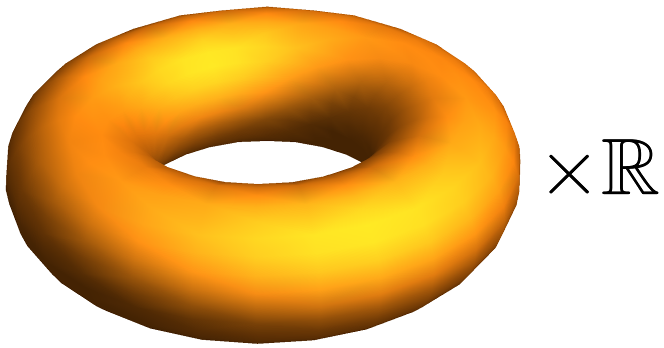
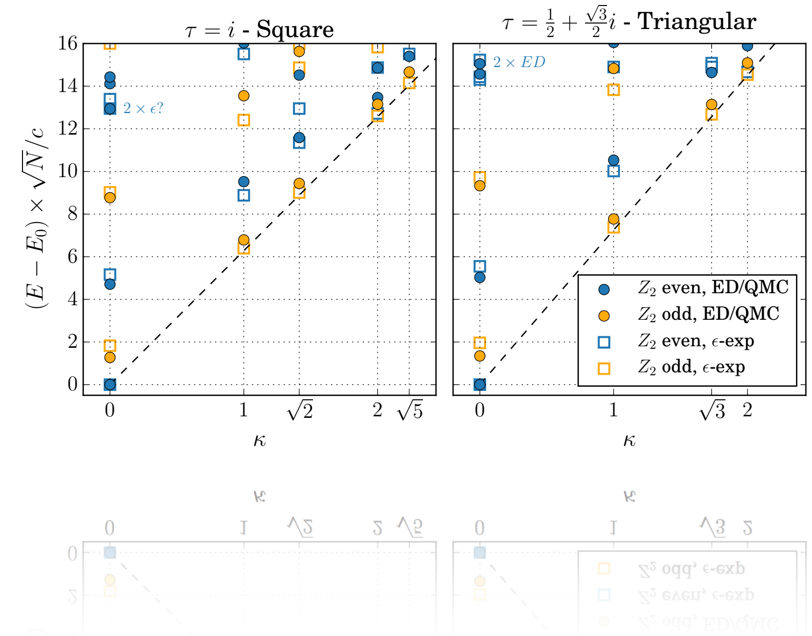


$\mathbb{T}^2 \times \mathbb{R}$



Spectroscopy of Conventional and Unconventional 2+1D Quantum Critical Points

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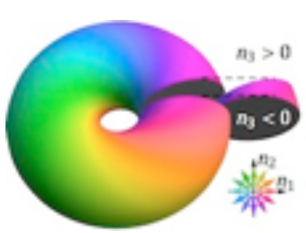
M. Schuler, S. Whitsitt, L.-P. Henry, S. Sachdev & AML
arXiv:1603:03042
to appear in PRL

L.-P. Henry, M. Schuler and AML
in preparation



“Topological Quantum Matter” Conference, KITP, 10/19/2016

Outline of this talk

- Torus Energy Spectra and CFT ?
- Spectrum of the standard 2+1D Ising transition (Ising)

  M. Schuler, S. Whittsitt, L.-P. Henry, S. Sachdev & AML
  arXiv:1603:03042
- Spectrum of the "Z_2 confinement" transition (Ising*)

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- Spectrum of the 3D XY* Transition

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- Outlook

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Operator Spectrum

- A local operator has a scaling dimension:

$$\mathcal{O}_i \rightarrow \Delta_i = \text{scaling dimension}$$

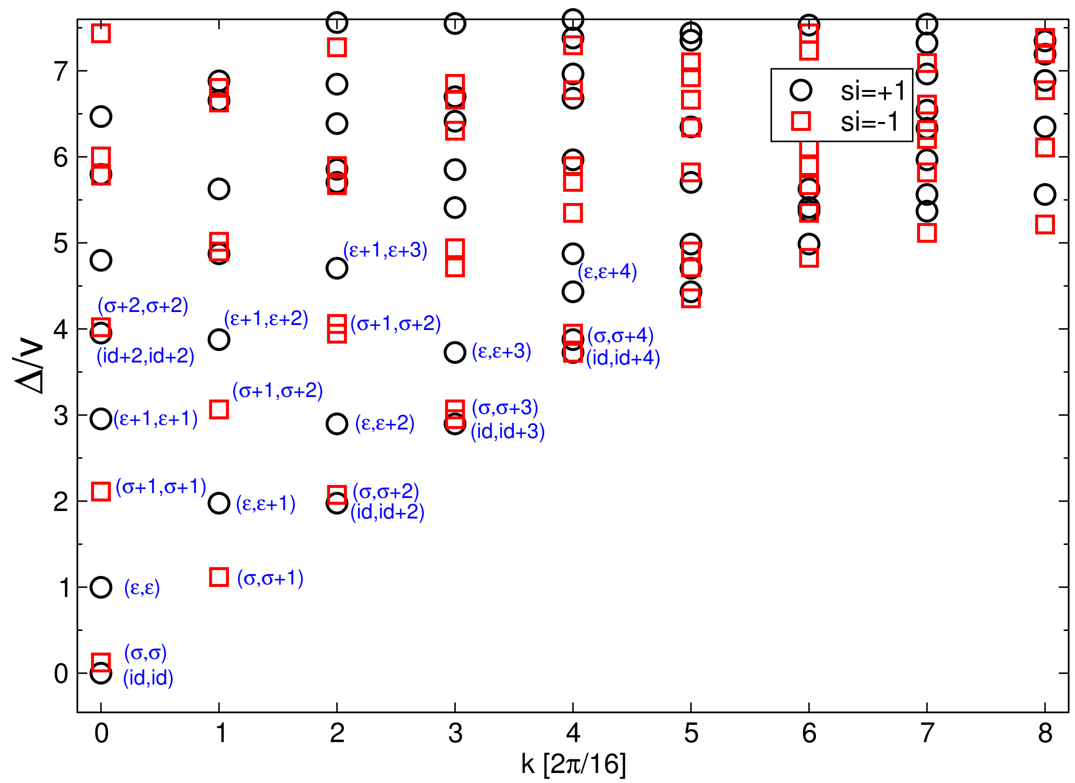
- The scaling dimension determines the decay of the 2-point correlation function:

$$\langle \mathcal{O}_i(x)\mathcal{O}_i(0)\rangle = \frac{c}{|x|^{2\Delta_i}}$$

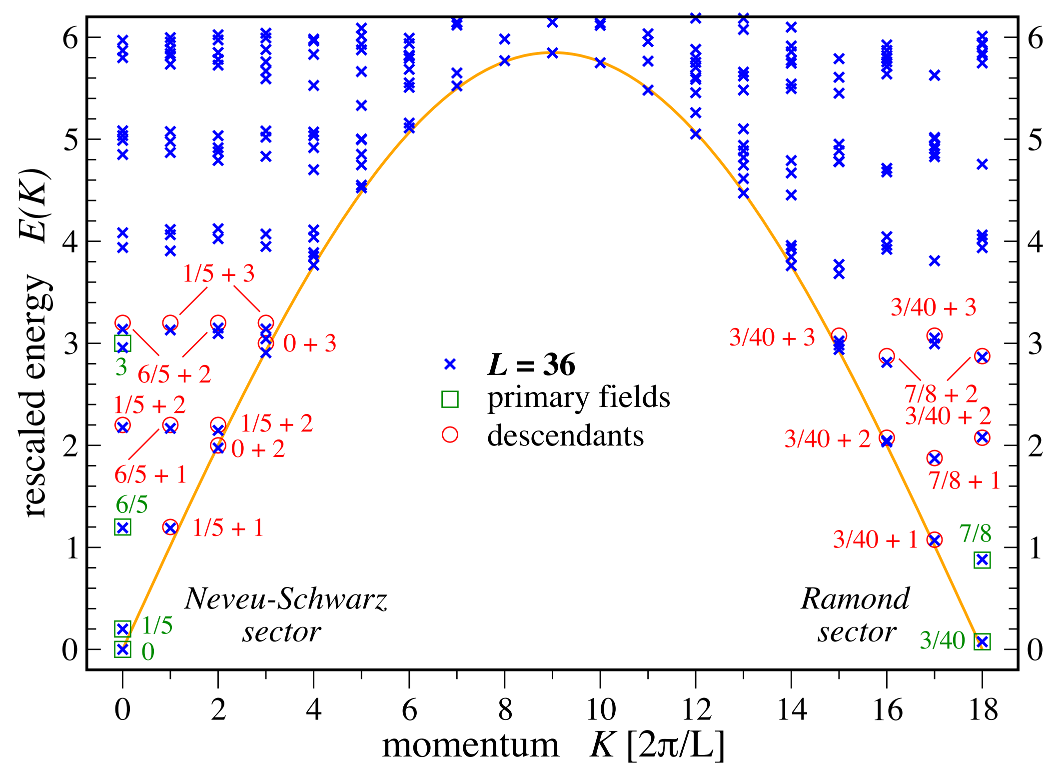
- It seems interesting and important to know the various fields with their corresponding scaling dimensions.

- Where can we find those in numerics ?

1D Torus (Circle) Energy Spectra

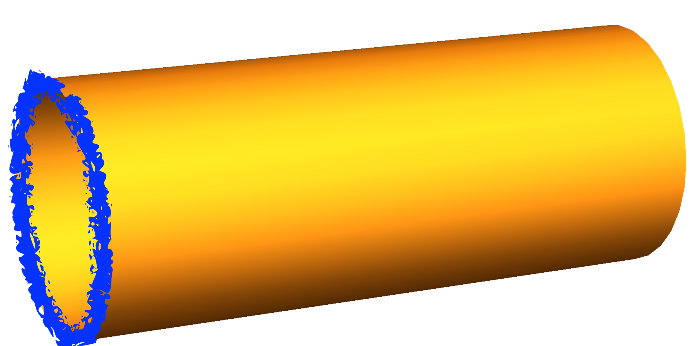


TFI chain L=16
2D Ising CFT Spectrum



A. Feiguin et al. PRL 2007
tricritical Ising CFT Spectrum in anyon chains

$\mathbb{S}^1$

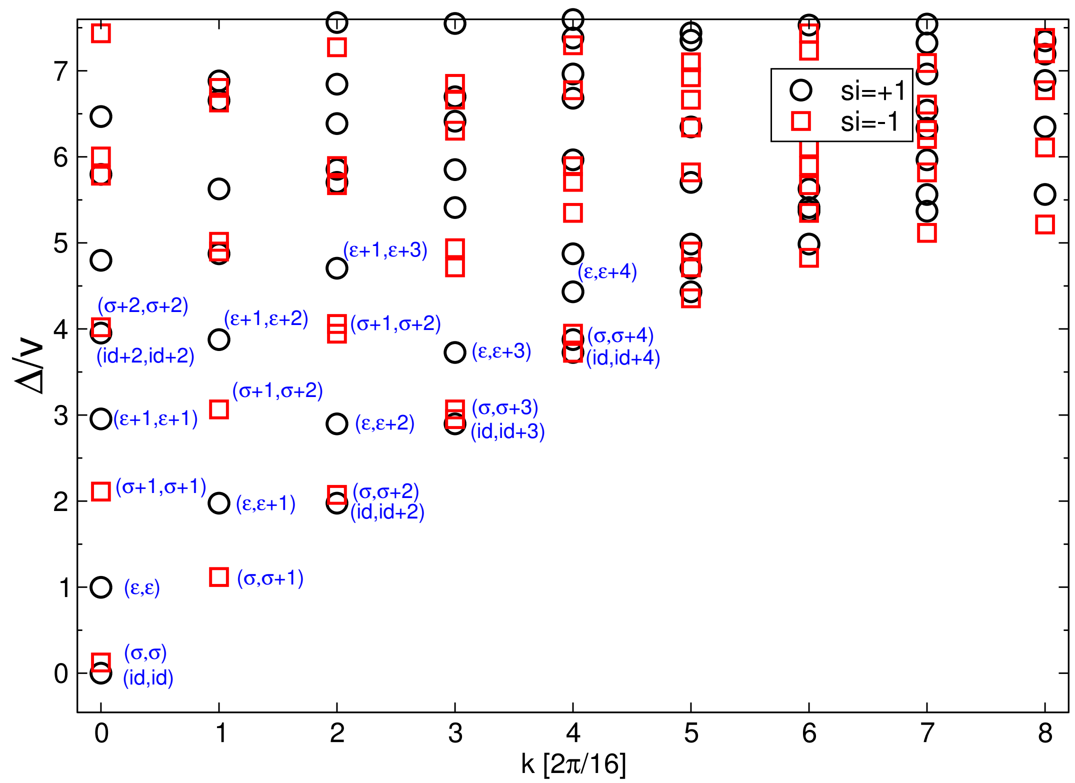


$\mathbb{S}^1 \times \mathbb{R}$

$$\mathbb{R}^2 \leftrightarrow \mathbb{S}^1 \times \mathbb{R}$$

1D Torus (Circle) Energy Spectra

- For CFTs energy spectra of finite size (1+1D) systems arrange into conformal towers !



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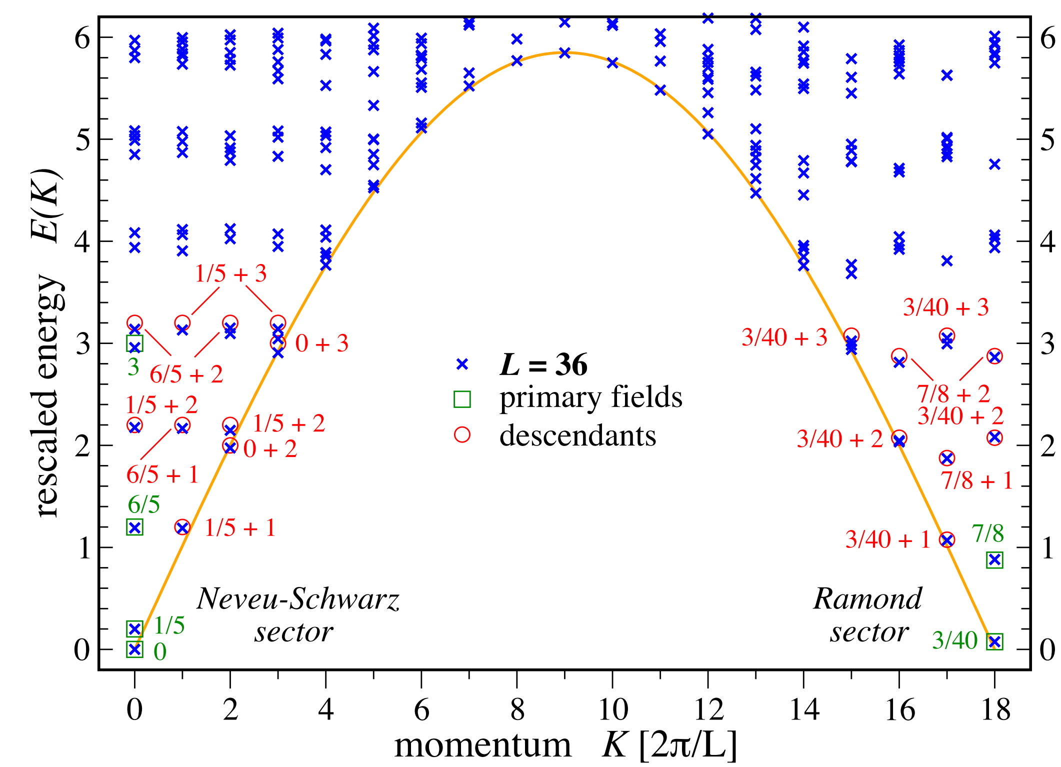
2D Ising CFT Spectrum



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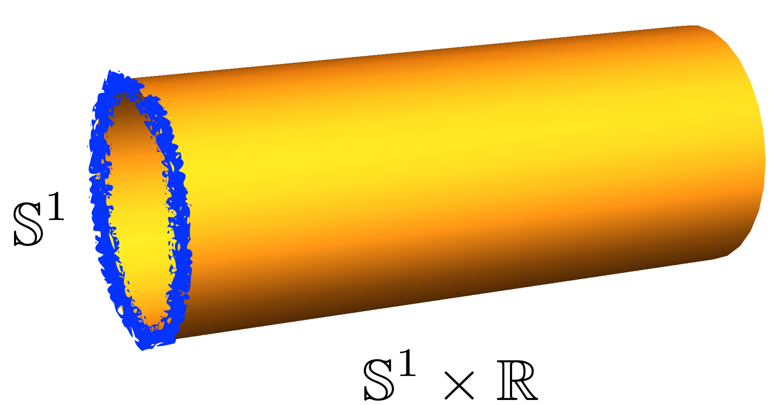


$$\mathbb{R}^2 \leftrightarrow \mathbb{S}^1 \times \mathbb{R}$$

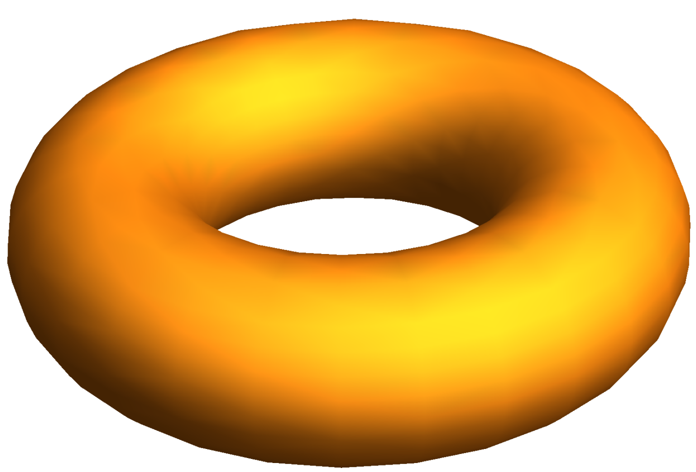
- Spectrum of scaling dimensions of CFT maps to Hamiltonian spectrum on a circle.

Energy spectra and CFTs in more than 1+1D ?

- In more than 1+1D, this relation does not hold for tori anymore, only for the sphere !

$$\mathbb{R}^d \leftrightarrow S^{d-1} \times \mathbb{R} \quad (\neq \mathbb{T}^{d-1} \times \mathbb{R},\ d > 2)$$

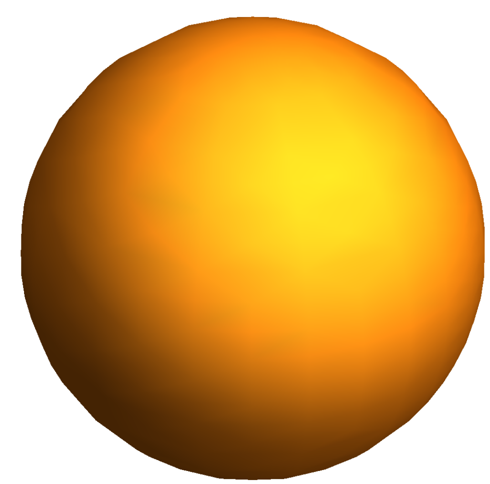
- First mapping: radial quantisation, can reveal scaling dimension in higher d, but not easily accessible to numerics (although several efforts over the decades).



$\times \mathbb{R}$

$\mathbb{T}^2 \times \mathbb{R}$

space x time



$\times \mathbb{R}$

$\mathbb{S}^2 \times \mathbb{R}$

space x time

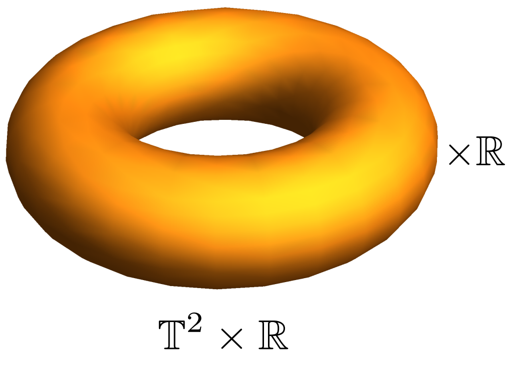
Energy spectra and CFT in more than 1+1D ?

- In more than 1+1D, this is not expected to hold anymore for tori !

$$\mathbb{R}^d \leftrightarrow S^{d-1} \times \mathbb{R} \quad (\neq \mathbb{T}^{d-1} \times \mathbb{R},\ d > 2)$$

- First mapping: radial quantization, can reveal scaling dimension in higher d, but not easily accessible to numerics (although several efforts over the decades).

- What about energy spectra on tori, which are numerically accessible?
  - Is there a universal low-energy spectrum (and is it accessible numerically) ?
  - How does it look like ?
  - Any analogy to the spectrum of scaling dimensions ?



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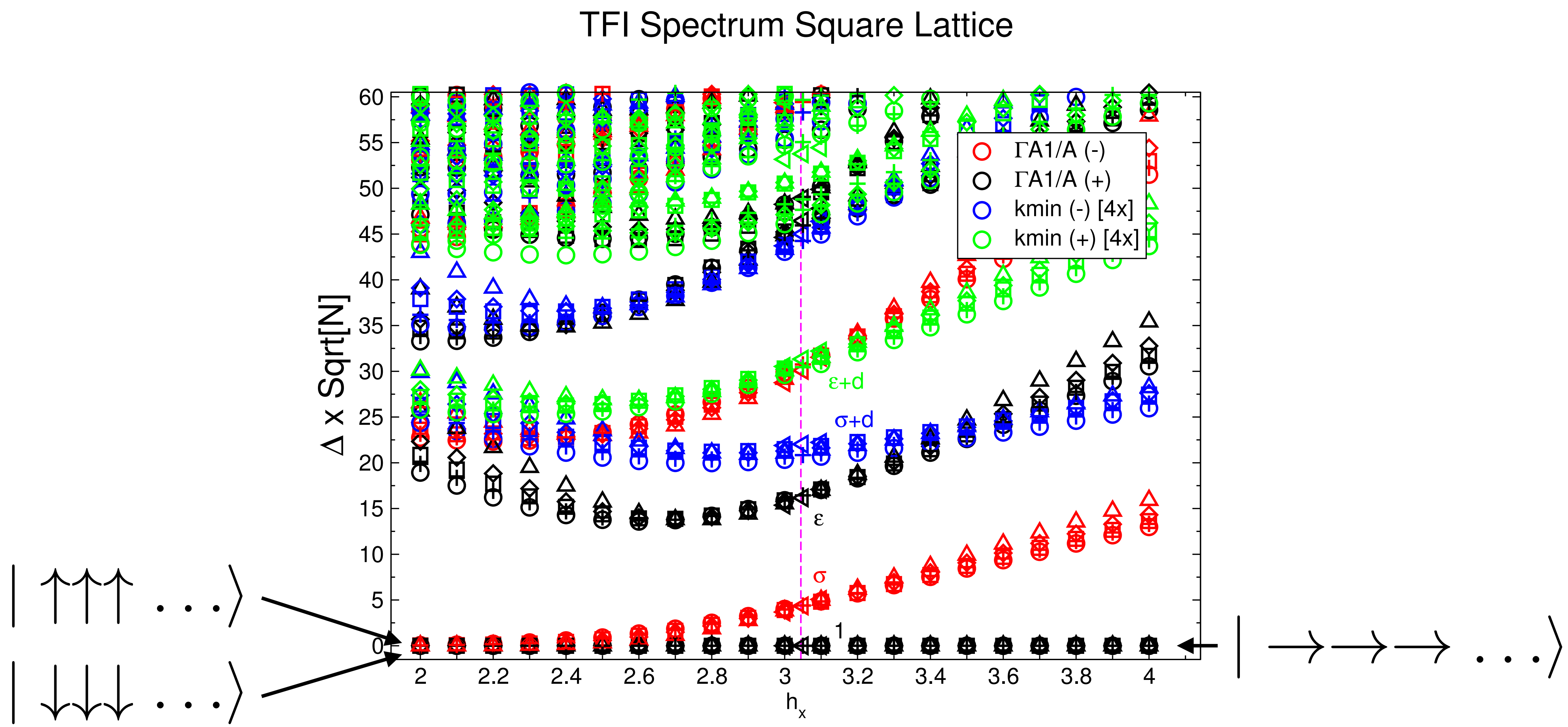
2+1D “standard” Ising CFT

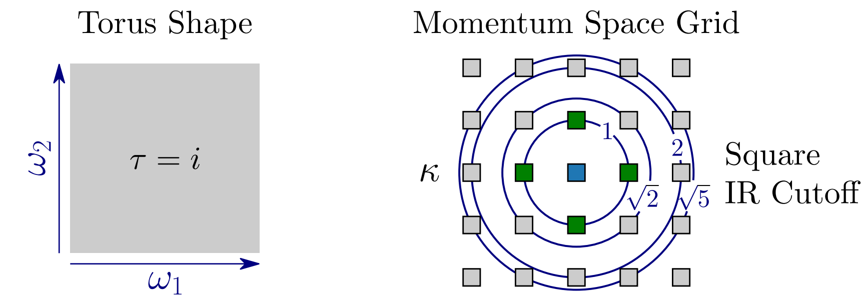
- We want to investigate the torus energy spectrum at a quantum critical point.
- While we do not expect to find the exact spectrum of scaling dimensions, the spectrum is still expected to be universal, i.e. UV cutoff independent.
- The spectrum could however depend on the IR-cutoff (shape of torus) (c.f. “hearing the shape of the drum”)
- We start with a Z_2 symmetry breaking transition, and consider the transverse field Ising (TFI) model as a particular microscopic realization

$$H_{\mathrm{TFI}} = -J \sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z - h \sum_i \sigma_i^x$$

“Raw” energy spectrum across the transition

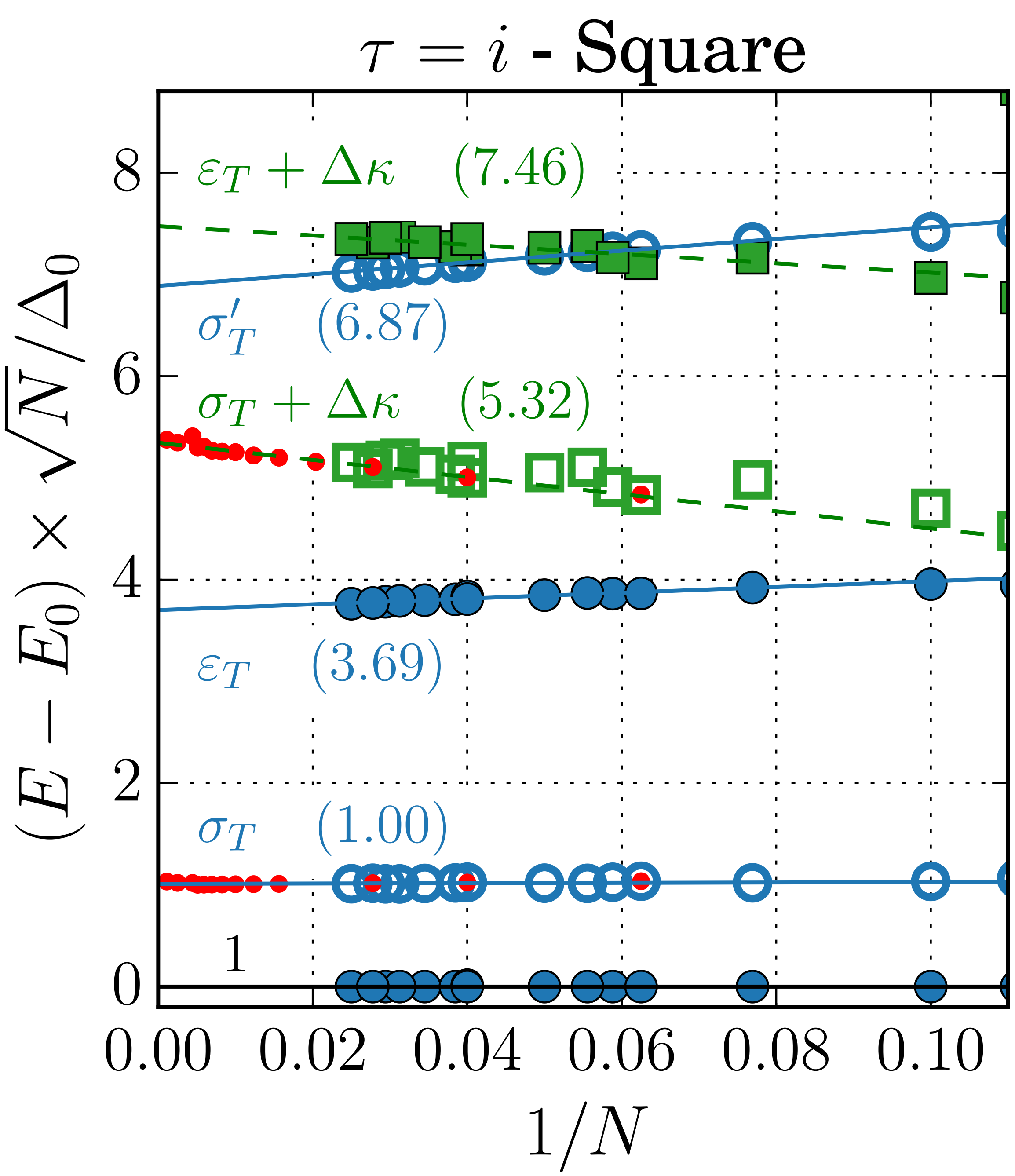
- small field: approx. 2-fold degeneracy due to Z_2-symmetry breaking.
- large field: unique ground state in paramagnetic phase.

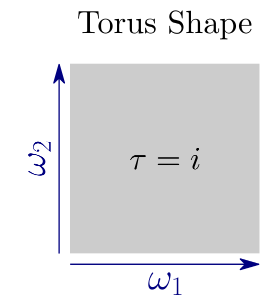


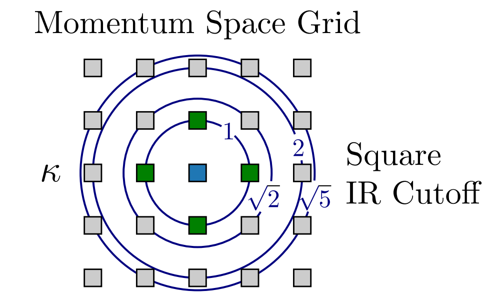


Detailed finite size scaling

- Square lattice at critical transverse field h_c:







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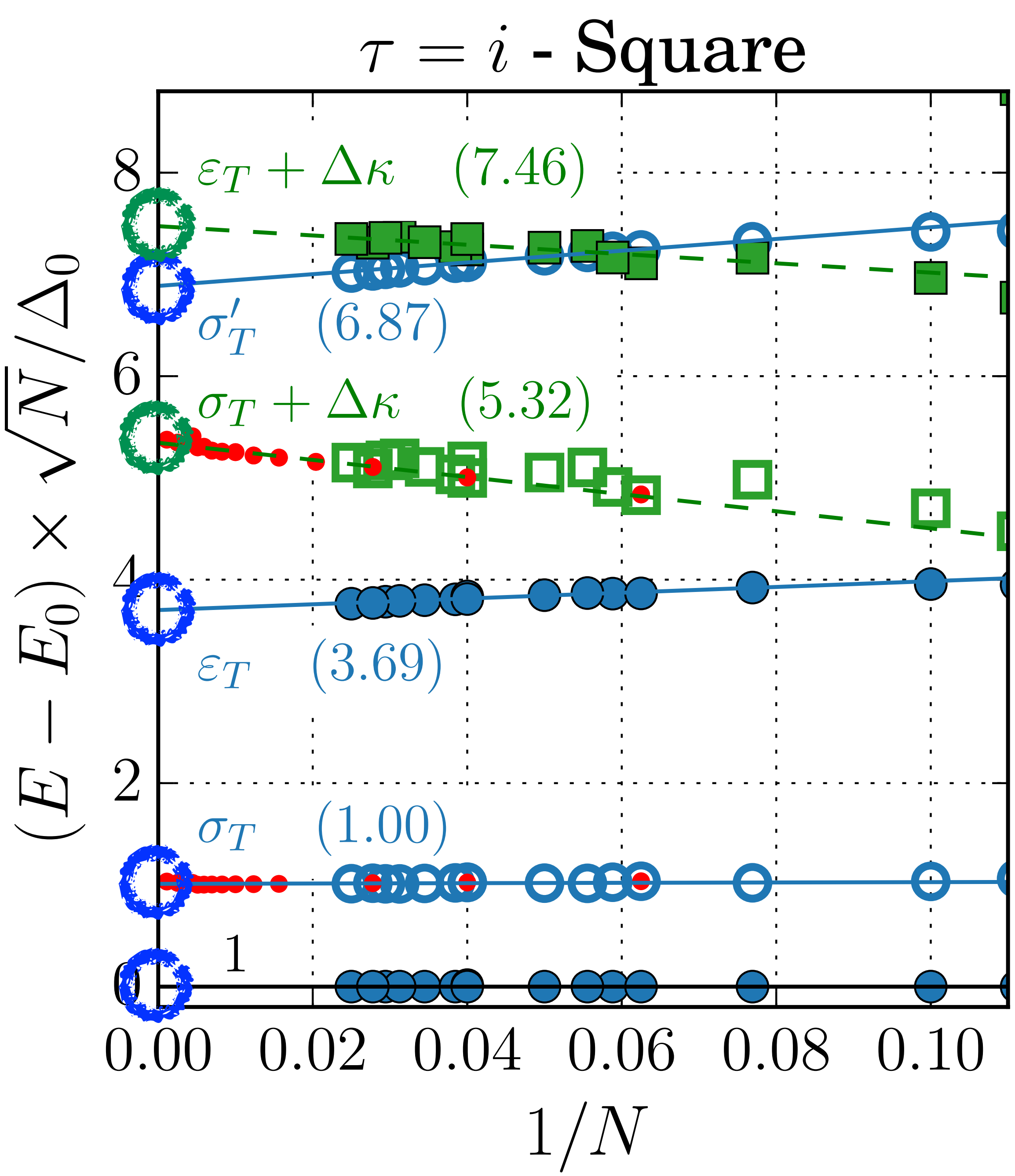
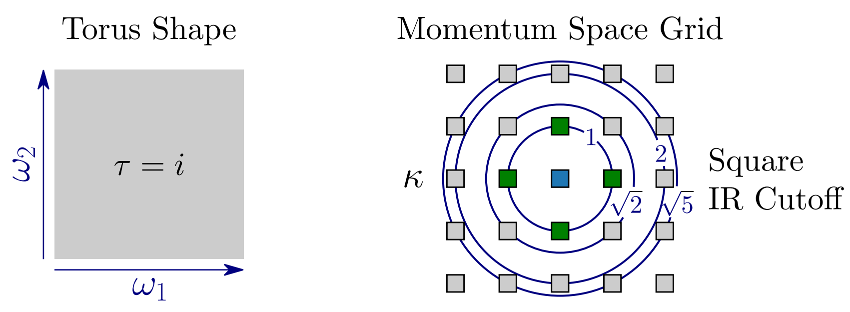
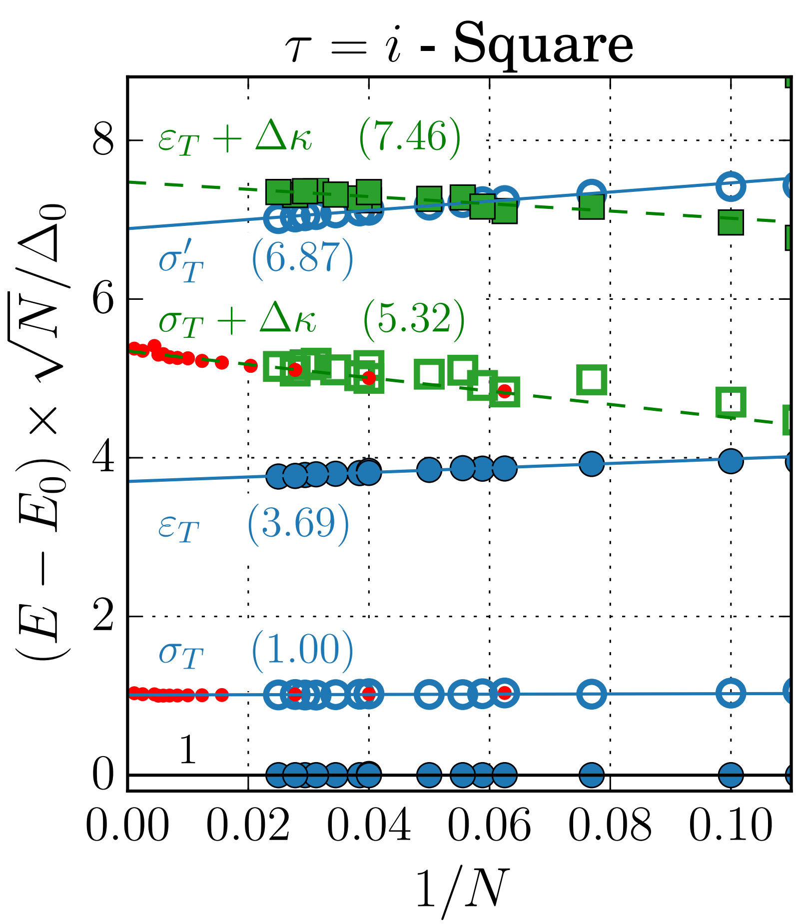
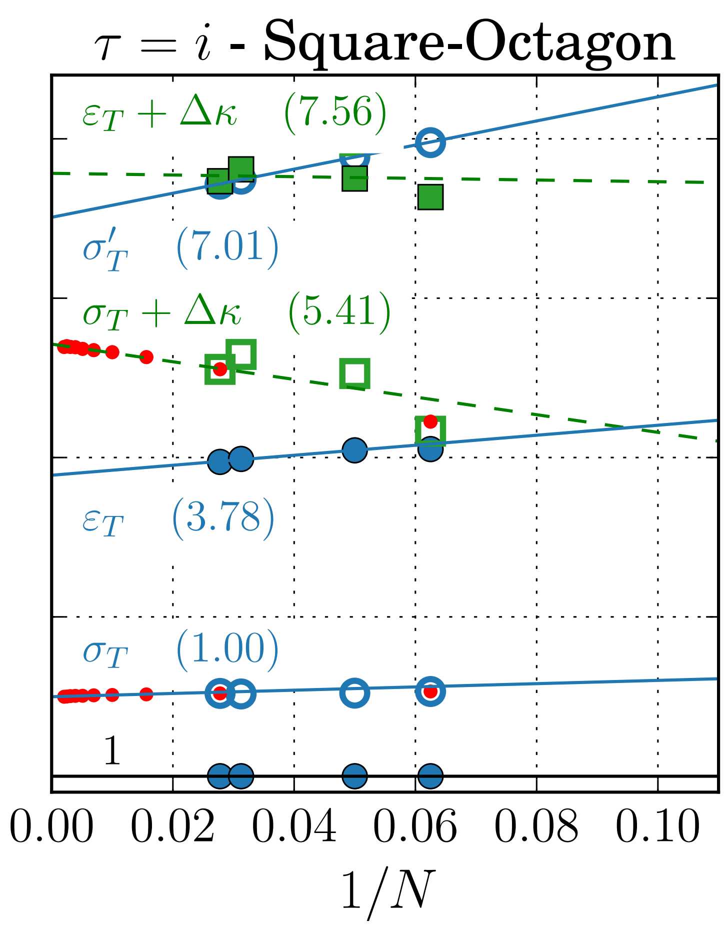


Comparison with a different lattice

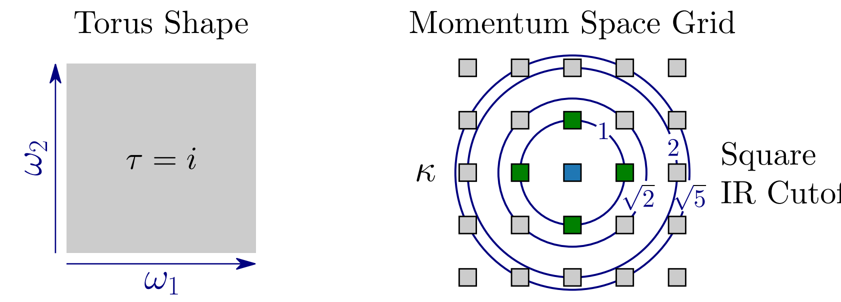


- Square lattice and Square-Octagon lattice at their critical point:



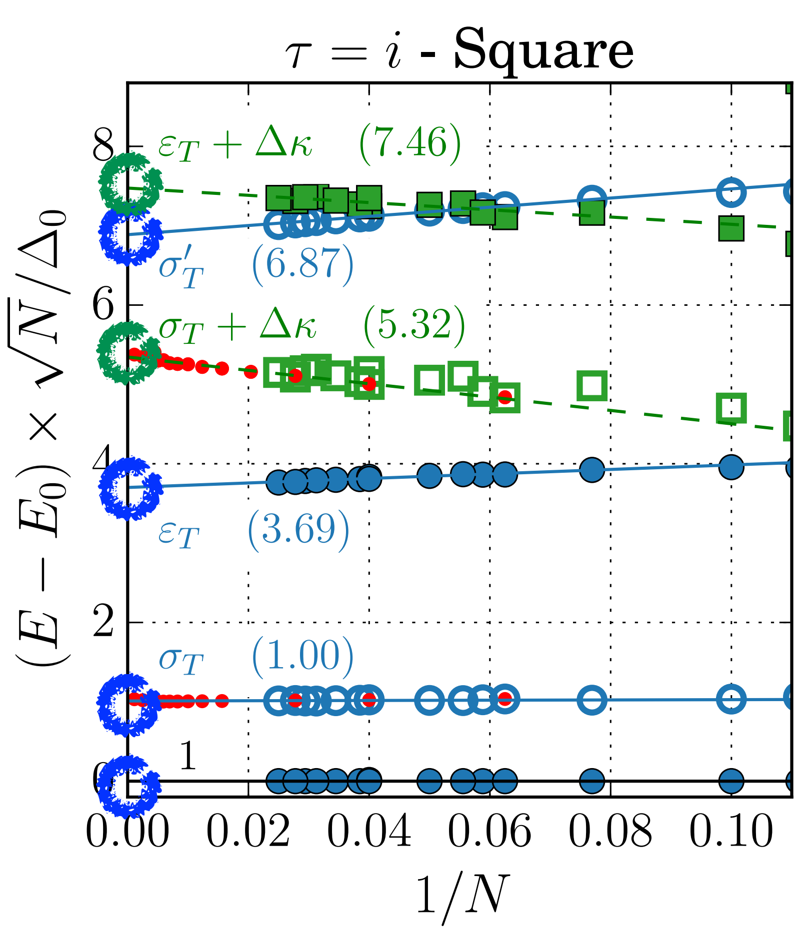


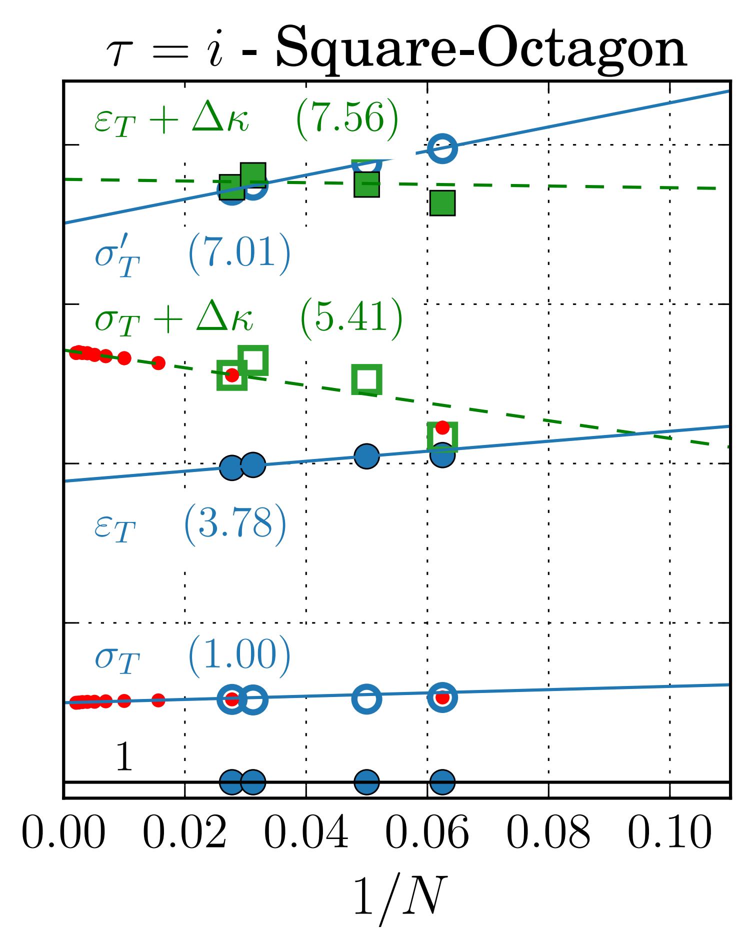
- The spectra are identical after finite-size extrapolation!
  This is thus the genuine Ising CFT spectrum on a square torus !



Comparison with a different lattice

- Square lattice and Square-Octagon lattice at their critical point:





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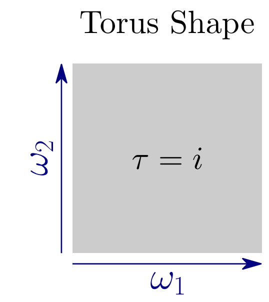

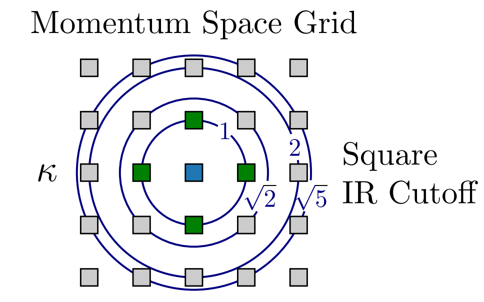

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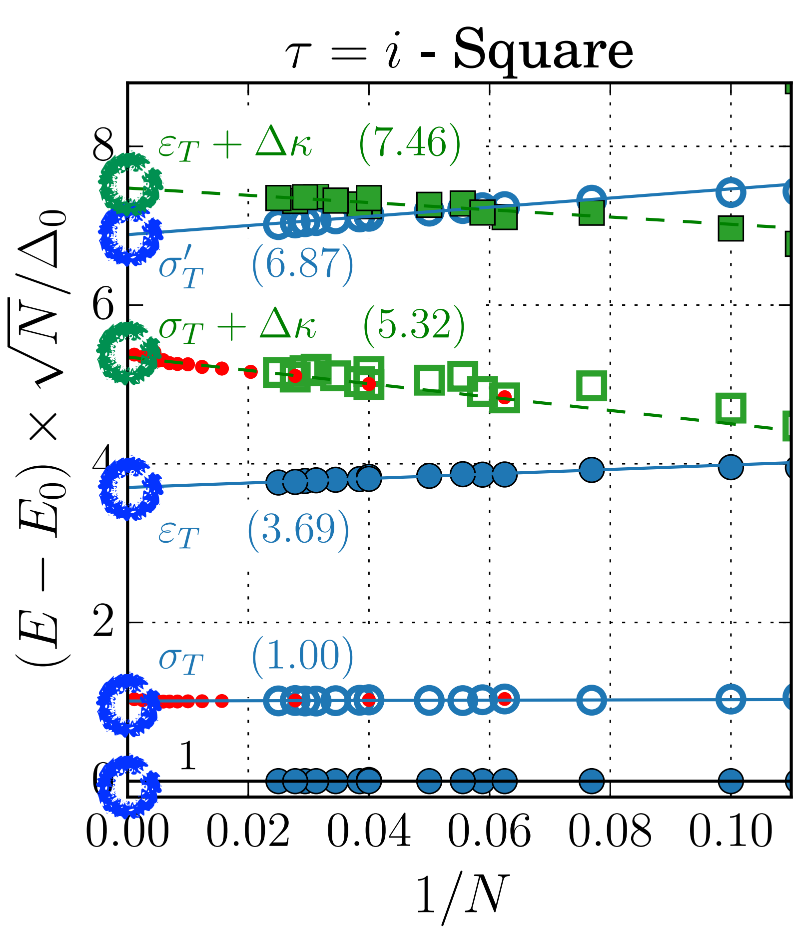
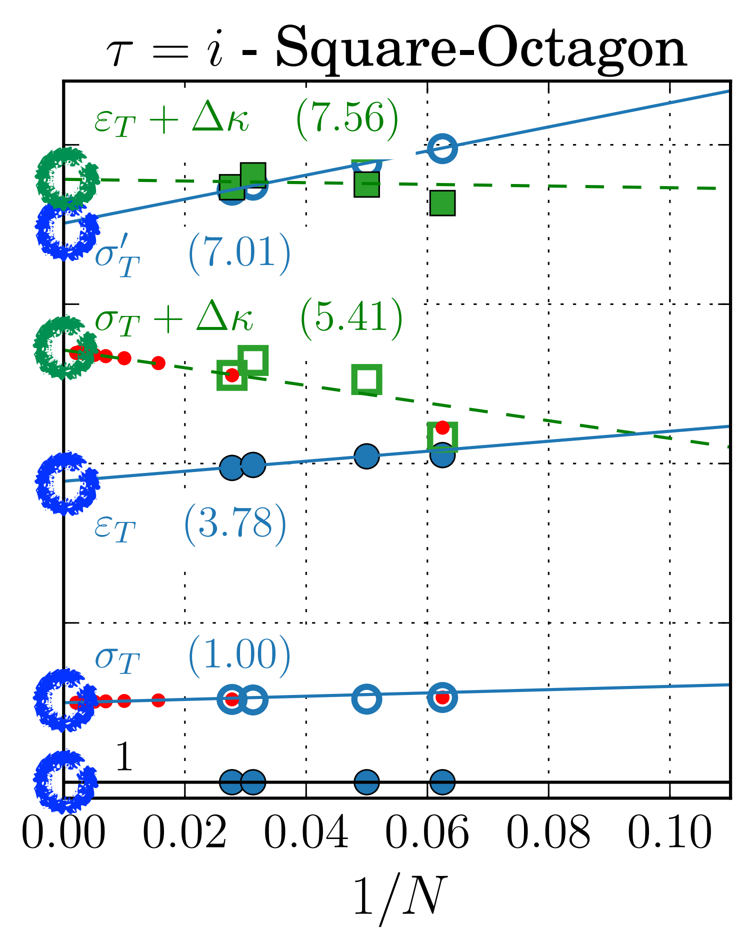
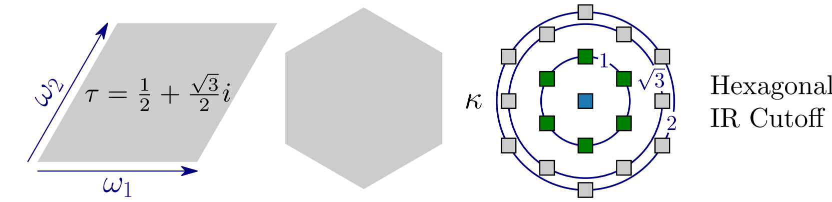


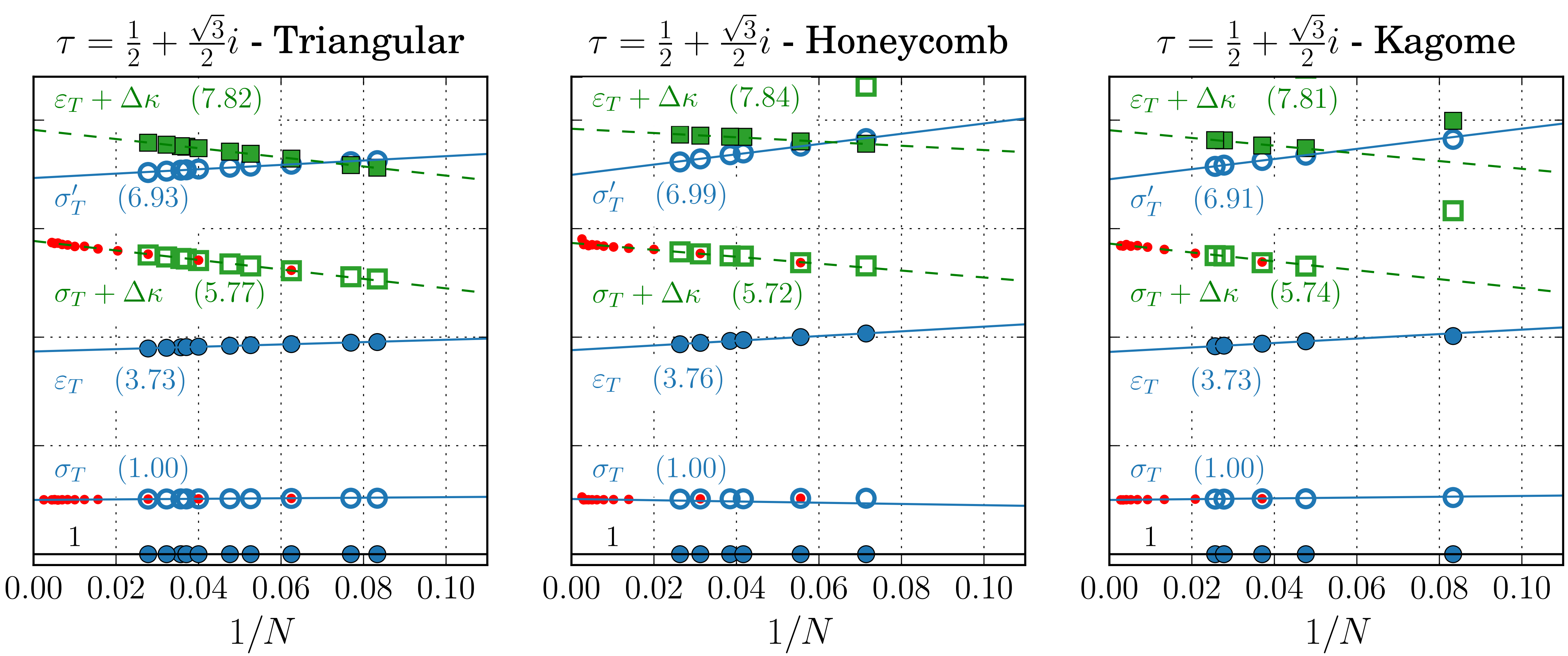


- The spectra are identical after finite-size extrapolation!
  This is thus the genuine Ising CFT spectrum on a square torus !



Comparison with different modular parameter

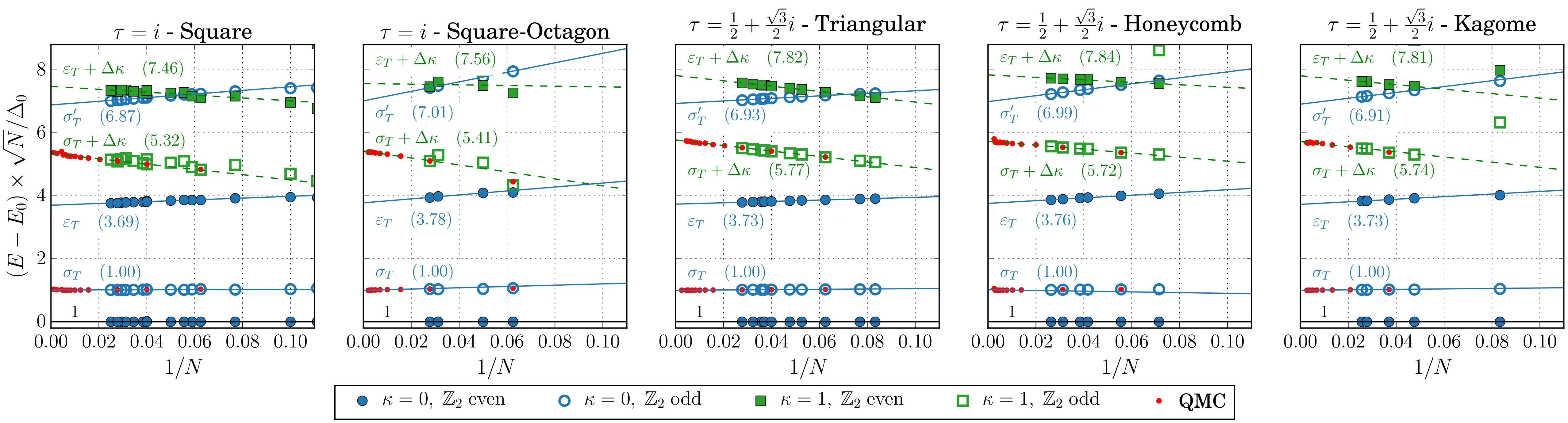
- Triangular, honeycomb and kagome lattice at their critical point:



- The spectra are identical after finite-size extrapolation!
  This is thus the genuine Ising CFT spectrum on a hexagonal torus !

Comparing the different geometries

- The “square” and the “hexagonal” tori have a slightly different spectrum.



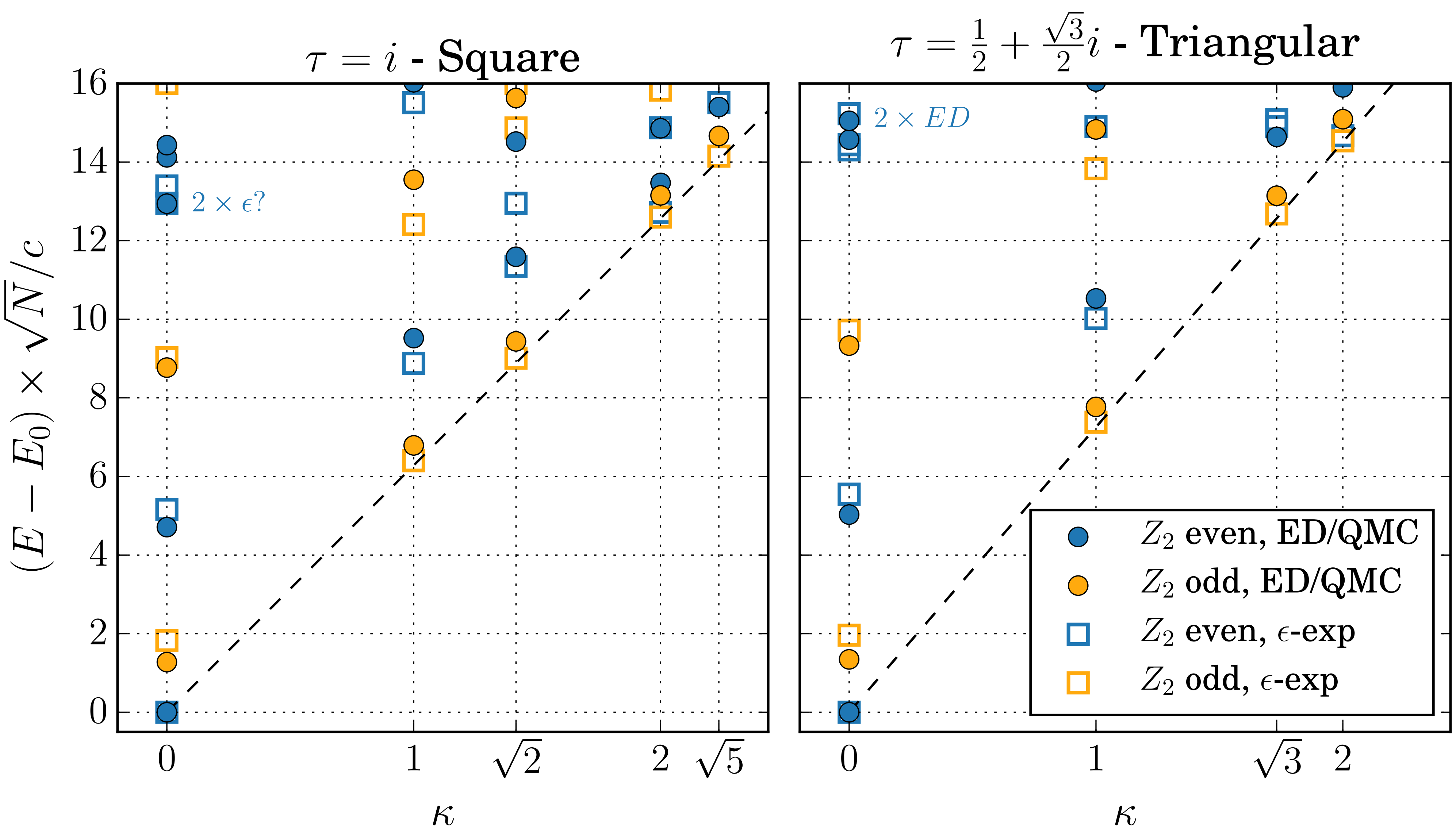
- The spectrum we see is the torus spectrum of the CFT describing the critical point.

Analytical approach: (4-epsilon)-expansion

- Work done by S. Withsett and S. Sachdev. Lowest non-trivial order in epsilon.

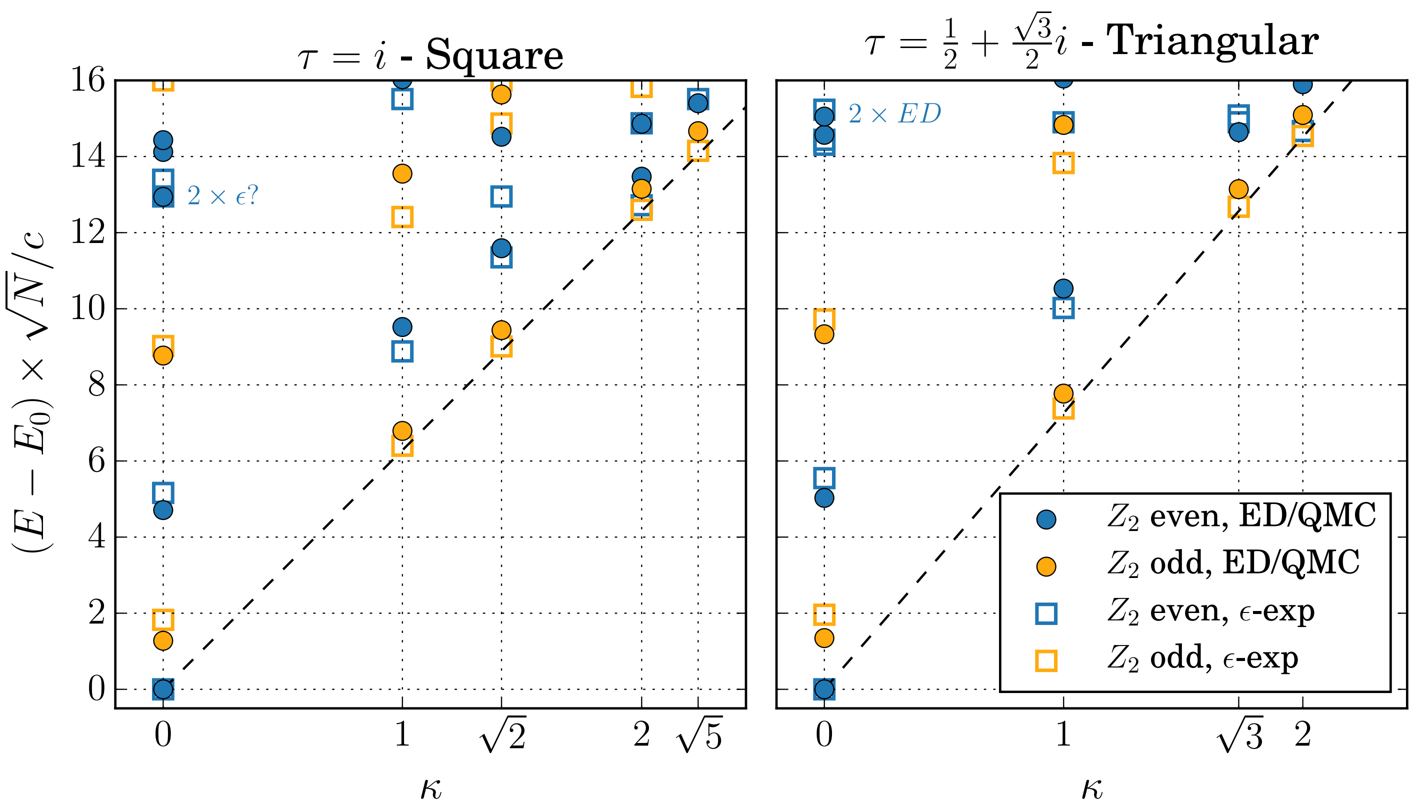
$$\mathcal{H} = \int d^d x \left[\frac{1}{2}\Pi^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{s}{2}\phi^2 + \frac{u}{4!}\phi^4\right]$$

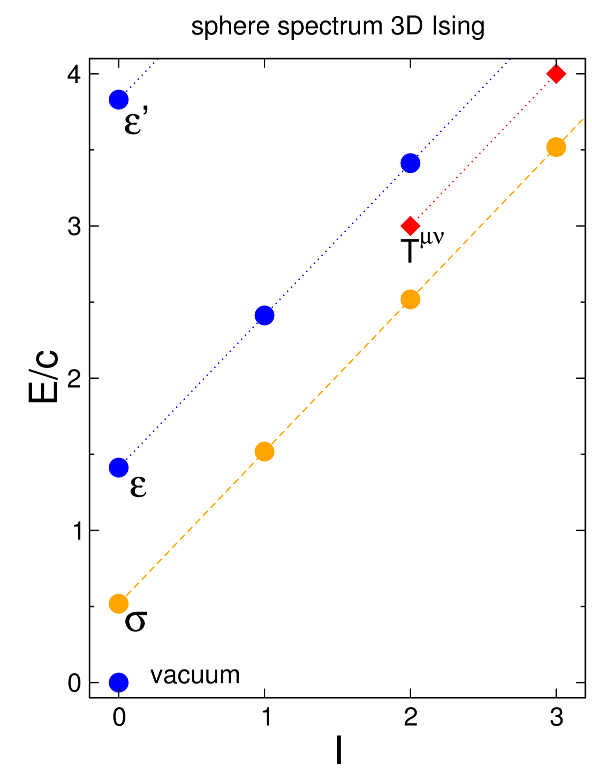
- Rather good agreement between analytics and numerics.



Comparison between torus and sphere spectra

- Torus spectra at low energy per sector resemble the spectrum on the sphere:





- We believe this handwaving resemblance might be more generally the case: "light states on the sphere have a light analogon on the torus"

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- Outlook

Confinement transition

- Z_2 spin liquids are rather fashionable these days.
- The are phases with a four-fold ground state degeneracy on a torus, but the degeneracy is topological, and not related to symmetry breaking.
- One of the simplest incarnations of this phase appears in the Toric Code model by Kitaev.
- By an appropriate perturbation the topological phase (“deconfined”) gives way to a simple paramagnetic phase (“confined”). The transition is a confinement transition and is expected to be in the 2+1D = 3D Ising universality class.
- Q: Is the torus spectrum at criticality identical to the symmetry breaking case ?

Toric code in a magnetic field

- We study the following microscopic model
  (but results will be independent of model):

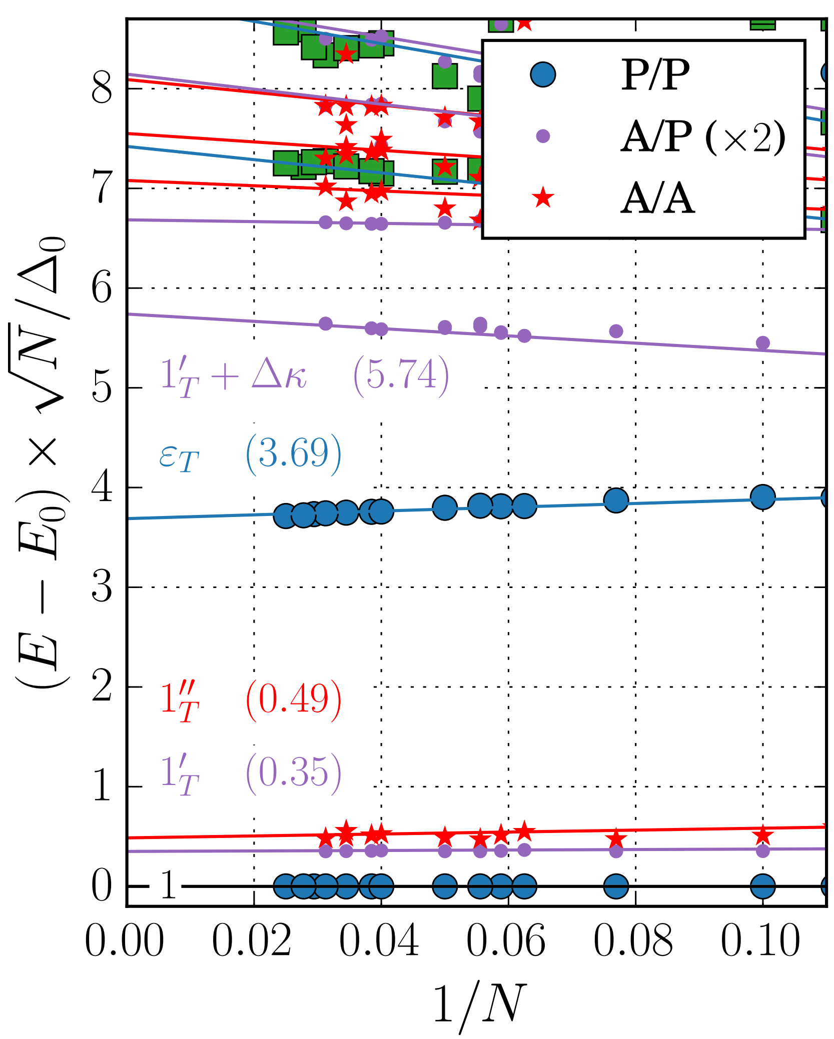
- Toric code with a longitudinal magnetic field (S. Trebst et al., …):

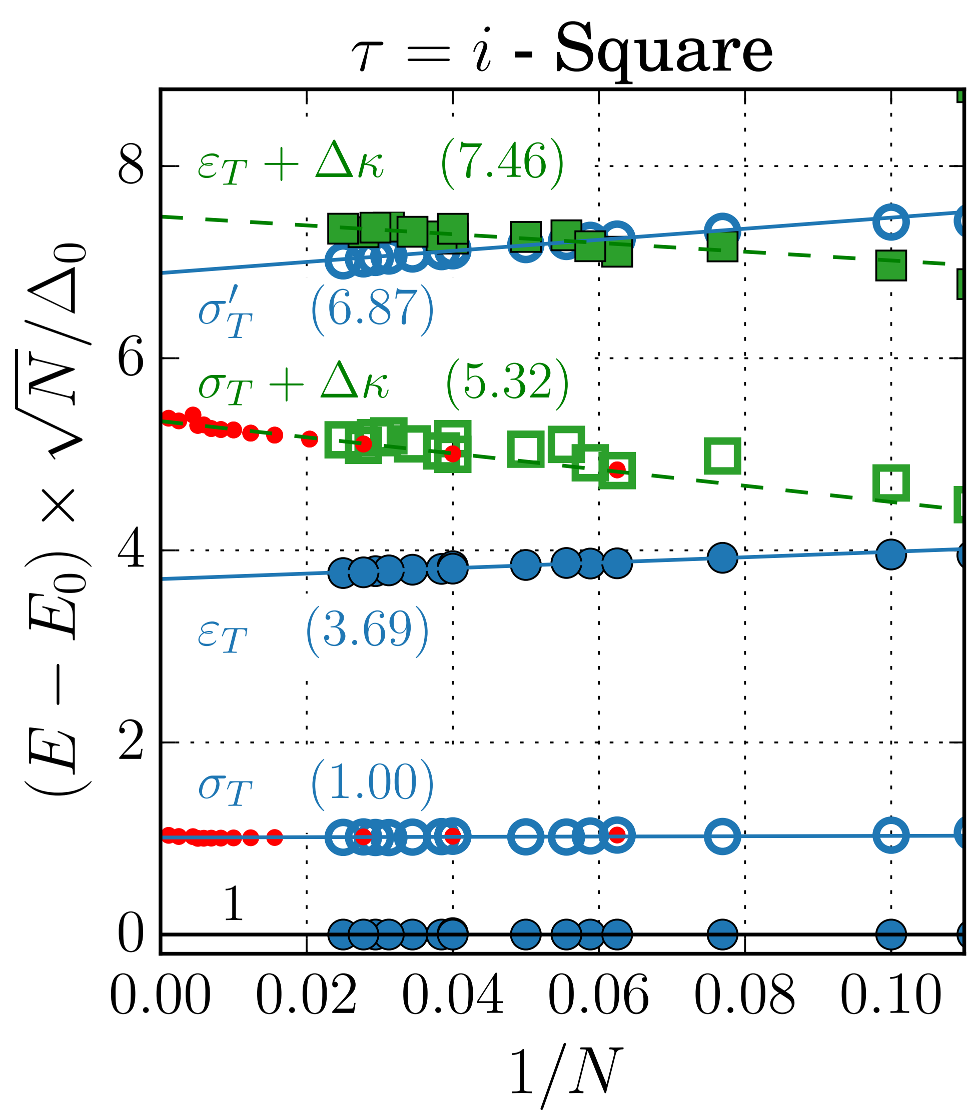
$$H = -J \sum_s A_s - J \sum_p B_p - h \sum_i \sigma_i^x$$

$$A_s = \prod_{i \in s} \sigma_i^x, \; B_p = \prod_{i \in p} \sigma_i^z$$

Numerics at criticality

- Left: data for the TC at criticality, Right: Symmetry breaking





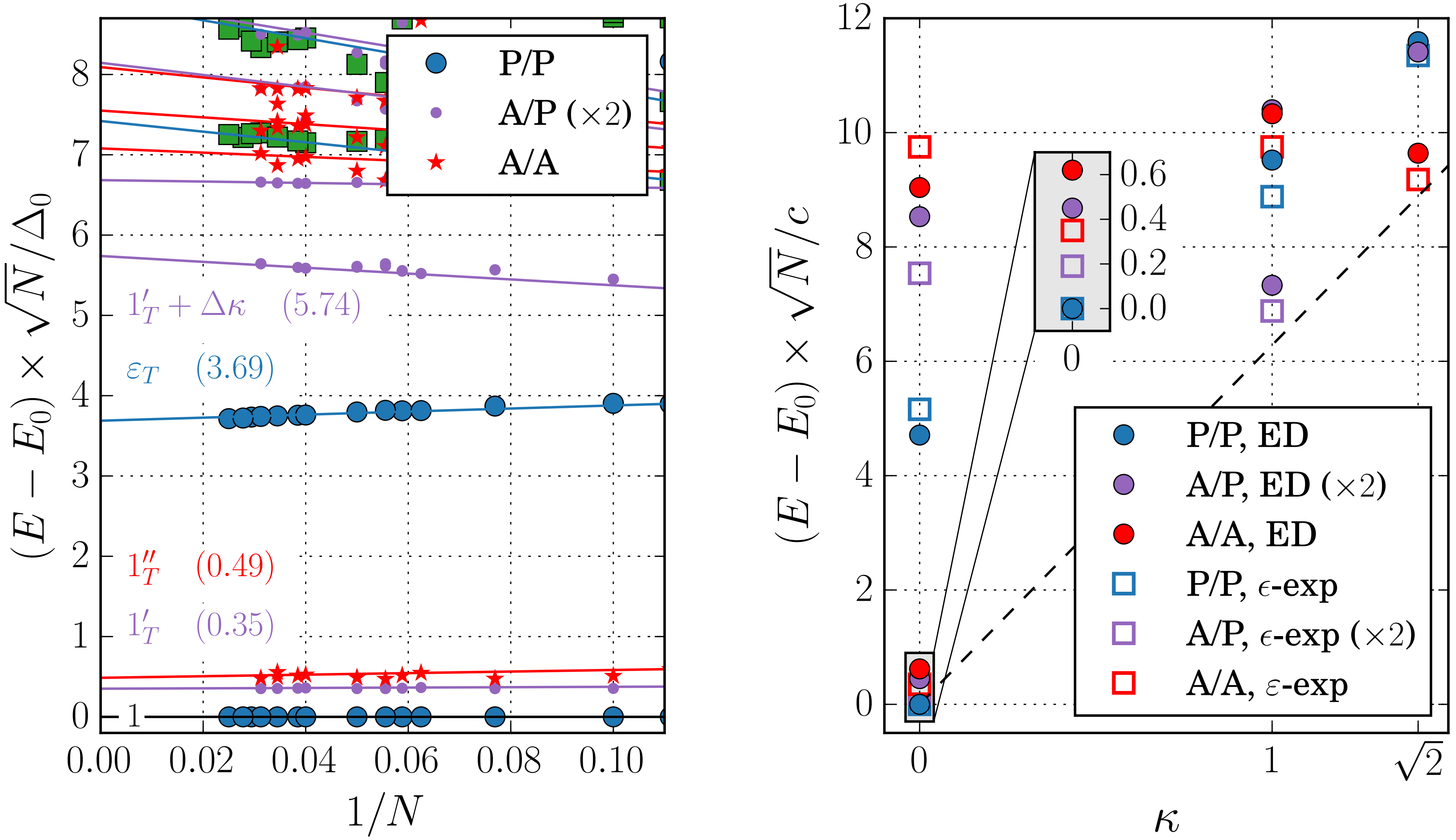
- The spectra at criticality do not agree ! What is going on ?

The Ising* transition

- The explanation is that the operator content of the two transitions are different:
- In the Z_2 symmetry breaking case we have Z_2 even and odd levels and only one set of boundary conditions (fixed by the lattice model).
- In the confinement transition (Ising*), only Z_2 even levels are allowed, and for periodic boundary conditions in the Toric Code, four different boundary conditions of the CFT become simultaneously apparent.
- This can be understood at the microscopic level in the Toric Code Hamiltonian and is supported by general field theoretical considerations.
- In the Ising* case the magnetic sector is completely absent, and the torus energy spectrum reflects this fact.

The Ising* transition

- comparison between numerics and epsilon-expansion:
- At criticality the 4 “topological sectors” scale also as 1/L , but are much closer together than the next level above them.



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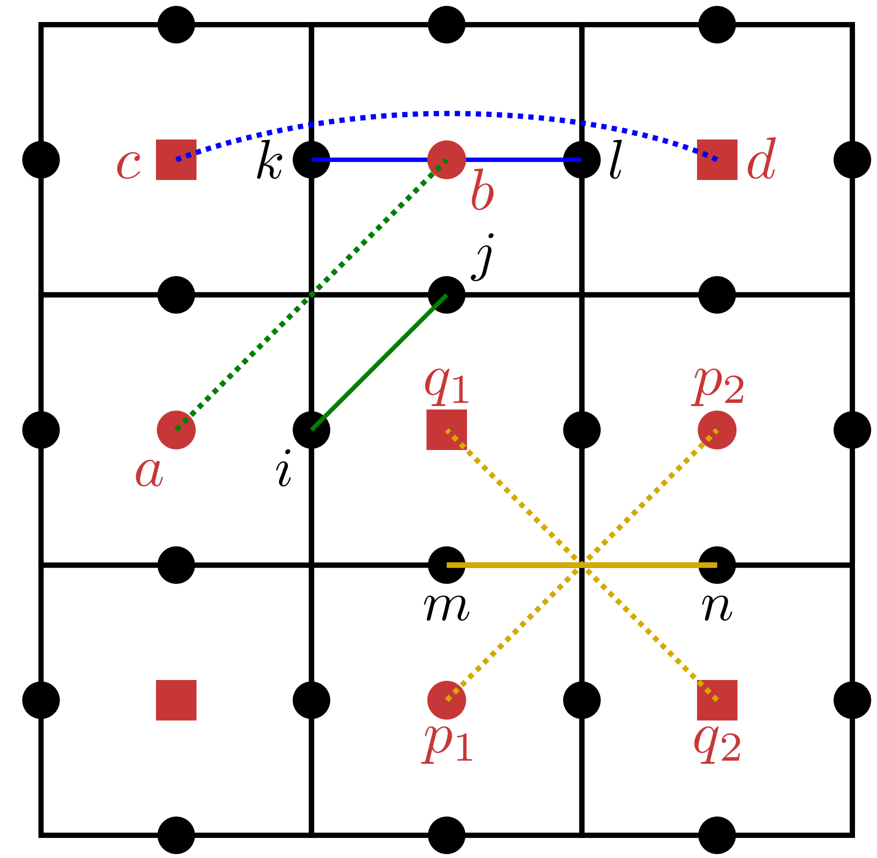
Toric code with Ising interactions

- Want to study a possible quantum phase transition between Z_2 topological order and spontaneous global Z_2 symmetry breaking.

- Toric code plus additional Ising interactions:

$$H = -J\sum_s A_s - J\sum_p B_p$$
$$-J_I \sum_{\langle i,j\rangle} \sigma_i^x \sigma_j^x - J_{I_2} \sum_{\langle\langle i,j\rangle\rangle} \sigma_i^x \sigma_j^x$$
$$A_s = \prod_{i\in s} \sigma_i^x \quad B_p = \prod_{i\in p} \sigma_i^z$$



$$\sigma_i^x \sigma_j^x \rightarrow 2\mu_a^x \mu_b^x$$
$$\sigma_k^x \sigma_l^x \rightarrow \mu_c^x \mu_d^x$$
$$\sigma_m^x \sigma_n^x \rightarrow 2\mu_{p_1}^x \mu_{p_2}^x \mu_{q_1}^x \mu_{q_2}^x$$

Toric code with Ising interactions

- Toric code plus additional Ising interactions:

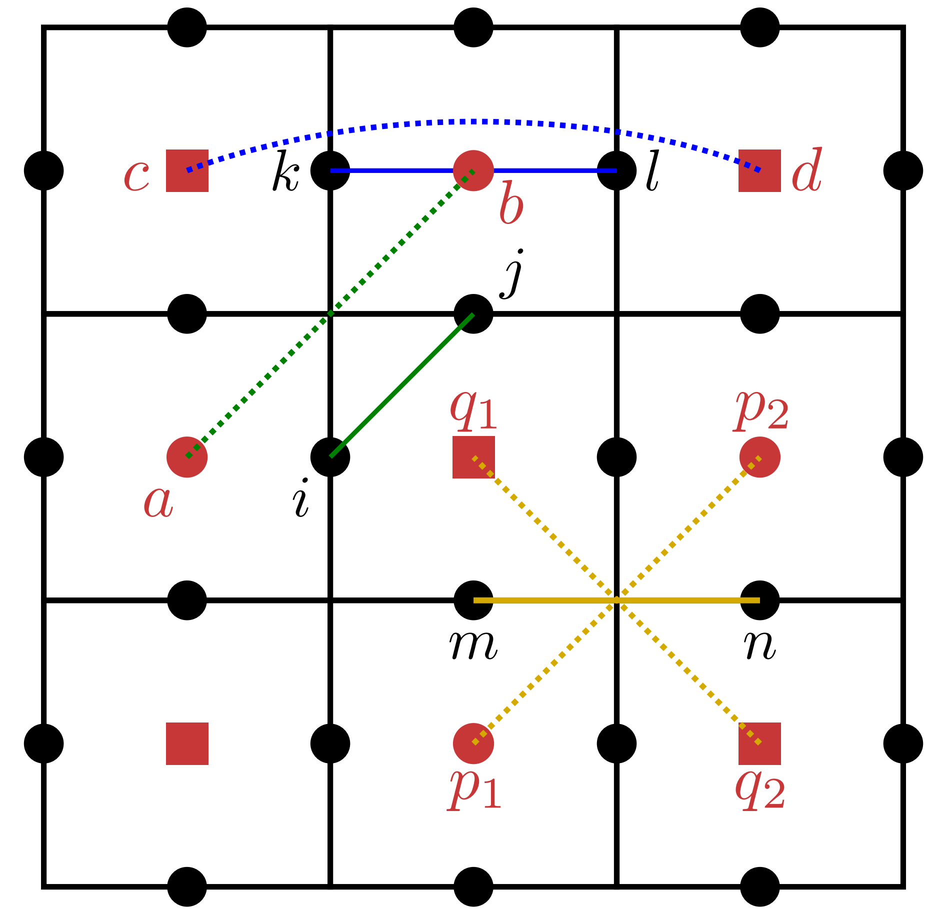
$$H = -J\sum_s A_s - J\sum_p B_p - J_I \sum_{\langle i,j\rangle} \sigma_i^x\sigma_j^x - J_{I_2}\sum_{\langle\langle i,j\rangle\rangle}\sigma_i^x\sigma_j^x$$

$$A_s = \prod_{i\in s}\sigma_i^x \quad B_p = \prod_{i\in p}\sigma_i^z$$

- Maps onto a particular 2+1D quantum Ashkin-Teller model:

$$H_{AT} = -J\sum_i \mu_i^z - 2J_I \sum_{\langle\langle i,j\rangle\rangle}\mu_i^x\mu_j^x - J_{I_2}\sum_{\langle\langle\langle i,j\rangle\rangle\rangle}\mu_i^x\mu_j^x - 2J_{I_2}\sum_i \mu_i^x\mu_{i+\hat{\mathbf{x}}}^x\mu_{i+\hat{\mathbf{y}}}^x\mu_{i+\hat{\mathbf{x}}+\hat{\mathbf{y}}}^x \quad \text{(A6)}$$

- This model has a two checkerboard lattice spatial structure, yielding the two AT-sublattices



$$\sigma_i^x\sigma_j^x \to 2\mu_a^x\mu_b^x$$

$$\sigma_k^x\sigma_l^x \to \mu_c^x\mu_d^x$$

$$\sigma_m^x\sigma_n^x \to 2\mu_{p_1}^x\mu_{p_2}^x\mu_{q_1}^x\mu_{q_2}^x$$

Toric code with Ising interactions

- Toric code plus additional Ising interactions:

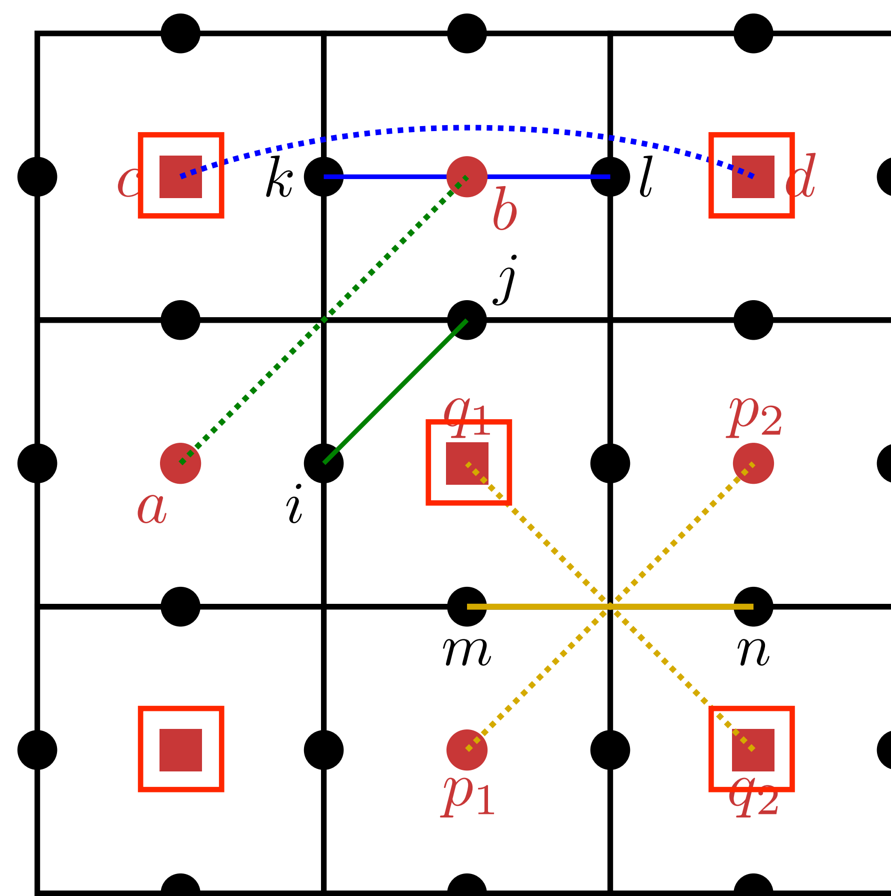
$$
\begin{aligned}
H = &- J\sum_s A_s - J\sum_p B_p \\
&- J_I \sum_{\langle i,j\rangle} \sigma_i^x \sigma_j^x - J_{I_2} \sum_{\langle\langle i,j\rangle\rangle} \sigma_i^x \sigma_j^x
\end{aligned}
$$

$$
A_s = \prod_{i\in s} \sigma_i^x \qquad B_p = \prod_{i\in p} \sigma_i^z
$$

- Maps onto a particular
  2+1D quantum Ashkin-Teller model:

$$
\begin{aligned}
H_{AT} = &- J\sum_i \mu_i^z - 2J_I \sum_{\langle\langle i,j\rangle\rangle} \mu_i^x \mu_j^x - J_{I_2} \sum_{\langle\langle\langle i,j\rangle\rangle\rangle} \mu_i^x \mu_j^x \\
&- 2J_{I_2} \sum_i \mu_i^x \mu_{i+\hat{\mathbf{x}}}^x \mu_{i+\hat{\mathbf{y}}}^x \mu_{i+\hat{\mathbf{x}}+\hat{\mathbf{y}}}^x \qquad \text{(A6)}
\end{aligned}
$$

- This model has a two checkerboard lattice spatial structure, yielding the two AT-sublattices



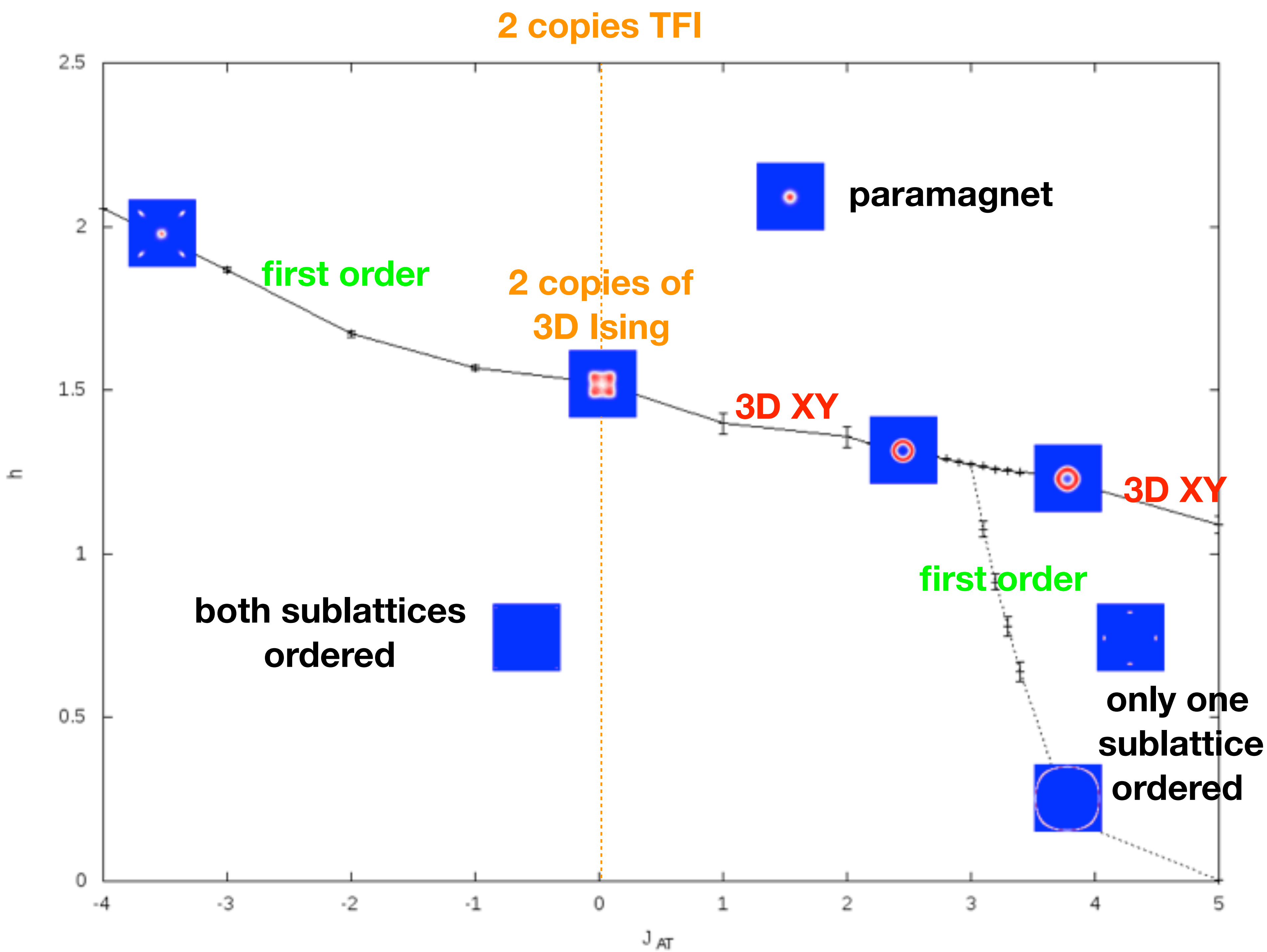
$$\sigma_i^x \sigma_j^x \rightarrow 2\mu_a^x \mu_b^x$$

$$\sigma_k^x \sigma_l^x \rightarrow \mu_c^x \mu_d^x$$

$$\sigma_m^x \sigma_n^x \rightarrow 2\mu_{p_1}^x \mu_{p_2}^x \mu_{q_1}^x \mu_{q_2}^x$$

Phase diagram of the Quantum Ashkin-Teller model

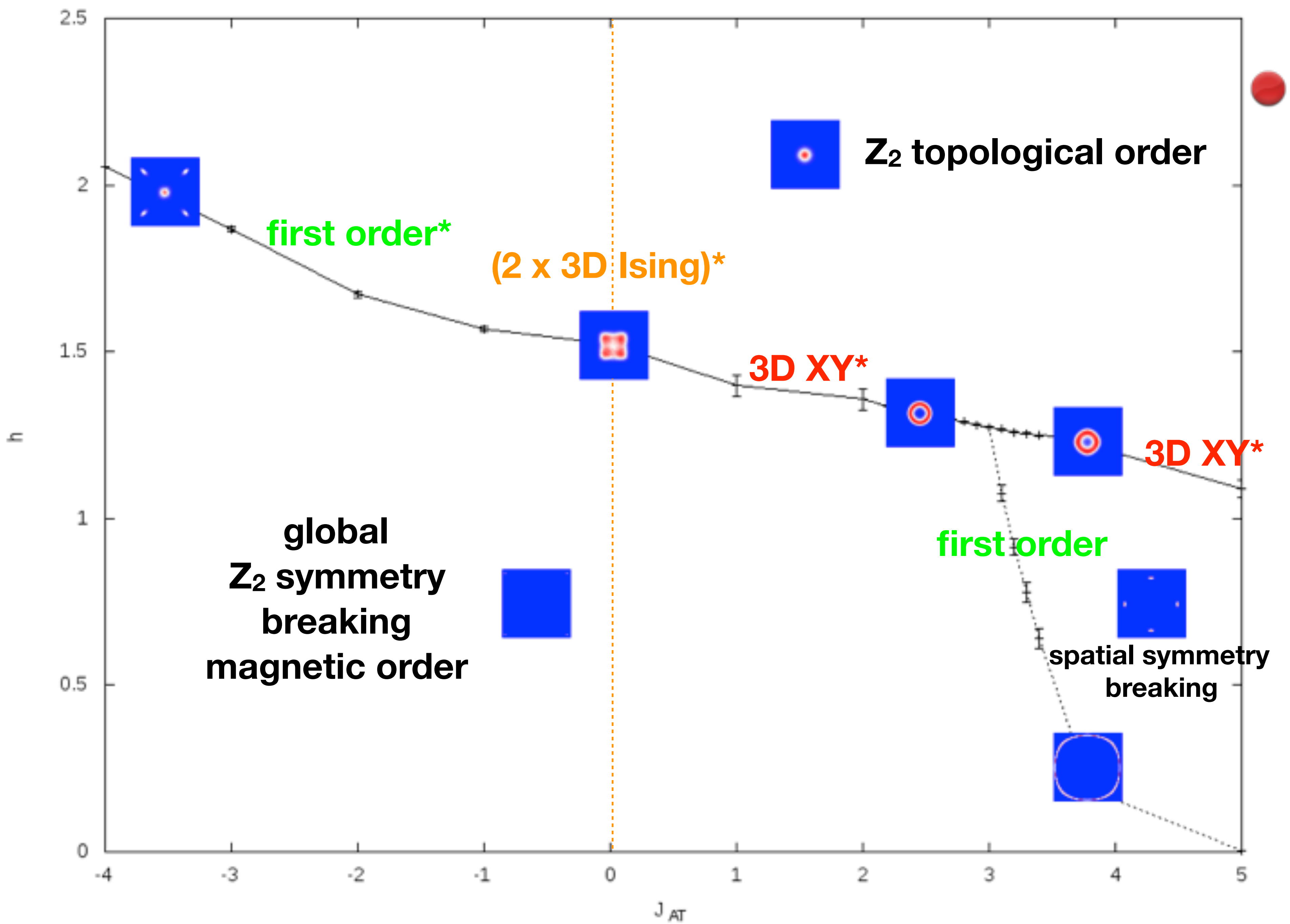
- Rather poorly studied in the past, so here we perform a new QMC study:



- Phase structure in agreement with QFT results of N_c=2 ϕ^4 theory with cubic anisotropy.

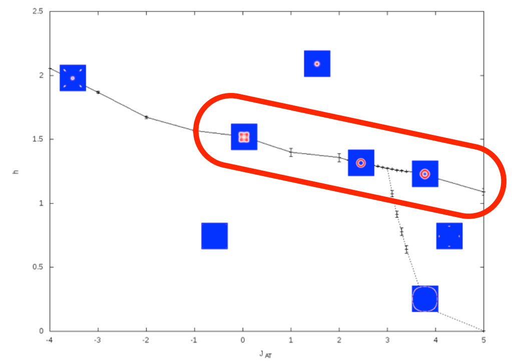
Phase diagram of the Toric Code + Ising interactions

- Translate the Ashkin-Teller results back to the Toric code + Ising:

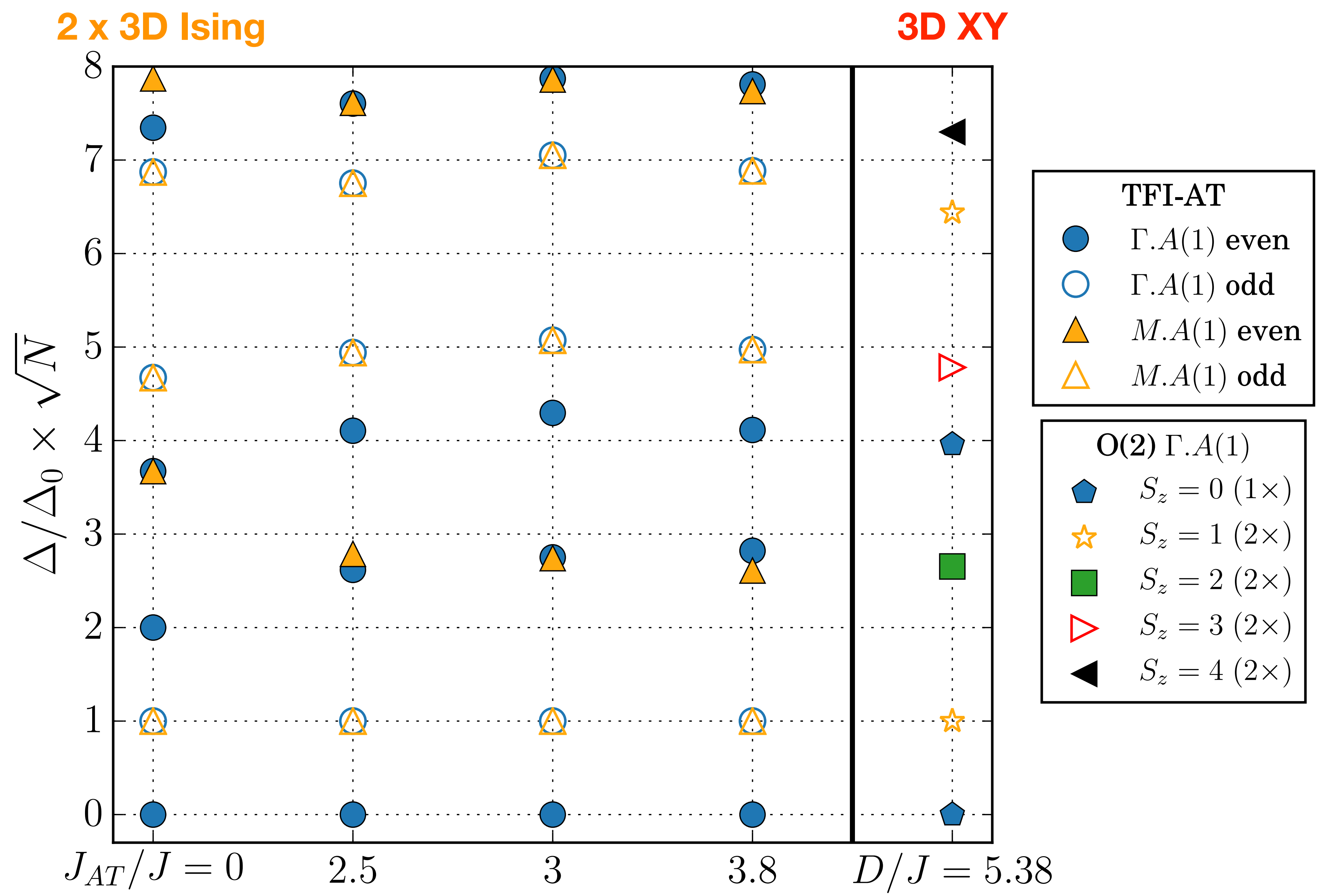


- The direct transition between Z_2 topological order and Z_2 symmetry breaking can be:
  - first order
  - (2 x 3D Ising)* unstable fixed point
  - 3D XY* !

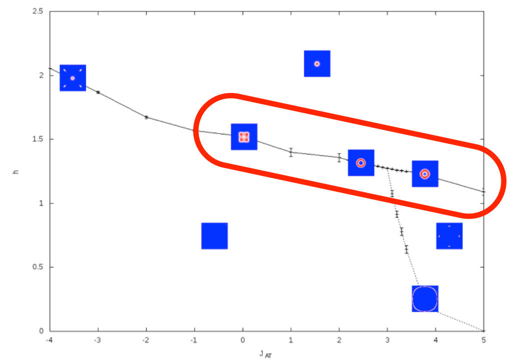
Spectroscopy of QCP



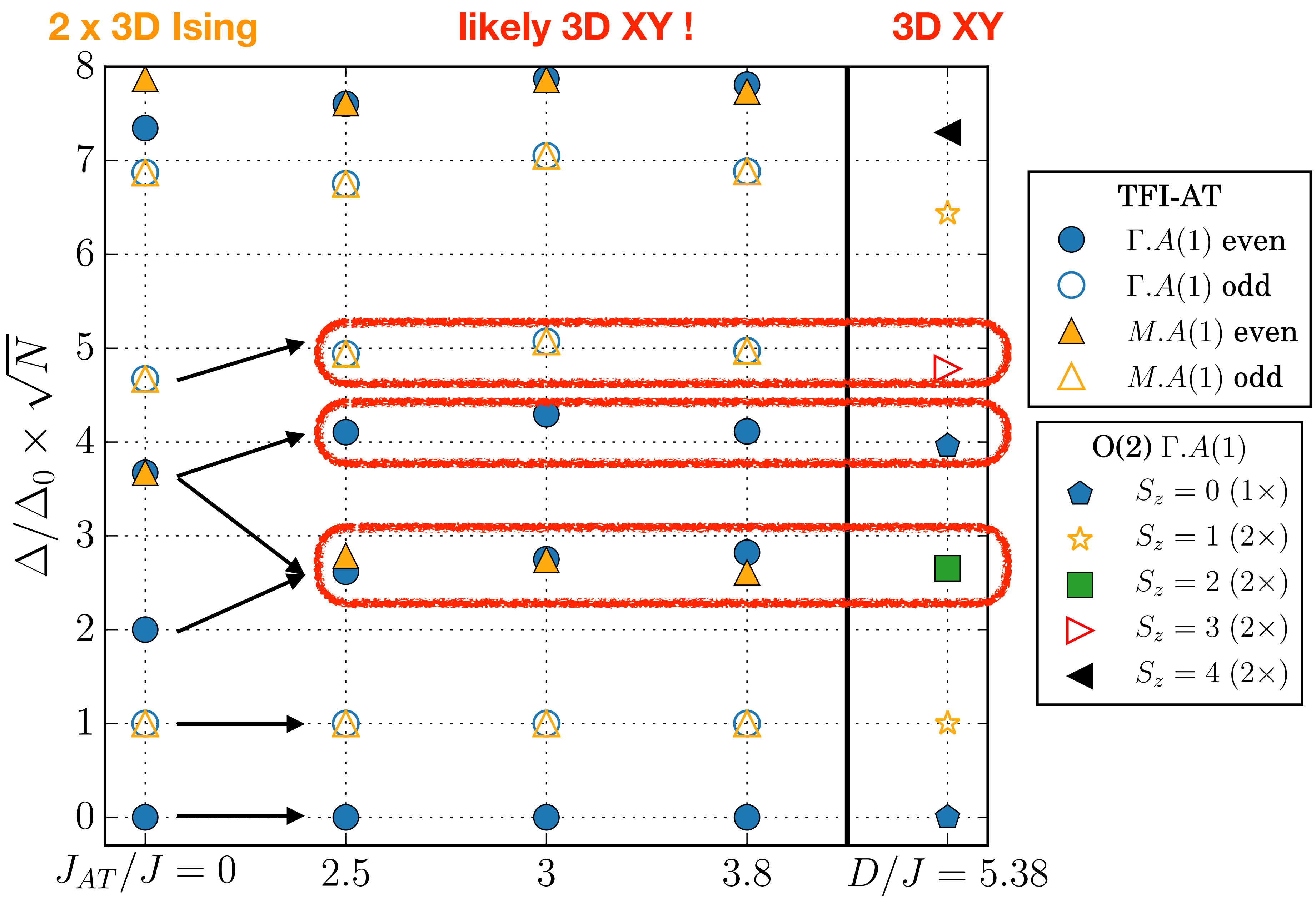
- ED Torus Spectra in the Quantum Ashkin-Teller model at criticality:



Spectroscopy of QCP

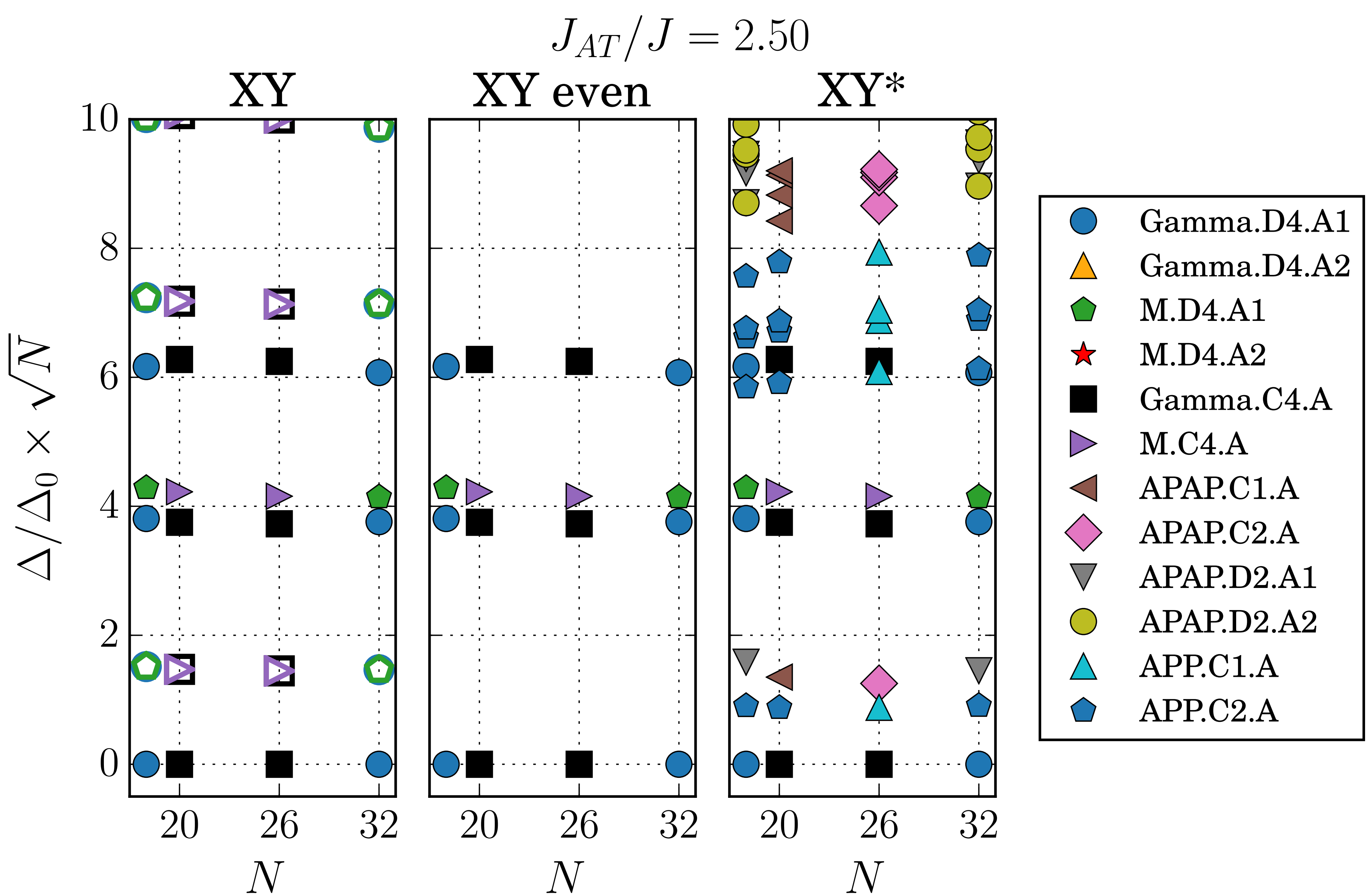


- ED Torus Spectra in the Quantum Ashkin-Teller model at criticality:



Torus energy spectrum of 3D XY*

- Remove all odd charge sectors in 3D XY but add all 4 BC PP/PA/AP/AA sectors:



Outline of this talk

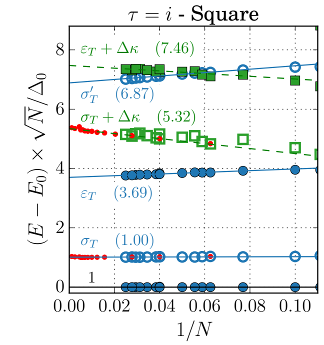
- Torus Energy Spectra and QFT ?
- Spectrum of the “standard” 2+1D Ising transition

  M. Schuler, S. Whittsitt, L.-P. Henry, S. Sachdev & AML
  arXiv:1603:03042
- Spectrum of the “Z_2 confinement” transition (Ising*)

  M. Schuler, S. Whittsitt, L.-P. Henry, S. Sachdev & AML
  arXiv:1603:03042
- Spectrum of the 3D XY* Transition

  L.-P. Henry, M. Schuler and AML
  in preparation
- Outlook

Conclusion / Outlook



- We have shown that the universal torus energy spectrum of the CFT describing quantum critical points is accessible numerically.

- The torus energy spectrum contains valuable information on the “operator content”. It is e.g. able to discriminate the Ising from the Ising* universality class, and 2 x Ising from 3D XY

- We have preliminary results for O(2)/O(3) Wilson-Fisher fixed points and some Gross-Neveu critical points.

- We believe that this technology could help to shed light on more advanced topics, such as the SO(5) symmetry claimed to appear at deconfined critical points by Nahum et al.

- Results from CFT side ?

Collaborators

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PhD Student

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- Harvard University

Seth Whitsitt
PhD Student

Subir Sachdev

Thank you for your attention !