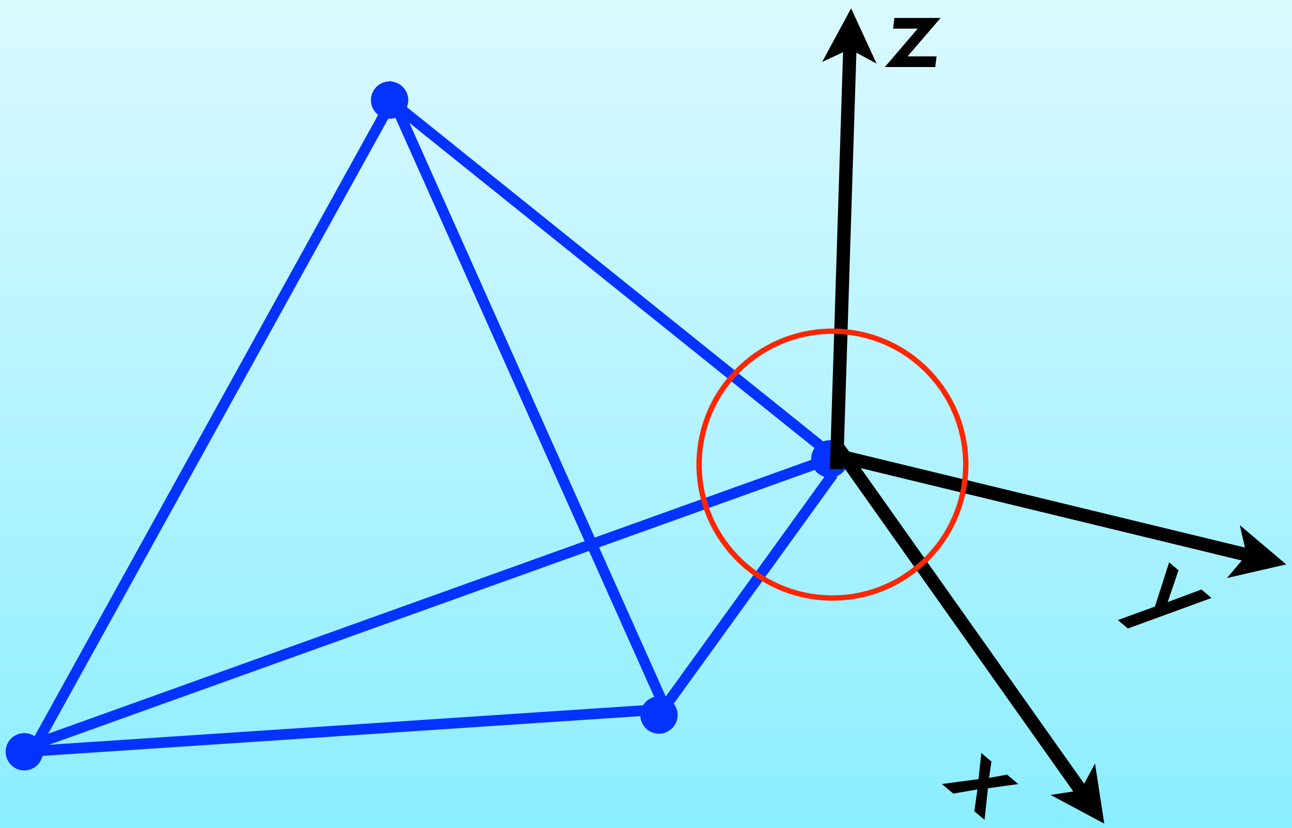
$$|\psi(\theta)\rangle = U(\theta)|\psi(0)\rangle$$

$$U(\theta) = e^{i(S-S_z)\theta}$$

It characterizes locality of the quantum object !

How to characterize local object ?

If decoupled, the twist by the transformation is gauged away !



$$|\psi(\theta)\rangle = U(\theta)|\psi(0)\rangle$$

$$U(\theta) = e^{i(S-S_z)\theta}$$

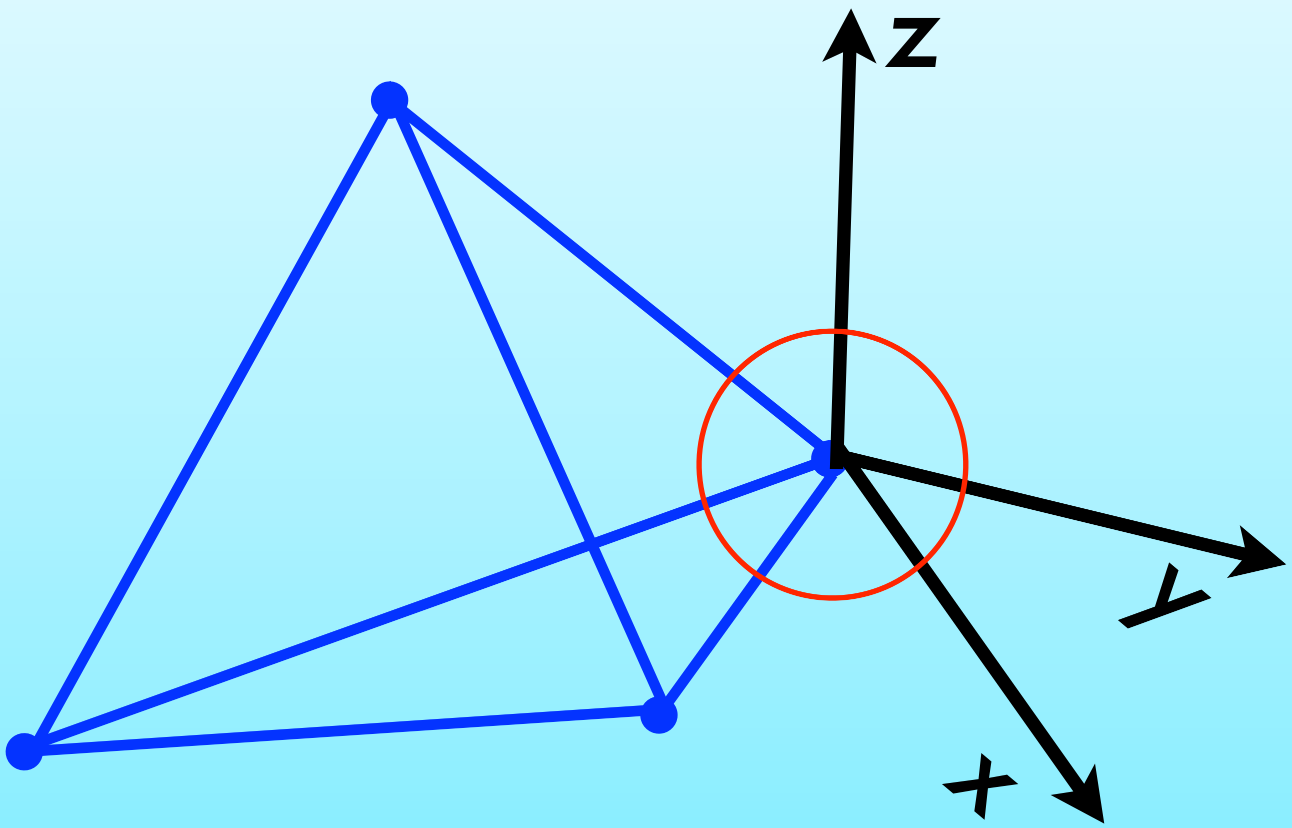
It characterizes locality of the quantum object !

Question ?

How to see this locality by skipping the adiabatic deformation ?

How to characterize local object ?

If decoupled, the twist by the transformation is gauged away !



$$|\psi(\theta)\rangle = U(\theta)|\psi(0)\rangle$$

$$U(\theta) = e^{i(S-S_z)\theta}$$

It characterizes locality of the quantum object !

Answer !

Calculate a topological invariant as an adiabatic invariant

- ☆ *Short range entanglement, symmetry & quantization*
  - ☆ *Adiabatic principle with symmetry*
  - ☆ *Gauge freedom for entangled state*
- ☆ *Two types of topological invariants as "order parameters"*
  - ☆ *Chern numbers in even dimensions*
  - ☆ *Quantized Berry phases in odd dimensions*

    (dim. of parameter space)
- ☆ *Examples in 1D, 2D, 3D and ...*
  - ☆ *Integer spin chains with dimerization*
  - ☆ *Random hopping models*
  - ☆ *Orthogonal dimers in 2D*
  - ☆ *Generalized dimers in Kagome, Pyrochlore ...*

    : d-Dim. fermions with frustration

Quantization for topological phases

Topological

≀

Quantization

$$\mathbb{Z} = \{\cdots, -2, -1, 0, 1, 2, , \cdots\}$$

$$\mathbb{Z}_Q = \{1, 2, \cdots, Q \ (\mathrm{mod}\ Q)\} \qquad Q \in \mathbb{Z}$$

$$\vdots$$

$$H|\psi\rangle = E|\psi\rangle$$

$$A = \langle\psi|d\psi\rangle$$
$$F = dA + A^2$$

Parameter dependent hamiltonian ⇨ Berry connection

☆ Intrinsically quantized (without boundary)

- Chern numbers: 1st, 2nd, 3rd,....

  QHE ... Z

$$C_1 = -\frac{1}{2\pi i}\int_{M^2} F$$

☆ Symmetry protected quantization

- Berry phases & generalization:

$$\gamma_1 = -\frac{1}{2\pi i}\int_{M^1} A$$

  Quantum spin chains, Spin-QHE ... Z_2

Topological quantities : Berry connection

collect M states gapped from the else

$$\Psi = (|\psi_1\rangle, \cdots, |\psi_M\rangle) \quad \langle \psi_j | \psi_k \rangle = \delta_{jk} \quad \Psi^\dagger \Psi = E_M$$

Berry connection & gauge transformation

$$A_g = \Psi_g^\dagger d\Psi_g = g^{-1} A g + g^{-1} dg \qquad F_g = dA_g + A_g^2 = g^{-1} F g$$

$$\Psi_g = \Psi g \qquad g \in U(M) \qquad g \in Sp(M)$$ with Kramers deg.

Chern numbers : intrinsically quantized

$$C_1 = -\frac{1}{2\pi i}\int_{S^2} \mathrm{Tr} F, \quad C_2 = -\frac{1}{8\pi^2}\int_{S^4} \mathrm{Tr} F^2, \cdots$$

2n dim.

$$\mathrm{Tr}\, F = d\omega_1 \qquad \mathrm{Tr}\, F^2 = d\omega_3$$

Berry phases & generalizations

$$\gamma_1 = -\frac{1}{2\pi i}\int_{S^1} \omega_1, \quad \gamma_3 = -\frac{1}{8\pi^2}\int_{S^3} \omega_3, \cdots$$

2n-1 dim.

any value

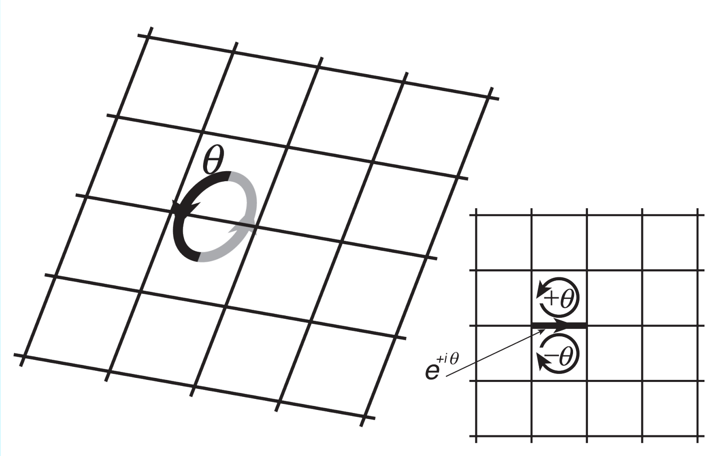
Gauge dependent : $\omega_1 = \mathrm{Tr}\, A, \quad \omega_3 = \mathrm{Tr}\,(AdA + \frac{2}{3}A^3), \cdots$

$\gamma_1 \equiv \gamma_1^g, \ (\mathrm{mod}\ 1)$ YH '06 $\quad \gamma_3 \equiv \gamma_3^g, \ (\mathrm{mod}\ 1)$ Qi-Hughes-Zhang '08 YH '09

Some constraint → Symmetry protected quantization

Example: Heisenberg model with local twist

Define a many body hamiltonian by local twist as a periodic parameter



$$H(x = e^{i\theta})$$

$$C = \{x = e^{i\theta} | \theta : 0 \to 2\pi\}$$

$$H_0 = \sum_{\langle ij \rangle} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

$$\boldsymbol{S}_i \cdot \boldsymbol{S}_j \to \frac{1}{2}(e^{-i\theta} S_{i+} S_{j-} + e^{+i\theta} S_{i-} S_{j+}) + S_{iz} S_{jz}$$

Only link <ij>

$$H(\theta)|\psi(\theta)\rangle = E(\theta)|\psi(\theta)\rangle$$

Lanczos diagonalization

Calculate the Berry phases using the many spin wave function

$$i\gamma_C = \int_C A = \int_0^{2\pi} \langle \psi | \frac{\partial \psi}{\partial \theta} \rangle \, d\theta = \pi, 0$$

Time-reversal

Z_2 Berry phase

Topological order parameter

YH, J. Phys. Soc. Jpn. 75, 123601, '06

Symmetry in physics

Symmetry protection for adiabatic process

- ☆Chiral symmetry
- ☆Particle-Hole symmetry
- ☆Time-reversal symmetry
- ☆Inversion symmetry
- ☆Z_Q symmetry: S_Q reduced into Z_Q with gauge twist

Symmetry in physics

Symmetry protection for Berry phases

- ☆ Chiral symmetry
- ☆ Particle-Hole symmetry
- ☆ Time-reversal symmetry
- ☆ Inversion symmetry
- ☆ Z_Q symmetry: S_Q reduced into Z_Q with gauge twist

Symmetry in physics

Symmetry protection for Berry phases

- Chiral symmetry Z_2
- Particle-Hole symmetry Z_2
- Time-reversal symmetry Z_2
- Inversion symmetry Z_2 Z_Q
- Z_Q symmetry: S_Q reduced into Z_Q with gauge twist

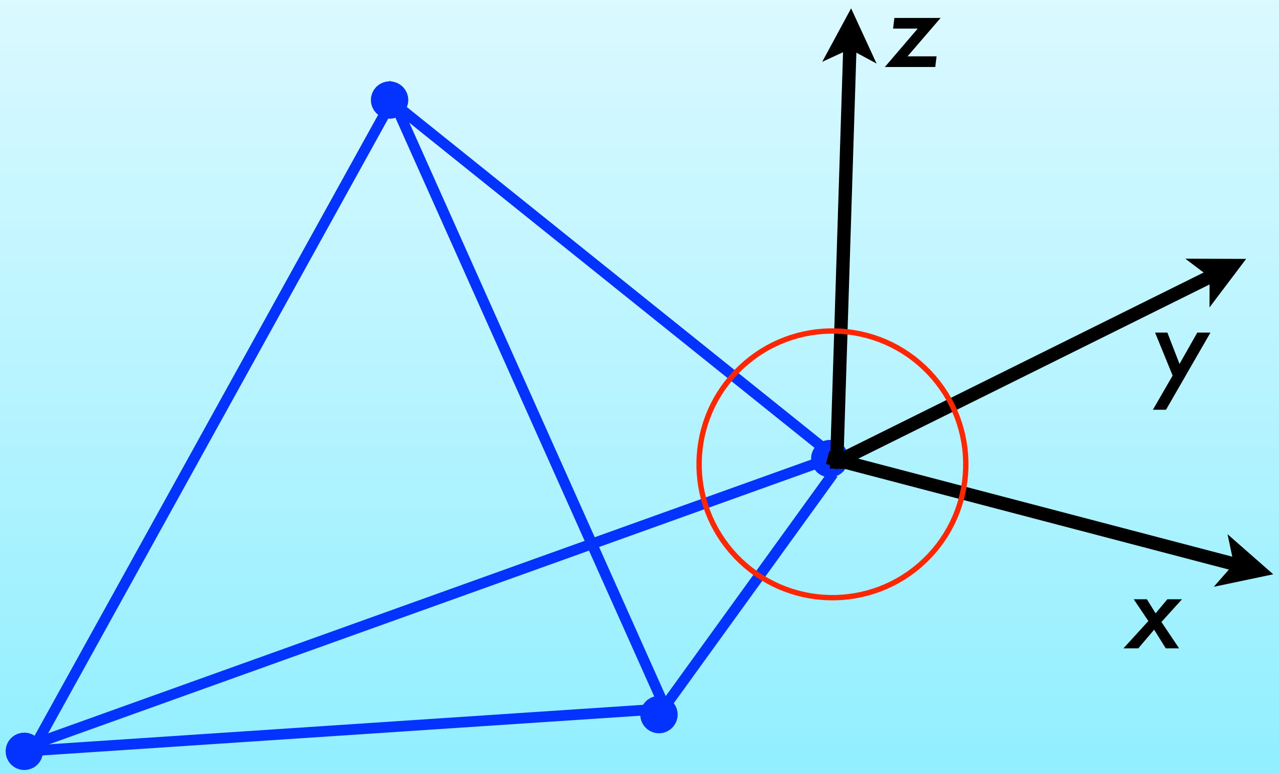
Quantization

Z_2 $\quad \gamma \equiv 0, \pi$

Z_Q $\quad \gamma \equiv 2\pi \frac{k}{Q}$ $\quad k = 0, 1, 2, \cdots, Q-1$

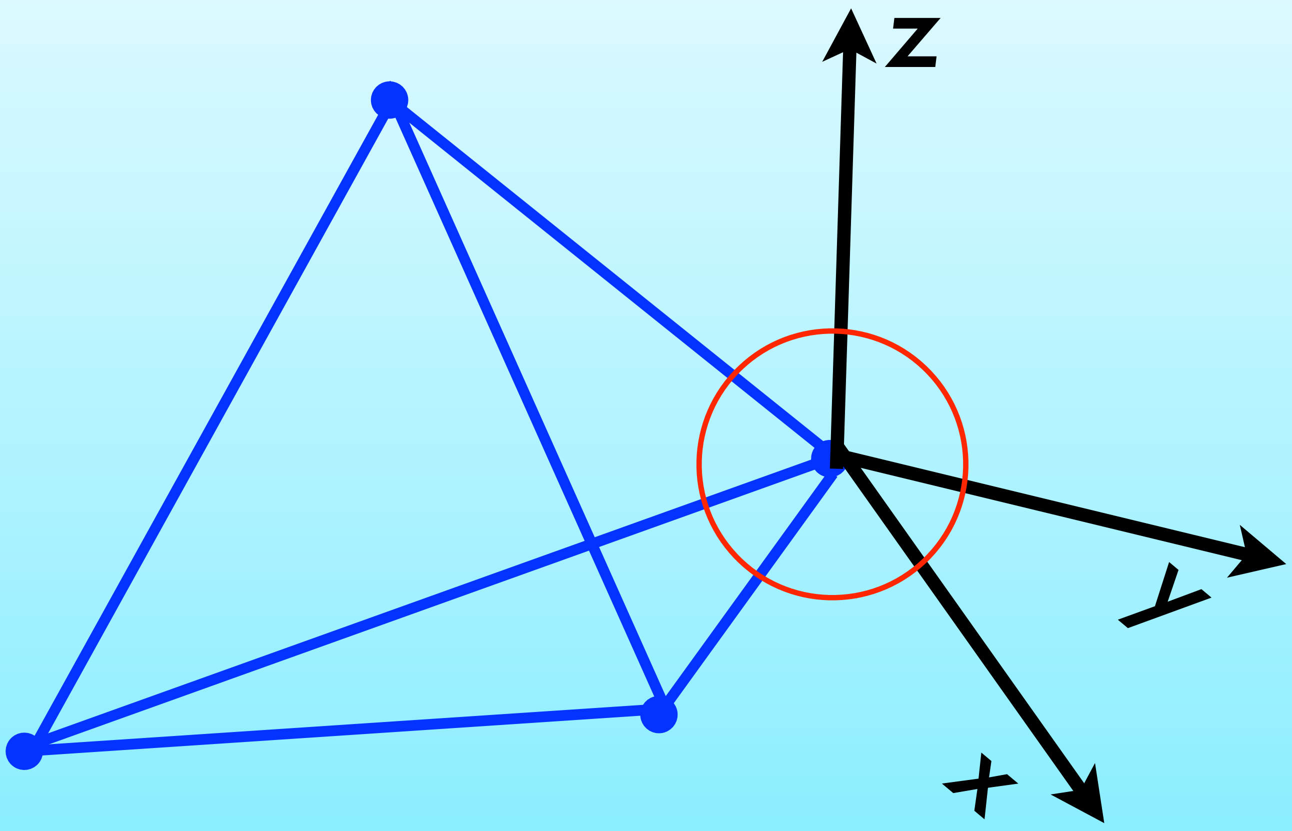
Gauge transformation & Berry phase

If gauged away, the Berry phase is trivially obtained



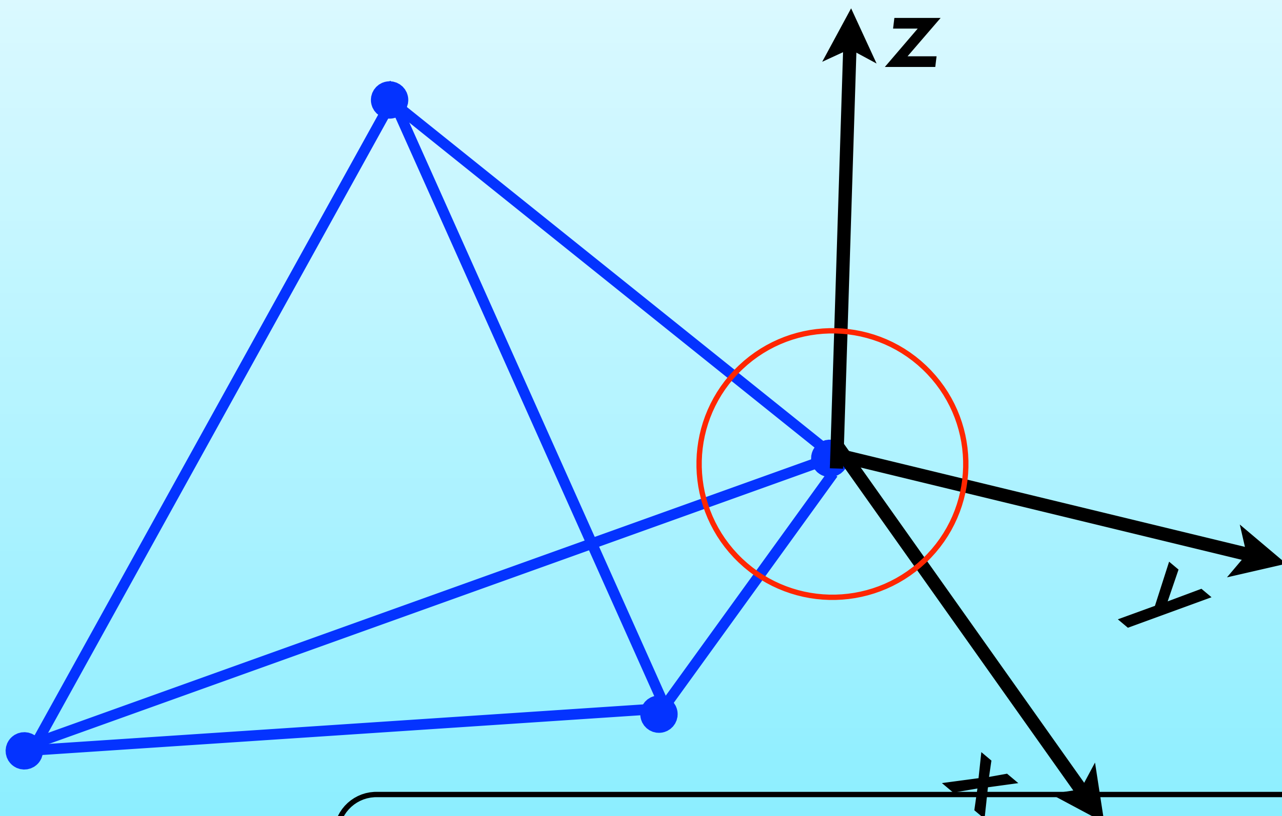
Gauge transformation & Berry phase

If gauged away, the Berry phase is trivially obtained



Gauge transformation & Berry phase

If gauged away, the Berry phase is trivially obtained



$$|\psi(\theta)\rangle = U(\theta)|\psi(0)\rangle$$

$$U(\theta) = e^{i(S-S_z)\theta}$$

$$A = \langle\psi|d\psi\rangle = Sd\theta$$

$$\gamma = 2\pi S$$

Spins

$$\gamma = 2\pi S = \pi$$

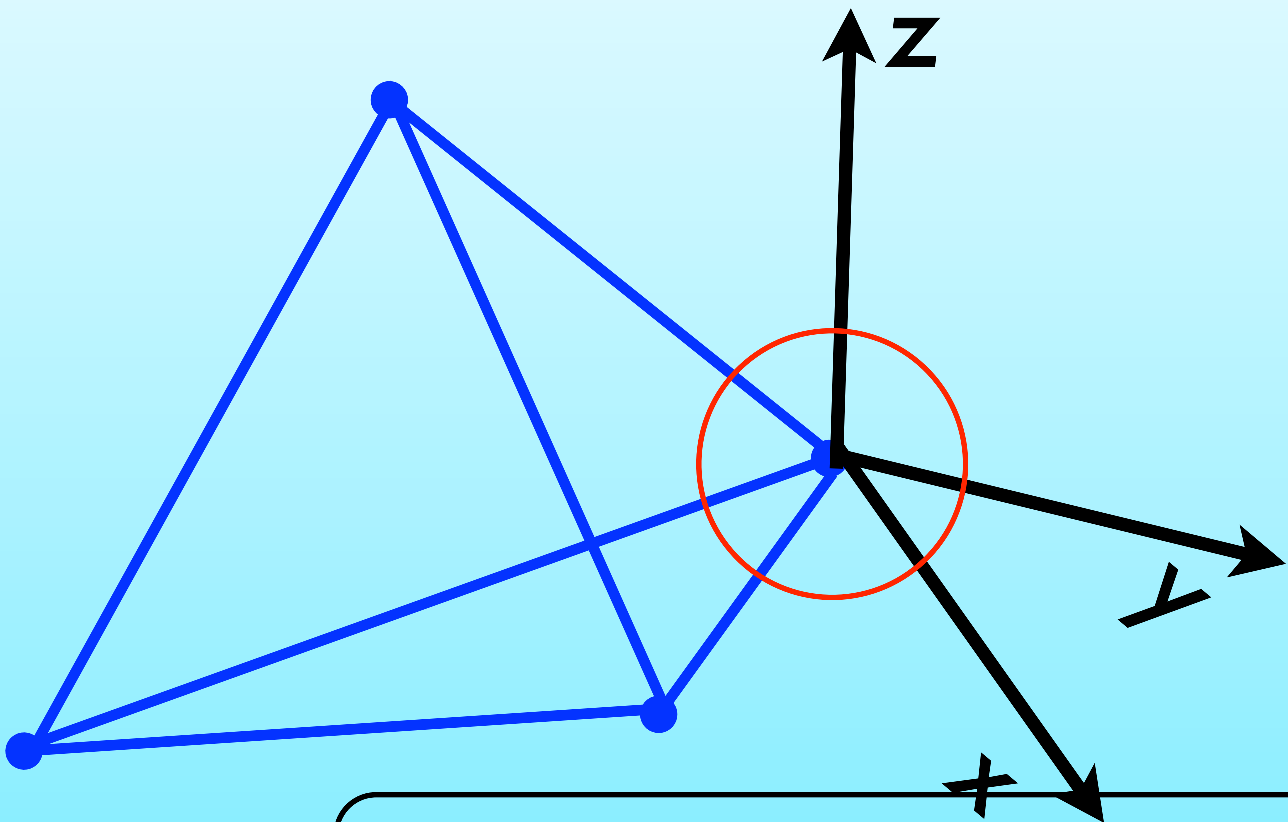
Z_2

$$S = 1/2$$

$$S = (\text{odd integer})/2$$

Gauge transformation & Berry phase

If gauged away, the Berry phase is trivially obtained



$$|\psi(\theta)\rangle = U(\theta)|\psi(0)\rangle$$

$$U(\theta) = e^{i(S-S_z)\theta}$$

$$A = \langle\psi|d\psi\rangle = S d\theta$$

$$\gamma = 2\pi S$$

Spins $\gamma = 2\pi S = \pi$ $S = 1/2$

Z_2 $S = (\text{odd integer})/2$

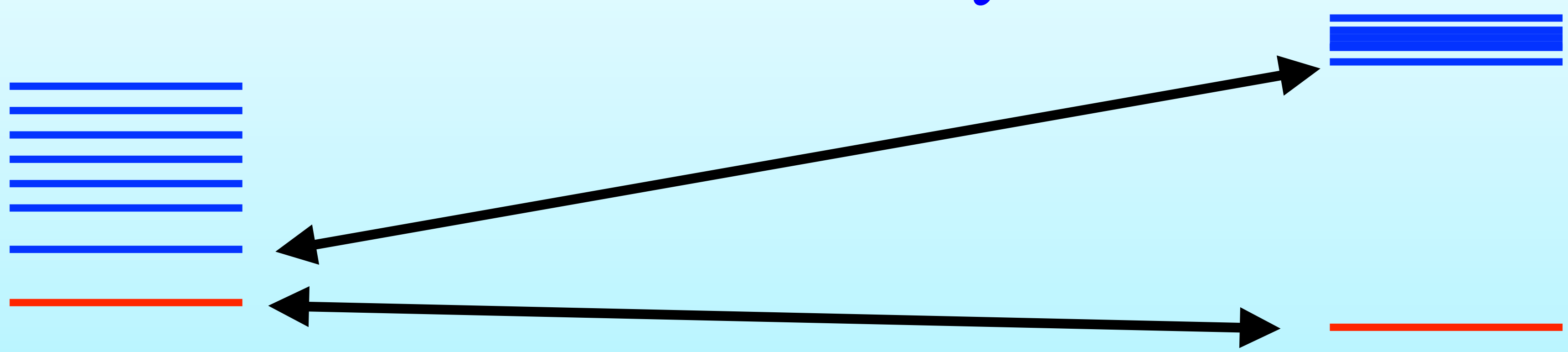
Fermions with filling $\rho = P/Q, \quad (P,Q) = 1$

$$\gamma = 2\pi\rho = 2\pi\frac{P}{Q}$$ Z_Q

Hirano-Katsura-YH, Phys. Rev. B 78, 054431 (2008)

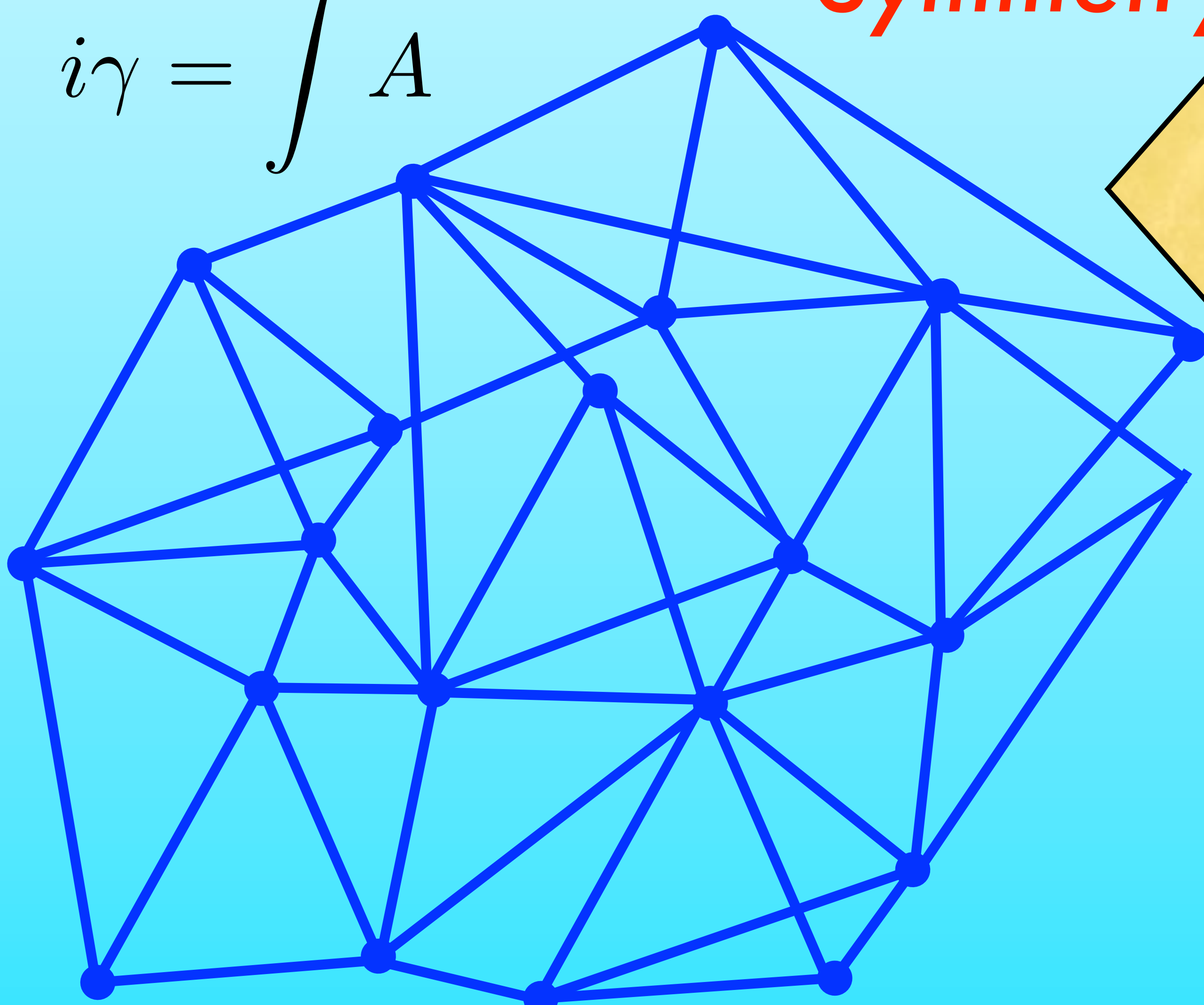
Spins $\gamma = 2\pi S = \pi$ Z_2

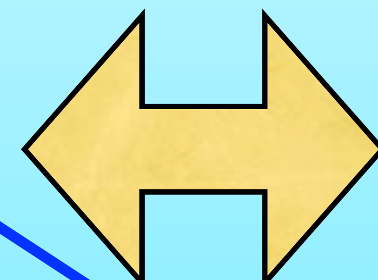
Fermions with filling $\gamma = 2\pi\rho = 2\pi\frac{P}{Q}$ Z_Q

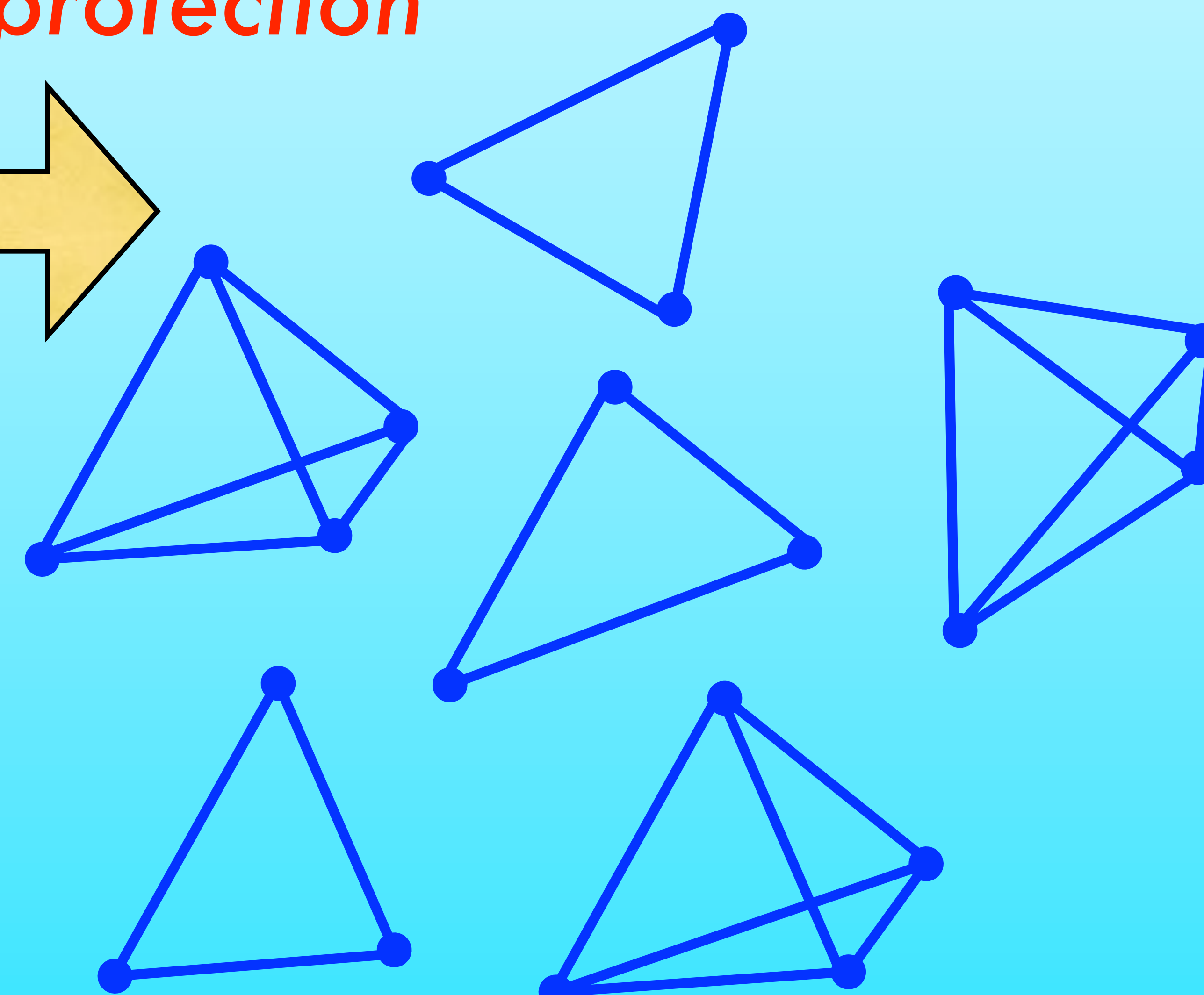


Symmetry protection

$$i\gamma = \int A$$



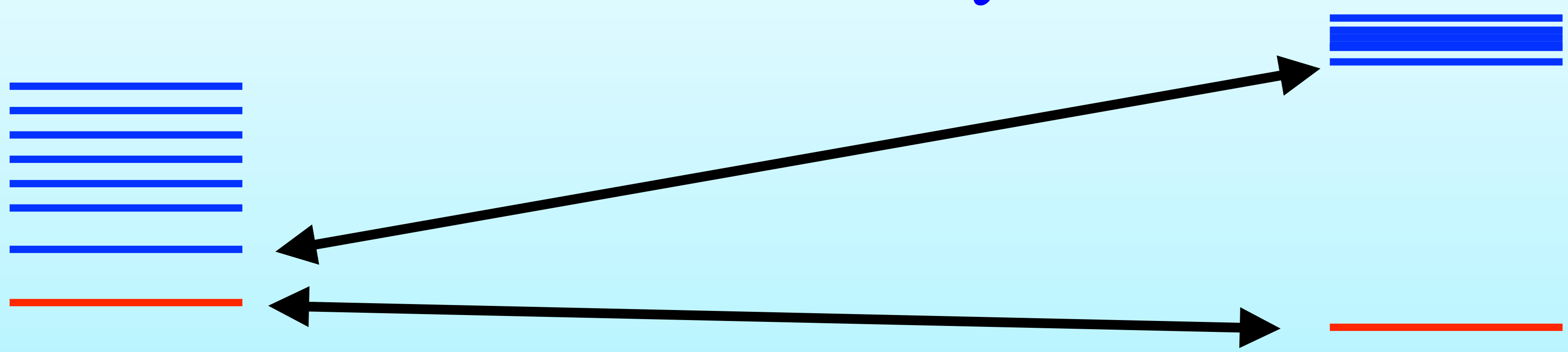




Quantized Berry phases for short range entangled states

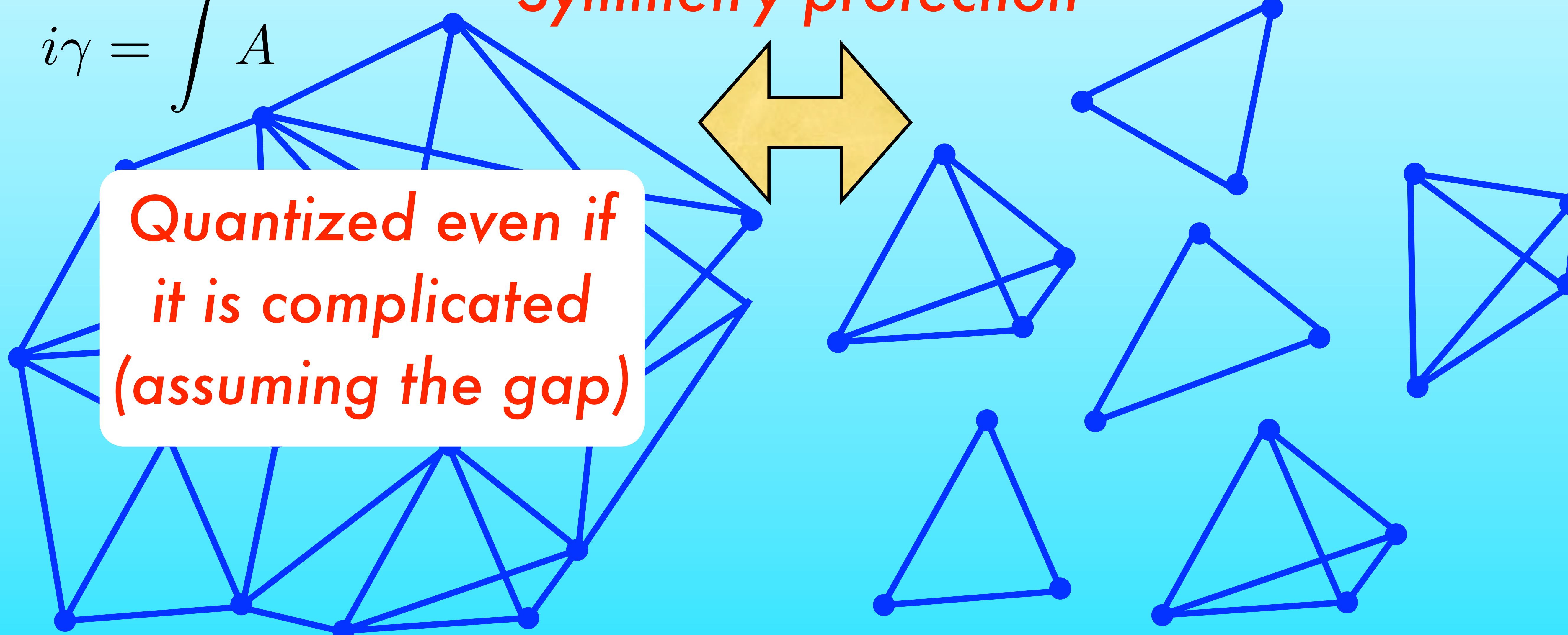
Spins $\gamma = 2\pi S = \pi$ Z_2

Fermions with filling $\gamma = 2\pi\rho = 2\pi \frac{P}{Q}$ Z_Q



Symmetry protection

$i\gamma = \int A$



Quantized Berry phases for short range entangled states

- Short range entanglement, symmetry & quantization
  - Adiabatic principle with symmetry
  - Gauge freedom for entangled state
- Two types of topological invariants
  - Chern numbers in even dimensions
  - Quantized Berry phases in odd dimensions
- *Examples in 1D, 2D, 3D and ...*
  - *Integer spin chains with dimerization*
  - *Random hopping models*
  - *Orthogonal dimers in 2D*
  - *Generalized dimers in Kagome, Pyrochlore ...*
    : d-Dim. fermions with frustration

1D S=1/2 chains with dimerization

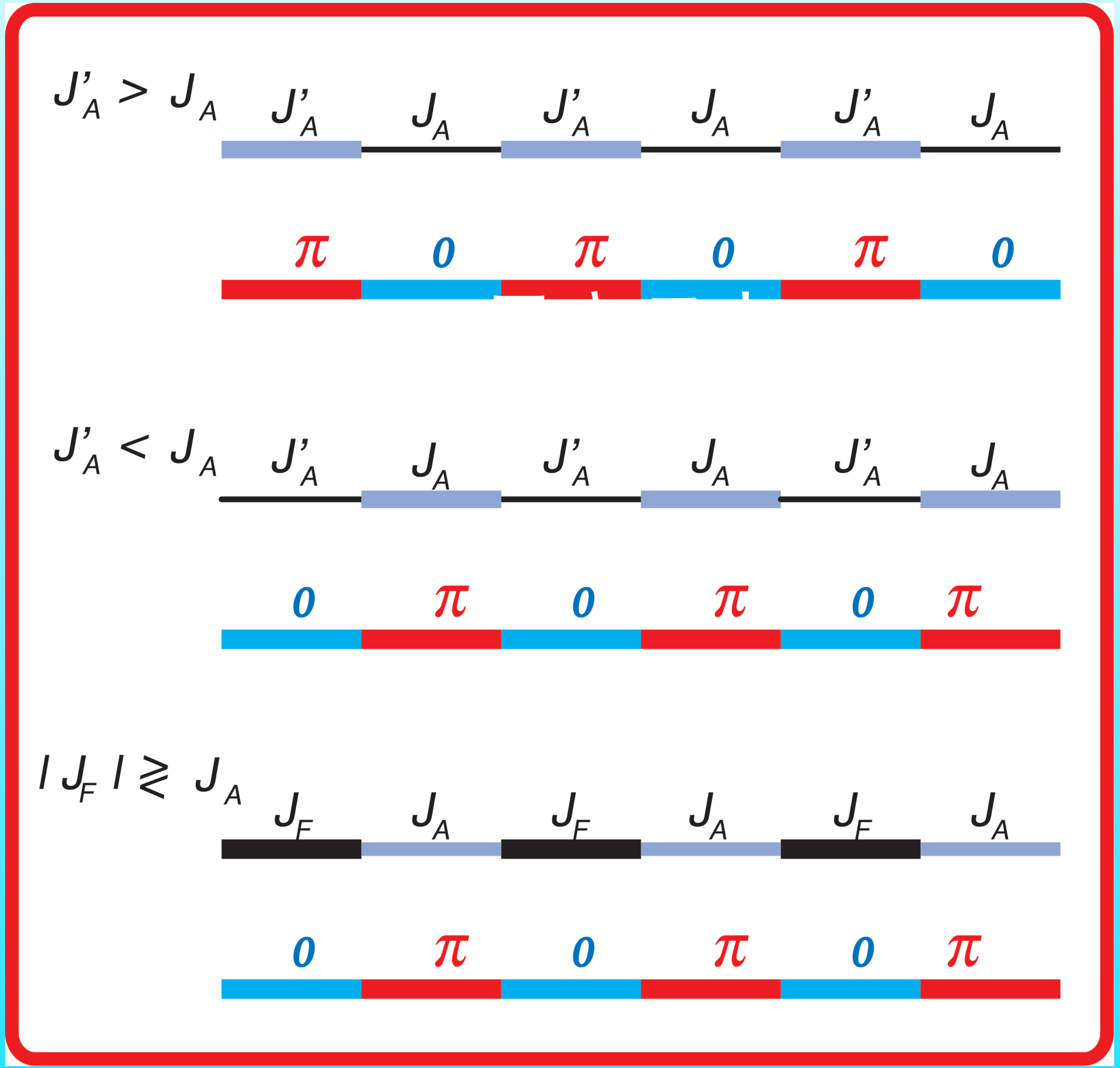
$$H = \sum_{\langle i \rangle} J_i \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1}$$

Y.H., J. Phys. Soc. Jpn. 75 123601 (2006)

AF-AF

Ferro-AF

Hida



AF-AF case

Strong bonds

: π bonds

F-AF case

AF bonds

: π bonds

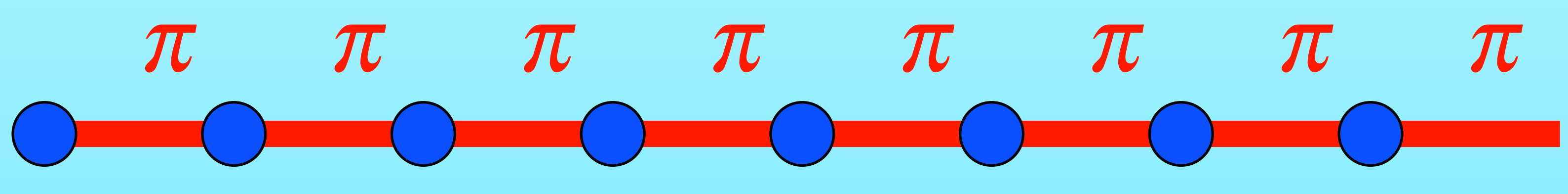
Heisenberg Spin Chains with integer S

S=1 $\quad (\boldsymbol{S}_i)^2 = S(S+1), \ S = 1$

$$H = J \sum_{\langle ij \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + D \sum_i (S_i^z)^2$$

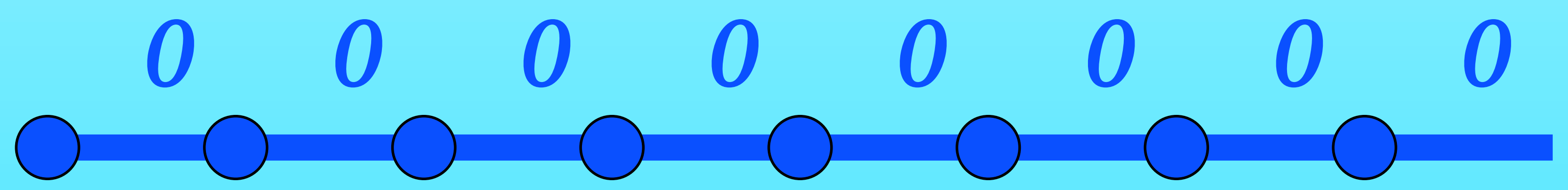
Y.H., J. Phys. Soc. Jpn. 75 123601 (2006)

Haldane phase



$D < D_C$

Large D phase



$D > D_C$

Characterize the Quantum Phase Transition

S=1,2 dimerized Heisenberg model

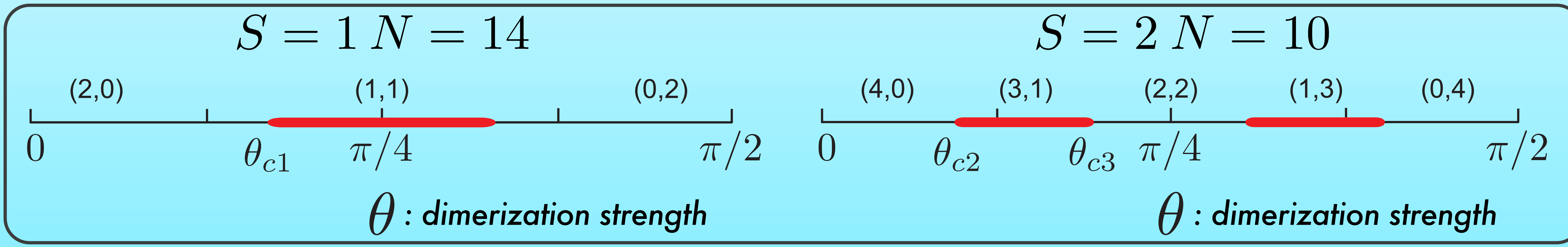
T.Hirano, H.Katsura &YH, Phys.Rev.B77 094431'08

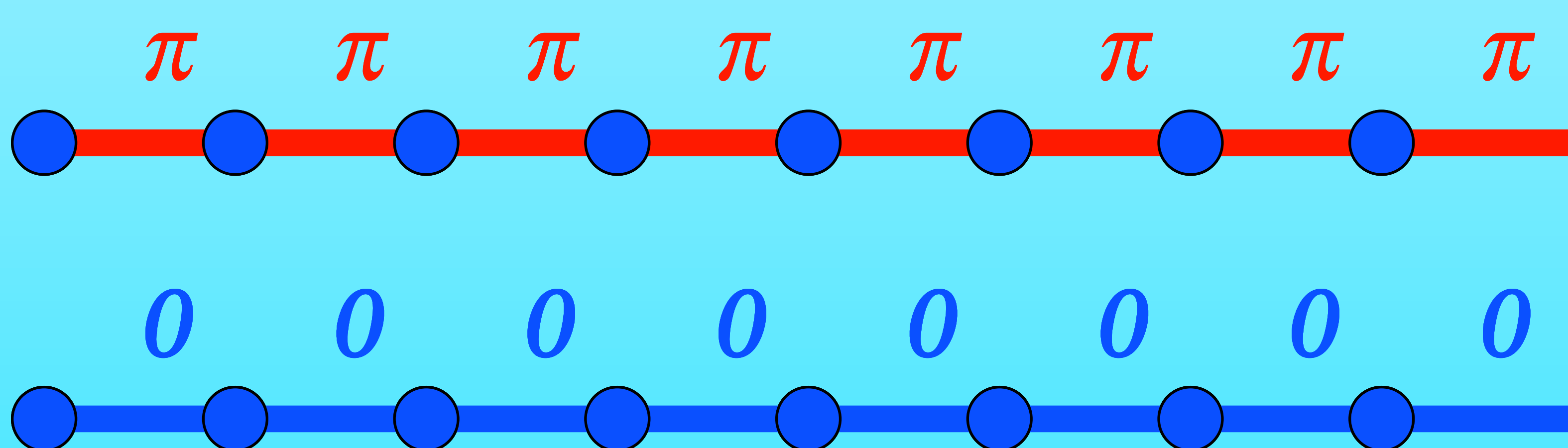
$$H = \sum_{i=1}^{N/2} (J_1 \boldsymbol{S}_{2i} \cdot \boldsymbol{S}_{2i+1} + J_2 \boldsymbol{S}_{2i+1} \cdot \boldsymbol{S}_{2i+2})$$

$$J_1 = \cos\theta, J_2 = \sin\theta$$

Z_2 Berry phase

Red line :Berry phase π





S=1 & 2

Sequential transitions among gapped phases

S=1,2 dimerized Heisenberg model

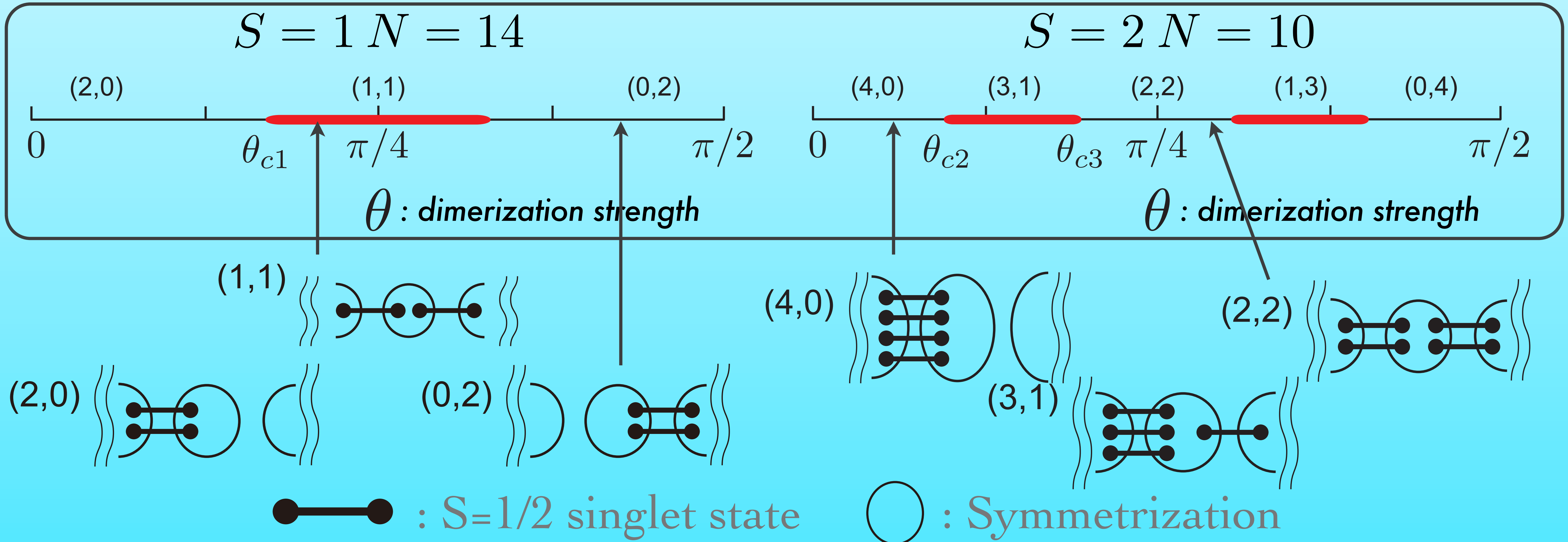
T.Hirano, H.Katsura &YH, Phys.Rev.B77 094431'08

$$H = \sum_{i=1}^{N/2} (J_1 \boldsymbol{S}_{2i} \cdot \boldsymbol{S}_{2i+1} + J_2 \boldsymbol{S}_{2i+1} \cdot \boldsymbol{S}_{2i+2})$$

$$J_1 = \cos\theta, J_2 = \sin\theta$$

Z_2Berry phase

Red line :Berry phase π



Reconstruction of valence bonds!

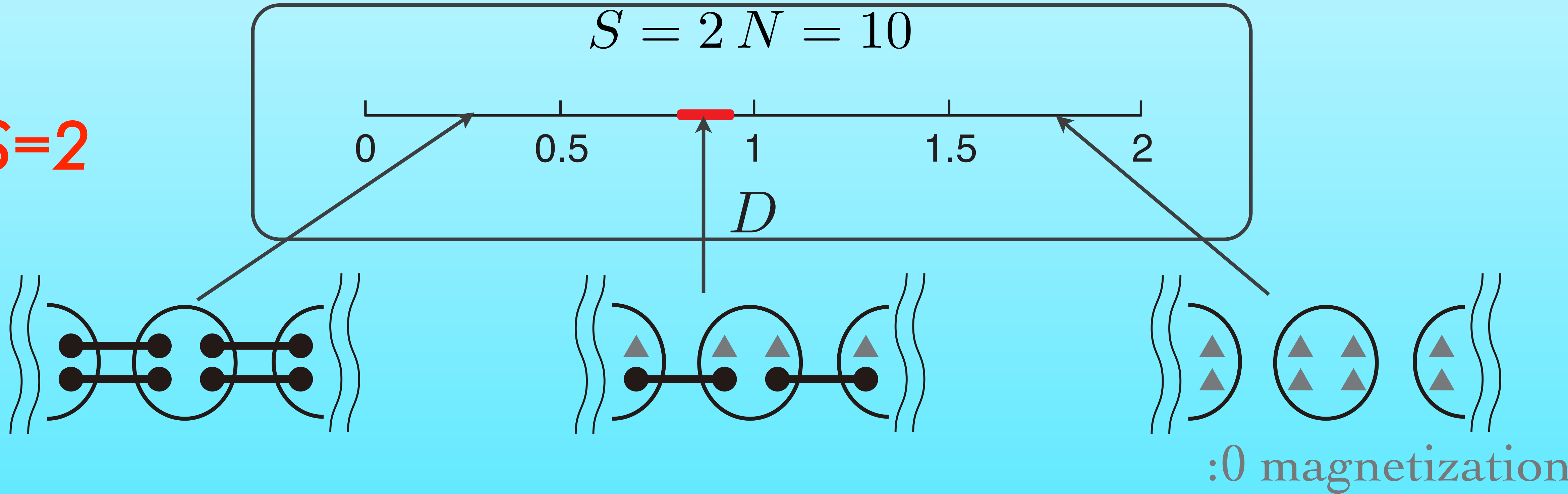
S=2 Heisenberg model with D term

T.Hirano, H.Katsura &YH, Phys.Rev.B77 094431'08

$$H = \sum_i^N \left[J \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1} + D \left(S_i^z \right)^2 \right]$$

Red line :Berry phase π

S=2



Reconstruction of valence bonds!

Generic AKLT (VBS) models

T.Hirano, H.Katsura &YH, Phys.Rev.B77 094431'08

Twist the link of the generic AKLT model

$$H(\{\phi_{i,i+1}\}) = \sum_{i=1}^{N} \sum_{J=B_{i,i+1}+1}^{2B_{i,i+1}} A_J P^J_{i,i+1}[\phi_{i,i+1}]$$

VB

$$|\{\phi_{i,j}\}\rangle = \prod_{\langle ij\rangle} \left(e^{i\phi_{ij}/2} a_i^\dagger b_j^\dagger - e^{-i\phi_{ij}/2} b_i^\dagger a_j^\dagger\right)^{B_{ij}} |\mathrm{vac}\rangle$$

Berry phase on a link (ij)

$$\gamma_{ij} = B_{ij}\pi \mod 2\pi$$

S=1/2

The Berry phase counts the number of the valence bonds!

S=1/2 objects are fundamental in integer spin chains

Random hopping model on bipartite lattice

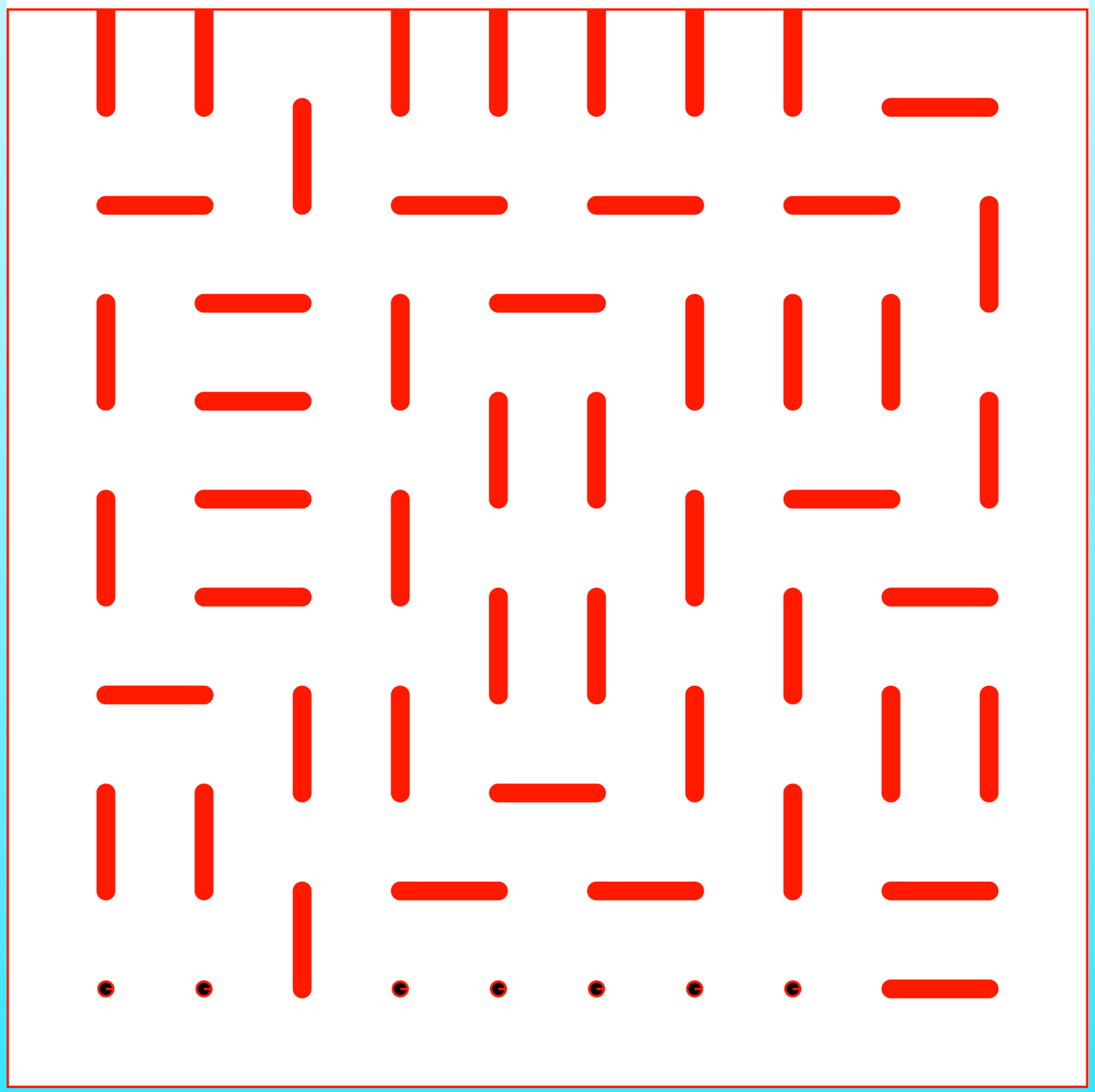
$$H = \sum_{\langle ij \rangle} t_{ij} c_i^\dagger c_i + h.c. + V_{ij} n_i n_j$$

P.H. symmetry in many body

Half-filled many body state

Chiral symmetry in one particle part

t′/t=0.6



$V_{ij} = 0$

$$\gamma_C = \pi$$

Y.H., J. Phys. Soc. Jpn. 75 123601 (2006)

Random hopping model on bipartite lattice

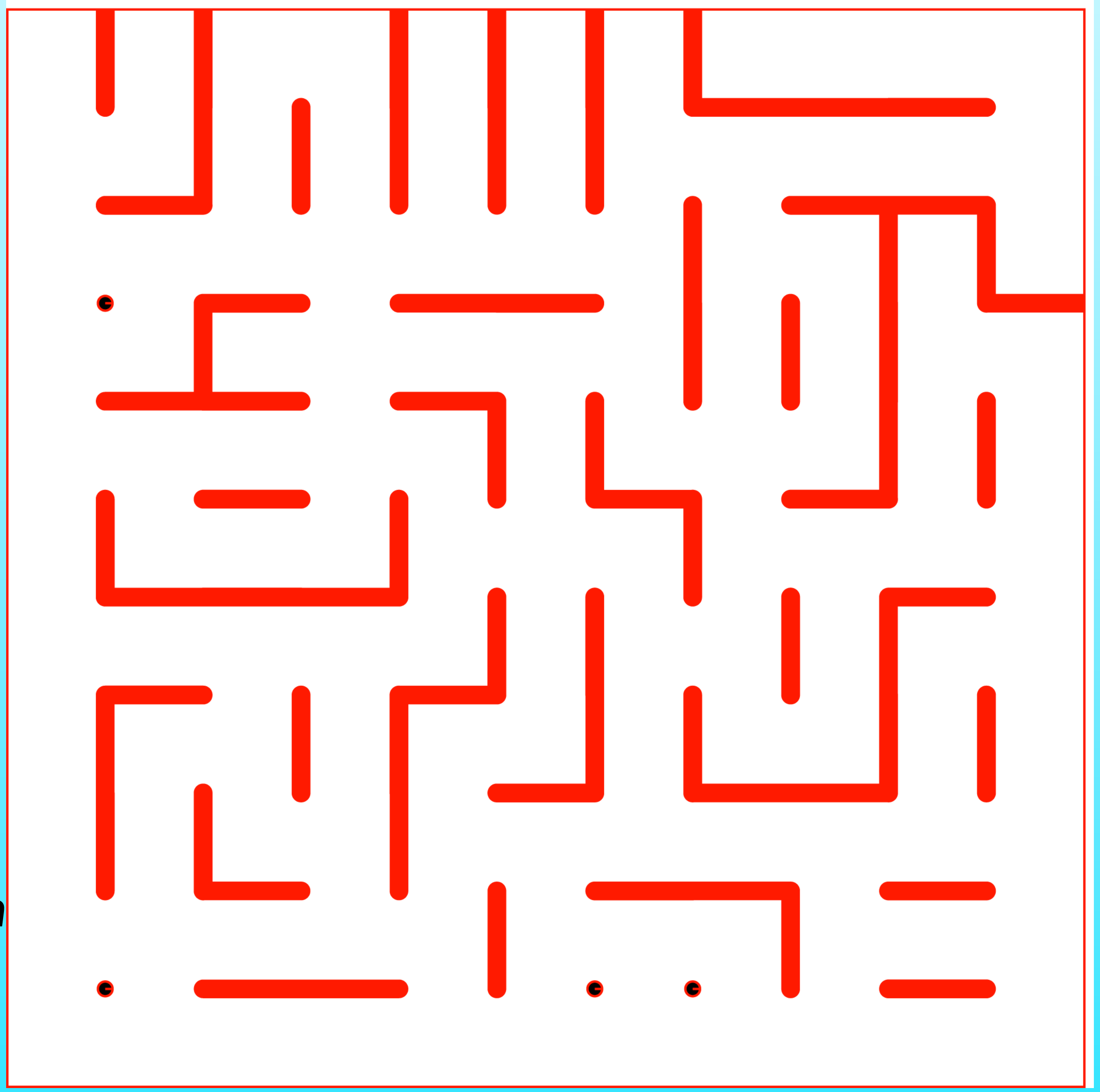
$$H = \sum_{\langle ij \rangle} t_{ij} c_i^\dagger c_i + h.c. + V_{ij} n_i n_j$$

P.H. symmetry in many body

Half-filled many body state

Chiral symmetry in one particle part

t′/t=0.7



$V_{ij} = 0$

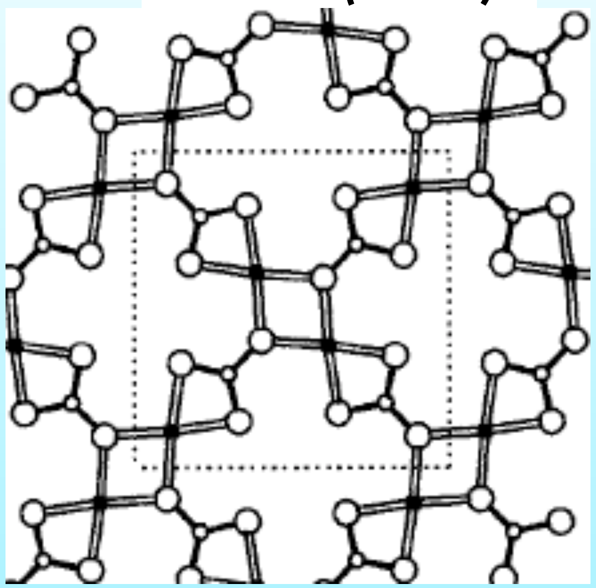
$\gamma_C = \pi$

Quantum Phase Transition
with (local) Gap Closing

Y.H., J. Phys. Soc. Jpn. 75 123601 (2006)

Orthogonal dimers

$SrCu_2(BO_3)_2$

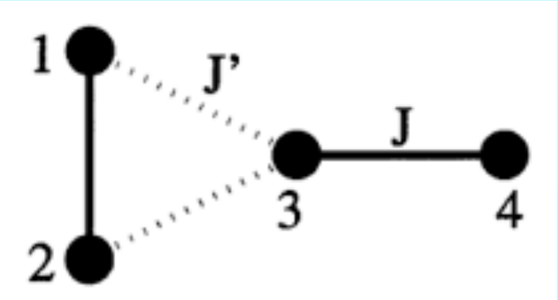


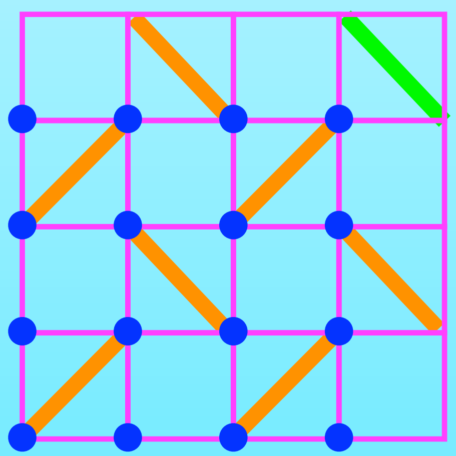
- discovery

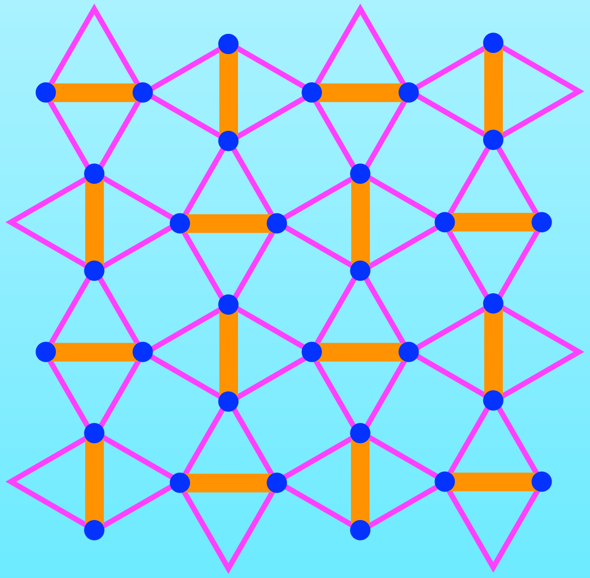
H. Kageyama et al. , Phys. Rev. Lett. 82, 3168 (1999)

- Theory: spin gap & magnetic plateaus

B. S. Shastry and B. Sutherland, Physica, 108B, 1069 (1981).







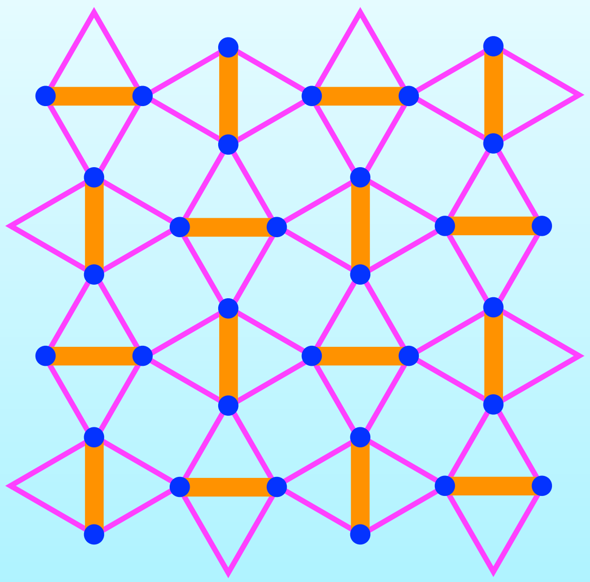
$$H = J \sum_{\langle ij \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + J' \sum_{\langle ij \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

S. Miyahara & K. Ueda , Phys. Rev. Lett. 82, 3701 (1999)

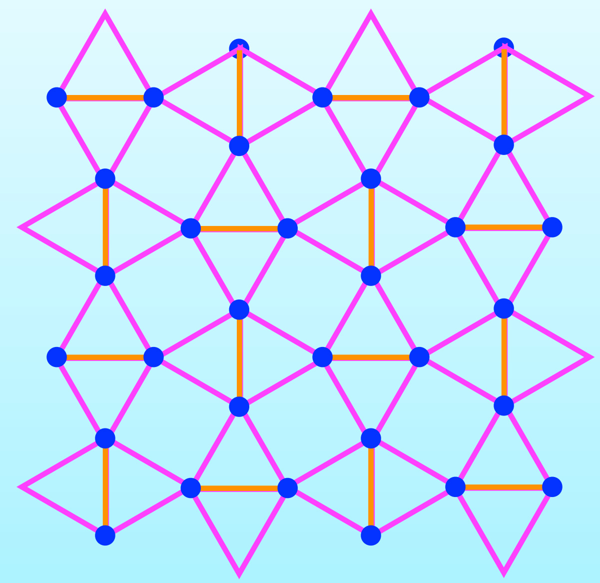
T. Momoi and K. Totsuka, Phys. Rev. B 61, 3231 (2000)

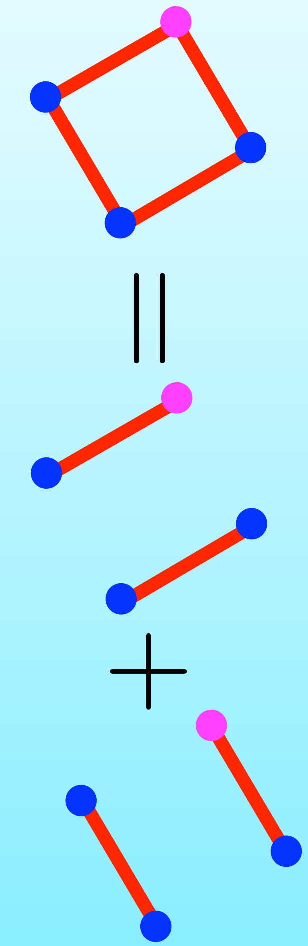
Gapped to gapped transition

Dimer phase



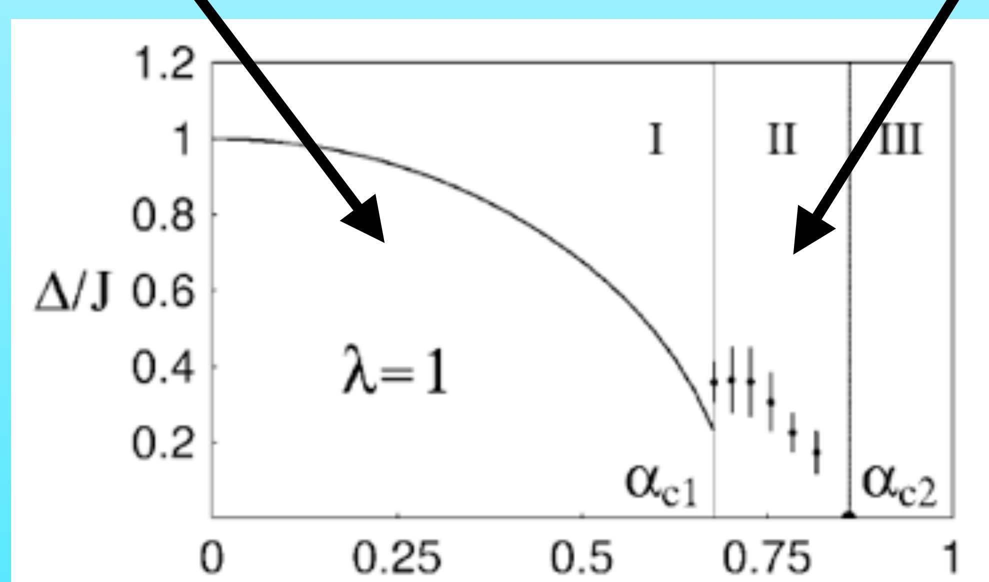
Plaquette singlet phase





$J >> J'$

$J \approx J'$



$$H = J \sum_{\langle ij \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + J' \sum_{\langle ij \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

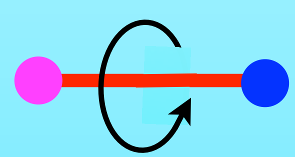
A. Koga & N. Kawakami, Phys. Rev. Lett. 84, 4461 (2000)

Orthogonal dimers

Dimer phase

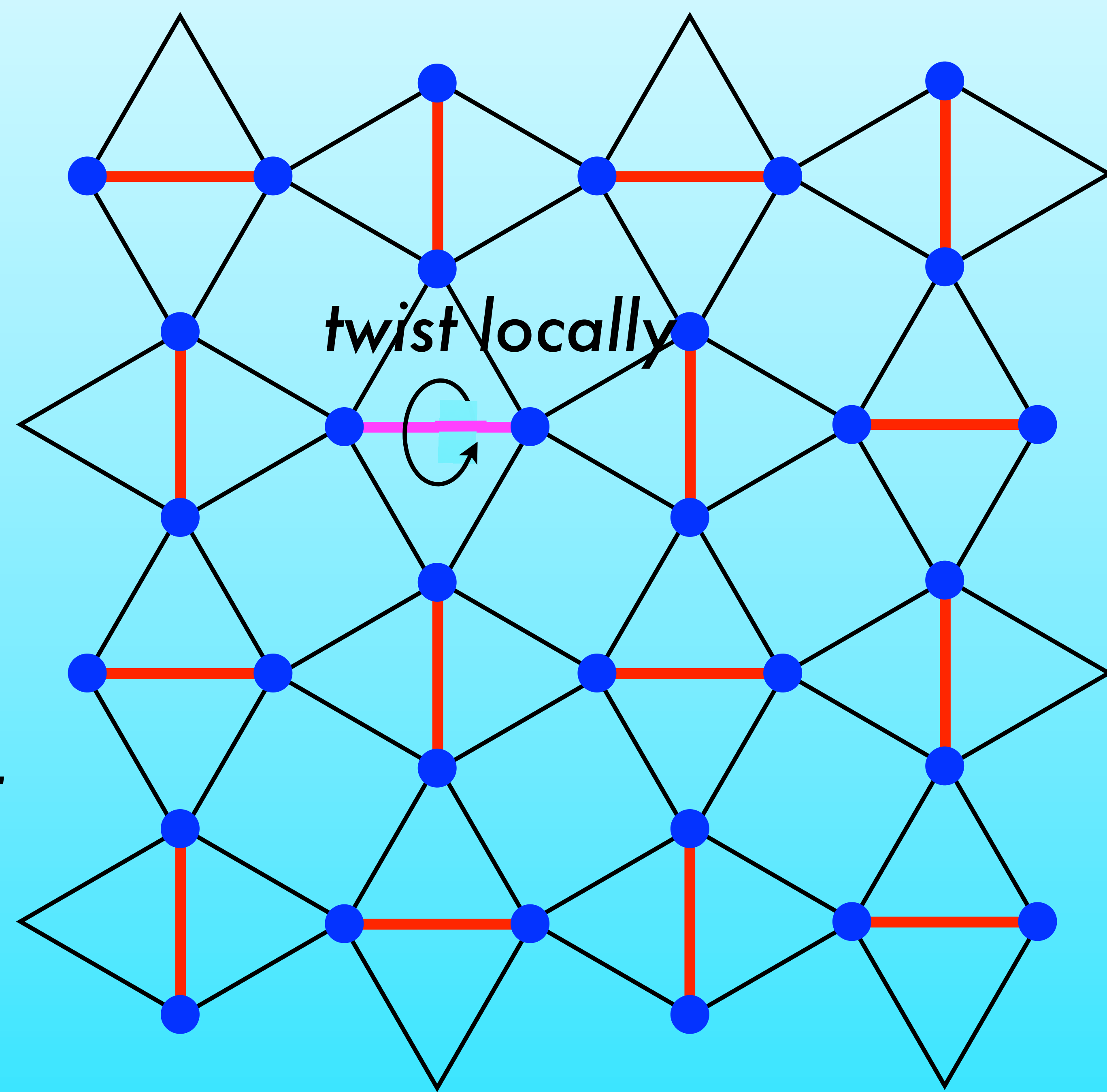
Gauge transform only at ●

$$U(\theta) = e^{i(S-\hat{S}_z)\theta}$$



It can be gauged out if decoupled

$$\gamma_{\mathrm{P}} = \pi$$

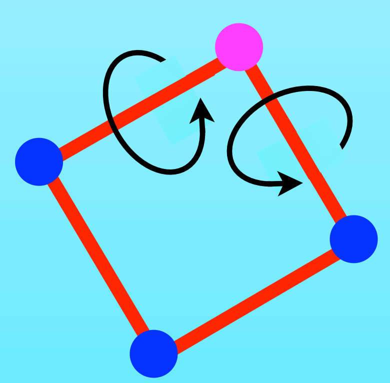


Orthogonal dimers

plaquette singlet phase

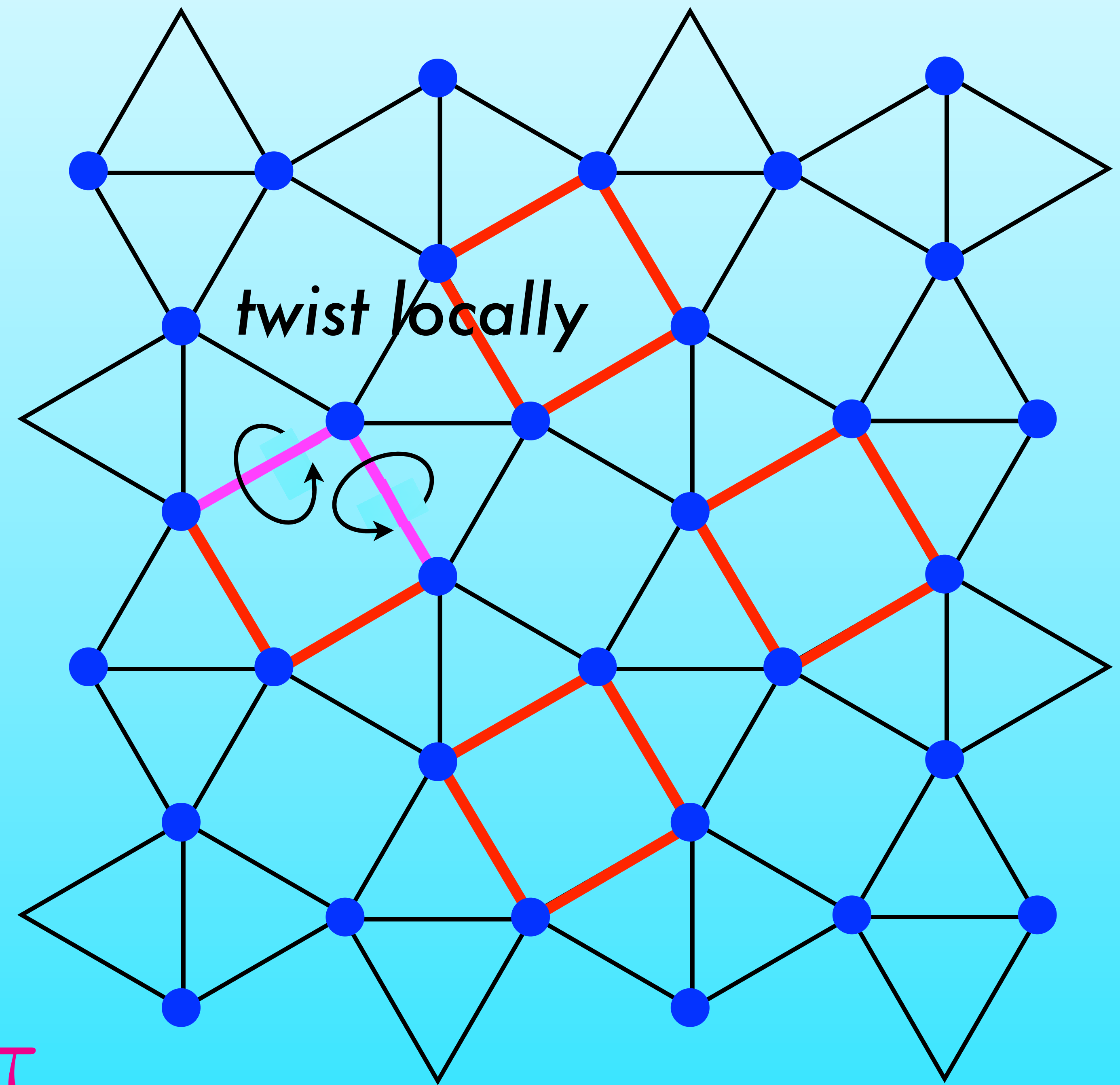
Gauge transform only at ●

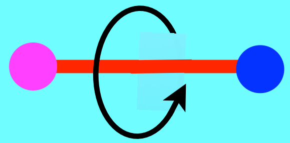
$$U(\theta) = e^{i(S - \hat{S}_z)\theta}$$



It can be gauged out if decoupled

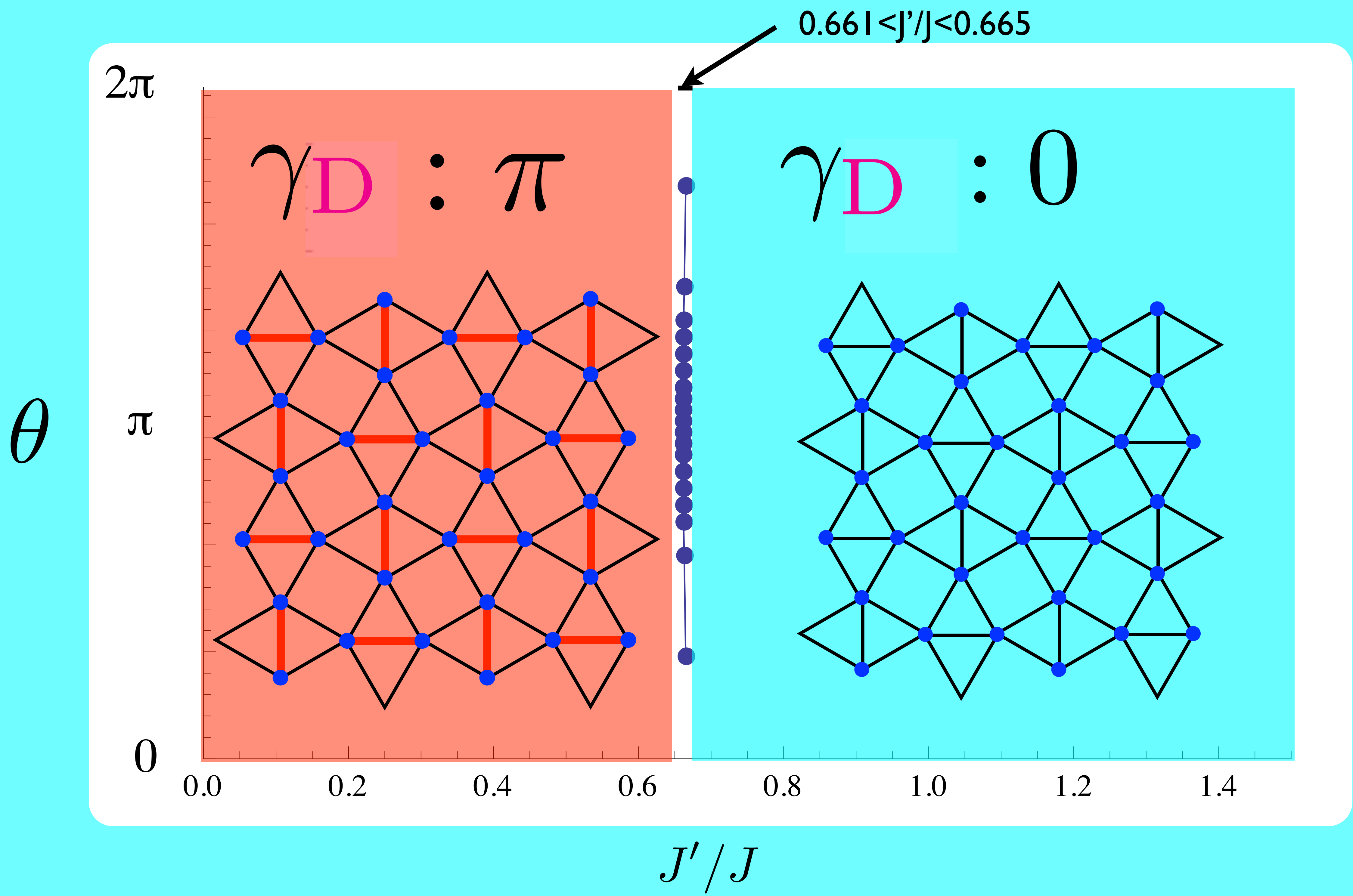
$$\gamma_{\mathrm{p}} = \pi$$



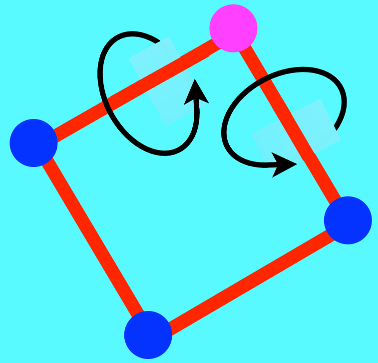


Z_2 Berry phase γ_D

gauge twist for singlet pair

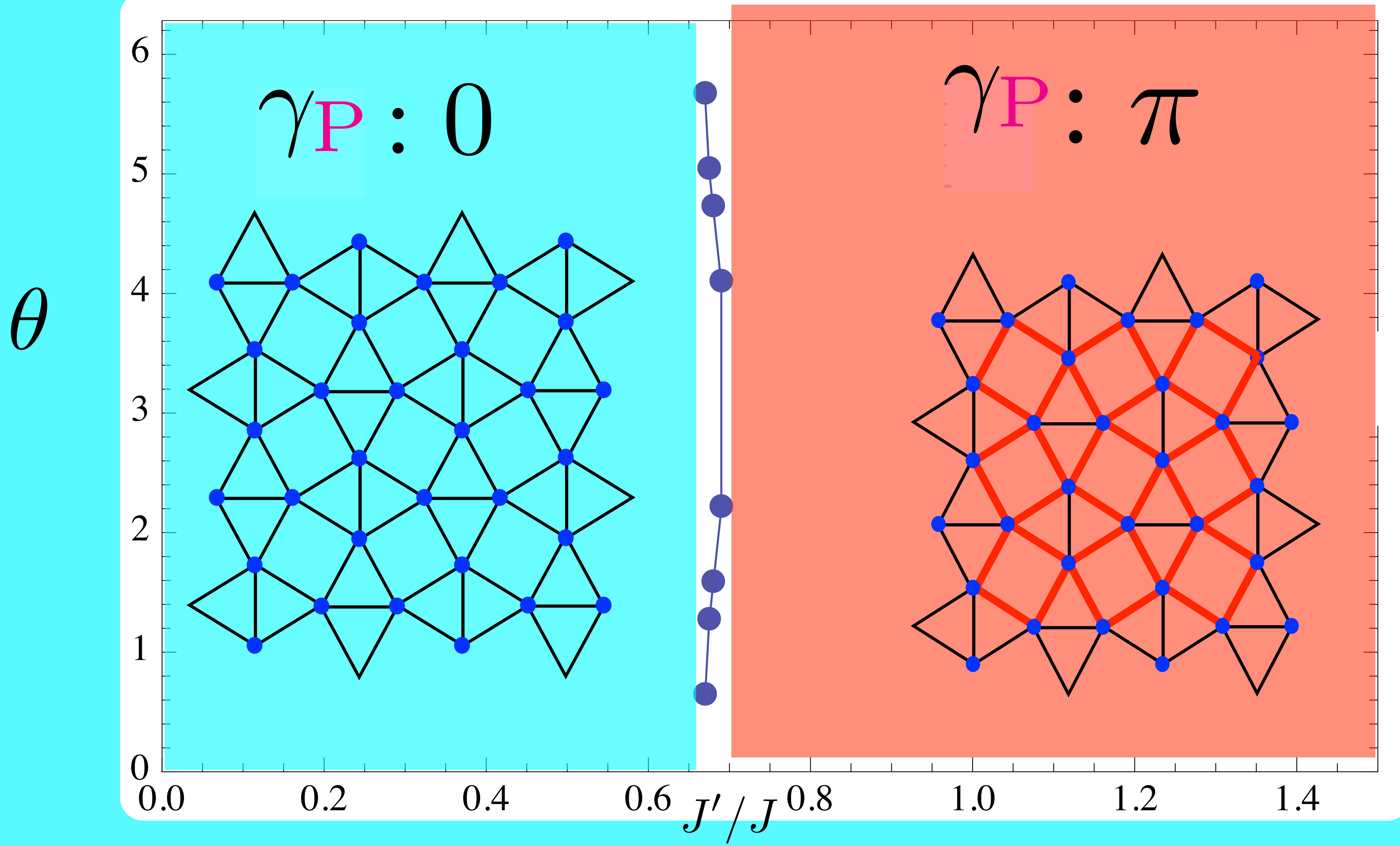


I. Maruyama, S. Tanaya, M.Arikawa & YH. , arXiv:1103.1226



Z_2 Berry phase γ_P

gauge twist for plaquette singlet



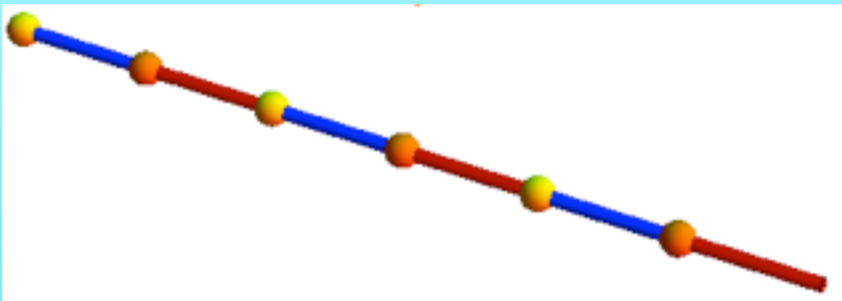
I. Maruyama, S. Tanaya, M.Arikawa & YH. , arXiv:1103.1226

⭐ Fermions with frustrated lattice

Generalized to Z_Q (Q=d+1)

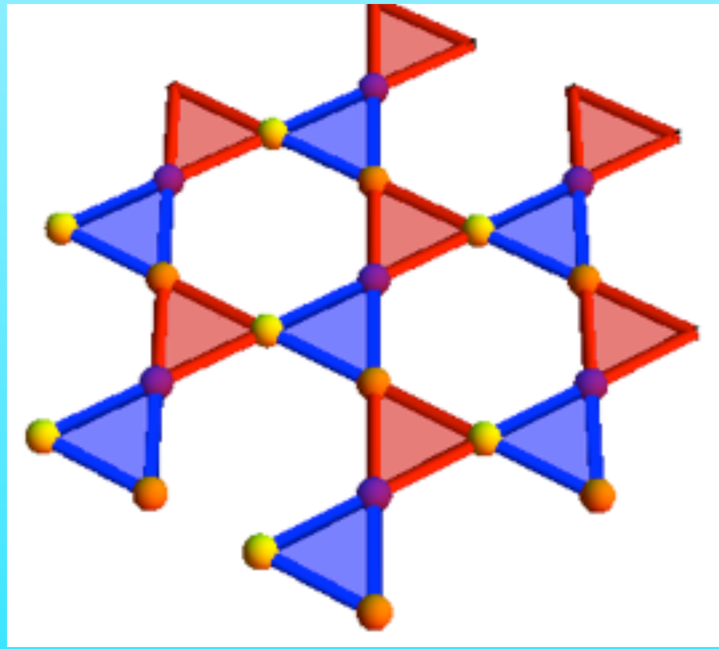
Z_Q Berry phases

Series of fermionic models in d-dimensions
Minimum model with frustration

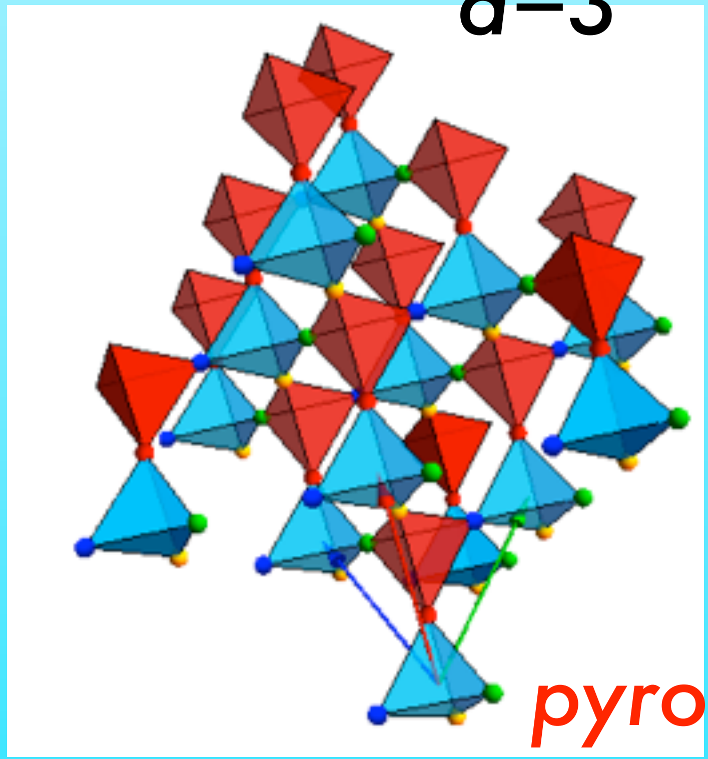


d=1

d=2



d=3



pyrochlore

d=4
...

Y. Hatsugai & I. Maruyama, EPL 95, 20003 (2011), arXiv:1009.3792

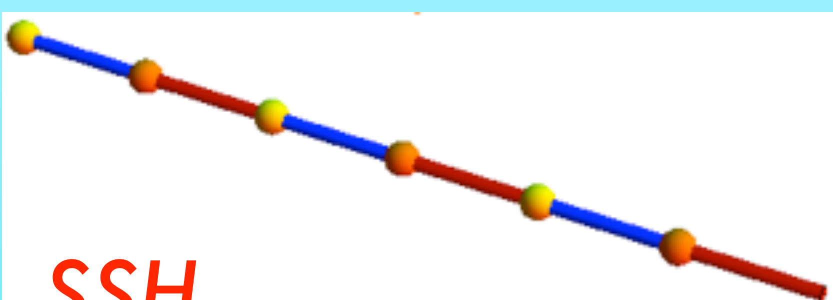
⭐ Fermions with frustrated lattice

Generalized to Z_Q (Q=d+1)

Z_Q Berry phases

Series of fermionic models in d-dimensions

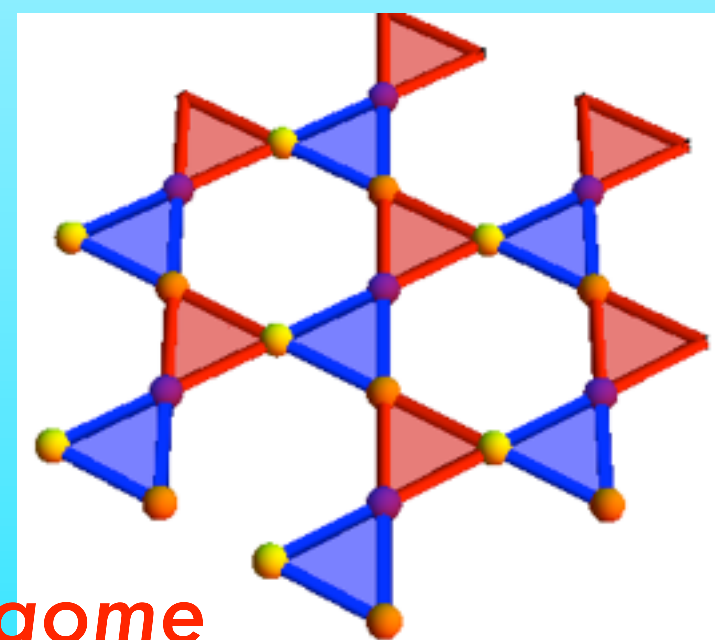
Minimum model with frustration



SSH

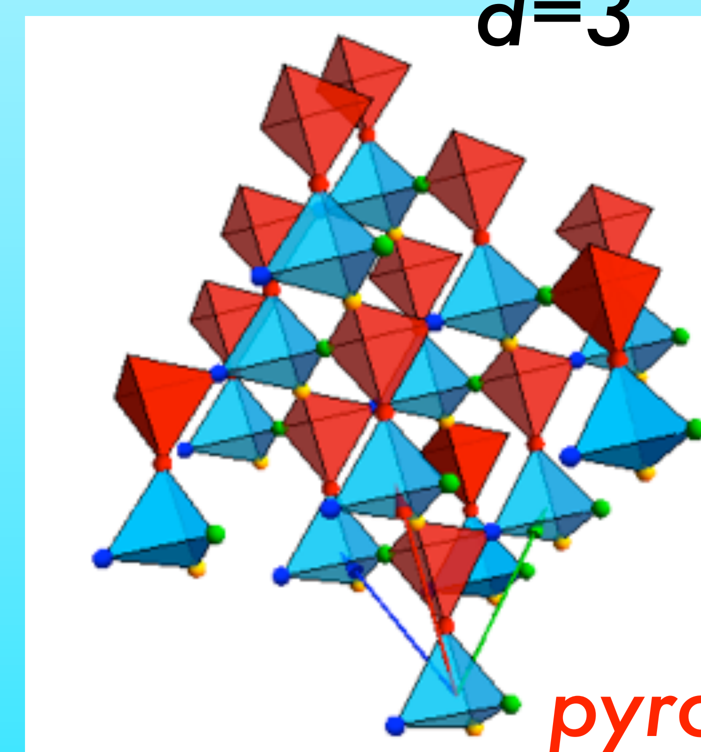
d=1

d=2



kagome

d=3



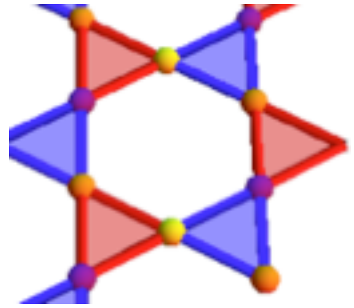
pyrochlore

d=4

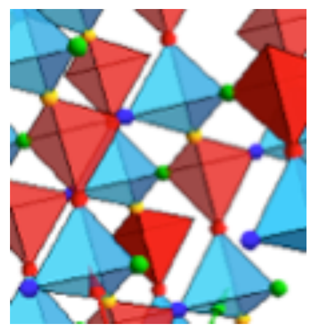
...

Y. Hatsugai & I. Maruyama, EPL 95, 20003 (2011), arXiv:1009.3792

Fermionic Hamiltonian with "dimerization"



$$H = \sum_{\langle ij \rangle} t_{ij} c_i^\dagger c_j + h.c. - \mu \sum_i n_i$$



3D pyrochlore

$$H = \sum_{\langle ij \rangle} t_{ij} c_i^\dagger c_j + h.c. - \mu \sum_i n_i$$

$$+V \sum n_i n_j$$

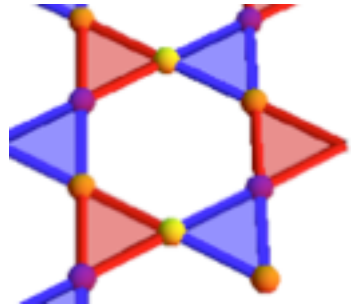
$$t_{ij} = \begin{cases} t_R & \langle ij \rangle \in \\ t_B & \langle ij \rangle \in \end{cases}$$

Tetramerization

One may include interaction if the energy gap remains open

d-D generic pyrochlore as well

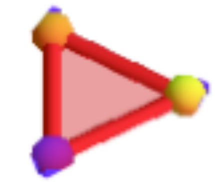
Fermionic Hamiltonian with "dimerization"

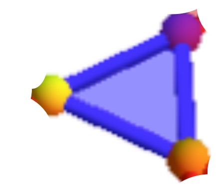


2D kagome

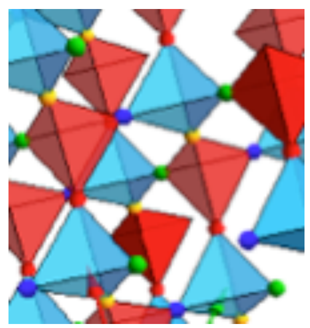
$$H = \sum_{\langle ij \rangle} t_{ij} c_i^\dagger c_j + h.c. - \mu \sum_i n_i$$

$$t_{ij} = \begin{cases} t_R & \langle ij \rangle \in \\ t_B & \langle ij \rangle \in \end{cases}$$





Trimerization



3D pyrochlore

$$H = \sum_{\langle ij \rangle} t_{ij} c_i^\dagger c_j + h.c. - \mu \sum_i n_i$$

$$+V \sum n_i n_j$$

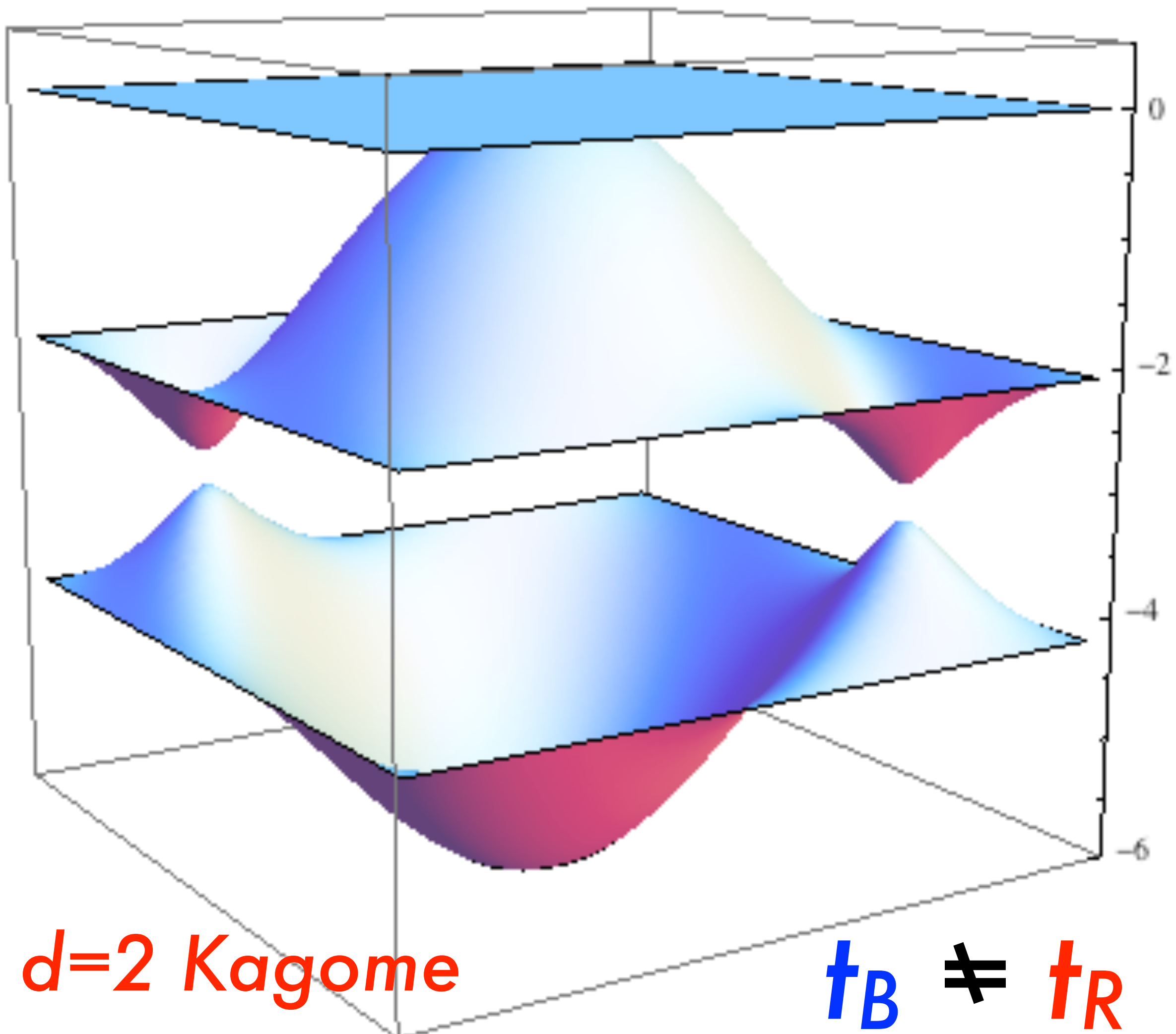
$$t_{ij} = \begin{cases} t_R & \langle ij \rangle \in \\ t_B & \langle ij \rangle \in \end{cases}$$

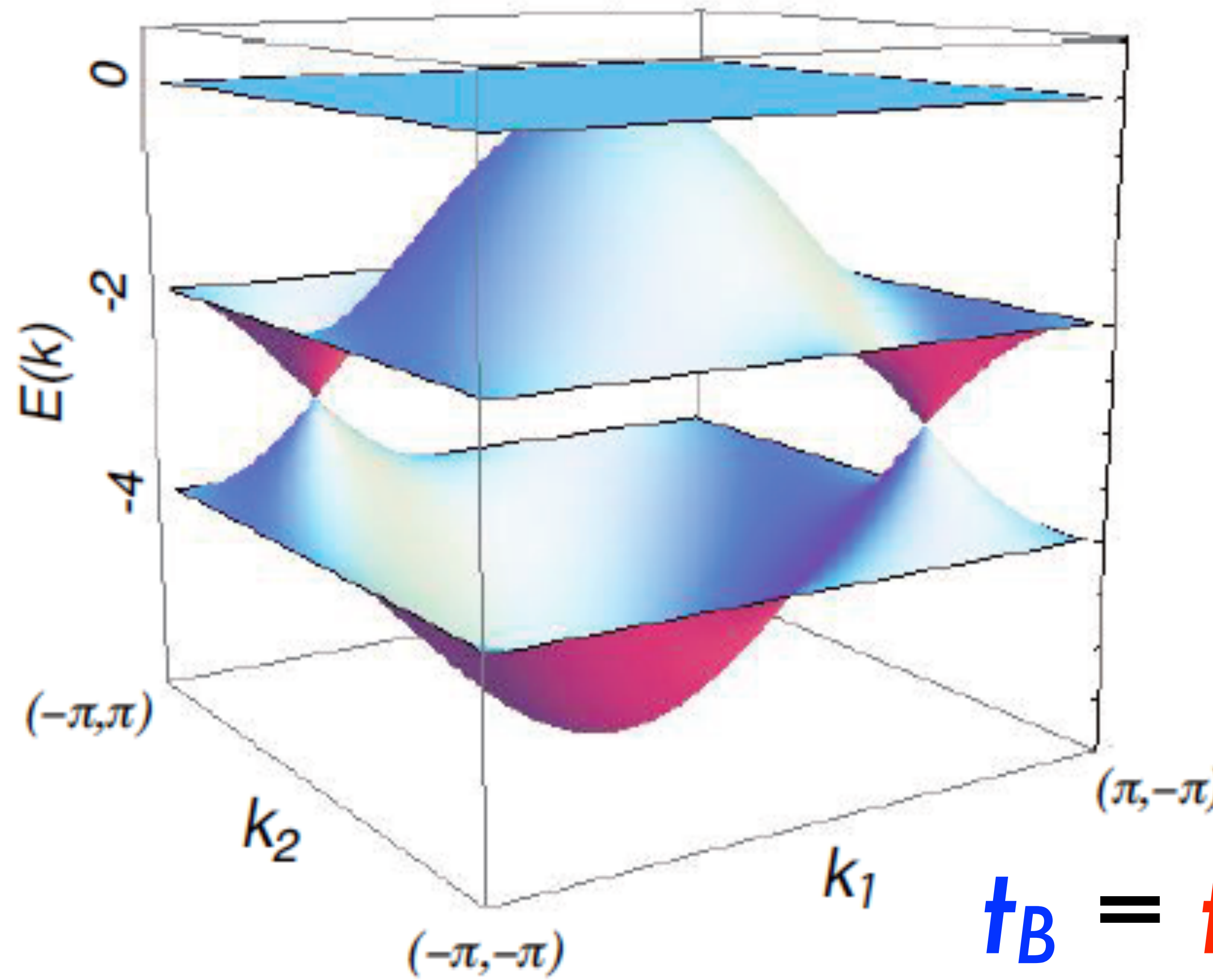
Tetramerization

One may include interaction if the energy gap remains open

d-D generic pyrochlore as well

Dirac fermions + flat bands with d-1 fold degeneracy





d=2 Kagome

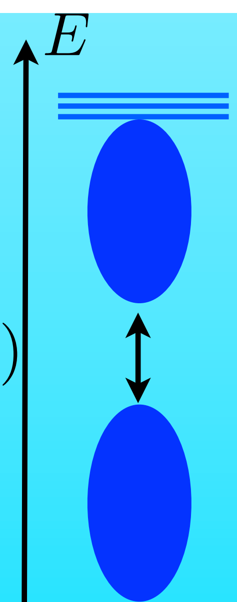
Q=d+1=3

$t_B \neq t_R$

$t_B = t_R$

$t_B \neq t_R$

$E_g = |t_B - t_R|(Q/2)$

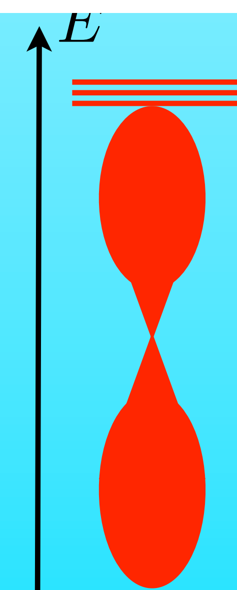


$t_B > t_R$

Quantum Phase Transition

$t_B < t_R$

$t_B = t_R$



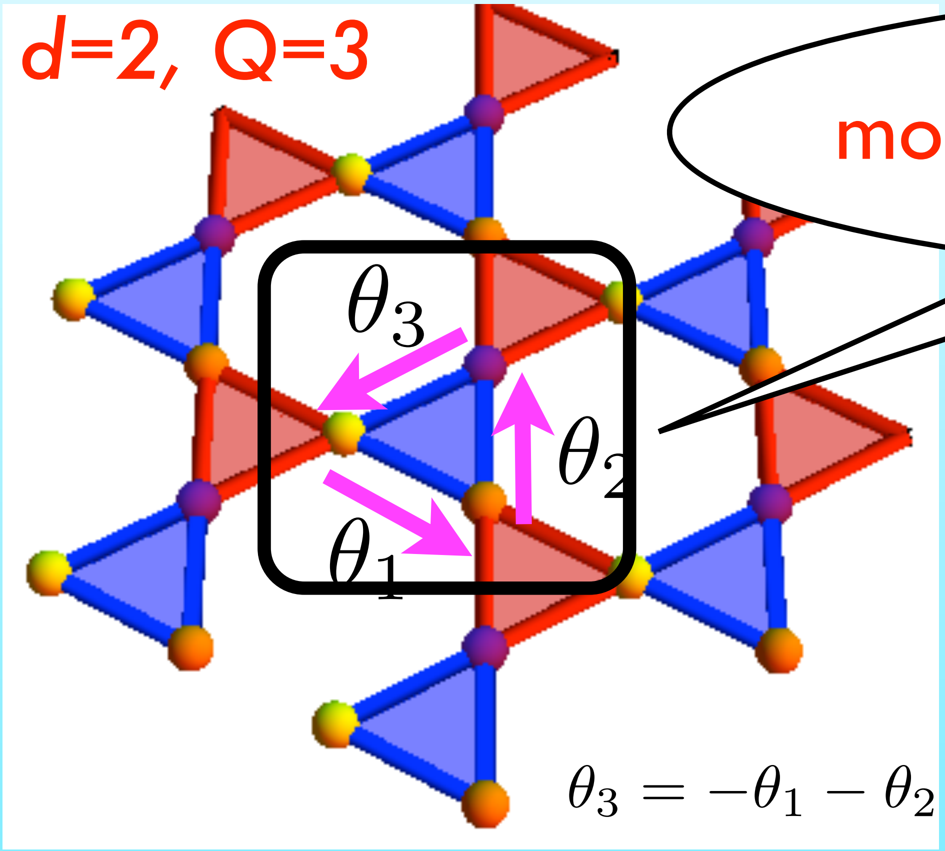
$k = 0$: touching momentum

Massless Dirac, Critical

Ex.) $Z_{Q=3}$ quantized Berry phases for fermions on Kagome

$\Theta = (\theta_1, \theta_2, \theta_3)$

periodic boundary condition



modify phases locally (in some way)

Global Z_Q symmetry with twists Θ

Many body state

$|\Psi(\Theta)\rangle$ filling 1/Q

$$A = \langle \Psi(\Theta) | d\Psi(\Theta) \rangle$$

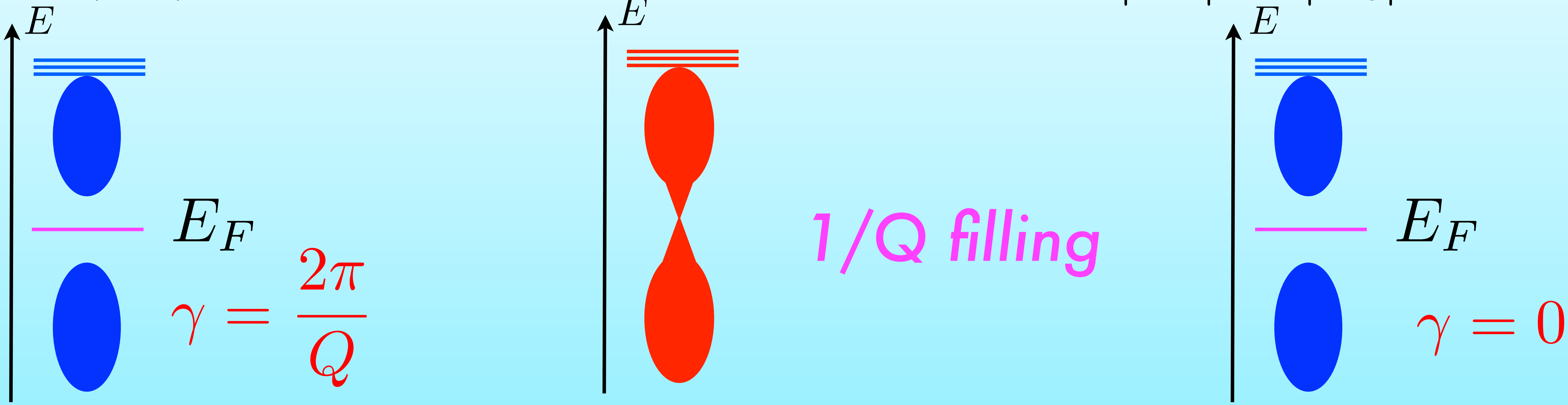
$$i\gamma = \int_L A$$

$$\gamma \equiv 2\pi \frac{n}{Q}, \ \mathrm{mod}\, 2\pi, \ n \in \mathbb{Z}$$

Z_Q quantization

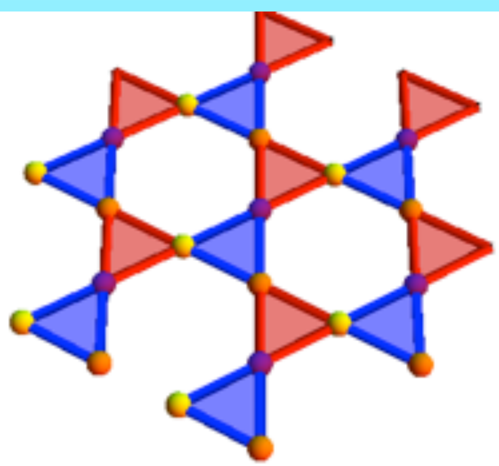
Topological order parameter for Q-Multimerization Transition

$|t_B| < |t_R|$ $\qquad$ $t_B = t_R$ $\qquad$ $|t_B| > |t_R|$

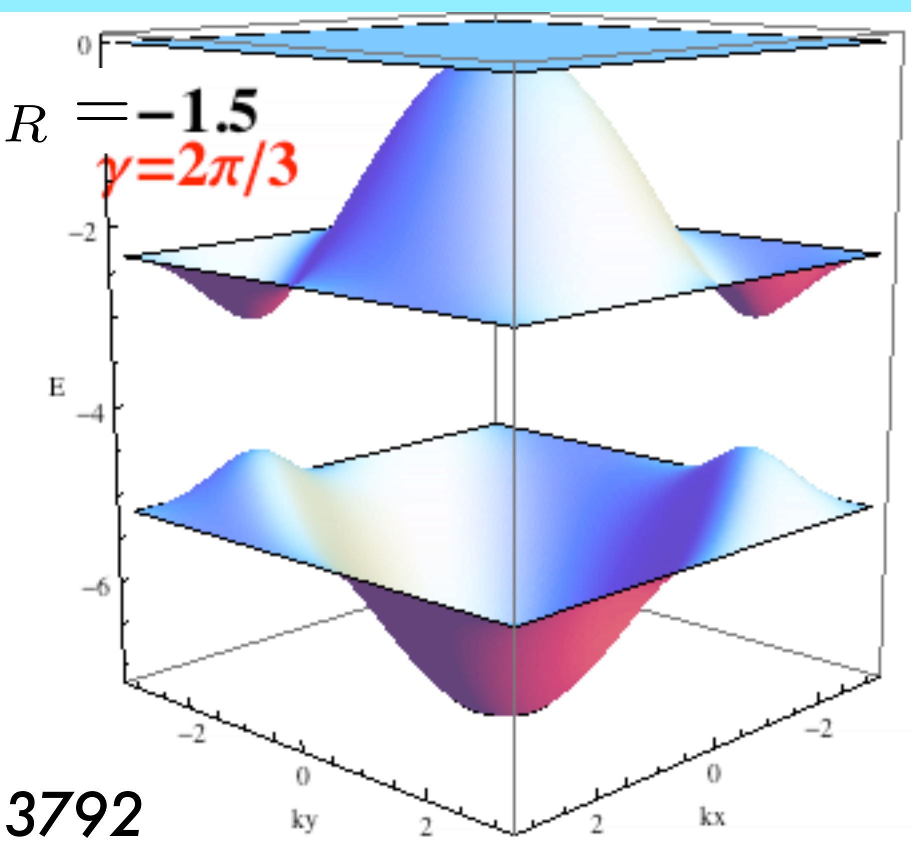


Massless Dirac, Critical

d=2, Q=3, Kagome



$t_B = -1, t_R = -1.5$

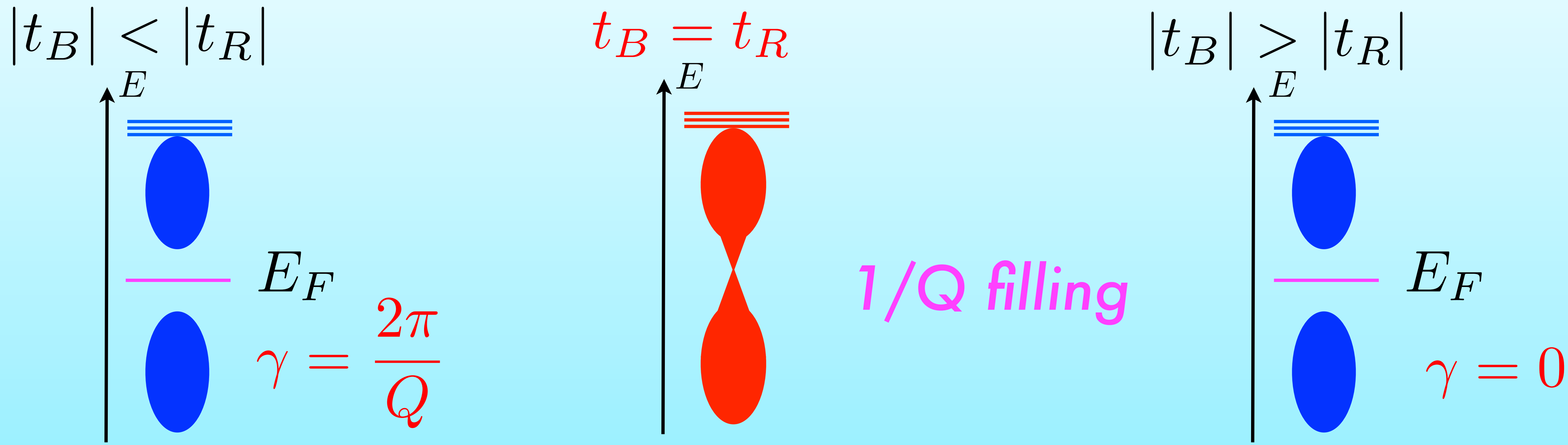


Quantum Phase Transition

Q-Multimerization

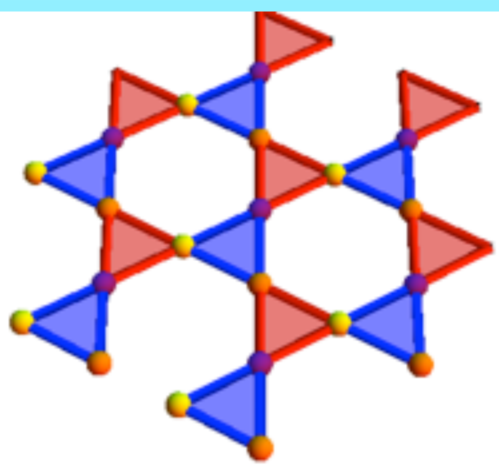
Y. Hatsugai & I. Maruyama, EPL 95, 20003 (2011), arXiv:1009.3792

Topological order parameter for Q-Multimerization Transition

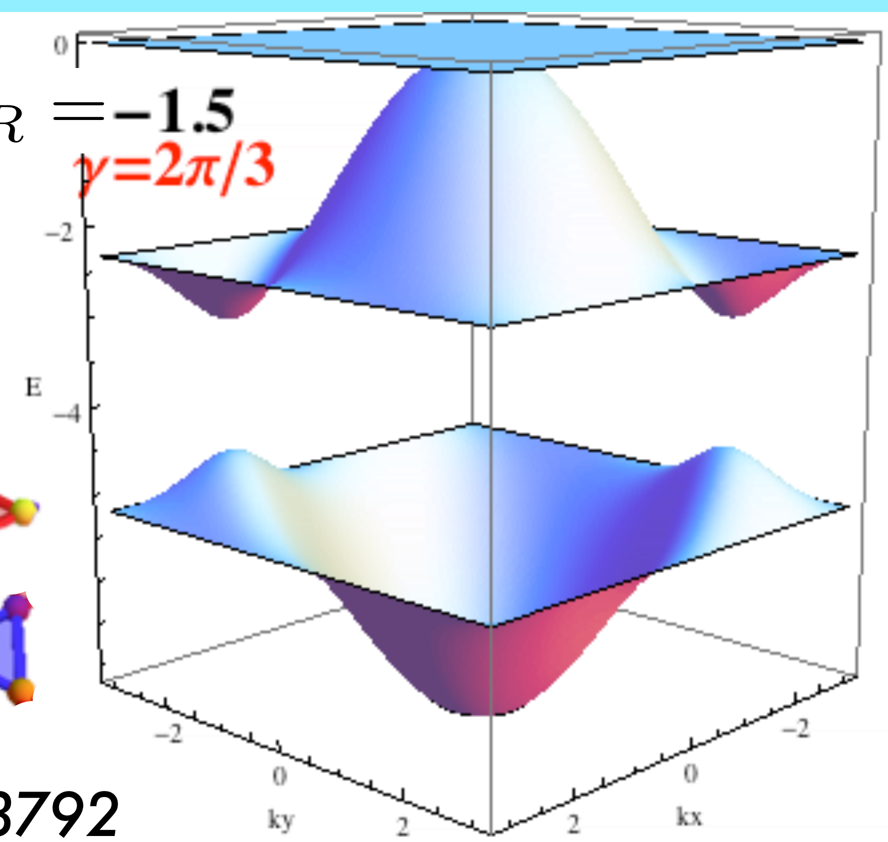


Massless Dirac, Critical

d=2, Q=3, Kagome

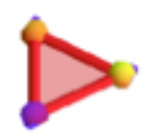
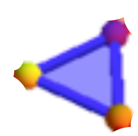


$t_B = -1, t_R = -1.5$



Quantum Phase Transition

$$t_{ij} = \begin{cases} t_R & \langle ij \rangle \in \\ t_B & \langle ij \rangle \in \end{cases}$$

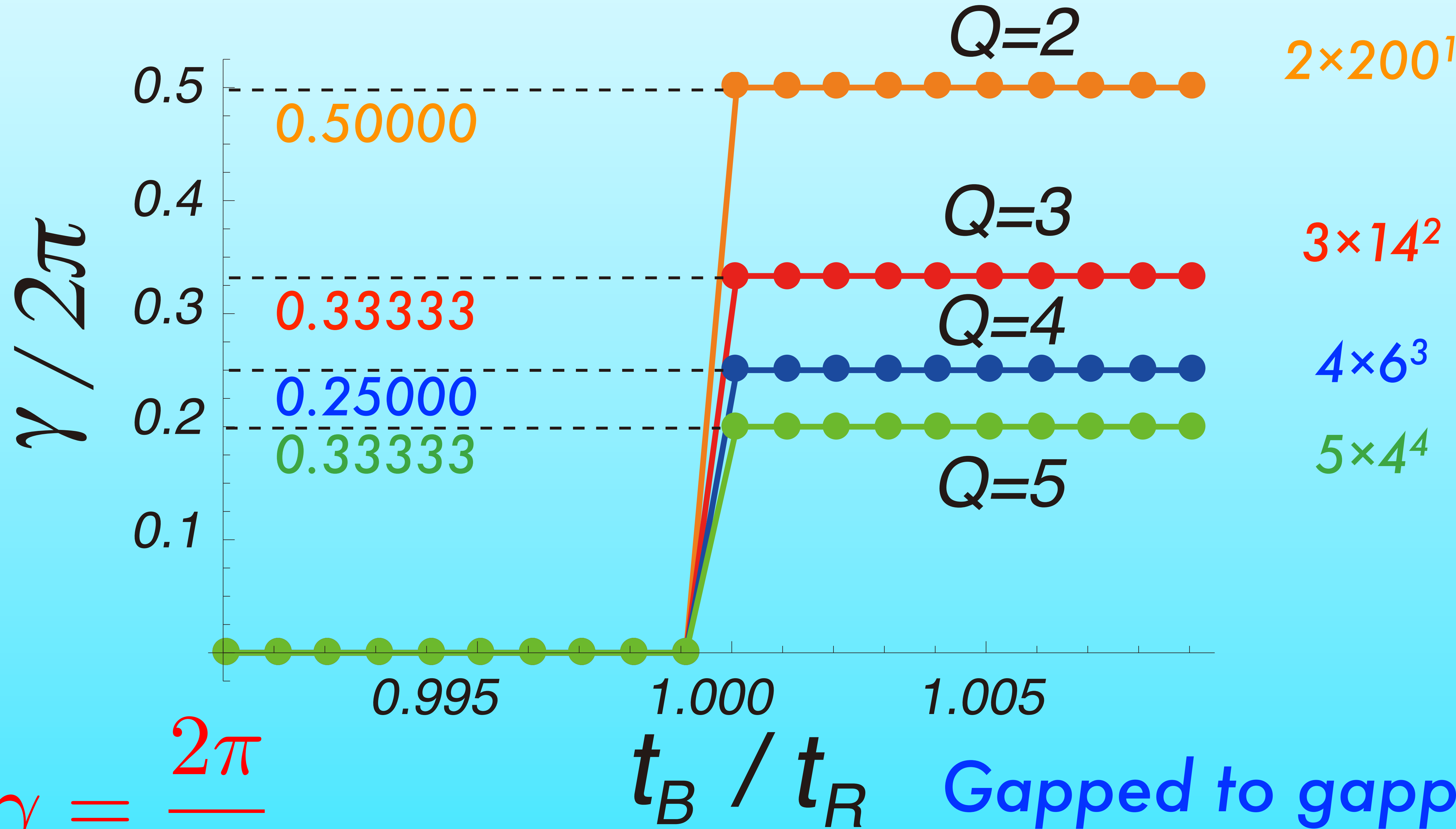
Q-Multimerization

Y. Hatsugai & I. Maruyama, EPL 95, 20003 (2011), arXiv:1009.3792

Numerical demonstration up to 4D

Quantized dimer order parameter

Quantum Phase Transition



Gapped to gapped

$$\gamma = \frac{2\pi}{Q}$$

$$Q = d + 1$$

1D, 2D, 3D, 4D, d-Dim

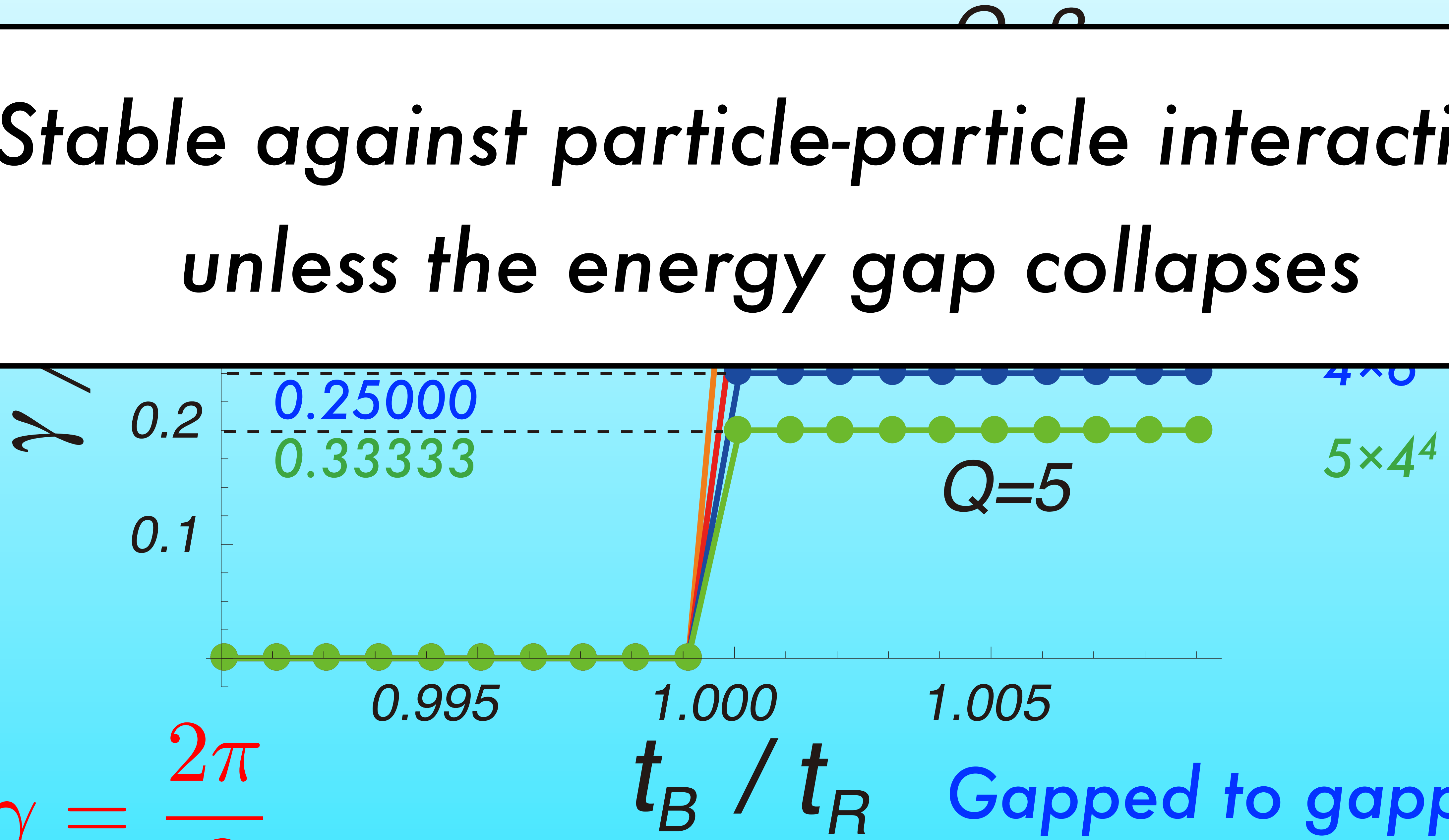
Y. Hatsugai & I. Maruyama, EPL 95, 20003 (2011), arXiv:1009.3792

Numerical demonstration up to 4D

Quantized dimer order parameter

Quantum Phase Transition

Stable against particle-particle interaction unless the energy gap collapses



Gapped to gapped

1D, 2D, 3D, 4D, d-Dim

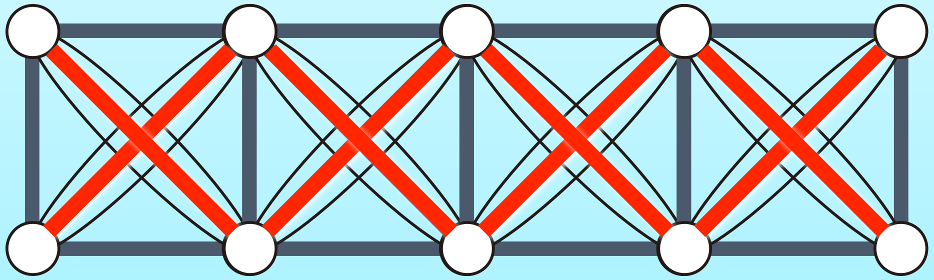
$$\gamma = \frac{2\pi}{Q}$$

$$Q = d + 1$$

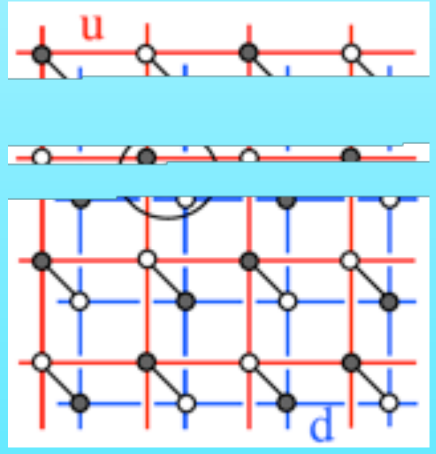
Y. Hatsugai & I. Maruyama, EPL 95, 20003 (2011), arXiv:1009.3792

Other systems applied

Spin ladders with ring exchange

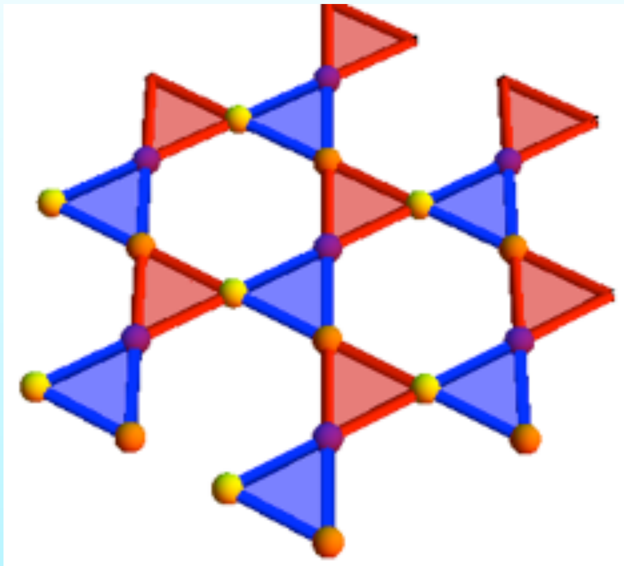


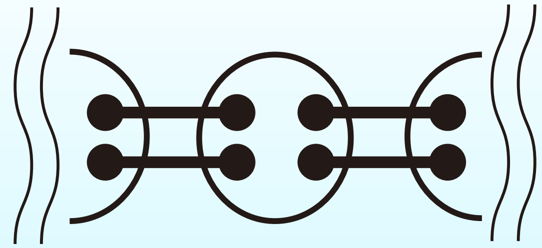
I. Maruyama, T. Hirano, and Y. H.,Phys. Rev. B 79, 115107 (2009)
M. Arikawa, S. Tanaya, I. Maruyama, Y. H.,Phys. Rev. B 79, 205107 (2009)

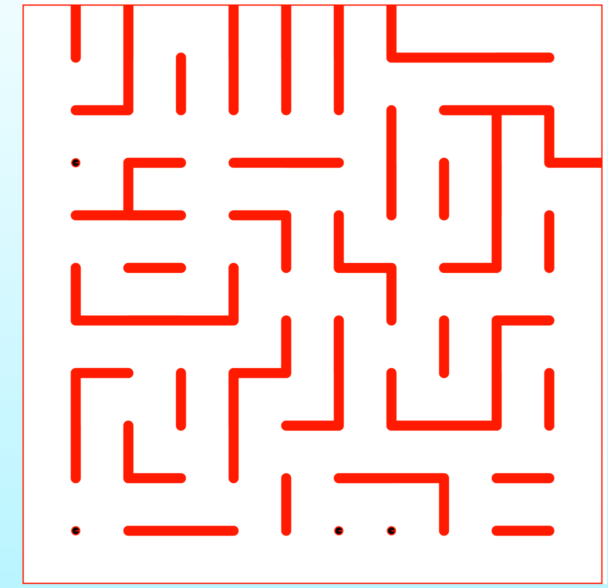


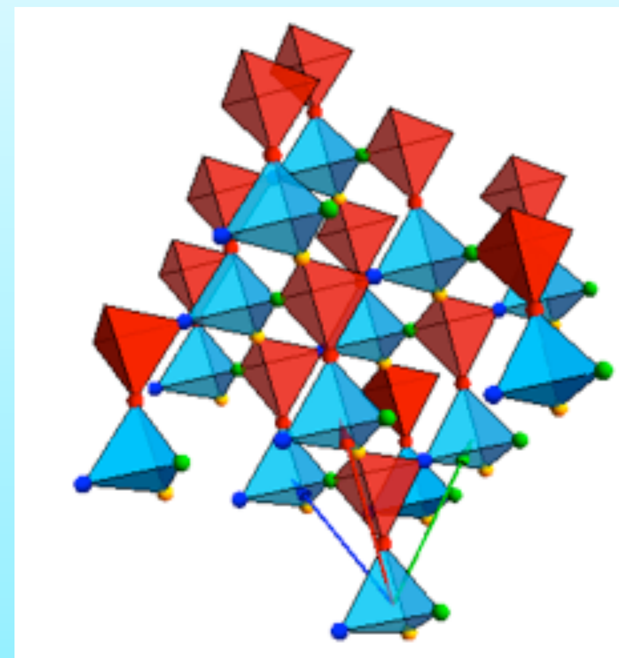
BEC-BCS crossover at half filling

M. Arikawa, I. Maruyama, and Y. H., Phys. Rev. B 82, 073105 (2010)

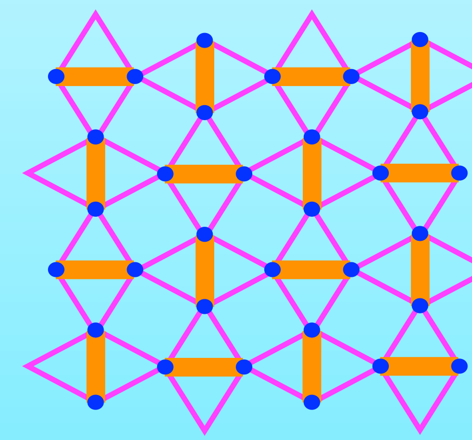




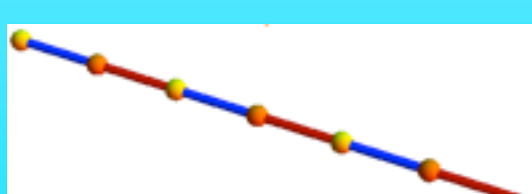


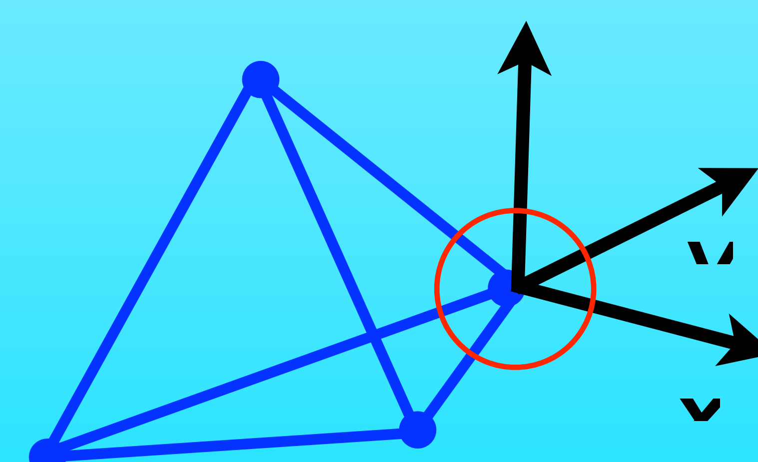


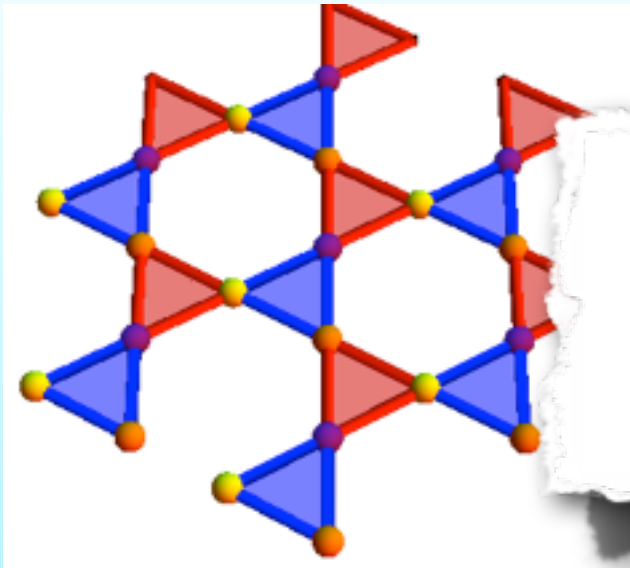
Summary

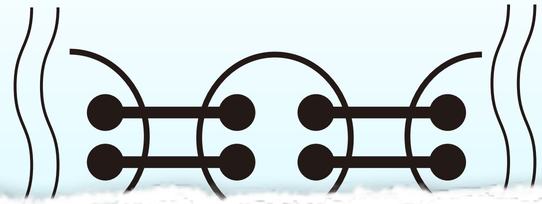


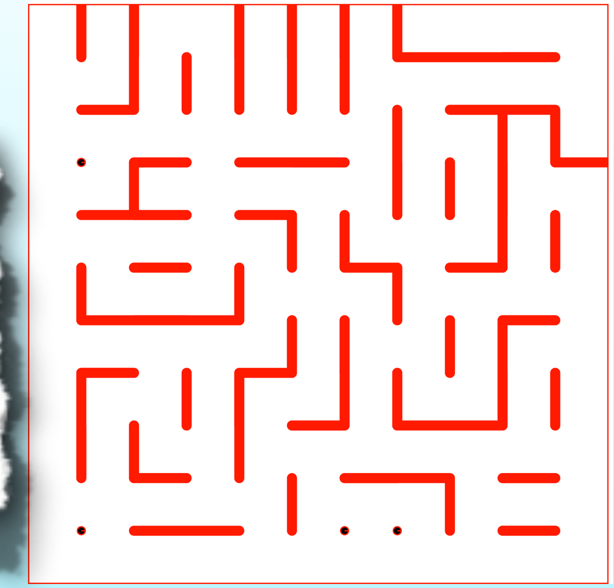
Topology is useful to classify short range entangled states with symmetry



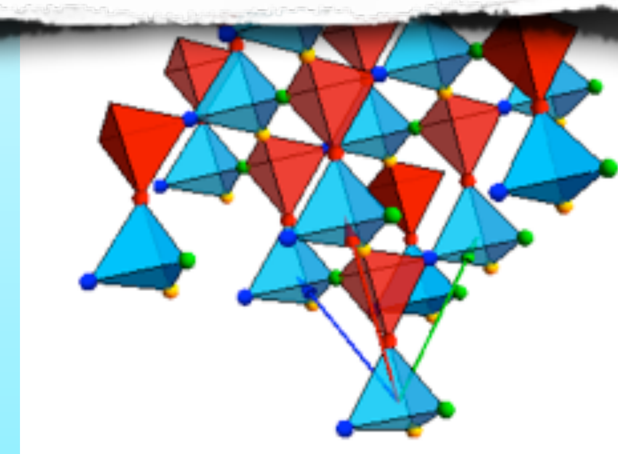




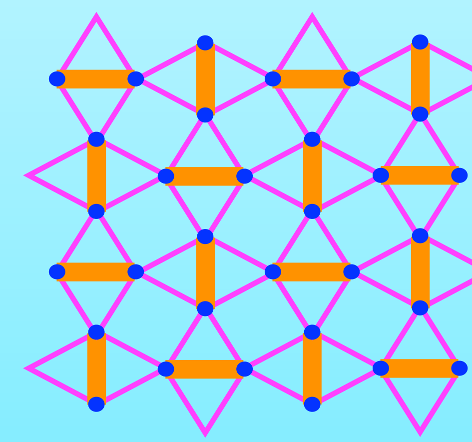




Thank you



Summary



Topology is useful to classify short range entangled states with symmetry