Chiral magnetic effect and Natural optical activity in metals with and without Weyl points

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Weyl Semimetal (WS) is a collection of nondegenerate band touchings in 3D k-space.

“3D Graphene”

Conduction band

\[-\nu \vec{\sigma} \vec{p} - \mu - \sigma_0\]

Weyl points

\[+\nu \vec{\sigma} \vec{p} - \mu + \sigma_0\]

Valence band

(Herring’37, Abrikosov&Beneslavsky’71, Wan et al’11, Burkov&Balents’11)

Two bands ➔ the spin degeneracy is lifted, either by T or I breaking.
Where does one get a WS?

**Weyl semimetals with inversion breaking:**

*Photonic crystal:* Lu et al, Science, 2015.

*TaAs:*
  
  S-Y. Xu et al., Science 2015
  
  B. Q. Lv et al., PRX 2015

*NbAs:*
  
  Y. Luo et al., PRB 2015

**Weyl semimetal with time-reversal breaking (?)**:

*Er2Ir2O7:*
  
  Sushkov et al., PRB (R) 2015

*YbMnBi₂:*
  
  Borisenko et al., arxiv:1507.04847
Weyl semimetals (WS) are gapless phases with nontrivial topology

(review: Turner&Vishwanath, cond-mat: 1301.0330)

1. Nodes are stable due to topology, not symmetry
2. There are protected surface states (‘arcs’), which cannot be realized in a 2DEG.
3. Hall response is determined by the distance between nodes, and nothing else (broken TR).
4. The chiral anomaly and the chiral magnetic effect are of geometric origin
Chiral anomaly in CM setting

$\vec{B} = (0, 0, B)$,  
$H = \pm v \left[ \hat{\sigma}_\perp (\vec{p}_\perp - e\vec{A}) + \sigma_z p_z \right]$  

$E_{n \neq 0}^{R,L}(p_z) = \pm v \sqrt{2|n|eB\hbar/c + p_z^2}$  
$E_0^{R,L}(p_z) = \pm vp_z, \quad n = 0$

$\Delta \mathcal{N}_R - \Delta \mathcal{N}_L = \frac{e^2}{2\pi^2\hbar^2c} \mathbf{E} \cdot \mathbf{B}$  

“3D chiral anomaly”

Brief intro into “Chiral magnetic effect”

\[
\begin{align*}
\mathbf{j}_{\text{CME}}^L &= \frac{e^2 \mu_L}{4\pi^2} \mathbf{B} \\
(\text{Vilenkin, 1980})
\end{align*}
\]

\[
\begin{align*}
\mathbf{j}_{\text{CME}}^R &= -\frac{e^2 \mu_R}{4\pi^2} \mathbf{B}
\end{align*}
\]

\[
\begin{align*}
\mathbf{j}_{\omega=0} &\propto \frac{e^2 (\mu_L - \mu_R)}{4\pi^2} \mathbf{B} \\
(\text{Kharzeev, Warringa, 2009; Son, Yamamoto, 2013})
\end{align*}
\]

\[
\begin{align*}
\mathbf{j}_{\omega \neq 0} &\propto \frac{e^2 (\mu_L - \mu_R)}{12\pi^2} \mathbf{B}
\end{align*}
\]
Is there anything interesting in optics?
“Unusual” electrodynamics in WS

\[ S_{WS} [A] = \frac{e^2}{32\pi^2} \int d^4 x \, \theta(r,t) \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta} \]

\[ \text{“(E \cdot B)”} \]

\[ = -\frac{e^2}{8\pi^2} \int d^4 x \, \epsilon^{\mu\nu\alpha\beta} \partial_{\mu} \theta A_{\nu} \partial_{\alpha} A_{\beta} \]

“3D Chern-Simons action”, (Goswami&Tewari’12, Zyuzin&Burkov’12)

Current: \[ j^\rho = -\frac{e^2}{4\pi^2} \partial_{\mu} \theta \epsilon^{\mu\rho\alpha\beta} \partial_{\alpha} A_{\beta} \]

\[ \mu \neq 0: \quad j^\rho = -\frac{e^2}{4\pi^2} \partial_{\mu} \theta \epsilon^{\mu\rho\alpha\beta} \partial_{\alpha} A_{\beta} = \text{Anomalous Hall effect} \]

\[ \mu = 0: \quad j^\rho = -\frac{e^2}{4\pi^2} \partial_0 \theta \epsilon^{0\rho\alpha\beta} \partial_{\alpha} A_{\beta} \]

\[ \vec{j} \propto \lambda_{inv} \vec{B} \quad \text{Implies } \vec{M} \propto \vec{A} \text{ in equilibrium} \]
Recap of “Chiral magnetic effect”

\[ j^{CME}_L = \frac{e^2 \mu_L}{4\pi^2} B \]

\[ j^{CME}_R = -\frac{e^2 \mu_R}{4\pi^2} B \]

(Vilenkin, 1980)

\[ j_{\omega=0}^{CME} = \frac{e^2 (\mu_L - \mu_R)}{4\pi^2} B \]

\[ j_{\omega \neq 0}^{CME} = \frac{e^2 (\mu_L - \mu_R)}{12\pi^2} B \]

(Kharzeev, Warringa, 2009; Son, Yamamoto, 2013)
There is only dynamic CME in equilibrium crystals

\[ j_{\omega=0}^{CME} = 0 \]  


\[ j_{\omega \neq 0}^{CME} = ? \]  

(Particular model, strong B, broken TR and \( I \): Chen, Wu, Burkov, PRB, 2013)
“Chiral magnetic effect” is natural optical activity

Natural optical activity:

\[ \sigma_{ab}(\omega, \mathbf{q}) = \sigma_{ab}(\omega) + \lambda_{abc}(\omega)q_c \]

Means different response (refractive index) for right/left circularly polarized light:

\[ \mathbf{q} = (0, 0, q_z) : \epsilon_{ij} = \left(1 - \frac{\omega_p^2}{\omega^2}\right) \delta_{ij} - \frac{\lambda q_z}{\omega \epsilon_0} (\tau_2)_{ij} \]

Chiral magnetic effect:

\[ \mathbf{j}(\omega, \mathbf{q}) = \eta(\omega, \mathbf{q}) \mathbf{B}(\omega, \mathbf{q}) \]

\[ \mathbf{B} = \frac{1}{\omega} \mathbf{q} \times \mathbf{E} \]

\[ \lambda_{metal}(\omega) = -\frac{\eta(\omega, \mathbf{q})}{\omega} \epsilon_{abc} \]

(non-analytic function of frequency)
CME for an arbitrary band structure: multiband calculation
CME is an interband coherence effect

Semiclassical corrections to intraband kinetics:

**Berry curvature:**

\[ \Omega_{np} = i \langle \partial_p u_{np} \rangle \times \partial_p u_{np}, \]

\[ \delta \vec{r} = -e \vec{E} \times \Omega_{np} - e(\vec{v}_{np} \cdot \Omega_{np})B \]

**Orbital magnetic moment:**

\[ m_{np} = \frac{ie}{2} \langle \partial_p u_{np} \rangle \times (h_p - \epsilon_{np}) \partial_p u_{np} \]

\[ E_{np} = \epsilon_{np} - m_{np} B, \quad v_{np} = \partial_p \epsilon_{np} - \partial_p (m_n B) \]

**NB:** non-uniform orbital magnetization leads to current!
Semiclassical corrections to kinetics

Standard single-band kinetic equation:

\[ \partial_t f_{np} + \dot{r} \partial_r f_{np} + \dot{p} \partial_p f_{np} = 0 \]

- no collisions,
  high frequency

Equations of motion:

\[ \dot{r} = v_{np} - \dot{p} \times \Omega_{np}, \]
\[ \dot{p} = eE + e\dot{r} \times B. \]

Expression for the current:

\[ j_{qp} = e \sum_n \int \left( d\mathbf{p} \right) \left( \partial_{\mathbf{p}} \epsilon_{np} - \partial_{\mathbf{p}} (m_n \mathbf{B}) - e\mathbf{E} \times \Omega_{np} - e(\partial_{\mathbf{p}} \epsilon_{np} \cdot \Omega_{np}) \mathbf{B} \right) f_{np} \]
\[ j_m = \partial_r \times \sum_n \int (d\mathbf{p}) m_{np} f_{np} \]
CME/NOA for an arbitrary band structure

**Details:** J. Ma, DP. Phys. Rev. B 92, 235205 (2015)

\[ \sigma_{ab}(\omega, \mathbf{q}) = \sigma_{ab}(\omega) + \lambda_{abc}(\omega)q_c \]

\[ \lambda_{abc} = -\frac{e^2}{\omega} \sum_n \int (d\mathbf{p}) \left( \epsilon_{np}(\partial_p f_{np} \cdot \Omega_{np})\epsilon_{abc} \right) \]

\[ + \frac{1}{e} m_{nd} \partial_a f_{np} \epsilon_{dcb} \]

\[ + \frac{1}{e} m_{nd} \partial_b f_{np} \epsilon_{adc} \]

Static CME, vanishes

Velocity renormalization

Magnetization current from non-uniform acceleration

\[ \sigma_{ab}(\omega, \mathbf{q}) = \sigma_{ba}(\omega, -\mathbf{q}) \Rightarrow \lambda_{abc}(\omega) = -\lambda_{bac}(\omega) \]
CME in simple models
Chiral magnetic effect without Weyl points

Model of a helical metal with $C_4$ point group:

$$H^{C_4} = \frac{p^2}{2m} + \sigma d^{C_{4v}}_p + \gamma_v p_x p_y (p_x^2 - p_y^2)$$

$C_{4v}$ SOC  $C_{4v} \rightarrow C_4$

$$j^{C_{ME}} = -\frac{e^2}{105\pi^2} \frac{\gamma p_F^5 \cdot \gamma_v p_F^4}{\mu} B$$
Chiral magnetic effect in an ideal WS

\[ H_w = E_w - \mu_w + Q_w v_w \sigma p \]

Results for “chiral conductivity”, \( j^{CME} / B \):

<table>
<thead>
<tr>
<th></th>
<th>( \mu_w = \mu )</th>
<th>( E_w = E )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static</td>
<td>0</td>
<td>( \frac{e^2}{4\pi^2} \sum_w Q_w \mu_w )</td>
</tr>
<tr>
<td>Dynamic</td>
<td>( \frac{e^2}{6\pi^2} \sum_w Q_w E_w )</td>
<td>( \frac{e^2}{12\pi^2} \sum_w Q_w \mu_w )</td>
</tr>
</tbody>
</table>

(see Chang, Yang, PRB 2015, Ma, DP, PRB 2015, Zhong, Moore, Souza, PRL 2016)
Chiral magnetic effect in a WS: numbers

\[ \mathbf{j} = \frac{e^2}{6\pi^2\hbar^2c} \mathbf{B} \sum_w Q_w E_w \]

Rotatory power: \[ \rho \sim \alpha \frac{\Delta E}{\hbar c} \sim 10^{-2}\text{rad/mm} \]

Skin layer \( \sim 100 \text{ nm} \): \[ \Delta \phi_{\text{skin}} \sim 10^{-6}\text{rad} \]
Real samples have boundaries…
The role of sample boundaries (or “the absence of polar Kerr effect”)
The role of sample boundaries (or "the absence of polar Kerr effect")

The solution: surface "Hall" current

\[ \dot{j}_{g,a} = \gamma_{abc} \partial_c E_b \]

(Agranovich, Yudson, 1973, via Onsager relations)

\[ \dot{j}_{g,a} = \frac{1}{2} \{ \gamma_{abc}(r), \partial_c \} E_b = \gamma_{abc} \partial_c E_b + \frac{1}{2} \partial_c \gamma_{abc} \cdot E_b \]

"Hall" conductivity,

\[ \gamma_{abc} = -\gamma_{bac} \]

(Cuprates: Hosur et al., 2015)

\[ \dot{j}_{surf} = -\frac{1}{2} \dot{j}_{bulk} \]

(compare to \( \dot{j}_{surf} = -2\dot{j}_{bulk} \)

Baireuther et al., 2016)
Conclusions

- Chiral magnetic effect is equivalent to natural optical in metals.

- Semiclassically, natural optical activity of metals is due to the orbital magnetic moment of quasiparticles. It has its roots in geometry, not topology.

- Weyl points are not required for CME/NOA

- 3D Chern-Simons action is not a property of WS, but of any helical metal with natural optical activity. Maxwell equations need not be revised.

- Experimentally, polarization rotation is confined to the skin layer, and is limited by a few $\mu$rad.