Anomalous Hall Effect in a Multiband Chiral Superconductor (e.g. Sr_2RuO_4)

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Ed Taylor and CK, arXiv:111.4471

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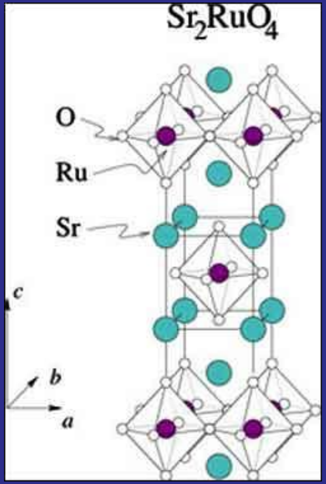
Anomalous or spontaneous Hall Effect

- AHE also occurs in topological insulators and metals
- Superconducting case is different. dc effect (in $Re(\sigma_H)$) is not quantized and not described by a Berry's curvature.
- Results apply in general to multiband chiral superconductors, but will focus on Sr_2RuO_4 and chiral p-wave, where results are relevant to Kerr effect at optical frequencies.

Experiments on Sr_2RuO_4

Evidence for odd-parity, triplet pairing:

- NMR [K. Ishida et al., *Nature* **396**, 658 (1998)]
- phase sensitive [Nelson et al., Science (2004)]
- half-quantum vortices [R. Budakian et al. (2011)]



Evidence for broken time-reversal symmetry:

- muSR [Luke et al., *Nature* **394**, 558 (1998).]
- polar Kerr effect [J. Xia et al., PRL 97, 167002 (2006.]
- Josephson [F. Kidwingira et al., Science 314, 1271 (2006)]

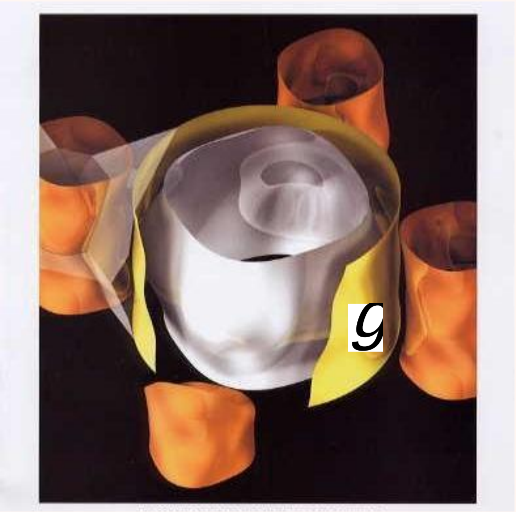
Triplet with BTRS + SRO xtal symmetry + energetics → chiral p-wave order

Chiral p-wave superconductivity

$$\mathbf{d}(k) = \mathrm{D}_0 \frac{\sin k_x \pm i \sin k_y}{k_F} \mathbf{z}$$

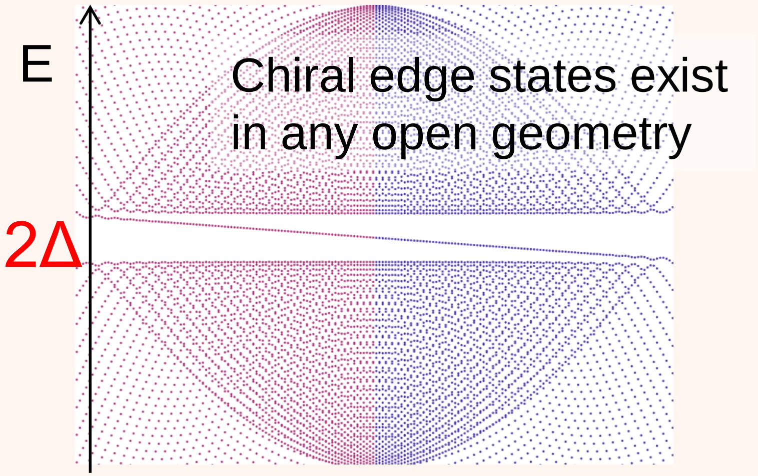
$$|\mathrm{D}(k)| = \mathrm{D}_o$$

d∥z (or c) ↔ S_z=0 or equal spin pairing in xy (ab) plane



Breaks time-reversal symmetry; chirality = ±1

k_x+ik_y degenerate with k_x-ik_y → can have domains

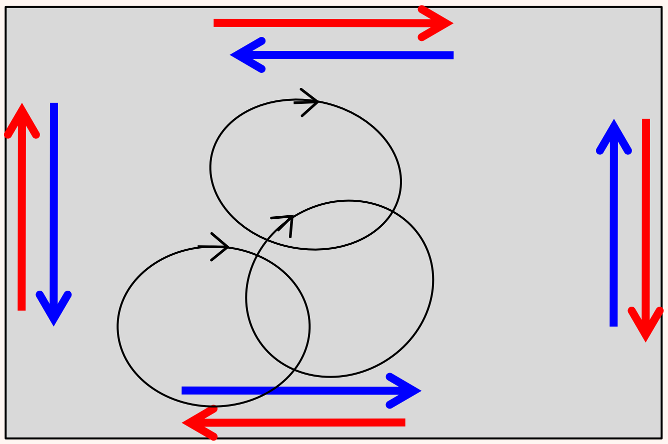


Chiral p-wave state has topological order, analogous to 5/2 Moore-Read QH state, characterized by Chern number = ±1. (Read & Green 2000)

Spontaneous supercurrents for chiral p-wave

Equilibrium supercurrent within ξ of surface

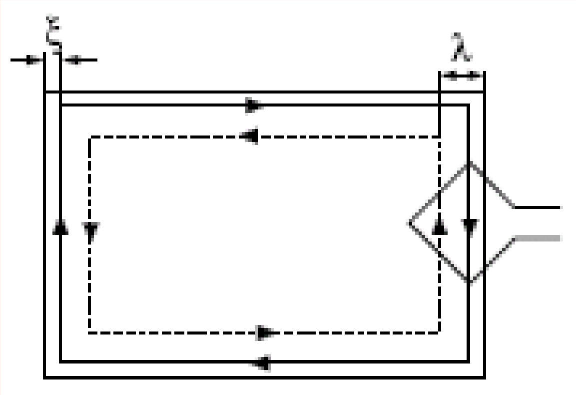
(for single domain)



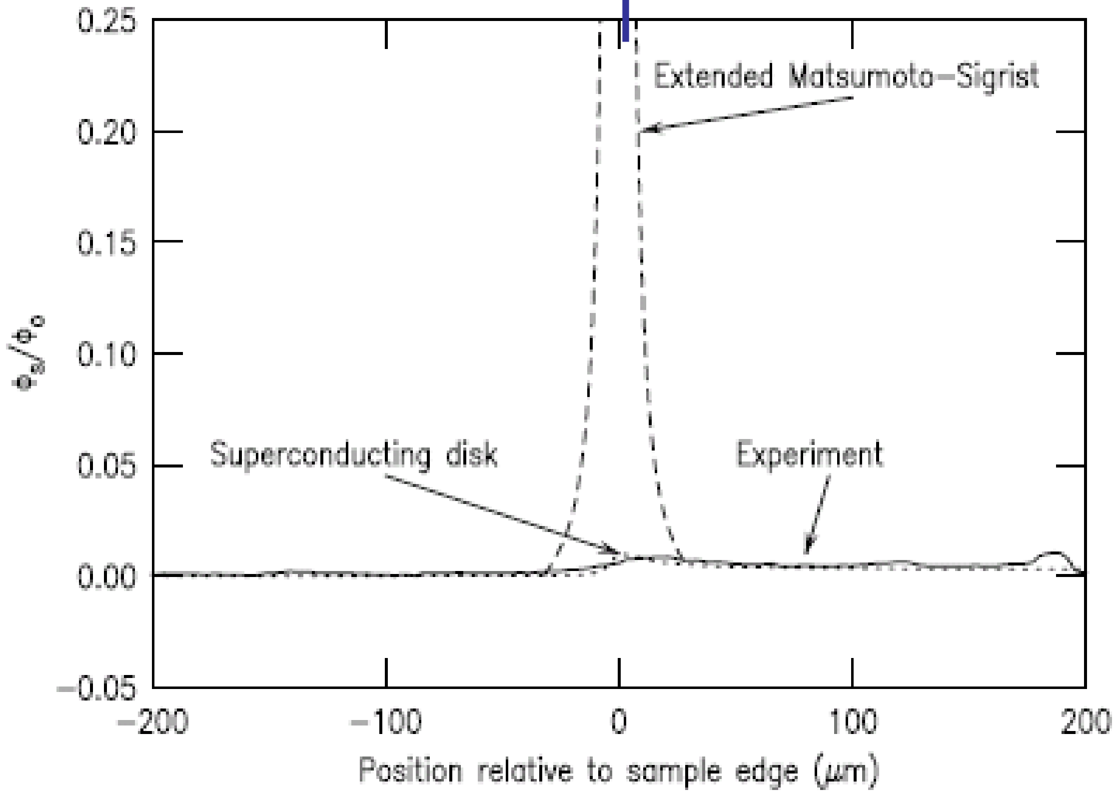
Stone and Roy (2004)
Matsumato and Sigrist (1999)

Screening current within λ+ξ of surface

→ Magnetic field B~10G within λ of surface and B~20G at domain walls.

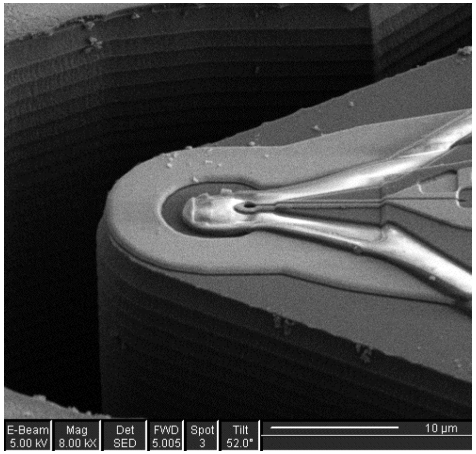


goes to 1



He3 scanning SQUID signal across ab face of Sr_2RuO_4 single crystal at T=0.27K

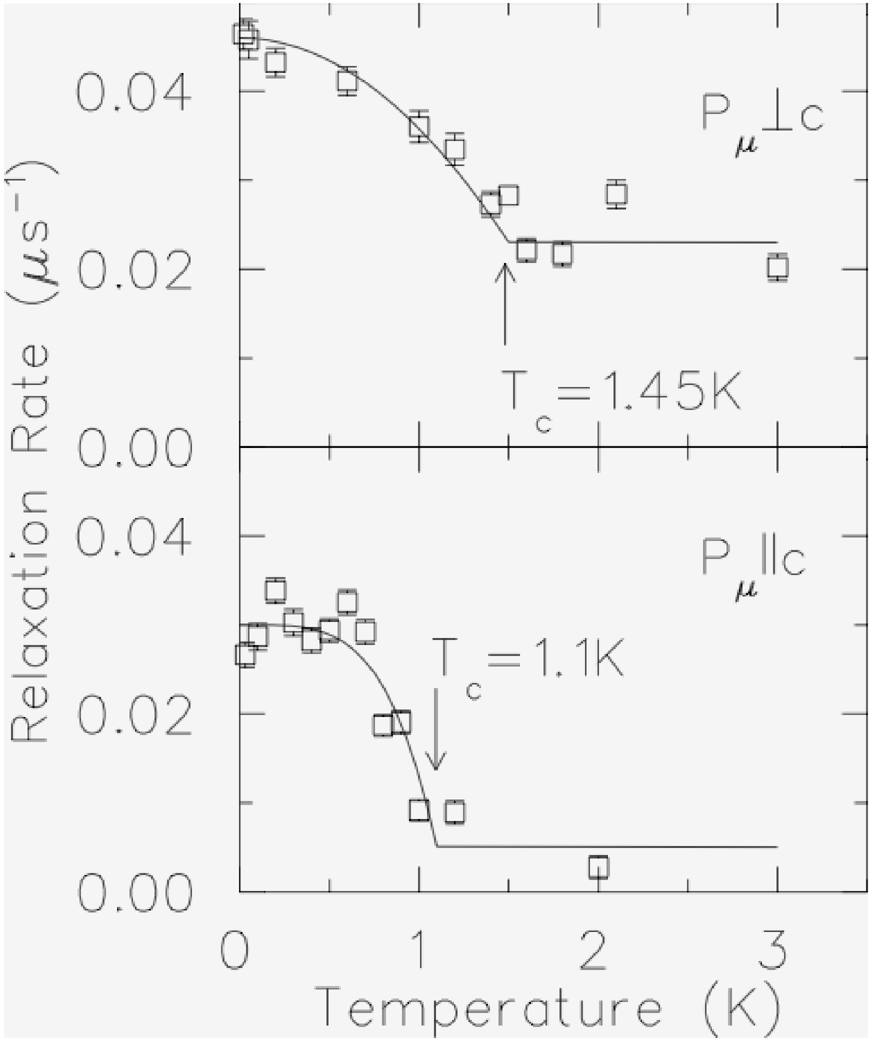
J.R. Kirtley, C. Kallin, C. Hicks, E.A. Kim, Y. Liu, K.A. Moler, Y. Maeno, PRB 76, 014526 (2007).



Smaller SQUIDs, Hall bar probes, micron samples → still no spontaneous fields observed

Experiments put upper bounds on edge currents which are ~ 3 orders of magnitude smaller than predicted.

Muon spin resonance sees internal fields below T_c



Interpreted as due to fields at domain walls → domains ~15 microns in size.

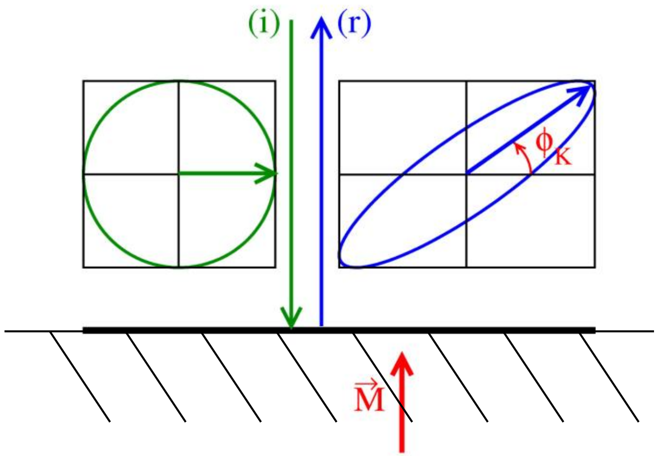
Other possibilities within chiral p-wave:

- impurities
- fields induced by muon

But difficult to reconcile with null measurements of edge and surface fields.

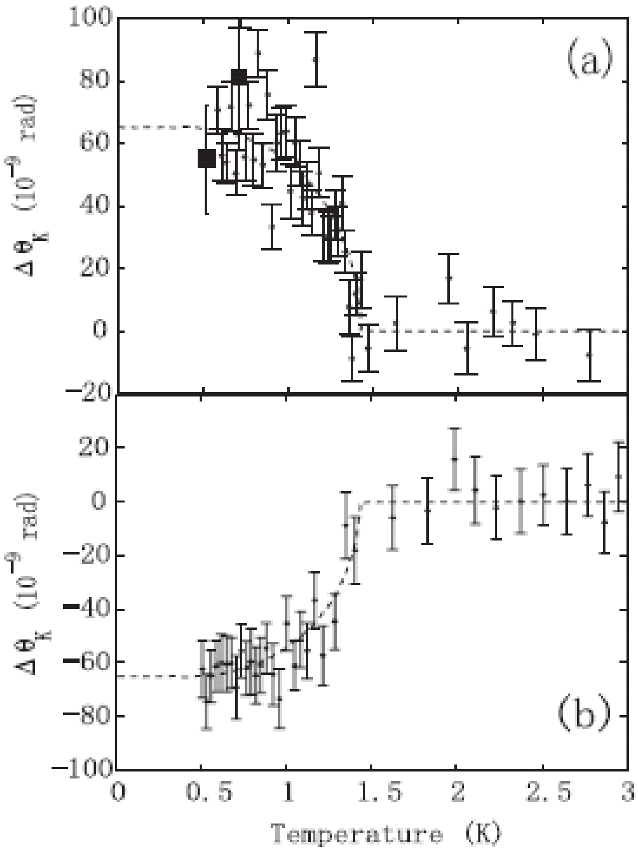
G.M. Luke et al., Physica B 289, 373 (2000).
[Also W. Higemoto et al. unpublished]

Polar Kerr effect



Linearly polarized light is reflected as elliptically polarized light, with rotation of polarization axis by Kerr angle

Measured Kerr angle, θ_K



Cooled in (a) 93 G (b) -43 G
[ω=0.8ev; Θ=65 nanorads]

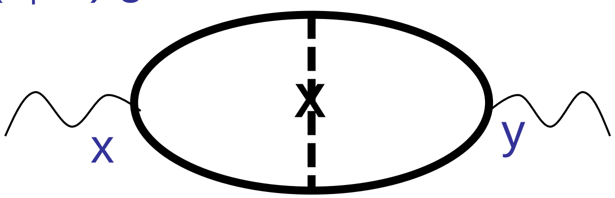
J. Xia, Y. Maeno, P.T. Beyersdorf, M.M. Fejer, A. Kapitulnik, PRL 97, 167002 (2006).

Kerr angle determined by $\sigma_{xy}(\omega)$. Expt (Sagnac interferometer) measures contribution from $\sigma_H=(\sigma_{xy}-\sigma_{yx})/2$.

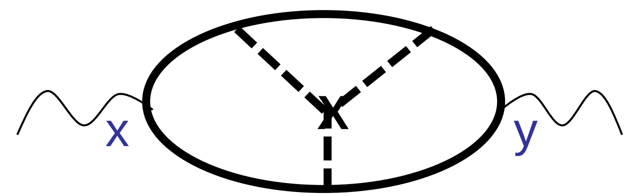
In system with Galilean invariance: $\mathbf{j}_s = \frac{ie^2 r_s / m}{w + id}\mathbf{E}$ Þ $s_{xy} = 0$

→ No Kerr effect without breaking translation symmetry
but broken translation symmetry and BTRS insufficient

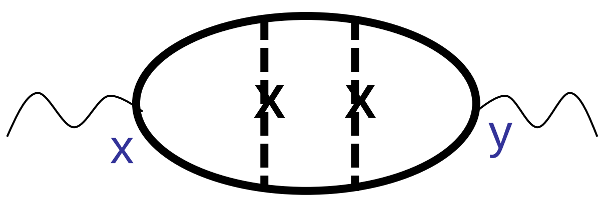
Lowest order Born scattering ($n_i U^2$) gives zero:



Goryo identified diagrams of order $n_i U^3$ (skew scattering) which contribute.



Higher order scattering can also contribute:



J. Goryo, PRB 78, 060501 (2008).

Thought to be dominant contribution in Sr_2RuO_4. Estimate: θ_K~40nrads for l_{imp}~1000Å.

Requires p-h asymmetry.
Lutchyn, Nagornykh, Yakovenko, PRB 80, 104508 (2009).

Intrinsic contributions to σ_H and θ_K

- $\sigma_H(q,\omega)$ at finite q – difficult to probe experimentally: Goryo and Ishikawa, Phys. Lett. A 246, 549 (1998).

- Effect due to edges (related to edge current): Furusaki, Matsumoto, Sigrist, PRB 64, 054514 (2001).

- Effect due to surface, particle-hole asymmetry and collective modes: Yip and Sauls, J. Low Temp. Phys. 86, 257 (1992).

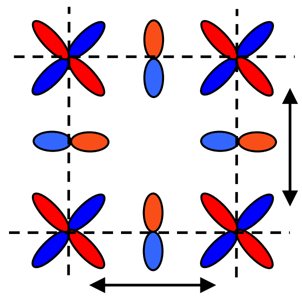
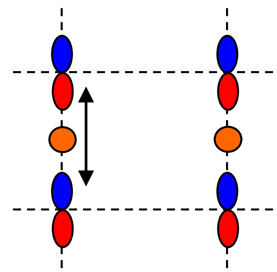
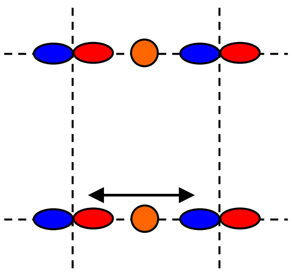
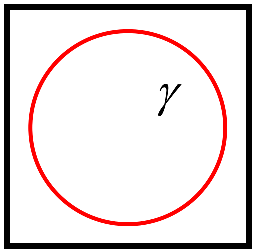
These are much too small to explain polar Kerr experiments on Sr_2RuO_4.

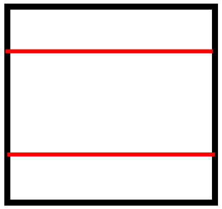
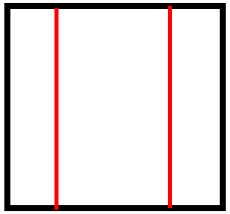
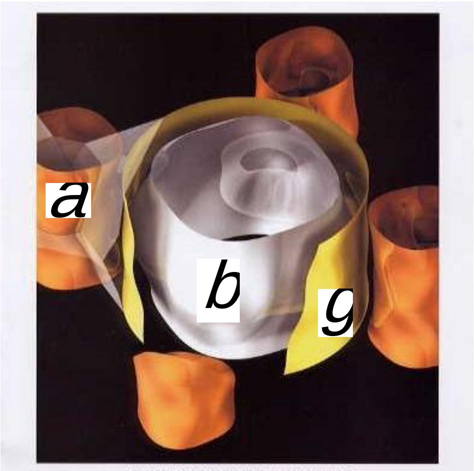
All the above, plus disorder calculations, used a single band model. Can multibands give an intrinsic effect?

Sr_2RuO_4 band structure

Ru d-orbitals

d_{xy} d_{yz} d_{xz}

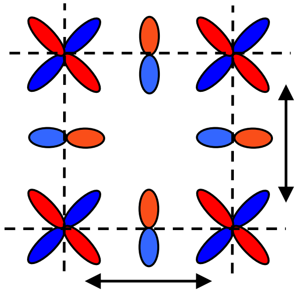





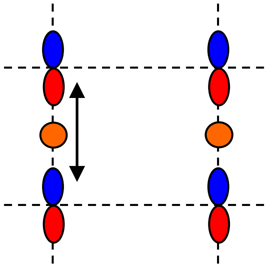
Sr_2RuO_4 band structure

Ru d-orbitals

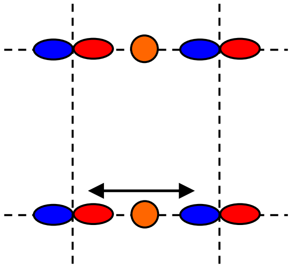
d_{xy}

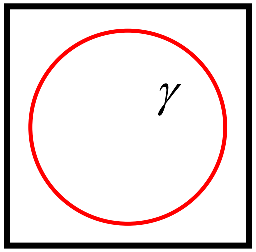


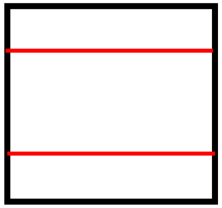
d_{yz}

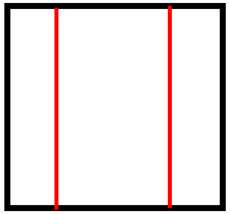


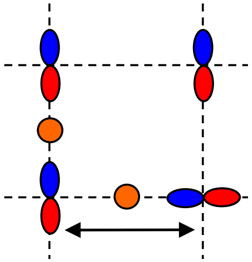
d_{xz}



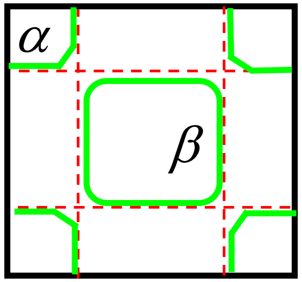


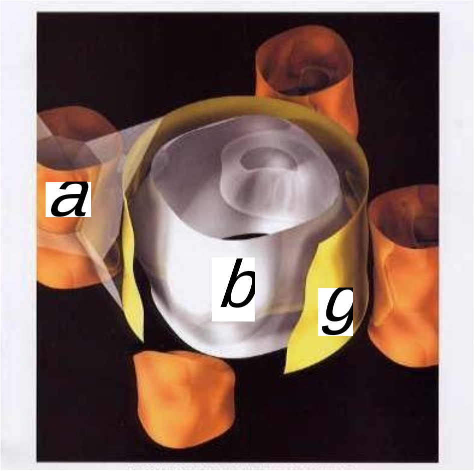






t” ~ 0.1t

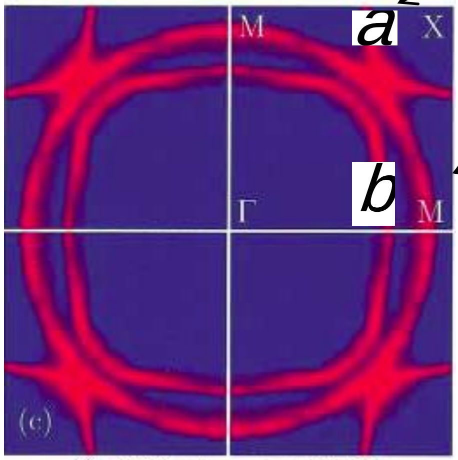




Can a multiband model resolve some of the puzzles?

Raghu, Kapitulnik and Kivelson studied chiral p-wave SC on the quasi 1-d bands, PRL 105, 136401 (2010).

hole

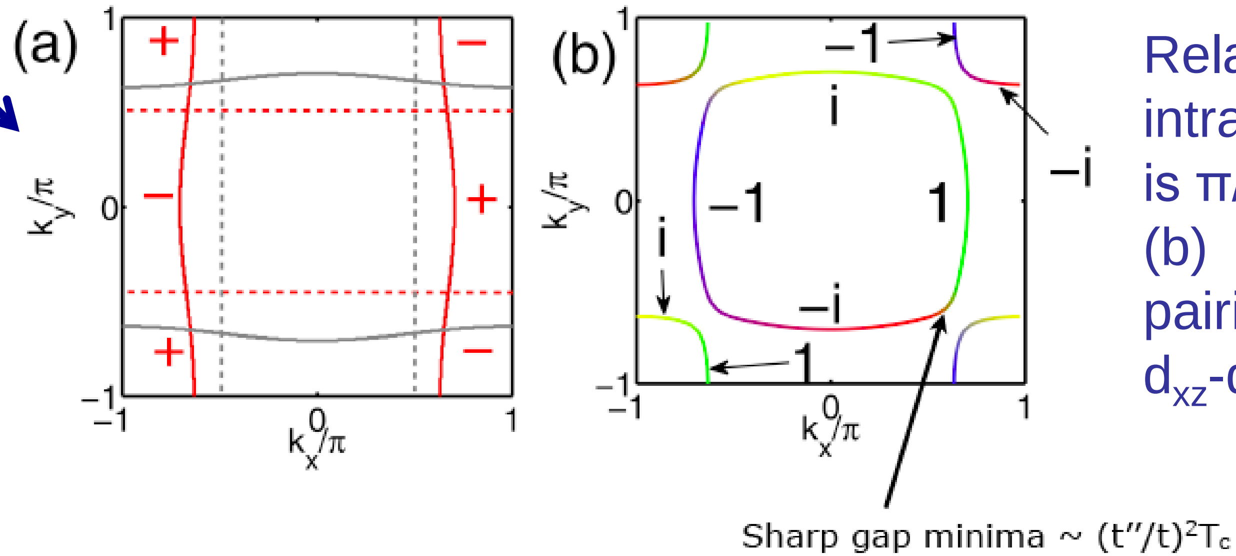


Sr_2RuO_4 cleaved at 180 K
T= 10 K hν=28 eV

electron

Find intraorbital p-wave pairing for d_{xz} and d_{yx}.

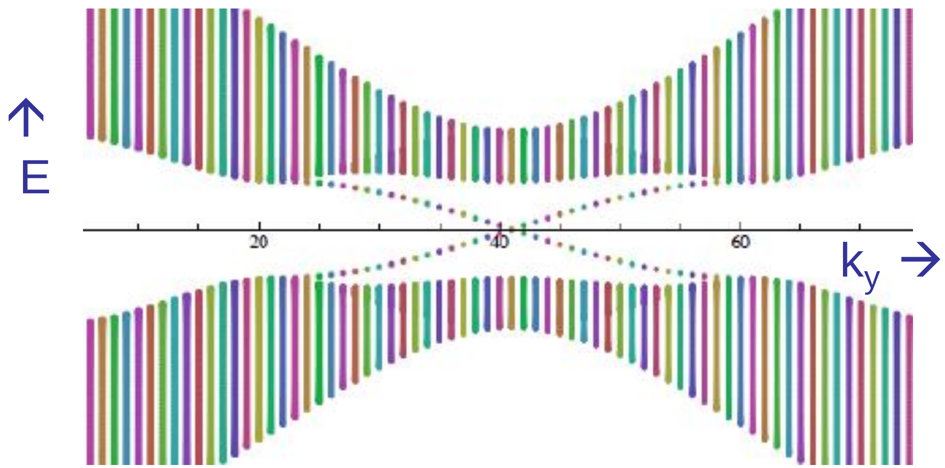
(a) Bands and pairing phases with no t".



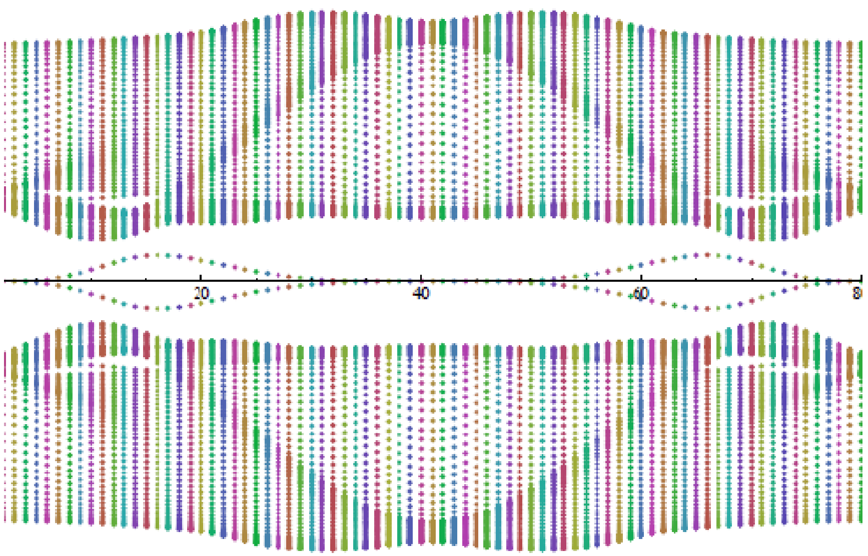
Relative phase of intraorbital pairing is π/2.

(b) Bands and pairing phases with d_{xz}-d_{yx} hopping, t".

Energy Spectra for Chiral p-wave SC on cylinder



Chiral p-wave SC on 2d xy band gives one chiral mode at each edge of cylinder.
Chern number is ±1.



Chiral p-wave SC on 1d xz and yz bands gives one non-chiral mode at each edge of cylinder.

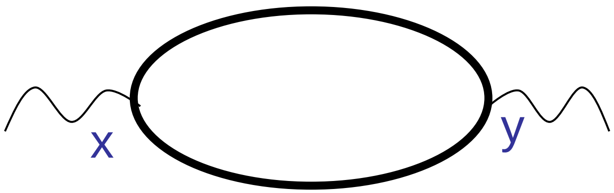
Hole and electron bands → Chern number is 0 → not topologically protected.

Could explain absence of observable edge/surface currents (and low-lying excitations)

Is there an intrinsic AHE in a multiband chiral superconductor?

$$\sigma_H(\omega) = \frac{\sigma_{xy}(\omega) - \sigma_{yx}(\omega)}{2}$$

$$\sigma_{xy}(\nu_n) = \frac{ie^2 T}{\omega} \sum_{\mathbf{k},\omega_n} \mathrm{tr}\left[\hat{v}_x G_0(\mathbf{k},\omega_n)\hat{v}_y G_0(\mathbf{k},\omega_n+\nu_n)\right]$$



$$\hat{v}_i = \frac{\partial \varepsilon(k)}{\partial k_i} \quad \left(\text{or } \frac{k_i}{m} \text{ for free electrons}\right)$$

$\Rightarrow$ σ_H vanishes for single band

Consider two-band (two-orbital) case

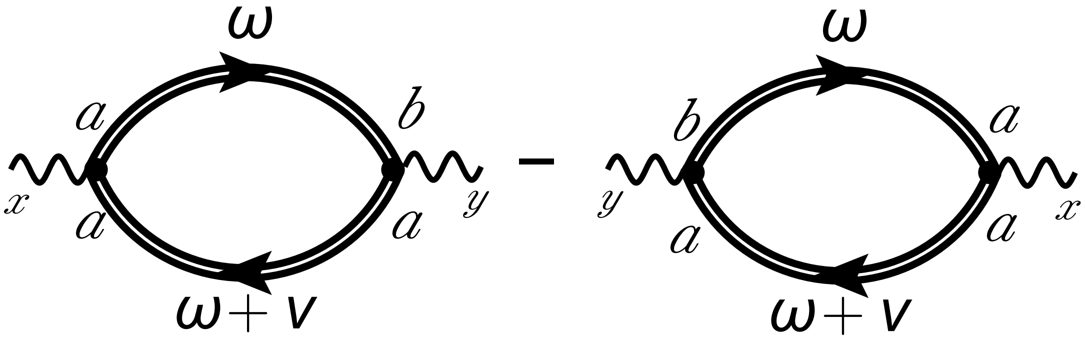
$$H_0 = \begin{pmatrix} \xi_1(k) & \varepsilon_{12}(k) \\ \varepsilon_{12}(k) & \xi_2(k) \end{pmatrix}; \quad \xi_i = \varepsilon_i - \mu$$

Intraorbital pairing: Δ_{11}, Δ_{22} $\Rightarrow 4\times 4\ G_0$

Interorbital pairing: Δ_{12}

The velocity matrix also has off-diagonal terms (interorbital transitions)

Is there an intrinsic AHE in a multiband chiral superconductor?



Nonzero contributions involve transitions between orbitals (ε_{12} or t”) or between bands (Δ_{12}) and require different relative OP phases.

I.e. $\Delta(k)=e^{i\theta}[\Delta'(k)+i\Delta''(k)]$ → relative phase=$\phi(k)=\tan^{-1}[\Delta''(k)/\Delta'(k)]$

Changes sign with chirality.

$$\sigma''_H(\omega)=\frac{\pi e^2}{\omega^2}\sum_{\mathbf{k}}\frac{((\mathbf{v}_{11}-\mathbf{v}_{22})\times\mathbf{v}_{12})_z}{E_- E_+}\left[\varepsilon_{12}\,\mathrm{Im}(\Delta^*_{11}\Delta_{22})+\xi_1\,\mathrm{Im}(\Delta^*_{22}\Delta_{12})-\xi_2\,\mathrm{Im}(\Delta^*_{11}\Delta_{12})\right]$$
$$\times\left[\delta(\omega-E_1-E_2)-\delta(\omega+E_1+E_2)\right]$$

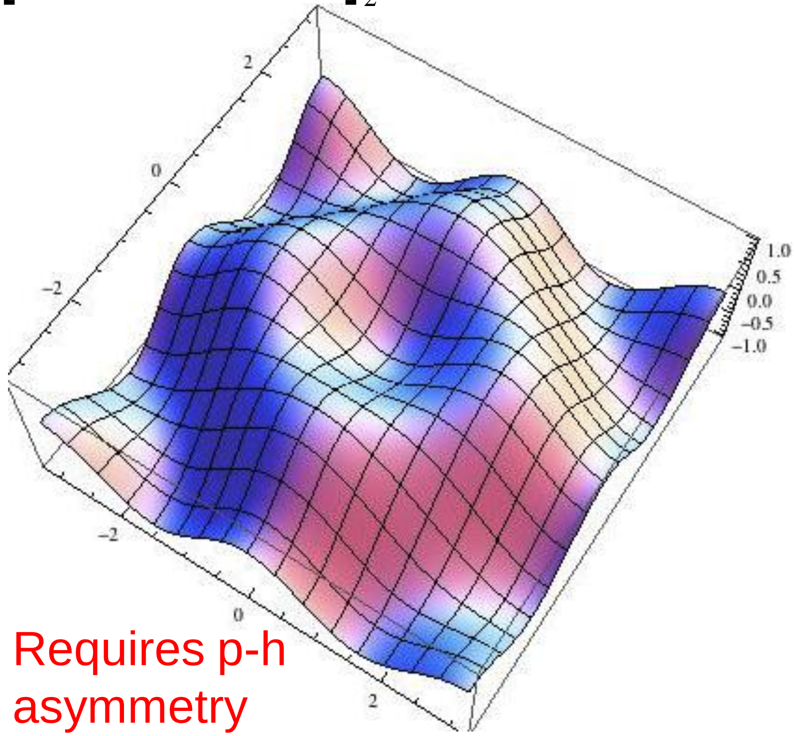
Applied to Raghu et al. quasi-1d model for Sr2RuO4

$$\sigma_H^{T=0}(\omega) = 2e^2 \sum_{\mathbf{k}} \frac{\mathrm{Im}\langle c^{+}_{\mathbf{k}1} c_{\mathbf{k}2}\rangle \left[(\mathbf{v}_{11} - \mathbf{v}_{22}) \times \mathbf{v}_{12}\right]_z}{(\omega + i\varepsilon)^2 - (E_- + E_+)^2}$$

$t = \mu = 10t'' = 1\mathrm{eV}$

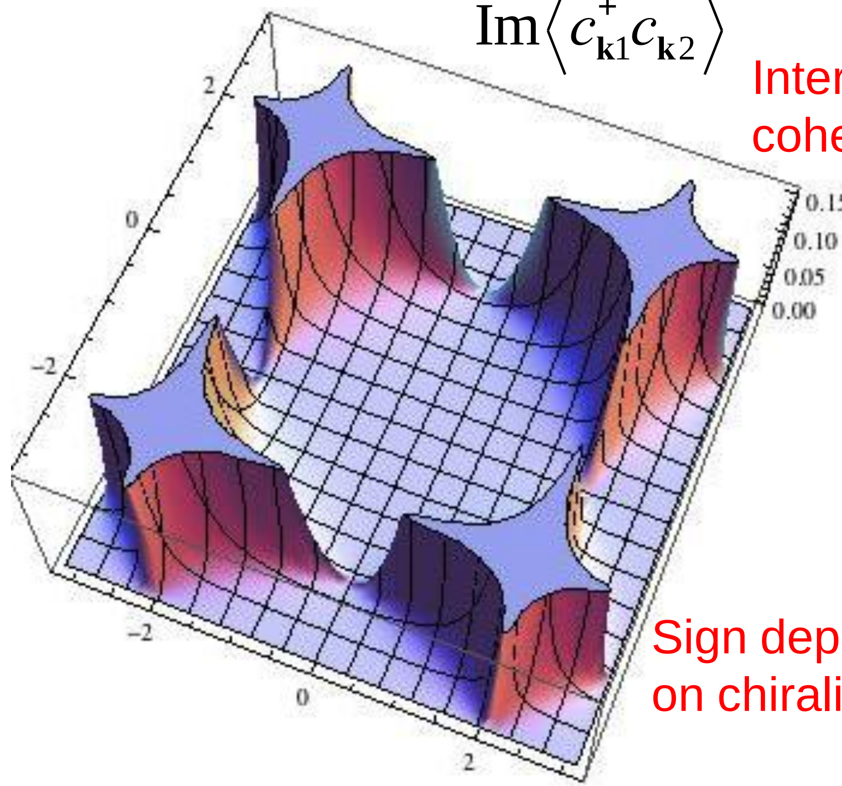
$\Delta_0 = 0.23\mathrm{meV}$

$\left[(\mathbf{v}_{11} - \mathbf{v}_{22}) \times \mathbf{v}_{12}\right]_z$



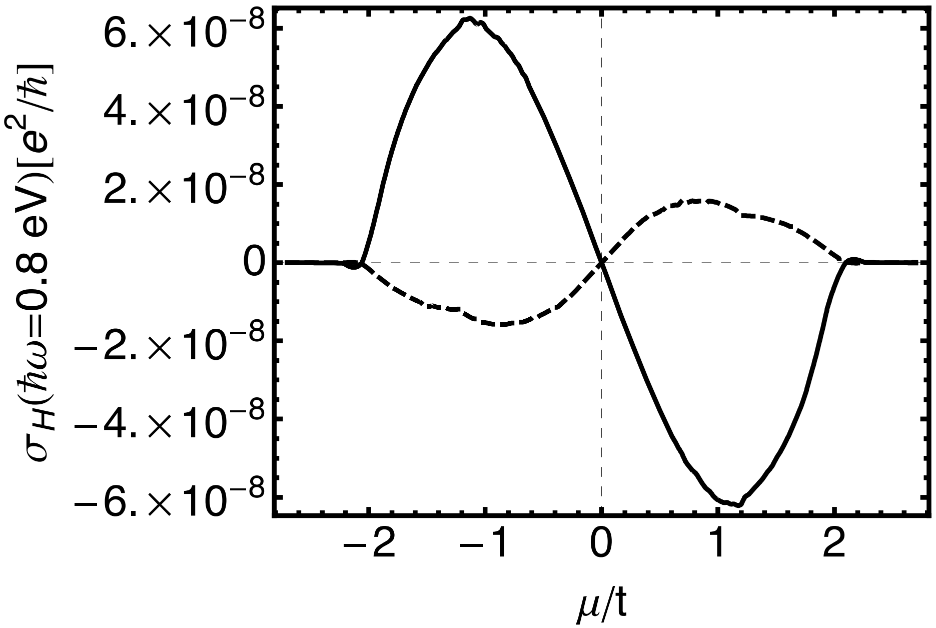
Requires p-h asymmetry

$\mathrm{Im}\langle c^{+}_{\mathbf{k}1} c_{\mathbf{k}2}\rangle$



Interobital coherence

Sign depends on chirality

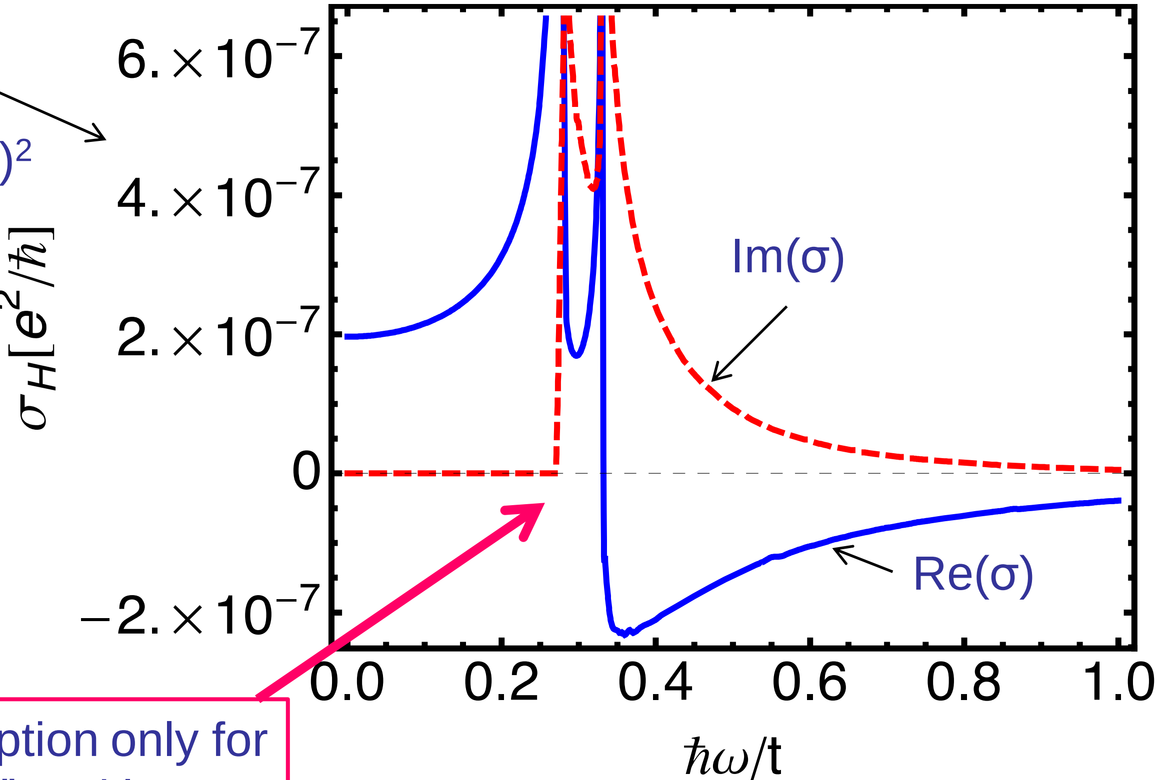


Effect requires particle-hole asymmetry.

Note: photon with polarization in ab (xy) plane cannot cause transitions involving d_{xy} orbitals → γ band only enters indirectly through coherence factors

Applied to Raghu et al. quasi-1d model for Sr2RuO4

Small numbers due to $(\Delta/t)^2$



$t = m = 10t'' = 1\text{eV}$
$D_0 = 0.23\text{meV}$

T=0

For T>0, also have qp scattering, i.e. $\delta(\omega-(E_1-E_2))$

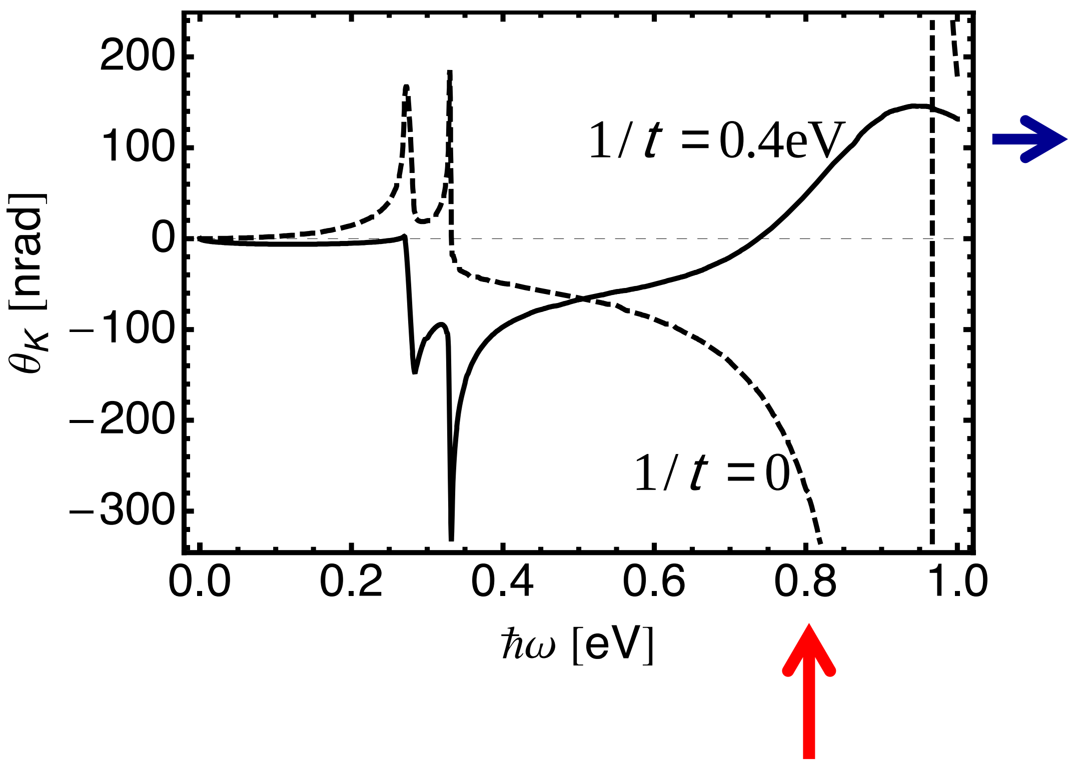
Absorption only for ω ~ 2t" and larger (not 2Δ)

$$s_H(w) = 2e^2 \sum_{\mathbf{k}} \frac{((\mathbf{v}_{11} - \mathbf{v}_{22})\times \mathbf{v}_{12})_z e_{12}\, \text{Im}(D_{11}^* D_{22})}{E_- E_+ (E_- + E_+)\left[(E_- + E_+)^2 - (w + ie)^2\right]}$$

$$= \frac{e^2}{h}\frac{16 D_0^2 t''^2 t}{p^2}\int_{-p}^{p} dx\, dy \frac{\sin^2 x \sin^2 y(\cos y \sin^2 x + \cos x \sin^2 y)}{E_- E_+ (E_- + E_+)\left[(E_- + E_+)^2 - (w + ie)^2\right]}$$

Polar Kerr Effect

$$\theta_K(\omega) = \frac{4\pi}{\omega}\,\mathrm{Im}\left[\frac{\sigma_H(\omega)}{n(n^2-1)}\right], \qquad n(\omega) = \sqrt{\epsilon_\infty - 4\pi i \sigma(\omega)/\omega}$$



~35 nrads at ω=0.8eV

[same parameters as used in skew-scattering estimate which gave 43 nrads.]

- Results very similar with further neighbor hoppings. Also whether there are low-lying excitations has little effect on T=0 result.
- Spin-orbit coupling does not give any new effect but can play the role of interorbital hopping.
- Adding γ band will not change much, as it plays a passive role. Need substantial superconductivity on quasi-1d bands.
- Quasi-1d model seems to maximize the effect. Much smaller effect if SC primarily on γ band.

In conclusion

- Identified an intrinsic contribution to the polar Kerr effect (and spontaneous Hall effect) which is generic to multiband chiral superconductors provided there is interband pairing with a different relative phase than intraband pairing [$\mathrm{Im}(\Delta^*_{aa}\Delta_{ab})$], and broken p-h symmetry.

- The quasi-1d model for Sr_2RuO_4 gives θ_K comparable to experiment. By contrast, if SC is primarily on the ϒ band, the Kerr angle would likely be reduced by more than an order of magnitude.

- Experiments with controlled disorder may determine if the observed Kerr angle is of intrinsic or extrinsic origin. Intrinsic would imply the SC is of a multiband nature. Other discrepancies point toward a multiband model of chiral p-wave.