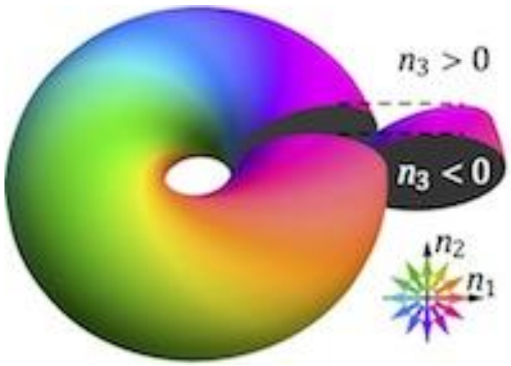
Duality in 2 + 1 Dimensions



KITP

Topological Quantum Matter

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IAS

Based on:

NS and E. Witten, arXiv:1602.04251;

NS, T. Senthil, C. Wang, and E. Witten, arXiv:1606.01989;

P.-S. Hsin, NS, arXiv:1607.07457

Recent related papers:

A. Karch and D. Tong, arXiv:1606.01893;

J. Murugan and H. Nastase, arXiv:1606.01912

Three (almost) independent lines of development – the unity of physics

- The condensed matter, $3d$ quantum field theory route
- The supersymmetric route
- The AdS/CFT, large N route

Unlike the standard CM literature

- Use (probably inessential)
  - Continuum QFT (no lattice)
  - Lagrangian (rather than Hamiltonian)
  - Lorentz invariance
- Significant
  - All gauge fields and Chern-Simons terms are properly normalized
  - Phase of the Fermion determinant is $-i\pi\eta/2$, rather than Chern-Simons at level 1/2. To compare with the imprecise treatment, replace

$$i\bar{\Psi}\not{D}_A\Psi \quad \rightarrow \quad i\bar{\Psi}\not{D}_A\Psi - 1/(8\pi)AdA$$

  - Spin/charge relation – A, a, … are spin$_c$ connections

Conjectured dualities

3 [Aharony] (+ 1 [Hsin, NS])

N_f scalars at $|\Phi|^4$ point coupled to ↔ N_f fermions coupled to

- $SU(N)_k \leftrightarrow U(k)_{-N+N_f/2,-N+N_f/2}$
- $U(N)_{k,k} \leftrightarrow SU(k)_{-N+N_f/2,-N+N_f/2}$
- $U(N)_{k,k+N} \leftrightarrow U(k)_{-N+N_f/2,-N-k+N_f/2}$
- $U(N)_{k,k-N} \leftrightarrow U(k)_{-N+N_f/2,-N+k+N_f/2}$

Fits the large N picture ($N, k \to \infty$ with finite N/k)

Fits the supersymmetric picture

Consistent with mass deformations

Related to level/rank duality (set $N_f = 0$)

Baryon and monopole operators match [Radicevic]

Puzzles

- Is it true for all N, k, N_f?
- How can a theory of bosons, which does not need a spin structure be dual to a theory of fermions, which needs it?
- What is the relation to various dualities in the condensed matter literature (with puzzles about quantization of coefficients, T-reversal invariance, etc.)?
- What is the precise statement of the dualities (including the coupling to background gauge fields and their Chern-Simons counterterms)?
- Are the assumptions independent? Can we assume some of these dualities and derive others?
- Are there other such dualities?

Particle/vortex duality
[Peskin; Dasgupta and Halperin]

- $$|D_B\Phi|^2 - |\Phi|^4 \quad \leftrightarrow \quad |D_{\hat{b}}\hat{\Phi}|^2 - |\hat{\Phi}|^4 + \frac{1}{2\pi}Bd\hat{b}$$
- B is a background field coupled to a global $U(1)_B$ symmetry.
- RHS is a gauged version of the theory in the LHS. $\hat{b}$ is a dynamical field.
- IR duality – two different theories flowing to the same IR fixed point.
- $\Phi \leftrightarrow \mathcal{M}_{\hat{b}}$ is a monopole operator of $\hat{b}$ (charged under $U(1)_B$).
- $|\Phi|^2 \leftrightarrow -\left|\hat{\Phi}\right|^2$. Upon deformation: unbroken $U(1)_B$ phase is Higgs phase in the RHS; broken $U(1)_B$ phase massless $\hat{b}$.

A new boson/boson duality

Assume the standard particle/vortex duality

$$|D_B\Phi|^2 - |\Phi|^4 \quad \leftrightarrow \quad |D_{\hat{b}}\hat{\Phi}|^2 - |\hat{\Phi}|^4 + \frac{1}{2\pi}Bd\hat{b}$$

Derive other dualities by changing the two sides:

- Add a Chern-Simons counterterm for the classical field B
- Gauge it by turning B into a dynamical field b and adding a new classical gauge field A.
- Use other dualities.
- Repeat.

A new boson/boson duality

$$|D_B\Phi|^2 - |\Phi|^4 \quad \leftrightarrow \quad |D_{\hat{b}}\hat{\Phi}|^2 - |\hat{\Phi}|^4 + \frac{1}{2\pi}Bd\hat{b}$$

Add to the two sides $\frac{1}{4\pi}BdB + \frac{1}{2\pi}AdB$ and turn B into a dynamical $U(1)$ gauge field b

$$|D_b\Phi|^2 - |\Phi|^4 + \frac{1}{4\pi}bdb + \frac{1}{2\pi}Adb \quad \leftrightarrow$$

$$|D_{\hat{b}}\hat{\Phi}|^2 - |\hat{\Phi}|^4 + \frac{1}{2\pi}bd\hat{b} + \frac{1}{4\pi}bdb + \frac{1}{2\pi}Adb$$

In the right hand side b represents an almost trivia theory $U(1)_1$ and it can be integrated out…

A new boson/boson duality

$$|D_b\Phi|^2 - |\Phi|^4 + \frac{1}{4\pi}bdb + \frac{1}{2\pi}Adb \quad \leftrightarrow$$

$$|D_{\hat{b}}\hat{\Phi}|^2 - |\hat{\Phi}|^4 - \frac{1}{4\pi}\hat{b}d\hat{b} - \frac{1}{2\pi}Ad\hat{b} - \frac{1}{4\pi}AdA$$

- For $A = 0$ we see that the Wilson-Fisher theory coupled to $U(1)_1$ is dual to itself with $U(1)_{-1}$; i.e. it is T-invariant.
  - This symmetry is not manifest as $|\Phi|^2 \leftrightarrow -\left|\widehat{\Phi}\right|^2$.
- The coupling to the background A needs an AdA counterterm; i.e. T-invariance is anomalous for nonzero A.
- A monopole operator $\mathcal{M}_b$ is not $U(1)_b$ invariant, but $\Phi^+\mathcal{M}_b$ is. It is charged under $U(1)_A$ and has spin $\frac{1}{2}$.

A new boson/boson duality

Deform the theory by $|\Phi|^2 \leftrightarrow -|\hat{\Phi}|^2$

- With one sign: gapped, low energy Lagrangian is $-\frac{1}{4\pi} AdA$
  - Φ is massive. The Φ particle is charged under $U(1)_b$ and therefore it is charged under $U(1)_A$. Its spin is transmuted to $\frac{1}{2}$.
  - $\langle\hat{\Phi}\rangle$ Higgses $U(1)_{\hat{b}}$. Vortex is charged under $U(1)_A$ and its spin is $\frac{1}{2}$.
- With the opposite sign (T-reversal) the description is exchanged between the two sides of the duality. The particles have spin $-\frac{1}{2}$. Low energy Lagrangian vanishes.

Suggests a new boson/fermion duality

This looks a lot like the theory of a free massless fermion

$$i\bar{\Psi} \not{D}_A \Psi \quad \leftrightarrow \quad |D_b\Phi|^2 - |\Phi|^4 + \frac{1}{4\pi} bdb + \frac{1}{2\pi} Adb$$

- Global $\mathrm{U(1)_A}$ symmetry
- T-invariance with an anomaly $\frac{1}{4\pi} AdA$
- $\Psi \ \leftrightarrow \ \Phi^{+}\mathcal{M}_b$ is charged under $U(1)_A$ and has spin $\frac{1}{2}$.
- With mass deformation $\bar{\Psi}\,\Psi \ \leftrightarrow \ |\Phi|^2$ depending on the sign
  - the low energy theory is trivial or $-\frac{1}{4\pi} AdA$
  - a particle with spin $\pm\frac{1}{2}$
- Can also find it by boldly substituting $N = k = N_f = 1$ in the conjectured duality with generic $N,\ k,\ N_f$.

Derive many other dualities

Starting with

$$i\bar{\Psi} \not{D}_A \Psi \quad \leftrightarrow \quad |D_b\Phi|^2 - |\Phi|^4 + \frac{1}{4\pi} bdb + \frac{1}{2\pi} Adb$$

we can derive other dualities by changing the two sides:

- Add a Chern-Simons counterterm for the classical field A
- Gauge it by turning A into a dynamical field a and adding a new classical field.
- Use other dualities.
- Repeat.

Another boson/fermion duality

For example, derive:

$$|D_B\Phi|^2 - |\Phi|^4 \quad \leftrightarrow \quad i\bar{\Psi}\not{D}_a\Psi + \frac{1}{2\pi}Bda - \frac{1}{4\pi}BdB$$

LHS Wilson-Fisher fixed point

RHS QED with a single fermion, a.k.a $U(1)_{1/2}$

- Similar to [Chen, Fisher, Wu; Barkeshli, McGreevy]
- Derived from the other duality
- Neither side needs a spin structure when a is a spin$_c$ connection
- Need a Chern-Simons counterterm for B
- Can map the operators and check the phases
- RHS is T-reversal invariant (quantum symmetry)

A fermion/fermion duality

Derive: $i\bar{\Psi}\not{D}_A\Psi \quad \leftrightarrow$

$$i\bar{\chi}\not{D}_a\chi - \frac{2}{4\pi}bdb + \frac{1}{2\pi}adb + \frac{1}{2\pi}Adb - \frac{1}{4\pi}AdA$$

RHS QED with a single fermion, coupled to $U(1)_{-2}$ of b.

- *Incorrectly* integrating out b, we find a Lagrangian with *improperly* quantized CS terms [Son; Wang, Senthil; Metlitski, Vishwanath]

$$i\bar{\chi}\not{D}_a\chi + \frac{1}{8\pi}ada + \frac{1}{4\pi}adA - \frac{1}{8\pi}AdA$$

- No need for a spin structure when a and A are spin_c connections.
- Can map the operators and check the phases
- T-reversal invariance (with anomaly) is manifest in LHS. It acts non-trivially in the RHS (quantum symmetry).

More

- ✓ Many more dualities and relations between them
- ✓ Add gravitational Chern-Simons counterterms (more checks)
- ✓ Relation to $4d$ S-duality in half-space with these $3d$ theories on the boundary (Witten's S and T operations on $3d$ field theories)
- ✓ Generalization to arbitrary N and k
  - Using a precise version of level/rank duality
  - Problem with large N_f
  - Leads to many more dualities
- ❑ Much more can be done