# Chiral magnetic effect
# and
# Natural optical activity
# in
# metals with and without Weyl points

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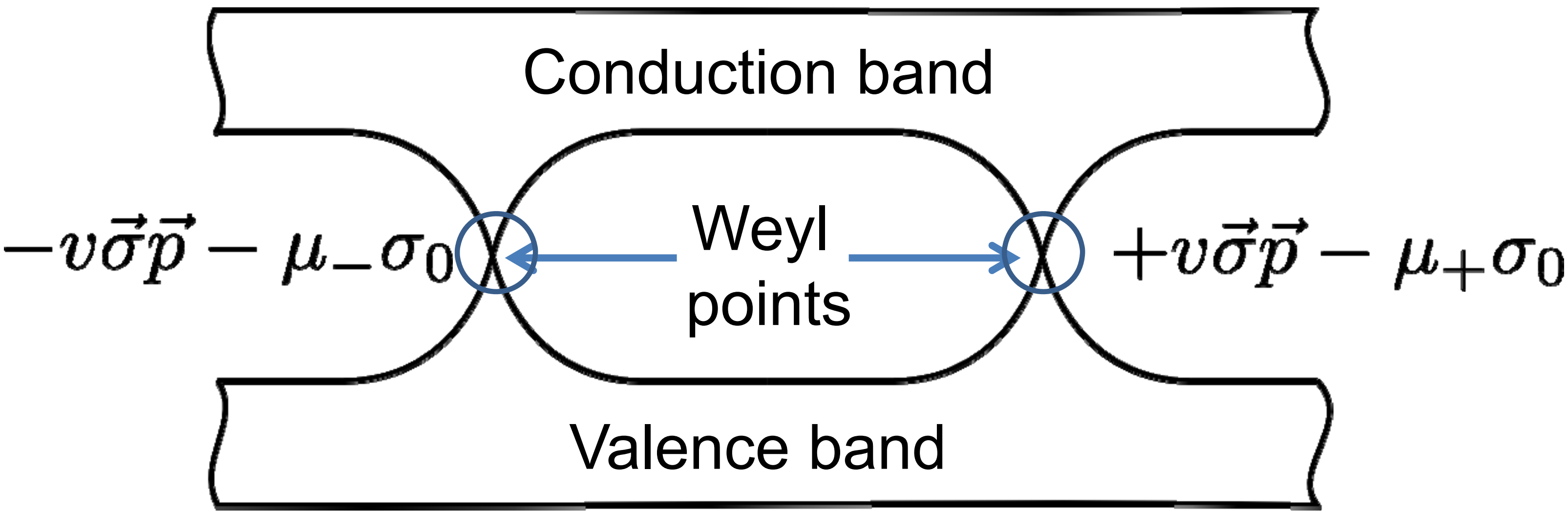
# Weyl Semimetal (WS) is a collection of nondegenerate band touchings in 3D k-space

**"3D Graphene"**




(Herring'37, Abrikosov&Beneslavsky'71, Wan *et al'11,* Burkov&Balents'11)

Two bands ⟹ the spin degeneracy is lifted, either by T or I breaking

# Where does one get a WS?

**Weyl semimetals with inversion breaking:**

*Photonic crystal:* Lu et al, Science, 2015.

*TaAs:*

S-Y. Xu et al., Science 2015

B. Q. Lv et al., PRX 2015

*NbAs:*

Y. Luo et al. , PRB 2015

**Weyl semimetal with time-reversal breaking (?):**

*Er2Ir2O7:*

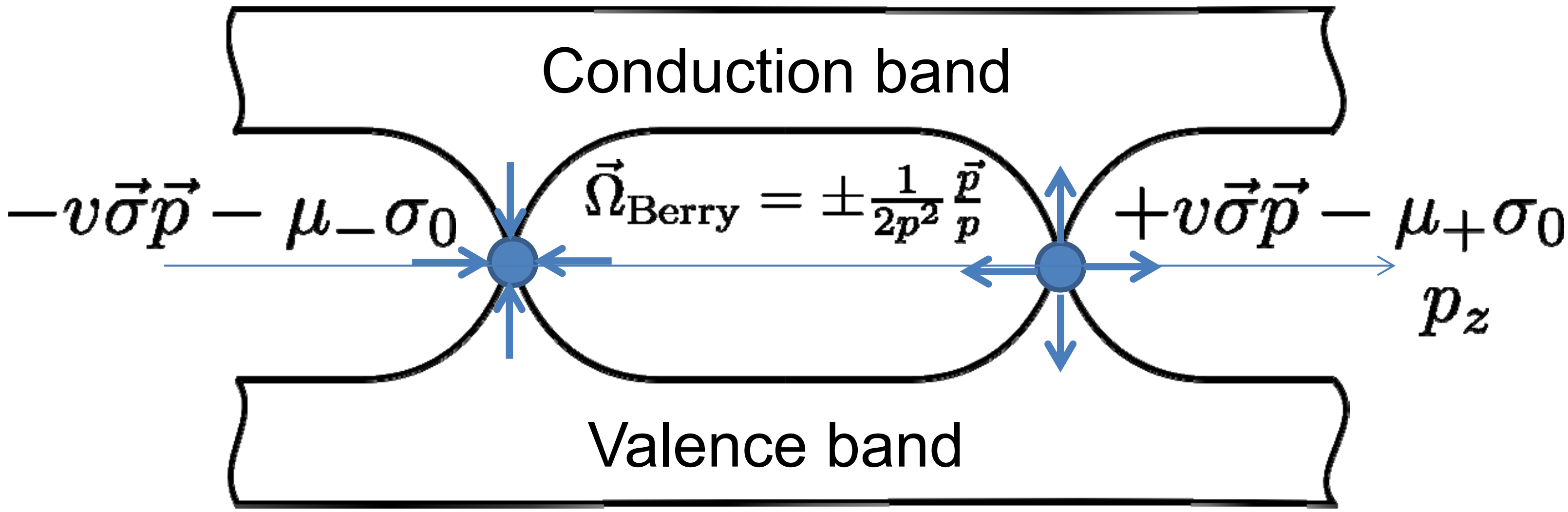
Sushkov et al., PRB (R) 2015

*$YbMnBi_2$:*

Borisenko et al., arxiv:1507.04847

# Weyl semimetals (WS) are gapless phases with nontrivial topology

(review: Turner&Vishwanath, cond-mat: 1301.0330)



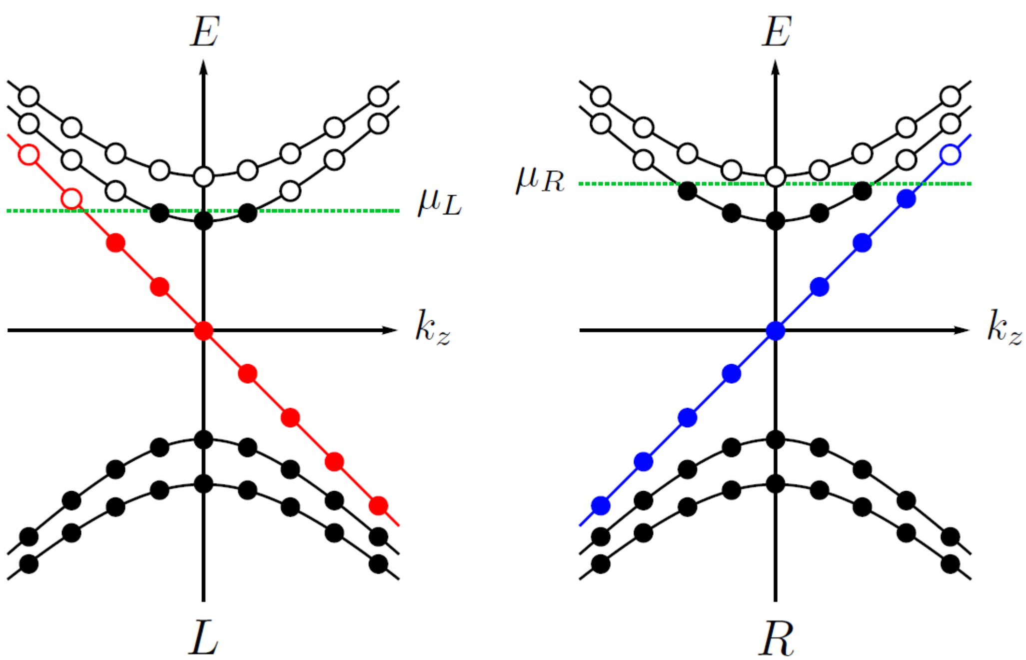

1. Nodes are stable due to topology, not symmetry
2. There are protected surface states('arcs'), which cannot be realized in a 2DEG.
3. Hall response is determined by the distance between nodes, and nothing else (broken TR).
4. The chiral anomaly and the chiral magnetic effect are of geometric origin

# Chiral anomaly in CM setting

$$\vec{B} = (0, 0, B),$$
$$H = \pm v \left[ \vec{\sigma}_\perp (\vec{p}_\perp - e\vec{A}) + \sigma_z p_z \right]$$

"graphene" "gap"

$$\Rightarrow \quad E^{R,L}_{n \neq 0}(p_z) = \pm v \sqrt{2|n|eB\hbar/c + p_z^2}$$
$$E^{R,L}_0(p_z) = \pm v p_z, \quad n = 0$$

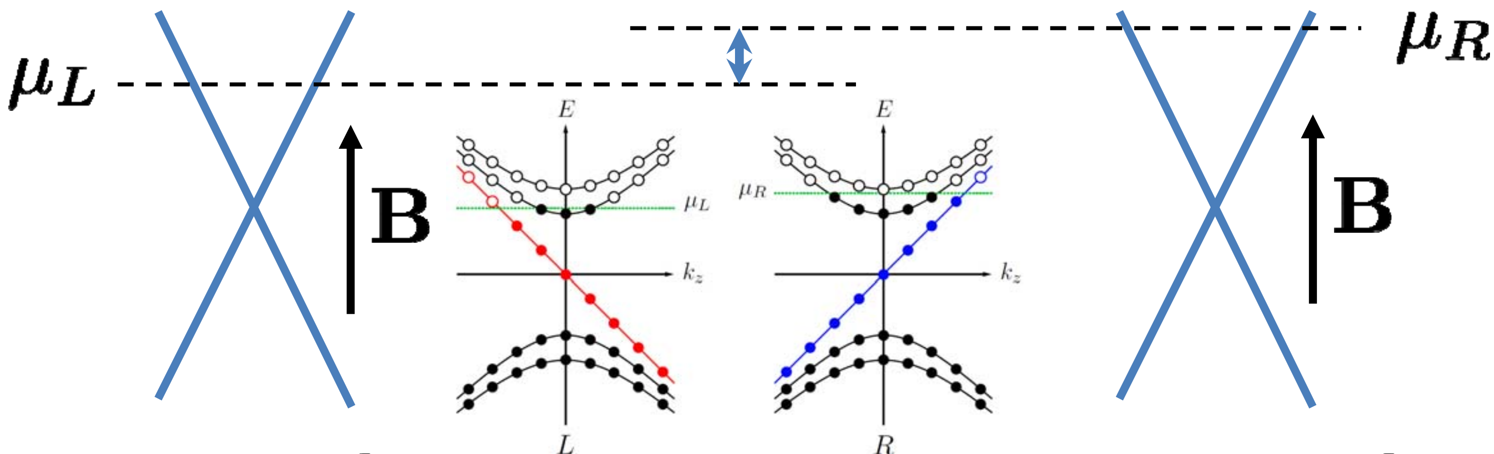



$$\dot{N}_R - \dot{N}_L = \frac{e^2}{2\pi^2\hbar^2 c} \mathbf{E} \cdot \mathbf{B}$$

**"3D chiral anomaly"**

(S. L. Adler, 1969 ; J. S. Bell and R. Jackiw, 1969; Nielsen&Ninomiya, 1983)

# Brief intro into "Chiral magnetic effect"




$$\mathbf{j}_L^{CME} = \frac{e^2 \mu_L}{4\pi^2}\mathbf{B}$$

(Vilenkin, 1980)

$$\mathbf{j}_R^{CME} = -\frac{e^2 \mu_R}{4\pi^2}\mathbf{B}$$

$$\mathbf{j}_{\omega=0}^{CME} = \frac{e^2(\mu_L - \mu_R)}{4\pi^2}\mathbf{B}$$

$$\mathbf{j}_{\omega\neq 0}^{CME} = \frac{e^2(\mu_L - \mu_R)}{12\pi^2}\mathbf{B}$$

(Kharzeev, Warringa, 2009;
Son, Yamamoto, 2013)

# Is there anything interesting in optics?

# "Unusual" electrodynamics in WS

$$S_{WS}[A] = \frac{e^2}{32\pi^2} \int d^4x\, \theta(\mathbf{r},t) \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}$$

"$(\mathbf{E}\cdot\mathbf{B})$"

$$= -\frac{e^2}{8\pi^2} \int d^4x\, \epsilon^{\mu\nu\alpha\beta} \partial_\mu \theta A_\nu \partial_\alpha A_\beta$$

"3D Chern-Simons action", (Goswami&Tewari'12, Zyuzin&Burkov'12)

Current: $j^\rho = -\frac{e^2}{4\pi^2} \partial_\mu \theta \epsilon^{\mu\rho\alpha\beta} \partial_\alpha A_\beta$

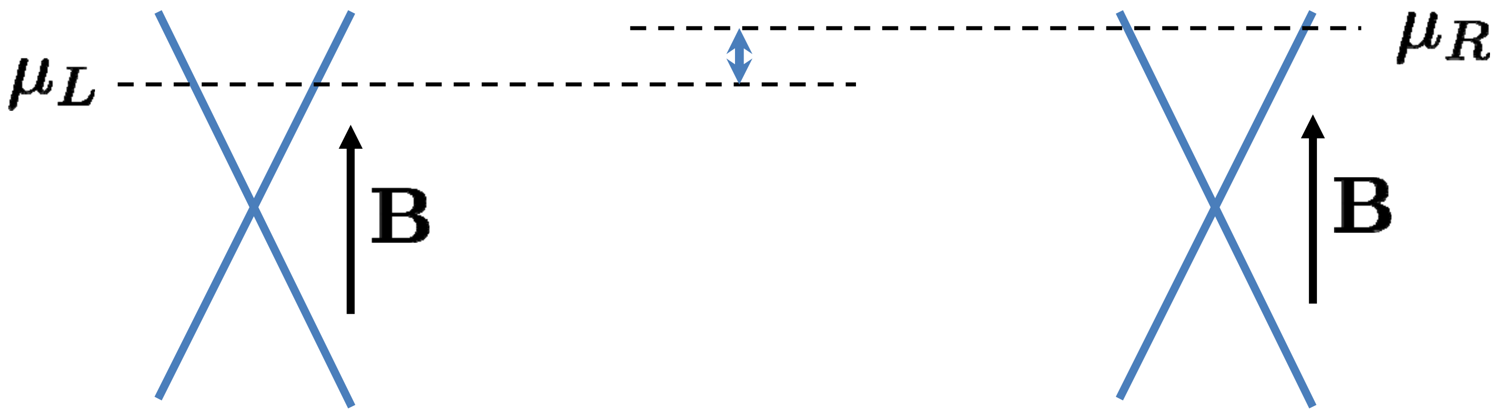
$\mu \neq 0: \; j^\rho = -\frac{e^2}{4\pi^2} \partial_\mu \theta \epsilon^{\mu\rho\alpha\beta} \partial_\alpha A_\beta$ = Anomalous Hall effect

$\mu = 0: \; j^\rho = -\frac{e^2}{4\pi^2} \partial_0 \theta \epsilon^{0\rho\alpha\beta} \partial_\alpha A_\beta$

Levitov, Nazarov
Eliashberg, 1985

$\vec{j} \propto \lambda_{inv} \vec{B}$ ??? Implies $\vec{M} \propto \vec{A}$ in equilibrium

# Recap of "Chiral magnetic effect"




$$\mathbf{j}_L^{CME} = \frac{e^2\mu_L}{4\pi^2}\mathbf{B}$$

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(Kharzeev, Warringa, 2009;
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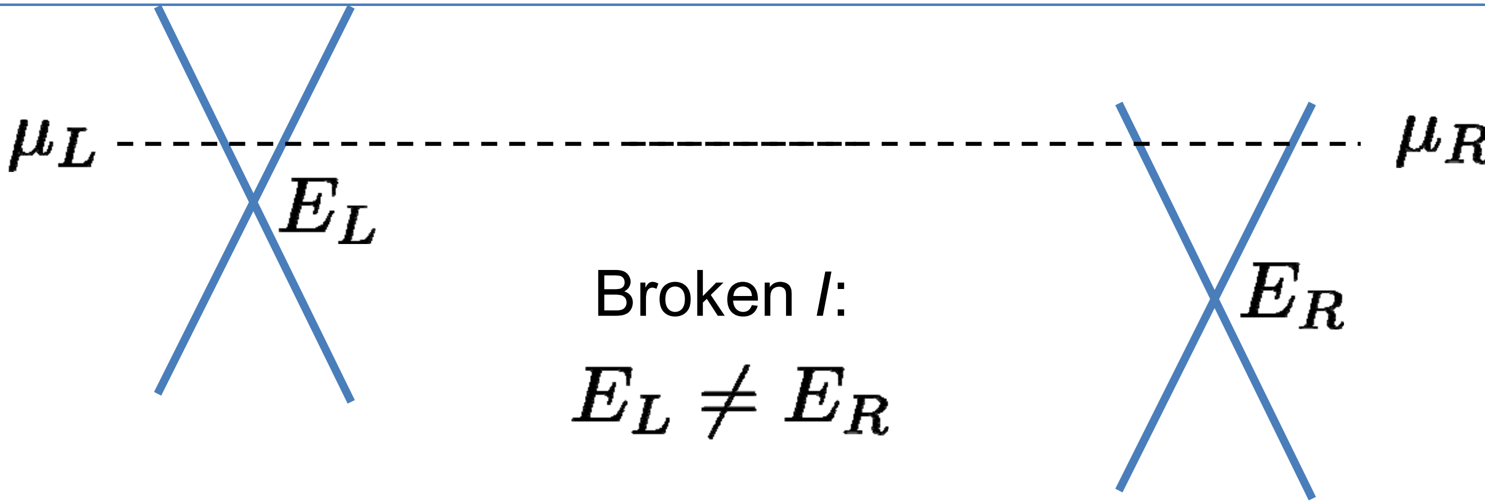
# There is only dynamic CME in equilibrium crystals




Broken *I*:

$$E_L \neq E_R$$

$$\mathbf{j}^{CME}_{\omega=0} = 0$$

(Zhou, Jiang, Niu, Shi, Chin. Phys. Lett., 2013; Vazifeh, Franz, PRL, 2013)

$$\mathbf{j}^{CME}_{\omega\neq0} = ?$$

(Particular model, strong B, broken TR and *I*: Chen, Wu, Burkov, PRB, 2013)

# "Chiral magnetic effect" is natural optical activity

**Natural optical activity**:

$$\sigma_{ab}(\omega, \mathbf{q}) = \sigma_{ab}(\omega) + \lambda_{abc}(\omega) q_c$$

Means different response (refractive index) for right/left circularly polarized light:

$$\boldsymbol{q} = (0, 0, q_z) : \epsilon_{ij} = \left(1 - \frac{\omega_p^2}{\omega^2}\right)\delta_{ij} - \frac{\lambda q_z}{\omega \epsilon_0}(\tau_2)_{ij}$$

**Chiral magnetic effect:**

$$\mathbf{j}(\omega, \mathbf{q}) = \eta(\omega, \mathbf{q}) \mathbf{B}(\omega, \mathbf{q})$$

$$\mathbf{B} = \tfrac{1}{\omega} \mathbf{q} \times \mathbf{E}$$

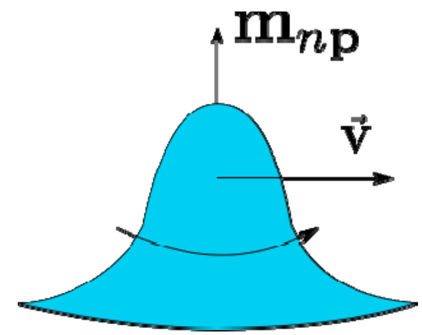
$$\lambda_{abc}^{metal}(\omega) = -\frac{\eta(\omega, \mathbf{q})}{\omega} \epsilon_{abc}$$

(non-analytic function of frequency)

# CME for an arbitrary band structure: multiband calculation

# CME is an interband coherence effect

Semiclassical corrections to intraband kinetics:




**Berry curvature:**

$$\mathbf{\Omega}_{n\mathbf{p}} = i\langle \partial_{\mathbf{p}} u_{n\mathbf{p}}| \times |\partial_{\mathbf{p}} u_{n\mathbf{p}}\rangle,$$

$$\delta\dot{\mathbf{r}} = -e\mathbf{E} \times \mathbf{\Omega}_{n\mathbf{p}} - e(\mathbf{v}_{n\mathbf{p}} \cdot \mathbf{\Omega}_{n\mathbf{p}})\mathbf{B}$$

**Orbital magnetic moment:**

$$\mathbf{m}_{n\mathbf{p}} = \frac{ie}{2}\langle \partial_{\mathbf{p}} u_{n\mathbf{p}}| \times (h_{\mathbf{p}} - \epsilon_{n\mathbf{p}})|\partial_{\mathbf{p}} u_{n\mathbf{p}}\rangle,$$

$$E_{n\mathbf{p}} = \epsilon_{n\mathbf{p}} - \mathbf{m}_{n\mathbf{p}}\mathbf{B}, \quad \mathbf{v}_{n\mathbf{p}} = \partial_{\mathbf{p}}\epsilon_{n\mathbf{p}} - \partial_{\mathbf{p}}(\mathbf{m}_n\mathbf{B})$$

**NB: non-uniform orbital magnetization leads to current!**

# Semiclassical corrections to kinetics

Standard single-band kinetic equation:

$$\partial_t f_{n\mathbf{p}} + \dot{\mathbf{r}}\partial_{\mathbf{r}} f_{n\mathbf{p}} + \dot{\mathbf{p}}\partial_{\mathbf{p}} f_{n\mathbf{p}} = 0$$

- no collisions, high frequency

Equations of motion:

$$\begin{aligned} \dot{\mathbf{r}} &= \mathbf{v}_{n\mathbf{p}} - \dot{\mathbf{p}} \times \mathbf{\Omega}_{n\mathbf{p}}, \\ \dot{\mathbf{p}} &= e\mathbf{E} + e\dot{\mathbf{r}} \times \mathbf{B}. \end{aligned}$$

$$f_{n\mathbf{p}} = f_{eq}(\epsilon_{n\mathbf{p}}) + \frac{1}{i(\omega - \mathbf{q}\partial_{\mathbf{p}}\epsilon_{n\mathbf{p}})} e\mathbf{E}\partial_{\mathbf{p}} f_{eq}$$

Expression for the current:

$$\mathbf{j}_{qp} = e\sum_n \int (d\mathbf{p}) \left(\partial_{\mathbf{p}}\epsilon_{n\mathbf{p}} - \partial_{\mathbf{p}}(\mathbf{m}_n\mathbf{B}) - e\mathbf{E} \times \mathbf{\Omega}_{n\mathbf{p}} - e(\partial_{\mathbf{p}}\epsilon_{n\mathbf{p}} \cdot \mathbf{\Omega}_{n\mathbf{p}})\mathbf{B}\right) f_{n\mathbf{p}}$$

$$\mathbf{j}_m = \partial_{\mathbf{r}} \times \sum_n \int (d\mathbf{p}) \mathbf{m}_{n\mathbf{p}} f_{n\mathbf{p}}$$

# CME/NOA for an arbitrary band structure

**Details:** J. Ma, **DP**. Phys. Rev. B **92**, 235205 (2015)

$$\sigma_{ab}(\omega, \mathbf{q}) = \sigma_{ab}(\omega) + \lambda_{abc}(\omega) q_c$$

$$\lambda_{abc} = -\frac{e^2}{\omega} \sum_n \int (d\mathbf{p})$$

$$\Big( \epsilon_{n\mathbf{p}} (\partial_{\mathbf{p}} f_{n\mathbf{p}} \cdot \mathbf{\Omega}_{n\mathbf{p}}) \epsilon_{abc}$$ Static CME, vanishes

$$+ \frac{1}{e} m_{nd} \partial_a f_{n\mathbf{p}} \epsilon_{dbc}$$ Velocity renormalization

$$+ \frac{1}{e} m_{nd} \partial_b f_{n\mathbf{p}} \epsilon_{adc} \Big)$$ Magnetization current from non-uniform acceleration

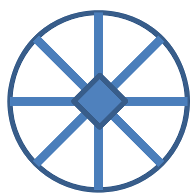
$$\sigma_{ab}(\omega, \mathbf{q}) = \sigma_{ba}(\omega, -\mathbf{q}) \Rightarrow \lambda_{abc}(\omega) = -\lambda_{bac}(\omega)$$

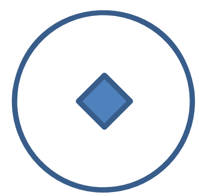
# CME in simple models

# Chiral magnetic effect without Weyl points

Model of a helical metal with $C_4$ point group:

$$H^{C_4} = \frac{p^2}{2m} + \underbrace{\boldsymbol{\sigma}\mathbf{d}_{\mathbf{p}}^{C_{4v}}}_{C_{4v}\ \mathrm{SOC}} + \underbrace{\gamma_v p_x p_y (p_x^2 - p_y^2)}_{C_{4v} \to C_4}$$

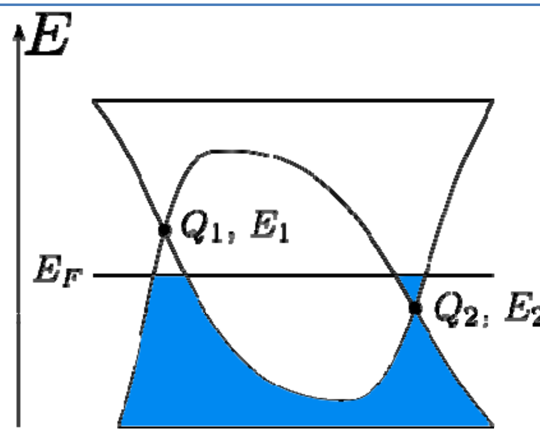




$$\mathbf{j}^{CME} = -\frac{e^2}{105\pi^2}\frac{\gamma p_F^5 \cdot \gamma_v p_F^4}{\mu}\mathbf{B}$$

# Chiral magnetic effect in an ideal WS

$$H_w = E_w - \mu_w + Q_w v_w \sigma \mathbf{p}$$

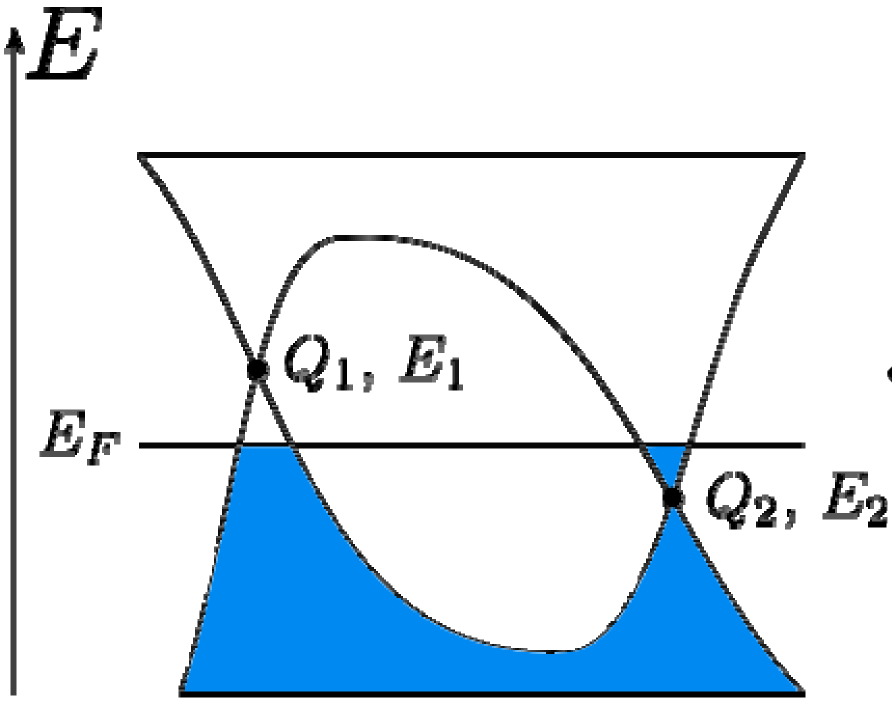



Results for "chiral conductivity", $j^{CME}/B$ :

| | $\mu_w = \mu$ | $E_w = E$ |
|---|---|---|
| Static | 0 | $\frac{e^2}{4\pi^2}\sum_w Q_w \mu_w$ |
| Dynamic | $\frac{e^2}{6\pi^2}\sum_w Q_w E_w$ | $\frac{e^2}{12\pi^2}\sum_w Q_w \mu_w$ |

(see Chang, Yang, PRB 2015, Ma, **DP**, PRB 2015, Zhong, Moore, Souza, PRL 2016)

# Chiral magnetic effect in a WS: numbers




$$\mathbf{j} = \frac{e^2}{6\pi^2\hbar^2 c}\mathbf{B}\sum_w Q_w E_w$$

Rotatory power:

$$\rho \sim \alpha\frac{\Delta E}{hc} \sim 10^{-2}\mathrm{rad/mm}$$

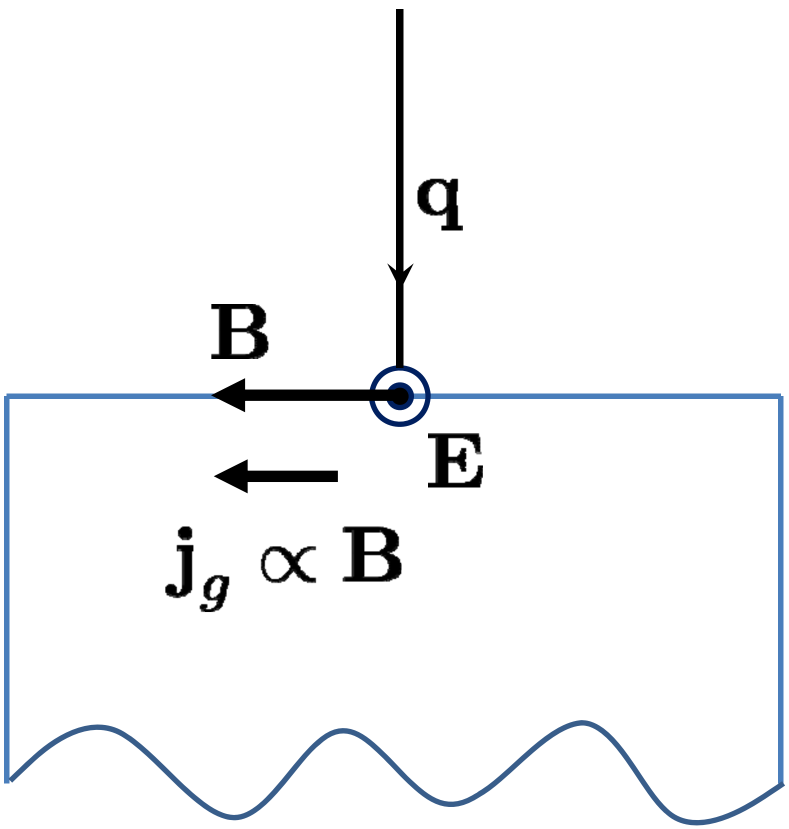
Skin layer $\sim 100$ nm:

$$\Delta\phi_{\mathrm{skin}} \sim 10^{-6}\mathrm{rad}$$

# Real samples have boundaries…

# The role of sample boundaries (or "the absence of polar Kerr effect")

**The problem:** **bulk gyrotropic current leads to polar Kerr effect. FORBIDDEN in a TR-inv crystal**




$\mathbf{j}_g$ radiates waves of polarization orthogonal to the incident one, hence polarization rotation, which is **not allowed**

(B. I. Halperin, 1992;
A. Shelankov, Pikus, 1992)

# The role of sample boundaries
# (or “the absence of polar Kerr effect”)

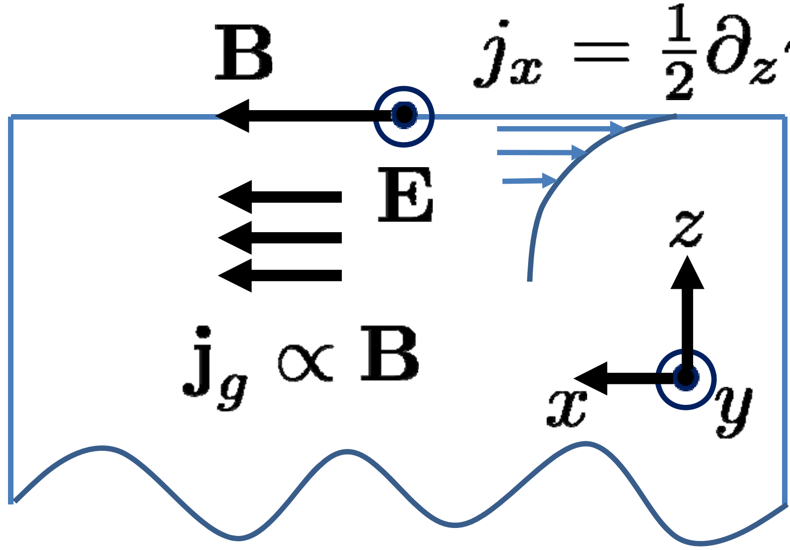
**The solution:** *surface “Hall” current*

$$j_{g,a} = \gamma_{abc} \partial_c E_b$$

(Agranovich, Yudson, 1973, via Onsager relations)

$$j_{g,a} = \frac{1}{2}\{\gamma_{abc}(\mathbf{r}), \partial_c\} E_b = \gamma_{abc}\partial_c E_b + \underbrace{\frac{1}{2}\partial_c \gamma_{abc}}_{} \cdot E_b$$

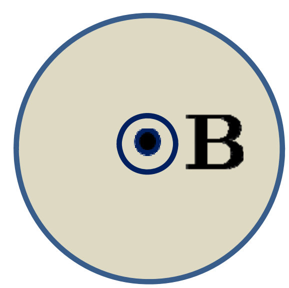
$$j_x = \frac{1}{2}\partial_z \gamma_{xyz} \cdot E_y$$




“Hall” conductivity,

$$\gamma_{abc} = -\gamma_{bac}$$

(Cuprates: Hosur et al., 2015)




$$j_{surf} = -\frac{1}{2} j_{bulk}$$

(compare to $j_{surf} = -2 j_{bulk}$
Baireuther et al, 2016)

# Conclusions

- Chiral magnetic effect is equivalent to natural optical in metals.
- Semiclassically, natural optical activity of metals is due to the orbital magnetic moment of quasiparticles. It has its roots in geometry, not topology.
- Weyl points are not required for CME/NOA
- 3D Chern-Simons action is not a property of WS, but of any helical metal with natural optical activity. Maxwell equations need not be revised.
- Experimentally, polarization rotation is confined to the skin layer, and is limited by a few $\mu\mathrm{rad}$