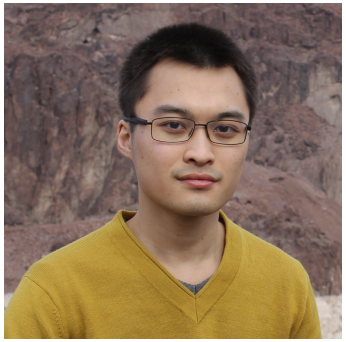
Twisted Gauge Theory in 3D Walker Wang Model

Xie Chen, Caltech, Oct. 2016

Joint work with Zitao Wang, to appear



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What 3D topological phases are there?

- Gapped
- Finite ground state degeneracy
- Fractional excitations in the bulk

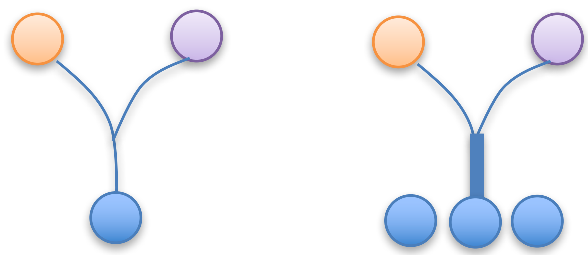
We know many topological phases in 2D, but not so many in 3D

2D topological phases

- Fractional quantum Hall
- Toric code (quantum double) Kitaev, 2003
- String-net models Levin, Wen, 2005

2D topological phases

- Fractional point excitations – anyons
- Fusion

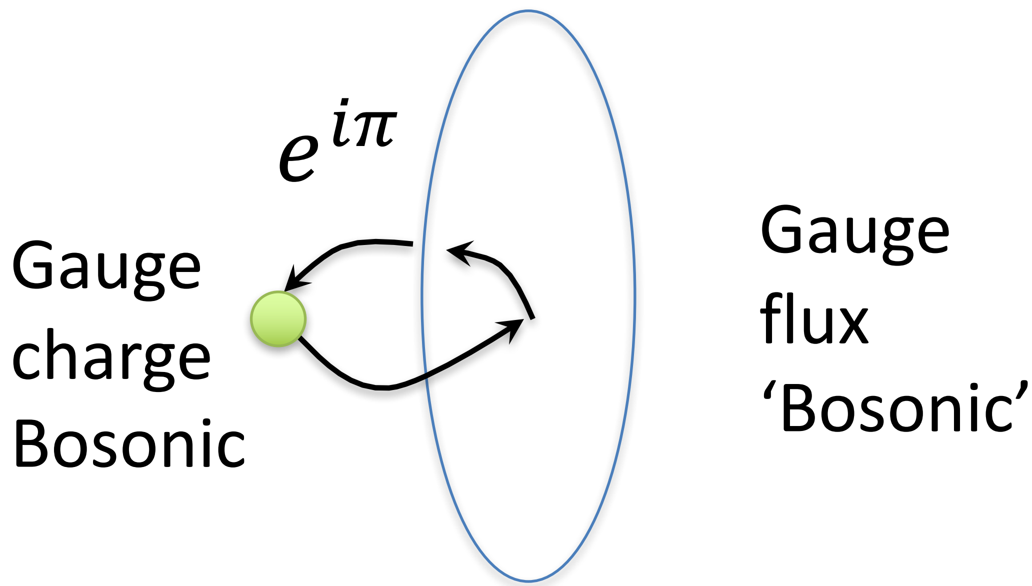


- Braiding

- Braided fusion category

3D?

- Toric Code – 3D Z_2 gauge theory

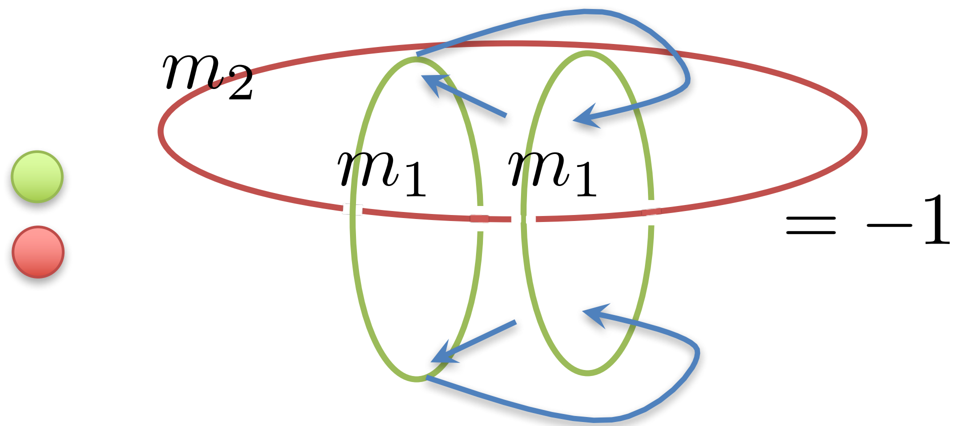


- 3D discrete gauge theory – Aharonov Bohm effect

3D twisted gauge theory

- $Z_2 \times Z_2$ gauge theory (for example)

Bosonic Gauge Charge



Anyonic Gauge Flux

- Dijkgraaf-Witten gauge theory, 1990
  - Bosonic gauge charge
  - Anyonic gauge flux

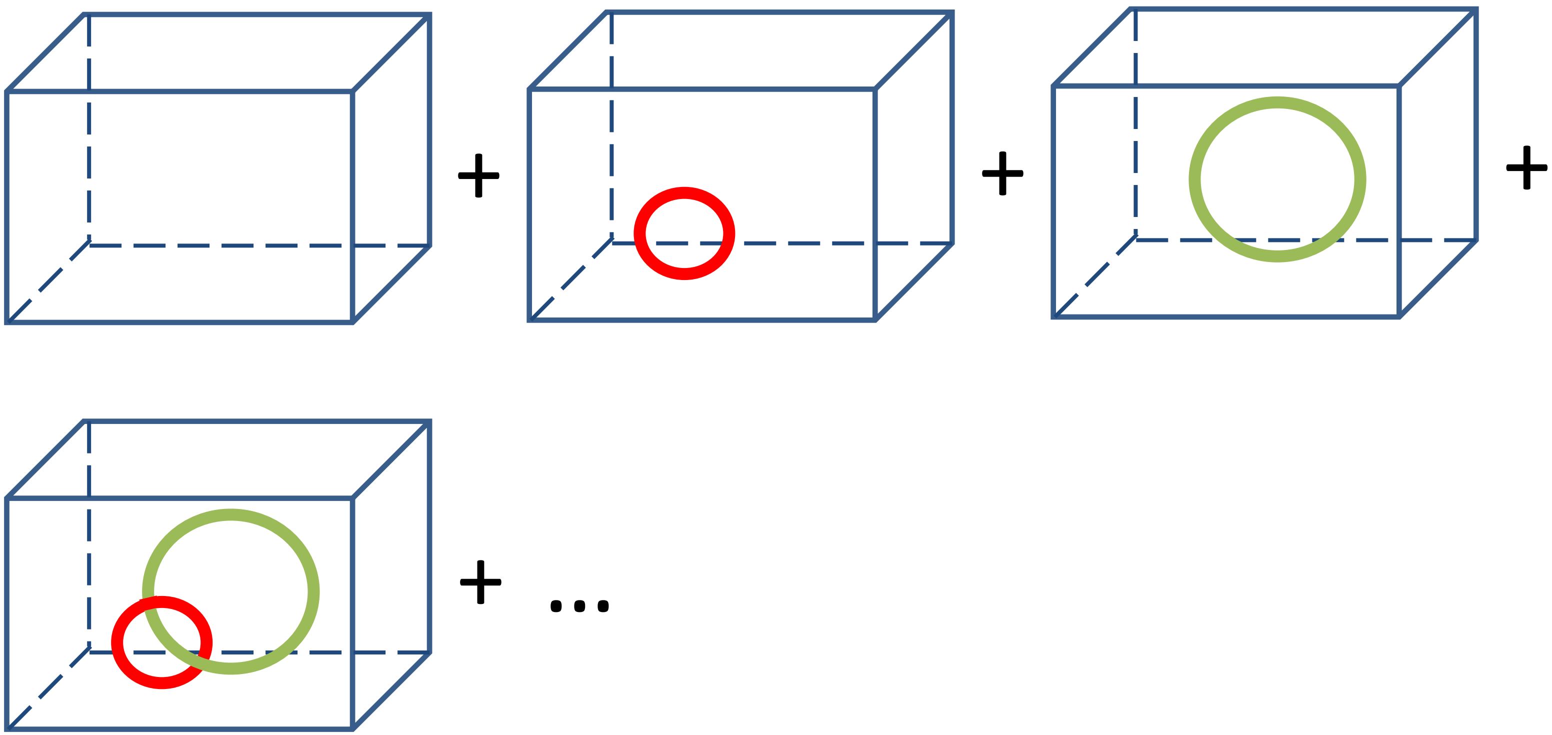
Wang, Levin, 2014; Jiang, Mesaros, Ran, 2014; Wang, Wen, 2015

3D topological phase

- A limited variety compared to 2D
- What are the phase that we do not know?
- What twisted gauge theory with fermonic gauge charge are there?
- Generalize Dijkgraaf Witten to include fermions
- Insight from a very different construction by Walker and Wang, 2011

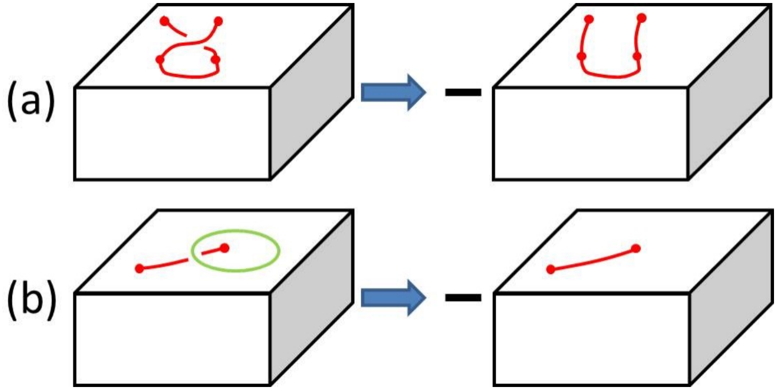
Walker Wang model

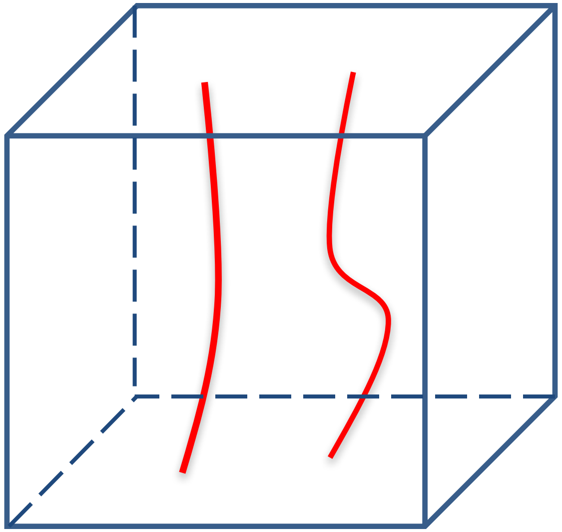
- Wave function represents space time trajectory of 2D anyons



Walker Wang model

- Input: a set of 2D anyons A, their fusion and braiding rule
- Output: a 3D gapped exactly solvable lattice model, with A appearing as fractional excitation on the surface

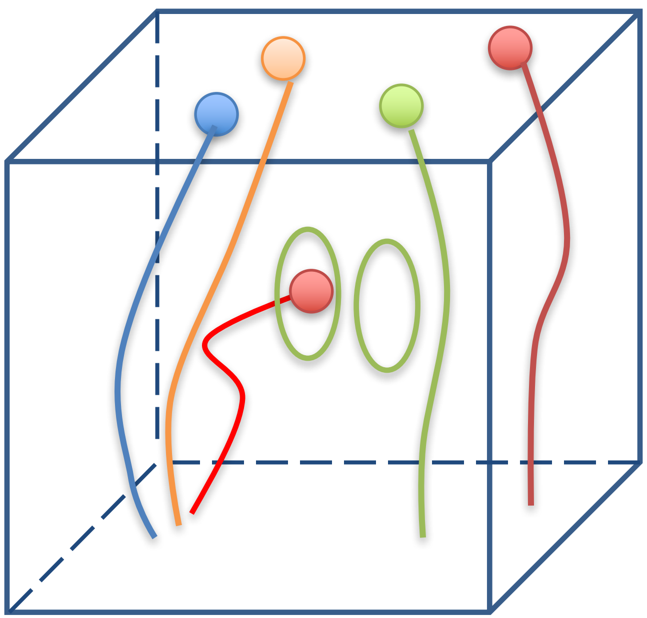




- What is the order in the bulk?

Walker Wang model

- If A is modular (every anyon can be detected from braiding), the 3D bulk has trivial topological order
- If A is non-modular (a subset C cannot be detected from braiding), the 3D bulk has nontrivial topological order, with quasi-particle excitation in C



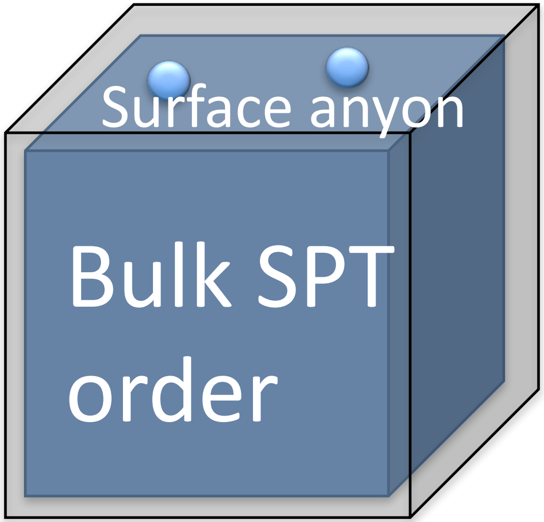
Walker Wang model

- Example I: $A = I, e; C = I, e.$ e is a boson
  - Bulk order: 3D Z2 gauge theory
  - Bulk quasiparticle: $C = I, e$.
  - Surface order: $A = I, e$.
- Example II: $A = I, s; C = I.$ s is a semion
  - Bulk order: trivial
  - Bulk quasiparticle: $C = I$.
  - Surface order: $A = I, s$.

Keyserlingk, Burnell, and Simon, 2012

Walker Wang model

- Ways WW model has been used:
  - Exactly solvable model of fractional quantum Hall (on the surface) Walker, Wang, 2011
  - Surface topological order of symmetry protected topological phase Burnell, Fidkowski, XC, Vishwanath, 2013-14
- Can have fractionalized bulk, quasi-particle content easy to determine

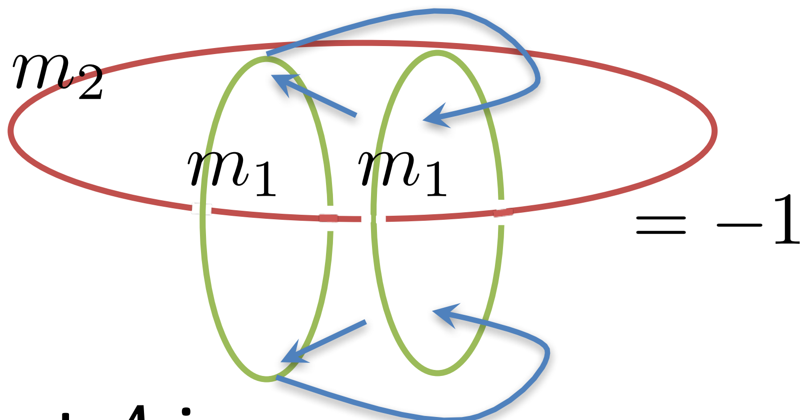


Walker Wang model

- What are the flux loops doing in the 3D bulk?
- Do they have nontrivial loop braiding statistics?
- Can they represented Dijkgraaf Witten twisted gauge theory?
- Yes

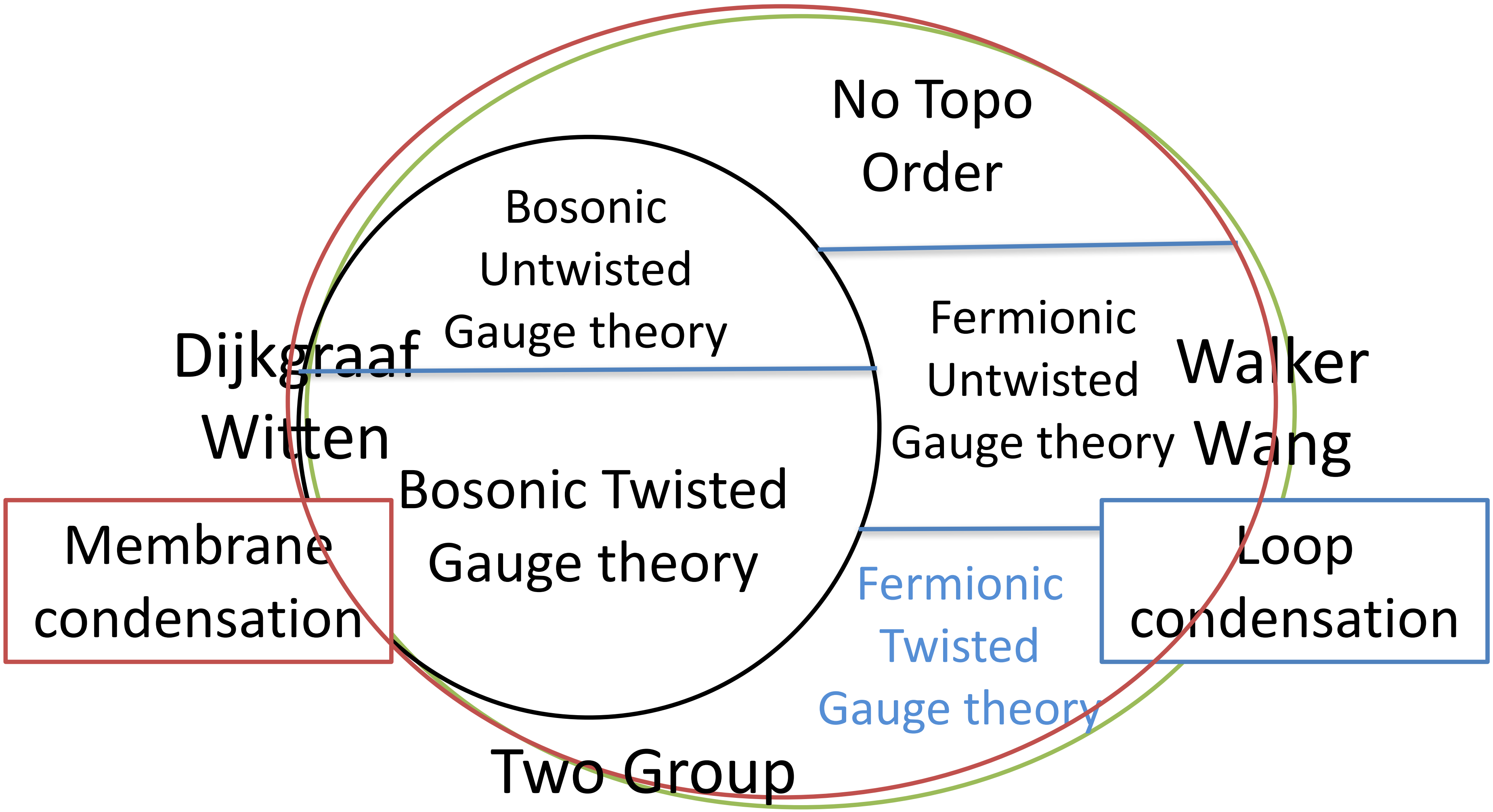
Summary of result

- The set of input 2D anyons such that the bulk has twisted $Z_2 \times Z_2$ gauge theory topo order



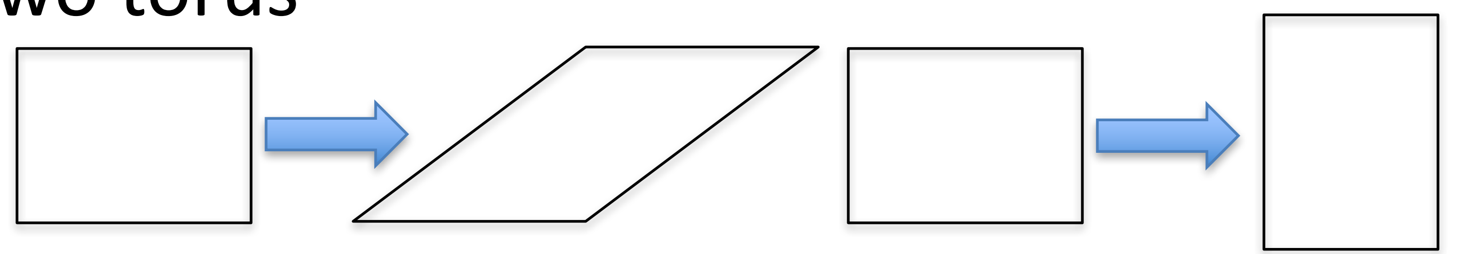
- What the input set A is
- How we demonstrate the twisted topo order
  - Loop braiding statistics from membrane operators
  - Modular transformation

Summary of Result



Modular transformation

- Large gauge transformation of the manifold the system lives on
- The corresponding transformation of the degenerate ground space.
- 2D, two torus

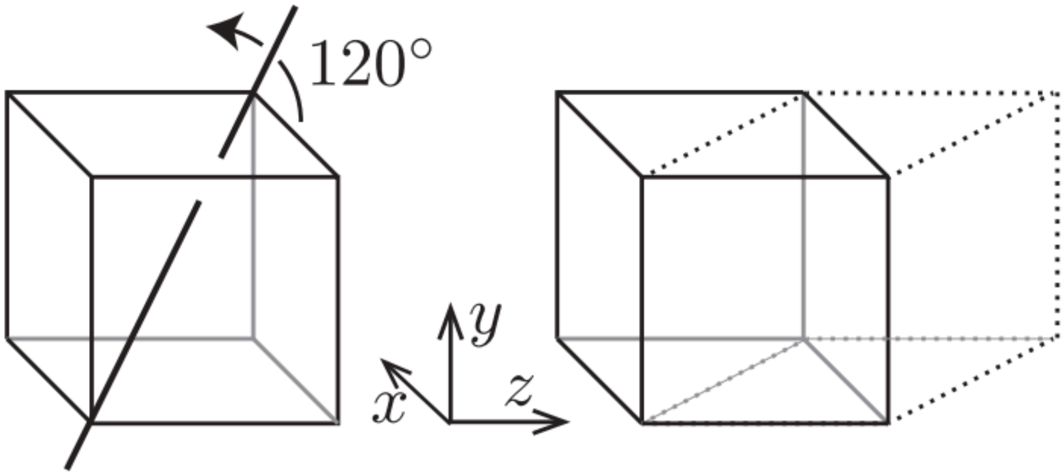


- Projective rep of $SL(2,Z)$
- Distinguishes different 2D topological order

Wen, 1990

3D modular transformation

- 3D, three torus



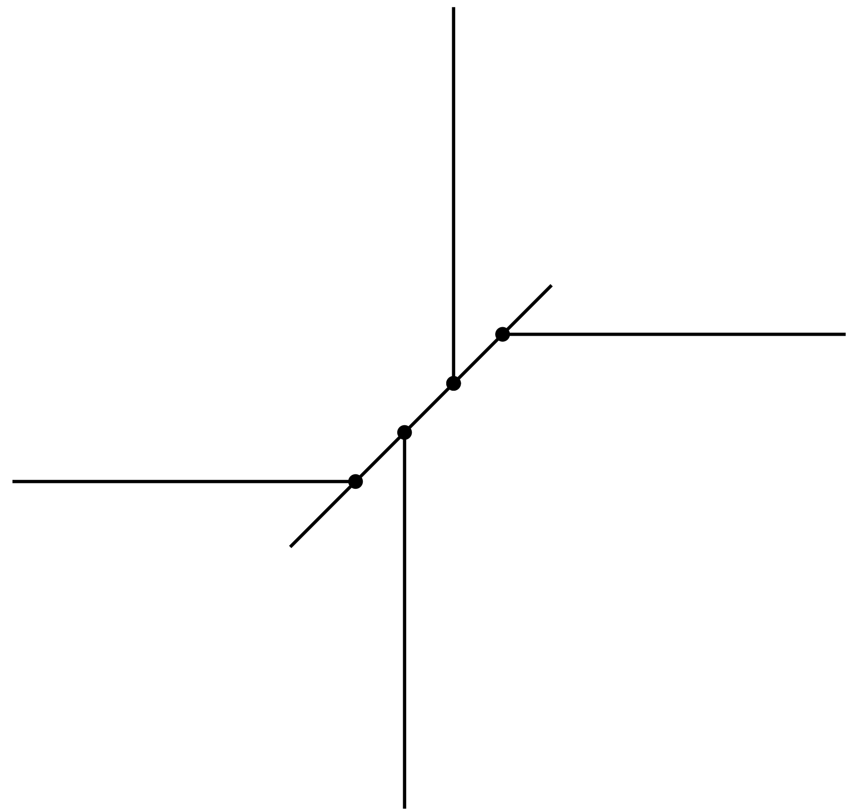
S T_{xz}

- Data calculated based on Dijkgraaf Witten theory for simple twisted gauge theories

Jiang, Mesaros, Ran, 2014; Wang, Wen, 2015; Wan, Wang, He, 2015

Mod. Trans. on WW model

- Minimal lattice model of Walker Wang
- Trivalent
- 3 torus
- 6 spins
- Following the 2D string-net procedure
  Liu,Wang,You,Wen,2013



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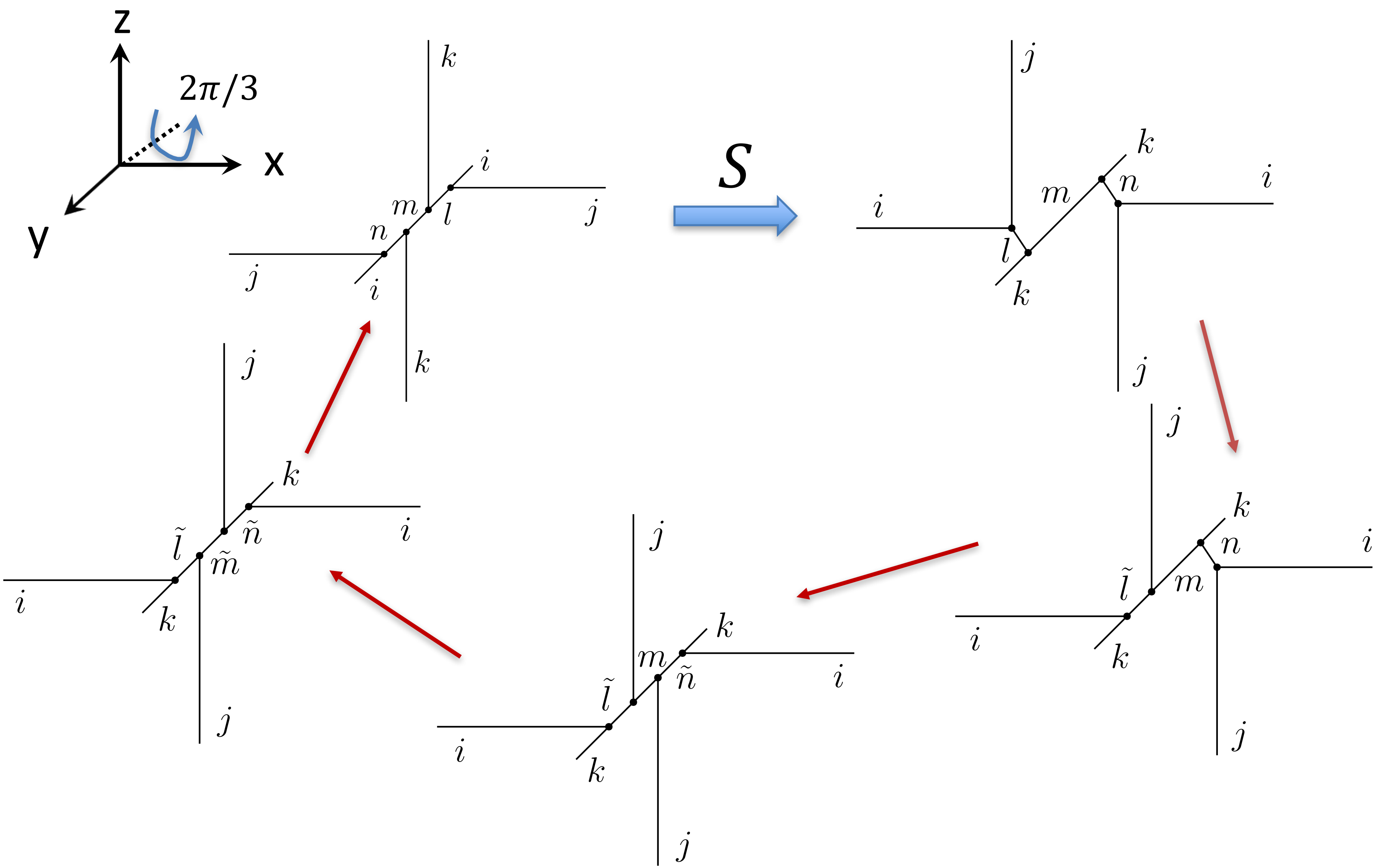

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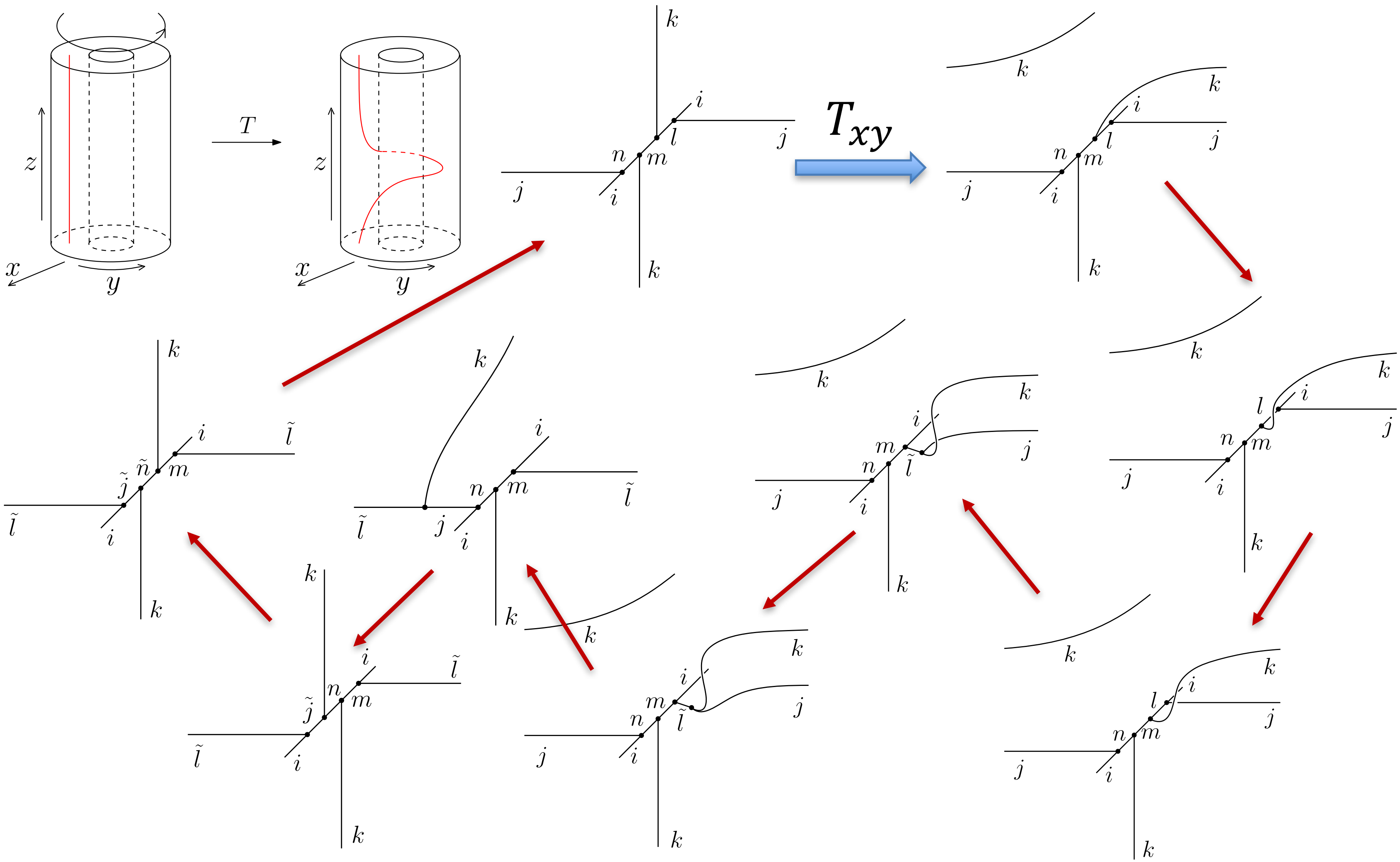
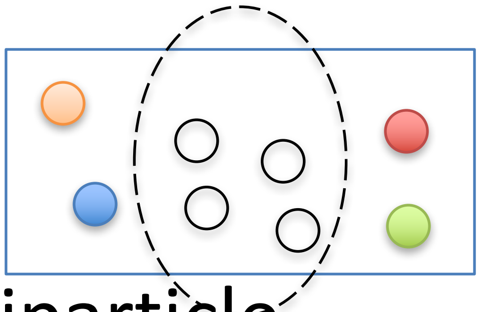


Mod. Trans. on WW model

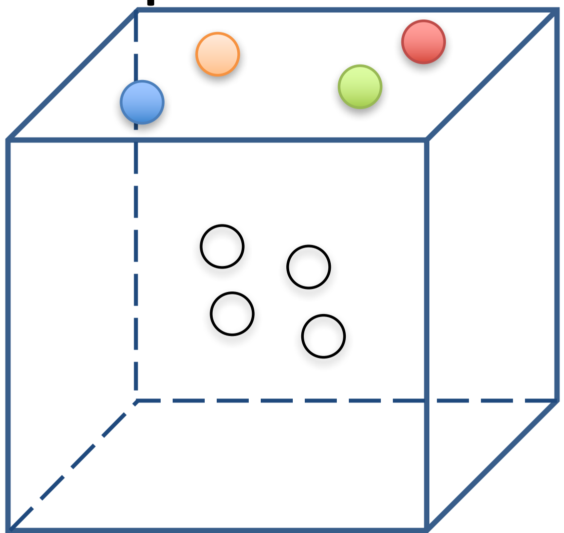
- S and T matrices thus calculated match that of Z_2 x Z_2 Dijkgraaf Witten twisted gauge theory
- Confirming the topological order
- What is the input set of 2D anyons A ?

The input

- General rule:
- Input: set of anyons A, with a central subset C (does not braid with everything)



- Output: 3D theory with C as quasiparticle excitations in the bulk



The input

- $A = I, e; C = I, e$. e is a boson
  - 3D Bulk quasiparticle content: $C = I, e$
  - 3D Bulk topo order: Z_2 gauge theory (toric code)
- $A = I, e_1, e_2, e_1 e_2; C = A$. e_1, e_2 are bosons
  - 3D Bulk q.p. content: $C = I, e_1, e_2, e_1 e_2$
  - 3D Bulk topo order: Z_2 x Z_2 gauge theory (two copies of toric code)
- This is too simple; we need something more 'twisted'

The input we choose

- Take the irreducible representations of the D_8 group (symmetry of a square)
- $I, e_1, e_2, e_3, (1D)\ \Omega\ (2D)$
- $e_1 \times e_2 = e_3, \Omega \times e_i = \Omega$
- $\Omega \times \Omega = I + e_1 + e_2 + e_3$
- I, e_1, e_2, e_3, Ω can all be self and mutual bosonic
- $C = I, e_1, e_2, e_3, \Omega$
- 3D bulk order: D_8 gauge theory (not what we want)

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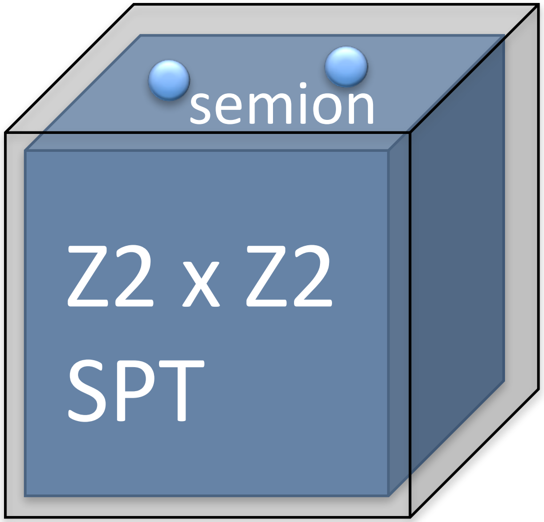
- $\Omega \times \Omega = I + e_1 + e_2 + e_3$
- Make Ω a semion, consistent with fusion rule
- Ω braids with itself, not a 3D quasi-particle
- 3D bulk order: Z_2 x Z_2 gauge theory and twisted!

In comparison

- Take the irreducible representations of the Q_8 group (the quaternion group)
- $I, e_1, e_2, e_3, (1D)\ \Omega\ (2D)$
- $e_1 \times e_2 = e_3, \Omega \times e_i = \Omega$
- $\Omega \times \Omega = I + e_1 + e_2 + e_3$
- Make Ω a semion, consistent with fusion rule
- Ω braids with itself, not a 3D quasi-particle
- 3D bulk order: Z_2 x Z_2 gauge theory and NOT twisted!

Where this is from

- Surface topological order of Z_2 x Z_2 symmetry protected topological phases
- A semion Ω transforms under Z_2 x Z_2 symmetry as $g_x = i\sigma_x$, $g_y = \sigma_y$, $g_z = \sigma_z$
- 2D projective rep of Z_2 x Z_2
- 2D regular rep of D_8



XC, Burnell, Vishwanath, Fidkowski, 2015

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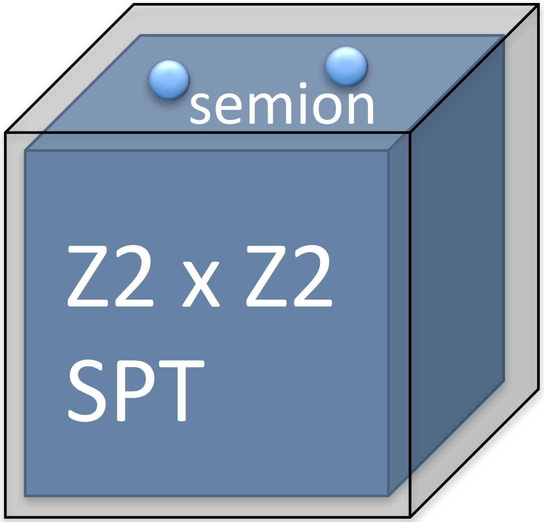
- Gauge Z_2 x Z_2, symmetry charges become frac. Excitation
- $\Omega \times \Omega = I + e_1 + e_2 + e_3$
- Bulk twisted Z_2 x Z_2 gauge theory



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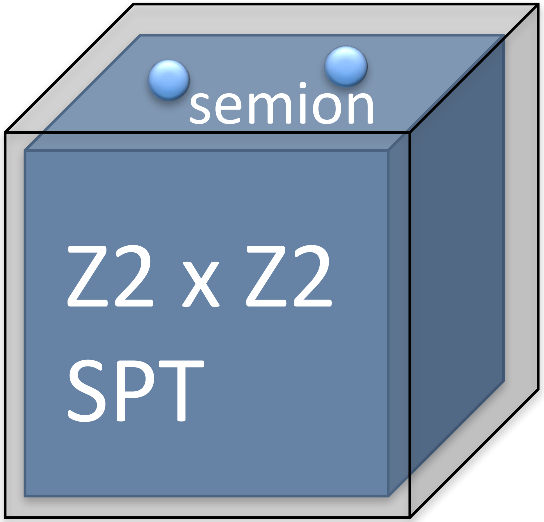

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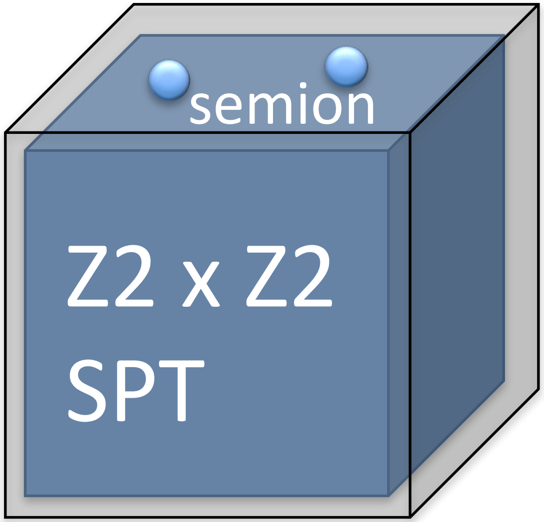
- A semion Ω transforms under Z_2 x Z_2 symmetry as $g_x = i\sigma_x$, $g_y = i\sigma_y$, $g_z = i\sigma_z$
- Gauge Z_2 x Z_2 , symmetry charges become frac. Excitation
- $\Omega \times \Omega = I + e_1 + e_2 + e_3$
- Bulk un-twisted Z_2 x Z_2 gauge theory



XC, Burnell, Vishwanath, Fidkowski, 2015

What we learned

- Walker Wang model can represent 3D gauge theory
- WW model can represent 3D twisted gauge theory
- WW model can have nontrivial loop braiding statistics (not directly shown)
- Modular transformation as a generic way to extract topological order of WW model

What we learned

- To find twisted gauge theory with fermion gauge charge
- Start from a set of anyons A, s.t.
- A contains a central subset C
- C includes all the irreps of gauge group G
- Some of the irreps are fermion, others are boson
- $A > C$, and probably has to be nonabelian