Light Induced
Topological Phenomena

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What this talk is about

$$H = \vec{x} \cdot \vec{\sigma}$$

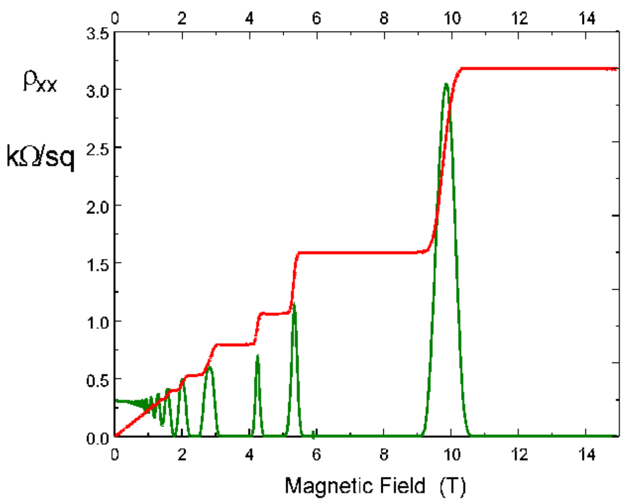
$$U(\vec{x}) = T_t\, e^{i \int dt\, H(\vec{x},t)}$$

Periodic Table of Topolgical Insulators/SCs

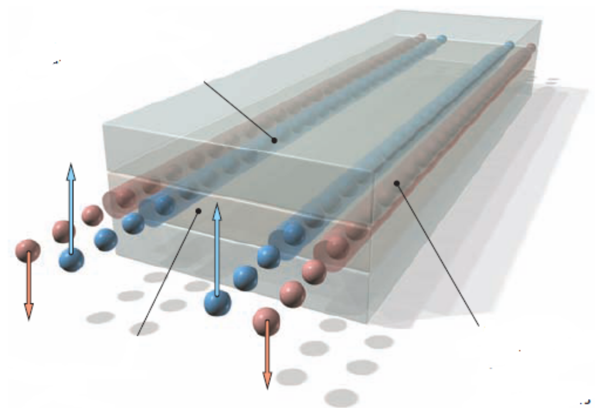
TRS	PHS	SLS	$d=1$	$d=2$	$d=3$
0	0	0	-	$\mathbb{Z}$	-
$+1$	0	0	-	-	-
-1	0	0	-	$\mathbb{Z}_2$	$\mathbb{Z}_2$
0	0	1	$\mathbb{Z}$	-	$\mathbb{Z}$
$+1$	$+1$	1	$\mathbb{Z}$	-	-
-1	-1	1	$\mathbb{Z}$	-	$\mathbb{Z}_2$
0	$+1$	0	$\mathbb{Z}_2$	$\mathbb{Z}$	-
0	-1	0	-	$\mathbb{Z}$	-
-1	$+1$	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
$+1$	-1	1	-	-	$\mathbb{Z}$

Shnyder et. al. (2009), Kitaev (2009)

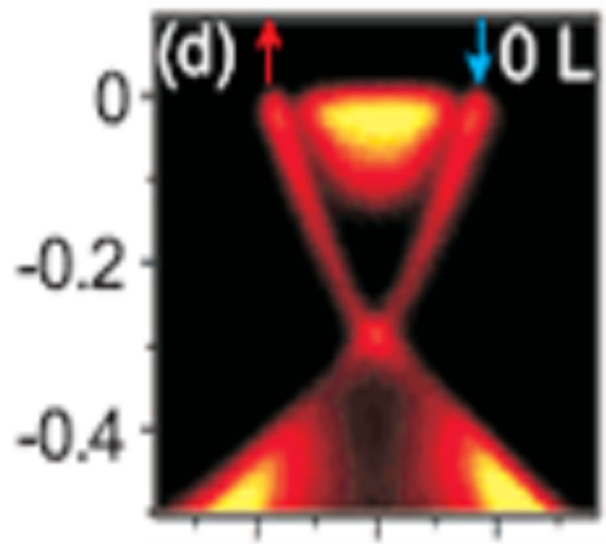
Topological Insulators



Integer quantum hall effect



Quantum spin hall effect



3D TRS TI

Outline

- 2D topological insulators
- Floquet theory
- How can we topologize the trivial?
- 3D Topological Floquet spectrum
- Edge states without chern numbers

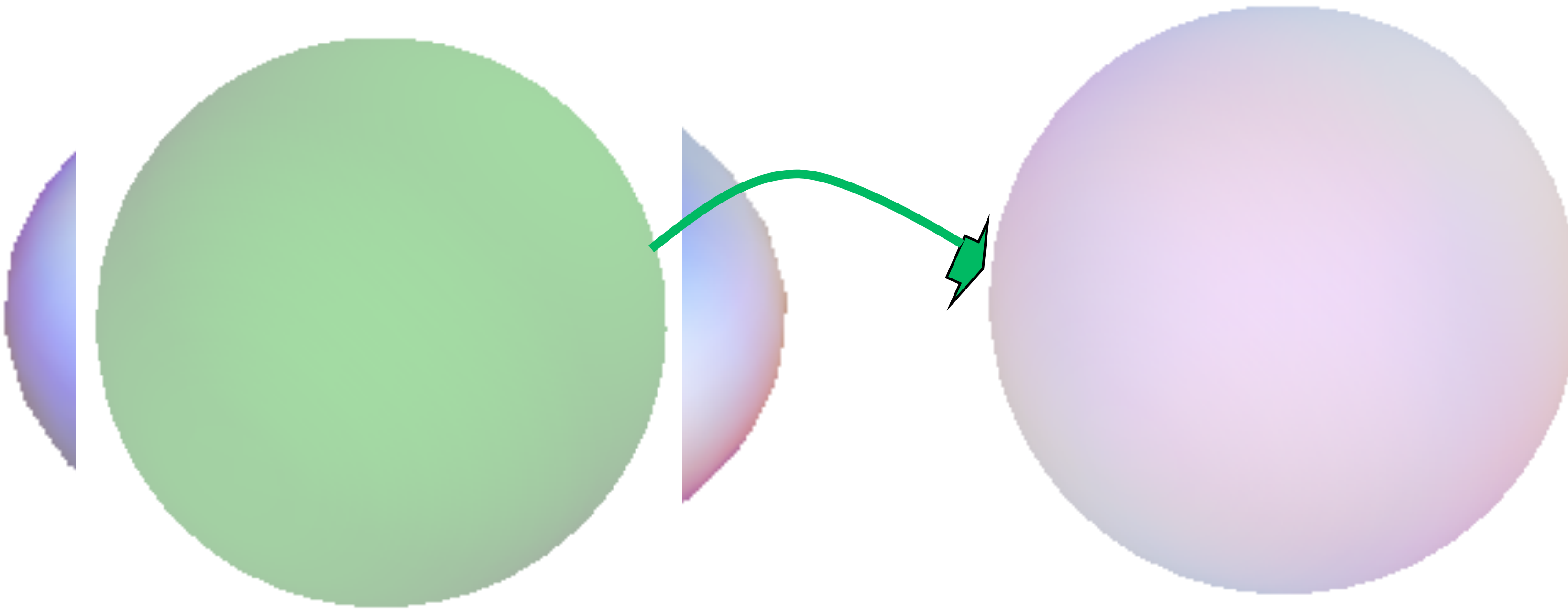
What makes an insulator topological?

$$H(\mathbf{k}) = \varepsilon_0(\mathbf{k}) + \vec{d}(\mathbf{k}) \cdot \vec{\sigma}$$

A map from the BZ to the unit sphere

Brillouin Zone

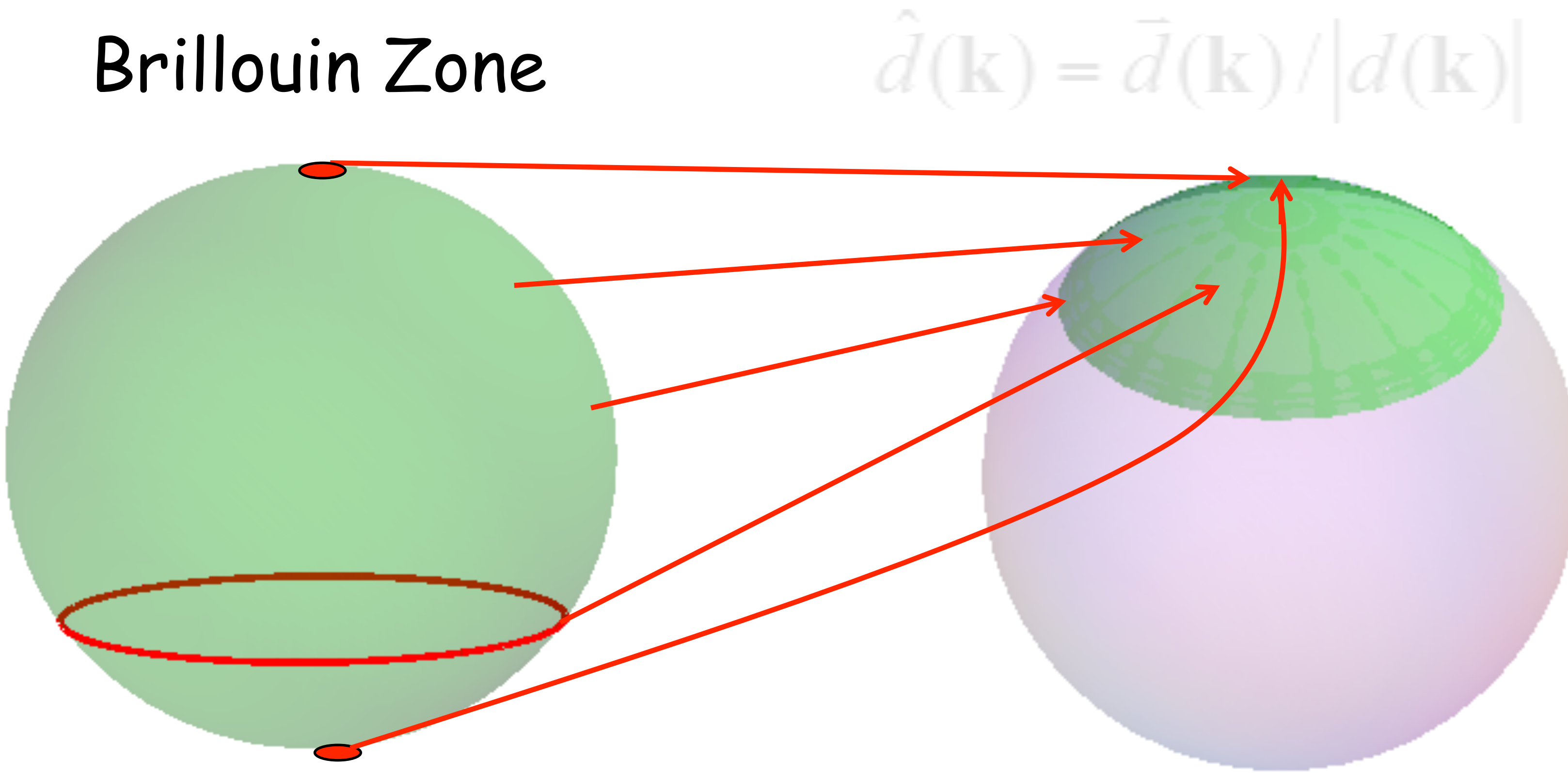
$$\hat{d}(\mathbf{k}) = \vec{d}(\mathbf{k}) / \left|\vec{d}(\mathbf{k})\right|$$



What makes a phase topological?

$$H = \varepsilon_0 + \vec{d}(\mathbf{k}) \cdot \vec{\sigma}$$

A "trivial" map



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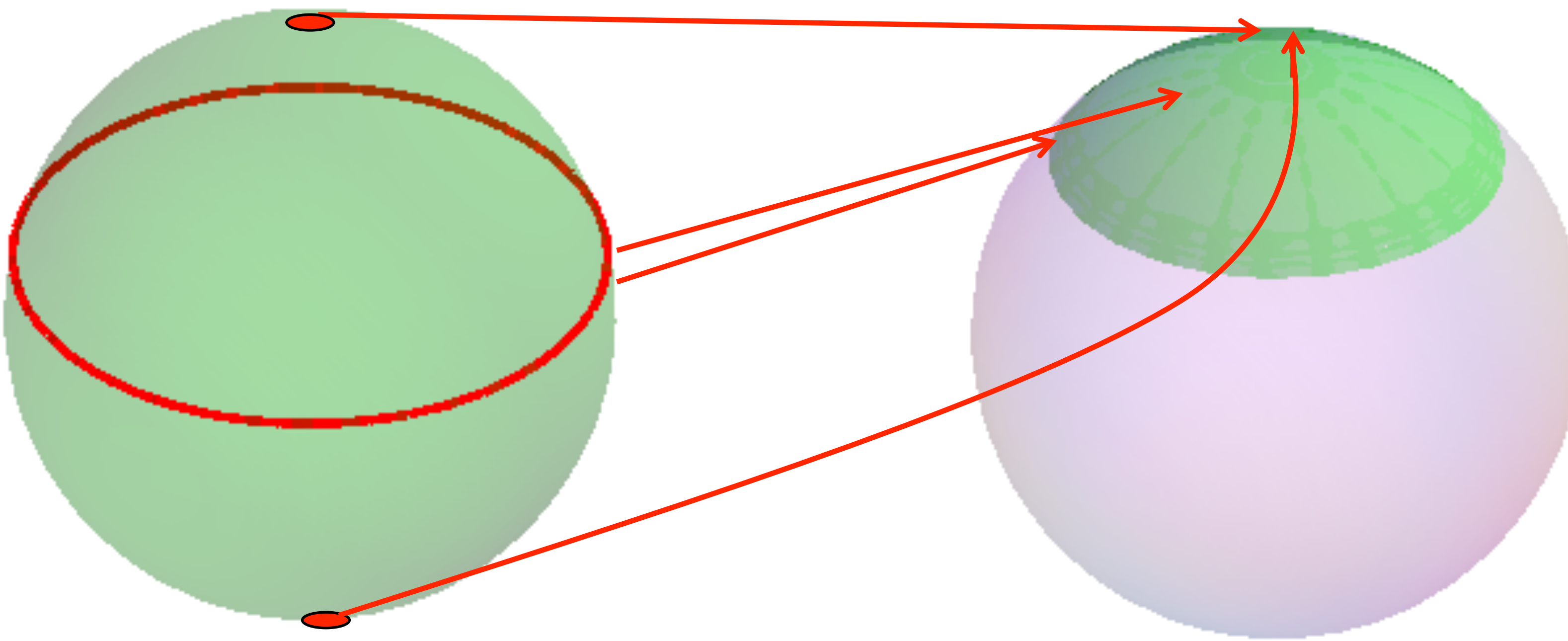

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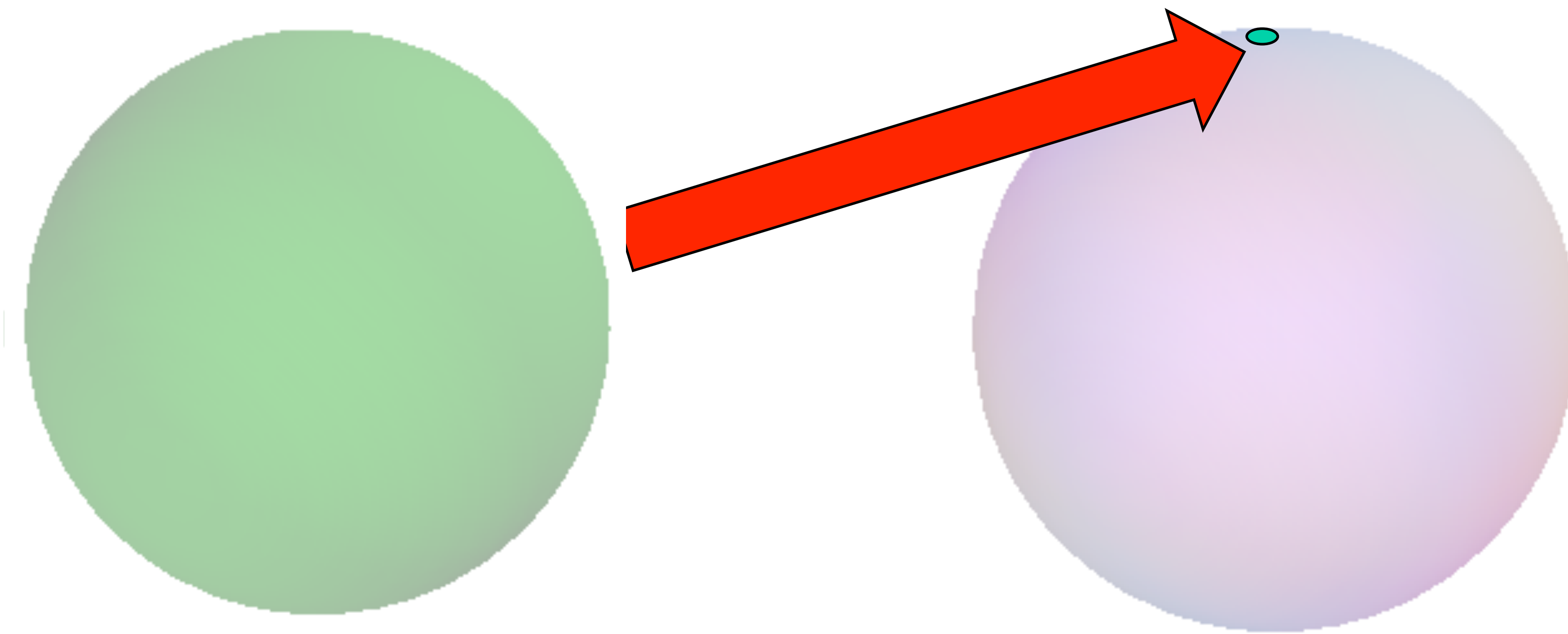

What makes a phase topological?

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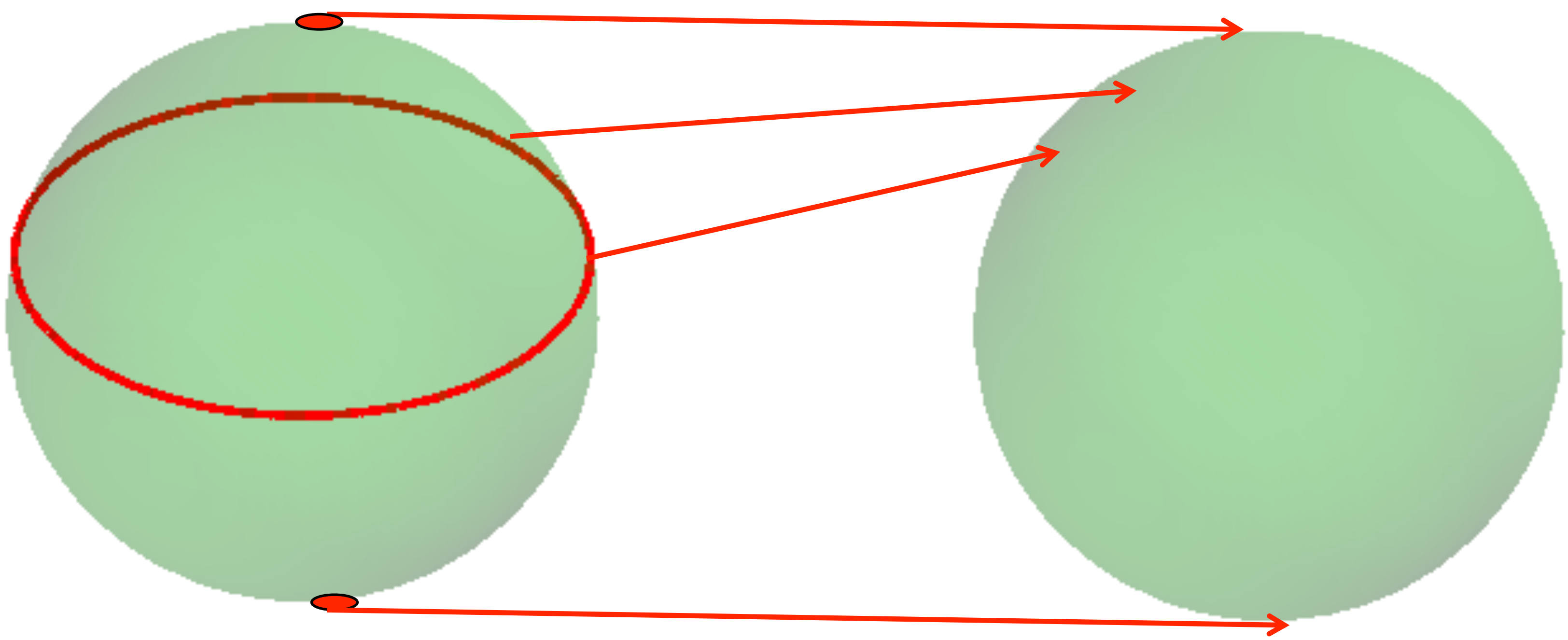
A "**non trivial**" map

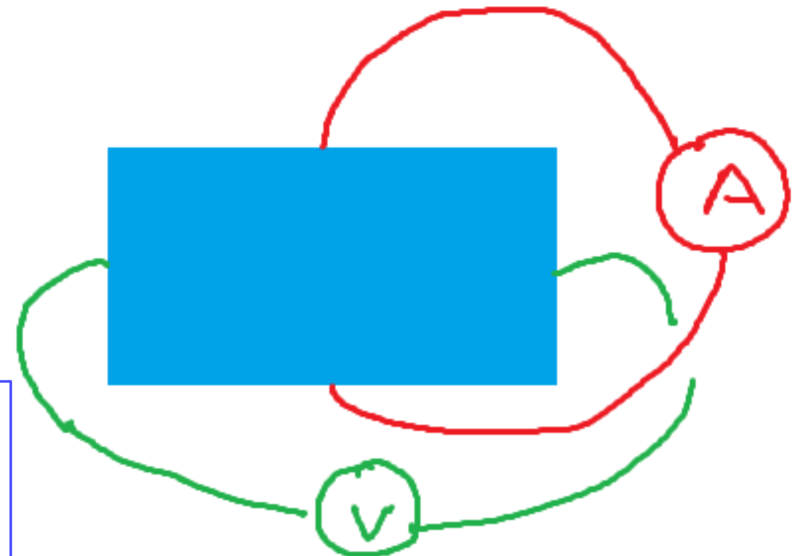
Brillouin Zone

$$\hat{d}(\mathbf{k}) = \vec{d}(\mathbf{k}) / |d(\mathbf{k})|$$

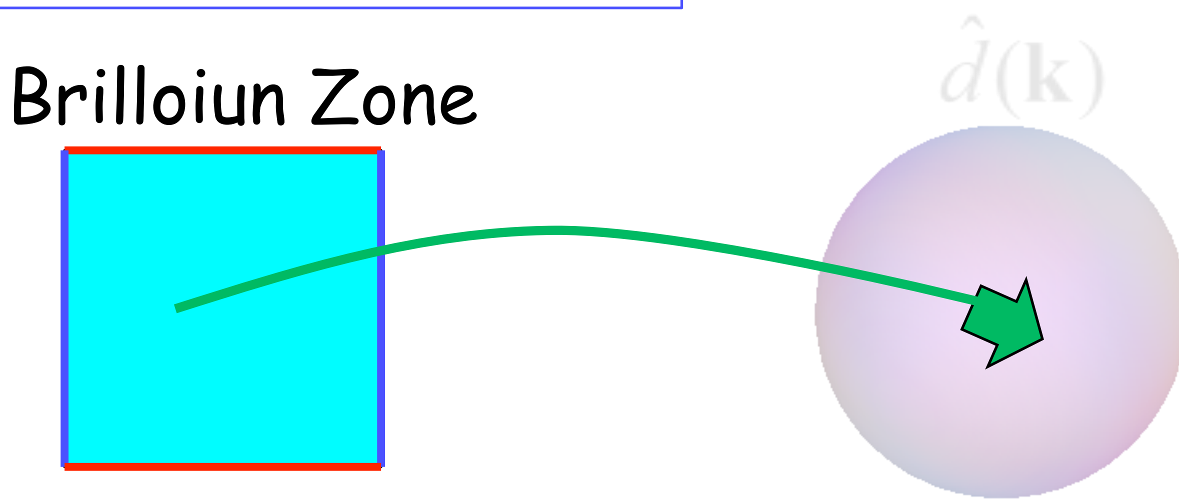


Quantized response

$$J_x = \sigma_{xy} E_y$$



$$\sigma_{xy} = \frac{e^2}{h} \text{ x (Number of wrappings)}$$

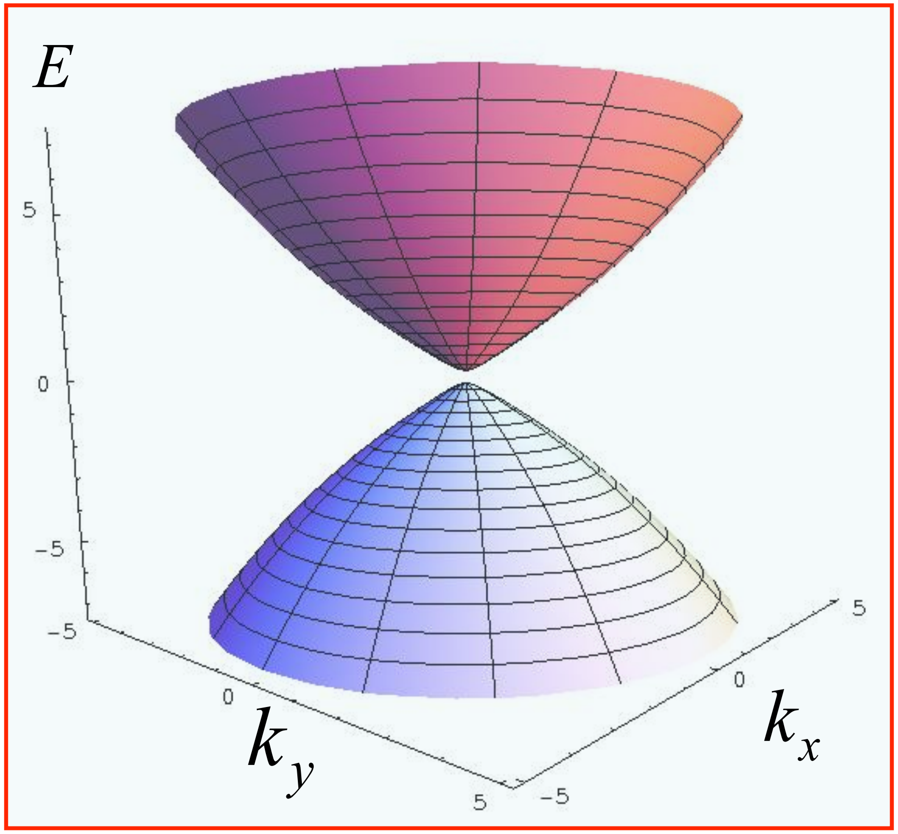


Dirac cone with narrow band gap

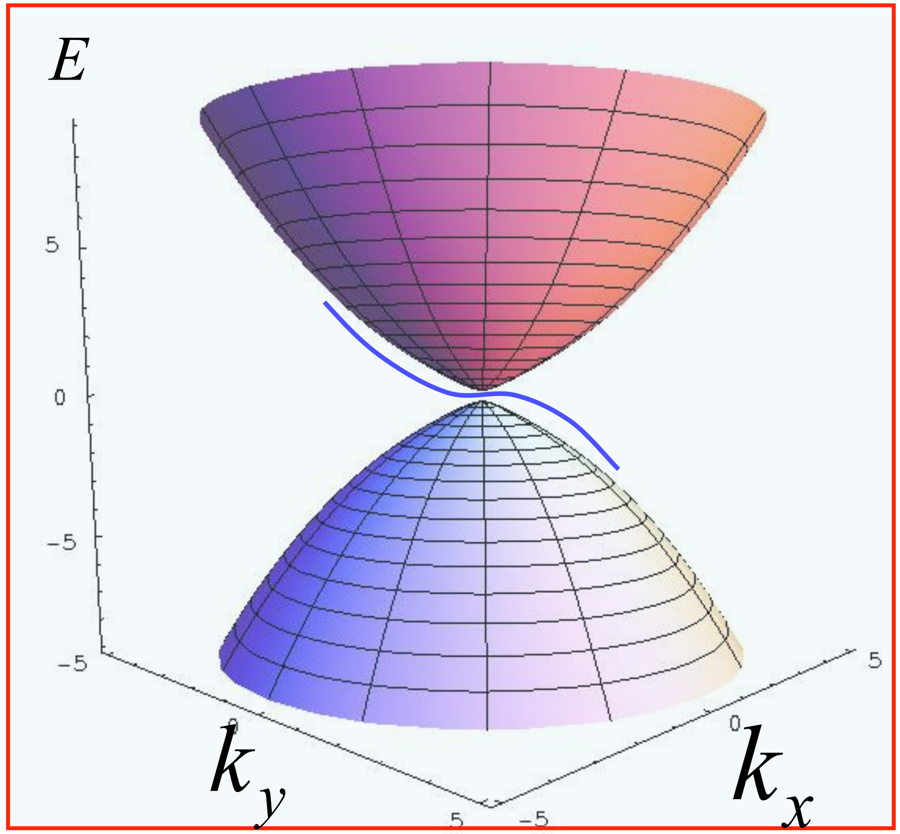
$$H = \varepsilon_0 + \vec{d}(\mathbf{k}) \cdot \vec{\sigma}$$

$$\vec{d}(\mathbf{k}) = (k_x, k_y, m + b\mathbf{k}^2)$$

$m > 0,\ b > 0$ (Trivial)



$m > 0,\ b < 0$ (Topological)



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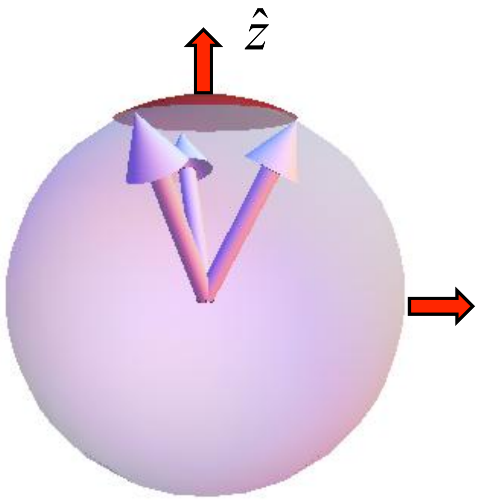
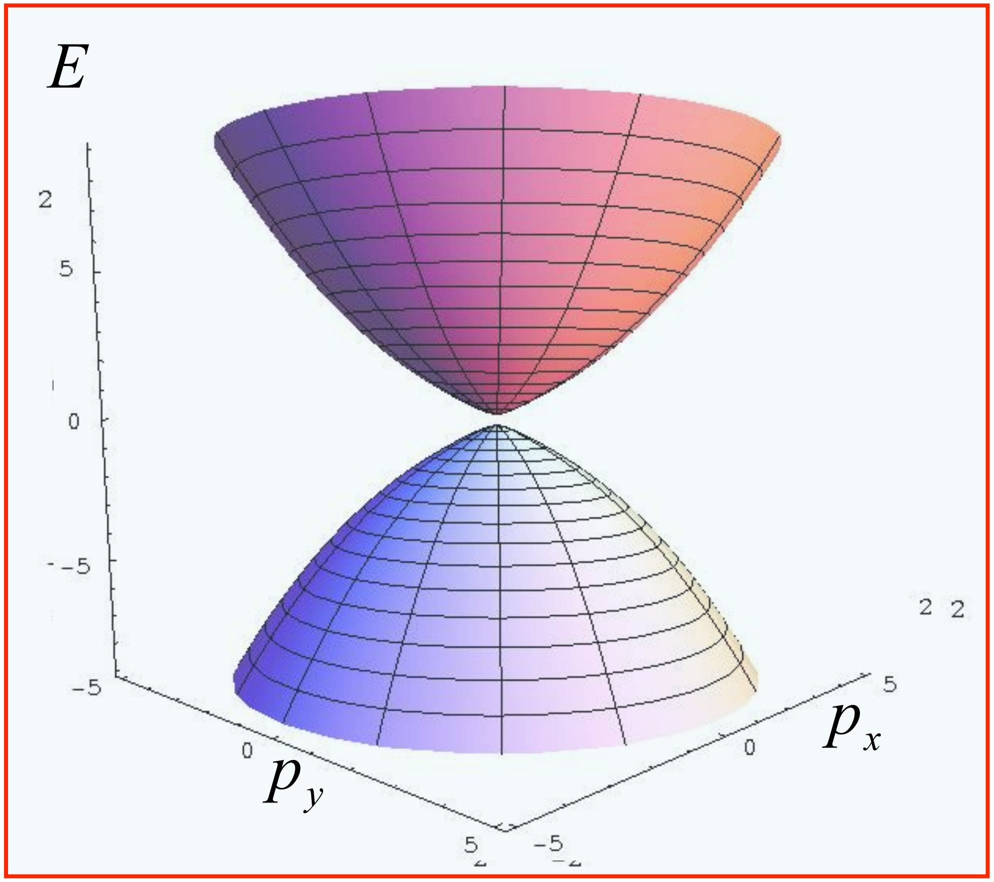
(map BZ to unit sphere)

$m = 0.2,\ b = 0.3 \quad (non-topo)$

Wrapping the unit sphere?

$\hat{\mathbf{d}} - sphere$





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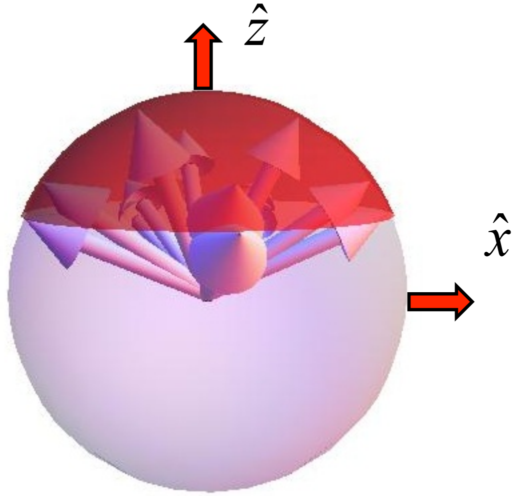

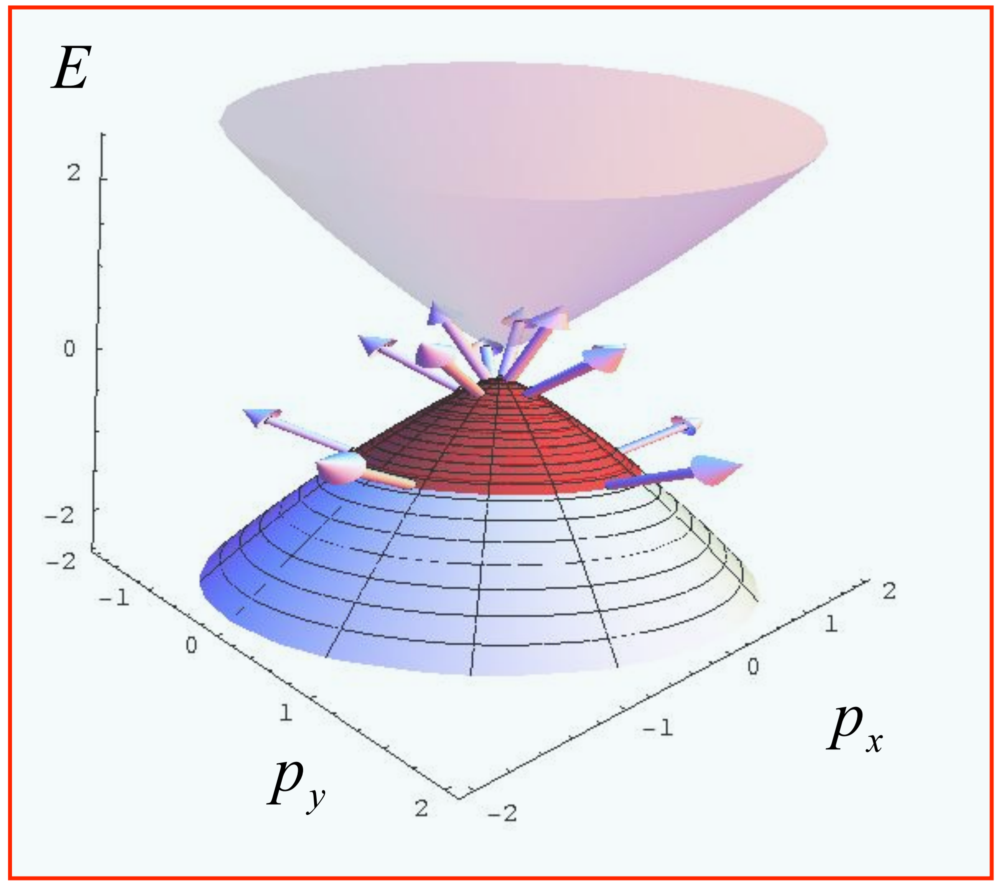

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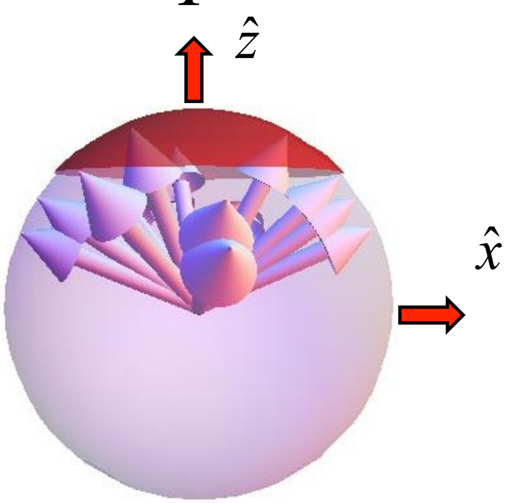

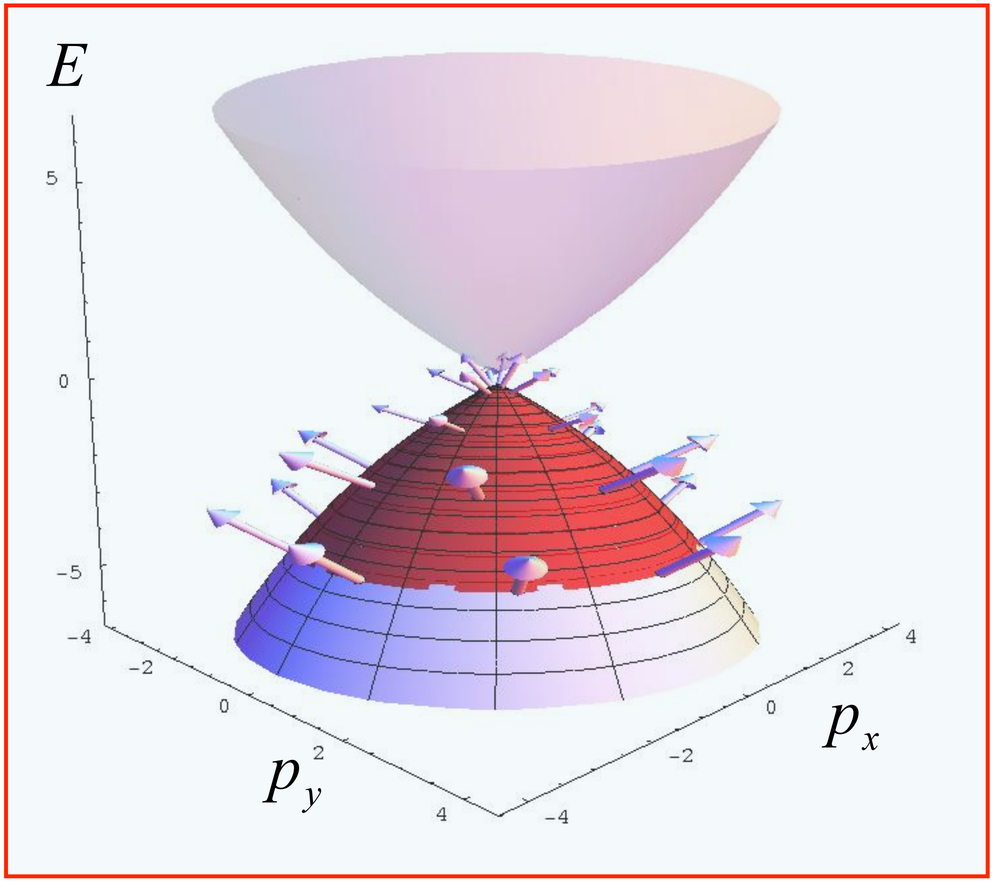

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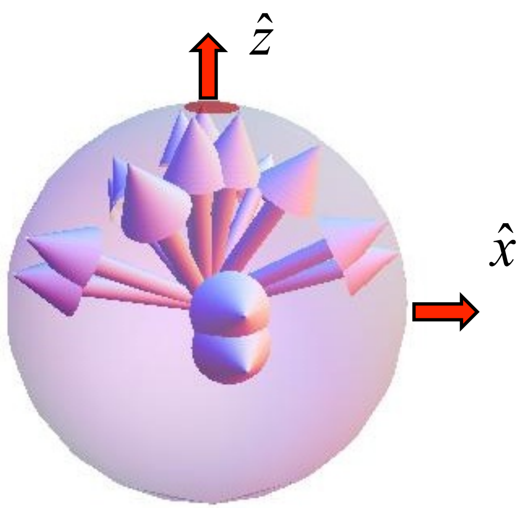

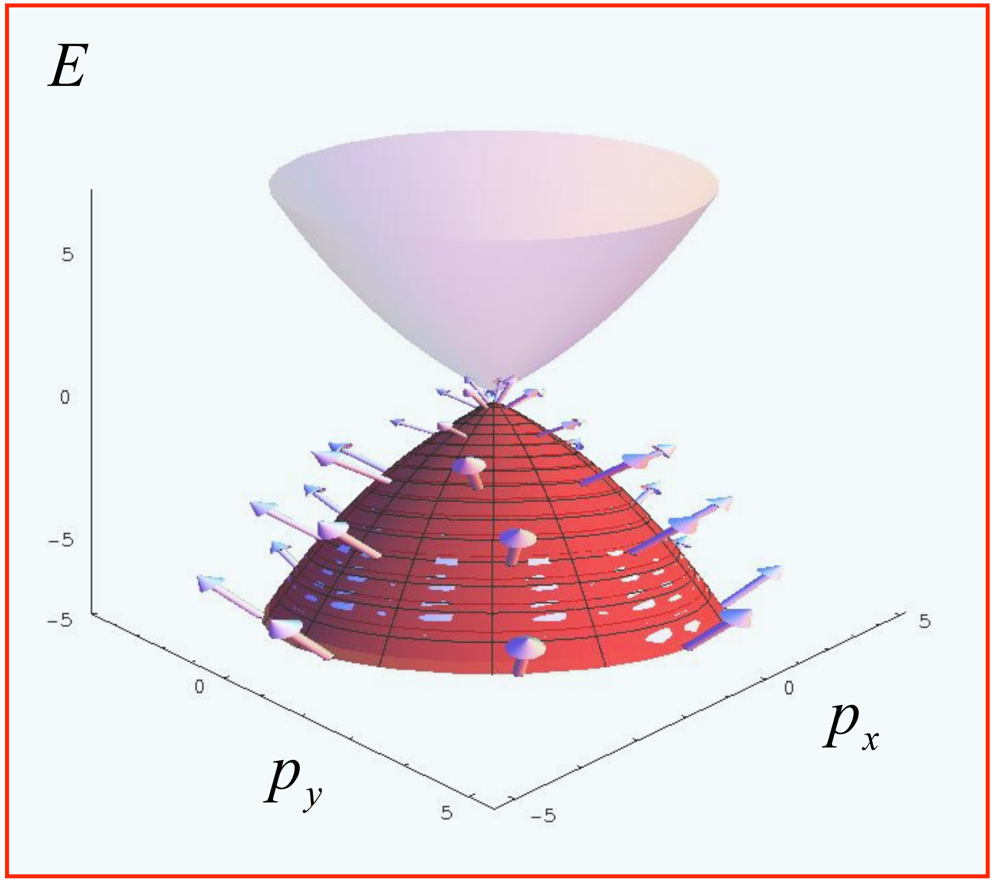

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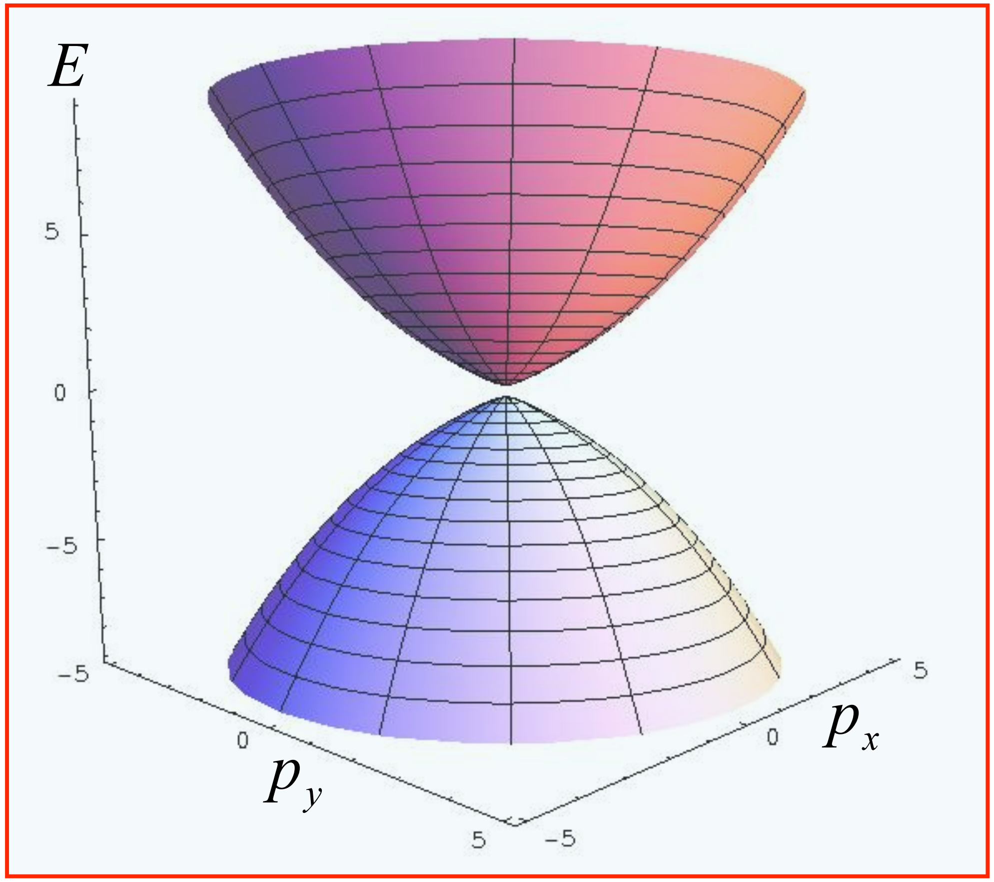

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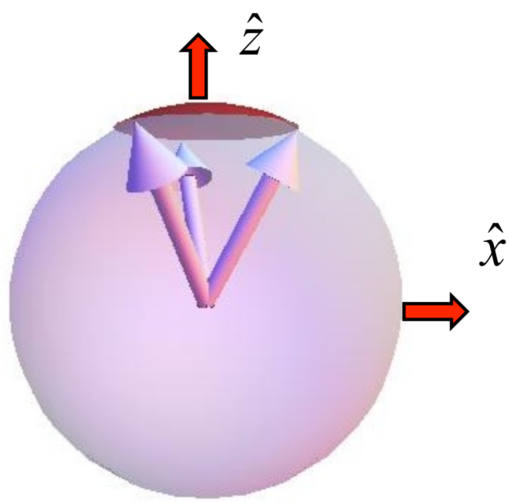

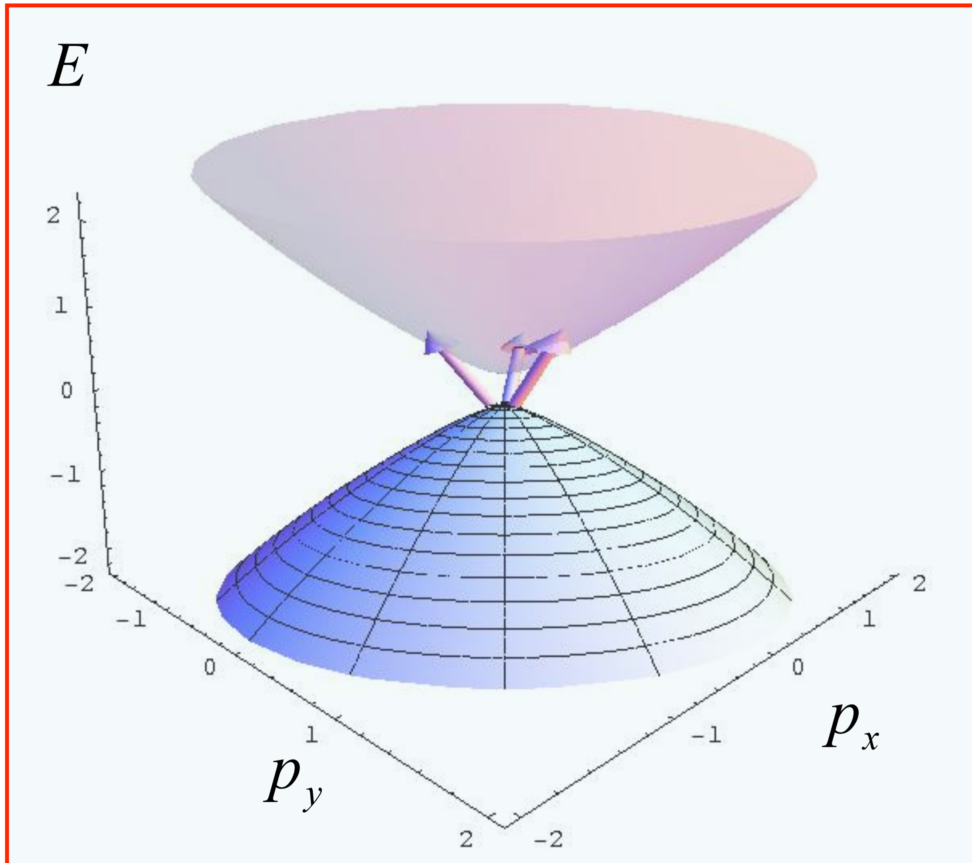

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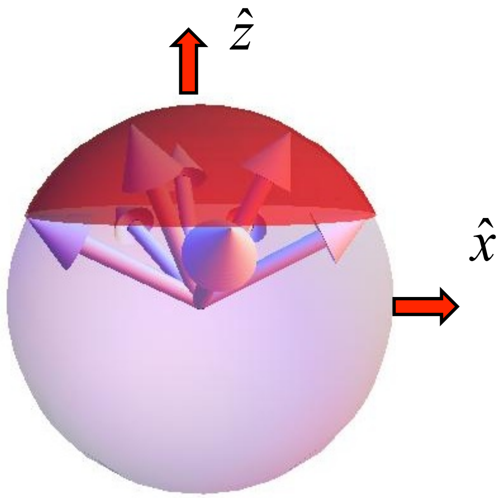

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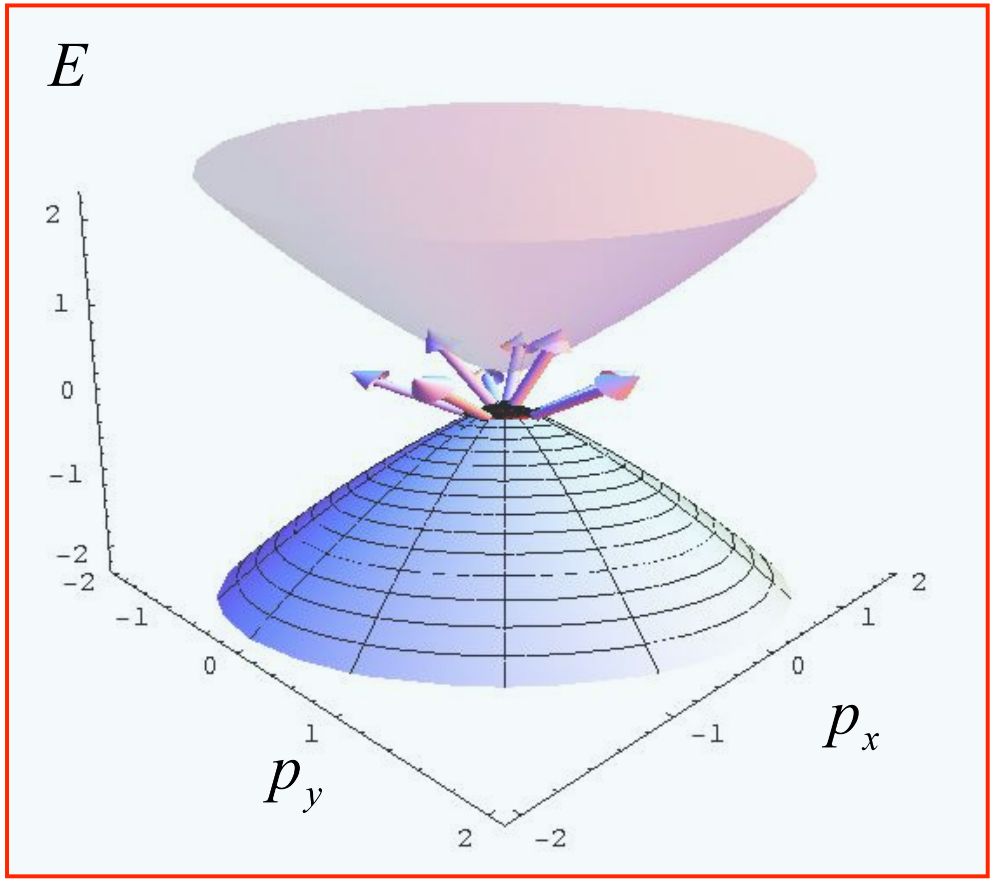

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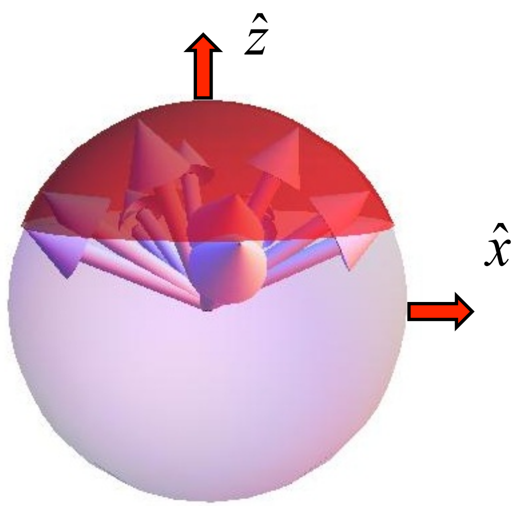

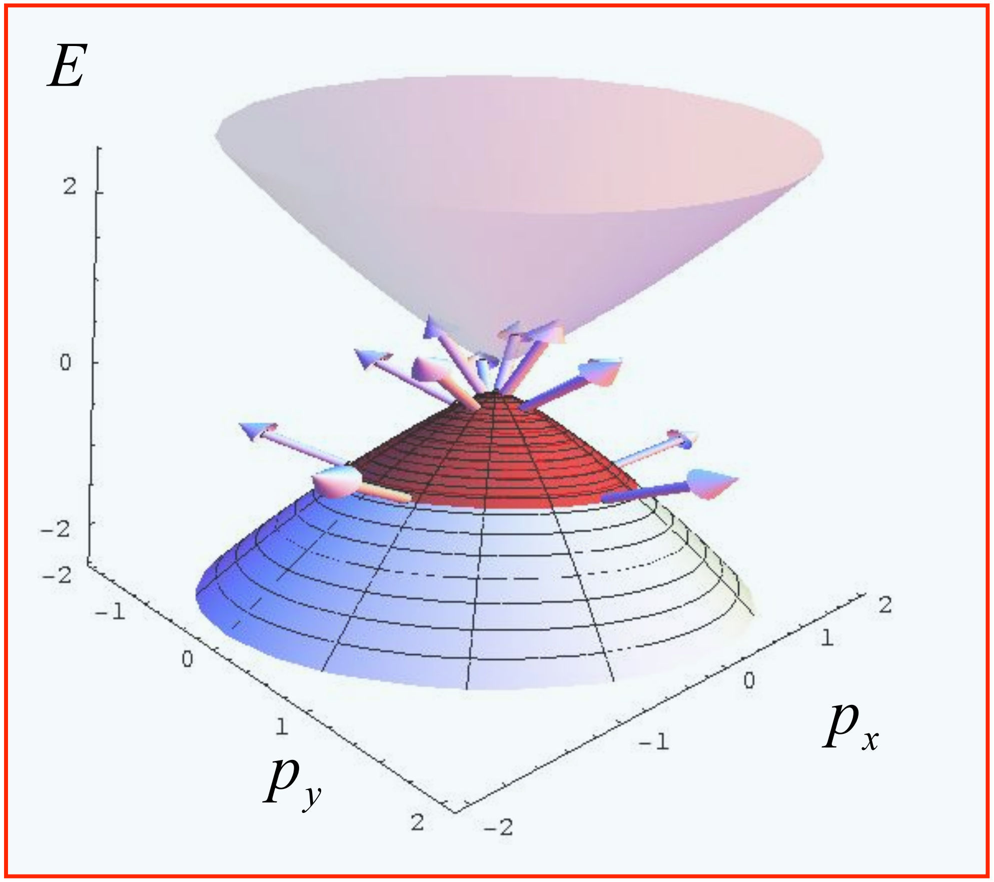

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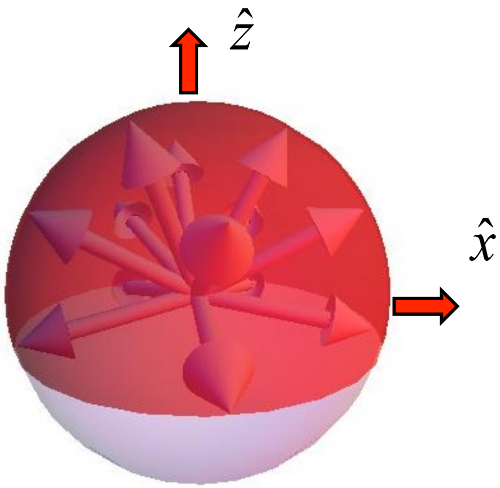

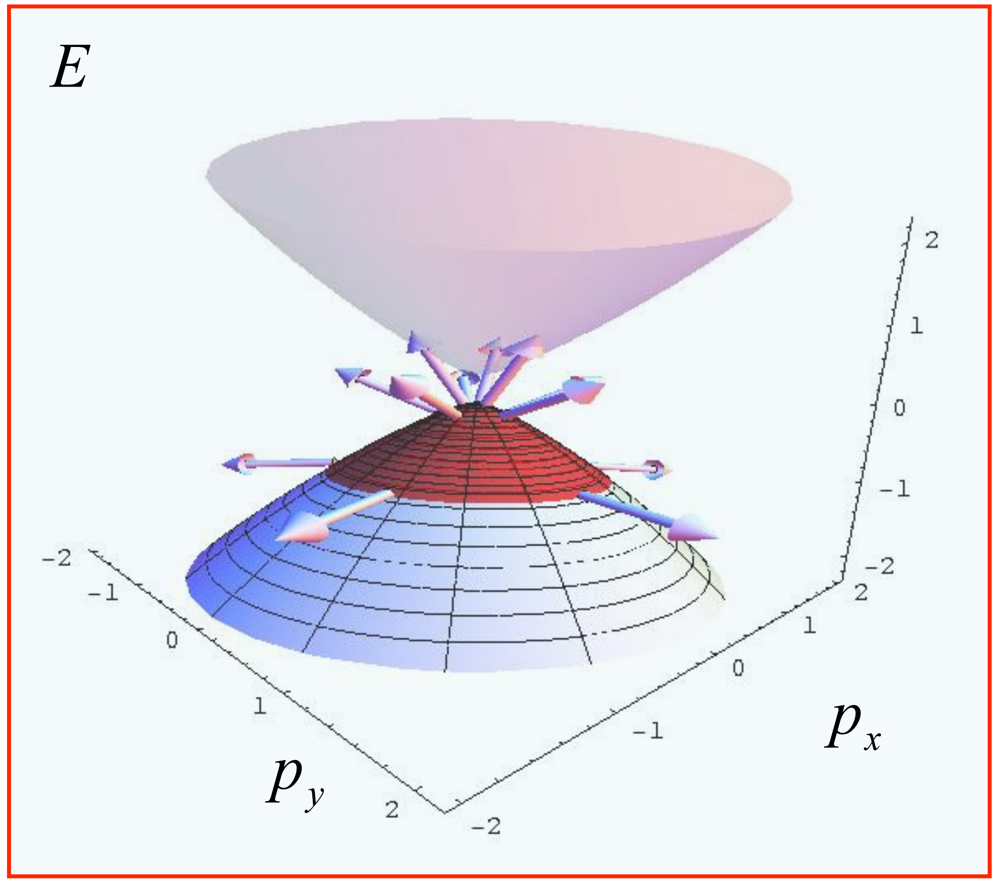

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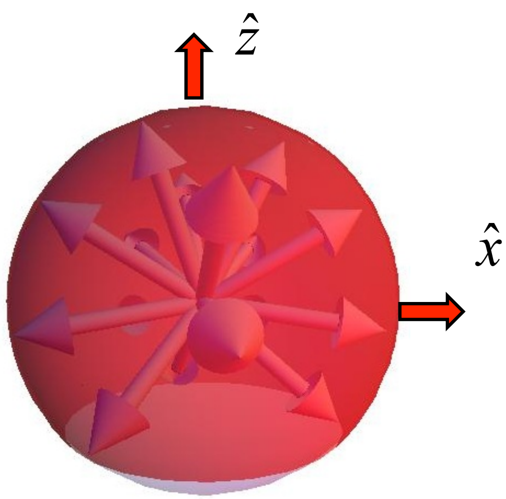

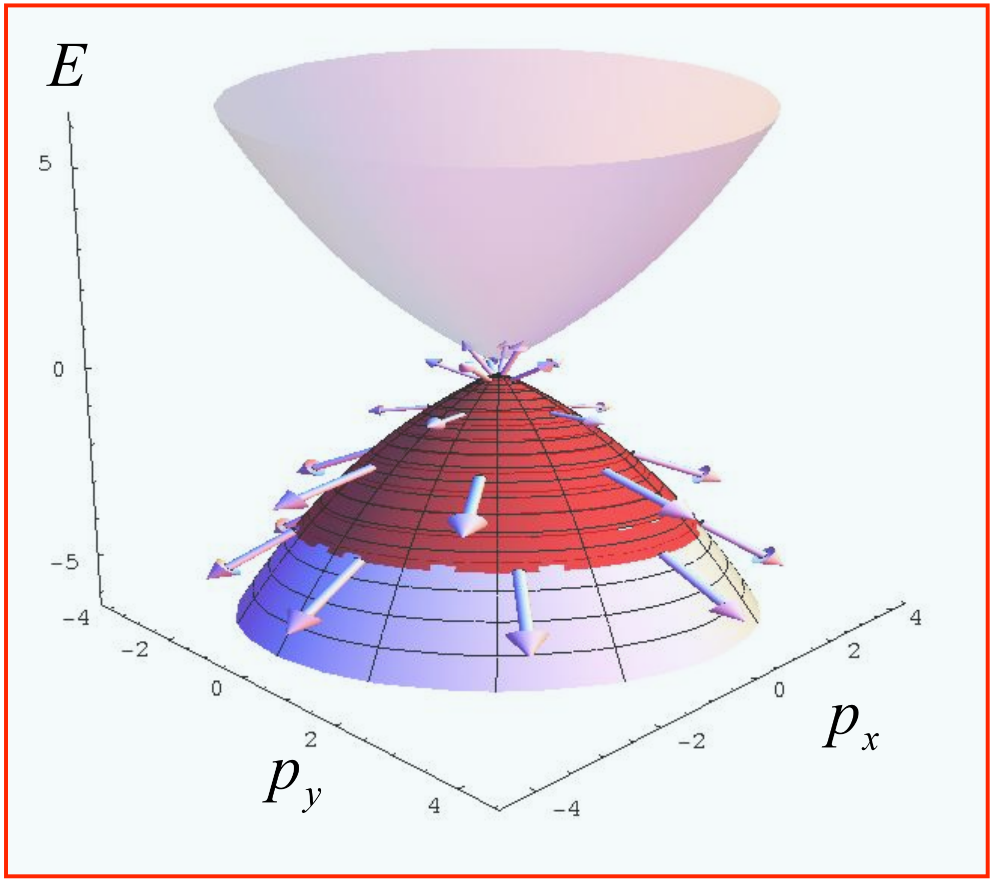

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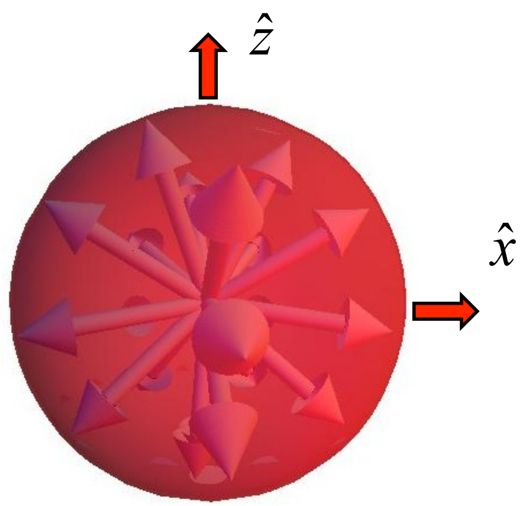
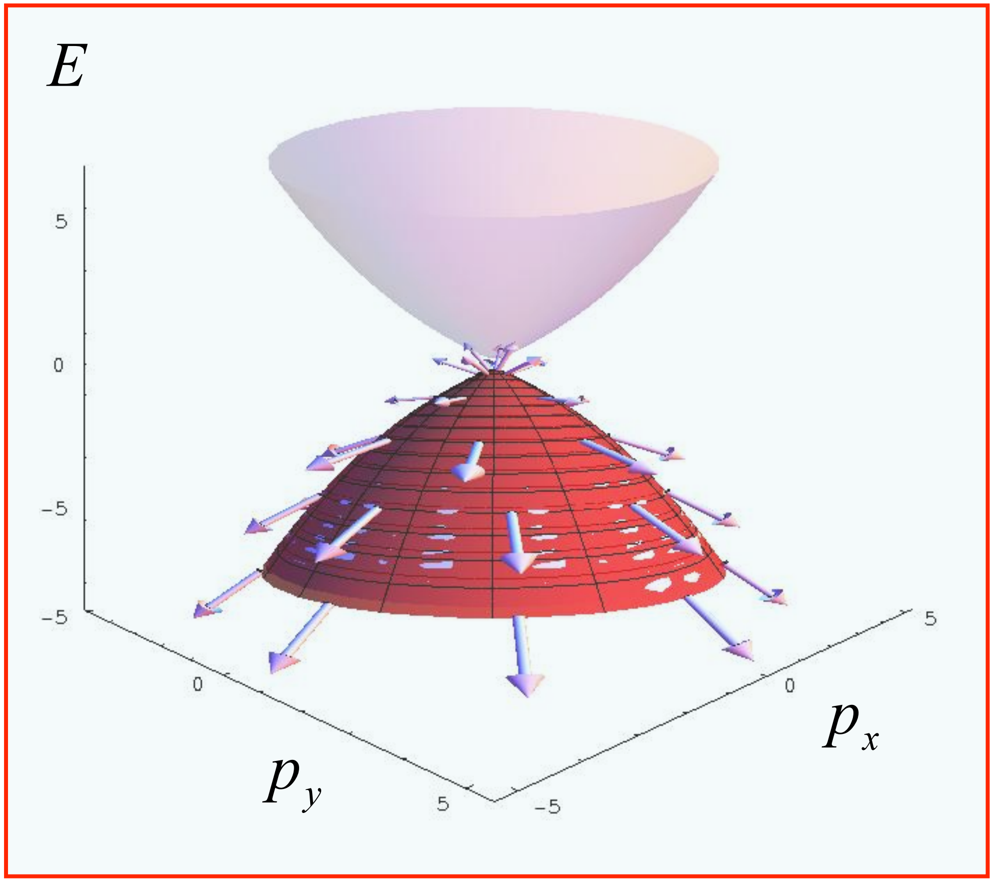




Outline

- 2D topological insulators
- Floquet theory
- How can we topologize the trivial?
- 3D Topological Floquet spectrum
- Edge states without chern numbers

Time Periodic Hamiltonians

- Topological classification for H(t)?
- For $H(t+T) = H(t)$ the evolution is given by:

$$U(t) = W(t)e^{-iH_F t} \qquad W(t+T) = W(t)$$

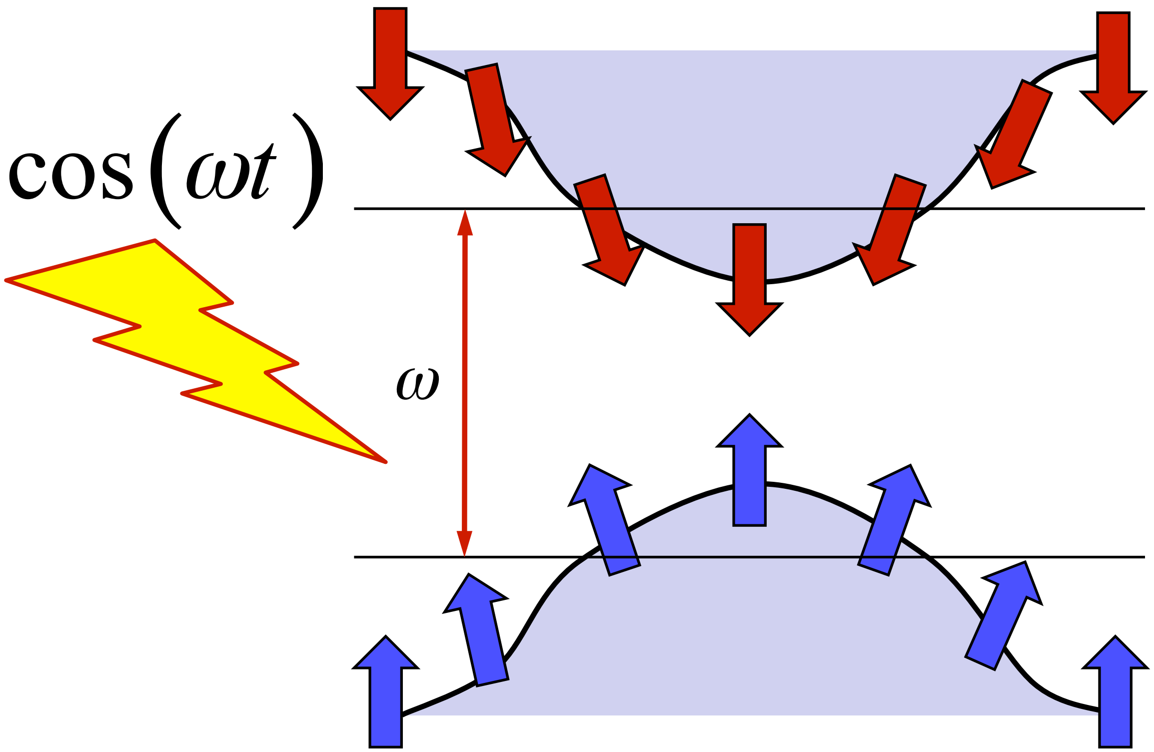
- Study the topological properties of

$H_F(\mathrm{k})$ = Floquet operator

Can we induce a topological phase with radiation effects?

- Start with non-topological spectrum:

$\hat{V}\cos(\omega t)$



$-\vec{d}/|\vec{d}|$

$\langle \psi_{\mathbf{k}} | \vec{\sigma} | \psi_{\mathbf{k}} \rangle = \vec{d}/|\vec{d}|$

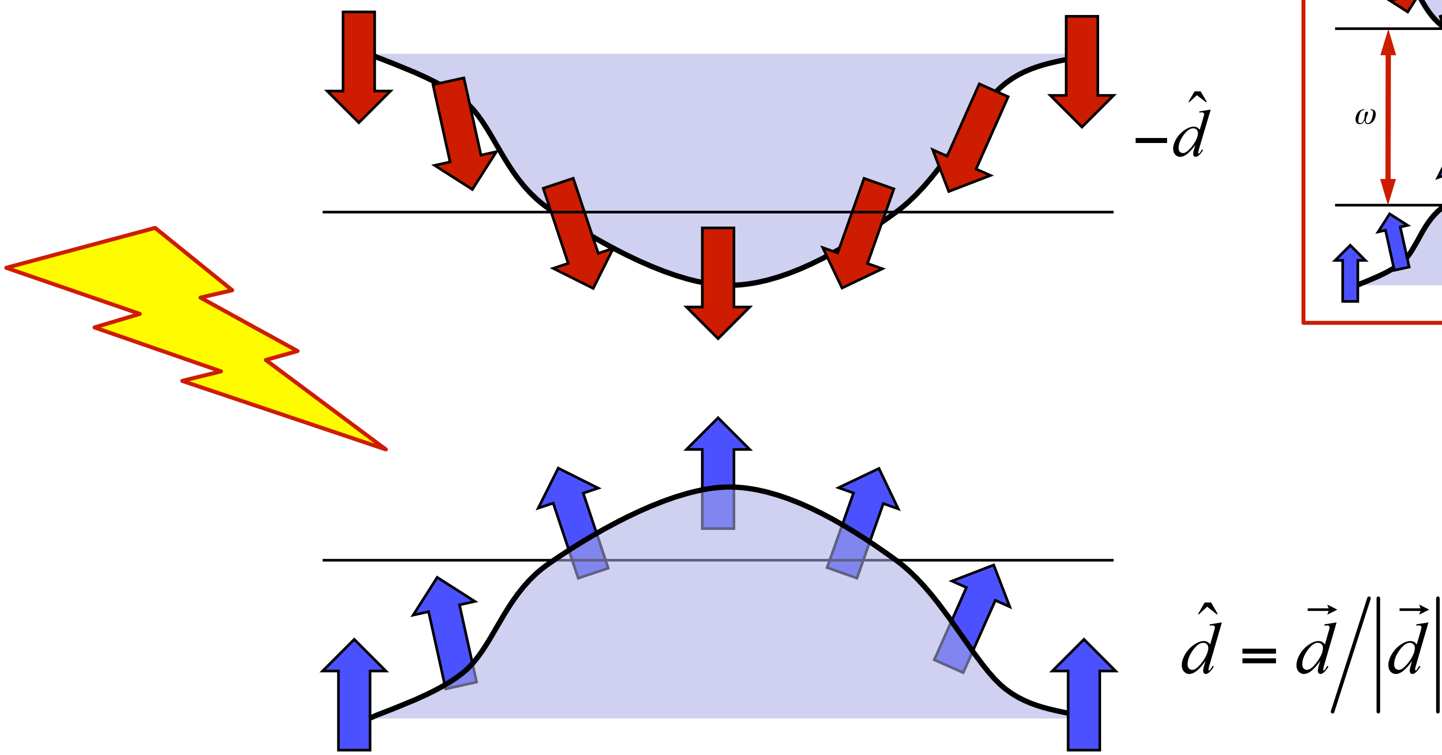
$H = \varepsilon_0 + \vec{d}(\mathbf{k})\cdot\vec{\sigma}$

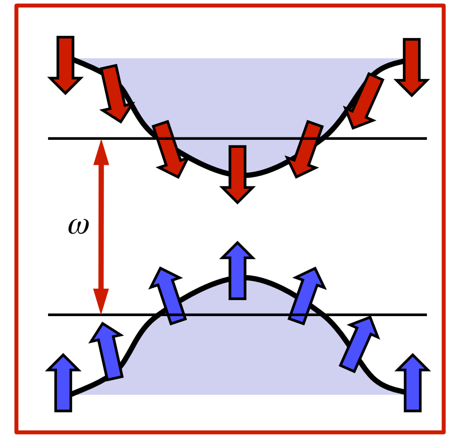
$\vec{d}(\mathbf{k}) = (k_x, k_y, m + b\mathbf{k}^2)$

$m,\ b > 0$

Interaction picture

- The same dynamics in a rotating frame:

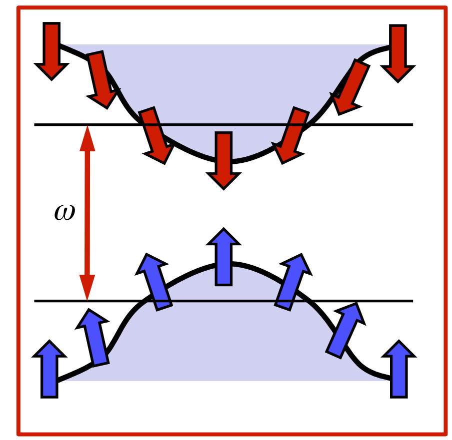


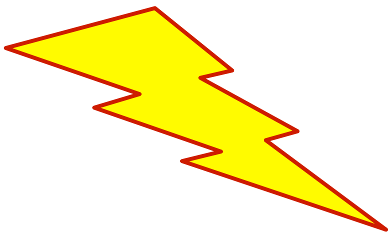


$$U(\mathbf{k},t) = P_+(\mathbf{k}) + P_-(\mathbf{k})e^{i\omega t}$$

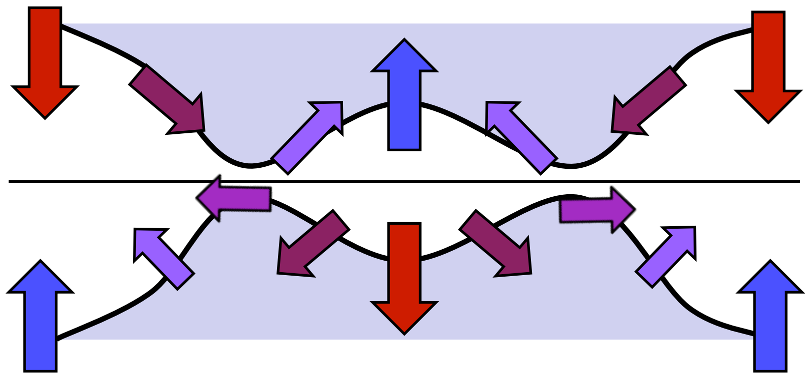
Interaction picture

- Generically, a gap is opened:

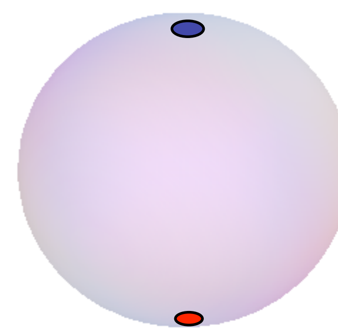




$$\hat{V}\cos(\omega t)$$



- A map from the south to the north pole:



Conditions for a Topological Spectrum

- In the rotating wave approximation

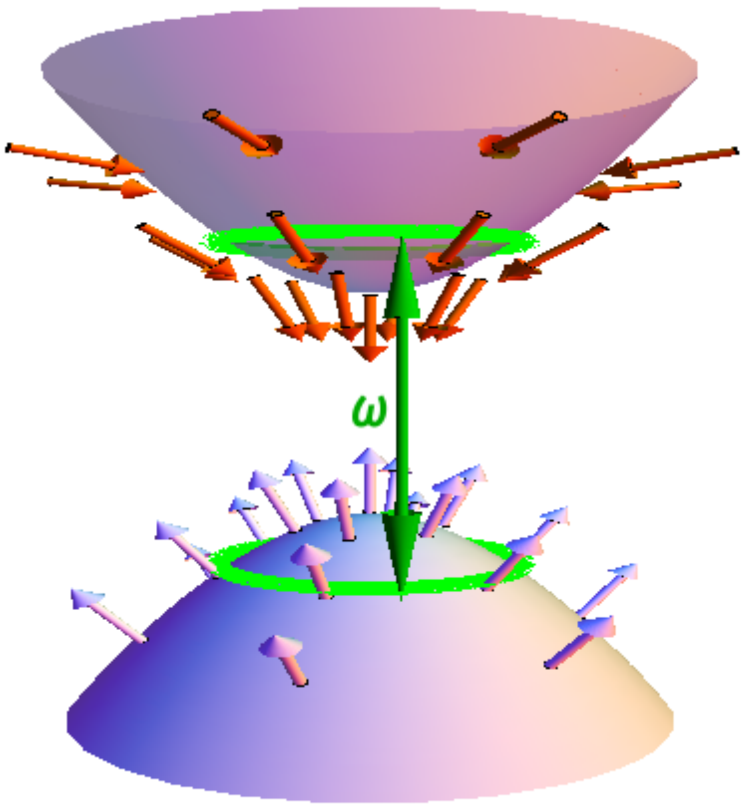
$$\hat{V}_{\mathrm{RWA}} = P_+(\mathbf{k})\hat{V}P_-(\mathbf{k}) + P_-(\mathbf{k})\hat{V}P_+(\mathbf{k})$$

- Consider a modulation of the form

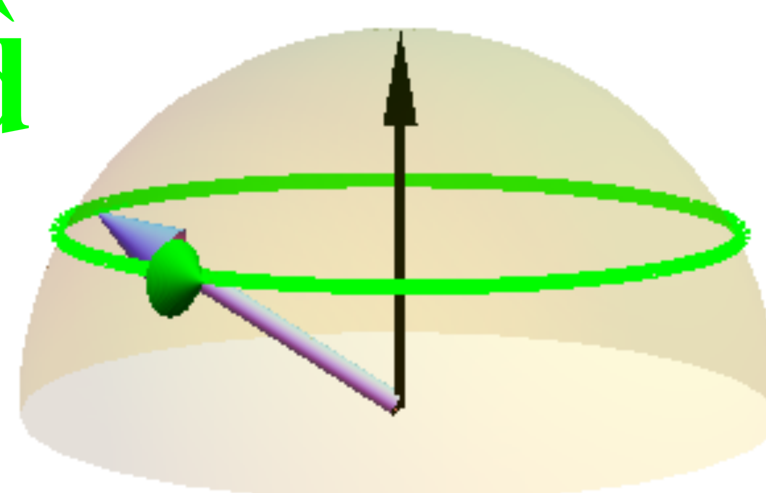
$$\hat{V} = \mathbf{V}\cdot\vec{\sigma}$$

$$\mathbf{V} = \left(\mathbf{V}\cdot\hat{\mathbf{d}}(\mathbf{k})\right)\hat{\mathbf{d}}(\mathbf{k}) + \mathbf{V}_\perp(\mathbf{k})$$

Condition on the modulation:



$\hat{\mathbf{d}}$



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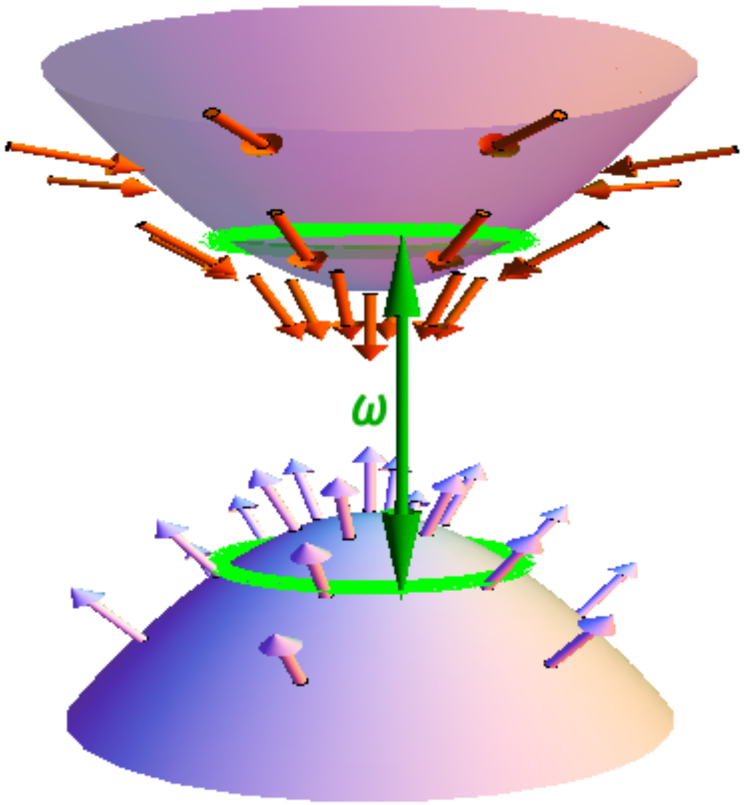
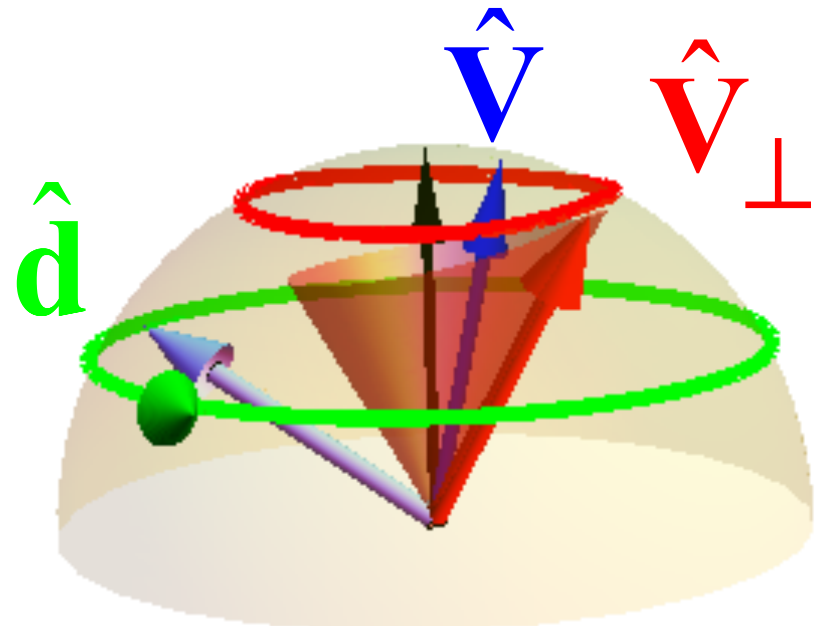
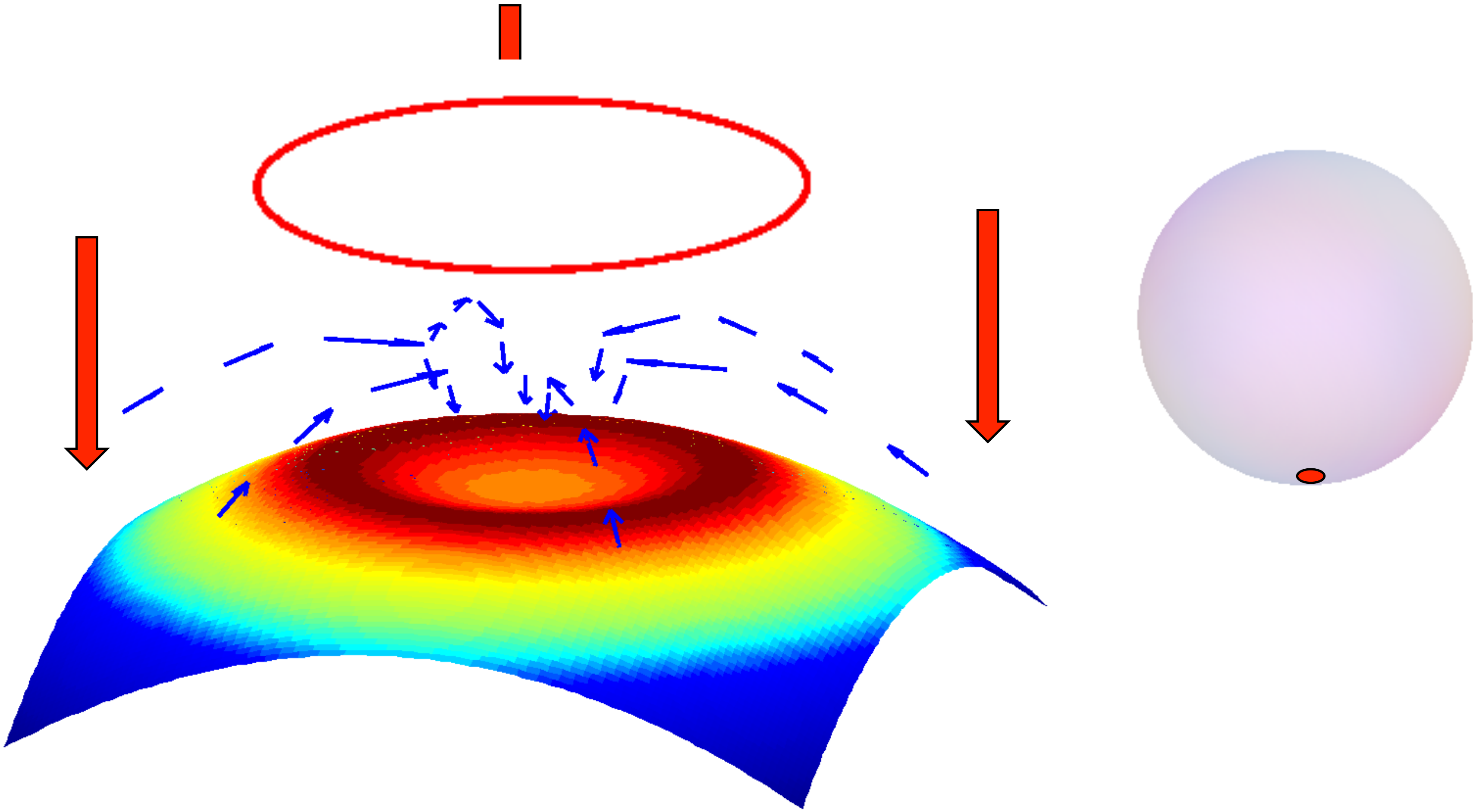


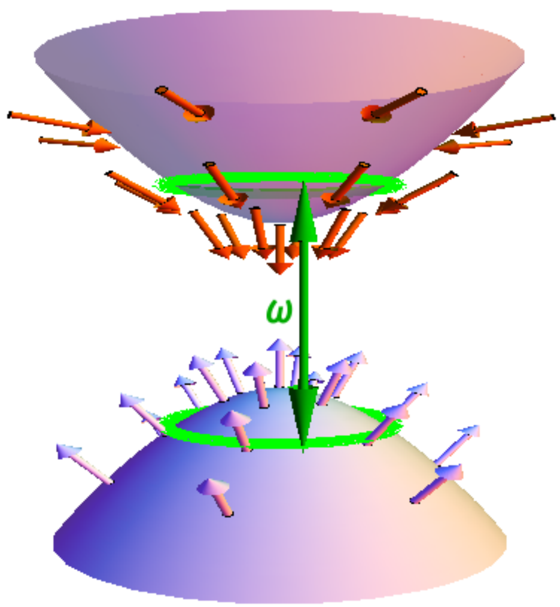


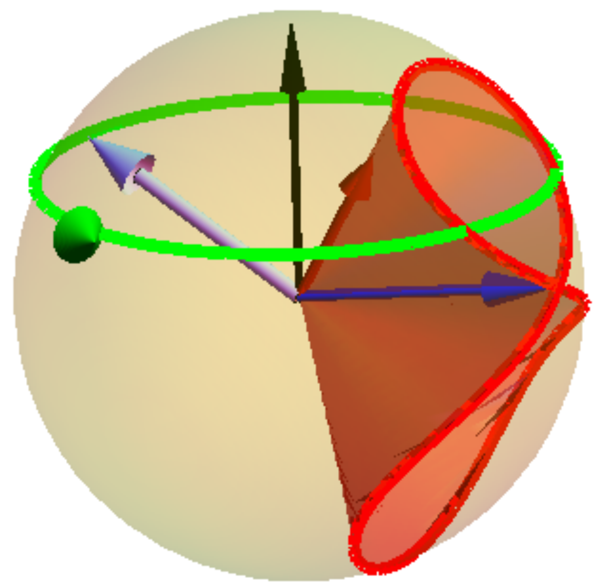
Topological Floquet Bands



Condition on the modulation

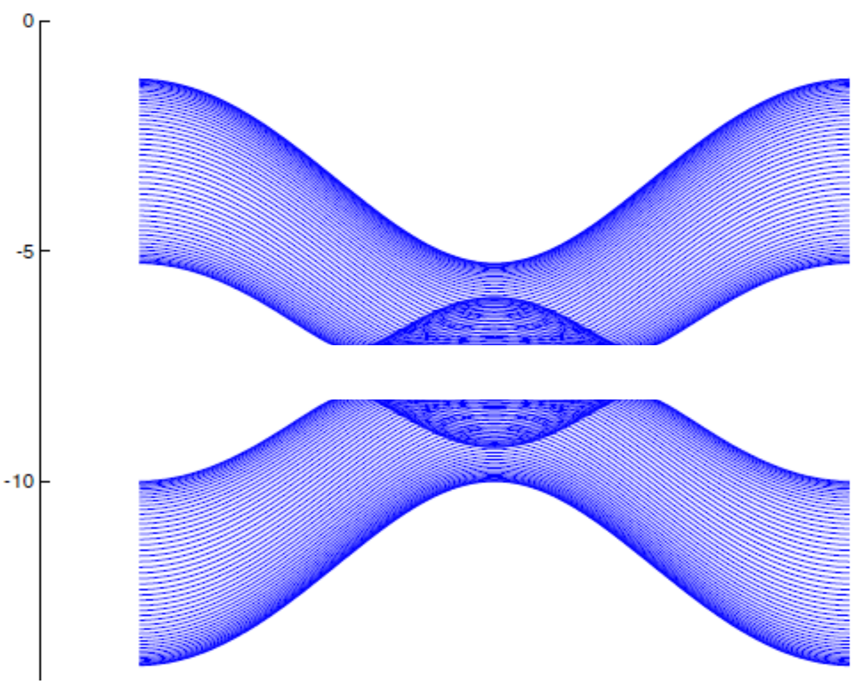
When the geometrical condition is
not satisfied:





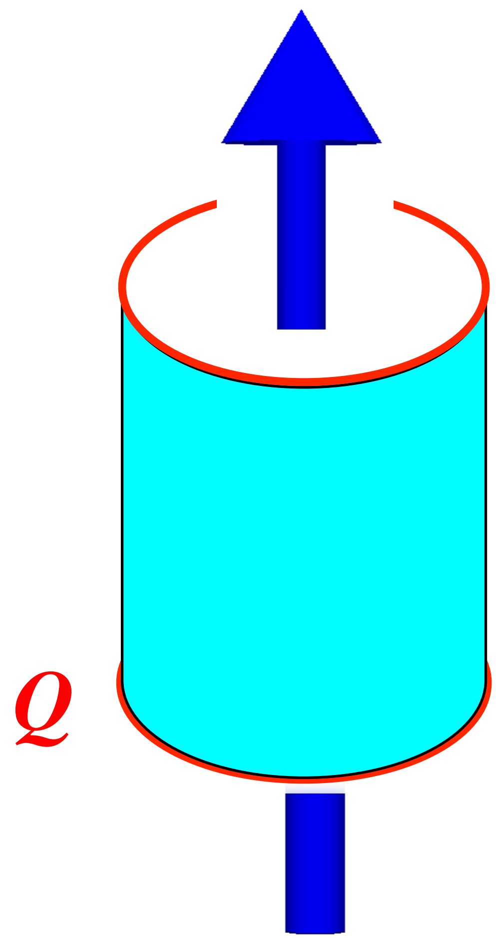
Topological Floquet Bands

$$\Phi : 0 \rightarrow \Phi_0$$



$= -1$

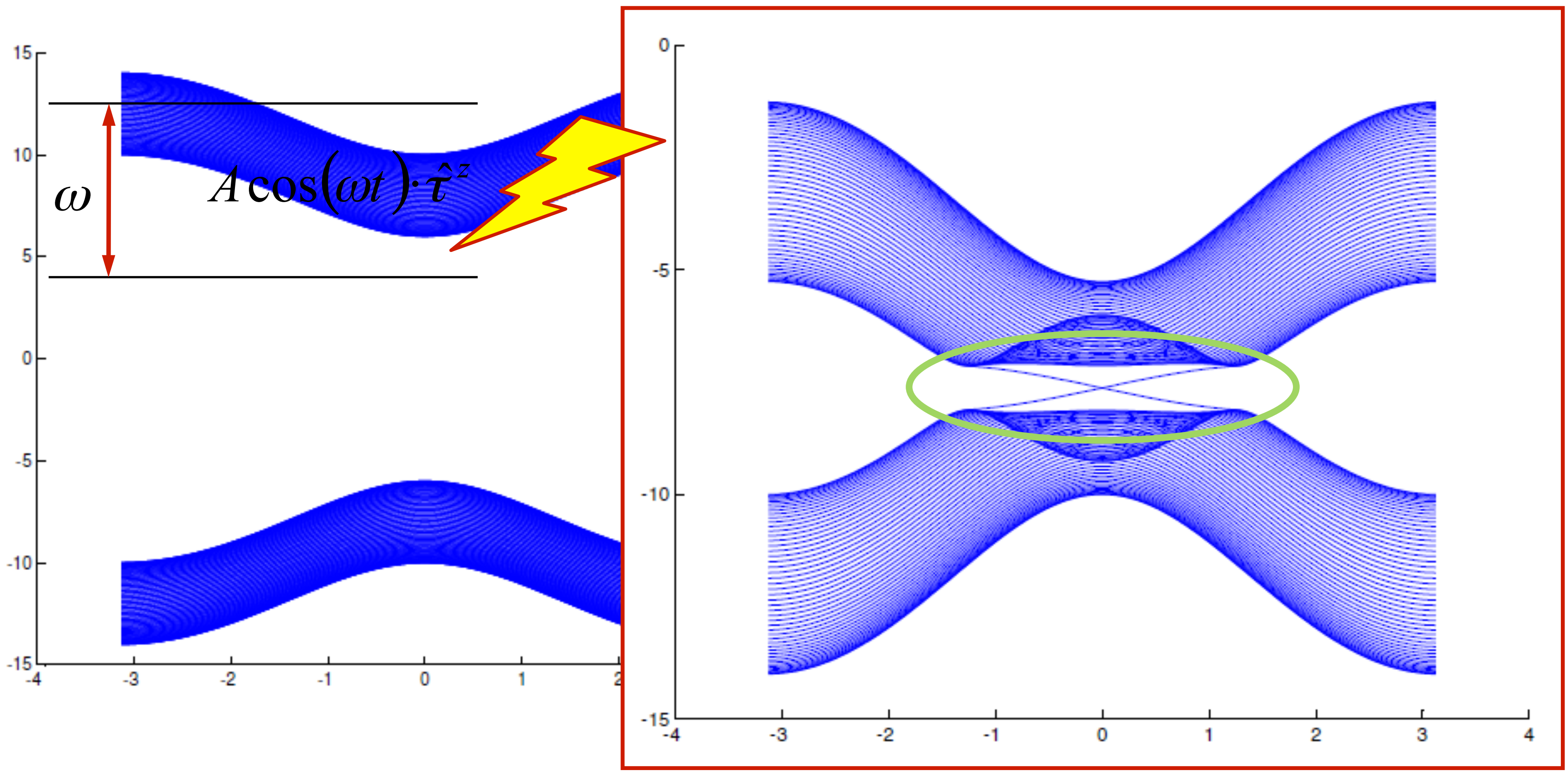
$= +1$



Edge modes

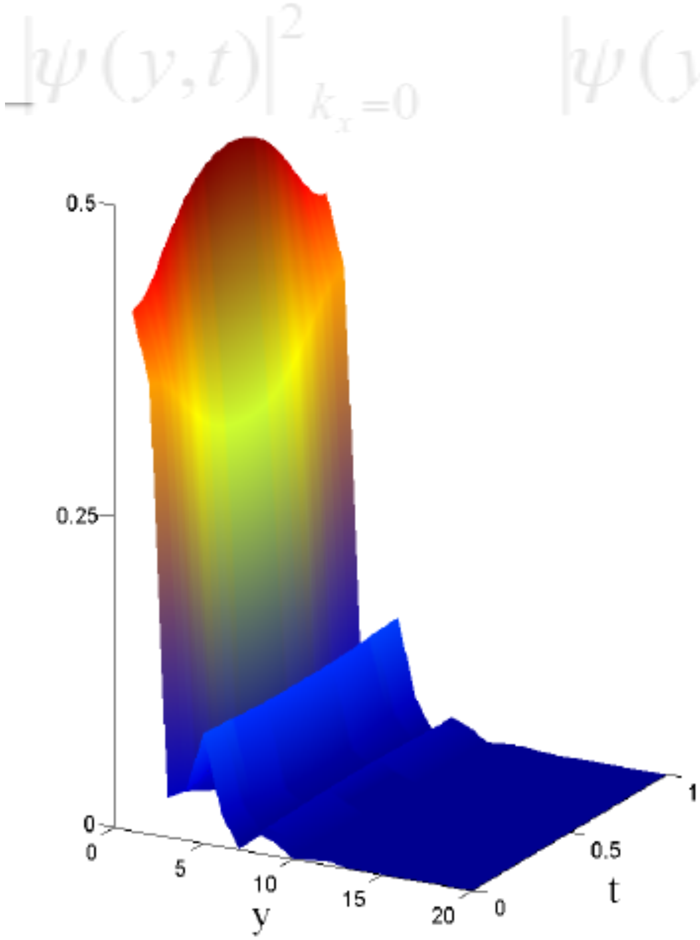
$$H = \varepsilon_0 + \vec{d}(\mathbf{k}) \cdot \vec{\sigma}$$

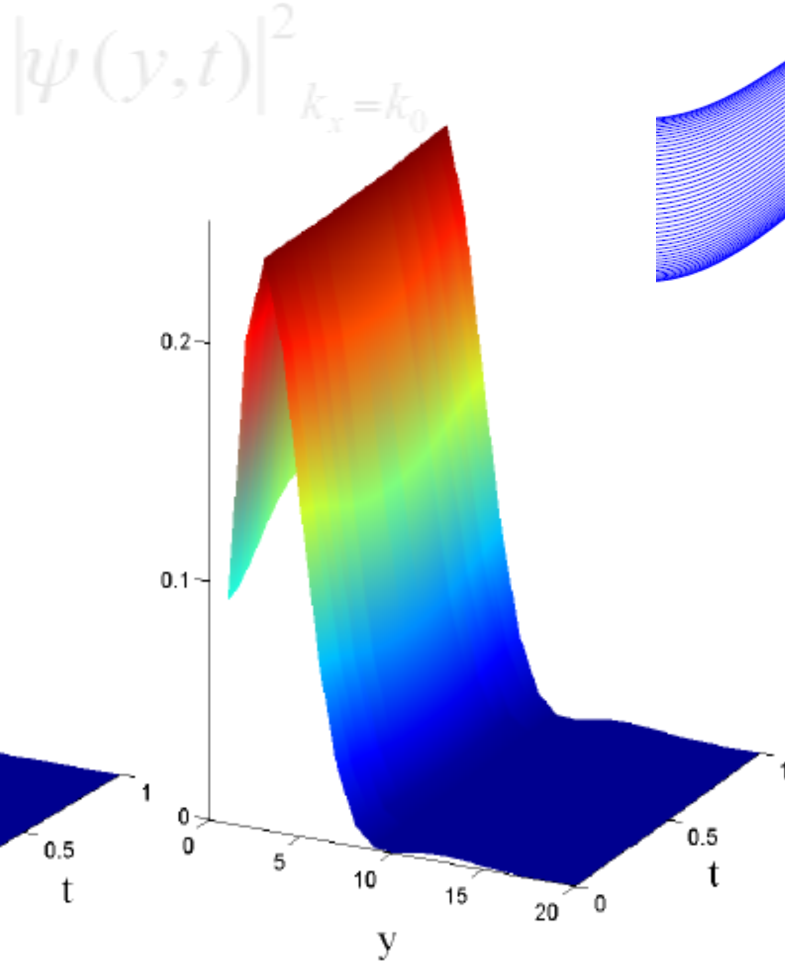
$$\vec{d}(\mathbf{k}) = (k_x, k_y, m + b\mathbf{k}^2)$$

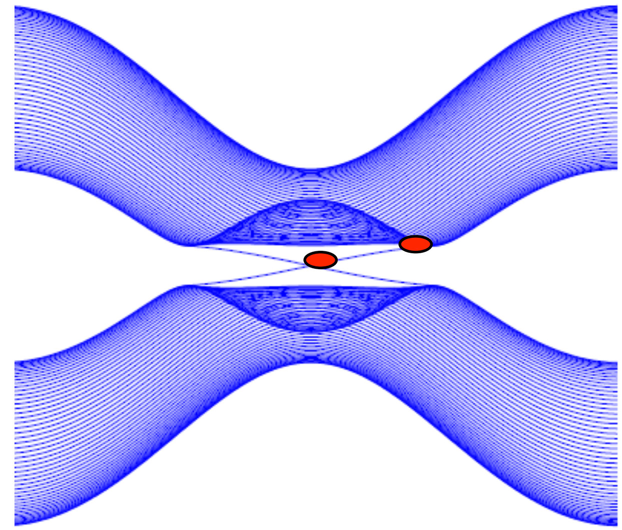


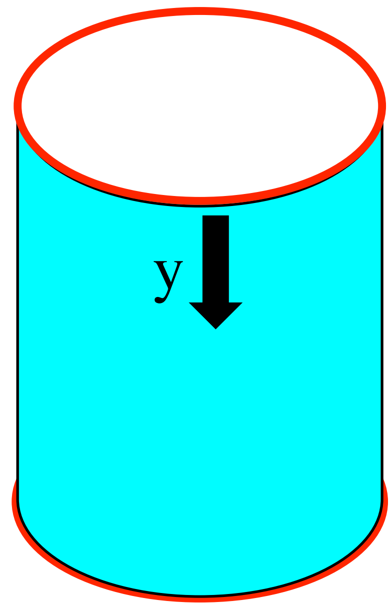
Edge modes

Density vs time:

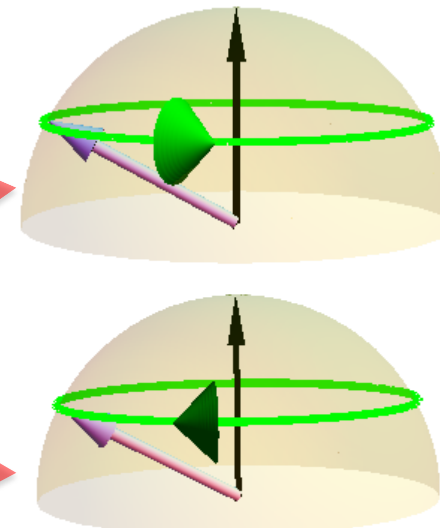




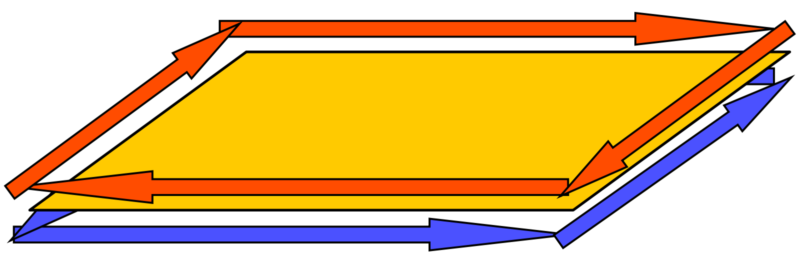


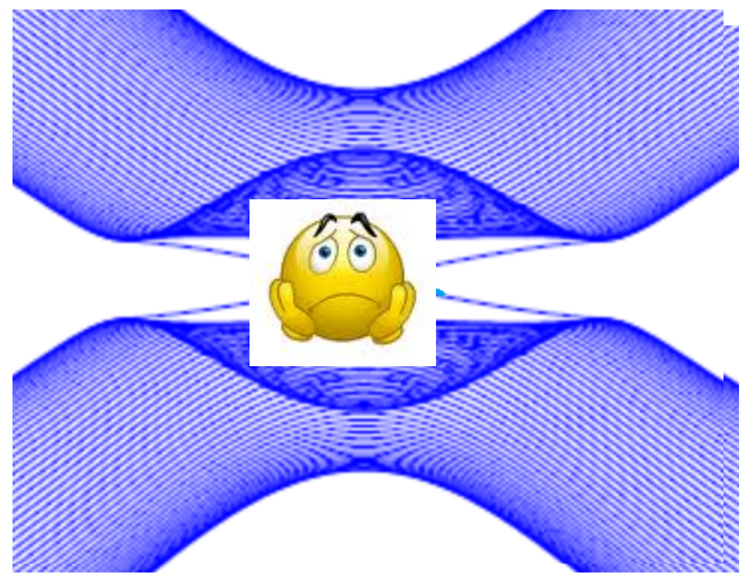


What about time reversal symmetry?



$$\hat{H}(\mathbf{k}) = \begin{pmatrix} H(\mathbf{k}) & 0 \\ 0 & H^*(-\mathbf{k}) \end{pmatrix}$$





Time reversal symmetry

- Original Hamiltonian has TRS

$$\Theta\hat{H}(\mathbf{k})\Theta^{-1} = \hat{H}(-\mathbf{k}) \qquad \Theta^2 = -1$$

- For any given time $\hat{H}(\mathbf{k},t)$ is **not** TRS.
- However, if

$$\boxed{\Theta\hat{H}(\mathbf{k},t)\Theta^{-1} = \hat{H}(-\mathbf{k},-t+\tau)}$$

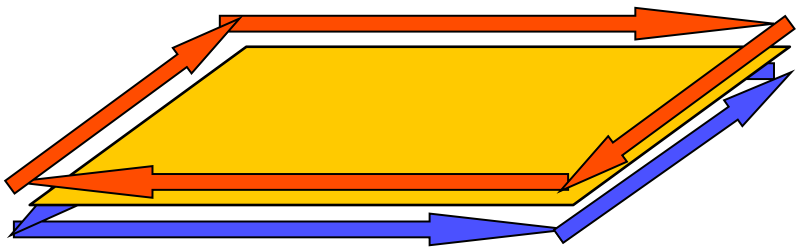
$$\Longrightarrow \quad \boxed{\tilde{\Theta}H_F(\mathbf{k})\tilde{\Theta}^{-1} = H_F(-\mathbf{k})} \qquad \boxed{\tilde{\Theta} = V^{\dagger}\Theta V}$$

Time reversal symmetry

$$\hat{H}(t) = \hat{H}_0 + \hat{V}\cos\left(\omega t + \phi\right)$$

- The condition: $\Theta \hat{H}(\mathbf{k},t)\Theta^{-1} = \hat{H}(-\mathbf{k},-t+\tau)$

is satisfied for: $\Theta \hat{V} \Theta^{-1} = \pm \hat{V}$



Kramer's pairs

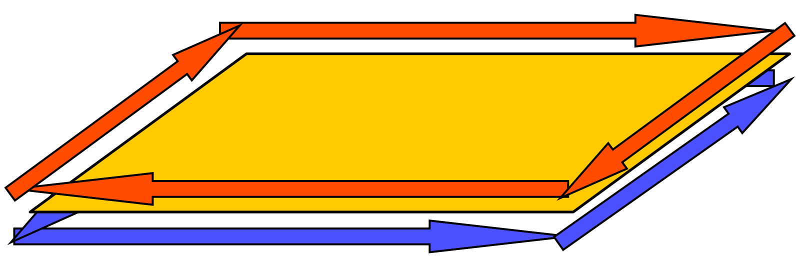
Two Classes of Topological Floquet States

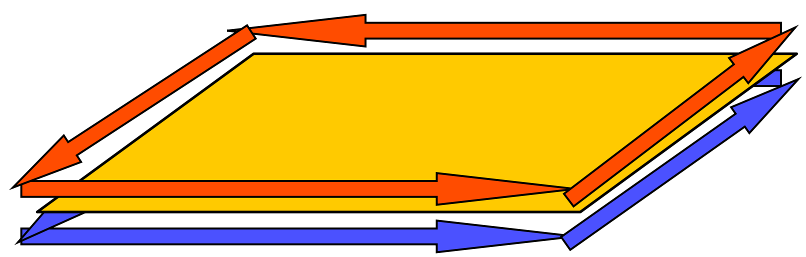
$\hat{V}\cos(\omega t+\phi)$

- Z_2 ,two counter propagating edge states, TRS

$\hat{E}_1\cos(\omega t)+\hat{E}_2\sin(\omega t)$

- Z ,co- propagating edge states, no TRS





Experimental realization in HgTe/CdTe wells?

- Electric field:

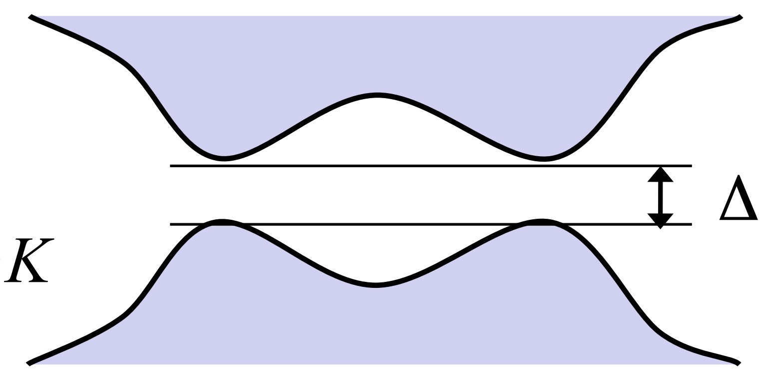
$$\mathbf{V}_{\perp} = \hat{\mathbf{d}}_{\mathbf{k}} \times \left(\operatorname{Re}\mathbf{E} \cdot \nabla_{\mathbf{k}}\right)\hat{\mathbf{d}}_{\mathbf{k}} - \left(\operatorname{Im}\mathbf{E} \cdot \nabla_{\mathbf{k}}\right)\hat{\mathbf{d}}_{\mathbf{k}}$$

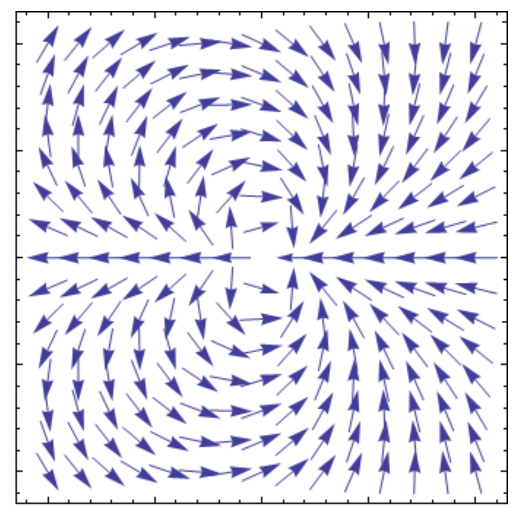
- Circular polarization: $E = E_0\left(\hat{x} + i\hat{y}\right)$

$$\mathbf{V}_{\perp} \propto \frac{1}{2}(k_x^2 - k_y^2)\hat{\mathbf{x}} + k_x k_y \hat{\mathbf{y}}$$

$E \sim 1\ V/m$

$\Delta \sim 10K$





Outline

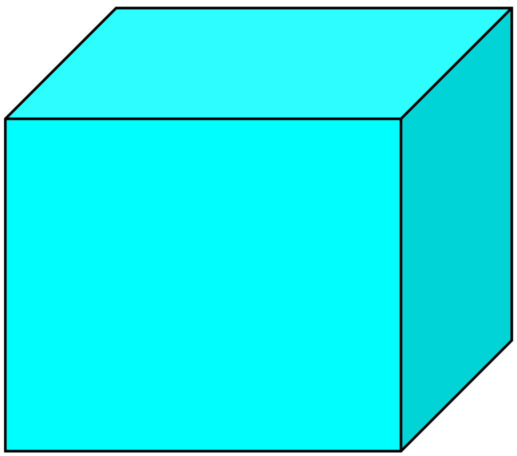
- 2D topological insulators
- Floquet theory
- How can we topologize the trivial?
- 3D Topological Floquet spectrum
- Edge states without chern numbers

Three dimensions

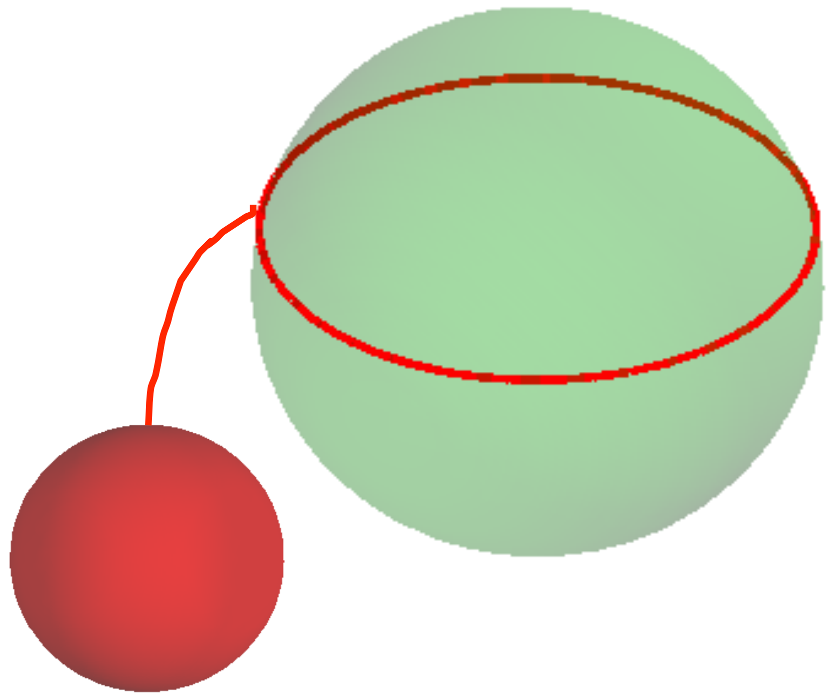
$$H(\mathbf{k}) = \varepsilon_0(\mathbf{k}) + \vec{D}(\mathbf{k}) \cdot \vec{\gamma}$$

$$\vec{D}(\mathbf{k}) = (d_1, d_2, d_3; d_5)$$

Brillouin Zone



Three - sphere



$$\pi_3(S^3) = \mathbb{Z}$$

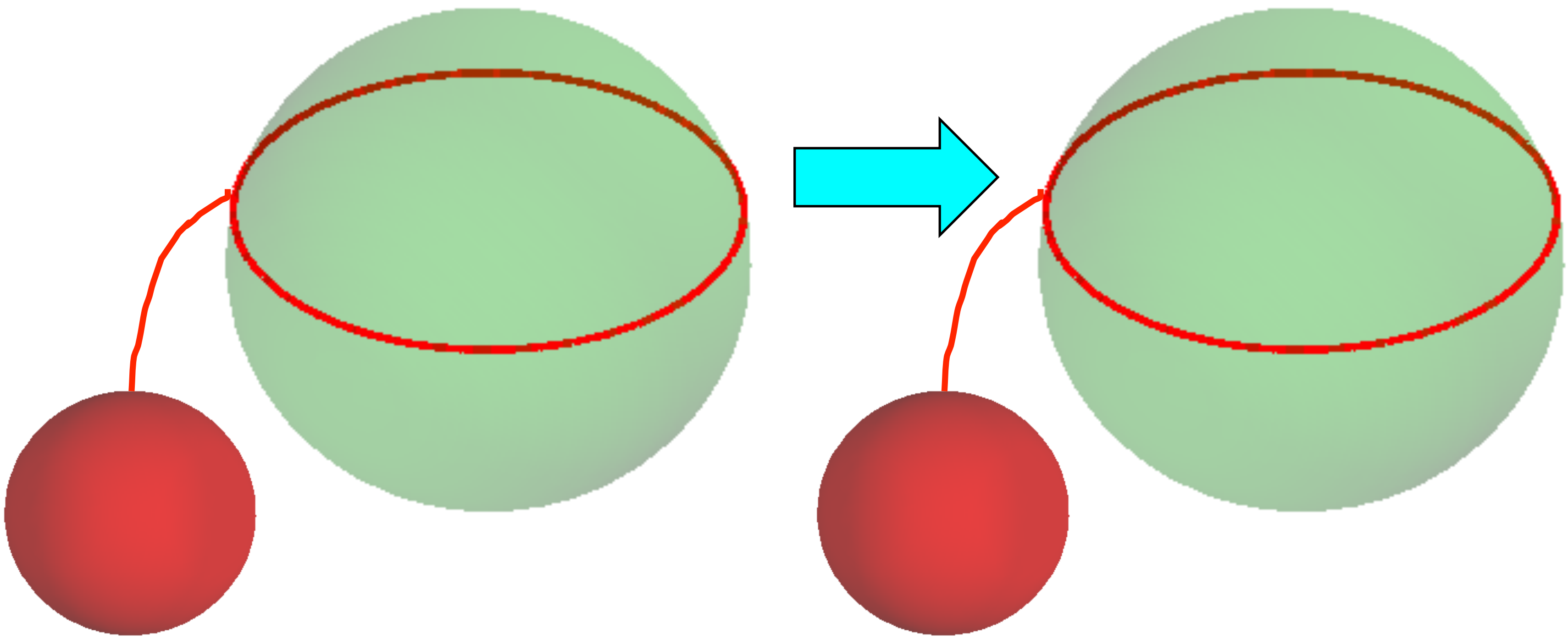
$$\to \mathbb{Z}_2$$

Three dimensions

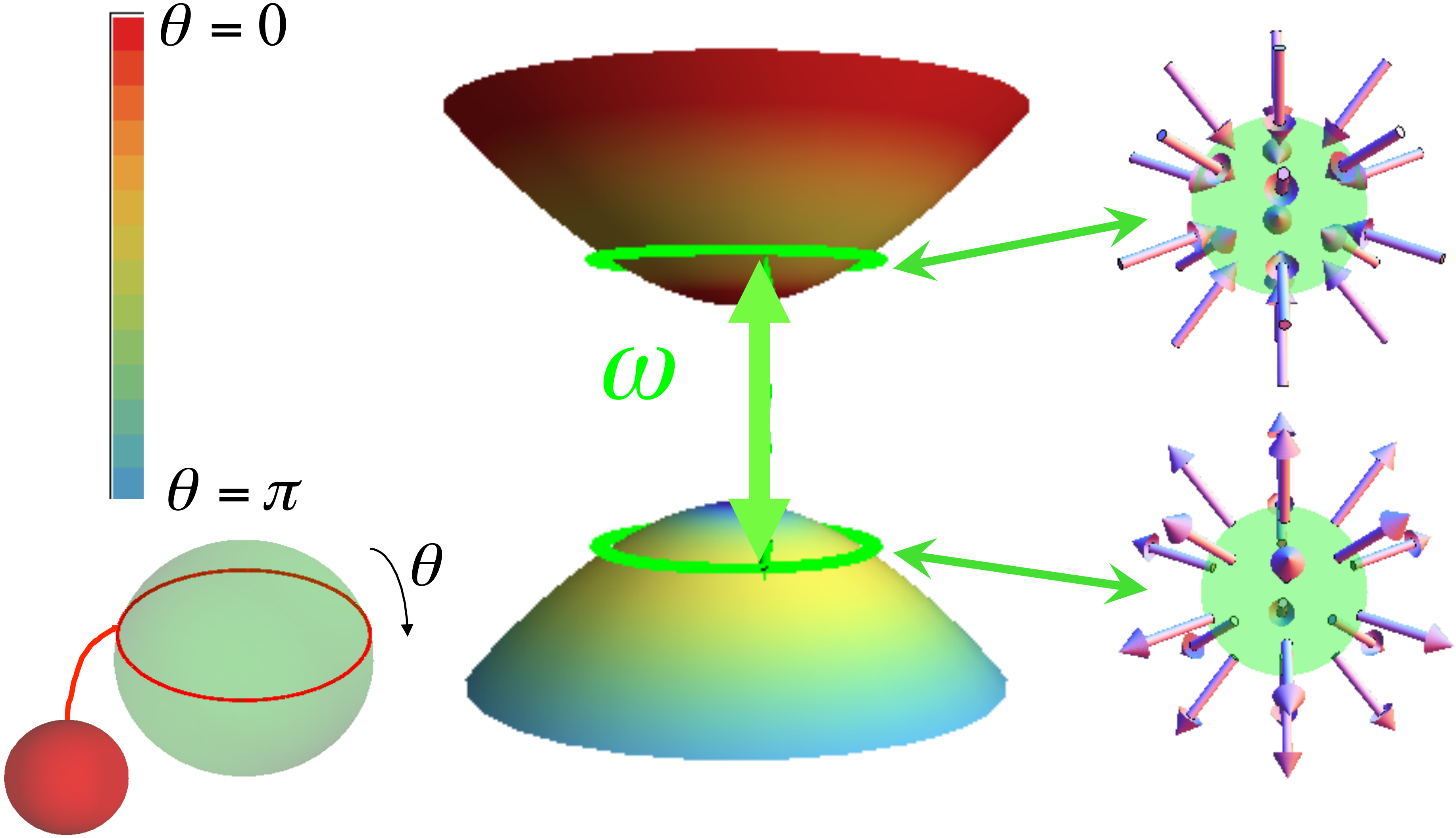
$$\vec{D}(\mathbf{k}) = (A\mathbf{k}; M - B\mathbf{k}^2)$$

Brillouin Zone

Three - sphere



Three dimensions



Conditions for a Topological Spectrum

$$H = H(\mathbf{k}) + \hat{V}\cos(\omega t)$$

- In the rotating wave approximation

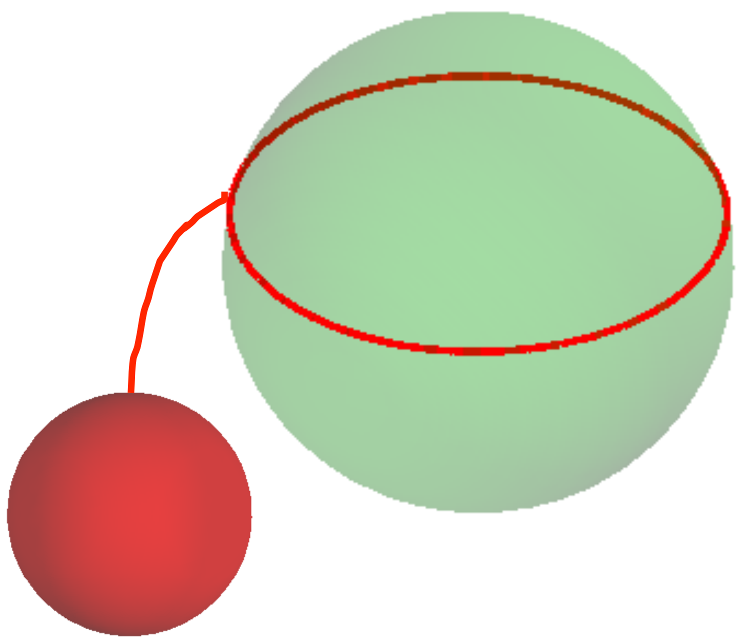
$$\hat{V}_{\mathrm{RWA}} = P_+(\mathbf{k})\hat{V}P_-(\mathbf{k}) + P_-(\mathbf{k})\hat{V}^\dagger P_+(\mathbf{k})$$

$$\hat{V} = \overrightarrow{V} \cdot \vec{\gamma} \quad \Longrightarrow \quad \boxed{\hat{V}_{\mathrm{RWA}} = \overrightarrow{V_\perp} \cdot \vec{\gamma}}$$

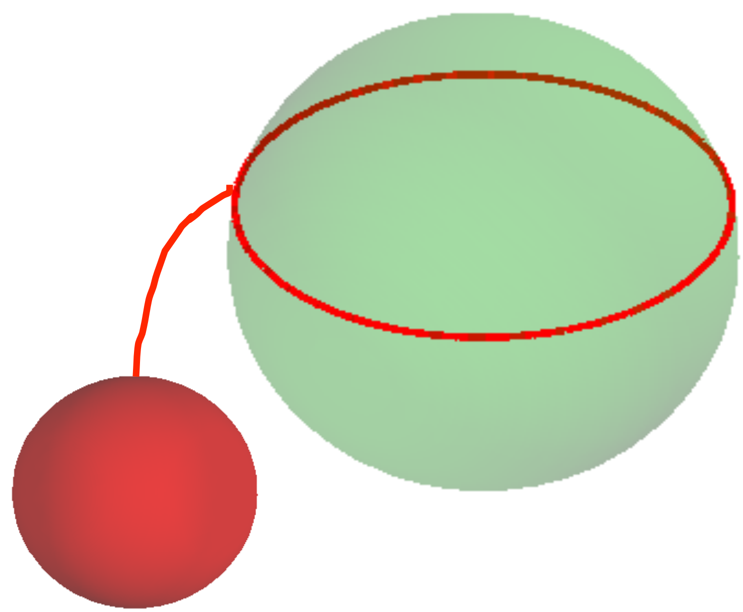
Conditions for a Topological Spectrum

$$\hat{V}_{\mathrm{RWA}} = \overrightarrow{V_{\perp}} \cdot \vec{\gamma}$$

Brillouin Zone



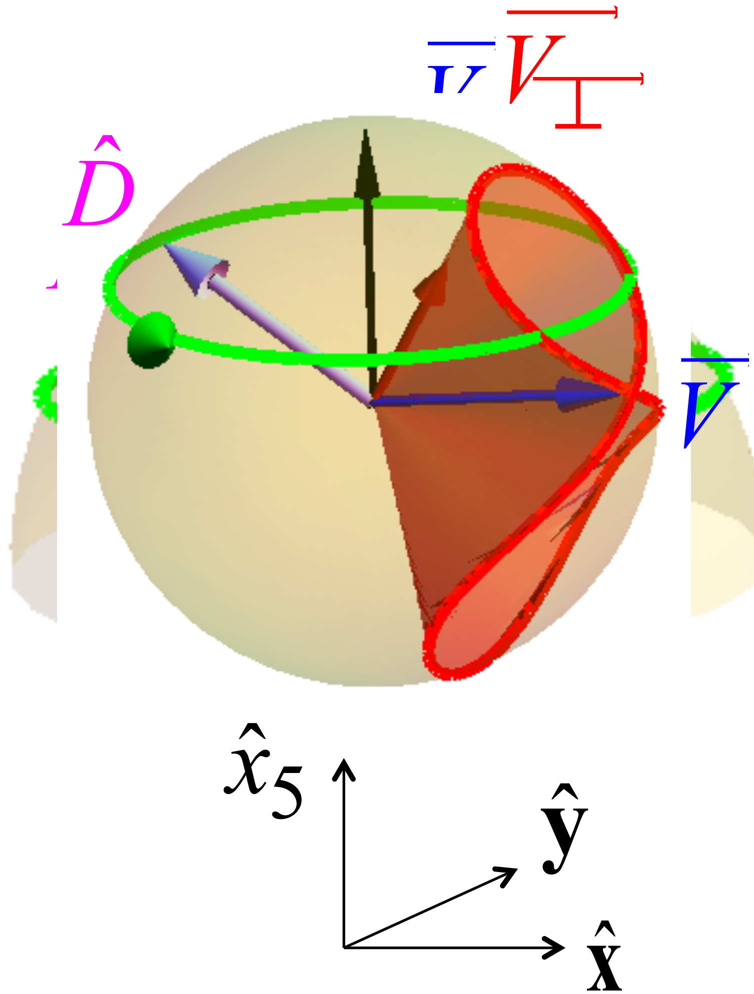
Three - sphere



Symmetry Considerations

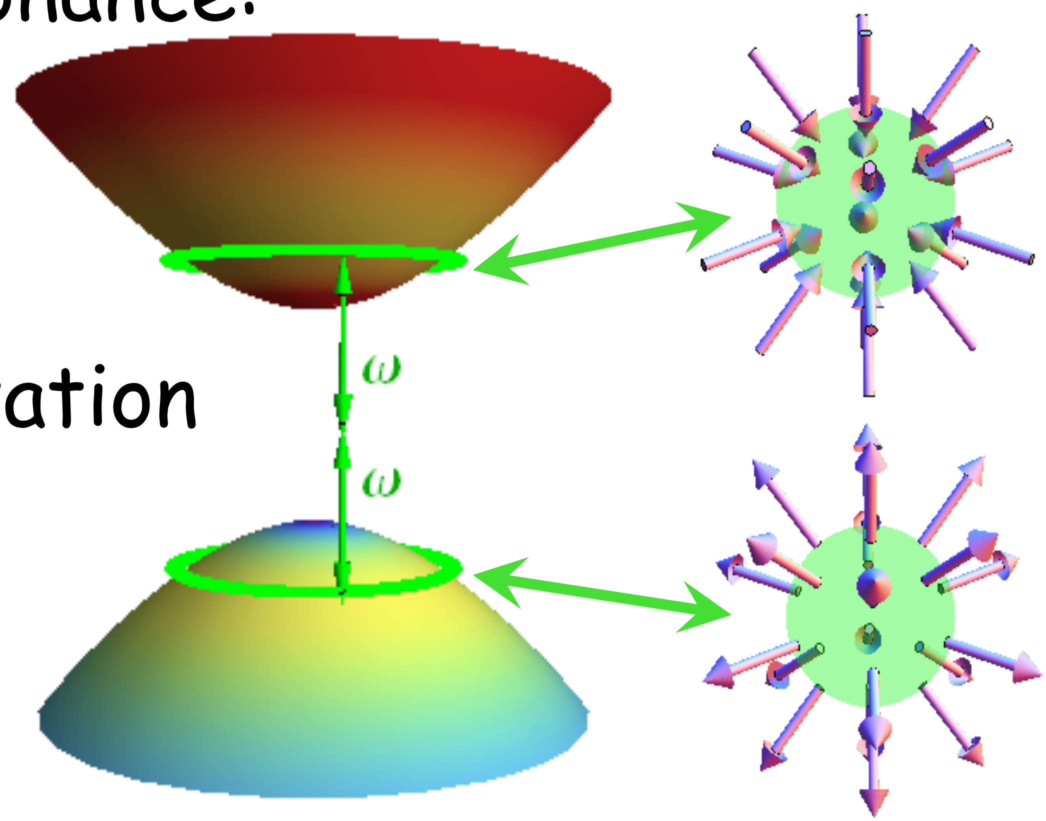
- $\hat{V}$ Scalar: $\vec{V} = V_5 \hat{x}_5$

$$\vec{V_\perp} = \left(-\frac{V_5 d_5}{D^2} A\mathbf{k}, V_5 - \frac{V_5 d_5}{D^2} \right)$$



Electromagnetic Fields

- $\overrightarrow{E}$ vector, $\mathrm{Tr}(E_i E_j)$ scalar
- 2-photon resonance:

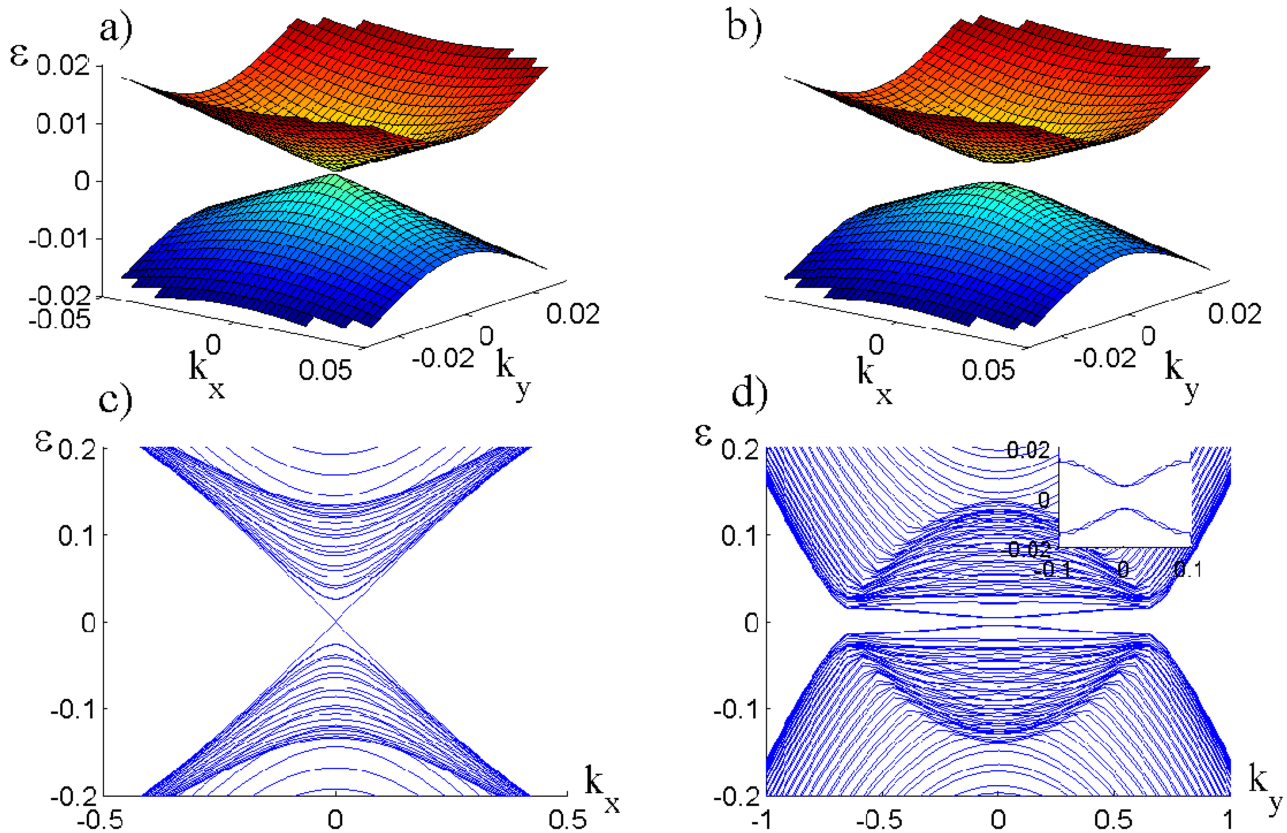


- TRS -> Linear polarization

arXiv:1111.4518

Surface spectrum

Linear polarization, "TRS"   Elliptic polarization, no "TRS"



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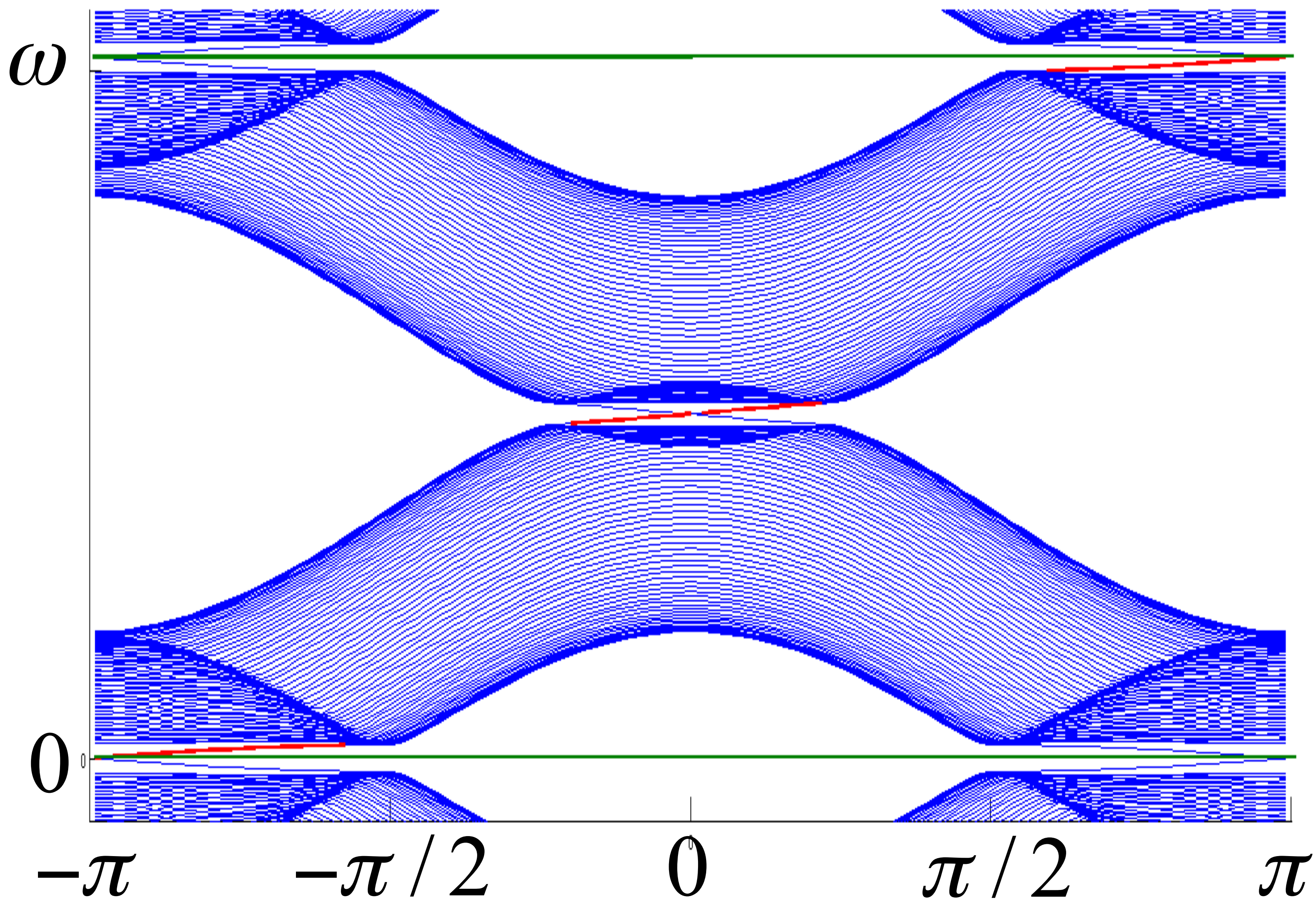
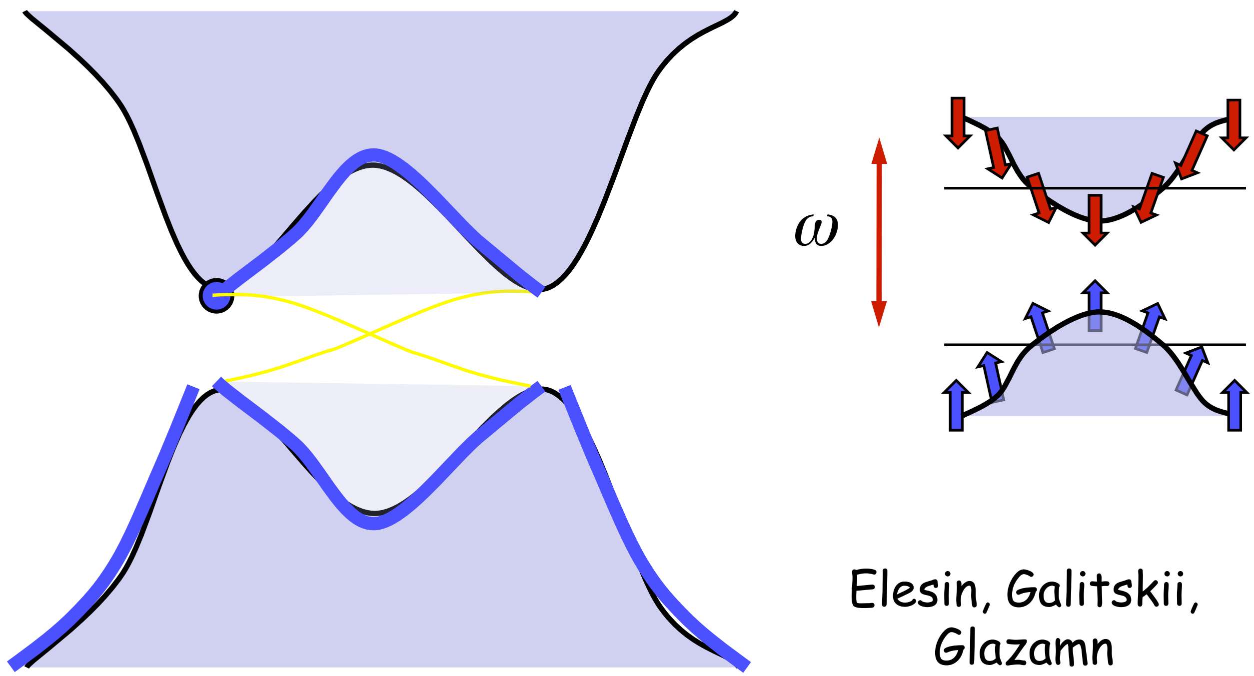
Edge states
without Chern numbers



Steady states

Phonons can provide bath for low-energy, low momentum relaxation.



Elesin, Galitskii, Glazamn

- Adiabatic loading of the floquet spectrum?

Summary

- Topological spectra for the Floquet operator in initially trivial medium.
- Different symmetry classes
- A lot more to explore:
  - Steady states
  - Quenched / Adiabatic onsets
  - Unique phenomena for driven systems
- Nature Physics **7**, 490–495 (2011) (2D Floquet Topological Insulator)
- arXiv:1111.4518 (Topological Floquet Spectrum in 3D)
- T. Kitagawa, et. al., Physical Review B 82, 235114 (2010).