Beyond parafermions:
Topological defects in non-Abelian systems

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Related work with:
G. Refael, J. Motruk,
F. Pollmann, A. Turner

Before we begin,
a short advertisement…

Modern quantum materials realize a remarkably rich set of electronic phases. This school will explore the many new concepts and methods which have been developed in recent years, moving beyond the traditional paradigms of Fermi liquid theory and spontaneous symmetry breaking. In particular, long-range quantum entanglement appears in topological and quantum-critical states, and the school will discuss new techniques required to describe their observable properties.

Application deadline: **31.10.2016**

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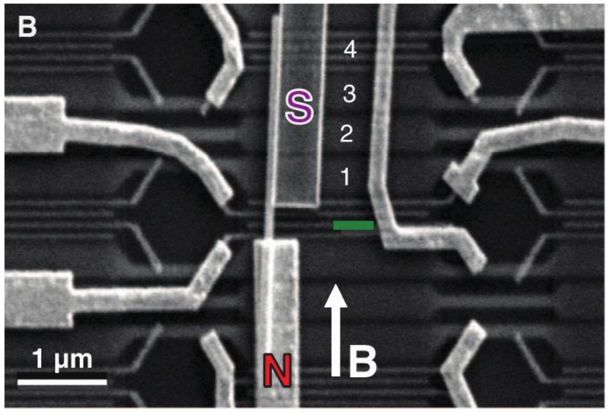
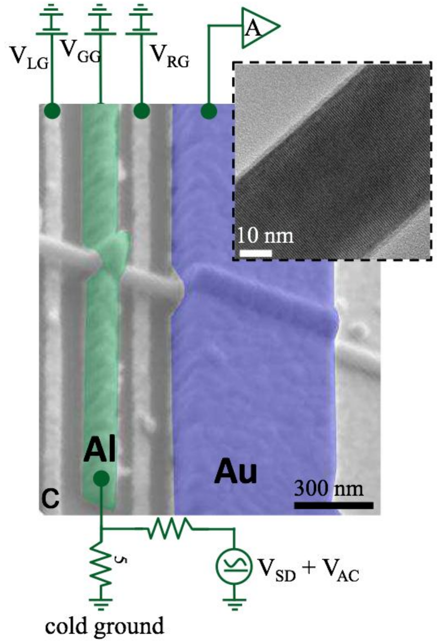
Majorana zero modes

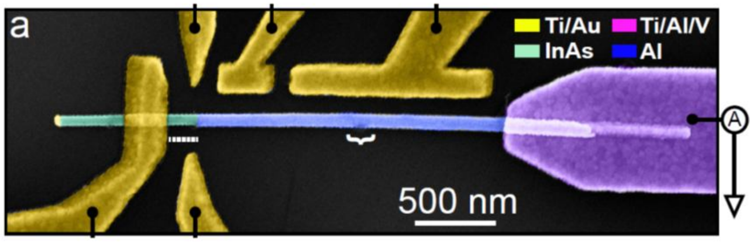


Mourik, Frolov, Kouwenhoven, et al. (2012)



Das, Ronen, Heiblum, et al. (2012)

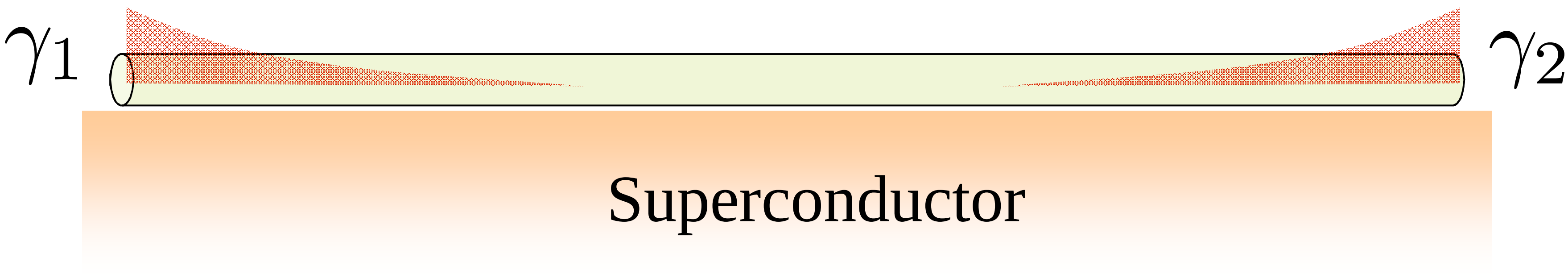
Epitaxial Majorana Device



Churchill, Krogstrup Albrecht, Marcus et al. (2013-2016)

Ferromagnetic atomic chains: Nadj-Perge, Yazdani et. al. (2014)

Majorana zero modes in a topological superconductor



- **Gapped system, two degenerate ground states,** **characterized by having a different fermion parity**
- **Defects (in this case, the edges of the system) carry protected zero modes**
- **Ground state degeneracy is "topological": no local measurement can distinguish between the two states!**

Kitaev (2001), Oreg et al. (2010), Lutchyn et al. (2010),...

Topological zero modes: What's next?

Gapped, local Hamiltonians of fermions or bosons in 1D, can give (at best) Majorana zero modes.

Fidkowski and Kitaev, 2010;
Turner, Pollmann, EB, Oshikawa, 2010;
Schuch, Perez-Garcia, and Cirac, 2011;
Chen, Gu, Wen, 2011

Outline

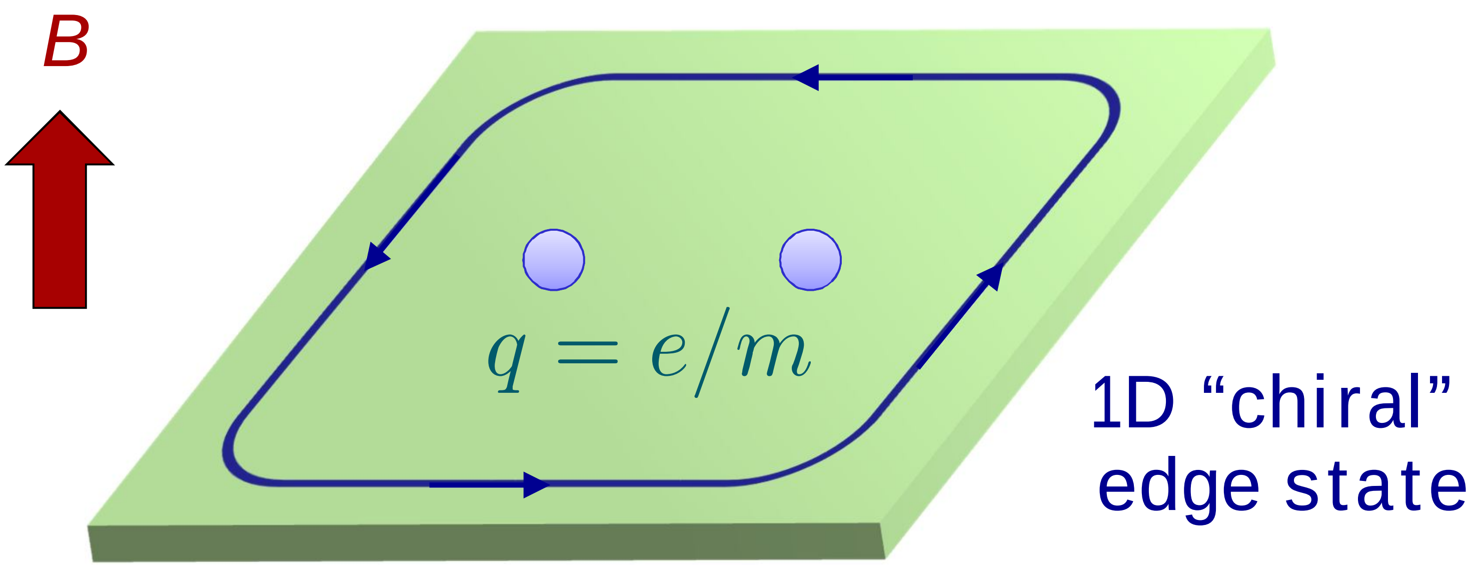
- “Fractionalized Majoranas” on fractional quantum Hall edges
  - Fractionalized 1D superconductors
  - Twist defects

- Twist defects in non-Abelian phases: Ising anyon systems
  - Setup: states and operators
  - Braiding
  - Chains of interacting defects

Beyond Majorana fermions

Consider the effectively 1D boundaries of 2D a topological phase which supports (abelian) anyons.

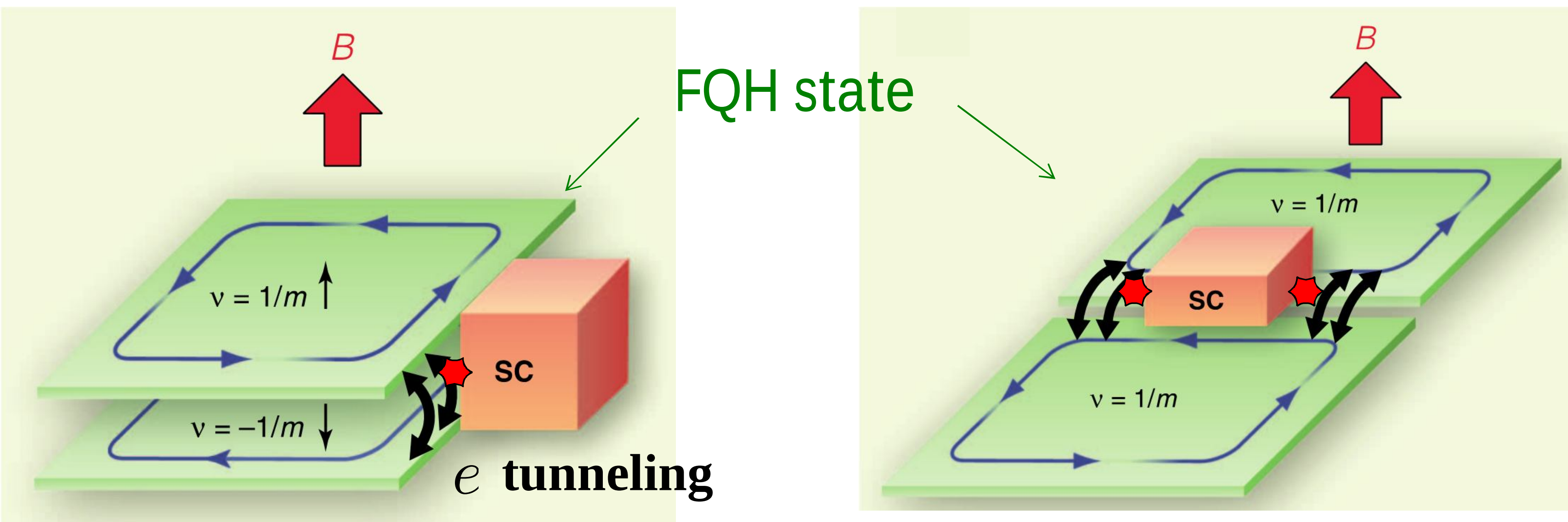
For example:
ν = 1/m Fractional Quantum Hall (Laughlin) state



Beyond Majorana fermions

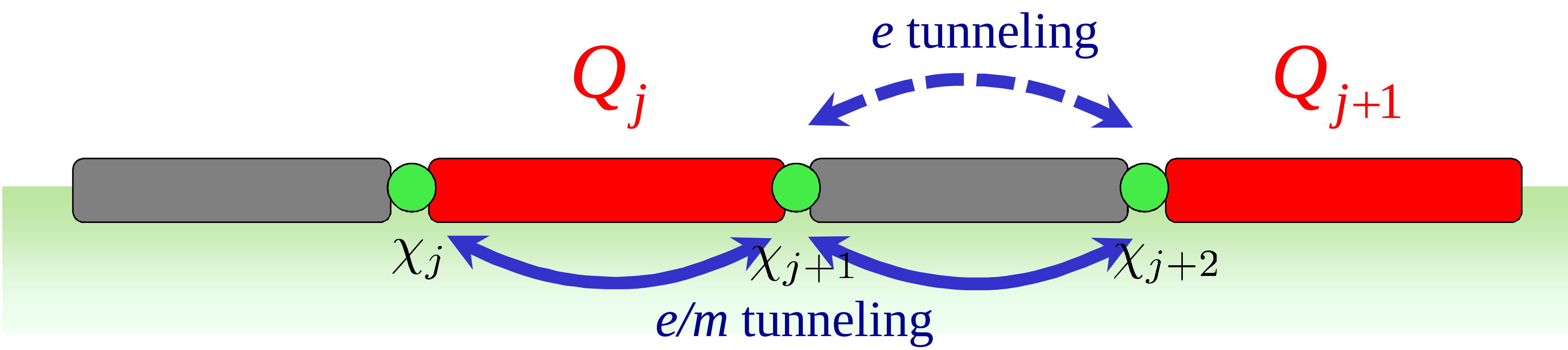
Setups for fractionalized Majorana zero modes:

Fractionalized Majorana (Parafermion) zero modes!



Lindner, EB, Stern, Refael (2012); Clarke, Alicea, Shtengel (2013); Cheng (2013)

Prafermion (fractionalized Majorana) zero modes



Topologically degenerate Hilbert space:
$2m$ states per SC domain

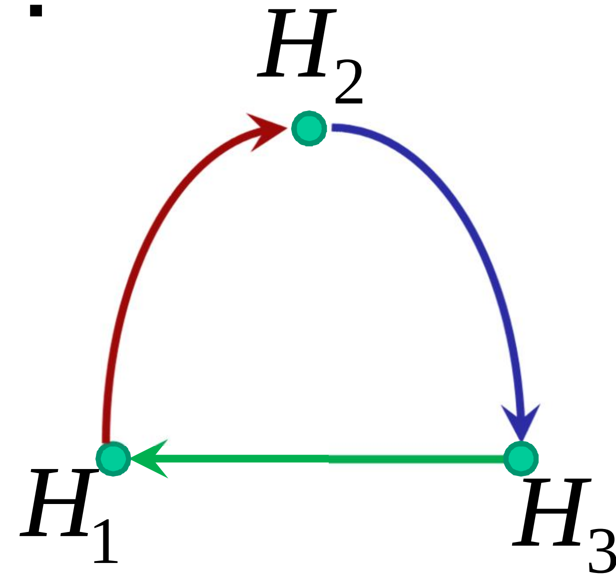
Physical operators:
Laughlin quasi-particle tunneling between "zero modes" described by unitary operators W_{ij}

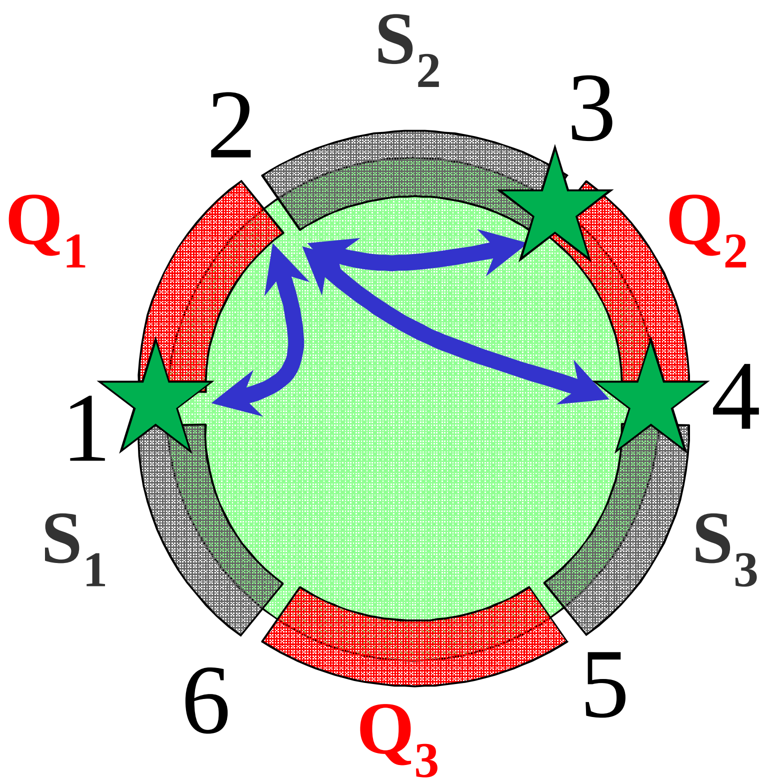
Commutation rule of physical operators:
$W_{ij}W_{jk} = e^{i\pi/m}W_{jk}W_{ij}$ ***("parafermion" exchange relation)***

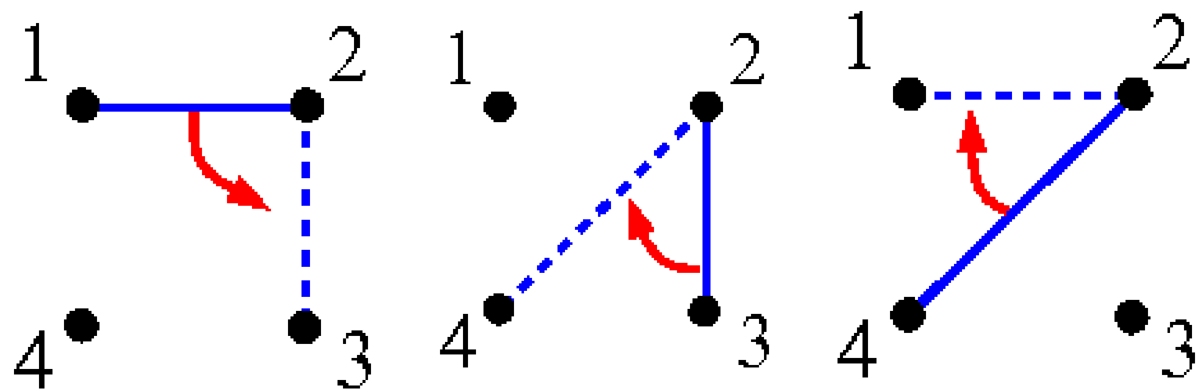
Braiding

Braiding interfaces ★ :

$$H(t) = -\sum_{ij} \lambda_{ij} W_{ij} + H.c.$$







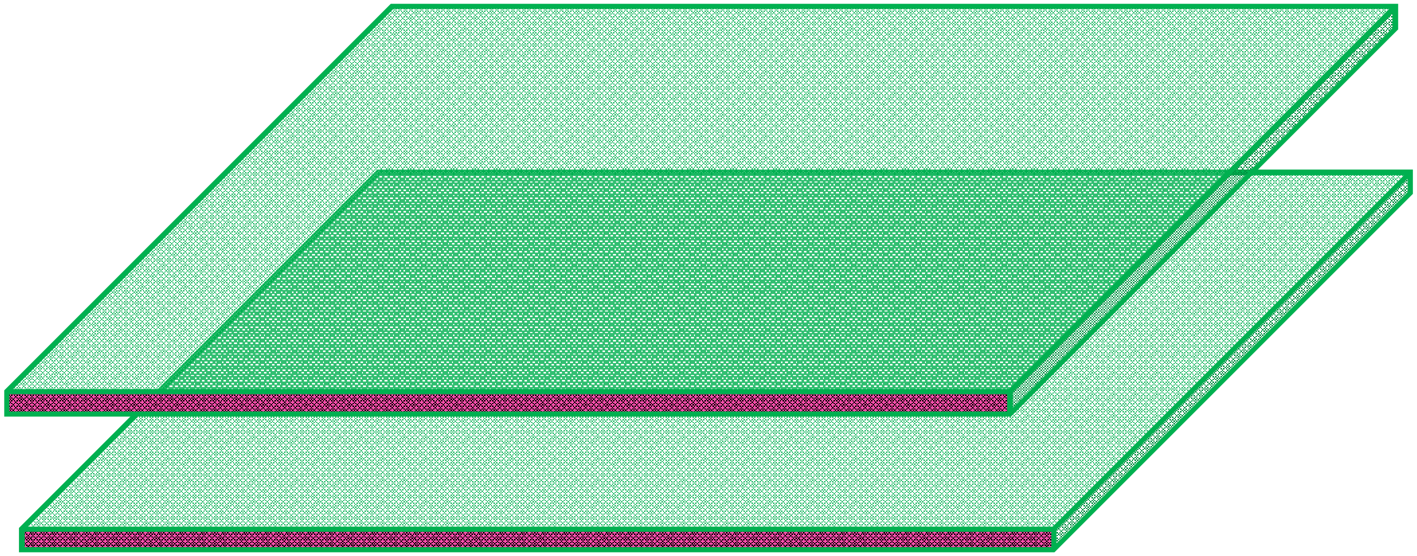
$$U_{34} = e^{\frac{i\pi m}{2}\hat{Q}_2^2}$$

Following braiding scheme for Majoranas: Alicea et al. (2010)

Fractionalized zero modes at "twist defects" in topological phases

Another example: ν=1/3 bilayer

Barkeshli, Jian, Qi (2013; 2014)

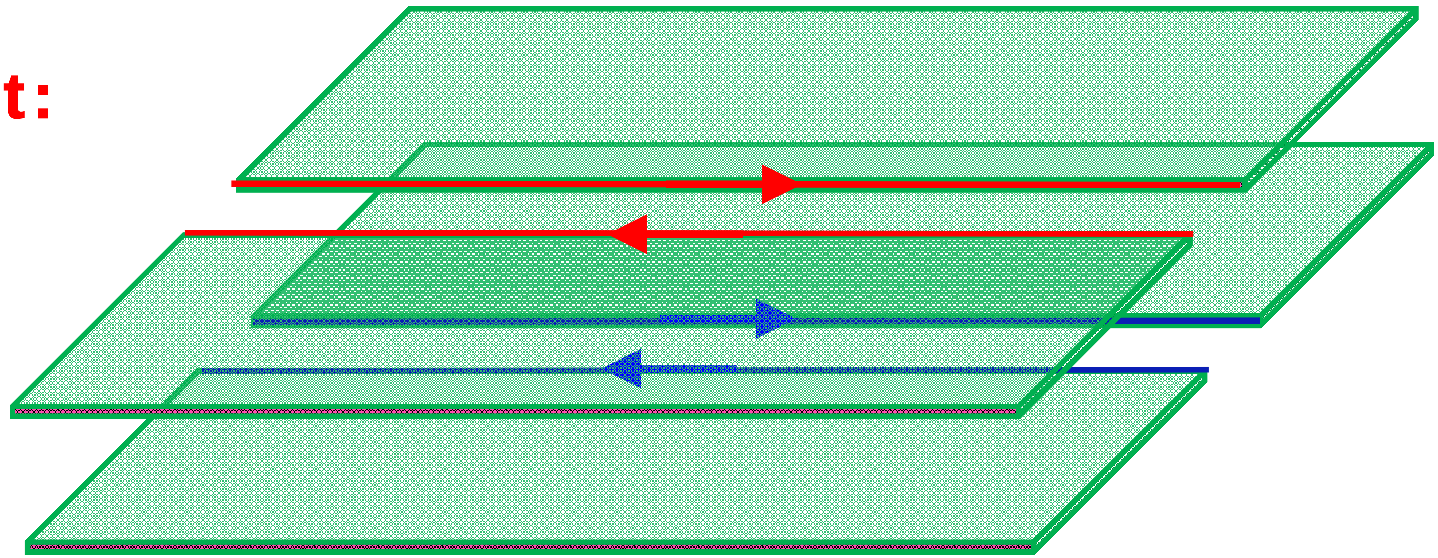


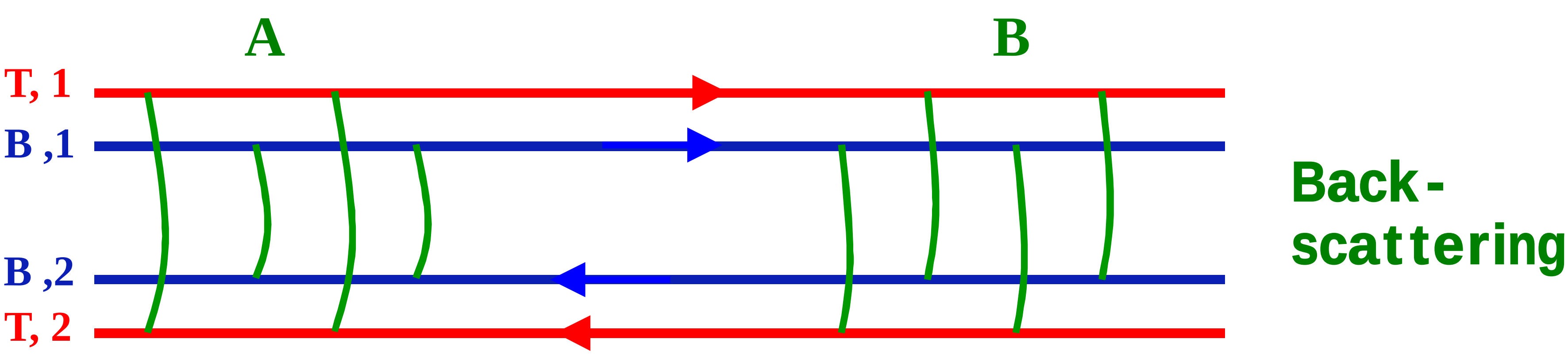
Fractionalized zero modes at “twist defects” in topological phases

Another example: ν=1/3 bilayer

Barkeshli, Jian, Qi (2013; 2014)

Cut:





Fractionalized zero modes at "twist defects" in topological phases

ν=1/3 bilayer

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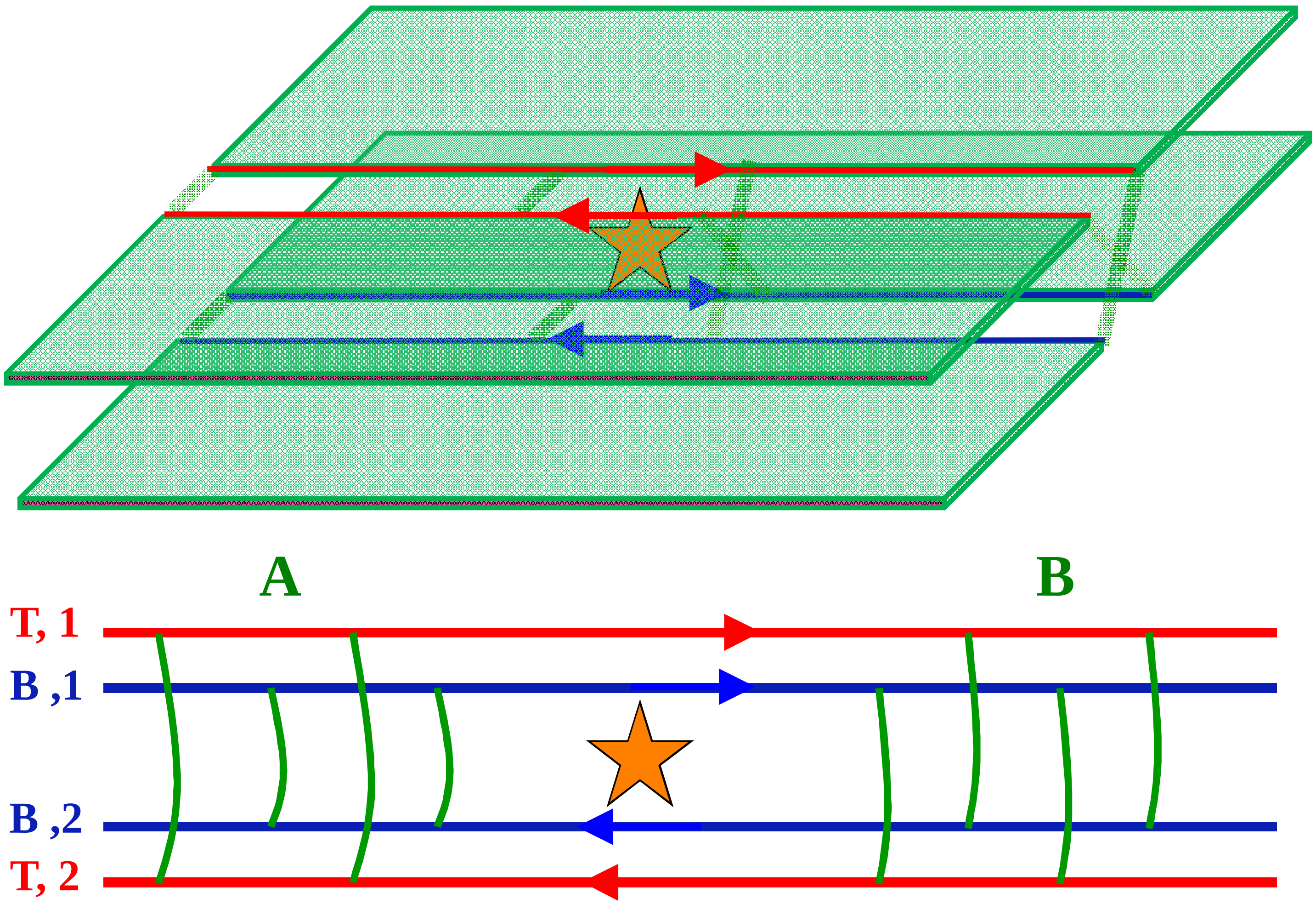
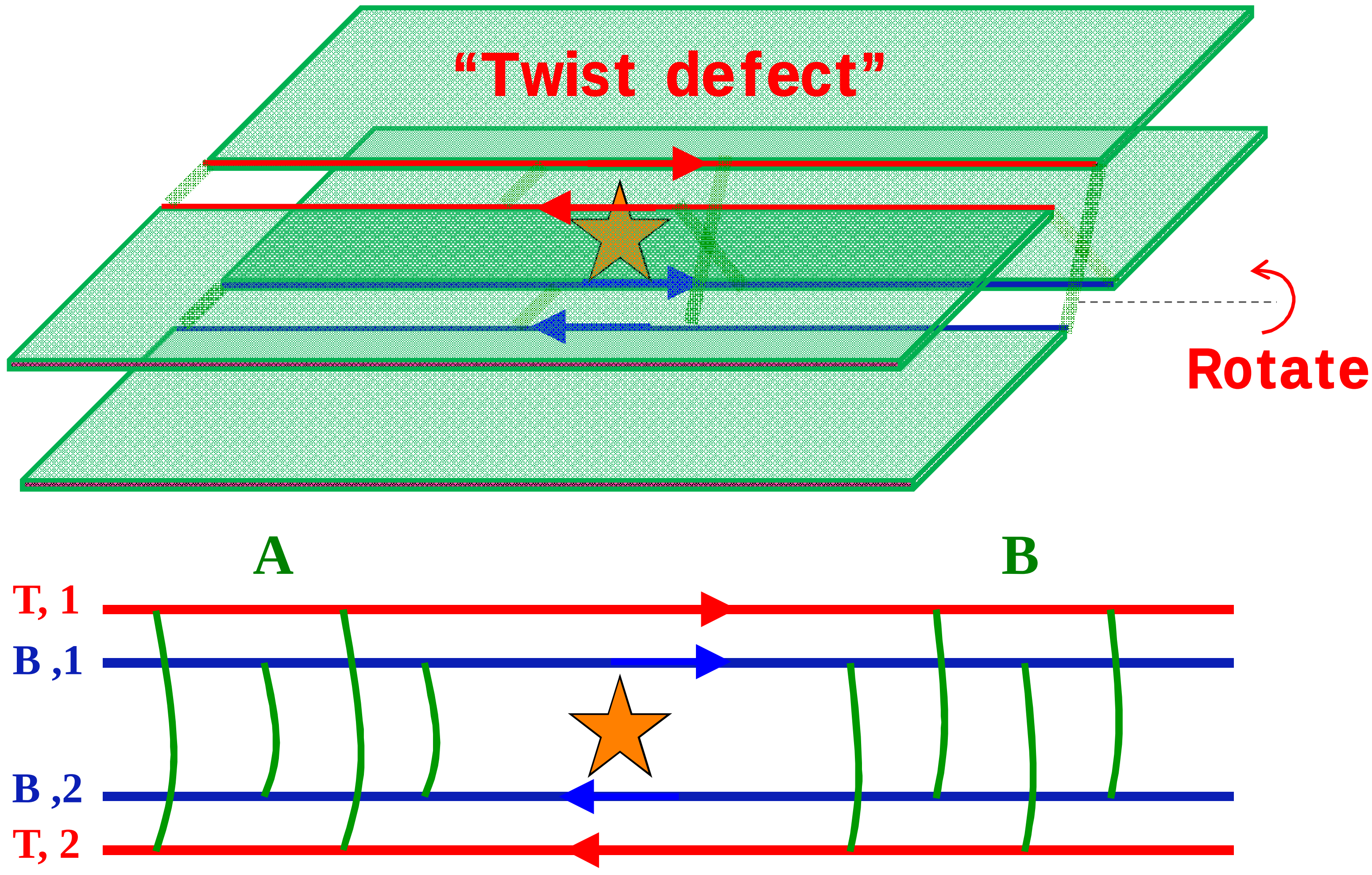


Fractionalized zero modes at "twist defects" in topological phases

$\nu = 1/m$ bilayer

Barkeshli, Jian, Qi (2013; 2014)



Fractionalized zero modes at "twist defects" in topological phases

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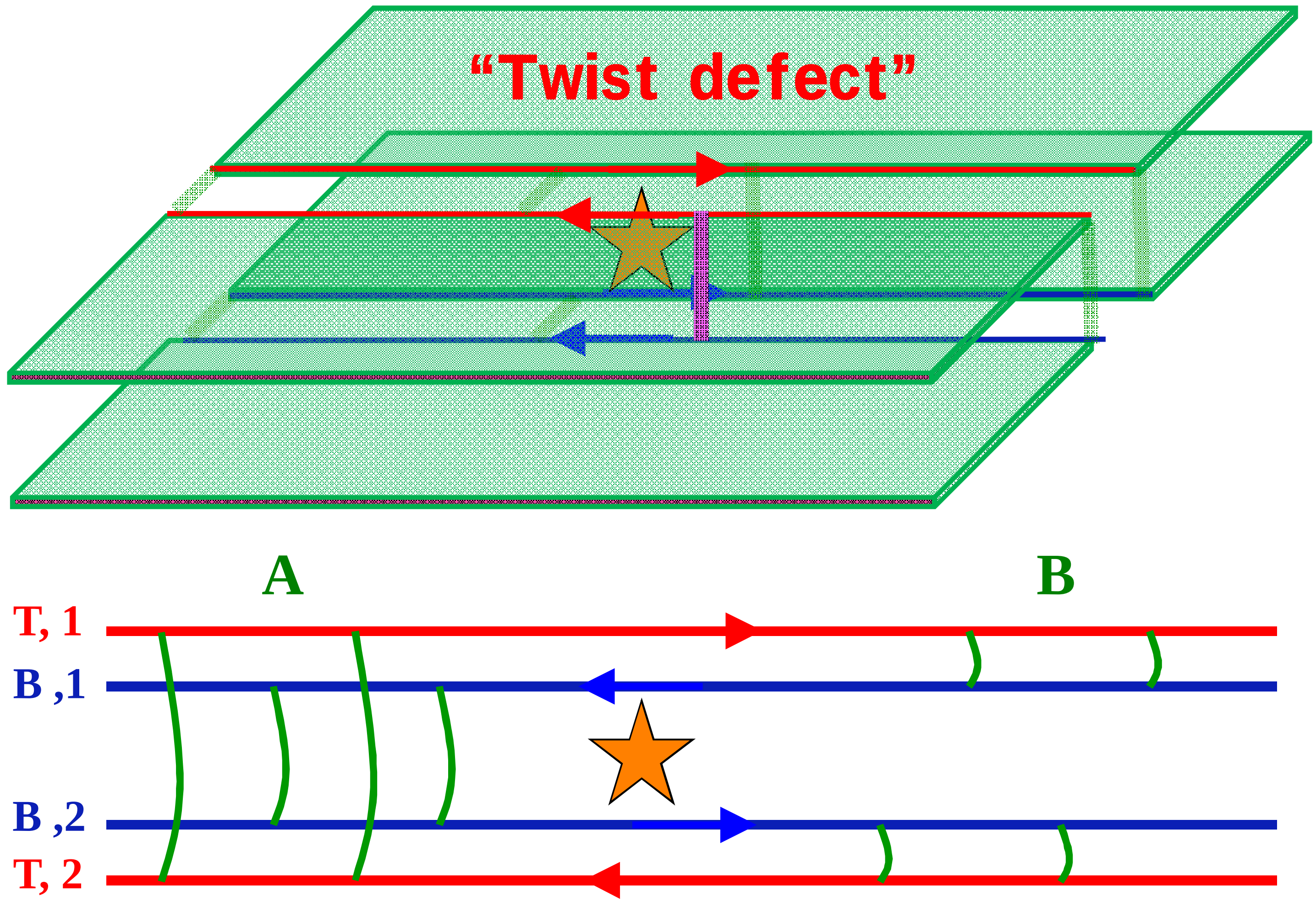
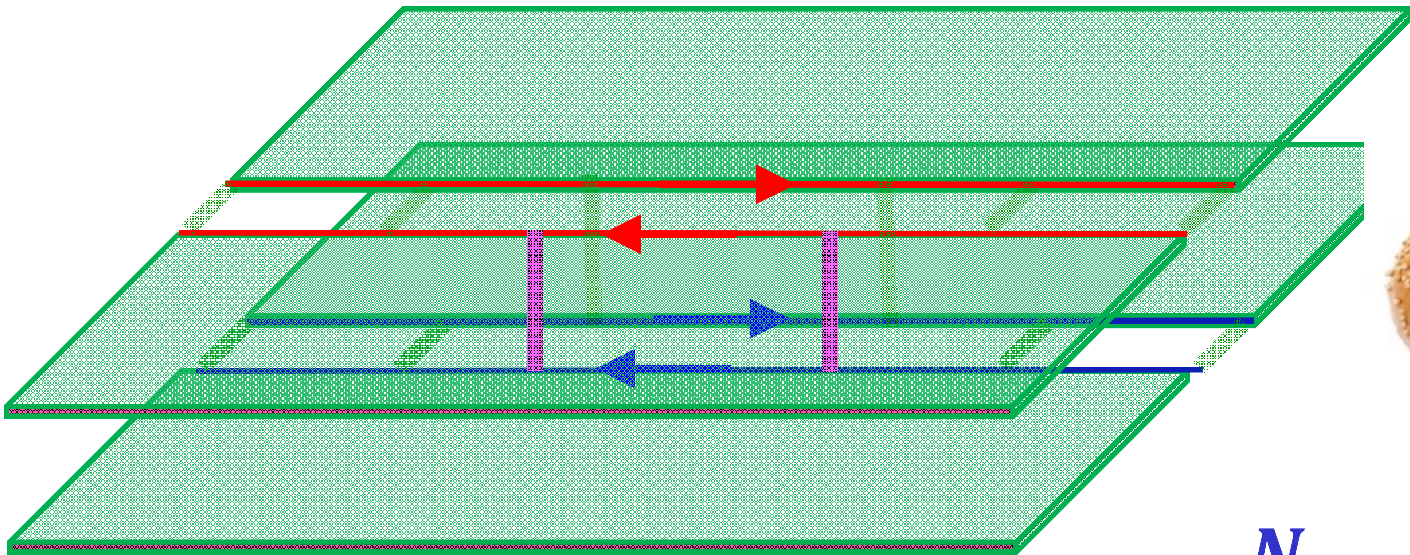


Fractionalized zero modes at "twist defects" in topological phases

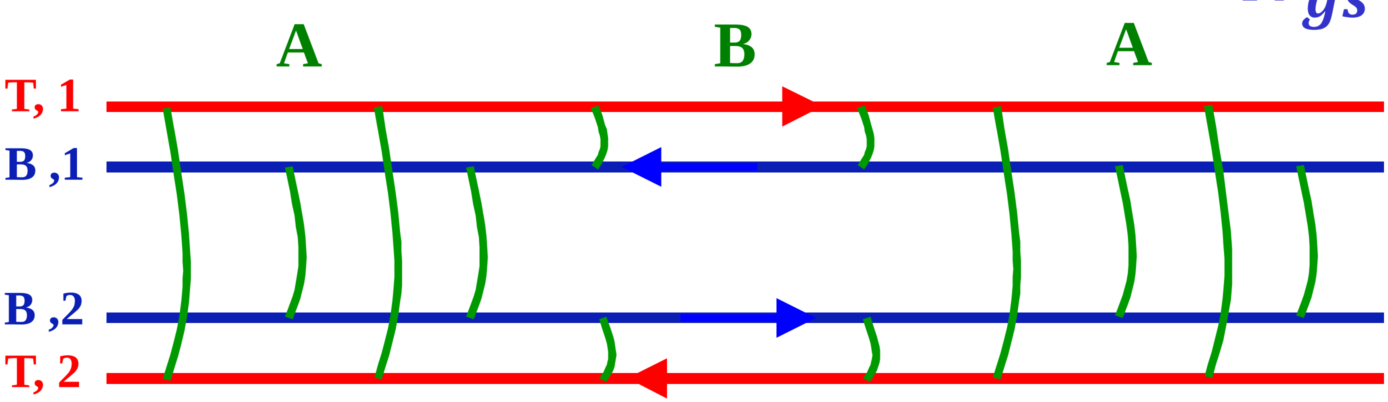
Alternate A,B domains:

Parafermions without superconductivity!



High genus surface

$$N_{gs} = m^{N_{holes}}$$



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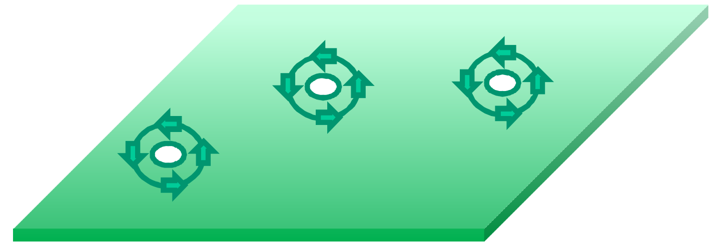
Ising anyons

$\nu = \frac{5}{2}$ **QHE**

$p_x + ip_y$ **Superconductors**

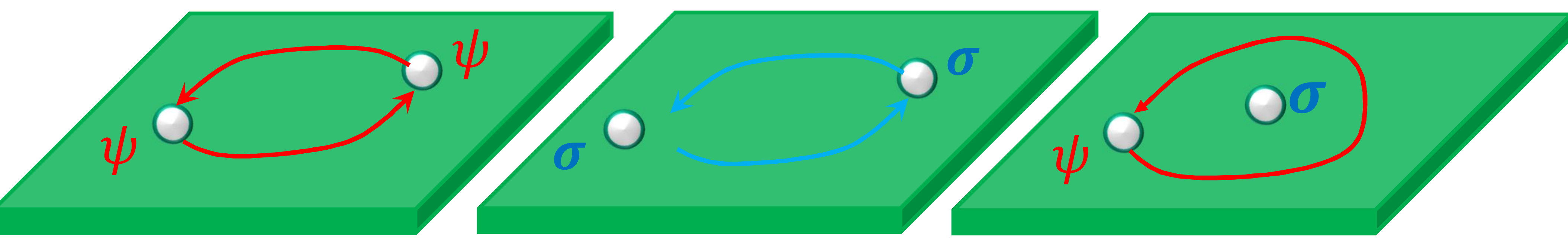
Kitaev's hexagonal spin model



Three types of particles: I (vacuum), ψ (fermion), σ (vortex)

Fusion rules: $\psi \times \psi = I \qquad \sigma \times \psi = I$

$$\sigma \times \sigma = I + \psi$$



$R^{I}_{\psi\psi} = -1$

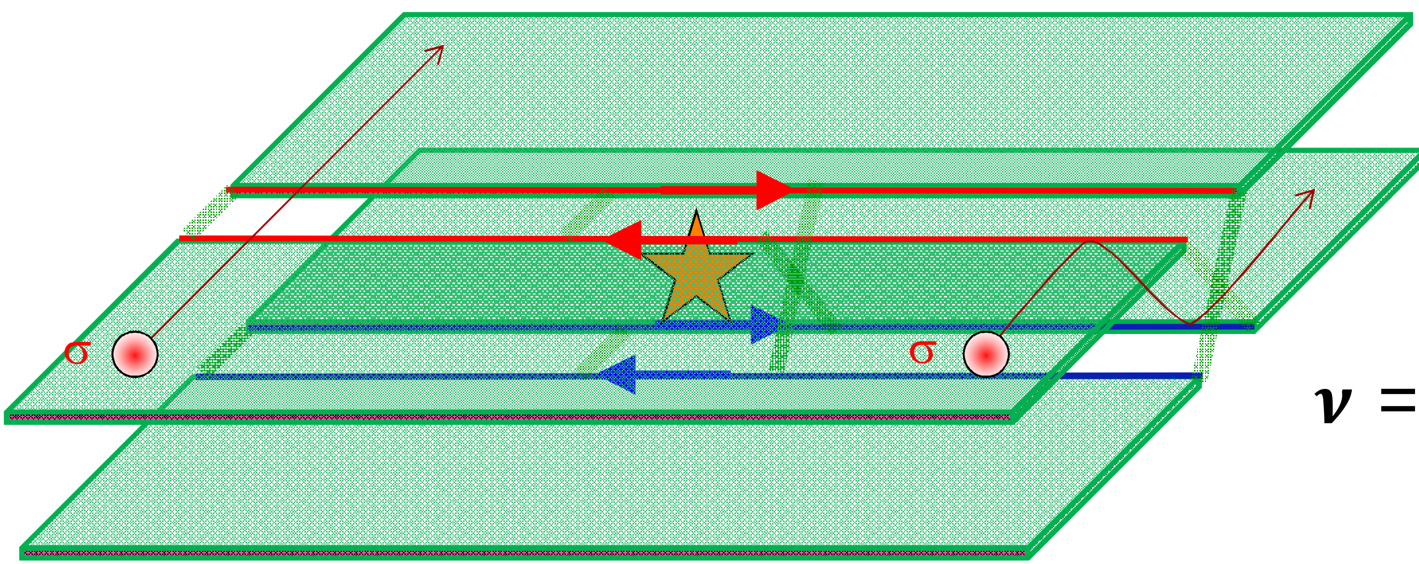
$R^{I}_{\sigma\sigma} = e^{-i\pi/8}$

$R^{\psi}_{\sigma\sigma} = e^{3i\pi/8}$

$R^{I}_{\psi\psi} = -1$

Defects in a bilayer Ising phase

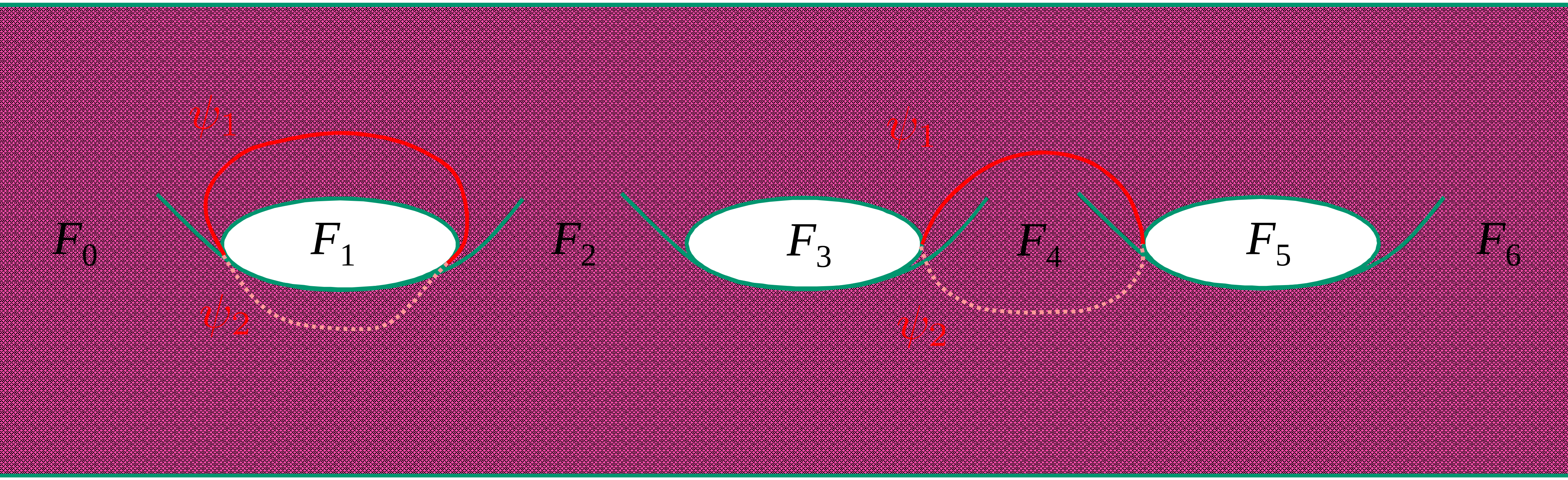
- **What is the mathematical description of the zero modes associated with the defects?**
- **Can the zero modes realize universal TQC even though the host Ising phase is not universal?**



Bilayer of $\nu = \frac{5}{2}$/Kitaev hex models/...

The Hilbert space

States can be described fluxes of holes, and measured by fermion loop operators

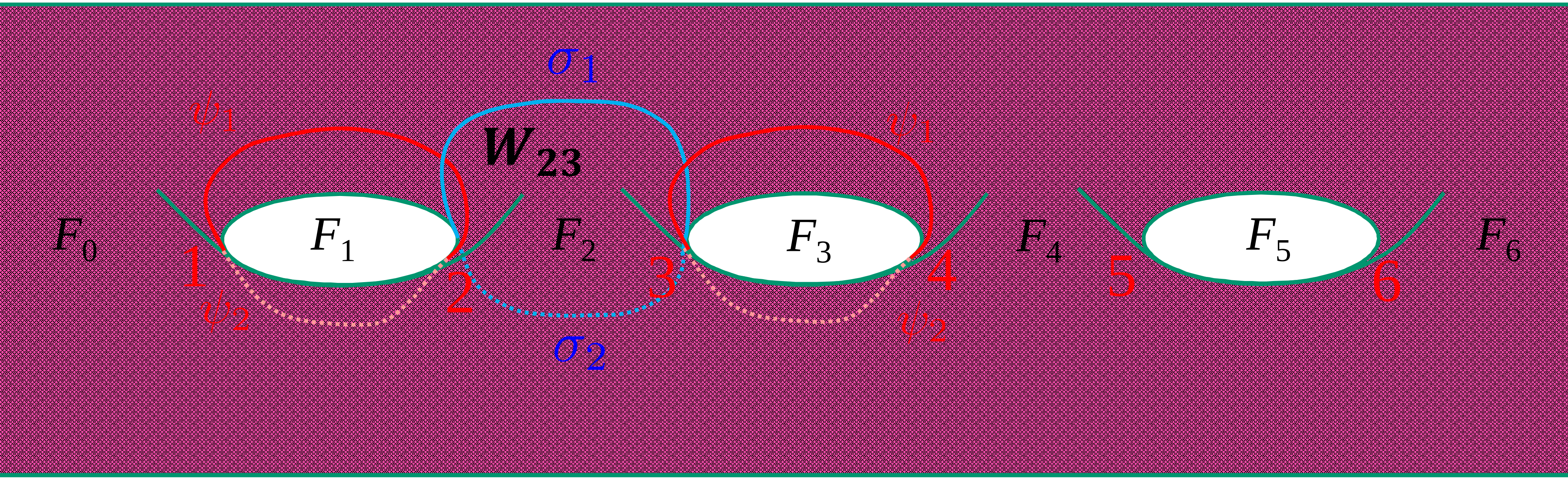


$F_j = 0, 1$ **Z_2 flux: represent as** $F_j = (1 + \sigma_j^z)/2$

Not all flux states are ground states

Creating flux states

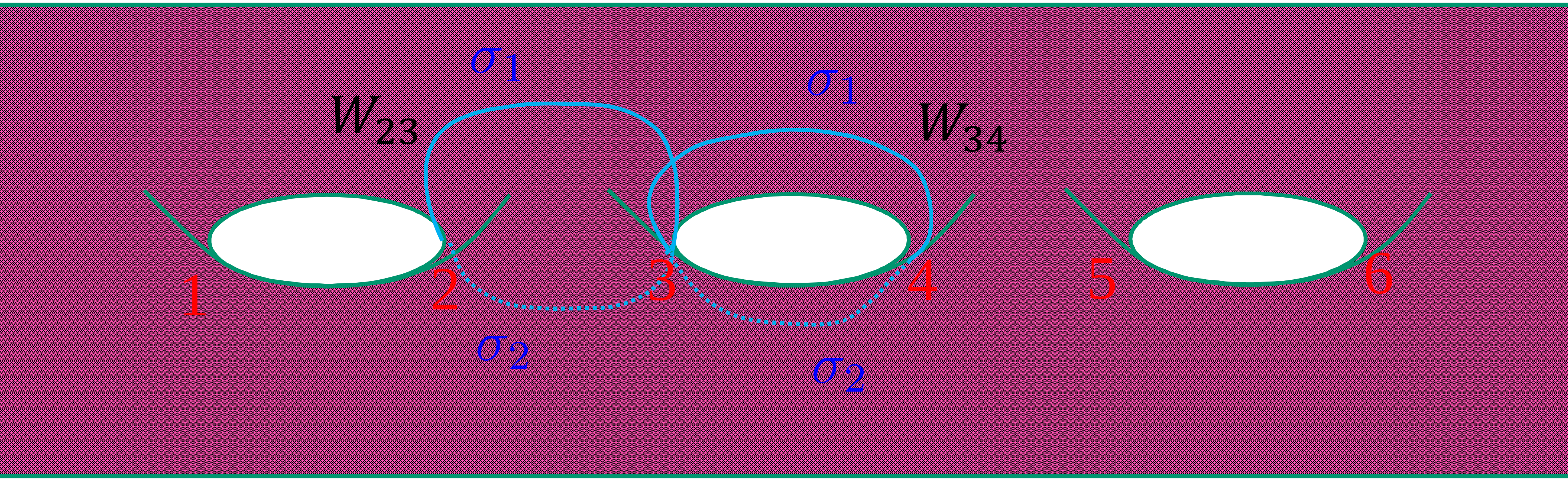
Flux states can be created by σ loops



σ loop operator W_{23} flips F_1 and F_3

Blocking rules

Final state has ψ excitation!

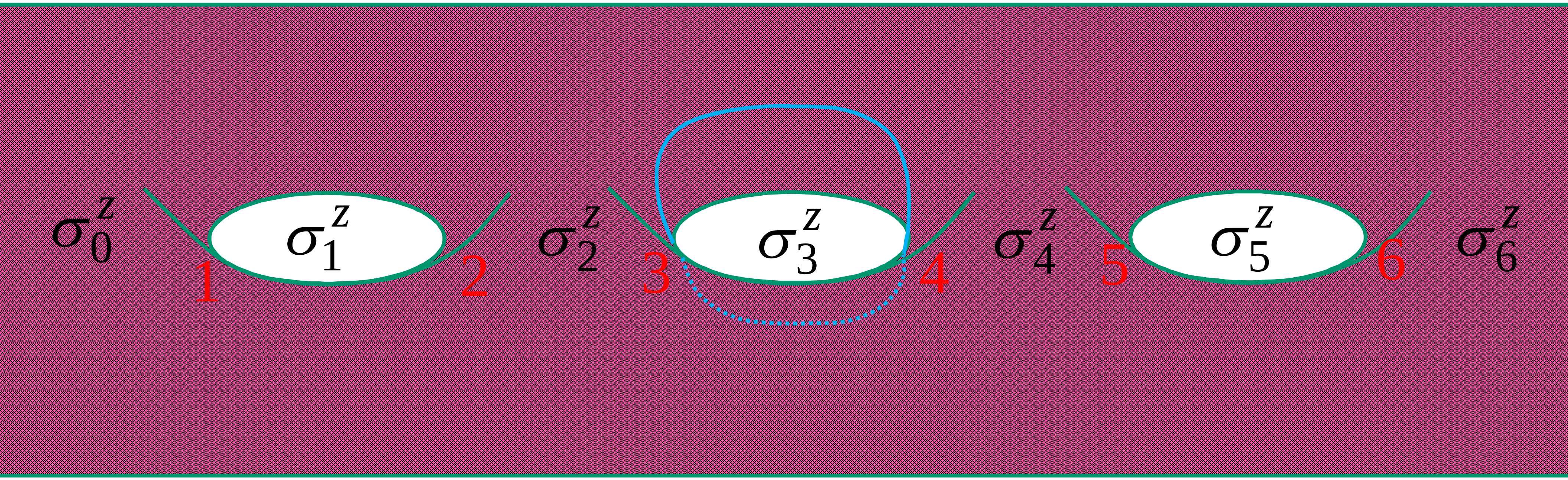


$$W_{23}^{-1}W_{34}^{-1}W_{23}W_{34} = 0$$

(projected to the ground state subspace)

Tunneling operators

Nearest neighbors: form in a **<u>convenient gauge</u>**:



$$W_{i,i+1} = \sigma_{i-1}^x \left(\frac{1+\sigma_i^z}{2} \right) \sigma_{i+1}^x$$

Hermitian
-not unitary
(projected)

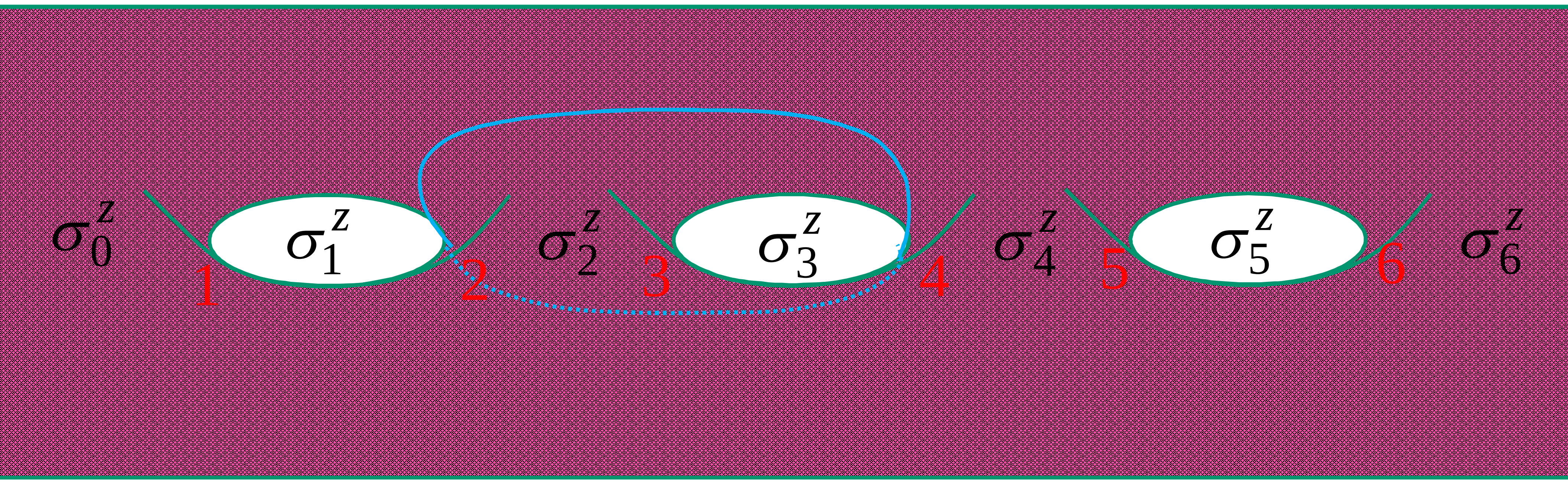
Tunneling operators

Tunneling of σ quasiparticles between zero modes: W_{mn}

Generalization of the parafermion algebra:

$$W_{mn} = e^{i\pi/8}\left(W_{mk}W_{kn} + h.c\right)$$

"tri-algebra"

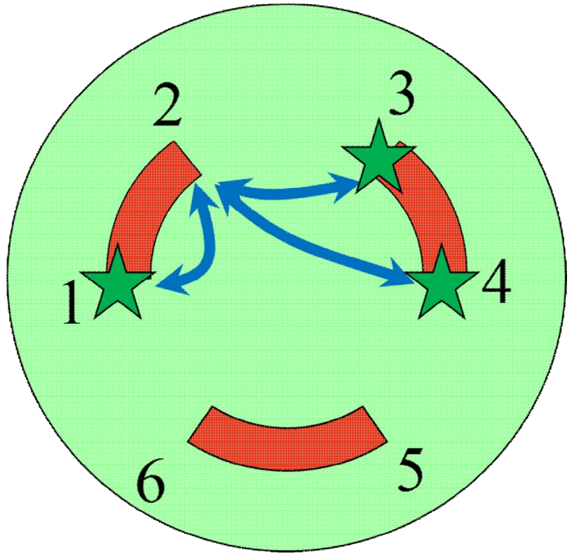


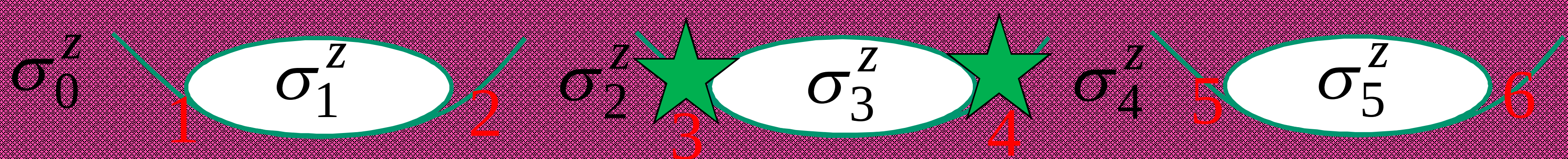
Braiding

EB, Lindner, Stern (in preparation)

$$U_{34} = \left(\frac{1+\sigma_3^z}{2}\right)\sigma_2^x\sigma_4^x + \left(\frac{1-\sigma_3^z}{2}\right)e^{i\pi/4}$$

Phase gate needed to make Ising theory universal!





Braiding implements a "dynamical topology change"

Freedman, Nayak, Walker (2006); Barkeshli, Jian, Qi (2013)

Chains of interacting defects

With Ari Turner, Netanel Lindner

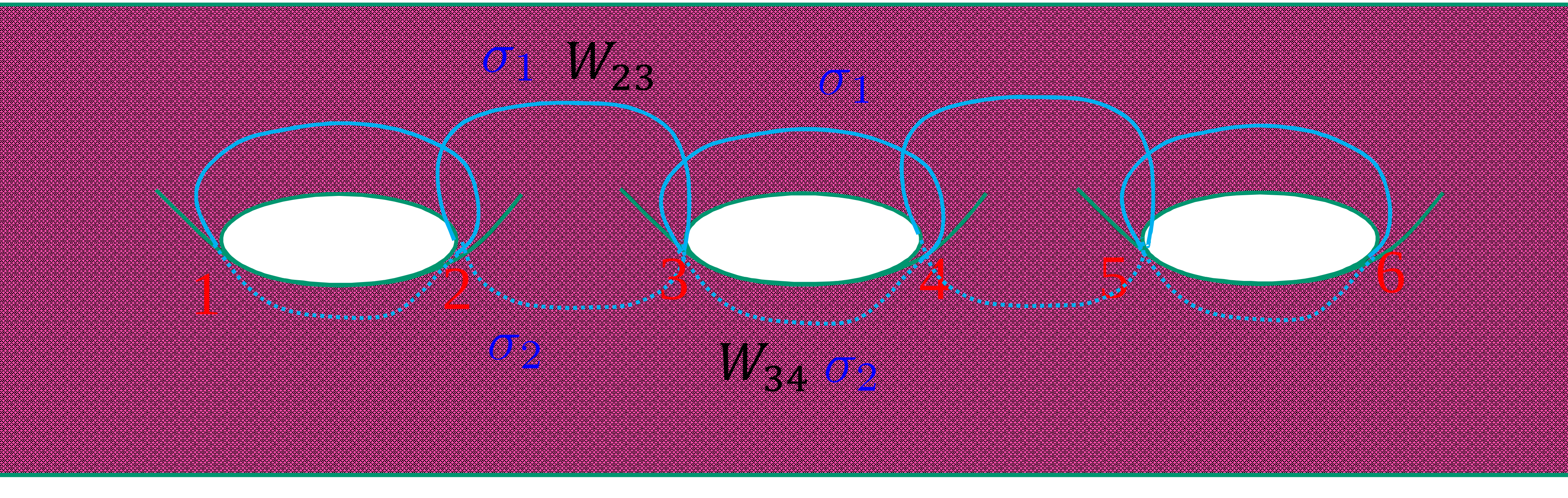
Chains of interacting defects

$$H = \sum_n J_n\, \sigma^x_{n-1} \frac{1+\sigma^z_n}{2} \sigma^x_{n+1}$$

Symmetries:

Z_2 flux conservation $\quad U_1 = \prod_n \sigma^z_{2n+1},\ U_2 = \prod_n \sigma^z_{2n}$

Fermion parity conservation $\quad P = e^{i\frac{\pi}{4}(\sigma^z_1 - \sigma^z_1\sigma^z_2 + \sigma^z_1\sigma^z_2\sigma^z_3 + \cdots)}$



Chains of interacting defects

$$H = \sum_n J_n \, \sigma^x_{n-1} \frac{1 + \sigma^z_n}{2} \sigma^x_{n+1}$$

Kramers-Wannier duality:

$$\tau^z_n \tau^z_{n-1} = -\sigma^z_n$$

$$\tau^x_n = \sigma^x_n \sigma^x_{n-1}$$

Symmetries become local!

$$A = e^{i\frac{\pi}{4}\sum_n \tau^z_n}, B = A = e^{i\frac{\pi}{2}\sum_n \tau^x_n}$$

D_4 group: $A^4 = 1$, $BA = A^{-1}B$

$$H = \sum_n \frac{J_n}{2} (\tau^x_n \tau^x_{n+1} + \tau^y_n \tau^y_{n+1})$$

Conclusions

In Abelian phases they harbor parafermion (fractional Majorana) zero modes.

In a non-Abelian Ising phase they realize new zero modes that enrich the non-Abelian statistics of the host phase.

Pure Ising anyons (Kitaev spin model) + defects: universal for TQC.

Bilayer of $\nu = \frac{5}{2}$ is not, however… Other physical realization? (E.g. $p + ip$ SC, networks of Kitaev chains: Barkeshli, Sau, 2015)

Thank you.