



Lev Levitin (RHUL)  
Andrew Casey (RHUL)  
John Saunders (RHUL)  
Robert Bennett (formerly RHUL, now at Cornell)

Fabrication: Jeevak Parpia (Cornell)  
SQUIDs: T Schurig, D Drung (PTB)

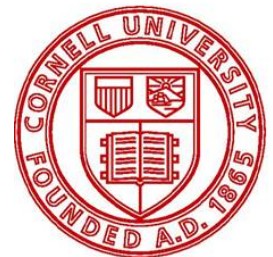
Theory:  
EV Surovtsev (Kapitza Institute)  
J Sauls (Northwestern)  
A Voronstov (Montana)  
GE Volovik (Helsinki)



EPSRC



**European Microkelvin  
Collaboration**

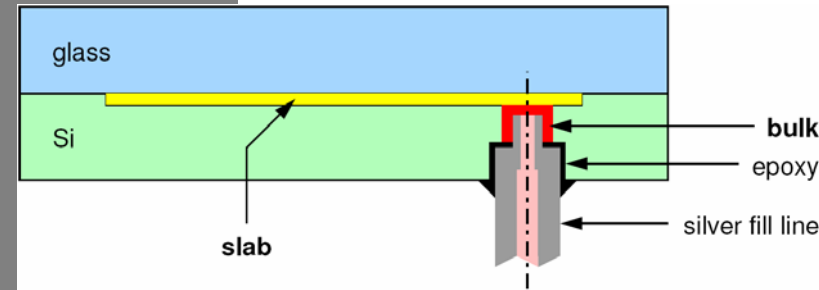




# Experiments on Superfluid $^3\text{He}$ in Controlled Nanofabricated Geometries: model systems for topological quantum matter

## Superfluid $^3\text{He}$ as a model material:

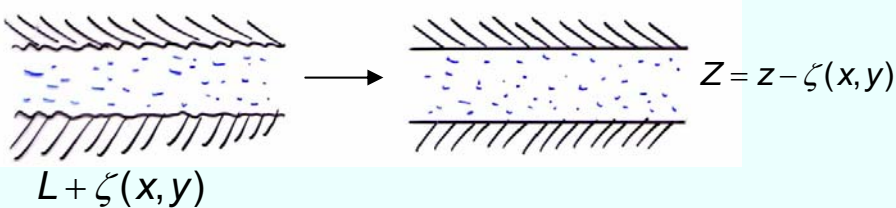
- Perfect purity
- Simple spherical Fermi surface
- Isotropic normal state (but size quantization on confinement)
- No crystal lattice
- Spin-orbit coupling via dipolar interaction
- Neutral: access to spin degrees of freedom and their manipulation by NMR
- Different superfluid ground states



## Confinement: Control parameters

- Precise control of sample dimensions (through design of nanofluidic sample chamber)
- Fully characterise surface roughness  $\rightarrow$  effective disorder potential
- Surface scattering tuneable by pre-plating with a  $^4\text{He}$  film
- Continuous pressure tuning of  $\xi(p)$  and hence of effective confinement

Understanding disorder [ theory: Tesanovic and Valls]



$\zeta(x,y)$  Determines an effective disorder potential within film, which is fully experimentally characterizable

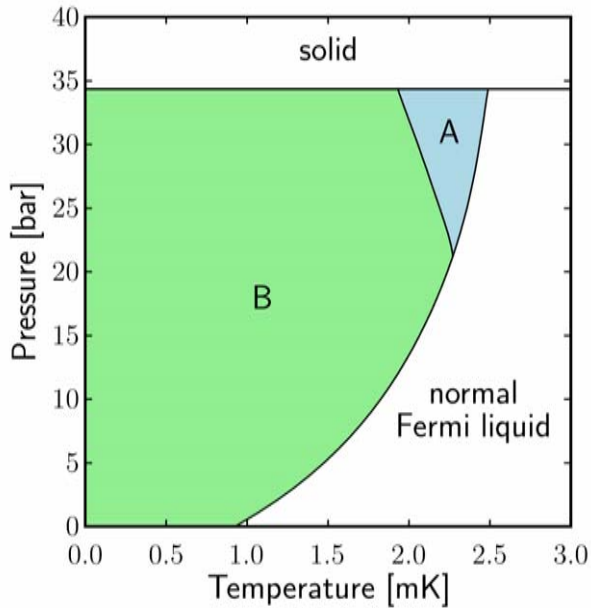
Recent expts. arXiv.1010.5993  
to appear Phys.Rev.Lett



# Outline

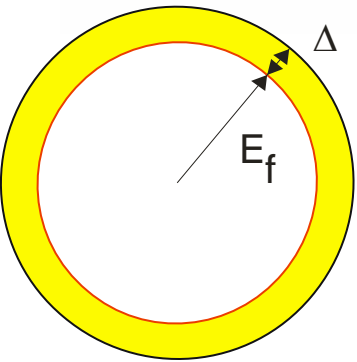
- Motivation
- Theoretical background: The order parameter at surfaces
- Superfluid  $^3\text{He}$  films: previous work
- Our experiments:
  - NMR fingerprint of superfluid  $^3\text{He}$  in a fully characterised nanofabricated slab geometry
- Future prospects:
  - Confinement as a control parameter
  - New p-wave states
  - Majorana excitations at surface, edges and defects

# Motivation: topological superfluids (with known order parameter)



G Volovik  
The Universe in a  $^3\text{He}$  Droplet

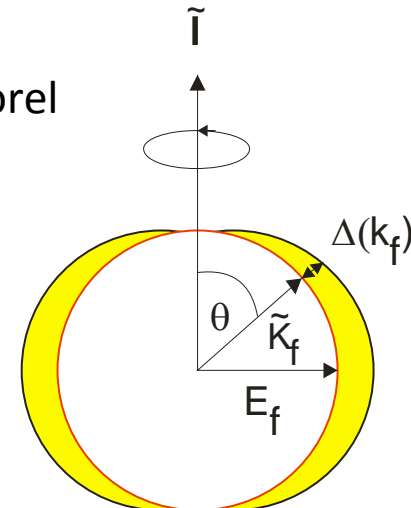
Pairs have  $L=1, S=1$   
 9 component order parameter  
 $L_z = -1, 0, +1$      $S_z = -1, 0, +1$   
 Break symmetry of spherical Fermi surface  
 Multiple superfluid phases



Balian-Werthamer  
(B) phase  
 Time reversal invariant  
 Preserves TRS

$$\Delta(\mathbf{p}) = \Delta \left[ (-\hat{p}_x + i\hat{p}_y) \left| \uparrow\uparrow \right\rangle + (\hat{p}_x + i\hat{p}_y) \left| \downarrow\downarrow \right\rangle + \hat{p}_z \left| \uparrow\downarrow + \downarrow\uparrow \right\rangle \right]$$

Anderson-Brinkman-Morel  
(A) phase  
 Chiral superfluid  
 Breaks TRS  
 Stabilised by strong  
 Coupling at high p

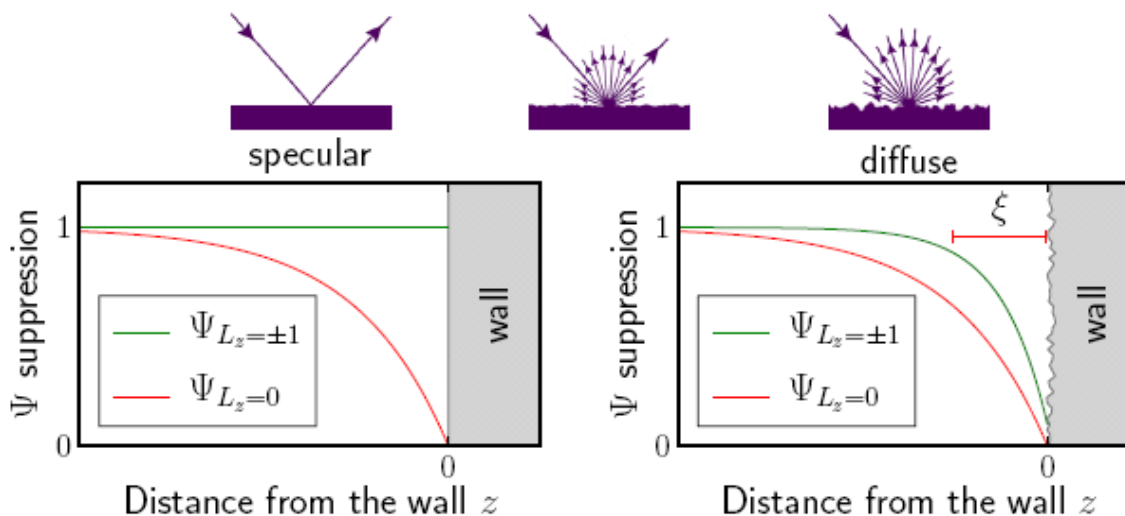


$$\Delta(\mathbf{p}) = \Delta (\hat{p}_x + i\hat{p}_y) \left| \uparrow\uparrow + \downarrow\downarrow \right\rangle$$



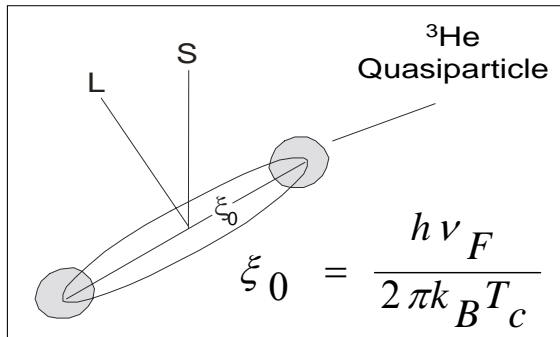
# What happens at a wall?

- Pairs with  $\hat{1} \perp$  wall:  $\Psi_{L_z=\pm 1}$ , pairs with  $\hat{1} \parallel$  wall:  $\Psi_{L_z=0}$  (wall in  $xy$  plane)
- Pairbreaking due to scattering of  $^3\text{He}$  quasiparticles off the walls
- ➔ ■  $\Psi_{L_z=0} = 0$  at the wall [AJ Leggett, *RMP* 47 (1975), V Ambegaokar, et al., *PRA* 9 (1974)]
- Nature of scattering determines boundary conditions for  $\Psi_{L_z=\pm 1}$

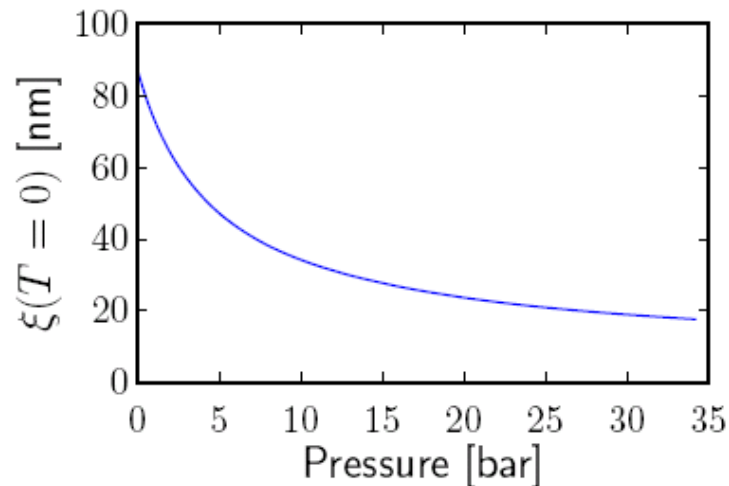
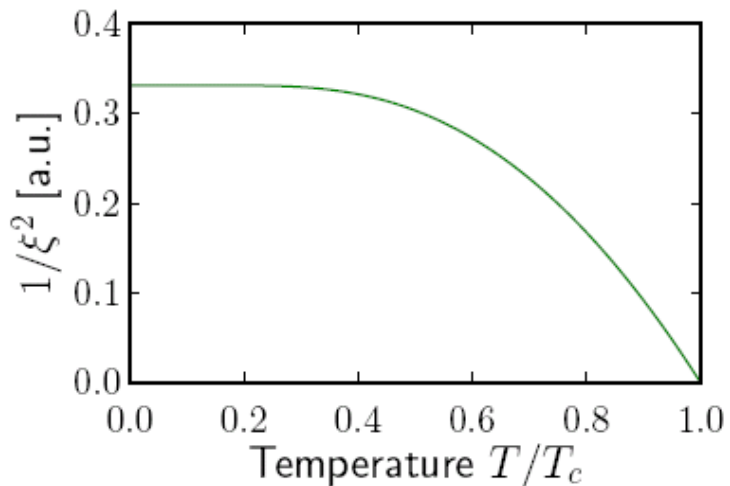


- Order parameter recovers over *coherence length*  $\xi$
- ➔ ■ Pairs with  $\hat{1} \perp$  wall are suppressed less than pairs with  $\hat{1} \parallel$  wall
- Specular scattering is enhanced by  $^4\text{He}$  preplating [MR Freeman, RC Richardson, *PRB* 41 (1990)]

.....alters energy balance of A and B phases under confinement



Zero temperature coherence length  
[Cooper pair diameter]



$$\xi_T = \left[ \frac{7\zeta(3)}{20} \right]^{1/2} \frac{\hbar v_F}{2\pi T_c^{\text{bulk}}} \left( 1 - \frac{T}{T_c^{\text{bulk}}} \right)^{-1/2} = \frac{\hbar v_F}{\Delta_B(T)\sqrt{10}}$$

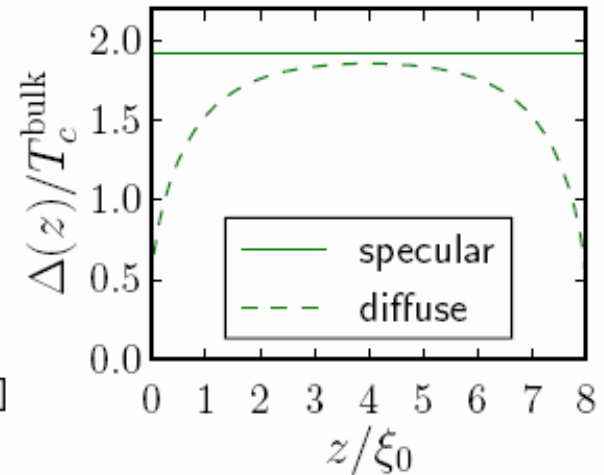


## A Phase

$$\Delta(\mathbf{p}) = \Delta(\hat{p}_x + i\hat{p}_y) |\uparrow\uparrow + \downarrow\downarrow\rangle$$

- Gap suppression is minimized by orienting  $\hat{\mathbf{I}} \perp$  slab (slab in  $xy$ -plane)
- No suppression, if walls are specular
- Otherwise energy gap  $\Delta(z)$  varies across the slab
- $T_c$  suppression, if diffuse scattering is present

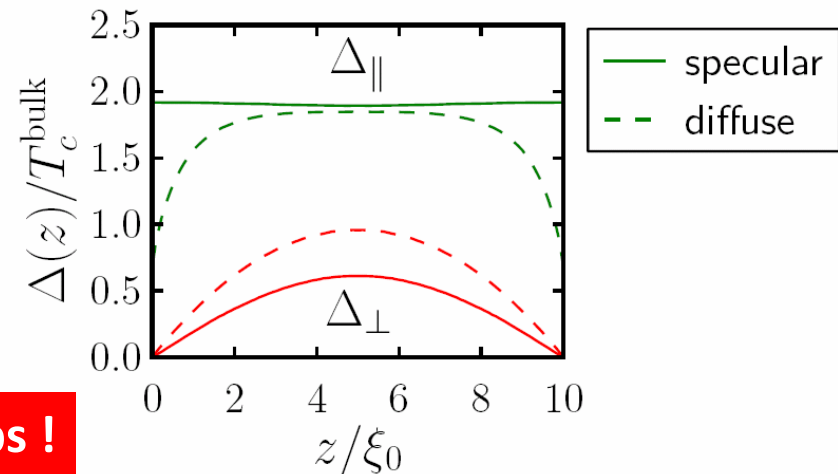
[AB Vorontsov and JA Sauls, *PRB* 68 (2003)]



## B phase

$$\Delta(\mathbf{p}) = [\Delta_{\parallel}(-\hat{p}_x + i\hat{p}_y) |\uparrow\uparrow\rangle + \Delta_{\parallel}(\hat{p}_x + i\hat{p}_y) |\downarrow\downarrow\rangle + \Delta_{\perp}\hat{p}_z |\uparrow\downarrow + \downarrow\uparrow\rangle]$$

- Planar distortion (spatially dependent)



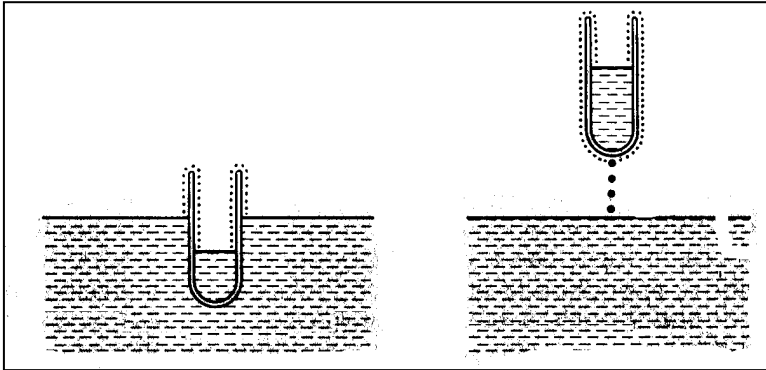
**A-phase energetically favoured in thin slabs !**

[Y Nagato and K Nagai, *Physica B* 284-288 (2000)]



# Motivation: Superfluid helium films.....<sup>4</sup>He

Superfluid <sup>4</sup>He film flow  
Daunt and Mendelssohn  
Proc. R. Soc A170, 423, 439 (1939)



Berezinskii-Kosterlitz-Thouless (vortex-unbinding) transition in a 2D <sup>4</sup>He film  
Bishop and Reppy 1978

Superfluidity survives in atomically thin films on disordered surfaces

Since coherence length is of order 0.1 nm

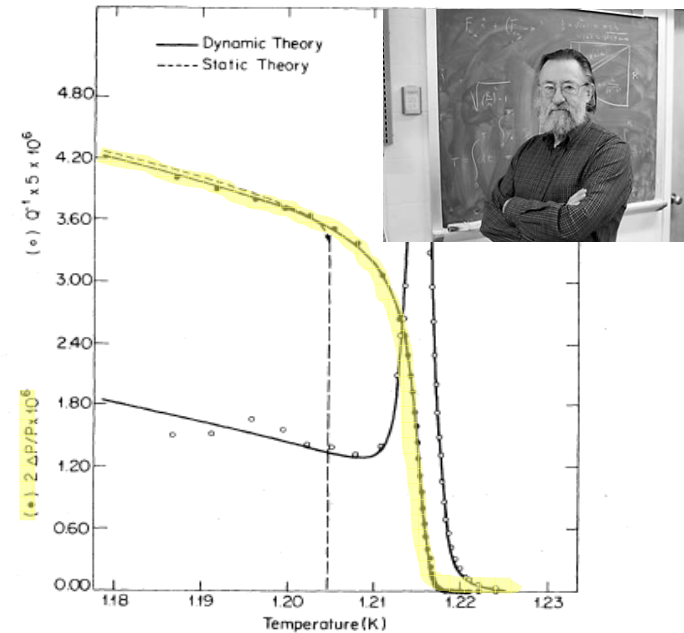


FIG. 2. The reduced period shift,  $2\Delta P/P$ , and dissipation  $Q^{-1}$  are shown for a superfluid transition temperature of 1.215 K. The solid lines are fits using the dynamic theory of AHNS (Ref. 6) and the dashed curve is the result of the static theory.

For atomically rough surfaces <sup>3</sup>He film must be relatively thick not to kill superfluidity





# Superfluid $^3\text{He}$ film flow experiments

Film flow:

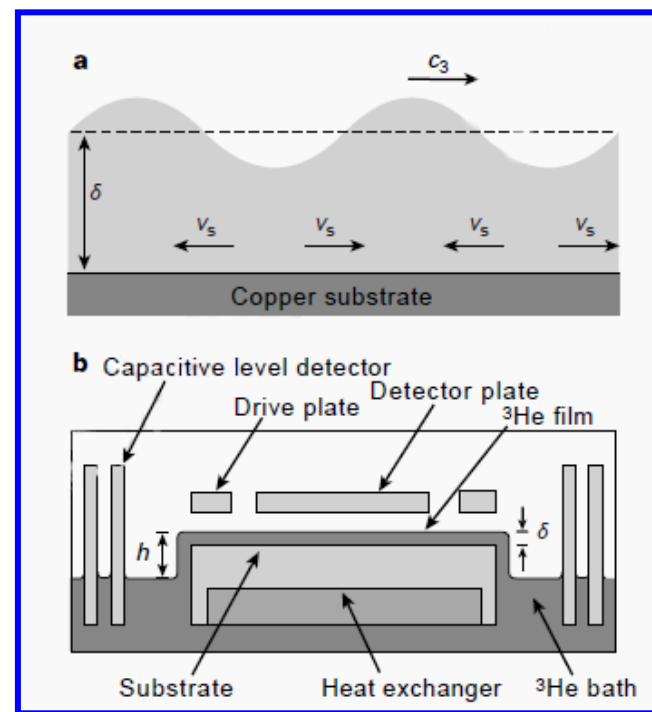
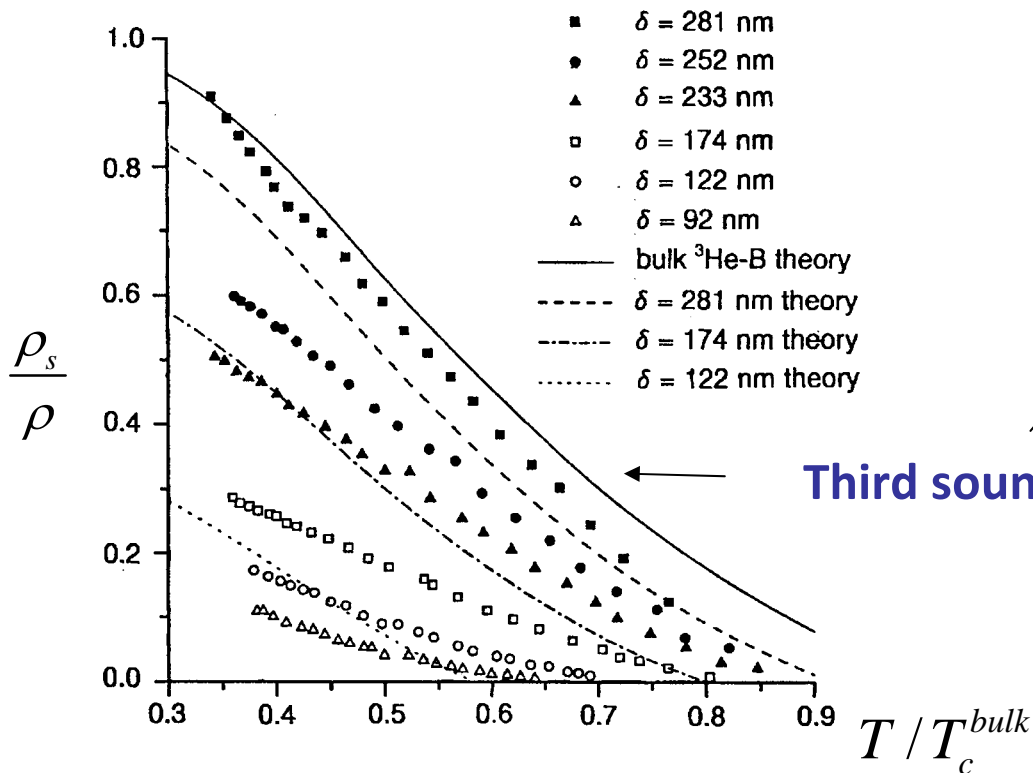
- S C Steel et al. J Low Temp. Phys. 95, 759 (1994)
- A Sachrajda et al. Phys. Rev. Lett. 55, 1602 (1985)
- J G Daunt et al. J Low Temp. Phys. 70, 547 (1988)

Third sound:

- A N R Schecter et al. Nature 554 (1998)

Torsional oscillator measurement of superfluid density:

- J Xu and C Crooker Phys. Rev. Lett. 65, 3005 (1990)
- A Corcoles et al. unpub.





# NMR on superfluid $^3\text{He}$

NMR fingerprints the equilibrium order parameter [A J Leggett *Ann. of Phys.* 85, 11 (1974)]

It determines the gap suppression and gap distortion arising from confinement.

Measure as a function of tip angle (non-linear response)

- Nuclear spins — degrees of freedom of cooper pairs
- Tipping the spin in NMR experiment modifies the order parameter:



- Relative rotation of spins with respect to orbital momenta of pairs
- Condensation energy is invariant, but dipolar coupling between the spins is affected
- Dipolar torque on the spins  $\rightarrow$  frequency shift [A J Leggett, *Ann. of Phys.* 85 (1974)]

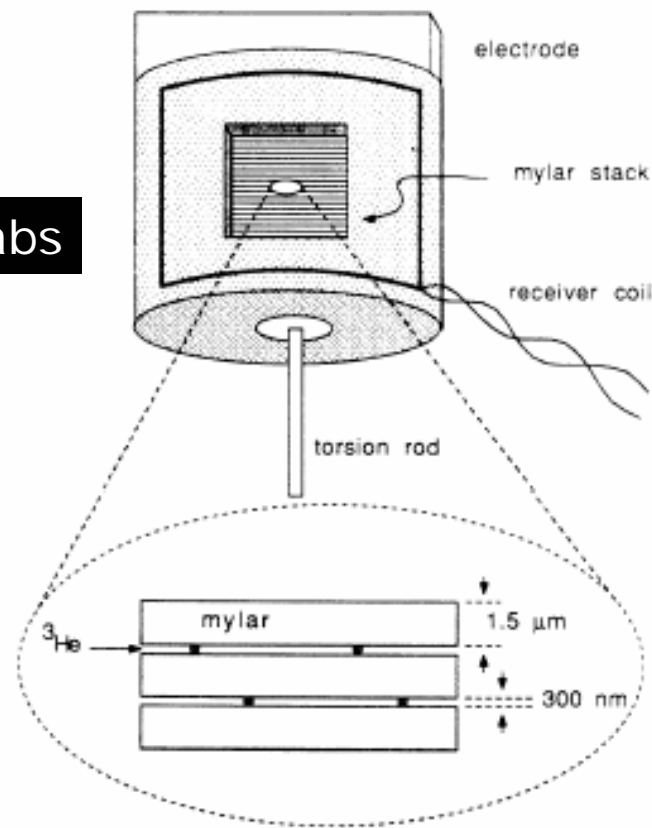
$$\Delta f \propto \Delta^2$$



# First NMR experiments on highly confined slabs

M R Freeman and R C Richardson  
Phys. Rev. B41, 11011 (1990)

3000 x 300 nm slabs



Main conclusions:

“A” phase at all pressures

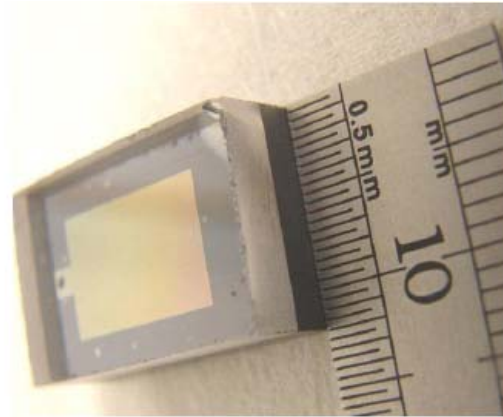
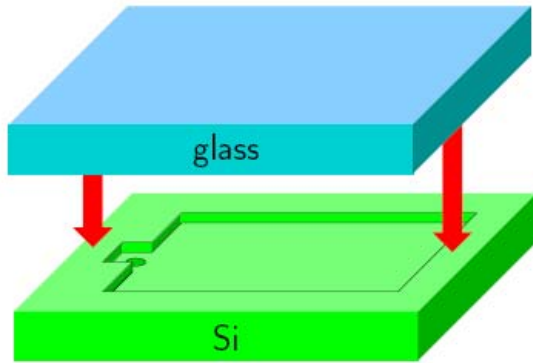
<sup>4</sup>He coating on boundary → specular scattering



# Confined superfluid $^3\text{He}$ (1)

Technical challenges:

- (1) controlled and well characterised confinement (single slab)
- (2) measurement sensitivity
- (3) cooling



650 nm x 10 mm x 7 mm

Fully characterise surface roughness (AFM)

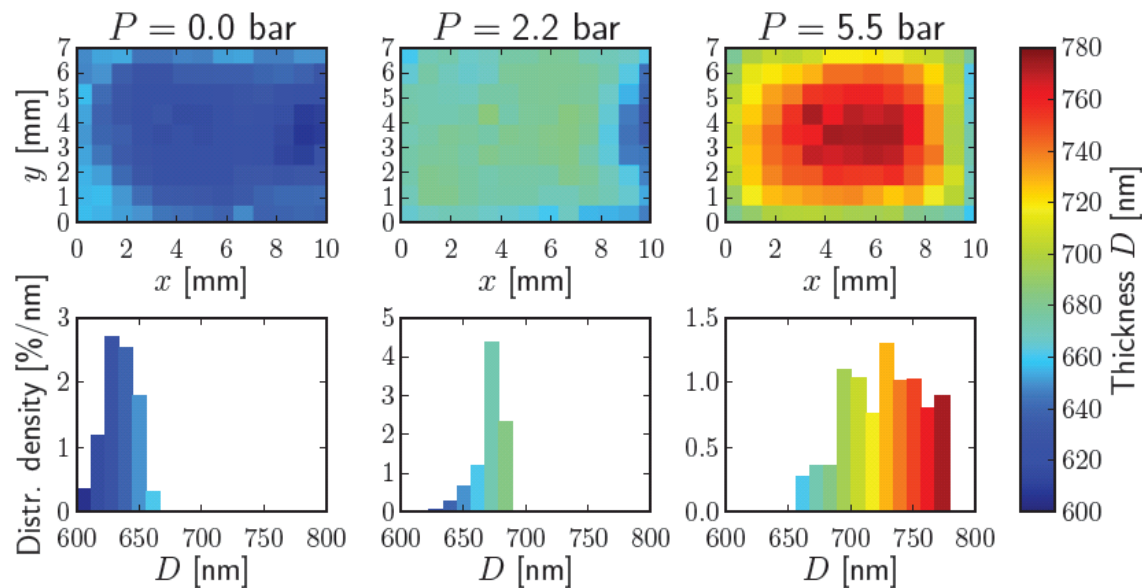
Optically characterise cavity thickness profile

[This cavity is unsupported:  
Search for “striped phase”]

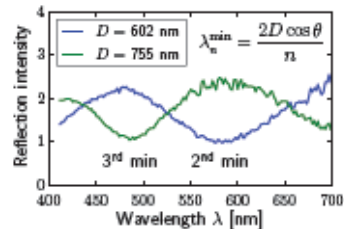
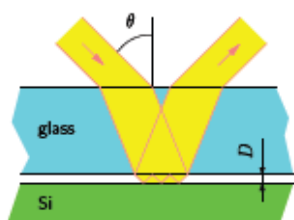
# Confined superfluid $^3\text{He}$ (1)



## Optical characterization of cavity profile at low temperatures

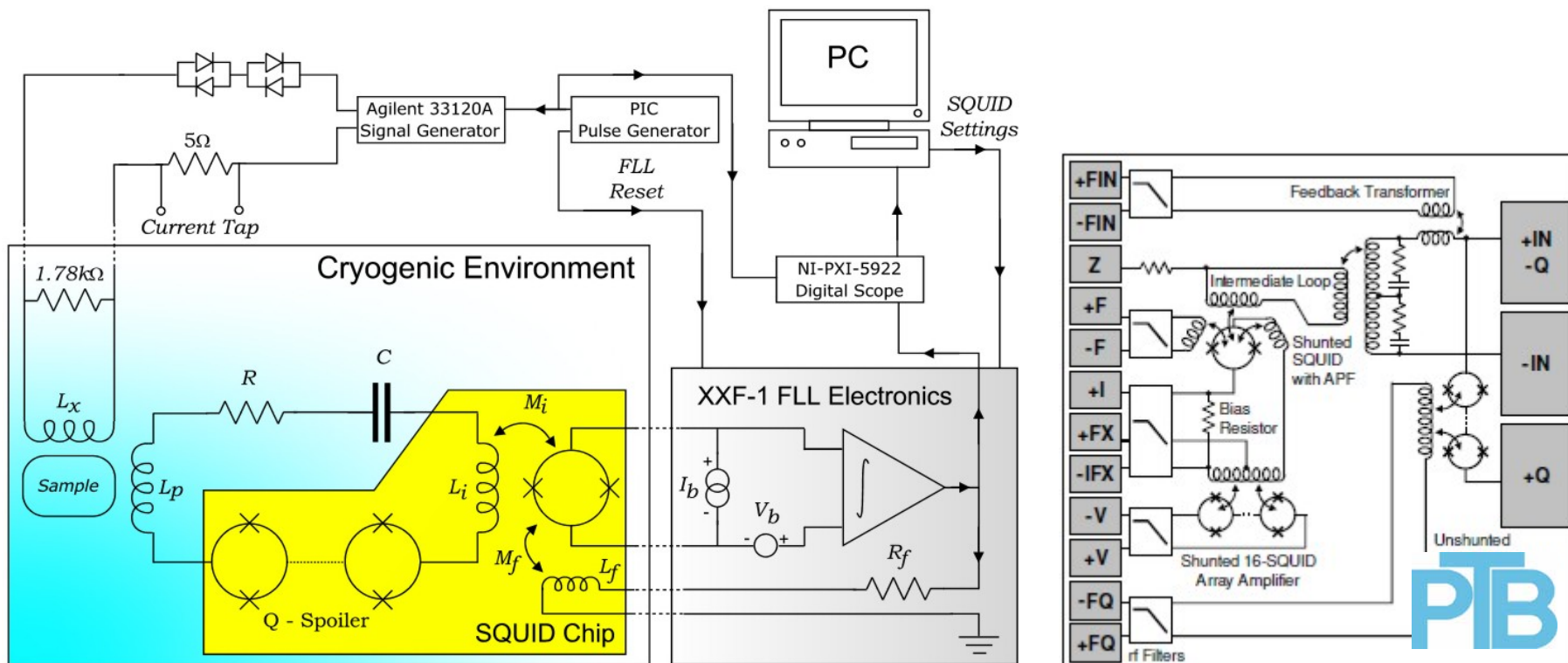


### Optical Thickness Measurement



- ▶ Interferometry in the cavity cooled below 7 K.
- ▶ Analyse reflection of  $\varnothing 0.3$  mm collimated white light beam
- ▶ Bowing due to differential thermal contraction. Stops at 30 K
- ▶  $T$ -indep. inflation under pressure. 28 nm/bar in the centre

# Detecting an NMR signal: Sensitive SQUID NMR spectrometer

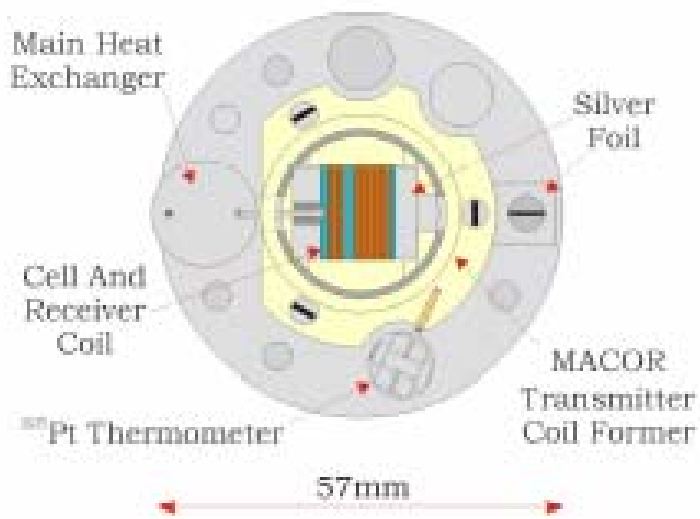
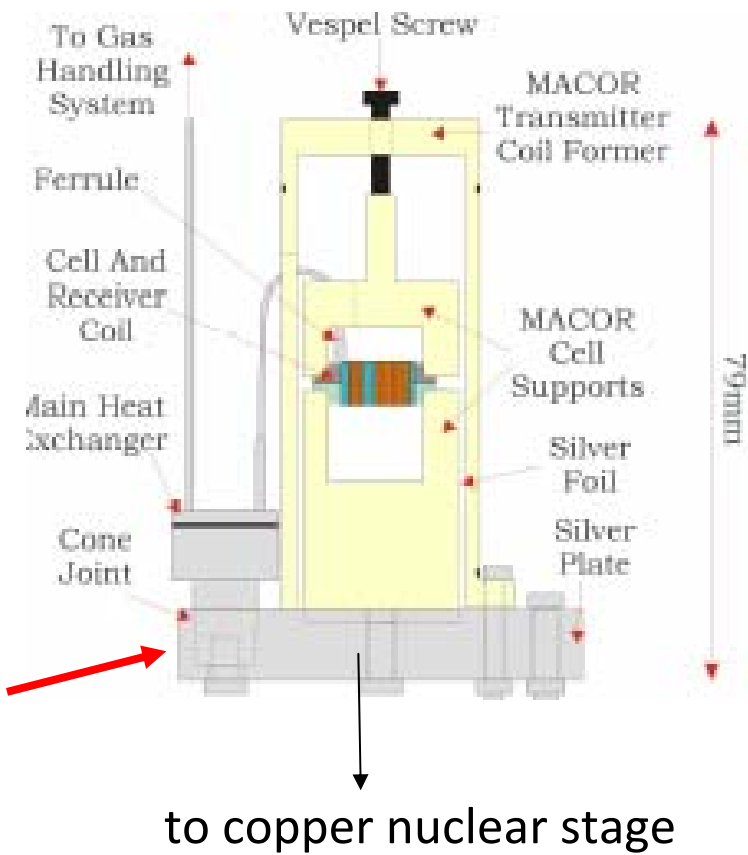


- NMR probes the order parameter of  $^3\text{He}$ : different phases show distinct shifts of the resonance away from the Larmor frequency
- Integrated two stage DC SQUID energy sensitivity **20 h** achieved in this spectrometer. [5 mK noise temperature at 1 MHz)

Levitin *et al.*, Appl. Phys. Lett. **91**, 262507 (2007)

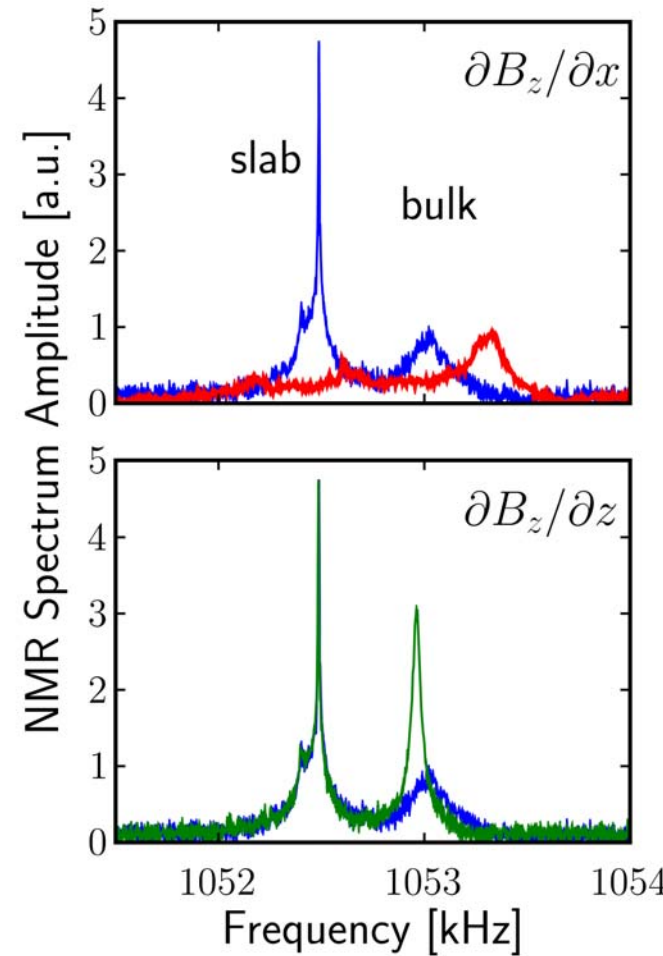
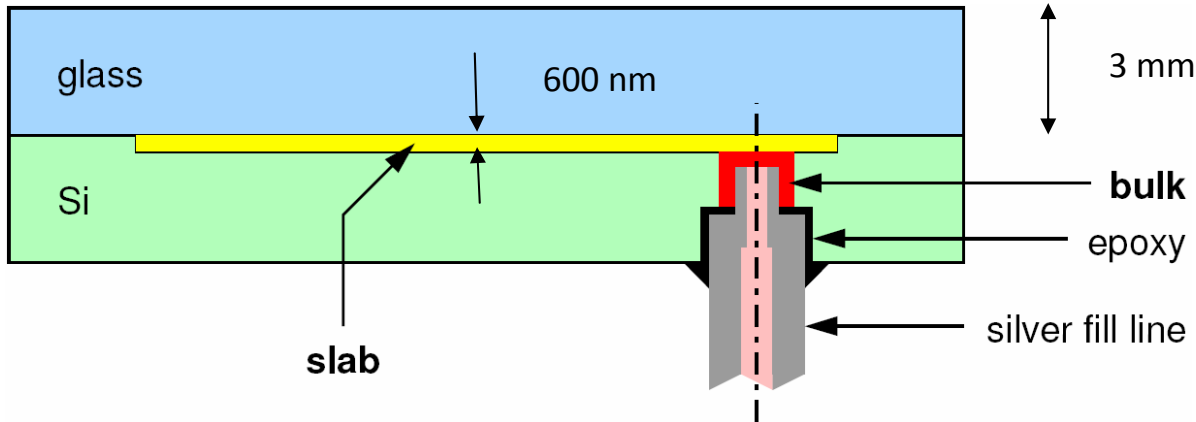


Sample is cooled through the "leads" =  $^3\text{He}$  in fill line





# NMR signals from slab



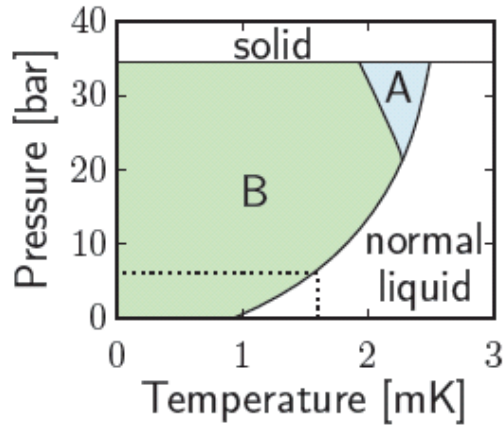
Apply field gradients, along or perpendicular to slab to clearly **image** slab and bulk signal



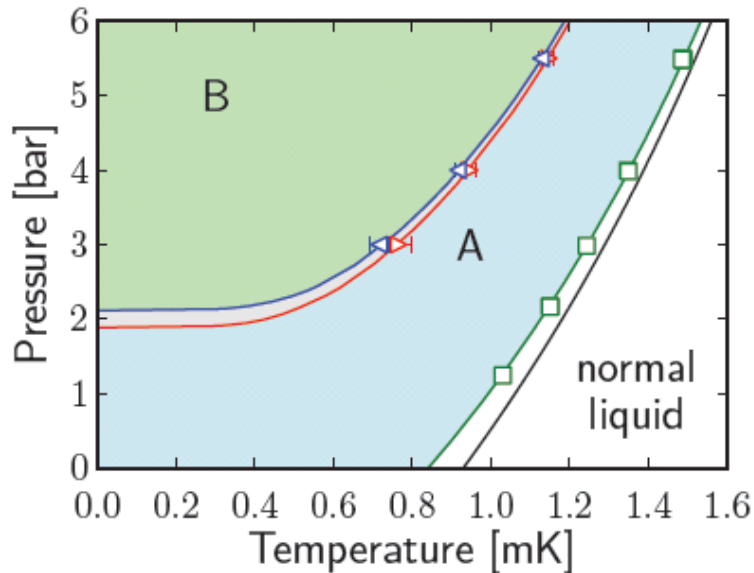


# Profound influence of confinement on the phase diagram

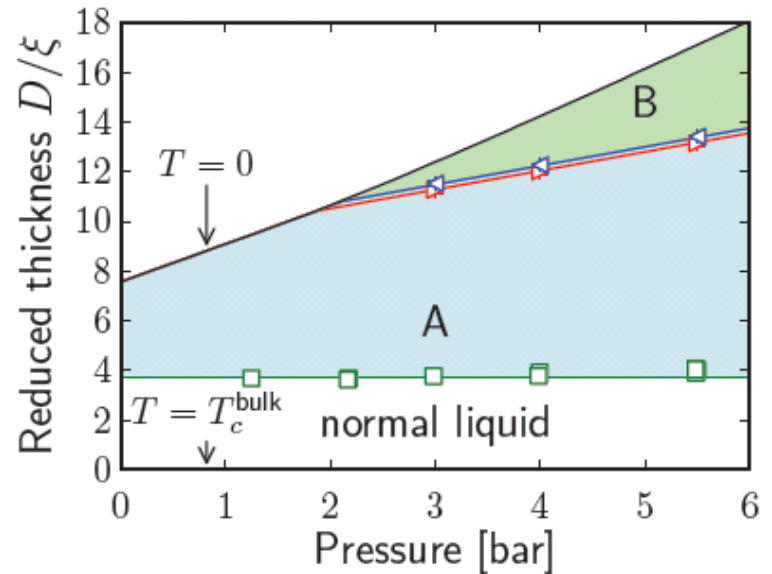
Bulk  $^3\text{He}$ :



0.7  $\mu\text{m}$  Slab:

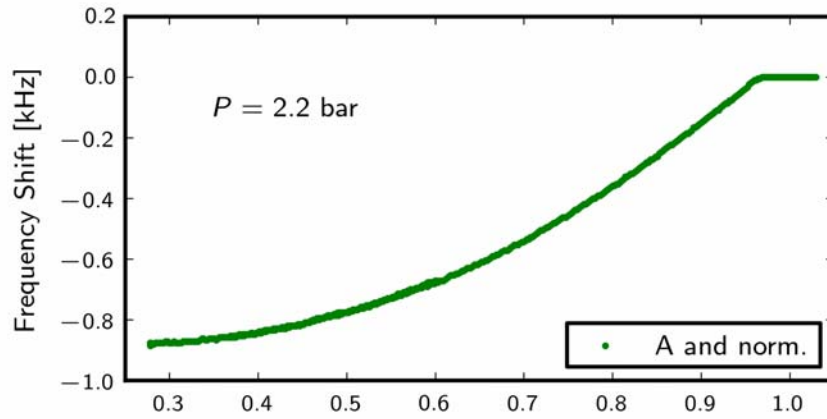


- Two superfluid phases:
  - ▶ the A phase and
  - ▶ the B phase with a planar distortion
- Suppressed  $T_c$
- Hysteresis at the AB transition
- Transitions at weakly  $P$ -dependent values of  $D/\xi$

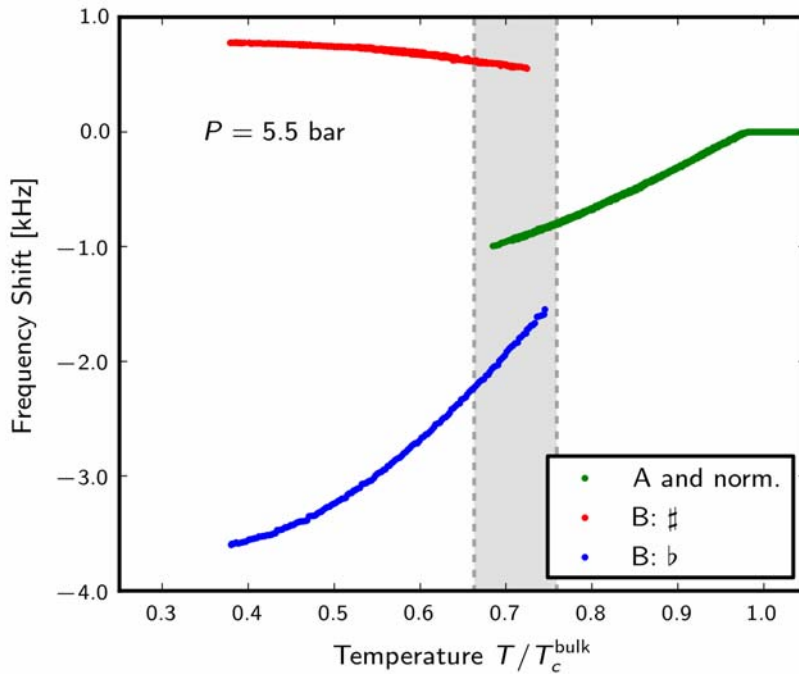




# NMR signatures of superfluid: frequency shift



Lower pressures:  
A phase



Higher pressures  
A phase  
then transition to  
B phase with planar distortion

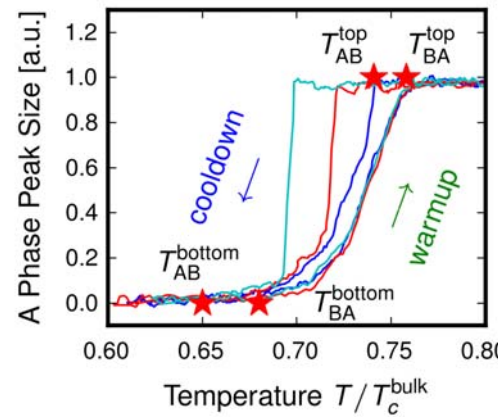
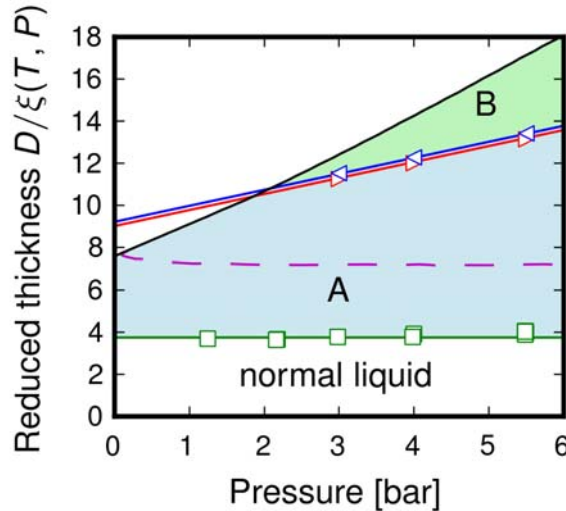
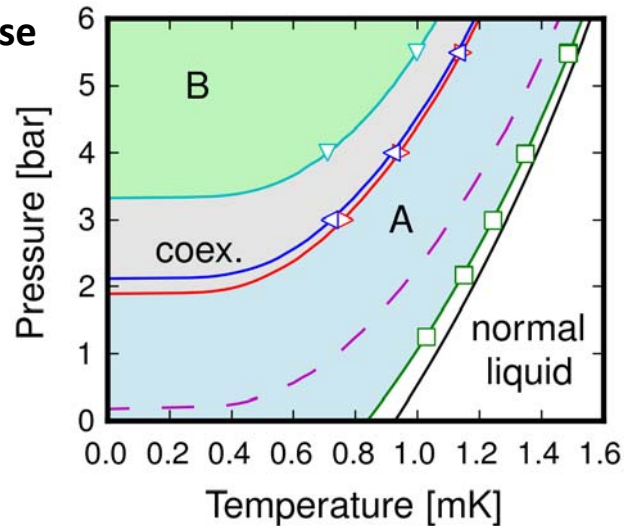
Pressure tunes effective confinement

$$\frac{D}{\xi_0}$$

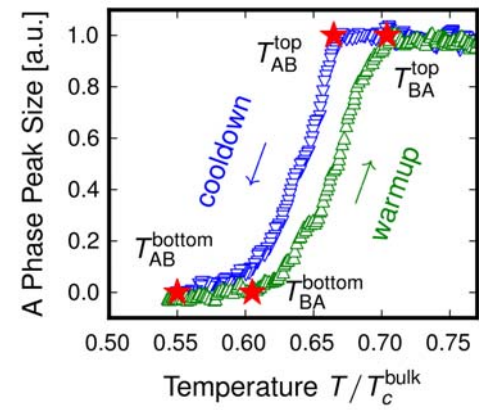
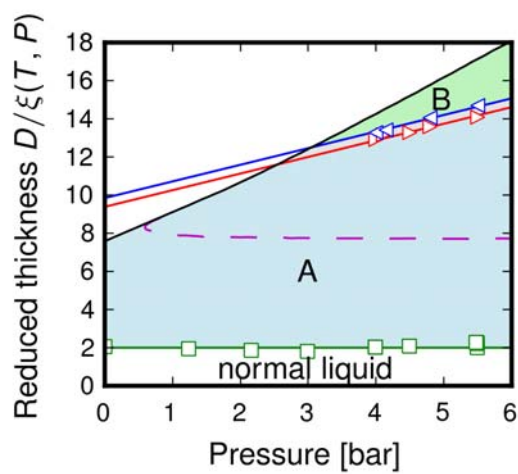
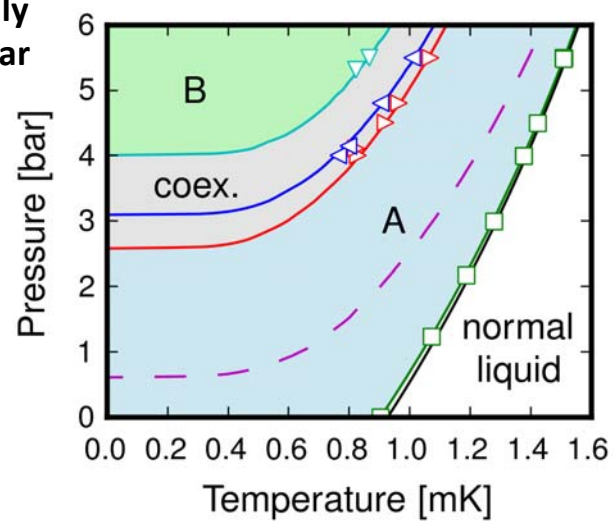


# Tuning boundary scattering in situ by coating surface with $^4\text{He}$ film

diffuse

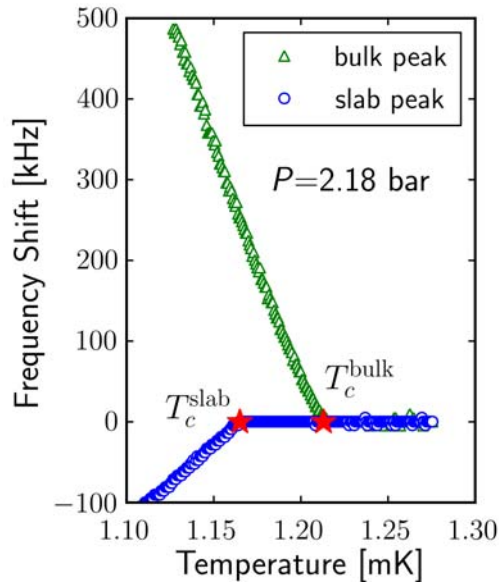


Partially specular

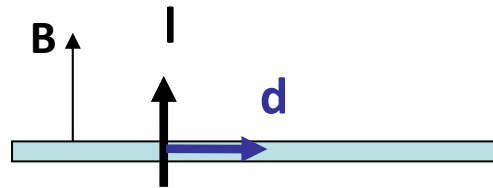




# A- Phase in slab: NMR fingerprint (i) small angle pulsed NMR

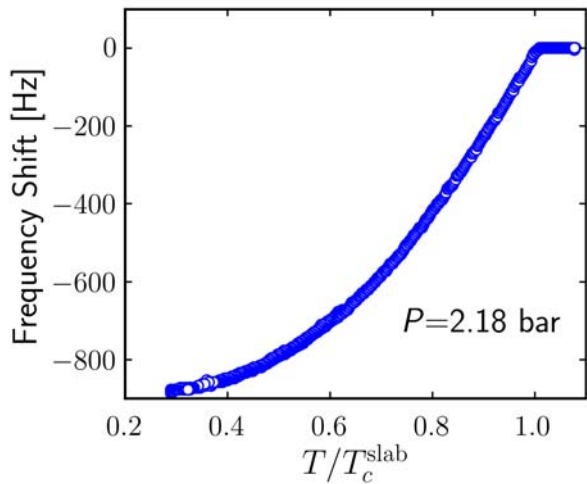
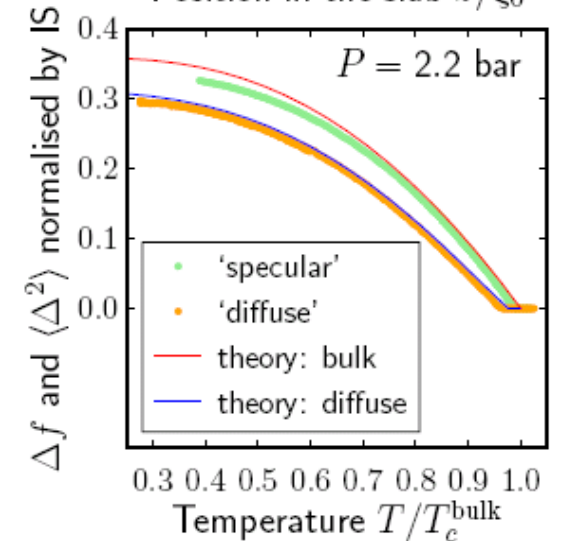
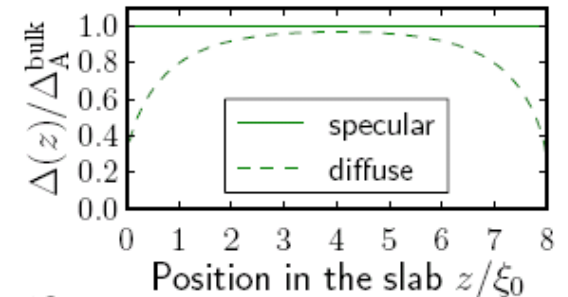


$$|\Delta f| = \frac{g^2 \lambda_D N_F}{2\pi^2 \chi_N f_L} \langle \Delta^2(z) \rangle$$

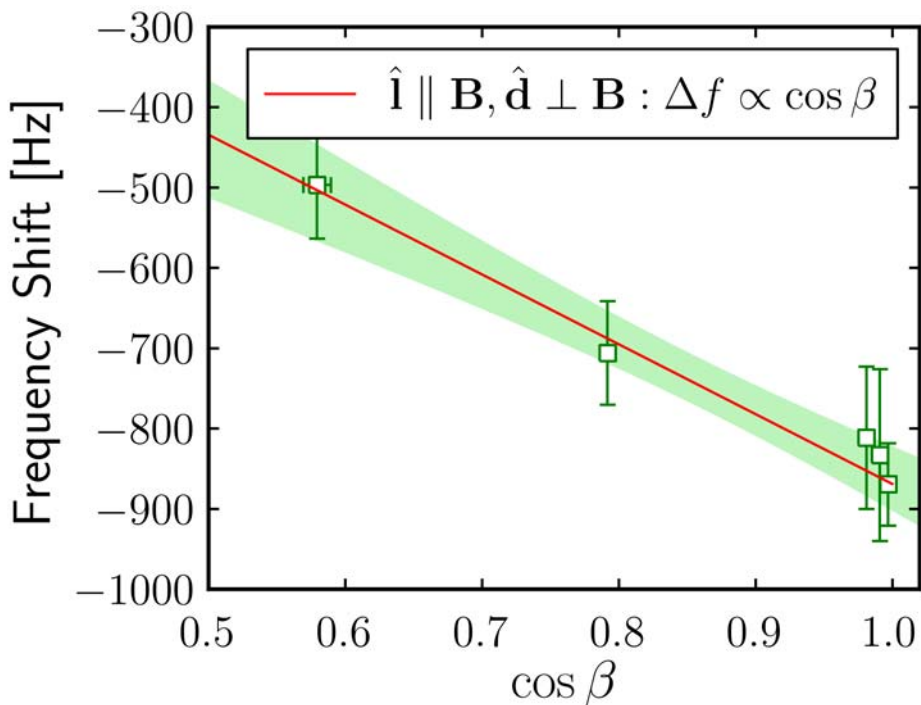


L normal to surface  
 Zeeman energy orients  
**d** ⊥ **B**  
 Dipole energy maximum  
 So -ve freq shift

Average gap:  
 Diffuse and partially specular boundary  
 Comparison with quasiclassical theory  
 (Vorontsov)



## A- Phase in slab: NMR fingerprint (ii) large tip- angle pulsed NMR



Use large tip angle NMR to confirm the observation of the “dipole-unlocked” state of the A phase  
Prediction: S Takagi *J Phys C* **8** (1975)

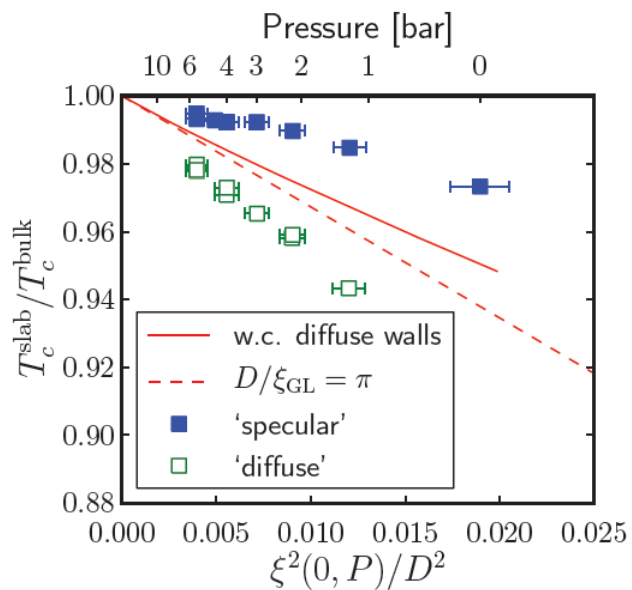
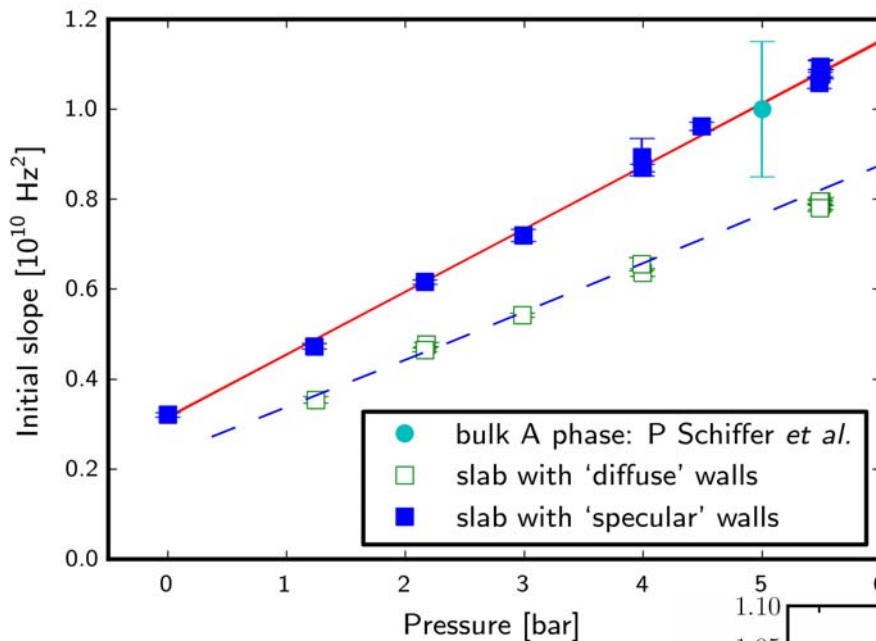
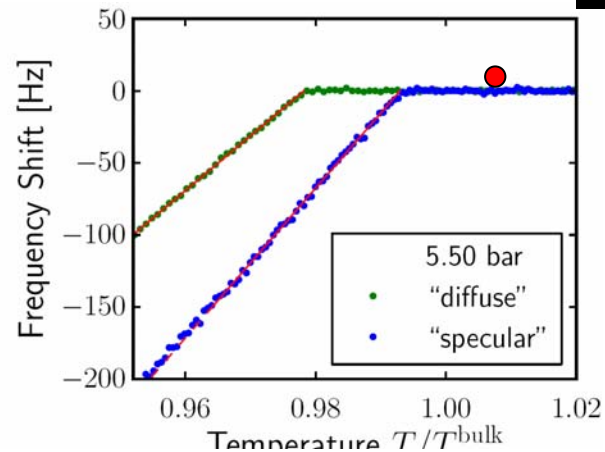
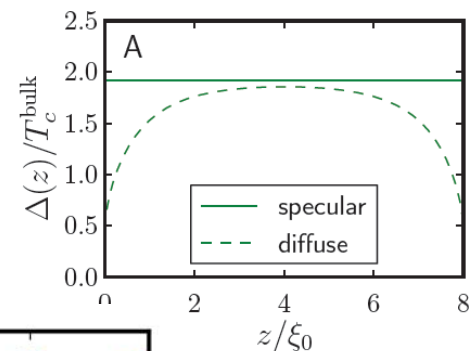
$$\Delta f(\beta) = -\frac{\nu_A^2}{2f_0} \cos \beta$$

**"A" phase:**

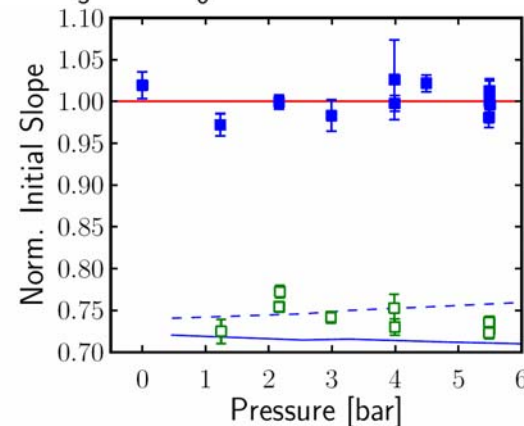
**diffuse and specular surface scattering, measuring the gap**

$$|\Delta f| = \frac{g^2 \lambda_D N_F}{2\pi^2 \chi_N f_L} \langle \Delta^2(z) \rangle$$

**"Tune" surface scattering by <sup>4</sup>He plating:  
alters  $T_c$  suppression and gap suppression**



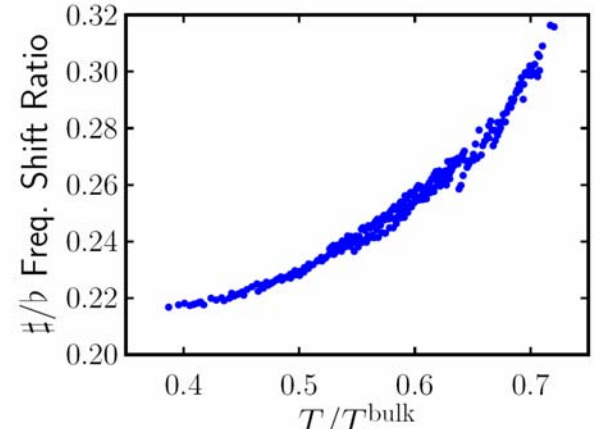
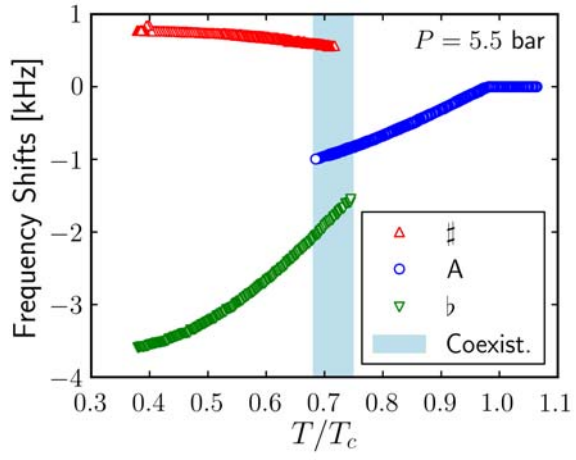
Comparison of A-phase gap suppression (diffuse walls) with *quasi-classical theory*





# NMR fingerprint of planar distorted B phase gap: (i) small angle tipping pulses

$$\Delta(\mathbf{p}) = \left[ \Delta_{\parallel}(-\hat{p}_x + i\hat{p}_y) \left| \uparrow\uparrow \right\rangle + \Delta_{\parallel}(\hat{p}_x + i\hat{p}_y) \left| \downarrow\downarrow \right\rangle + \Delta_{\perp} \hat{p}_z \left| \uparrow\downarrow + \downarrow\uparrow \right\rangle \right]$$



Indicates temperature dependence of

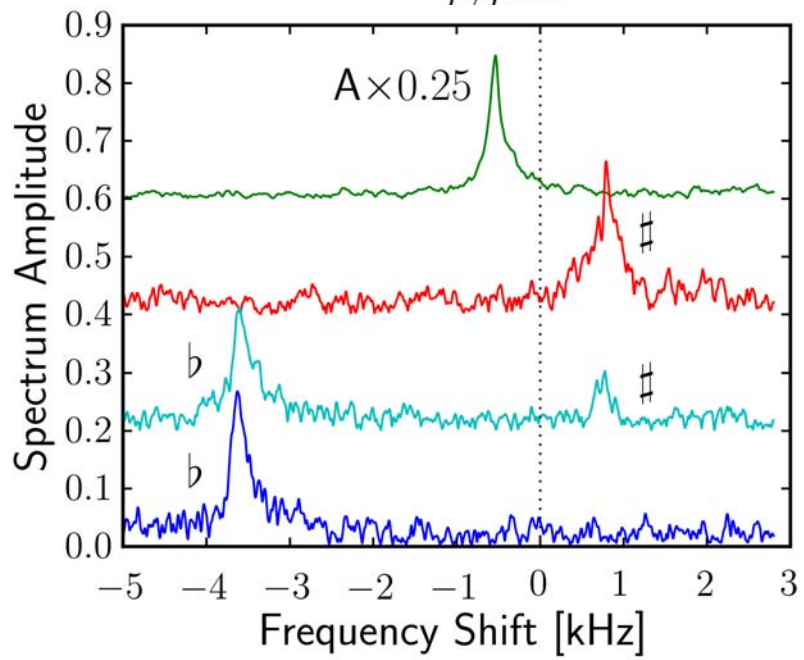
$$q = \frac{\Delta_{\perp}}{\Delta_{\parallel}}$$

Stable and metastable state

Gap anisotropy axis along z.

Susceptibility anisotropy axis is either parallel or antiparallel

These two states have different dipole energy,





The gap anisotropy axis  $\mathbf{l}$  is fixed  $\parallel \mathbf{z}$  by the walls of the slab

Order parameter with  $\mathbf{l} \parallel \mathbf{z}$ :

$$\mathbf{A} = \mathbf{R} \begin{pmatrix} \Delta_{\parallel} & & \\ & \Delta_{\parallel} & \\ & & \Delta_{\perp} \end{pmatrix}$$

with a rotation matrix  $\mathbf{R}$ , determined by Zeeman and dipolar energy

Anisotropic magnetic susceptibility, axis  $\mathbf{w} = \mathbf{R} \mathbf{l}$  due to planar distortion

Our experimental geometry:  $\mathbf{H} \parallel \mathbf{z}$ , slab in  $xy$  plane, so  $\mathbf{l} \parallel \mathbf{z}$

Zeeman energy:

$$F_Z = -\frac{1}{2} \Delta \chi (\hat{\mathbf{w}} \cdot \mathbf{H})^2$$

has two minima:  $\mathbf{w} = -\mathbf{l}$ ,  $\mathbf{w} = +\mathbf{l}$ .

Dipolar energy tends to be minimised with  $\mathbf{w} = \mathbf{l}$ , hence **positive frequency shift**

The state  $\mathbf{w} = -\mathbf{l}$  is metastable, does not minimise dipole energy, and gives rise to a **negative frequency shift**. [confirmed by field cycling experiments]





## (i) large angle tipping pulses

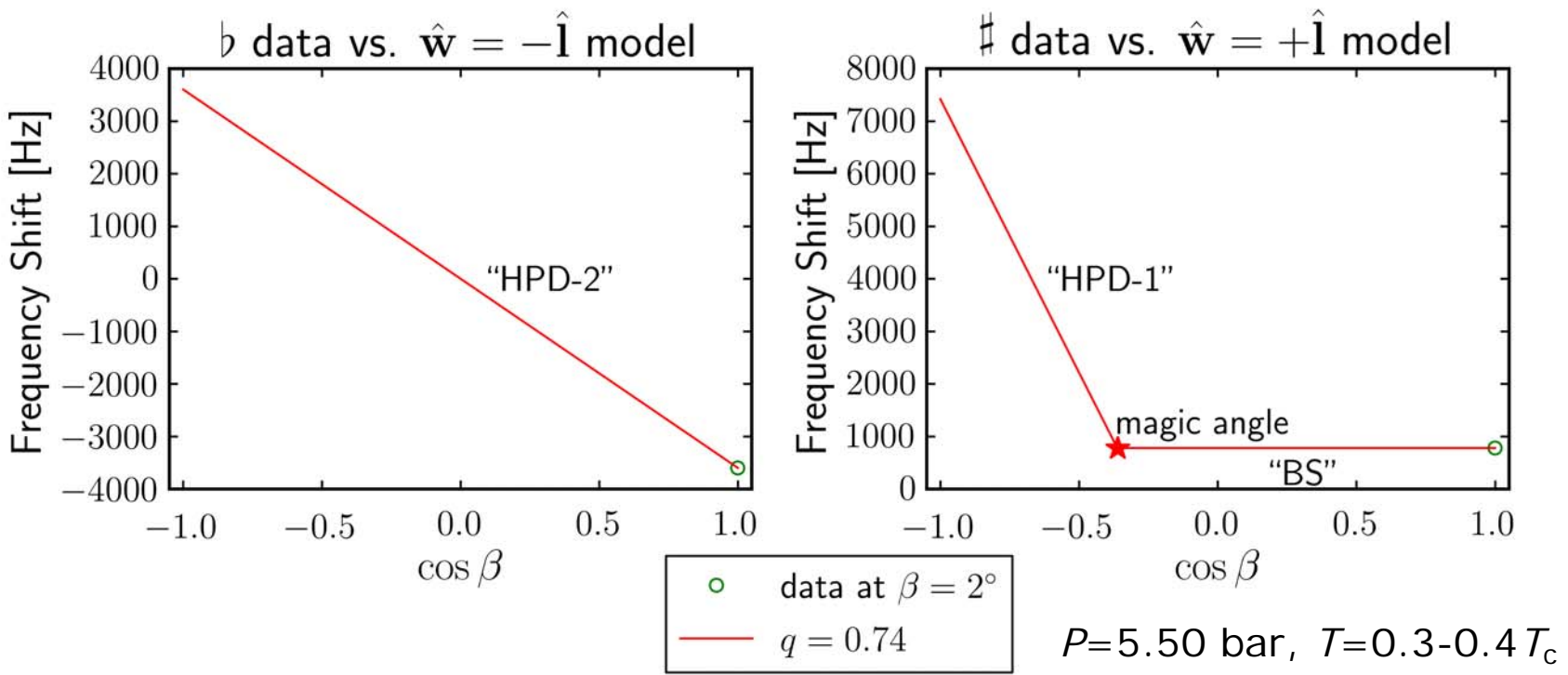
Theoretical **prediction** of NMR in the B phase with a uniform planar distortion by YuM Bunkov and GE Volovik, *JETP* **76** (1993)

Parameter of the theory, gap distortion parameter:

$$q = \frac{\Delta_{\perp}}{\Delta_{\parallel}}$$

Frequency shifts at small tip angle  $\beta$  determine  $q$  and yield a prediction for  $\Delta f(\beta)$

GE Volovik JETP LETTERS Vol. 91 No. 4 (2010)





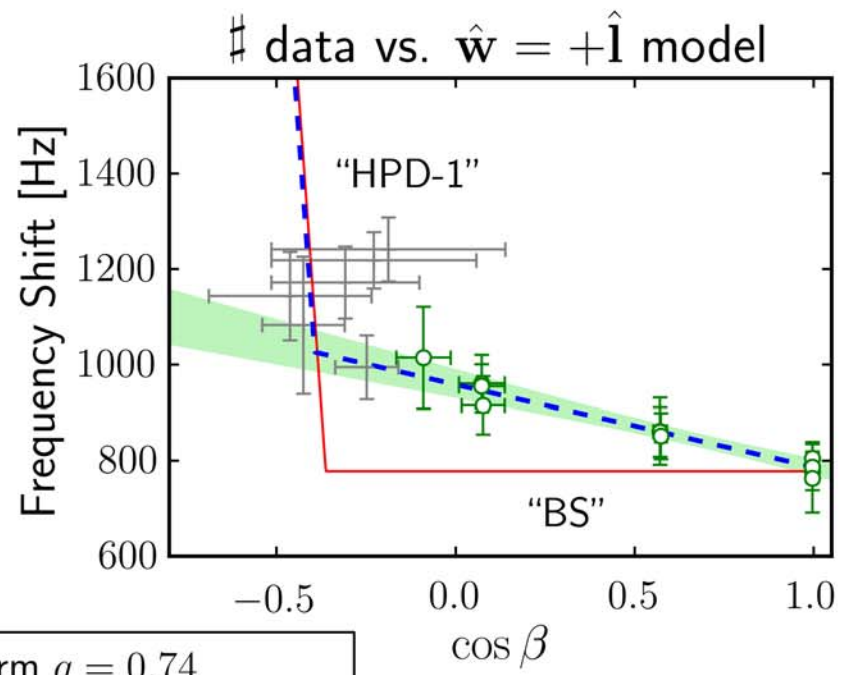
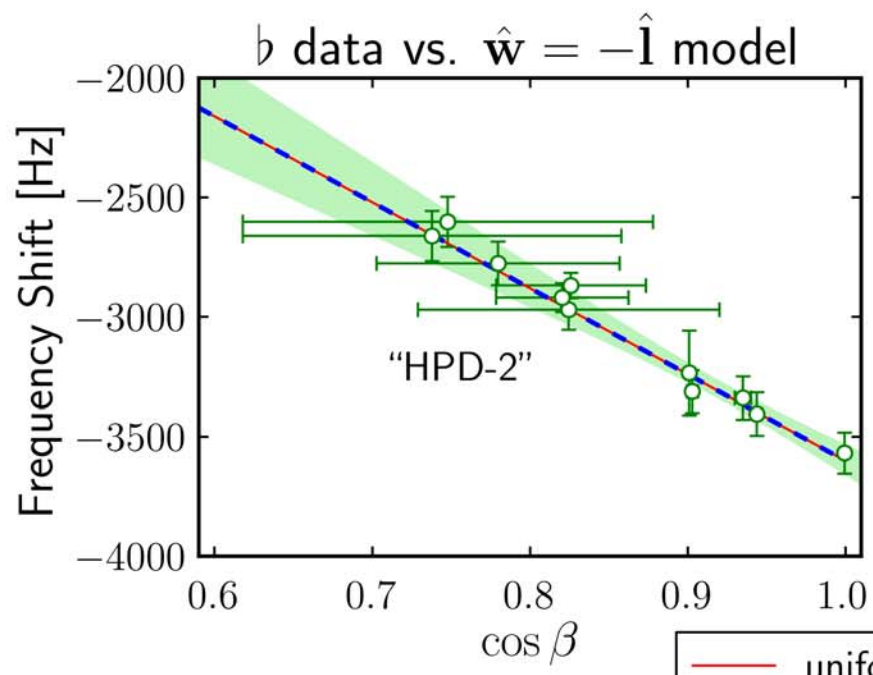
# Large Tip Angle NMR on the confined B Phase: determining spatial variation of planar distortion

Modify theory to include spatial dependence of planar distortion:

instead of just

$$q = \frac{\Delta_{\perp}}{\Delta_{\parallel}}$$

$$q = \frac{\langle \Delta_{\perp} \Delta_{\parallel} \rangle}{\langle \Delta_{\parallel}^2 \rangle}, \quad Q = \sqrt{\frac{\langle \Delta_{\perp}^2 \rangle}{\langle \Delta_{\parallel}^2 \rangle}}$$



— uniform  $q = 0.74$   
 - - - mean  $q = 0.68, Q = 0.71$



1. Determine profound modification of superfluid phase diagram due to confinement.
2. “A” phase (not planar phase) stable at zero pressure
3. Determine  $T_{AB}$ , and conclude that strong coupling corrections exist at all  $\rho$
4. No evidence, as yet, for striped phase (for good reason).
5. Identify both a stable and a metastable state of planar-distorted B-phase. Fully characterize by NMR.
6. Under confinement the gap is spatially dependent [due to wall suppression]. We can use **NMR** to **measure averages of the gap** and compare to predictions of quasi-classical theory.
7. We have **in situ control** of surface scattering by  $^4\text{He}$  pre-plating.  
We can study this through (i)  $T_c$  suppression (ii) gap suppression

- **Discovering new order parameters**

Prediction of spatially modulated B-phase (striped phase)

Gapped  $p+ip$  phase, arising from confinement (quantum size effect)

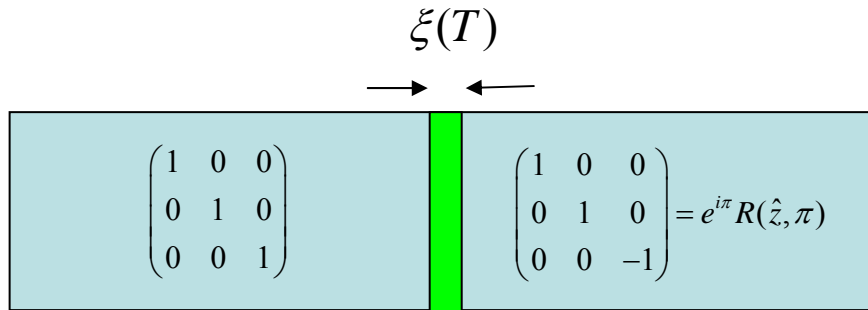
- **Probing surface and edge excitations: the search for majorana fermions**

Propagating majorana fermions at the surface of the B-phase

Chiral edge mode in the gapped A-phase (if stable)

Majorana zero-modes at core of HQV (if stable) in gapped A-phase

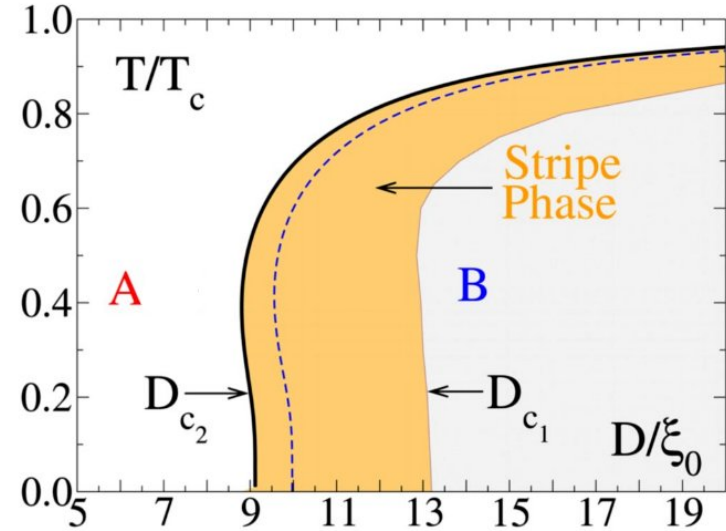
Domain wall (planar phase) between different B phase states



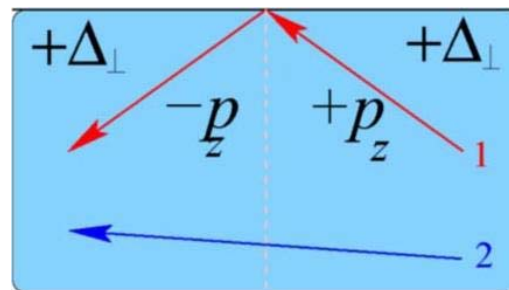
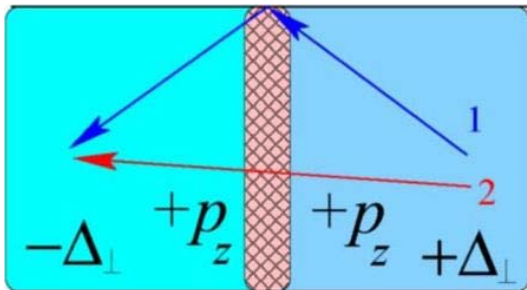
Salomaa and Volovik  
PRB. 37, 9298 (1988)

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Vorontsov and Sauls PRL 98, 045301 (2007)



Superfluid with spontaneously broken translational symmetry



# Towards the two dimensional A phase

Gapped  $p + ip$  superfluid  
 Breaks TRI  
 cf candidate  $\text{Sr}_2\text{RuO}_4$

Requires specular boundaries  
 Plate surfaces with superfluid  $^4\text{He}$  film  
 Diagnostic is  $T_c$  suppression

slab geometry: Size quantization

$$\varepsilon = \frac{1}{2m^*} [p_z^2 + q^2] \quad p_z = \frac{\hbar\pi j}{L}$$

Number of discs  $j_0 = \frac{k_F L}{\pi}$

$k_F = 7.9 \text{ nm}^{-1}$  so for  $L = 100 \text{ nm}$ : number of discs = 250

Note:

QSE show up clearly in transport in a 100 nm film. The transport relaxation time shows a  $1/T$  dependence.

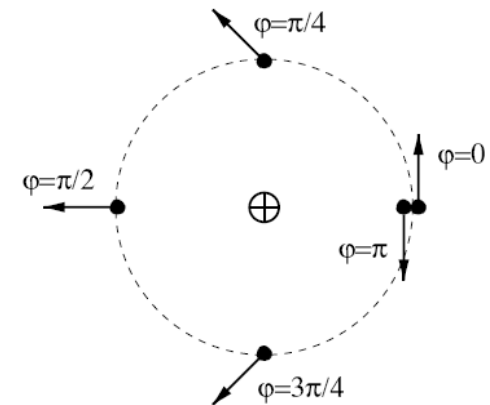
Search for this state in slabs of thickness  $< 100\text{nm}$

Consequences:

Quantum Hall analogues in transport of heat, spin, (mass?)

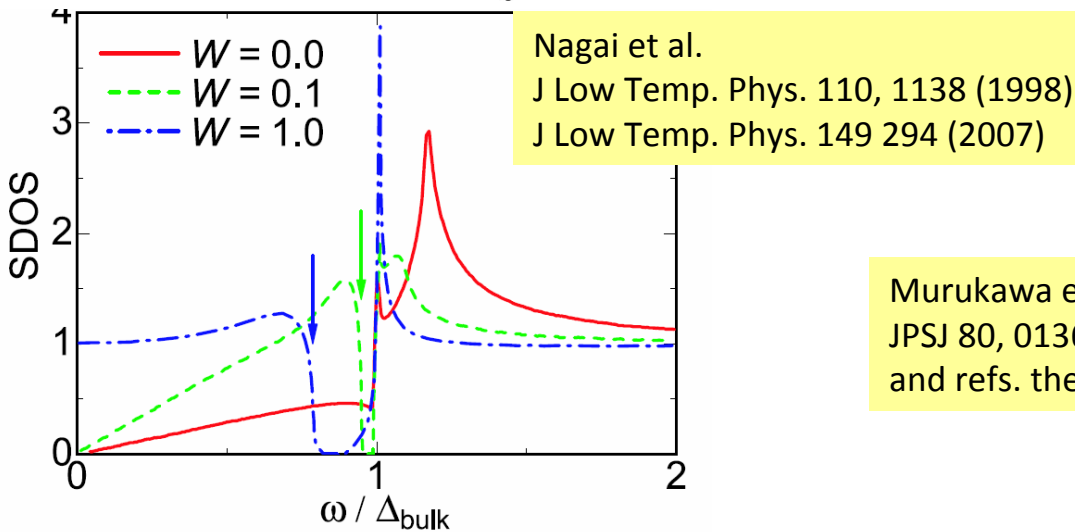
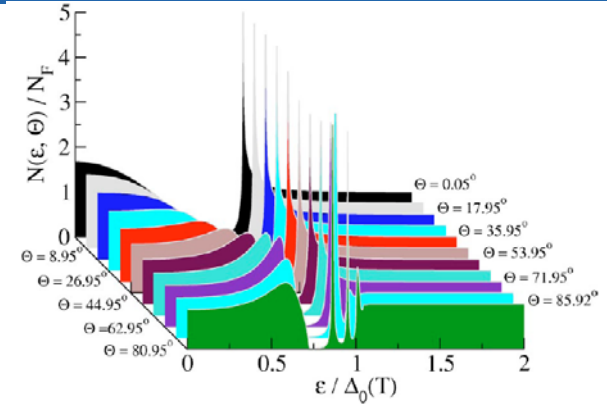
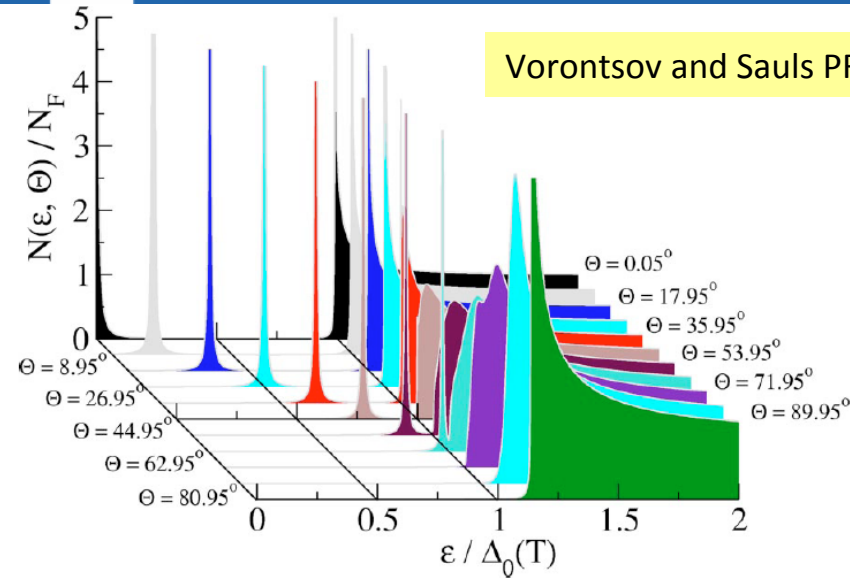
HQV in a gapped  $p + ip$  superfluid (detection and manipulation?)

$$\kappa_{xy} = \frac{1}{2} N \frac{\pi^2 k_B^2}{3h} T$$



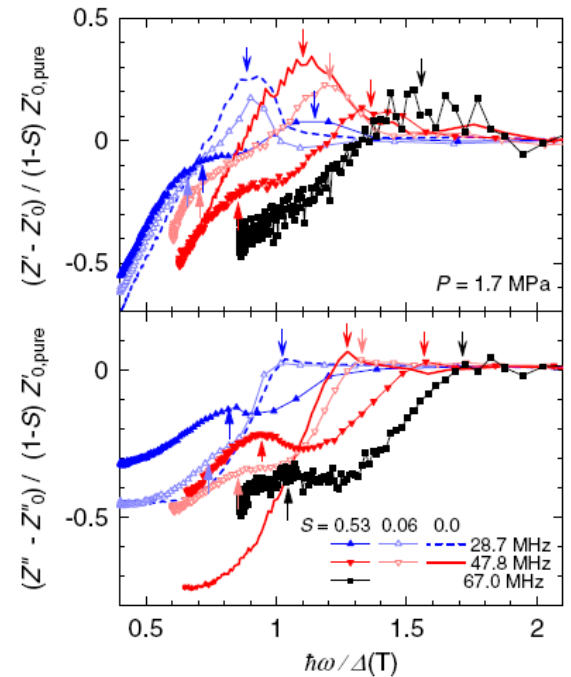
But...is this state stable relative to TRI planar phase?

# Surface Andreev Bound States: B phase Mid gap states and Majorana fermions

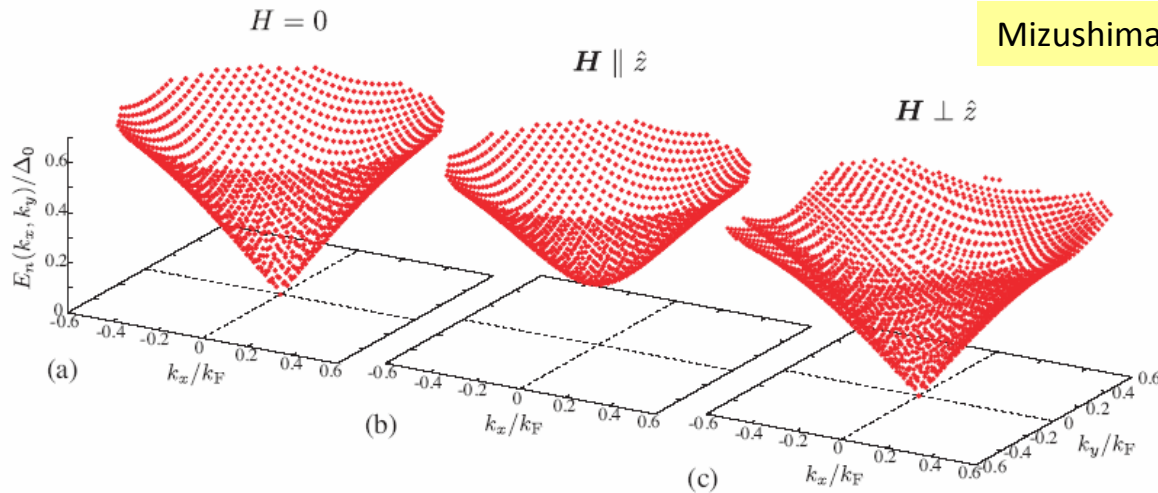


Murukawa et al.  
 JPSJ 80, 013602 (2011)  
 and refs. therein

## Transverse acoustic impedance

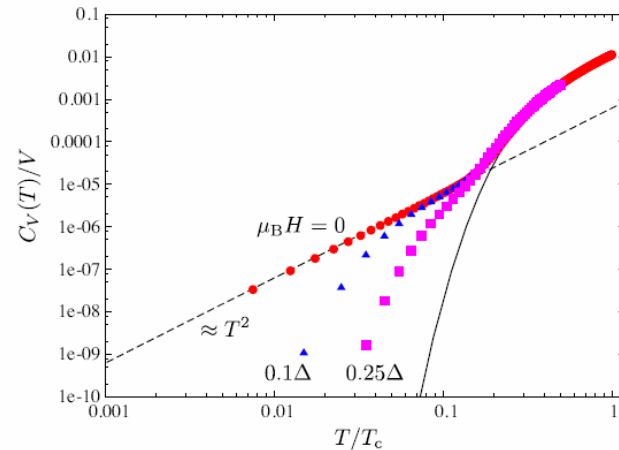


Mizushima and Machida J. Low Temp.Phys.



**Fig. 1** (Color online) Low energy quasiparticle spectra in a plane of  $\mathbf{K} = (k_x, k_y)$  parallel to the wall for the case of  $H=0$  (a),  $\mathbf{H} \parallel \hat{z}$  (b), and  $\mathbf{H} \perp \hat{z}$  (c). The zero energy corresponds to the Fermi energy and  $z$ -axis is set to be normal to the wall. Here, we use  $k_F \xi = 5$ . In (b) and (c), the Zeeman field is set to be  $\gamma H/2 = 0.25\Delta_0$ .

Thermodynamic signature:  
Power law heat capacity  
Thermal conductivity





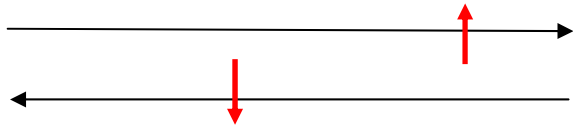


# Distinctive feature of Majorana: spin-orbit coupling

S-B Chung and S-C Zhang Phys. Rev. Lett. 103, 235301 (2009)

QSH and TI analogue

$$H = v_F \boldsymbol{\sigma} \cdot (\hat{\mathbf{z}} \times \mathbf{p})$$

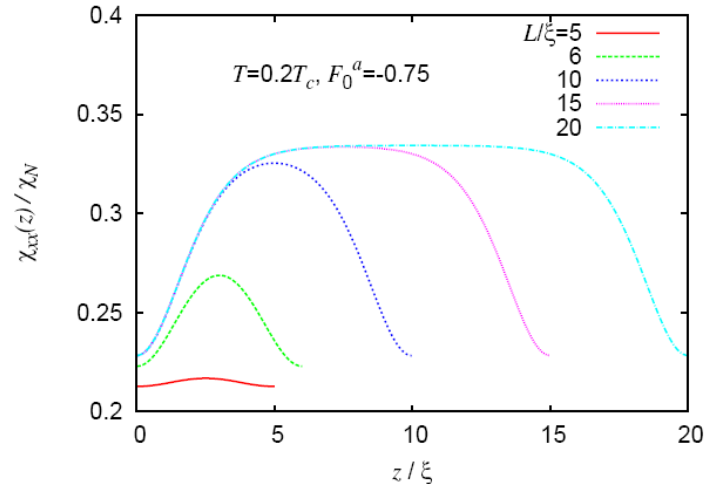
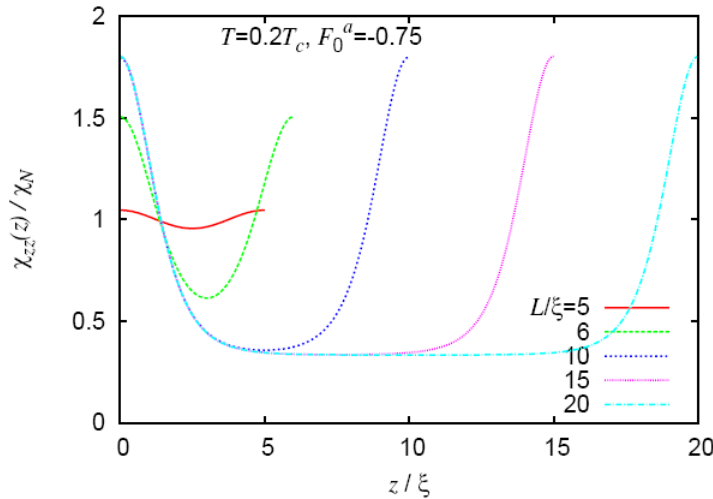


$$E = ck \quad c = \frac{\Delta}{p_F}$$

They move slowly, at half Landau critical velocity

→ Anisotropy of spin susceptibility

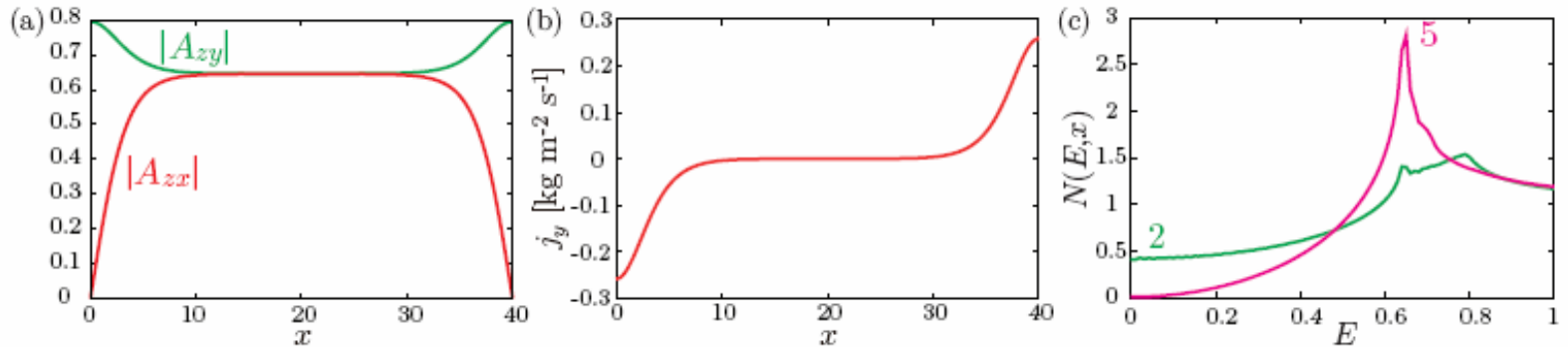
Nagato, Higashitani, Nagai JPSJ 78, 123604, 2009  
Volovik JETP Lett. 2010  
Kawakami, Tsutsumi, Ichioka, Machida





# Edge states in A phase

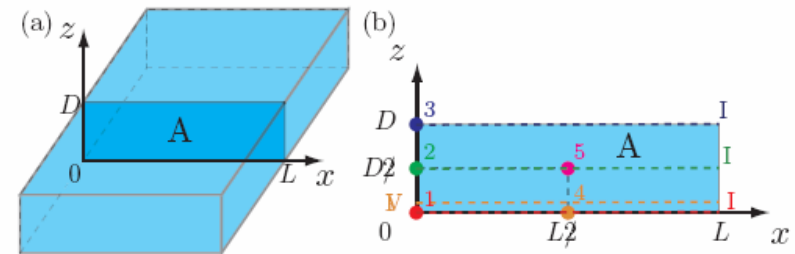
Tsutsumi, Ichioka, Machida 2010



For specular walls:  
No surface states on top and bottom surface of slab

Polar phase at edges

Counter-propagating mass currents



Hence intrinsic angular momentum in edge state of a mono-domain sample of <sup>3</sup>He-A.  
Create mono-domain by cooling under rotation.  
Detect edge state gyroscopically in the quietest vibrational environment in the world.

Also: can we stabilize HQVs in mono-domain sample?



# Conclusion



Rob Bennett



Lev Levitin



Andrew Casey

## New direction.....Quantum nanofluidics

Confinement as a control parameter (+ pressure, magnetic field)

Explore richness of p-wave superfluid film

Search for new order parameters

Surface and edge excitations....majorana search

Develop new experimental probes

