Topological, gapless, and critical matter:
More and the Same

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Illustrate some general principles through specific simple examples.

Emphasis: Physical pictures/intuition.

Topological insulators 1.0

Free electron band theory:

two distinct insulating phases of electrons in the presence of time reversal symmetry.

(i) Conventional Band Insulator

(ii) Topological Band Insulator (TBI)

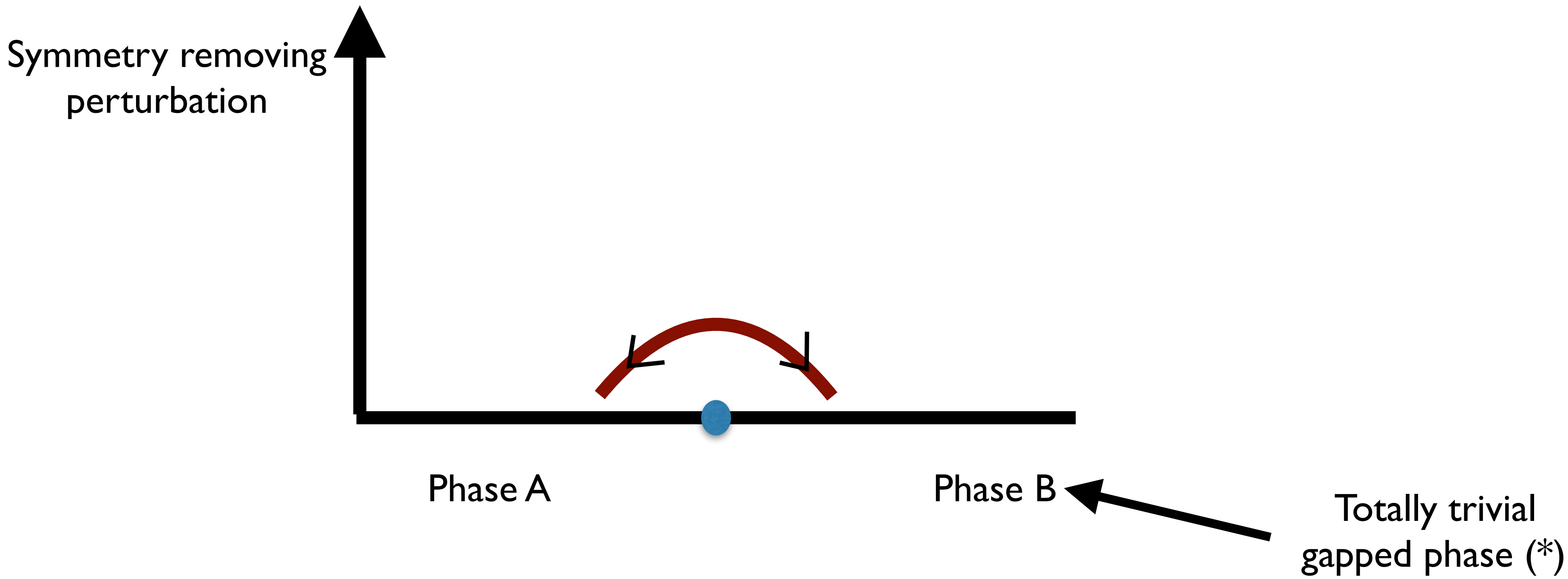
TBI occurs in materials with strong spin-orbit coupling.

Modern theoretical work:
Topological insulators beyond band theory

Topological Insulators: special case of more general class of phases of matter

``Symmetry Protected Topological'' phases

- may occur even with electron-electron interactions.



(*)More precise meaning: smoothly connected to wave function with no quantum entanglement between local degrees of freedom.

Topological insulators beyond band theory:

Some obvious conceptual questions

Spin-orbit coupled insulators of ordinary electrons with no exotic (i.e fractional charge/statistics) excitations

1. Are topological insulators stable to inclusion of electron-electron interactions?

2. Are there new interaction-enabled topological insulators with no band theory counterpart?

3. Similar questions for electrons with other symmetries? Generalizations to bosons/spin systems?

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Focus on space dim d = 3.

Some trivial observations about electronic insulators

Insulator with no exotic elementary excitations:

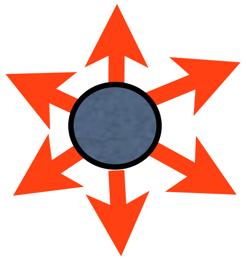
All excitations carry integer charge ne (e = electron charge).

n odd: fermion (eg: n = 1 is electron)
n even: boson. (eg: n= 2 is Cooper pair)

A powerful ***conceptual*** tool

A `gedanken' experiment:

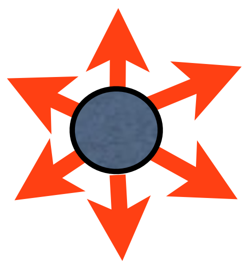
Probe the fate of a magnetic monopole inside the material.

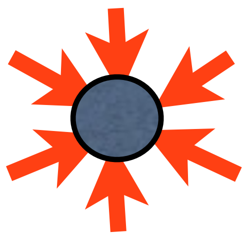


Thinking about the monopole is a profoundly simple way to non-perturbatively constrain the physics of the material.

Monopoles and symmetries

Time-reversal: Magnetic charge is odd
Electric charge is even





Suppose the monopole M has electric charge q.

Time reversed partner TM also has electric charge q but opposite magnetic charge.

Monopoles and symmetries

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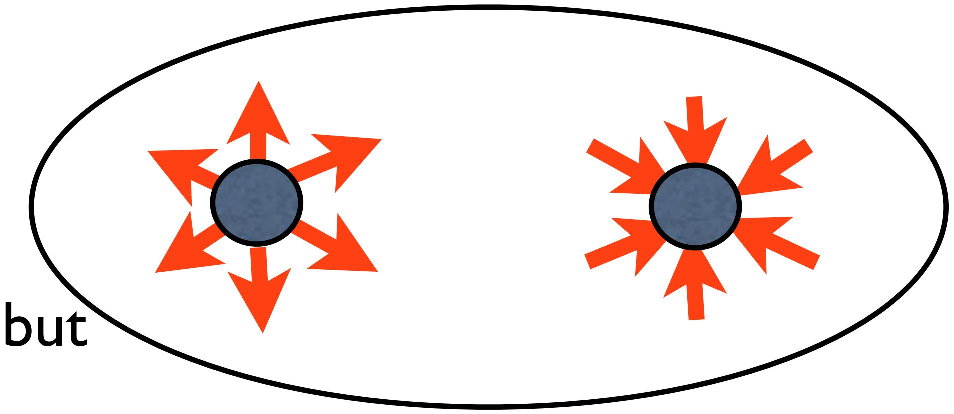


Suppose the monopole M has electric charge q.

Time-reversed partner TM also has electric charge q but opposite magnetic charge.

Bring M and TM together: result must be an excitation of the underlying material.

=> $2q = ne$ (n = integer)

Only two distinct possibilities consistent with time reversal
$q = 0, e, 2e, \ldots\ldots$ or $q = e/2, 3e/2, \ldots\ldots\ldots$

Fundamental distinction: $q = 0$ or $q = e/2$ (obtain rest by binding electrons)

Monopoles and topological insulators

Ordinary insulator: Monopoles have q = 0.

Topological band insulator: Monopoles have q = e/2.
(Qi, Hughes, Zhang 09)(*)

Fractional charge on probe monopole cannot be shifted by any perturbations (which preserve symmetry).

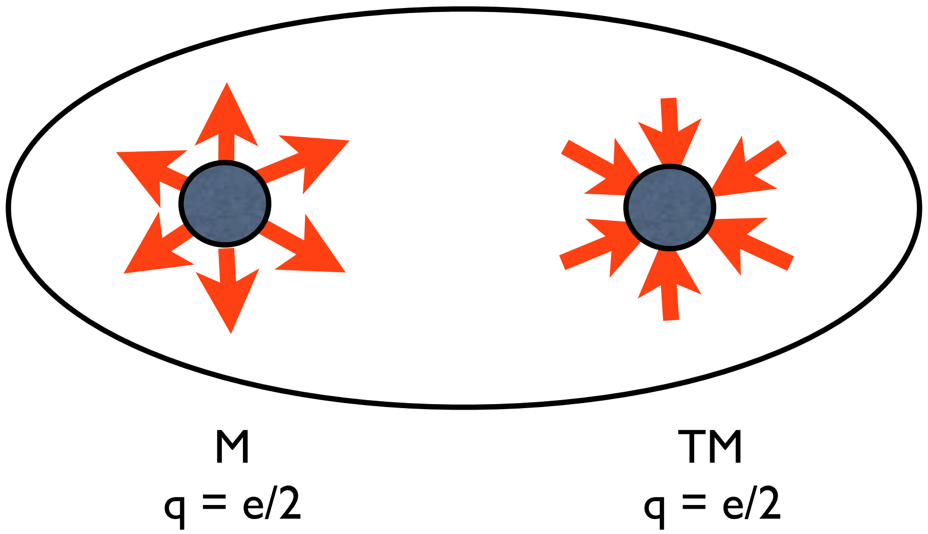
Topological band insulator stable to interactions.

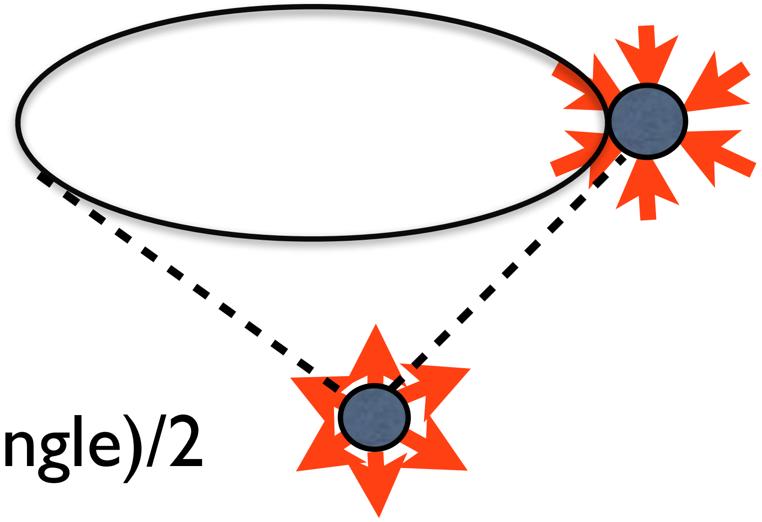
(*) Proof: Solve surface Dirac equation on sphere in presence of monopole in the bulk, or alternately use $\theta = \pi$ electromagnetic response and it's ``Witten effect''

Role of Fermi statistics and Kramers

Study, quantum mechanically, 2-particle problem for the bound state of M and TM.

TM and M see each other as a charge sees a monopole.





S_B = (solid angle)/2

=> Bound state (i.e, the basic charge e particle) is a ***Kramers doublet fermion***

Wang, Potter, TS, 2014; Metlitski, Kane, Fisher, 2014.

A converse question

Is fractional electric charge q = e/2 on a probe monopole in an **electronic** insulator only possible for the standard topological band insulator?

No!

q = e/2 on a probe monopole is a property of 4 distinct time reversal symmetric SPT insulators (with spin-orbit coupling) of electrons in 3d (Wang, Potter, TS, 2014; Freed, Hopkins, 2016).

Only one of them is standard topological band insulator; others require interactions.

Rather than just an isolated curiosity, symmetry protected topological insulators are centrally connected to other frontiers of modern condensed matter physics.

Orientation

Conventional ordered phases of matter:

Concepts of broken symmetry/ Long Range Order (LRO)

Characterize by Landau order parameter.

Examples

$$|\uparrow\uparrow\uparrow\uparrow\,\ldots\ldots\ldots\rangle \qquad\qquad |\uparrow\downarrow\uparrow\downarrow\,\ldots\ldots\ldots\rangle$$

Ferromagnet Antiferromagnet

Landau Order

Known for several millenia

Non-Landau order I: Topological quantum matter

Examples

(i) Symmetry Protected Topological (SPT) matter

- topological band insulators/superconductors, Haldane S = 1 chain, …….

(ii) ``Topological ordered" matter

- Quantum Hall states, gapped quantum spin liquids, ….

Emergence of **sharp** quasiparticles with long range statistical interactions, and possibility of fractional quantum numbers.

Low energy effective theory: a topological field theory

Known since 1980s

Non-Landau order II: Beyond topological order.
Gapless phases

Most familiar: Landau fermi liquid

Interesting variants: Dirac, Weyl,…… materials (Monday afternoon session)

Other known examples: Gapless quantum spin liquids, 1/2 filled Landau level
Yet others: non-fermi liquid metals , bose metals, ,,,,,,,

(Talks: Yinchen He, Thursday 2 pm, Haldane, Monday 11 am, Young, Metlitski, Motrunich, Wang, Tuesday morning)

Non-Landau order II: Beyond topological order
Gapless phases

Protected gapless excitations:

In simple cases these can be given a quasiparticle description (eg, Fermi liquids, Dirac liquids, some gapless spin liquids, etc)

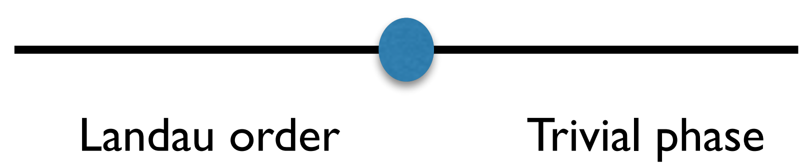
But in other such phases **elementary** excitations may not exist, i.e, no quasiparticle description of excitation spectrum.
(eg, other gapless spin liquids, composite fermi liquids, many other non-fermi liquids)

Slowly evolving understanding in last 25 years.

Critical quantum matter

Continuous T = 0 quantum phase transitions

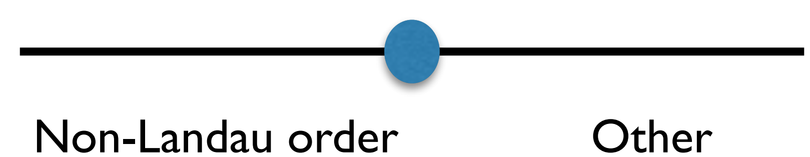
Textbook examples:



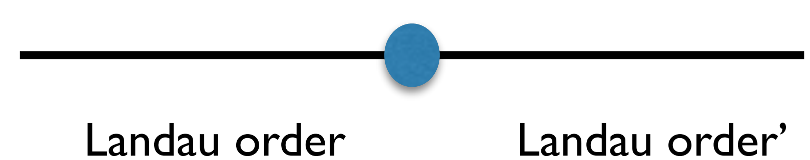
Quantum Landau-Ginzburg-Wilson (LGW) theory of fluctuating order parameter

All other examples: Standard LGW theory fails

Eg: 1. One or both phases have non-Landau order



2. Landau-forbidden continuous transitions between Landau allowed phases



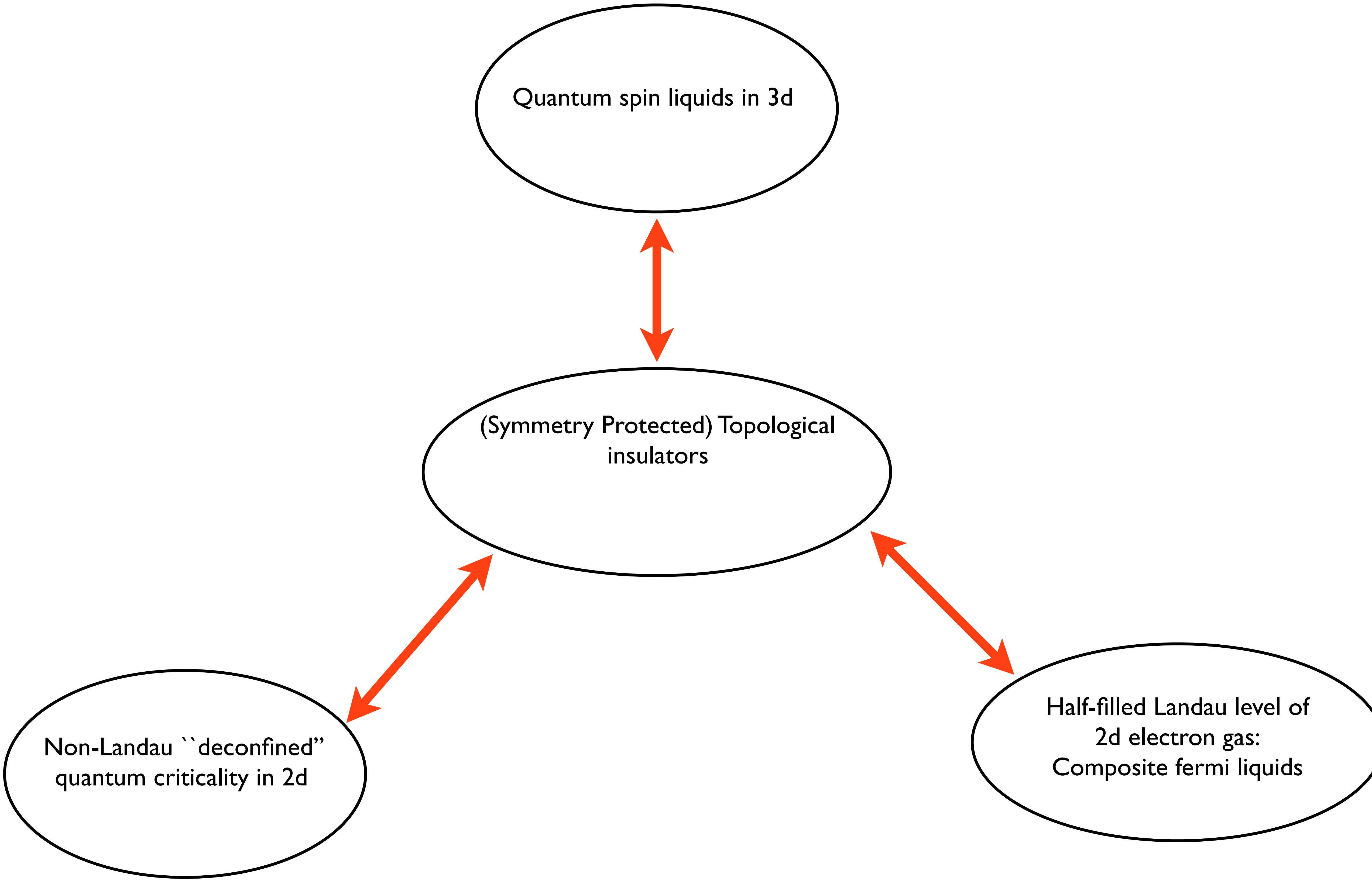
``Deconfined quantum criticality”

Remarkably intimate connections between theories of these different kinds of phases/phase transitions

- spectacular new progress in various directions.

I will illustrate some of these connections through examples.

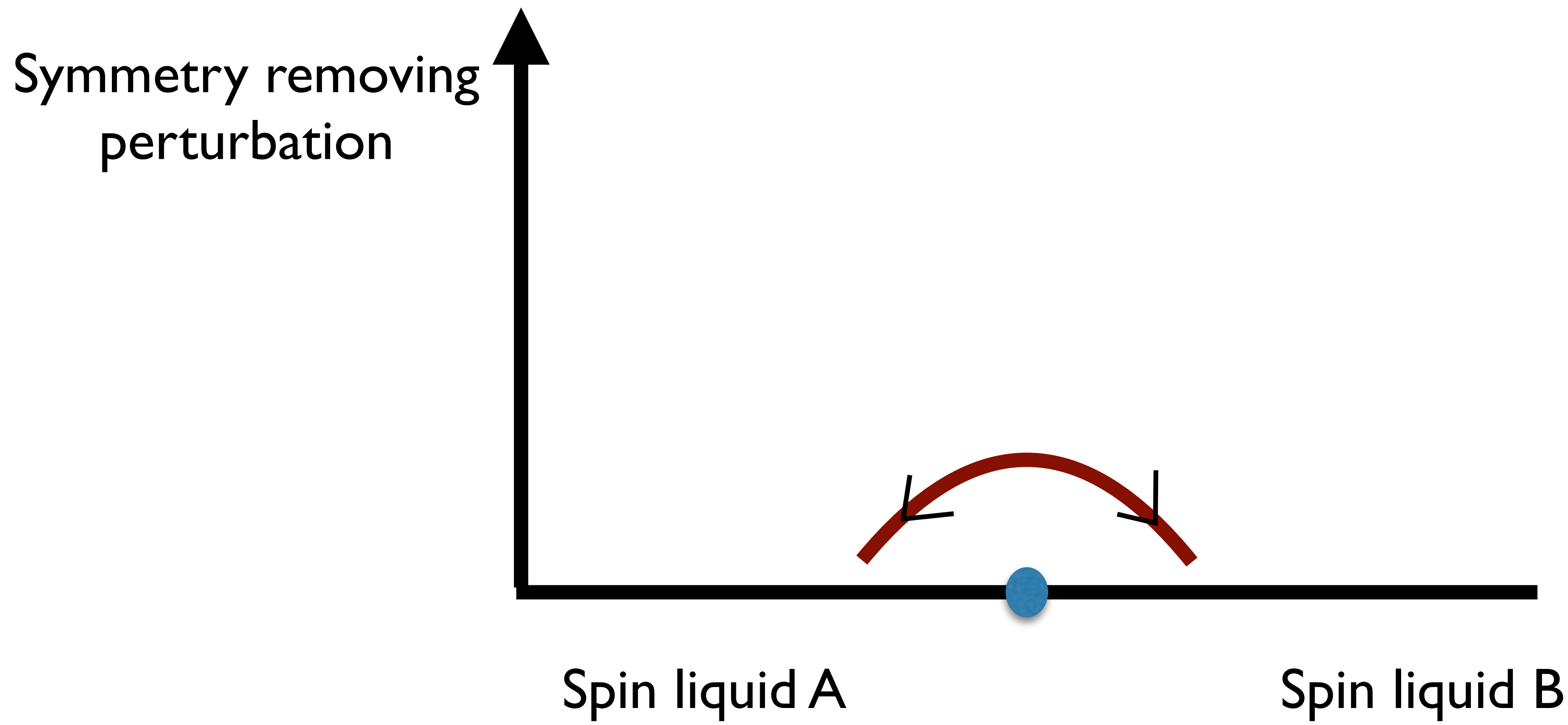
Deep connections between many apparently different problems



Quantum spin liquids and symmetry

Broad question:

Quantum spin liquid phases distinguished purely by (unbroken) symmetry?



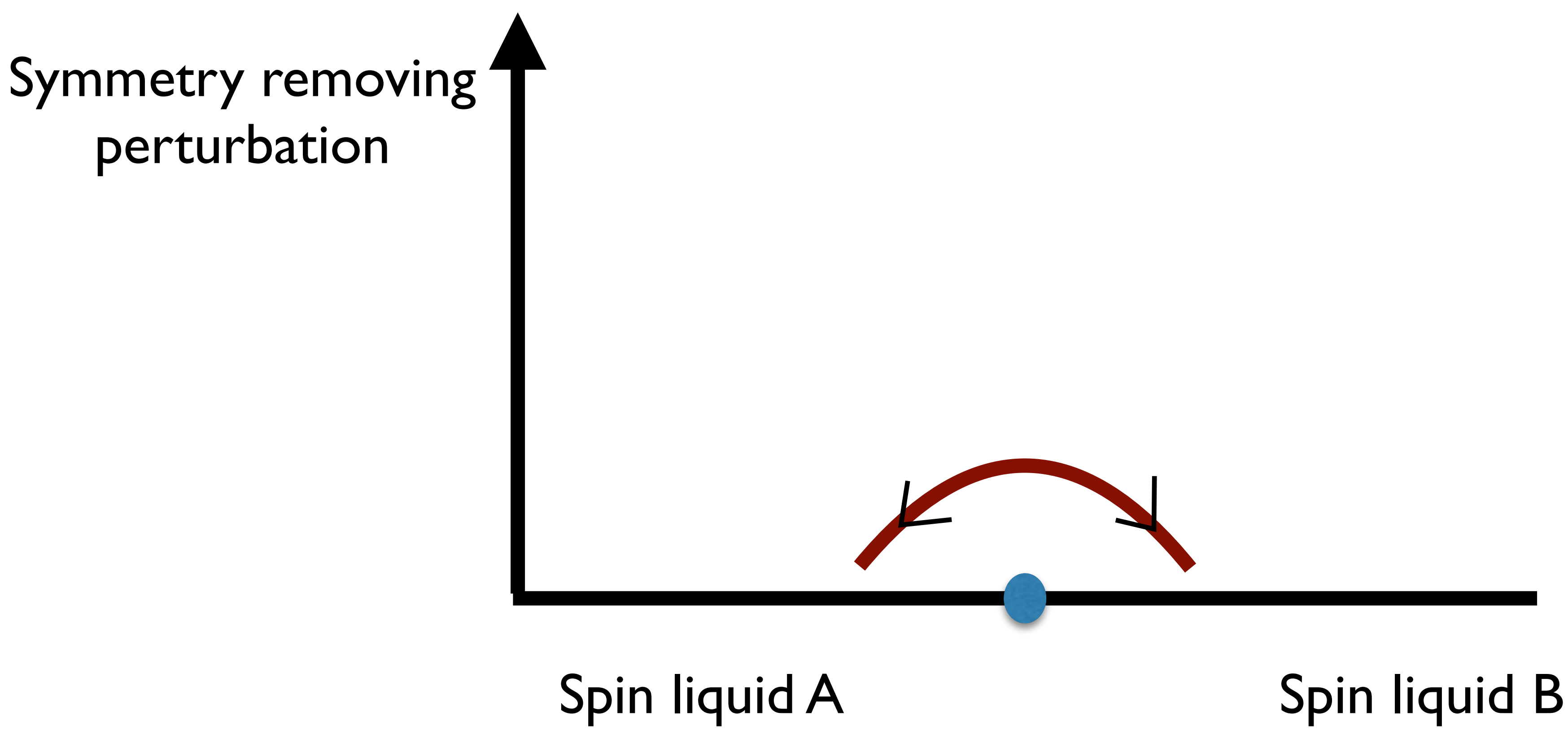
Symmetry can be ``fractionalized'' : symmetry distinctions more severe than for band insulators !

Talks: Barkeshli, Burnell, Hermele,.......

Quantum spin liquids and symmetry

A key idea: Understand spin liquid B as a topological insulator formed out of a quasiparticle of spin liquid A.

=> Topological insulator theory a crucial ingredient in describing role of symmetry in spin liquids.



Example: quantum spin liquids in 3d with an emergent ``photon".

Let there be (artificial) light.....

19th century dream:
Light as a collective mode of some material (``ether'')

21st century* question (Wen,…..):
??Quantum phases of spin/boson systems with an emergent excitation that behaves like a photon??

Such phases can indeed exist as `quantum spin liquids' of spin/boson systems in 3d.

Terminology: U(1) quantum spin liquid

*See however Foerster, Nielsen, Ninomiya, 1983

Excitations of U(1) quantum liquids

Excitations:

1. Gapless artificial photon

2. `Magnetic monopole' - the `M' particle

3. `Electric monopole' - the `E' particle

Emergence of photon ***necessarily*** accompanied by emergence of E and M particles.

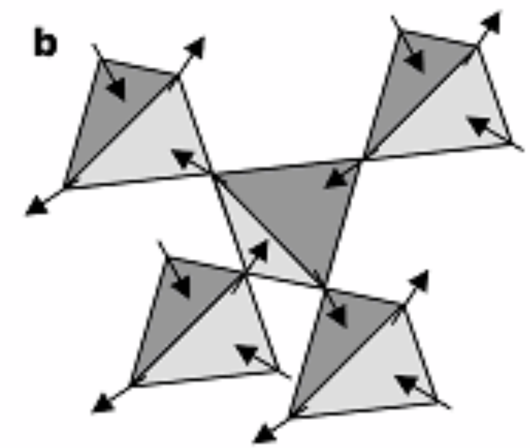
Microscopic models (many in last 10+ years): Motrunich, TS, 02; Hermele et al, 04,
Moessner 03, Banerjee, Isakov, Damle, Kim 08

Experiment?

Possibility of U(1) quantum spin liquids in some rare earth oxide materials

$Yb_2Ti_2O_7$, $Pr_2Zr_2O_7$, $Pr_2Sn_2O_7$, $Tb_2Ti_2O_7$..?

Expt: Gaulin, Broholm, Ong, …..
Theory: Gingras, Balents, Savary,…..



Spin ice

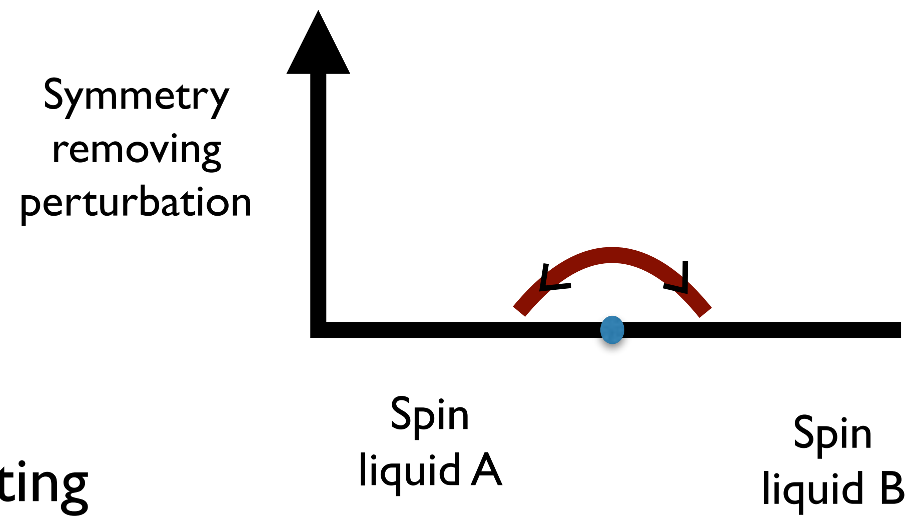
1. Complicated microscopic models! Hard to reliably solve even numerically except in special limits.

2. No symmetry except time reversal (and space group if no impurities). .

Time reversal symmetric U(1) quantum spin liquids

An interesting conceptual question for theory:

What distinct kinds of U(1) quantum spin liquids with time reversal symmetry are possible?



Solved recently (Wang, TS 16)

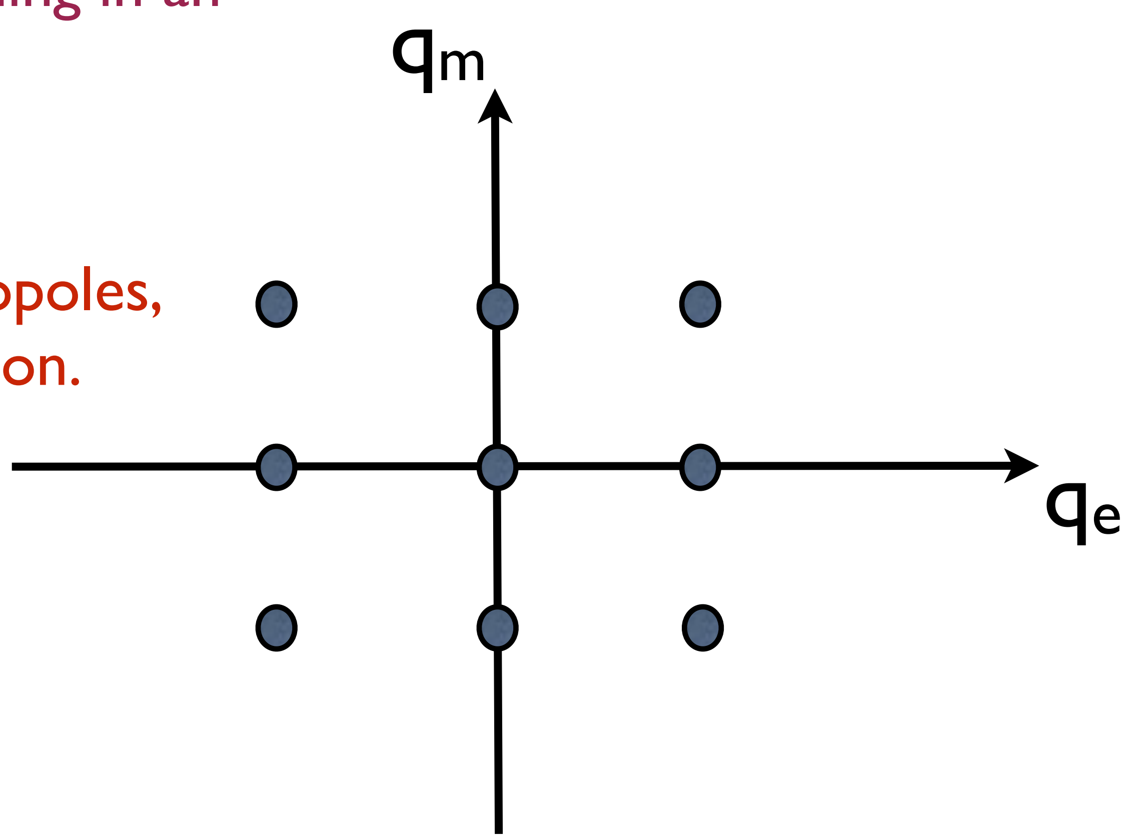
7 families of phases related to each other by putting emergent E or M particles in topological insulators.

Time reversal symmetric U(1) quantum spin liquids

Distinct families - distinct symmetry transformations/statistics of E and M particles.

An example: Both elementary electric charge E, and elementary magnetic charge M are bosons, transforming in an obvious way under time reversal.

In general must consider full lattice of charges, monopoles, and their bound states as allowed by Dirac quantization.

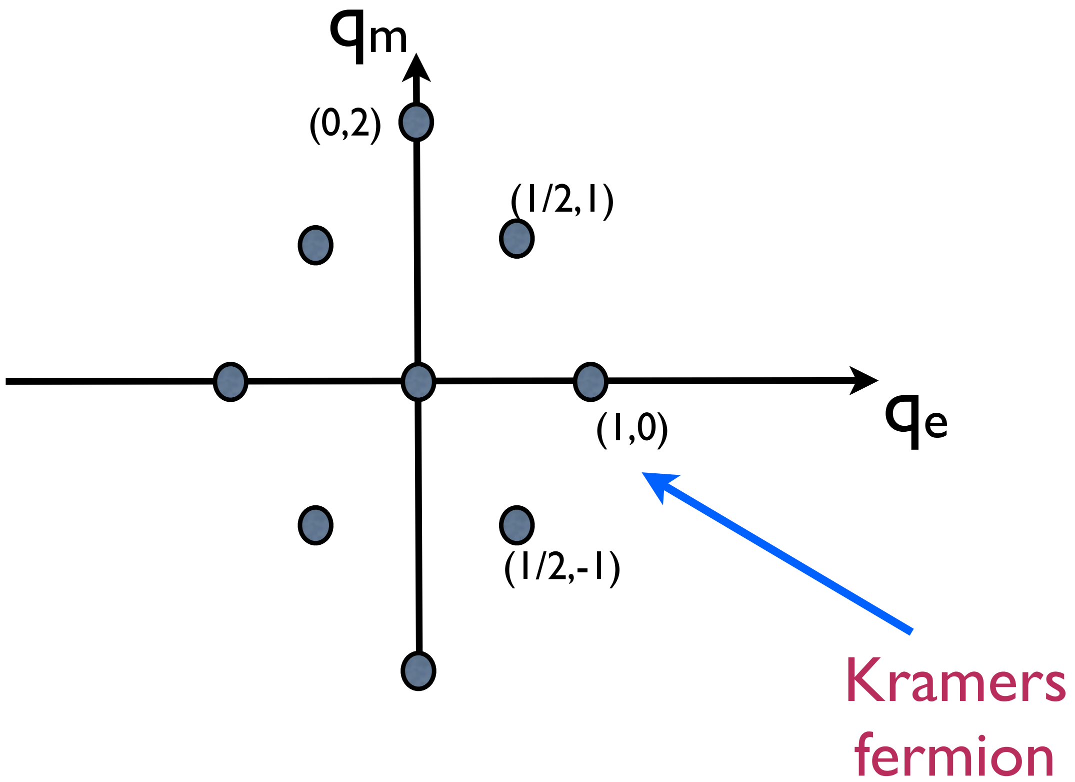


An interesting example (*)

E is a fermion, Kramers under T.

Put E in a topological band insulator.

=> M has q_e = 1/2 (+ integer), and charge-monopole lattice has different geometry.



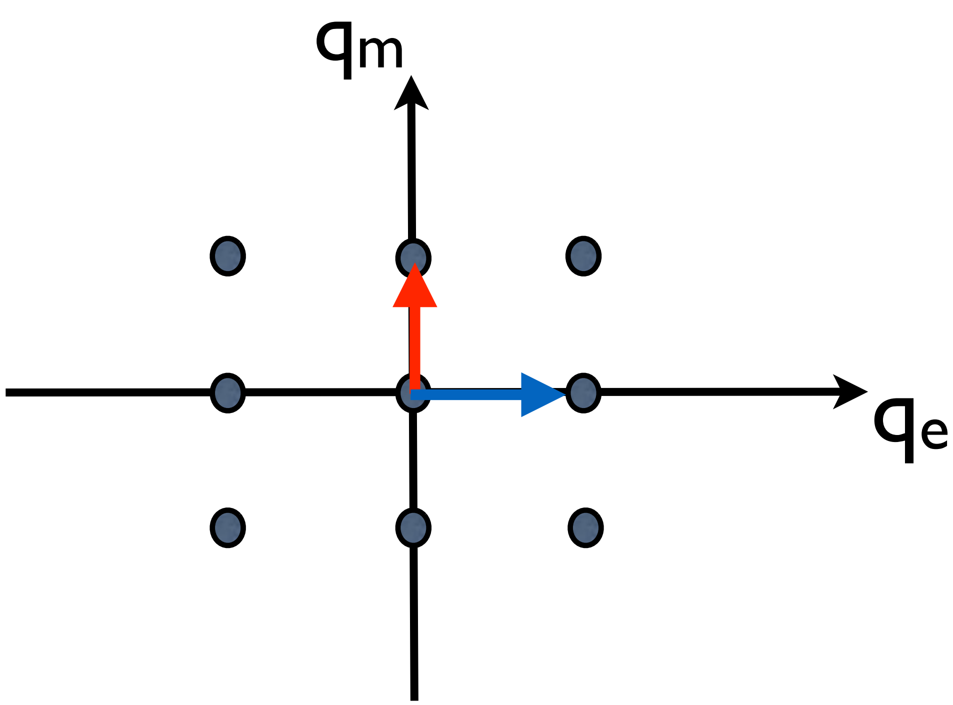
(*) ``Topological Mott insulator'', Pesin, Balents 2011.

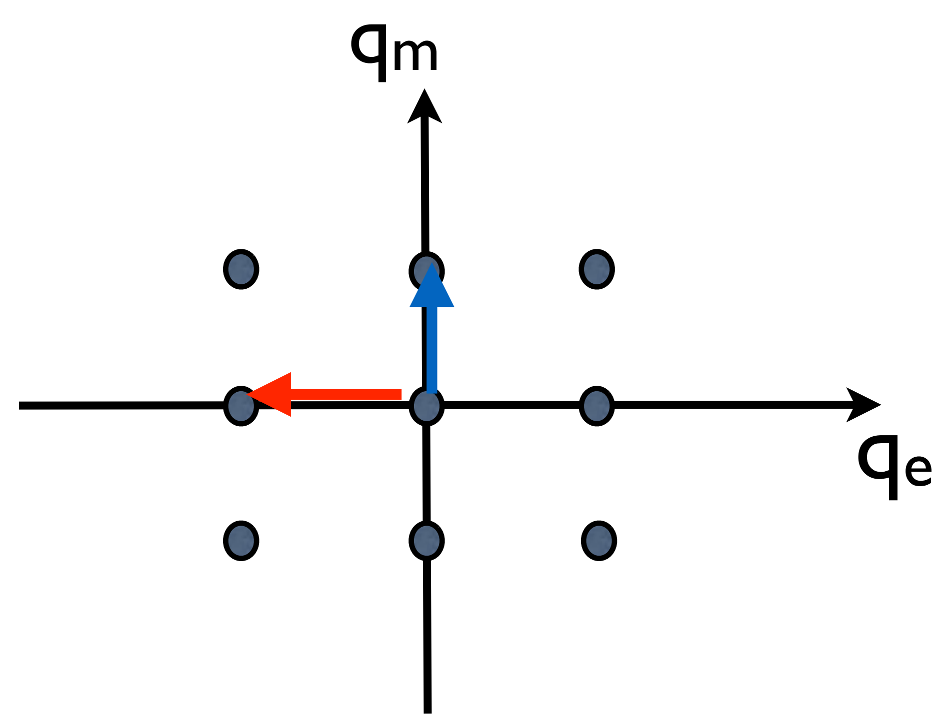
Many equivalent descriptions
(Dualities)

In principle can use any basis to describe the lattice.

Pick a basis and couple the basic particles to the U(1) gauge field with appropriate coupling constant to reproduce charge-monopole lattice.

Caveat: T-reversal symmetry action may be non-trivial.





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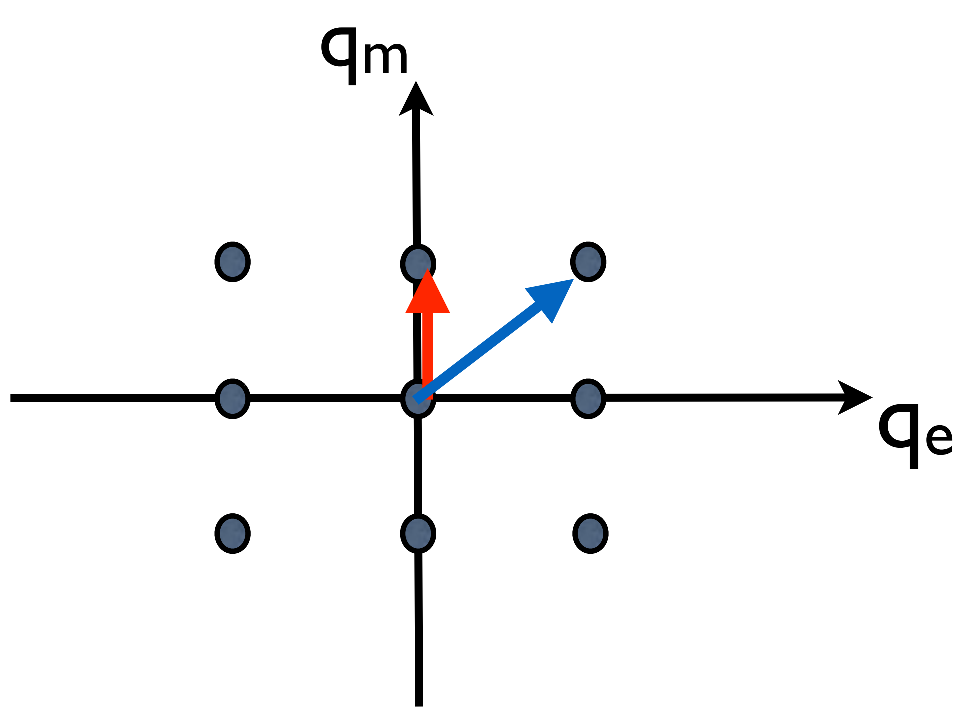
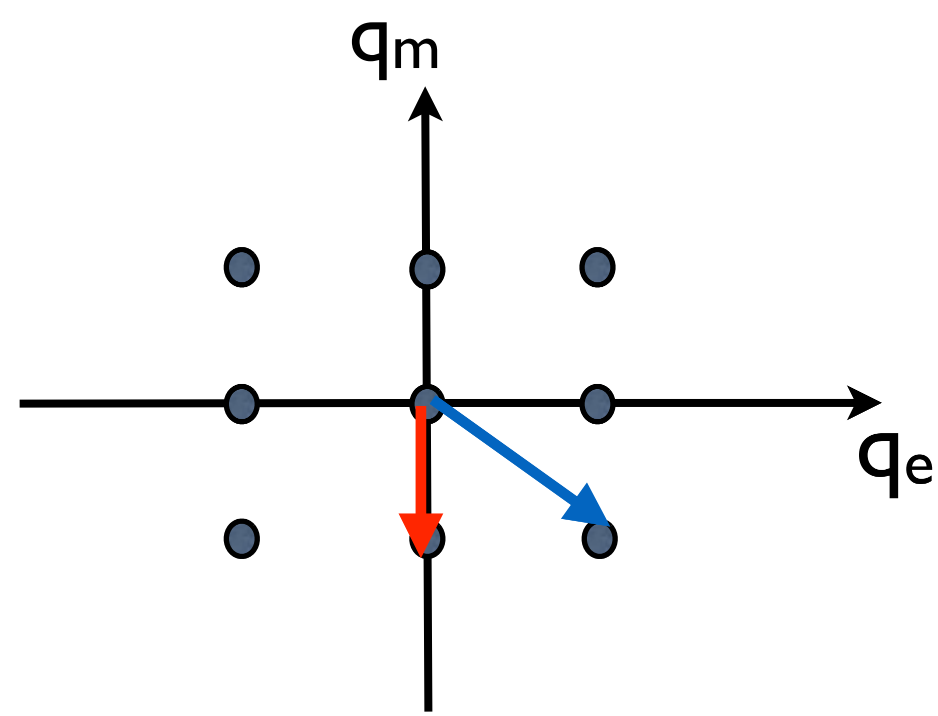
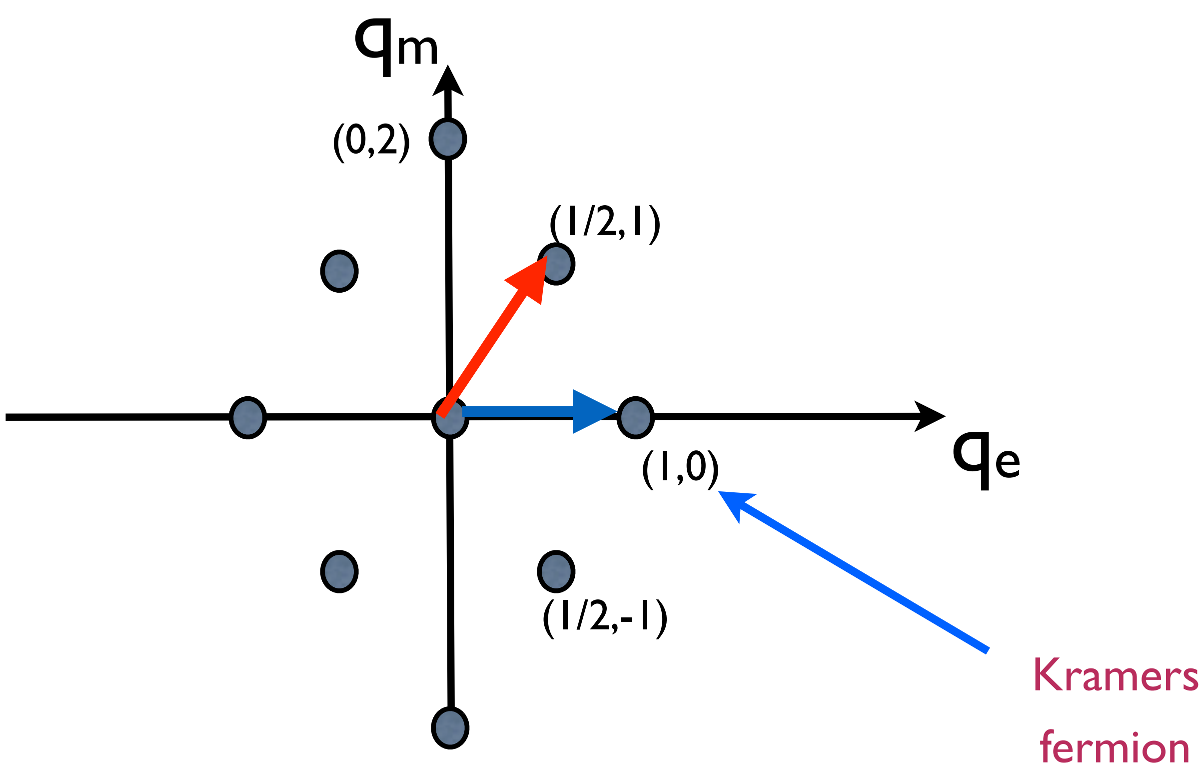


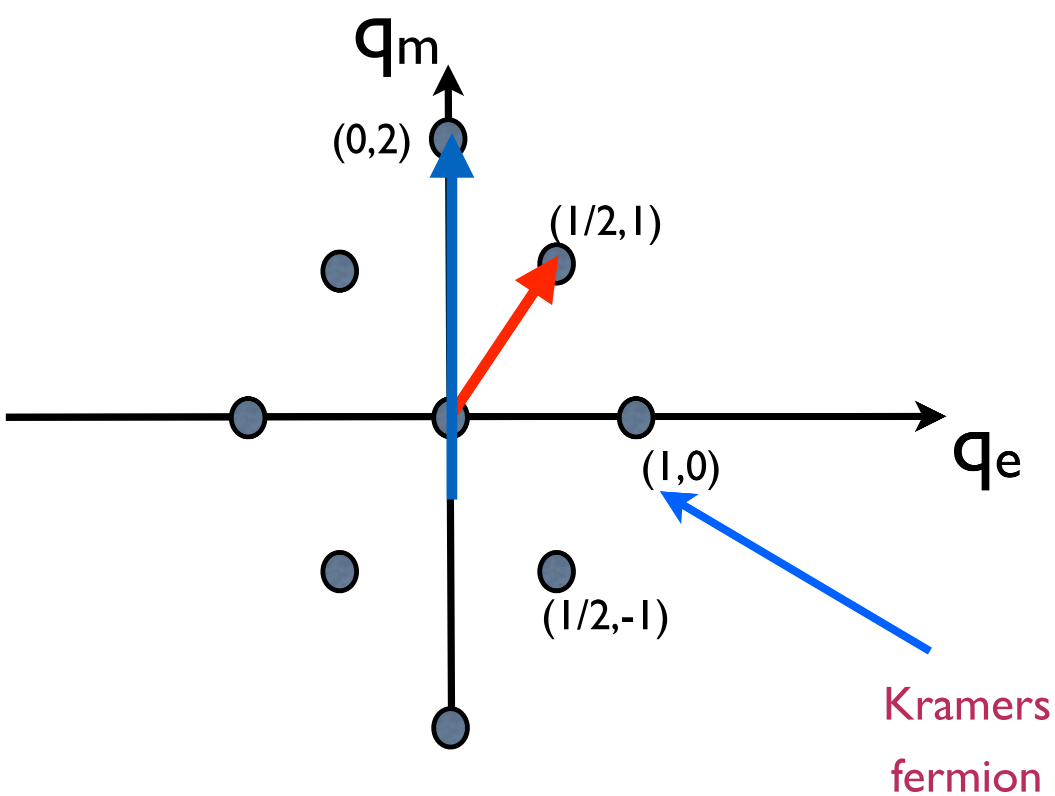


An interesting example

(More in talks by Metlitski, Chong Wang).



Standard basis



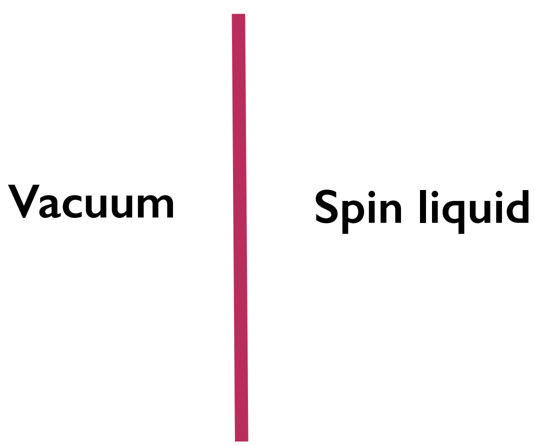
Interesting alternate basis

Can show (0,2) particle is also a fermion => Two points of view on same U(1) spin liquid.

``Electric'': Put (1,0) E fermion in topological insulator.
``Magnetic'': Put (0,2) M fermion in topological insulator.

Surface states

What happens if there is a surface to the vacuum?



Different choices of basis particles in bulk give different descriptions of the same surface

=> Theory of surface states admits many equivalent dual descriptions.

Can use this to argue for many new dualities of 2+1-d quantum field theories.

Non-trivial T-reversal action in some basis => corresponding non-trivial action in surface theory.

Talks: Seiberg (Monday 9.45 am), Metlitski (Tuesday 9.45 am), Wang (Tuesday 11.30 am)

Some applications of these kinds of dualities

1. Particle-hole symmetric theory of composite fermi liquids in lowest Landau level

2. Understand correlated surface states of topological insulators

3. Deconfined quantum critical points: Understand emergent symmetries. (Wang, Nahum, Metlitski, Xu, TS, to appear).

How is half-filled Landau level related to 3d topological insulators and spin liquids?

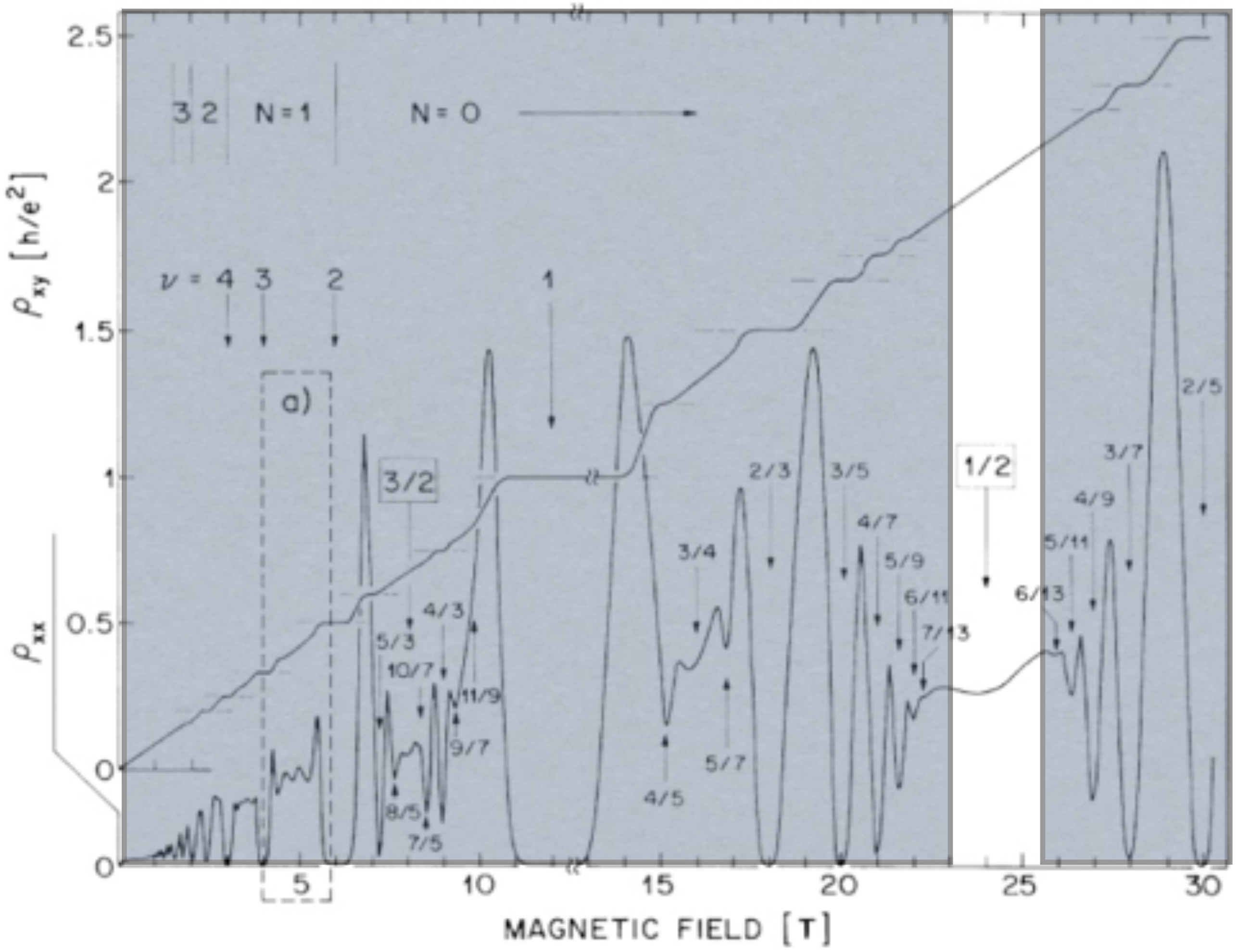
1/2-filled Landau level: the problem

What happens at ν = 1/2 ???

Experiment: Metal

``Unquantized quantum Hall effect"

How do interactions in the half-filled Landau level produce a metal?



p/h symmetric LL as a surface of a 3d fermionic topological insulator: Preliminaries

Consider (initially free) fermions with ``weird" action of time-reversal (denote C):

$C \rho C^{-1} = - \rho$

ρ = conserved ``charge" density.

Full symmetry = U(1) x C

p/h symmetric LL as a surface of 3d fermion SPT (cont'd)

Surface: Single massless Dirac fermion

C symmetry guarantees that surface Dirac cone is exactly at neutrality.

$$\mathcal{L} = \bar{\psi}\left(-i\not{\partial} + \not{A}\right)\psi + \dots.$$

external probe gauge field

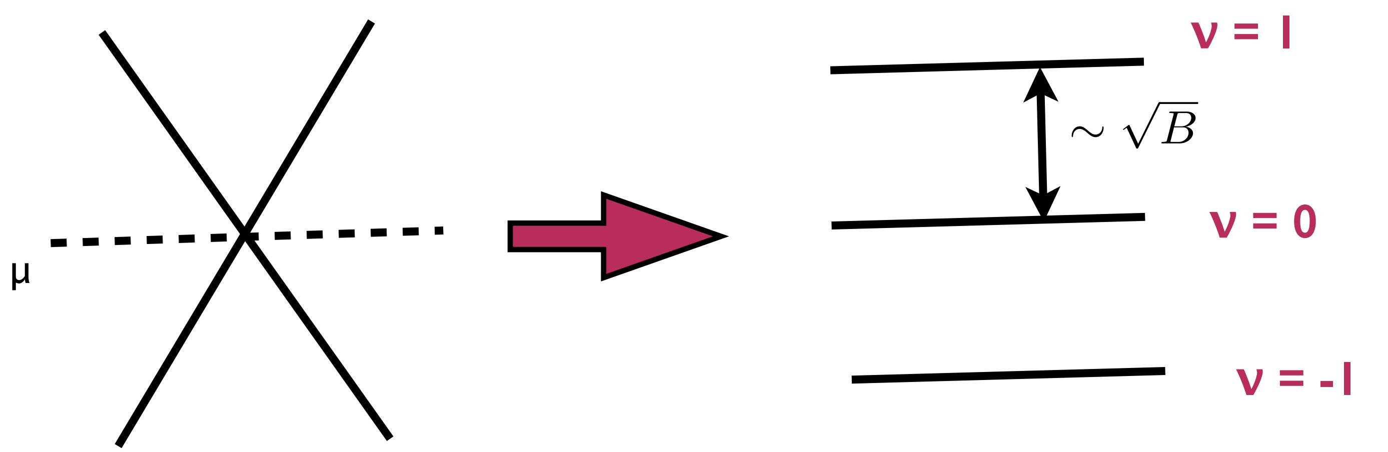
2-component fermion

p/h symmetric LL as a surface of 3d fermion SPT (cont'd)

ρ is odd under C => `electric current' is even.

External E-fields are odd but external B-fields are even.

=> Can perturb surface Dirac cone with external B-field.



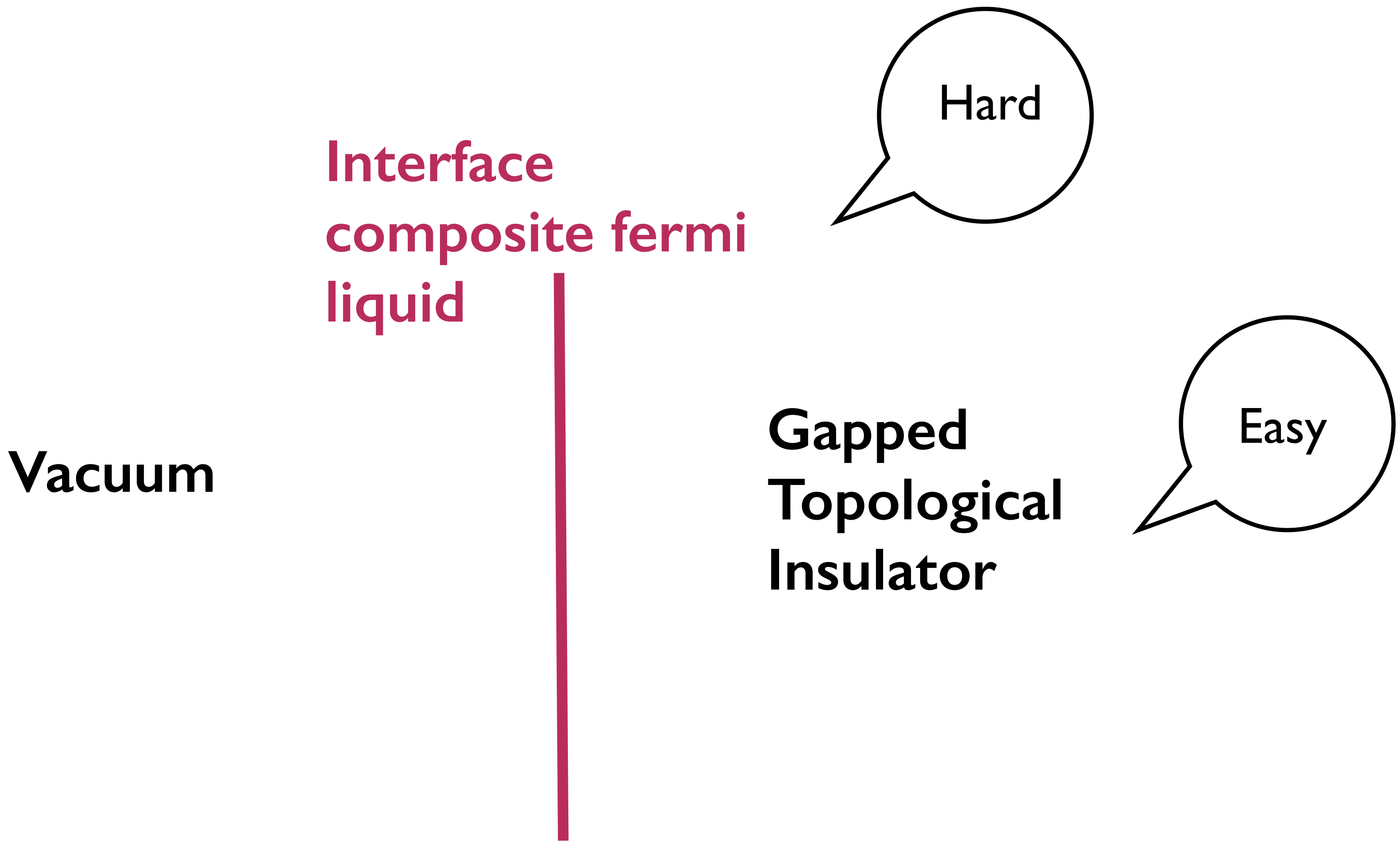
C-symmetry: ν = 0 LL is exactly half-filled.

Low energy physics: project to 0LL

With interactions => map to usual half-filled LL

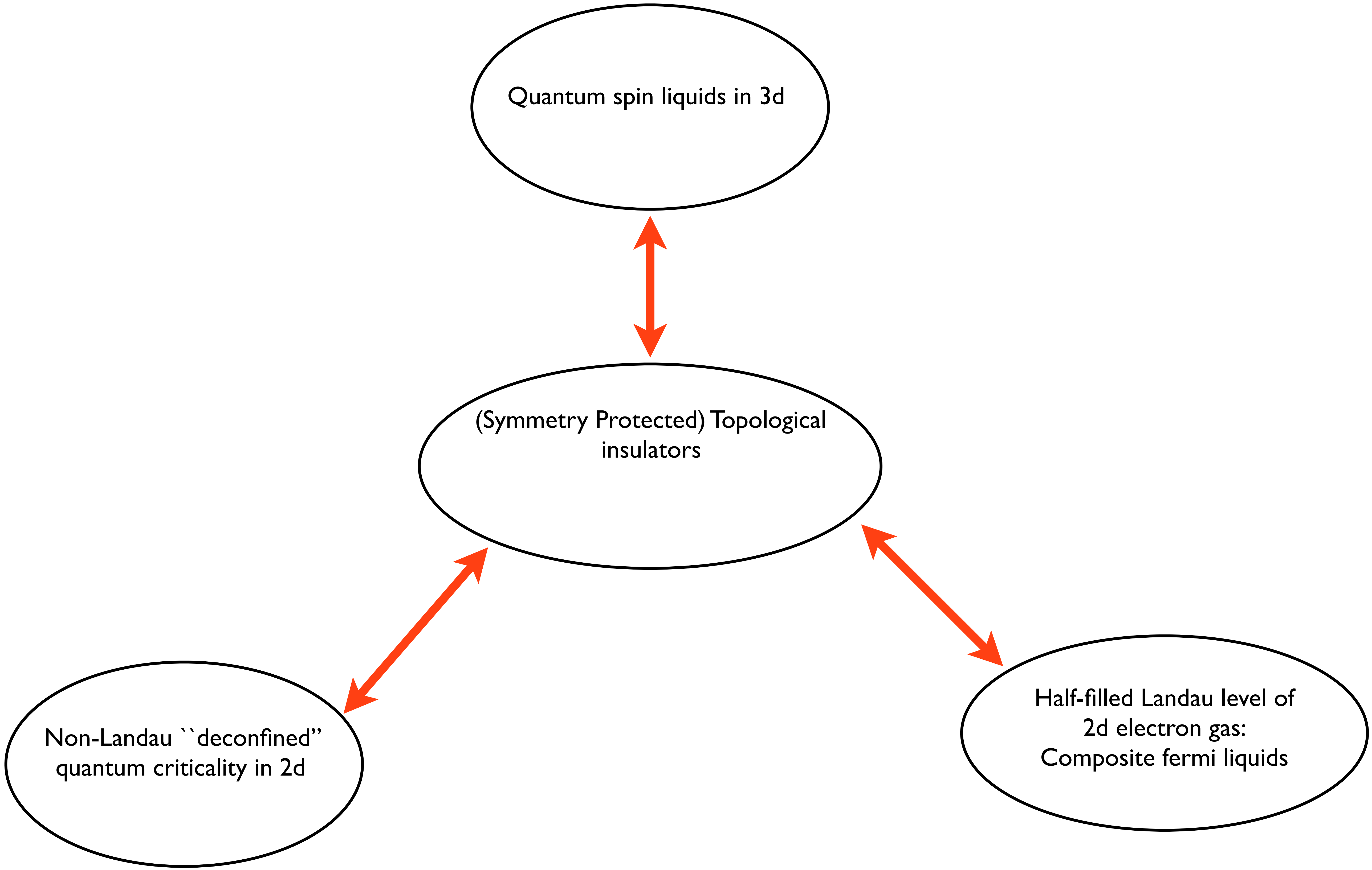
Comments

Implication: Study p/h symmetric half-filled LL level by studying correlated surface states of such 3d fermion topological insulators.



Exploit understanding of relatively trivial bulk TI to learn about non-trivial correlated surface state.

Deep connections between many apparently different problems



Summary

Topological, gapless, and critical matter: different but very intimate connections.

Deep understanding of any one feeds into the others.

Many interesting/fruitful interactions condensed matter with field (string) theory/math.

Look forward to a great conference!

Recent Collaborators

Chong Wang,
(MIT => Harvard)

L. Zou (MIT student), I. Sodemann (MIT post-doc), I. Kimchi (MIT post-doc), N. Seiberg (IAS), E. Witten (IAS)

Many others in past years……