Topological Photonics
Landau Levels in Curved Space

Jon Simon
University of Chicago
Chicago, Illinois

Topological Quantum Matter
KITP @ UCSB
October 18th, 2016

quantum.uchicago.edu & simonlab.uchicago.edu

Support: AFOSR YIP, DOE YIP, DARPA YFA, DARPA FP060494, ARO MURI, AFOSR MURI, UofC MRSEC
Synthetic Material Systems

Solid State

Interacting Electrons

Ionic Lattice
Topological Cavity QED

Twisted Optical Resonators

- **Bulk** Landau Levels
- **Atoms** Break T
- **Rydberg EIT** for Interactions
- **Flux threading** to build manybody states

**μ-wave Lattices**

- **Lattice** $Z_2$ & Chern Bands
- **Ferrites** Break T
- **Transmons** for Interactions
- **Chem pot’l** to build manybody states
Synthetic Materials

Photon Bl

1D

Exciton Polaritons [2]

[3] Peyronel et al., Nature


[3] Peyronel et al., Nature
Central Premise
Photons in Multimode Resonator
Massive Particles in Harmonic Trap

• This is a real space—not Fock space—analogy
A photon in a curved-mirror optical resonator behaves like a massive particle in a harmonic trap.
Understanding photons in optical resonators

[1] Sommer et. al., *NJP* 18, 035008 (2016)
Comparing Harmonic Oscillators & Optical Resonators

Hermite Gauss Wavefunctions

Uniform Frequency Spacing

Optical Modes
Understanding photons in optical resonators

Derived quantity (from waves/QM)

Intrinsic quantity (ray optics/indep of $\lambda$)

$n$

$e^{i \Delta \omega t}$
Formal Picture: Floquet Theory

Sommer et al., New J. Phys. 18 035008 (2016)
Formal Picture: Floquet Theory
Formal Picture: Floquet Theory

\[ H_{\text{floquet}} = \frac{1}{iT_{RT}} \log \left( e^{-\frac{iL}{2k} \left( \frac{\partial}{\partial x} \right)^2} e^{-\frac{iL}{2} \left( \frac{\partial}{\partial x} \right)^2} e^{-\frac{iL}{2k} \left( \frac{\partial}{\partial x} \right)^2} e^{-\frac{iL}{2} \left( \frac{\partial}{\partial x} \right)^2} \right) \]

Sommer et al., New J. Phys. 18 035008 (2016)
Synthetic Magnetic Fields for Photons

is there a (simple) way to add a synthetic magnetic field??

\[ H_{floquet} = \frac{1}{i T_{RT}} \text{Log} \left[ e^{i \theta L_z} e^{-i \frac{k x^2}{2 R}} e^{i \frac{L}{2 \hbar^2 k} p^2} e^{-i \frac{k x^2}{2 R}} e^{i \frac{L}{2 \hbar^2 k} p^2} \right] \]

Synthetic Gauge Fields for Resonator Photons

Twisting the resonator out of the plane makes the lab frame a rotating frame!

\[ H_{floquet} = \frac{1}{iTR_T} \text{Log} \left[ e^{i\theta L_z} e^{-i\frac{kx^2}{2R}} e^{i\frac{L}{2\hbar^2 k} p^2} e^{-i\frac{kx^2}{2R}} e^{i\frac{L}{2\hbar^2 k} p^2} \right] \]

Flattening Photonic Landau-Levels

\[ \Psi_L(z \equiv x + iy) = \frac{z^L}{\sqrt{\pi L!}} e^{-\frac{z^2}{L}} \]
Photonic Centrifugal Barrier

- Twisted Cavity $\rightarrow$ Lab Frame Coriolis Force + Anti-Trapping
- Slightly Astigmatic Harmonic Confinement (drives $\Delta l = \pm 2$ transitions)

Mirror-Induced Trapping

Twist-Induced Coriolis + Anti-Trapping

Stabilizing the System

• Floquet instability at degeneracy due to $\Delta l = \pm 2$ transitions
• Modify LLL to include only $l=0,3,6,9\ldots$ removes instability

$$\Psi_M(z \equiv x + iy) = \frac{z^{3M}}{\sqrt{3\pi M!}} e^{-z\bar{z}}$$

• How? Keep twisting resonator!

LLL for different Floquet manifolds, (longitudinal modes)

What is this new, stable “LLL,” physically?
Landau Levels on a Cone

All Gaussian Curvature
Concentrated Here

Schine et al., Nature doi:10.1038/nature17943 (2016)
Spectroscopy of Weakly Trapped Landau Levels on a Cone

No Centrifugal Barrier!

THREE-FOLD SYMMETRY DOES NOT COUPLE TO ASTIGMATISM

Schine et al., Nature doi:10.1038/nature17943 (2016)
Exploring the Lowest Landau Level (on a Cone)

Sensitive to Conical Geometry

Insensitive to Conical Geometry

Increasing Angular Momentum
Orbiting the Cone Tip

Biswas et al., arXiv 1412.3809 (2014)
Orbiting the Cone Tip

Biswas et al.
Interplay of Topology and Manifold Curvature

Prediction (Wen-Zee)

\[ \rho(x) = \frac{1}{2\pi l^2 B} + \frac{1}{2\pi R(x)} + \frac{e\delta B(x)}{\hbar} \]

- Mean orbital spin \( \langle S \rangle \)
- Cone Tip
- \( l = 0, 3, 6, 9, \ldots \)
- \( l = 1, 4, 7, 10, \ldots \)
- \( l = 2, 5, 8, 11, \ldots \)

Can et al., Phys. Rev. Lett

Can et al., Phys. Rev. Lett

Ev. B 90, 014435 (2014)

0.31 ± 2 states

-0.35 ± 2 states

-0.02 ± 1 states
Adding Photon-Photon Interactions

• Rydberg atoms interact $\rightarrow$ photons interact…
• *Now what? How do we make manybody ground states?*

Ningyuan *et al.*, PRA 93, 041802 (2016), Parigi *et al.*, PRL 109, 233602 (2016)
Peyronel *et al.*, Nature 488, 57-60 (2012)
Blockade-Assembly of Small Laughlin States
the “Quantum Optics” way

\[ \Psi_L(z = x + iy) = \frac{z^L}{\sqrt{\pi L!}} e^{-z \bar{z}} \]

No Interactions \rightarrow 2 photon transmission
LAUGHLIN FILTER


Thouless-Pumping to Laughlin States the “Quantum Optics” way

Blockaded injection of a single particle at the cone tip

Lohse et al., Nat. Phys. 3584, (2016)
T-Breaking in Twisted Optical Cavities
Next-Generation Resonator

Ostracizing the Rydbergs

Convex Mirrors

Smaller Waist (12 microns) @ 1cm to atoms stable (confocal)
Outlook

• Chern Number (Kitaev’s Real-Space Formalism):
  Holographic Reconstruction of Band Projectors

• Strong Interactions:
  (0-d) Polariton Quantum Dots to (2-d) Laughlin Puddles
Engineering μWave Quantum Materials

Z₂ TI (QSH)  Lattice QH  Chemical Potential

schuster.lab
The Analogy

**Cold Atoms**

- *Hopping* in an optical lattice
- *S*-wave contact *interactions*

**Microwave photons**

- *Hopping* btwn cap.-coupled SC resonators
- Qubit-mediated *interactions*
Two Approaches to Synthetic Magnetic Flux in Circuits

Weave Tunnel Couplers to Create Peierls Phase

Employ Chiral Orbitals to Create Peierls Phase

Ningyuan et al., PRX 5 (2), 021031

Anderson et al., arXiv: 1605.03177, in press PRX
(Braided) Topological Circuits
“Braiding” the Planes @ α = π/2
“Braiding” the Planes

Unit Cell

$|\uparrow\rangle = A + iB$

$|\downarrow\rangle = A - iB$

$|\uparrow\rangle \rightarrow -i |\uparrow\rangle = e^{i\pi/2} |\uparrow\rangle$

$|\downarrow\rangle \rightarrow -i |\downarrow\rangle = e^{-i\pi/2} |\downarrow\rangle$

So $\alpha = \pm \pi/2$: Spin-Hall
A Topologically Insulating Circuit

Transmission

frequency, $\Omega_{\sqrt{LC}}$

$-\pi$  $-\pi/2$  0  $\pi/2$  $\pi$

quasi-momentum, $q$

$0.5$  $1.0$  $1.5$

$\text{top}$  $\text{bottom}$  $\text{bottom}$  $\text{top}$  $C=+1$

$C=-2$

Transmission

Drive Frequency, $\nu_{\text{drive}}$ (kHz)

$100$  $150$  $200$  $250$  $300$

Bulk-Bulk

Y-site

$1$  $3$  $5$  $7$  $9$  $11$

X-site

$1$  $3$  $5$  $7$  $9$  $11$

c) Edge-Edge

Y-site

$1$  $3$  $5$  $7$  $9$  $11$
Spin- and Time-Resolved Dynamics

- Topologically Protected Mode
- Unprotected Gap

Excite One Spin State

Excite Both States
Exotic Global Topologies: Möbius TI

- Large wavelength compared to material size
- Useful for making tori, cylinders, etc.. for FQH physics
Breaking $T$ Symmetry
Microwave Chern Insulators
Building a Superconducting Photonic Lattice: A Chiral \((p_x + ip_y)\) Site

Isolated Chiral Mode (unspoiled by YIG)

Frequency

Transmission

Phase

Magnetic Field
A 7x7 $\alpha=\frac{1}{4}$ Harper-Hofstadter Lattice
Temporal Dynamics in an $\alpha=\frac{1}{4}$ Hofstadter Lattice
How do we build manybody states?

- Add interactions via J-J qubits on each site … but how do we create correlated, interacting phases?
- Connect the lattice to a T=0 chemical potential—populates the system with particles!

Hafezi et al., PRB 92, 174305 (2015)
Kapit et al., PRX 4, 031039 (2016)
Building a Particle Reservoir

Autonomously Stabilized

\[ |0⟩_l \]
\[ |1⟩_l \]
\[ |2⟩_l \]

\[ |0⟩_r \]
\[ |1⟩_r \]

Stabilized
\[ n=1 \]

Anharmonic Oscillator

Lossy Cavity

Coherent
Towards Photonic Materials

Experiment

Theory
• Sommer et al., “Engineering Photonic Floquet Hamiltonians through Fabry–Pérot resonators,” NJP 18 (3), 035008 (2016).
• Anderson et al., “Engineering topological materials in microwave cavity arrays,” accepted to PRX.
• Ma et al., “A Simple Chemical Potential for Photons,” In Preparation.
Thanks!

Bulk Polaritonic Materials
Ariel Sommer (PD)
Nathan Schine (PhD)
Alex Georgakopolous (PhD)
Jia Ningyuan (PhD)
Albert Ryou (PhD)
Michelle Chalupnik (UG)

Lattice Microwave Materials
Alex Ma (PD)
Clai Owens (PhD)
Brendan Saxberg (PhD)
Aman LaChapelle (UG)

Optical/mmWave Hybrid
Aziza Suleymanzade (PhD)
Mark Stone (PhD)
Scott Eustice (UG)

quantum.uchicago.edu & simonlab.uchicago.edu