Stellar convective penetration: Context, theory, and simulations

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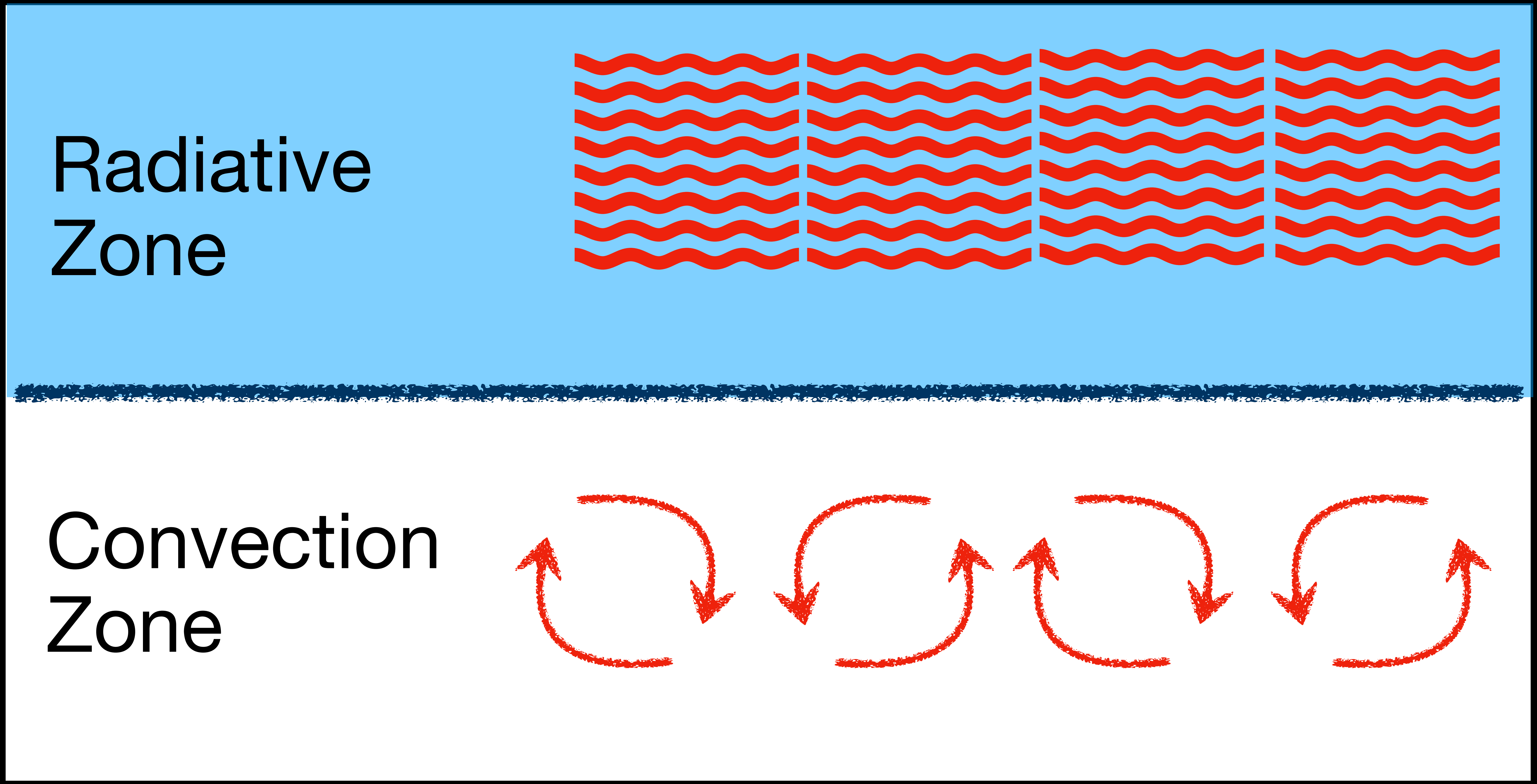
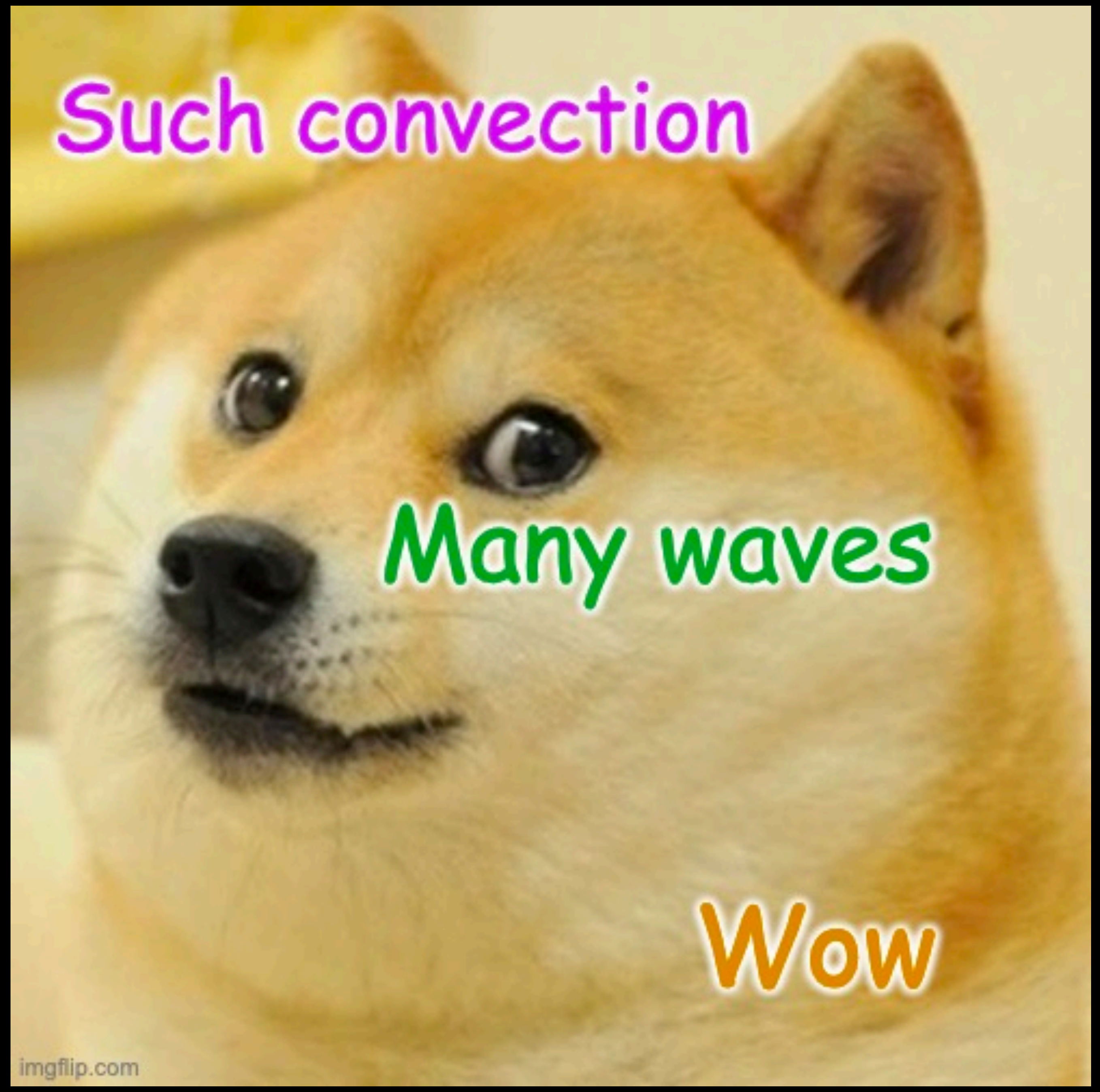
[Anders et al 2021 / arxiv: 2110.11356]

A simple model of a radiative-convective boundary





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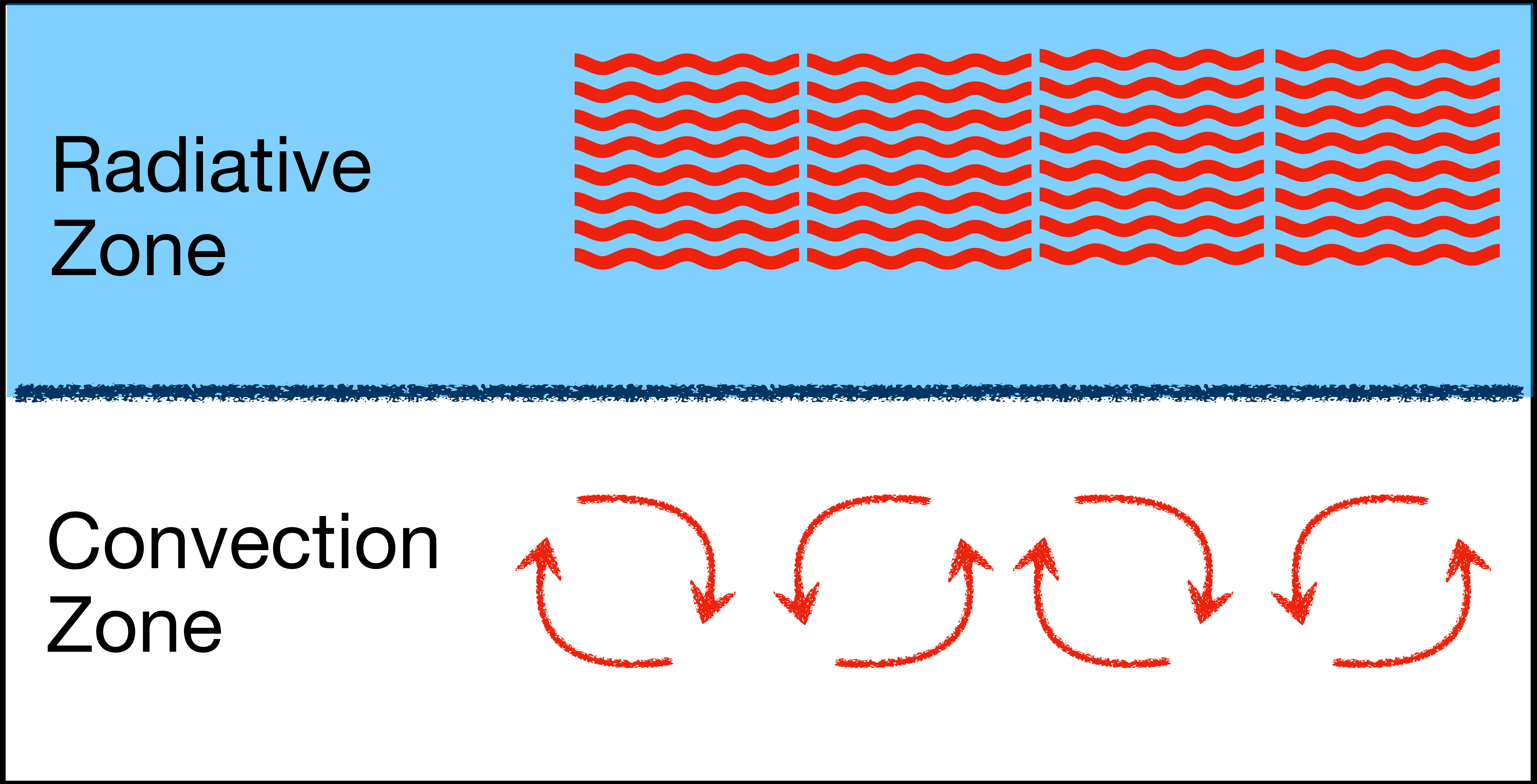
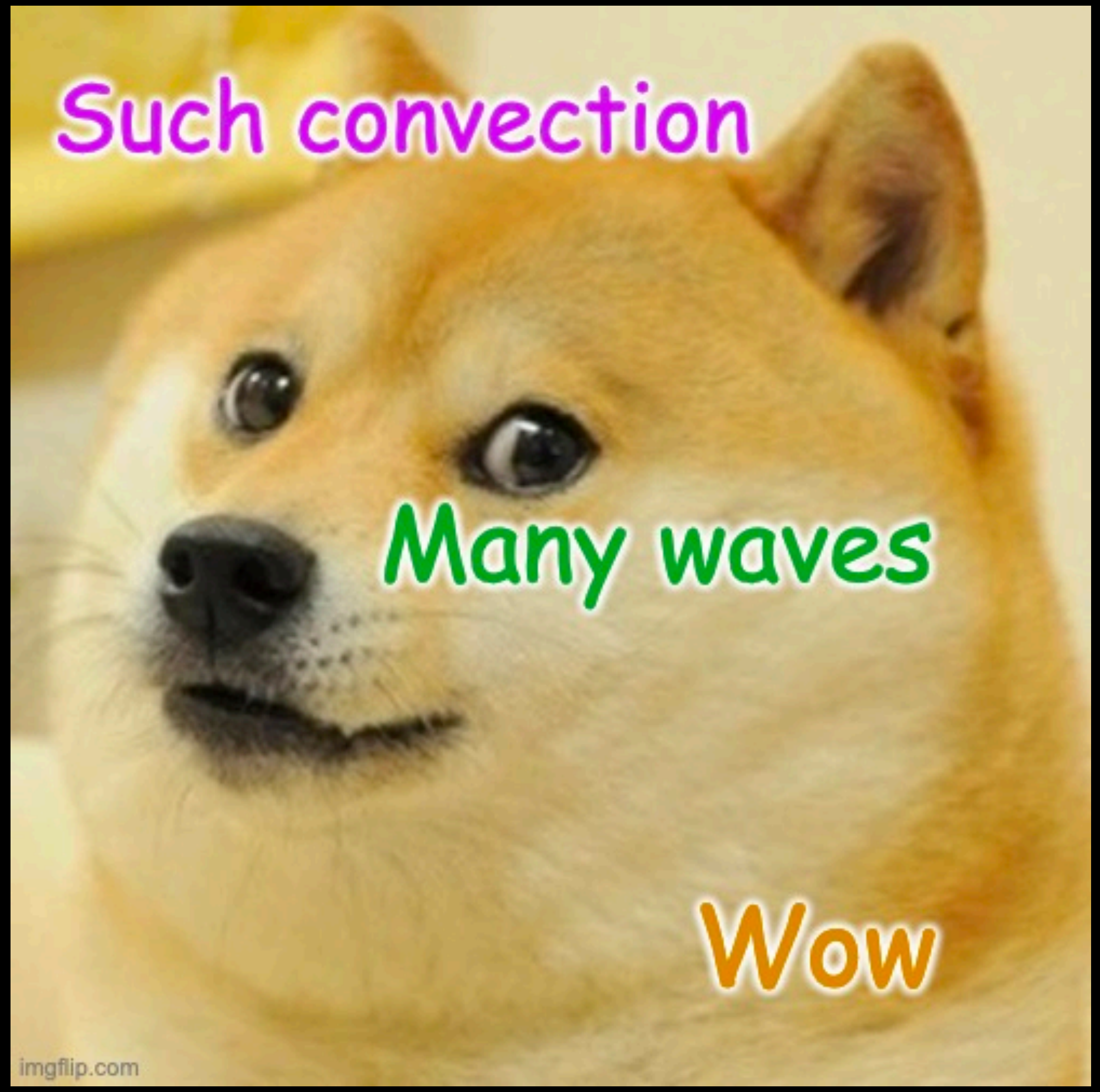
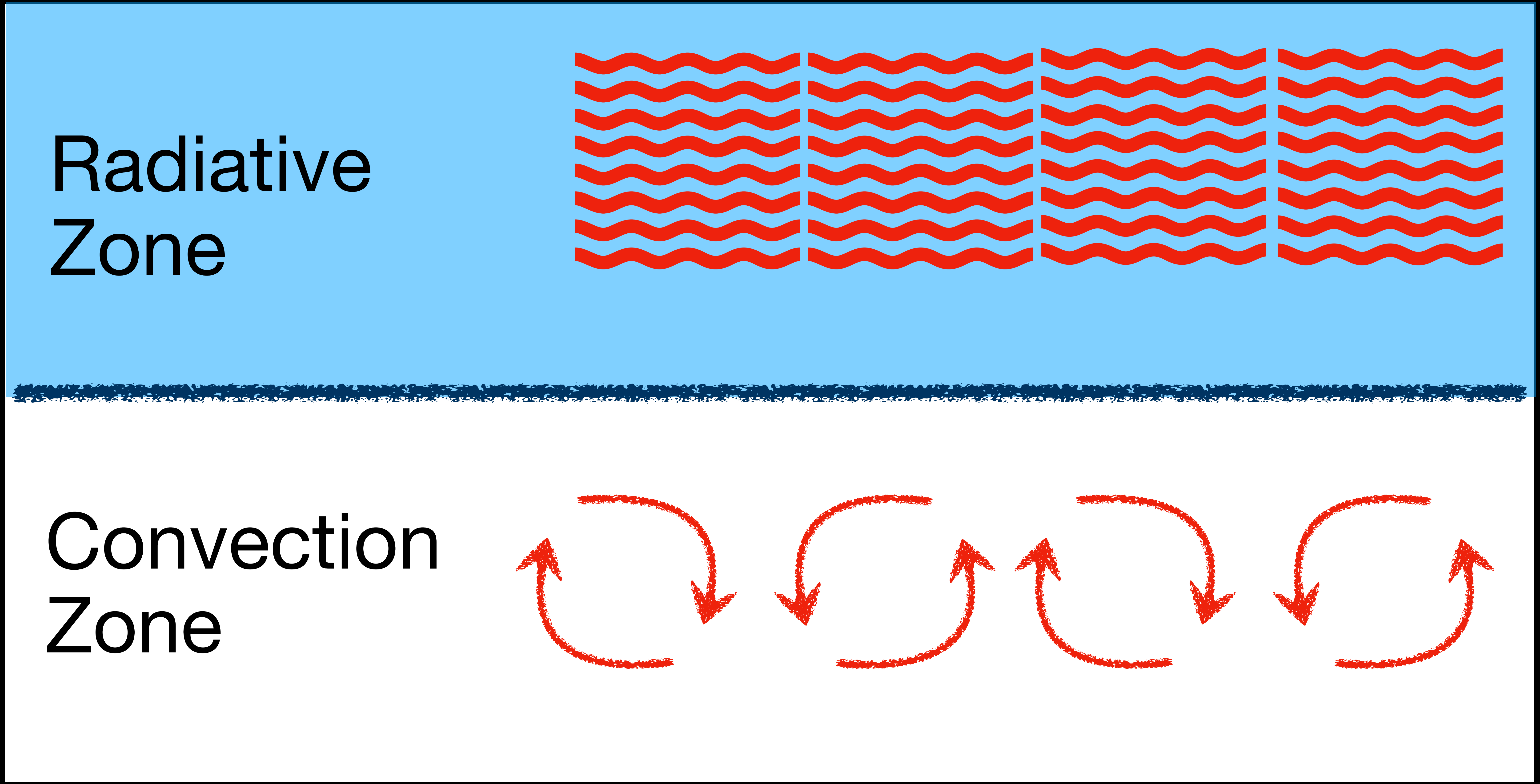
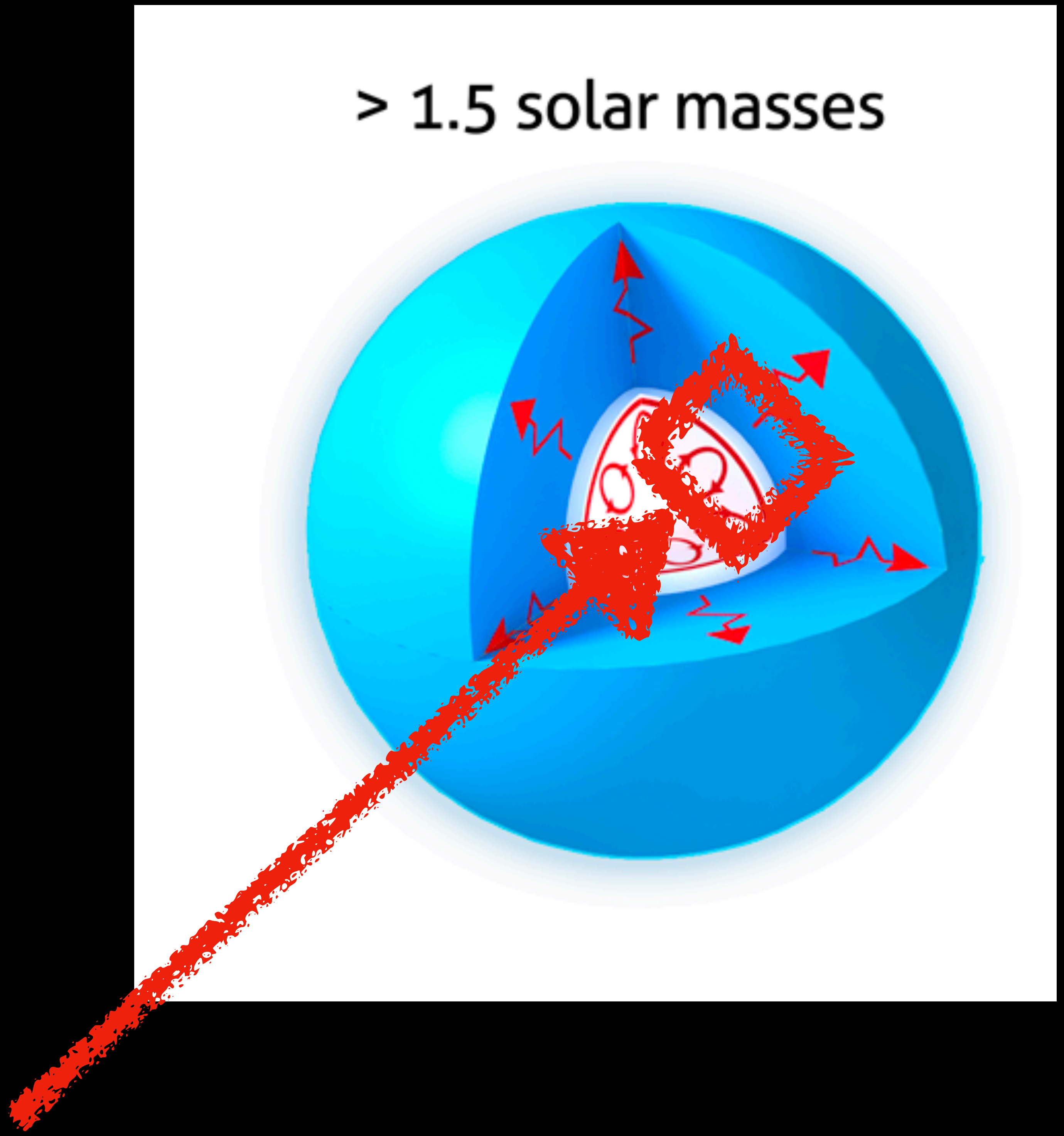




But: convective boundary mixing perpetually confounds us.

A simple model of a radiative-convective boundary





But: convective boundary mixing perpetually confounds us.

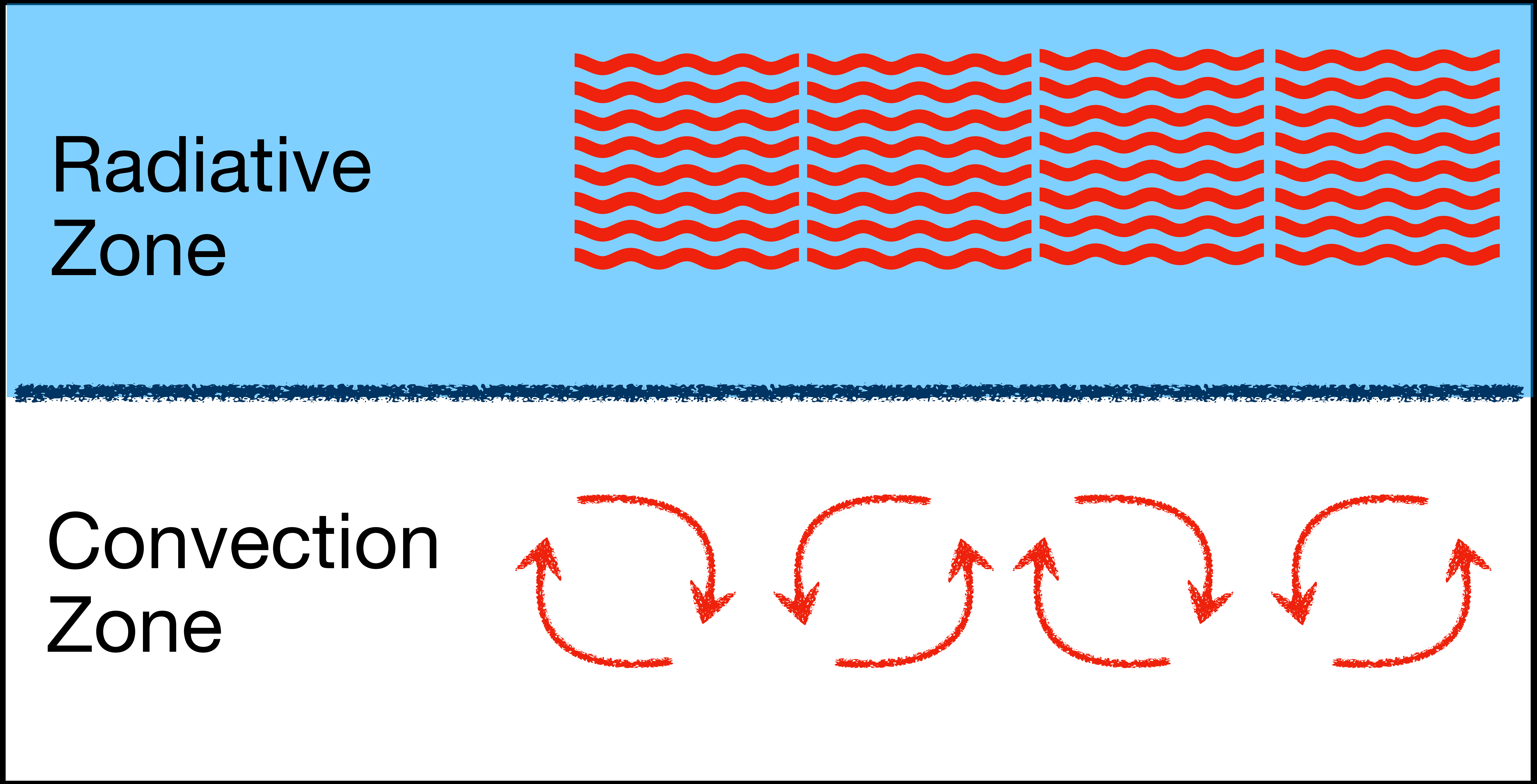
Three linked processes in convective boundary mixing

A fluid dynamical perspective

1. **Convective overshoot** (mechanical overshoot)
2. Entrainment
3. Convective penetration

[Zahn 1991, Hurlburt et al 1994, Brummel et al 2002, Korre et al 2019, many others….]

Convective overshoot

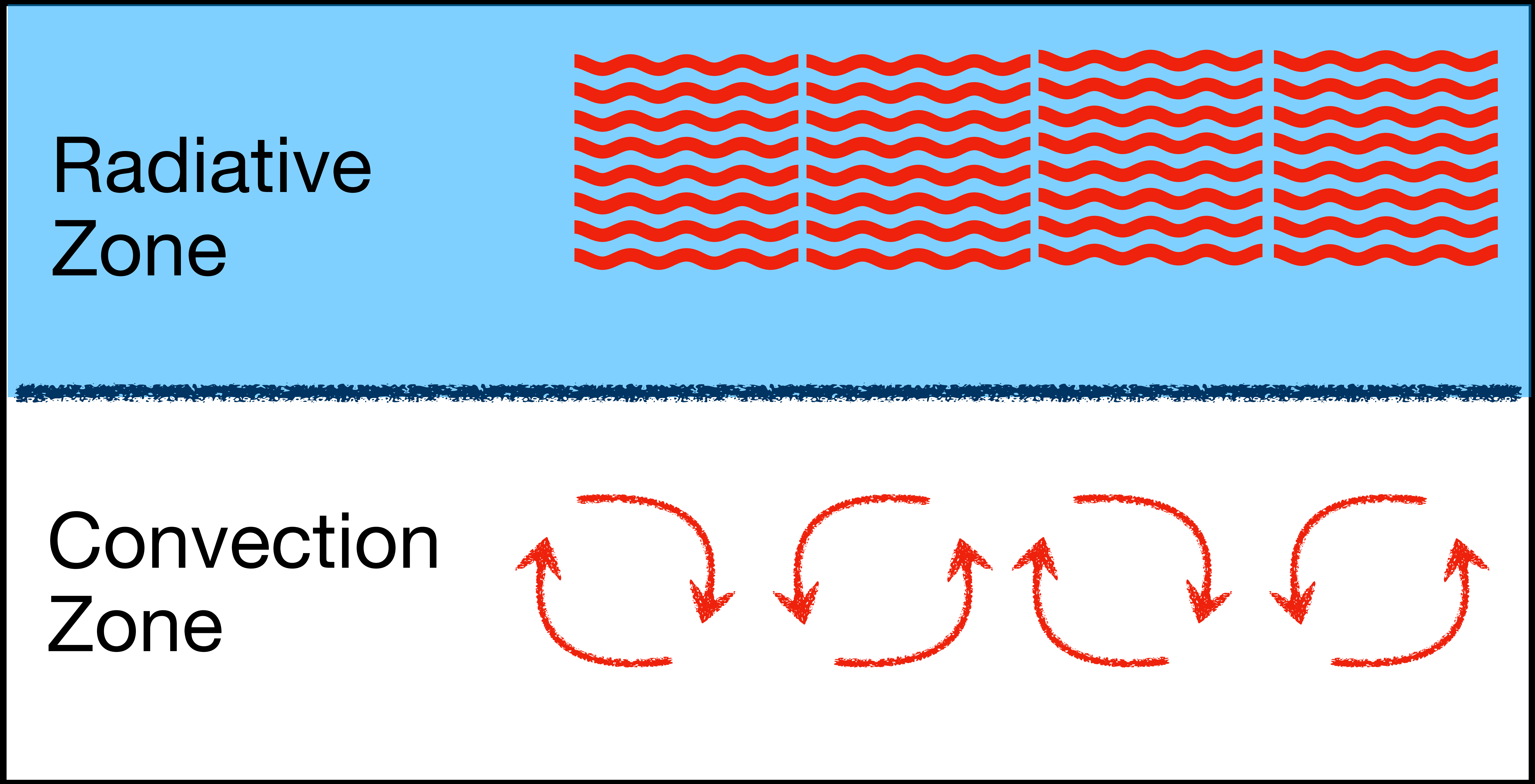


Characteristic buoyancy frequency N_{RZ}

Characteristic convective velocity u_{conv}

Convective overshoot

The edge of the convection zone is where the ***acceleration*** of convective blobs changes sign.

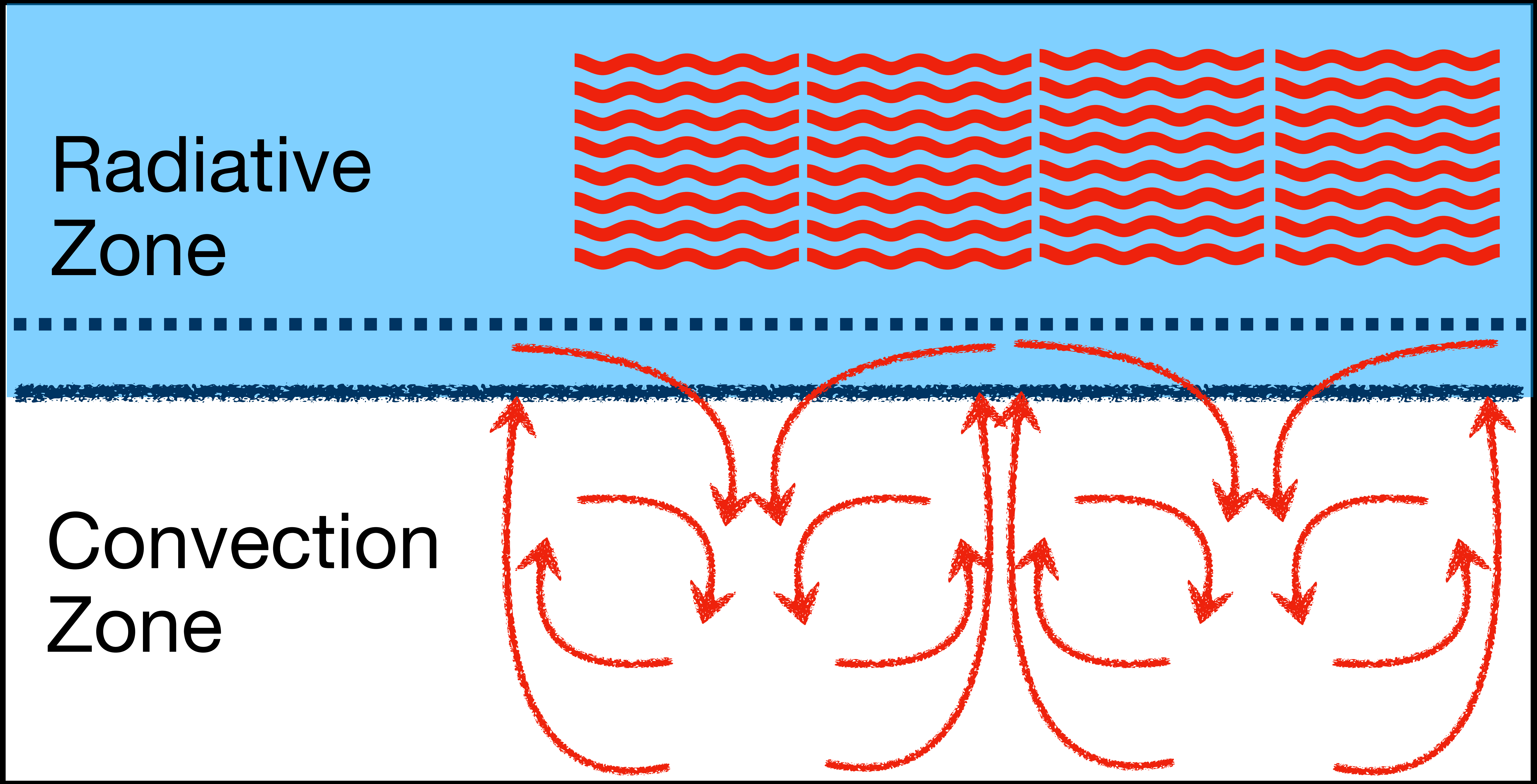


Characteristic buoyancy frequency N_{RZ}

Characteristic convective velocity u_{conv}

Convective overshoot

…so motions go above the nominal edge of the convection zone.

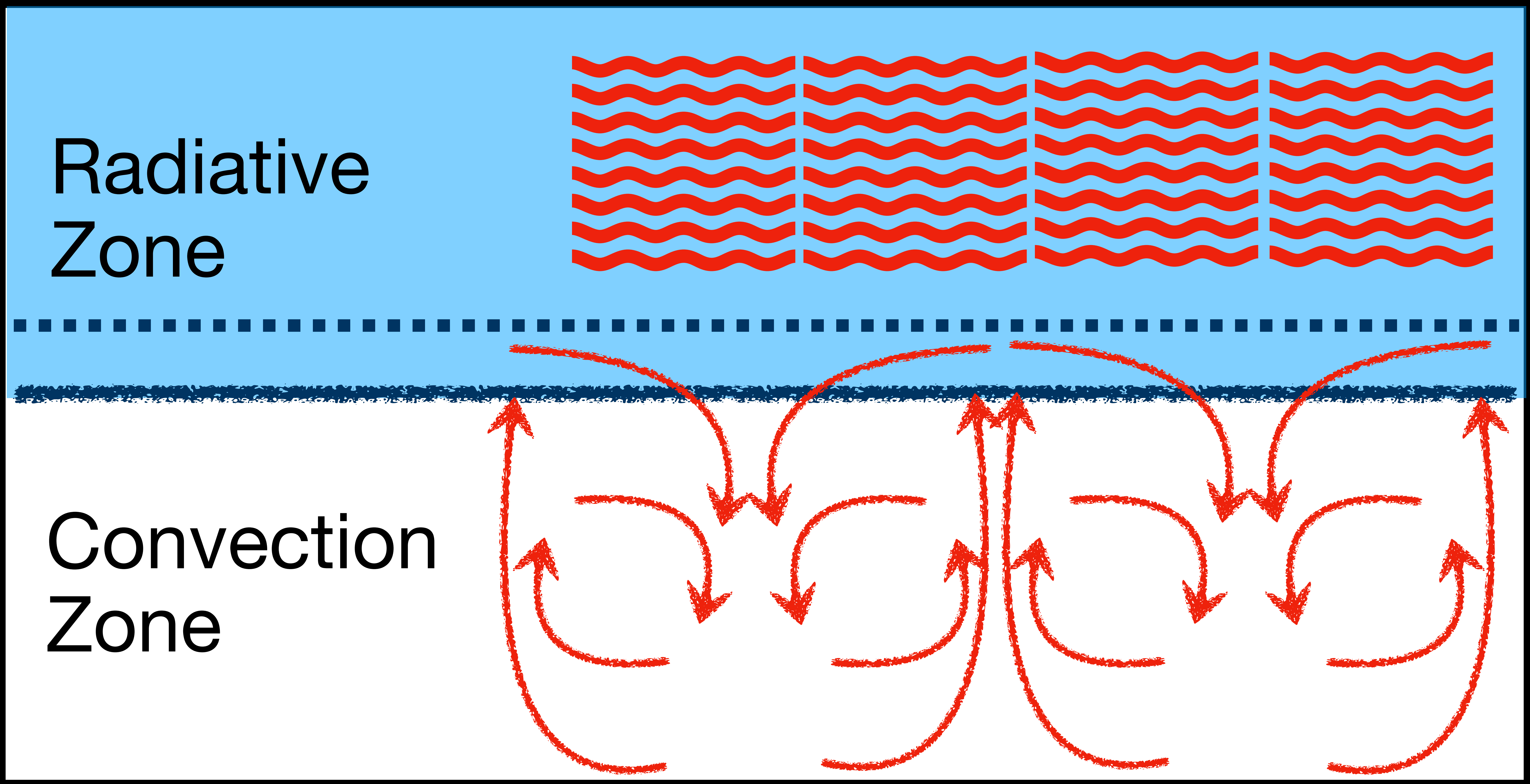


Characteristic buoyancy frequency N_{RZ}

Characteristic convective velocity u_{conv}

Convective overshoot

(Think $\Delta x = u \Delta t$)

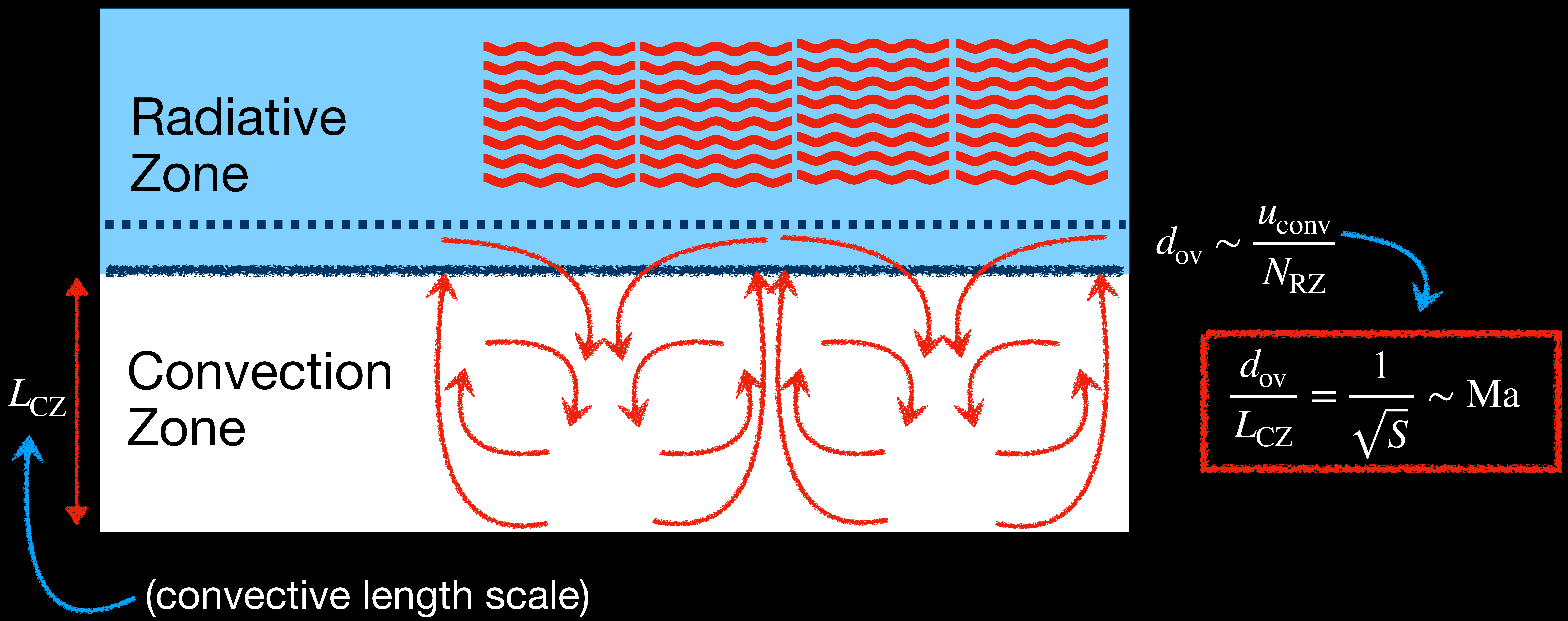


Characteristic buoyancy frequency N_{RZ}

$$d_{\mathrm{ov}} \sim \frac{u_{\mathrm{conv}}}{N_{\mathrm{RZ}}}$$

Characteristic convective velocity u_{conv}

Convective overshoot



Convective overshoot

$$\frac{d_{\mathrm{ov}}}{L_{\mathrm{CZ}}} = \frac{1}{\sqrt{S}} \sim \mathrm{Ma}$$

This is a persistent effect, and occurs in disequilibrium and equilibrium states.

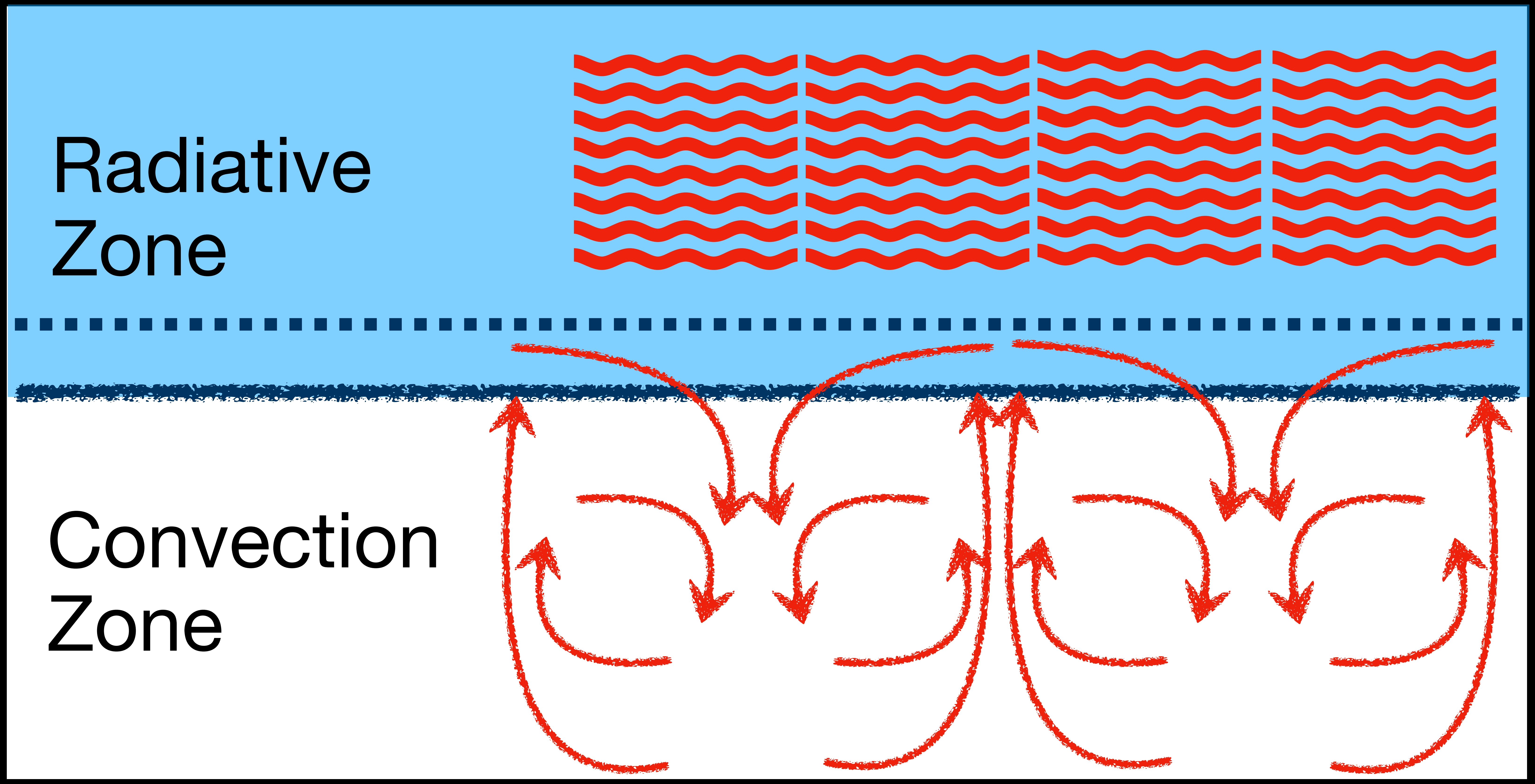
Three linked processes in convective boundary mixing

A fluid dynamical perspective

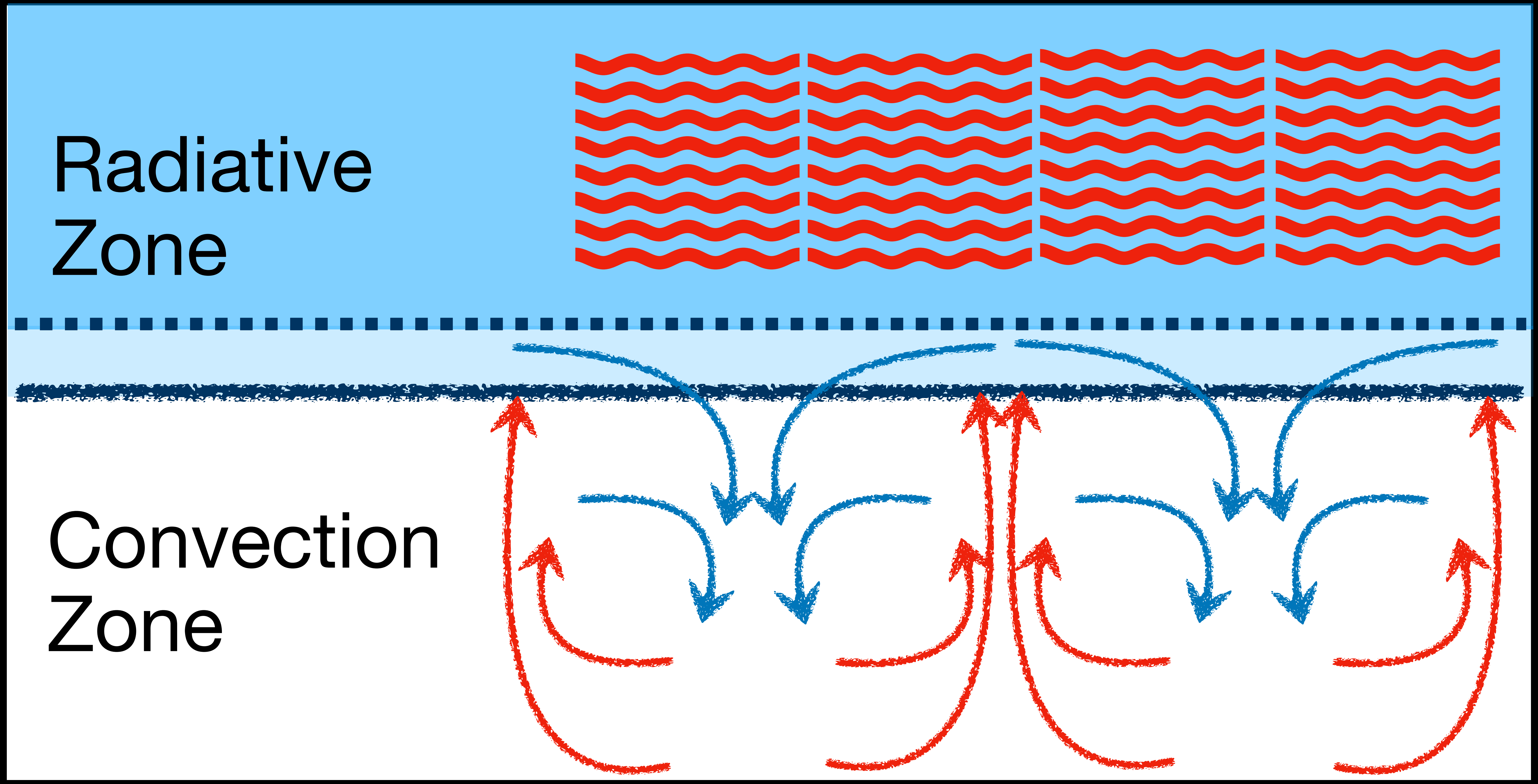
1. Convective overshoot (mechanical overshoot)
2. **Entrainment** (a *transient* process in a disequilibrium state)
3. Convective penetration

[Meakin & Arnett 2007, Viallet et al 2013, Jones et al 2017, Cristini et al 2017, Fuentes & Cumming 2020, many others….]

Convective overshoot causes *entrainment*.



Entrainment



Motions that overshoot mix fluid from the RZ into the CZ.

[Turner JFM 1968]

Three linked processes in convective boundary mixing

A fluid dynamical perspective

1. Convective overshoot (mechanical overshoot)
2. Entrainment (a *transient* process in a disequilibrium state)
3. **Convective penetration** (affects the *structure* of a convective interface)

Three linked processes in convective boundary mixing

A fluid dynamical perspective

1. Convective overshoot (mechanical overshoot)
2. Entrainment (a *transient* process in a disequilibrium state)
3. **Convective penetration** (affects the *structure* of a convective interface)

[Sims that probably had penetration: Hurlburt et al 1994, Saikia et al 2000, Brummell et al 2002, Rogers & Glatzmaier 2005, Rogers et al 2006, Kitiashvili et al 2016]

[Penetration depends on energy fluxes?: Singh et al 1998, Kapyla et al 2007, Tian et al 2009, Hotta 2017, Kapyla 2019]

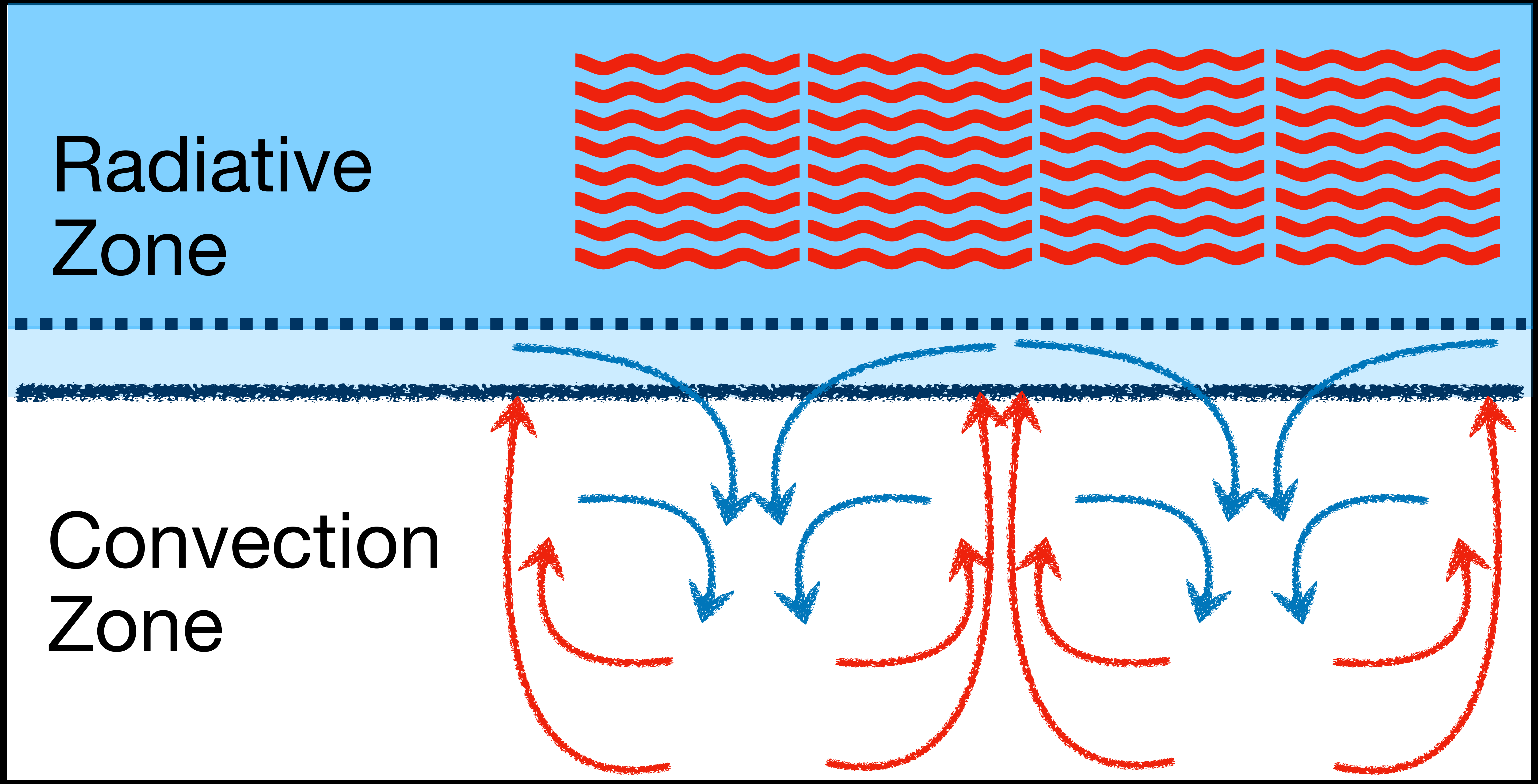
Three linked processes in convective boundary mixing

A fluid dynamical perspective

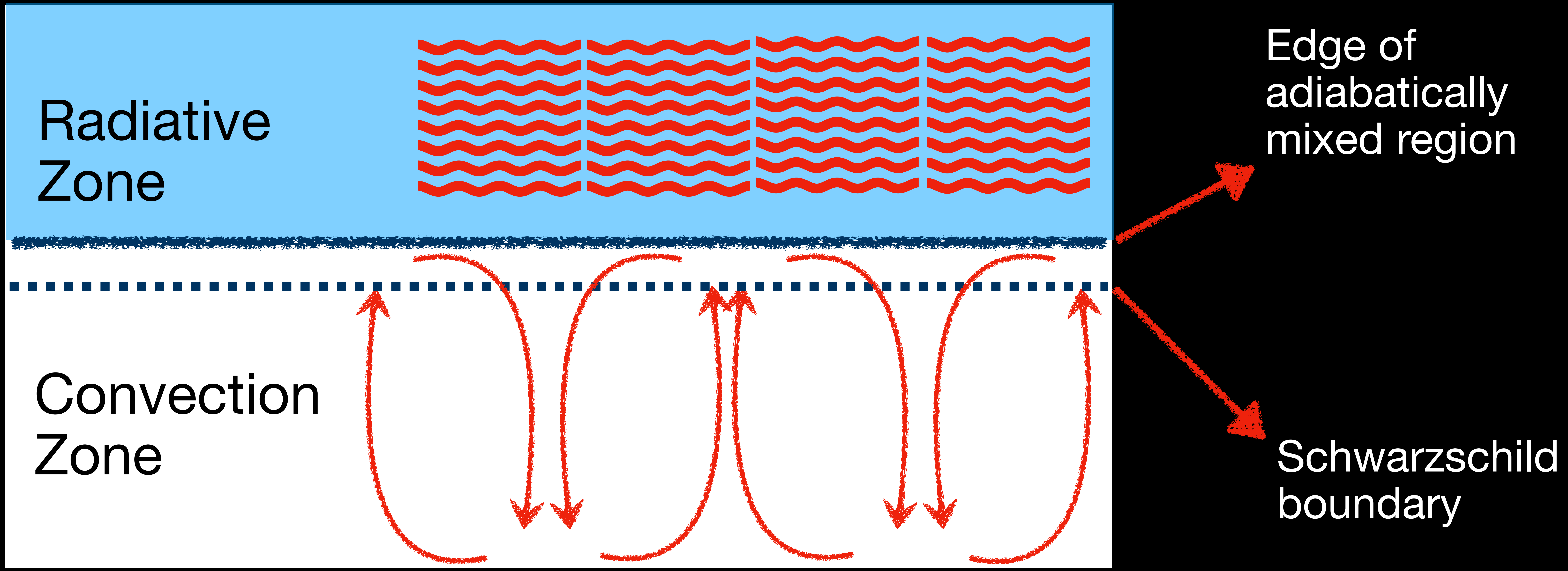
1. Convective overshoot (mechanical overshoot)
2. Entrainment (a *transient* process in a disequilibrium state)
3. **Convective penetration** (affects the *structure* of a convective interface)

[Theory: Roxburgh 1989, Zahn 1991, Anders et al 2021]

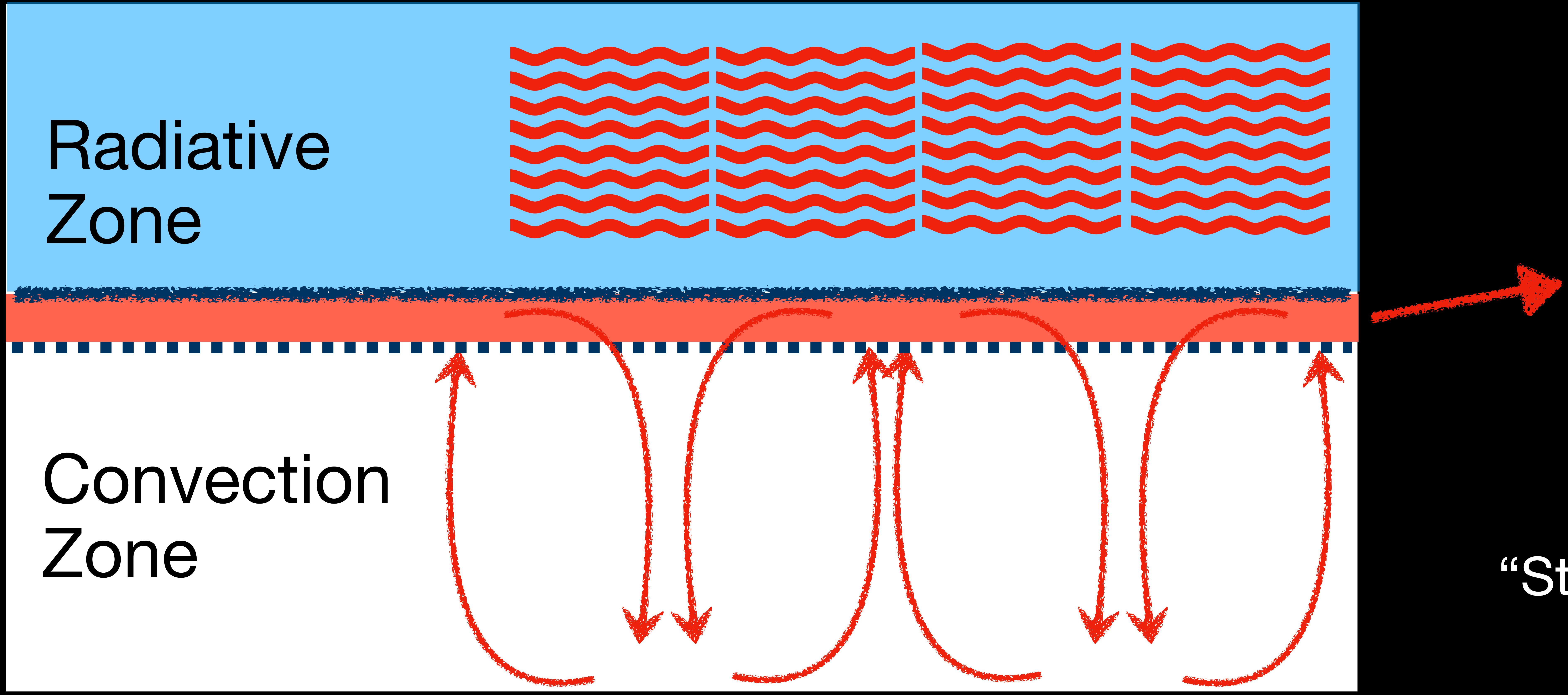
Entrainment builds a penetration zone



Convective Penetration



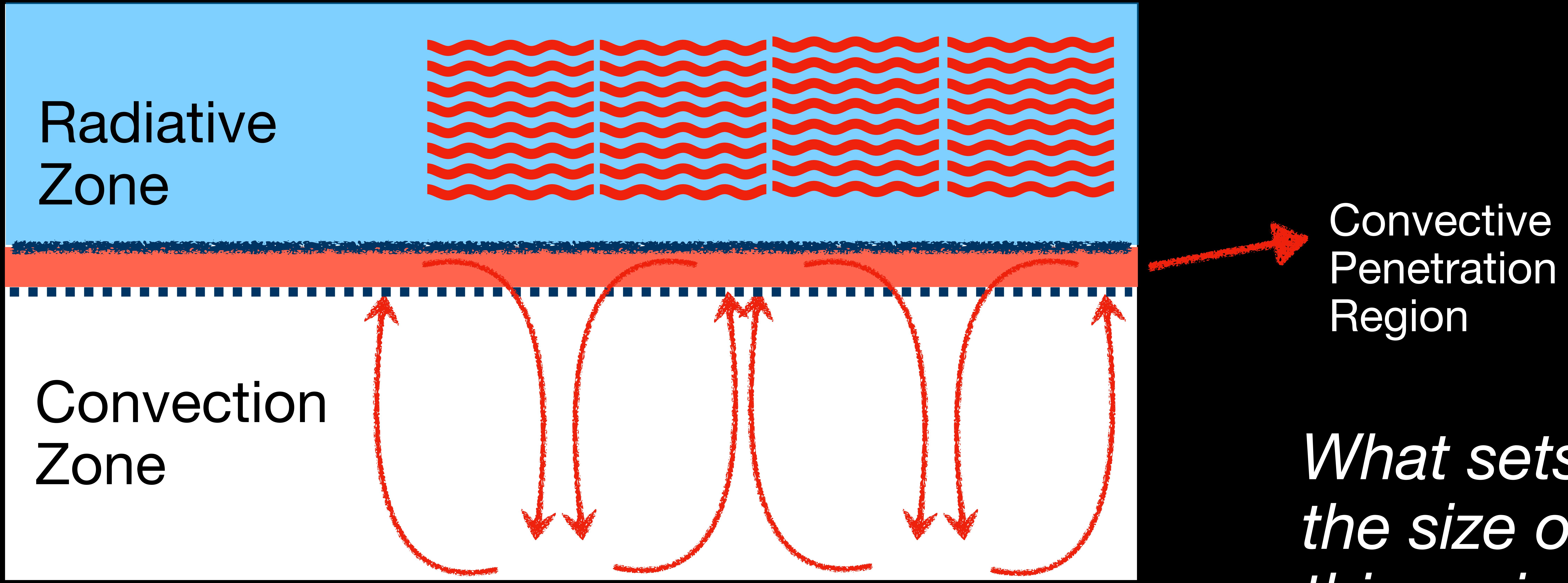
Convective Penetration



Convective Penetration Region

"Step overshoot"

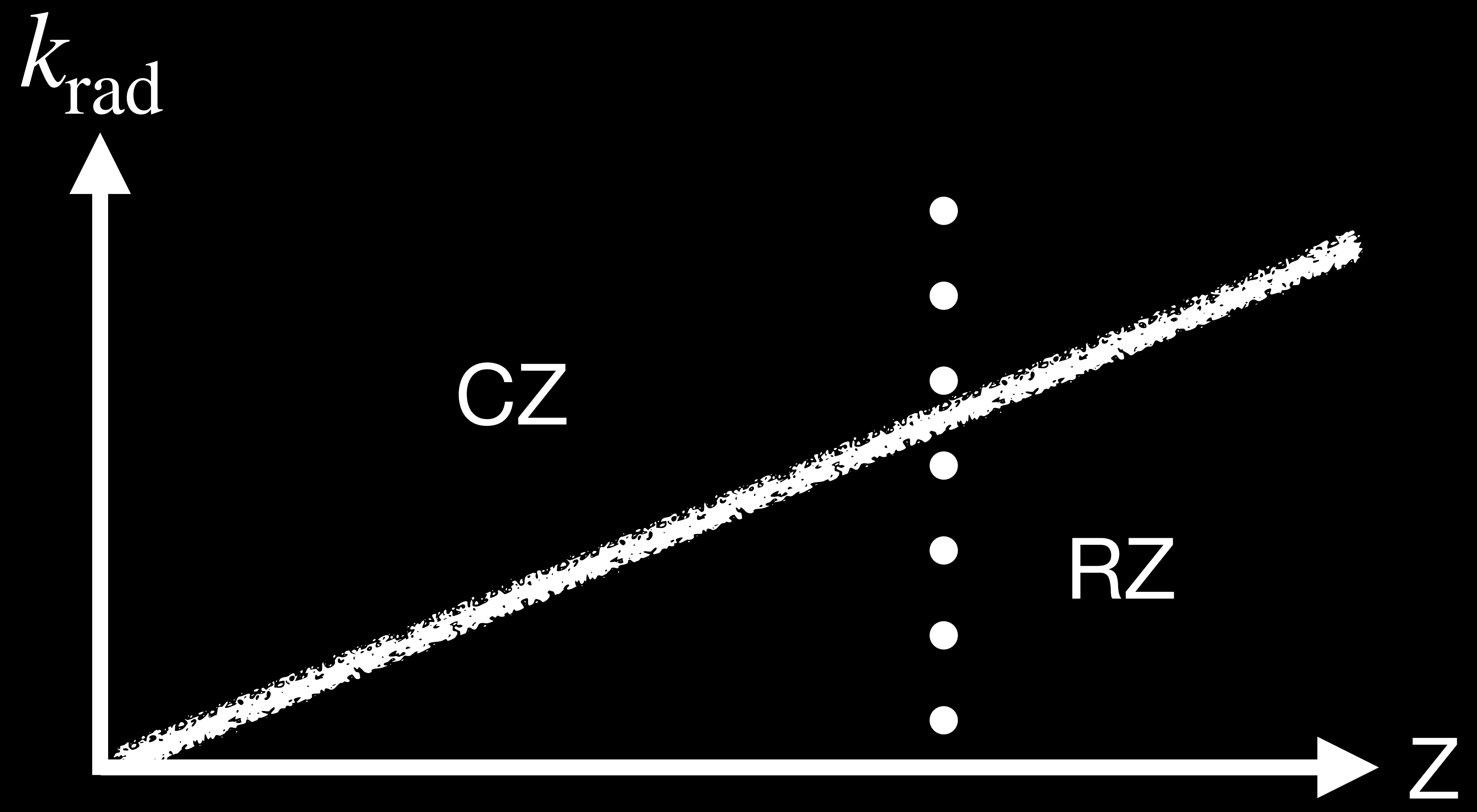
Convective Penetration



What sets the size of this region?

A toy model of fluxes in stellar convection

$$F_{\rm rad} = -\, k_{\rm rad} \nabla T$$



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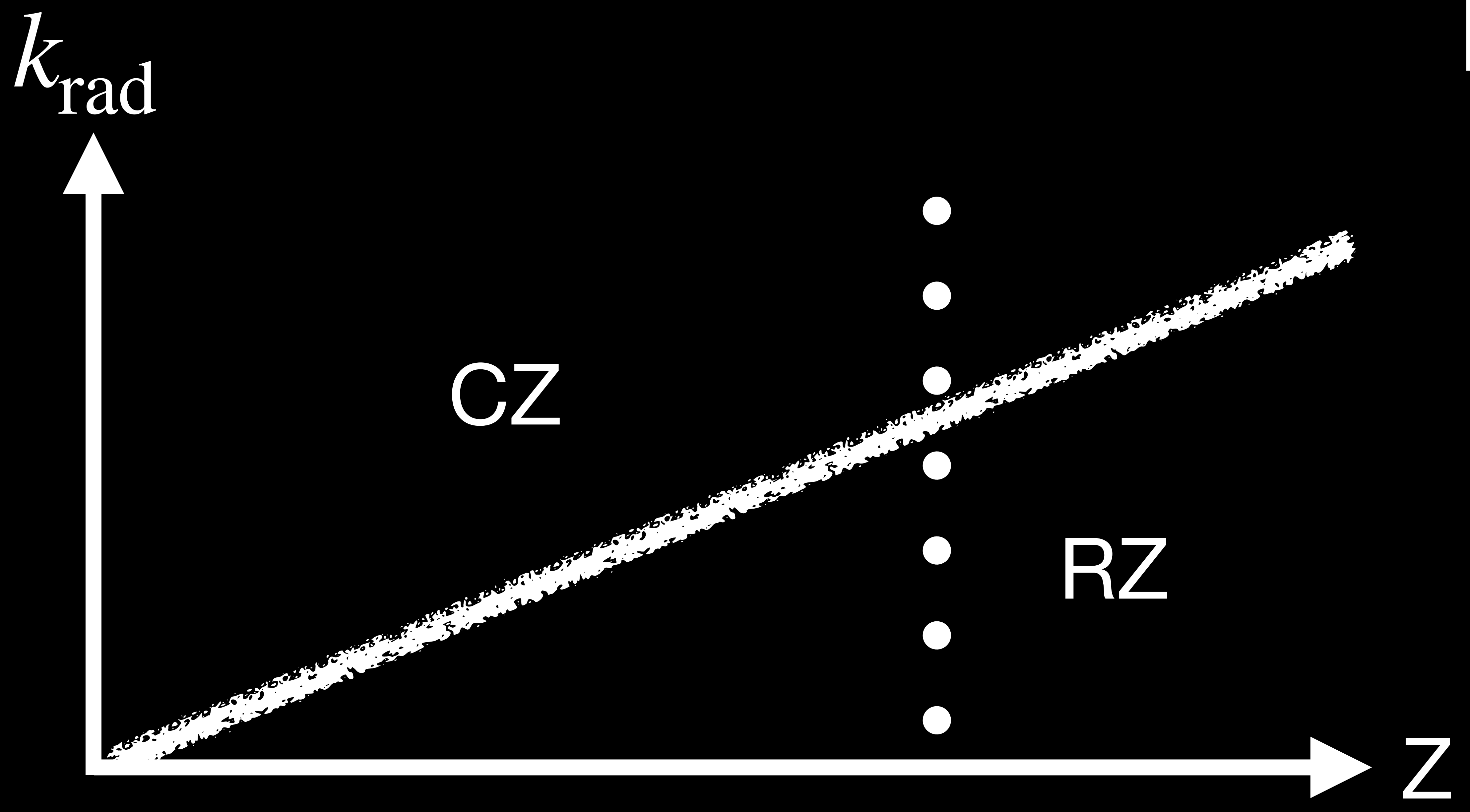

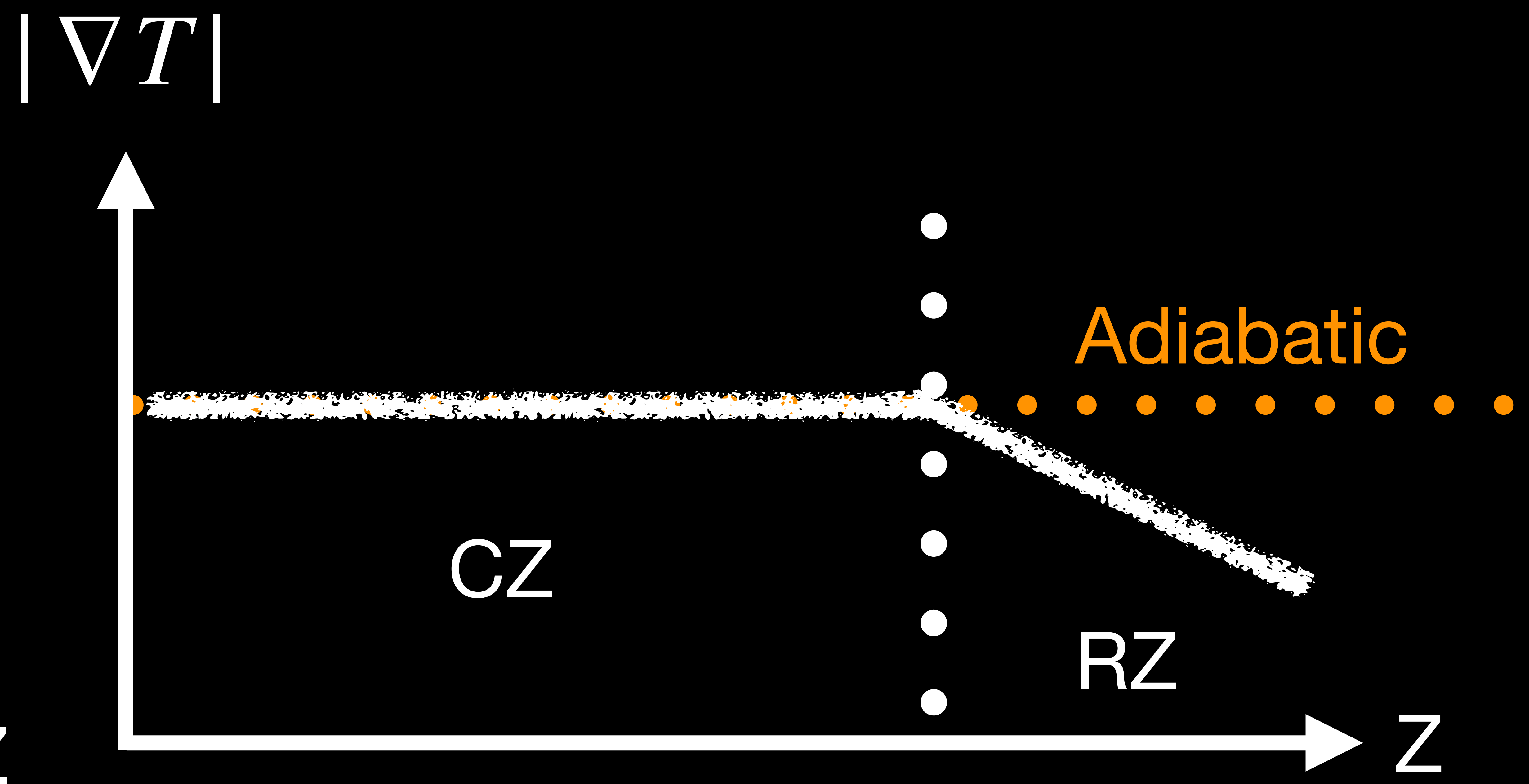

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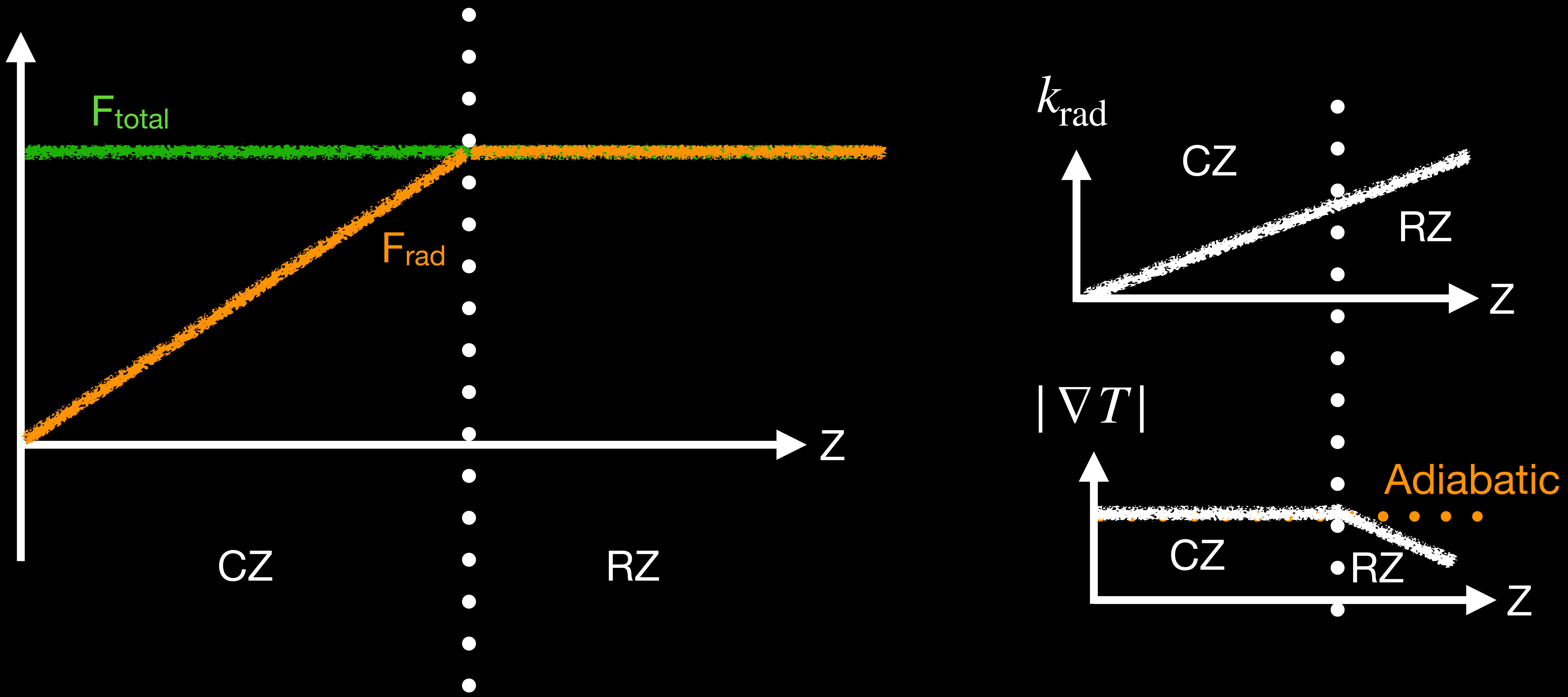

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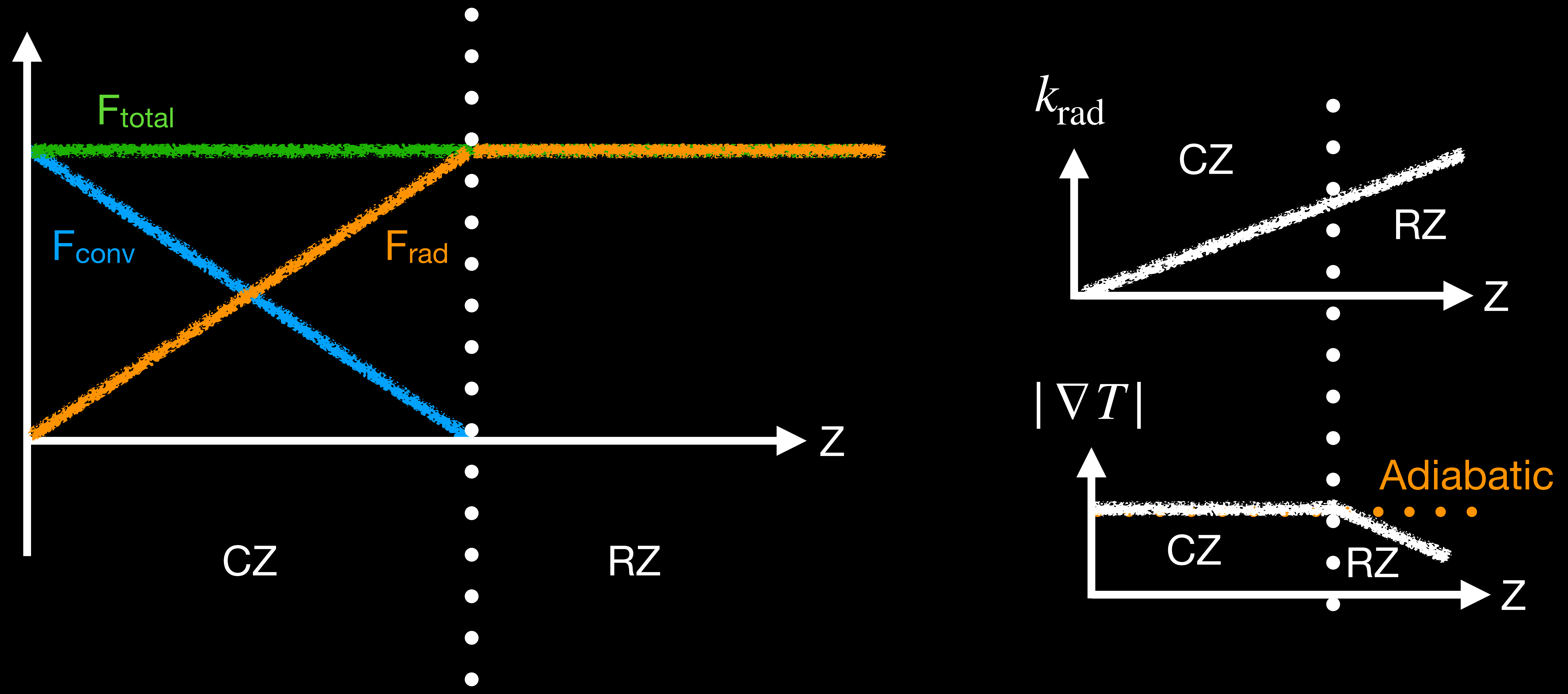
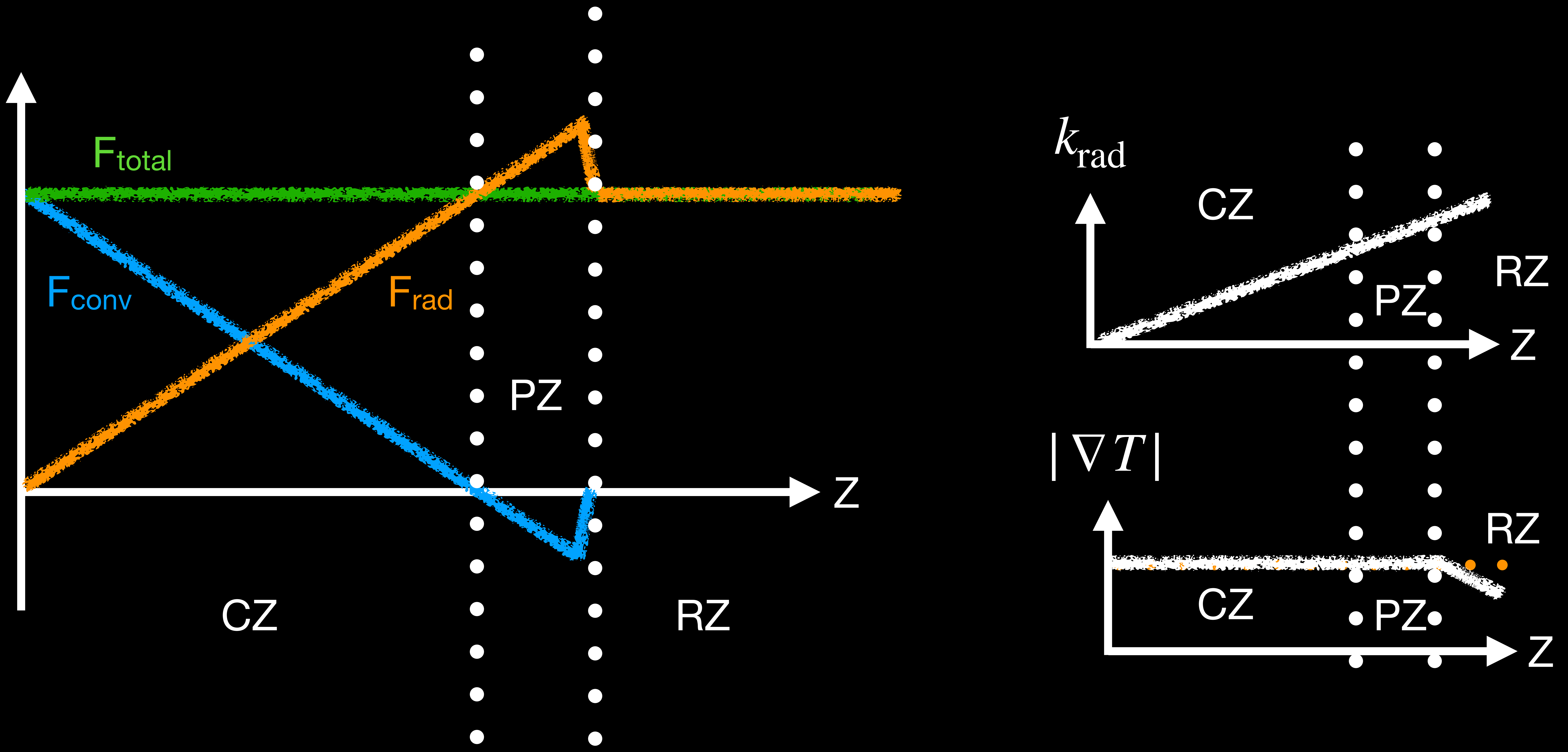


A toy model of fluxes in *convective penetration*



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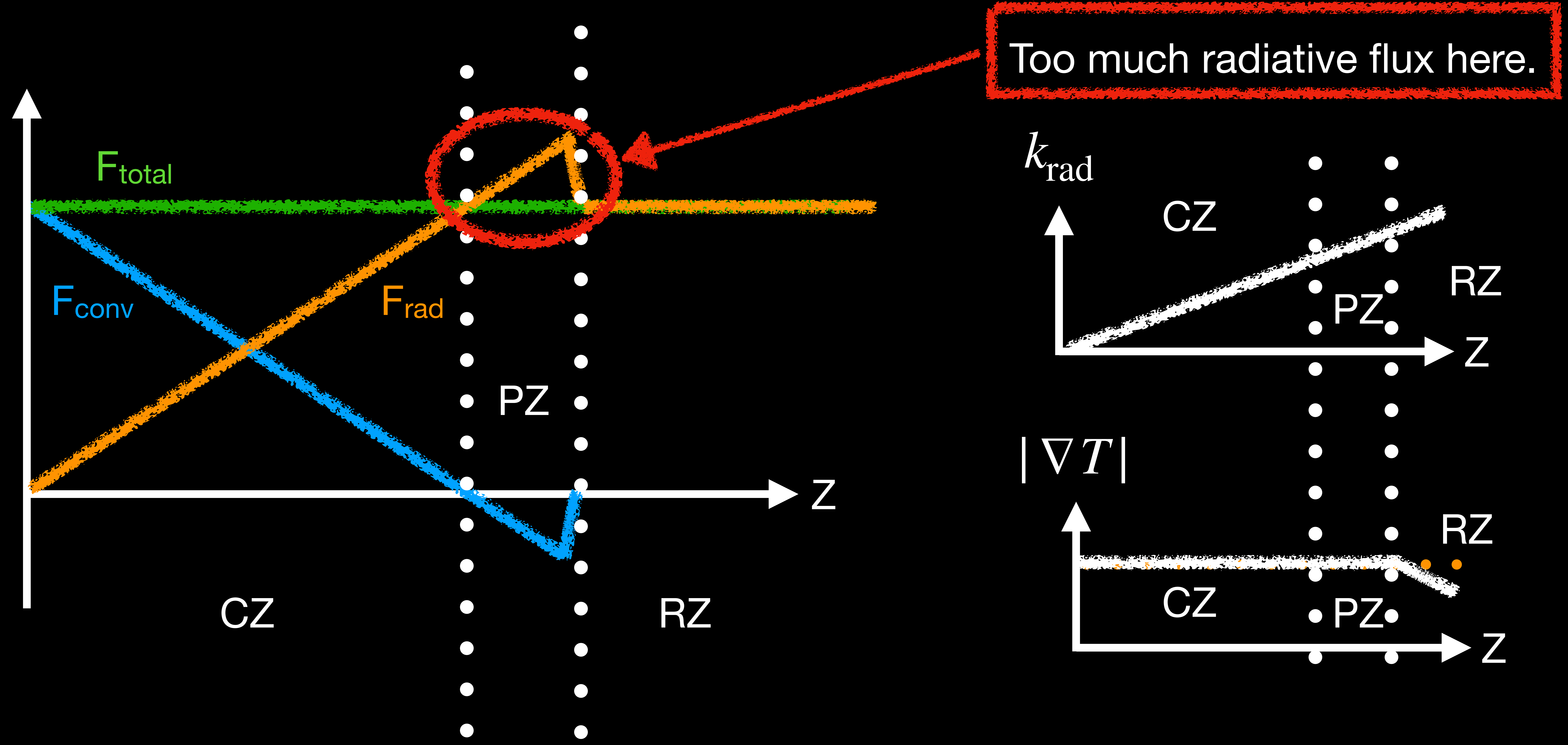

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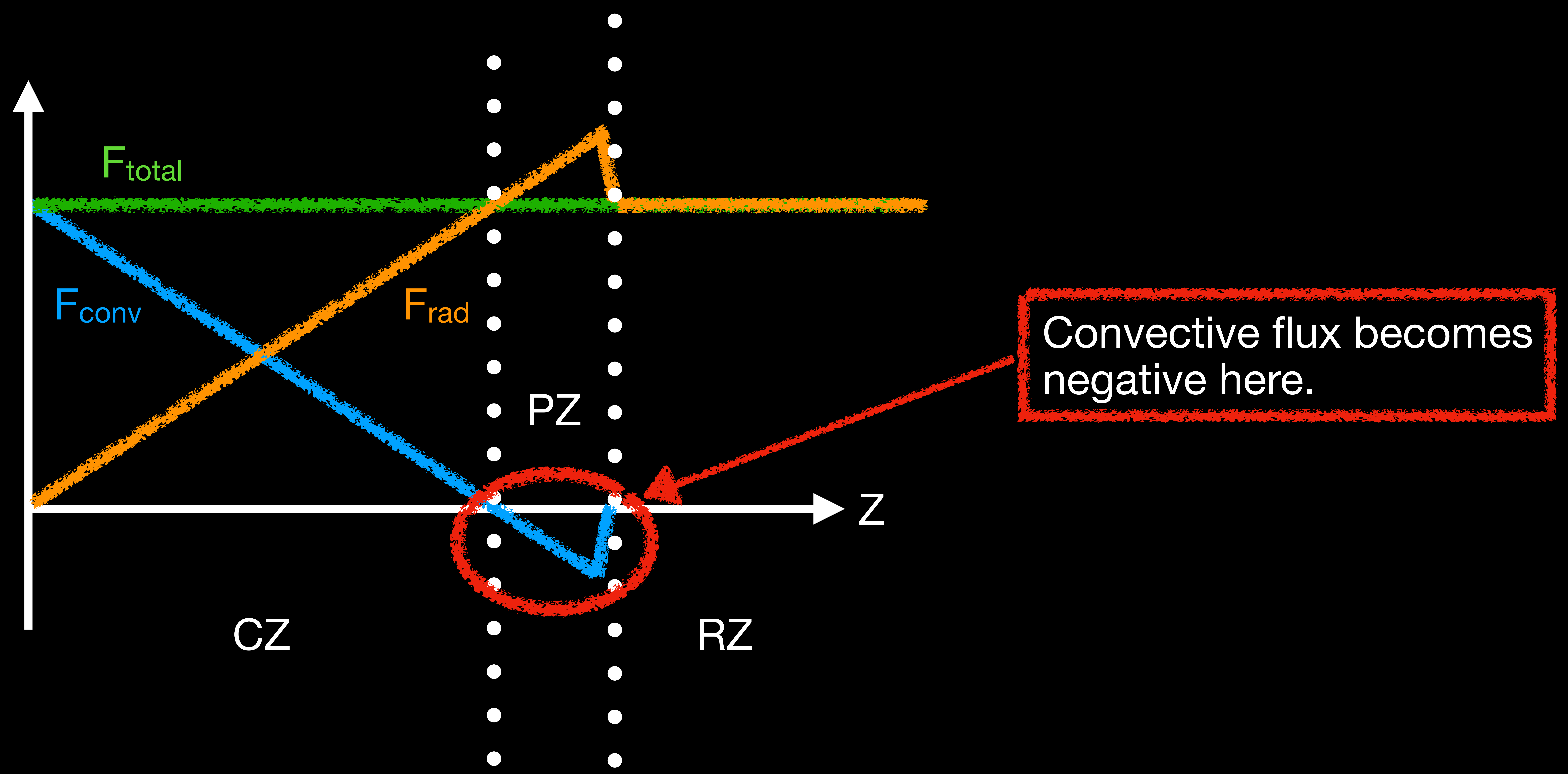
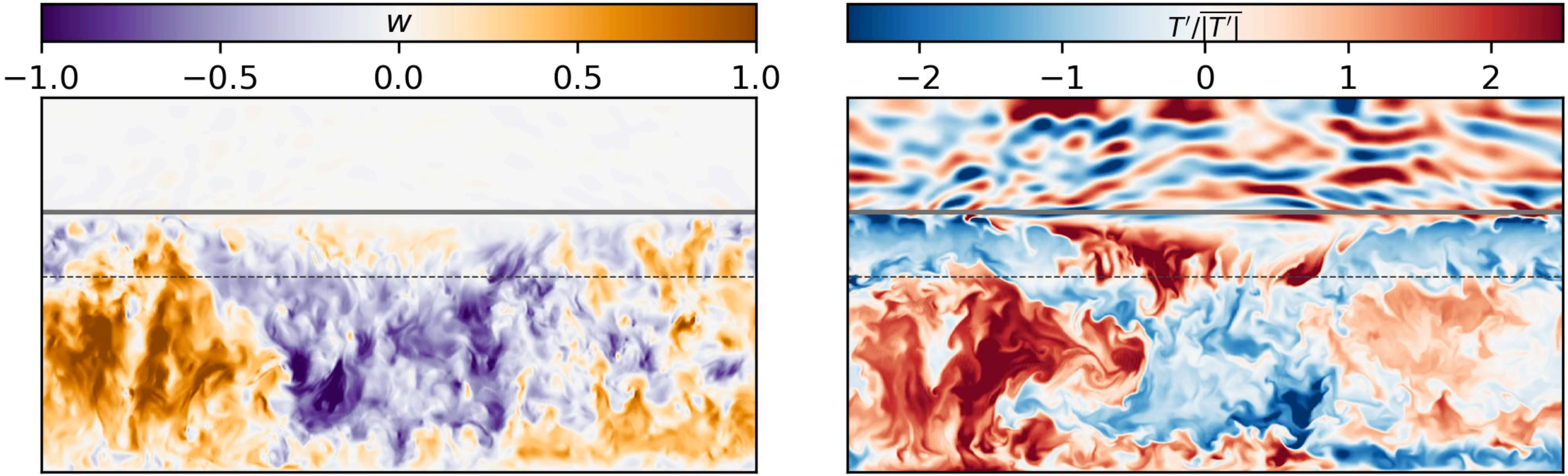


Dynamics of convective penetration

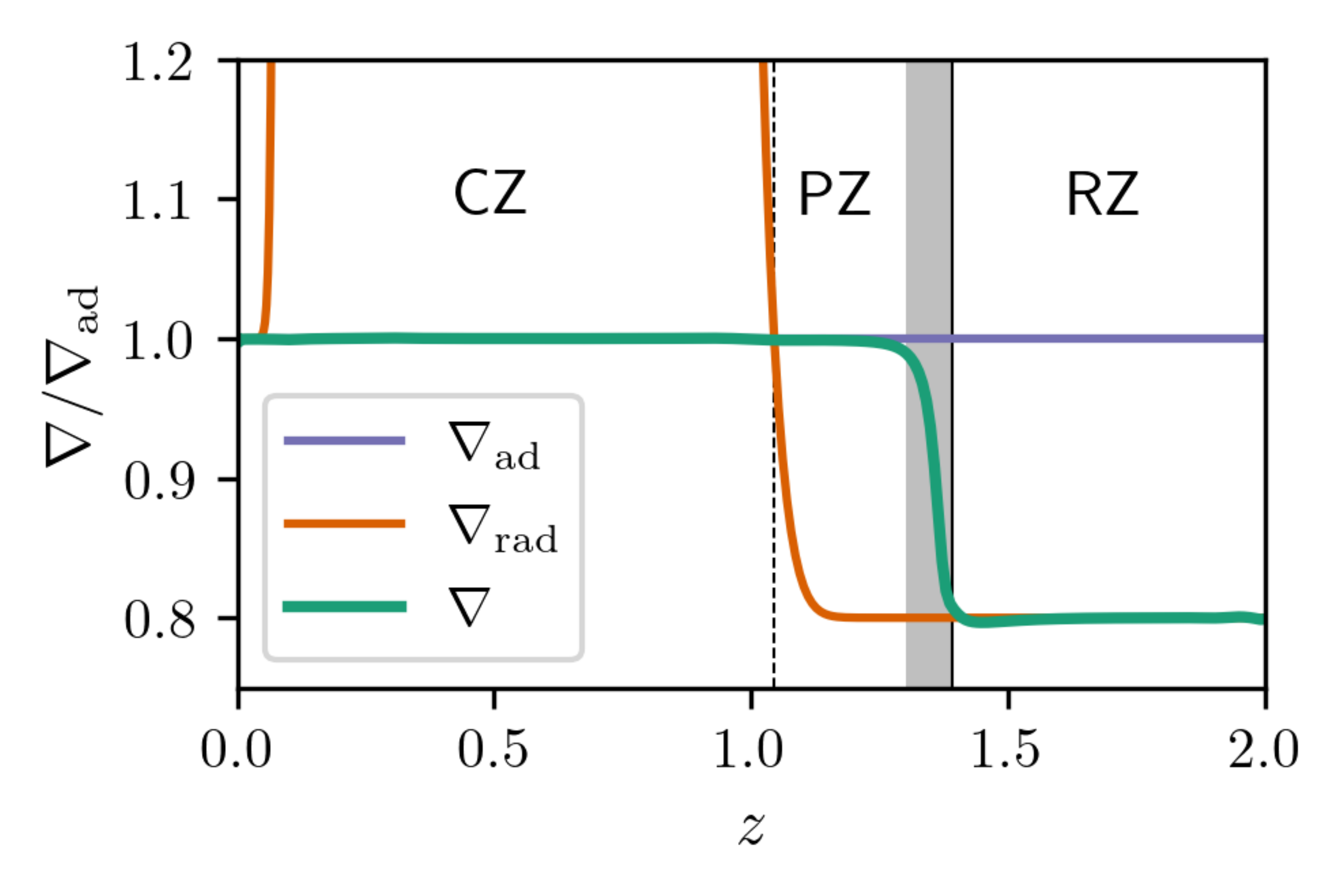


Vertical velocity

Temperature perturbations
(scaled with depth)

Dashed line = Schwarzschild boundary,
Solid line = dynamical boundary

Measured *output* profiles of a simulation of penetration



[Anders et al 2021]

What sets the size of a convective penetration region?

Kinetic energy equation

$$\frac{\partial \mathscr{K}}{\partial t} + \nabla \cdot \left(\overrightarrow{\mathscr{F}} \right) = \mathscr{B} - \Phi$$

[Roxburgh 1989]

What sets the size of a convective penetration region?

Kinetic energy equation

$$\frac{\partial \mathscr{K}}{\partial t} + \nabla \cdot \left(\vec{\mathscr{F}} \right) = \mathscr{B} - \Phi$$

Kinetic energy (KE)

KE fluxes

Buoyant KE generation

Viscous losses (dissipation)

[Roxburgh 1989]

What sets the size of a convective penetration region?

Volume average (over CZ + PZ) & assume time stationary

$$\frac{\partial \mathscr{K}}{\partial t} + \nabla \cdot \left(\overrightarrow{\mathscr{F}} \right) = \mathscr{B} - \Phi$$

[Roxburgh 1989]

What sets the size of a convective penetration region?

Volume average (over CZ + PZ) & assume time stationary

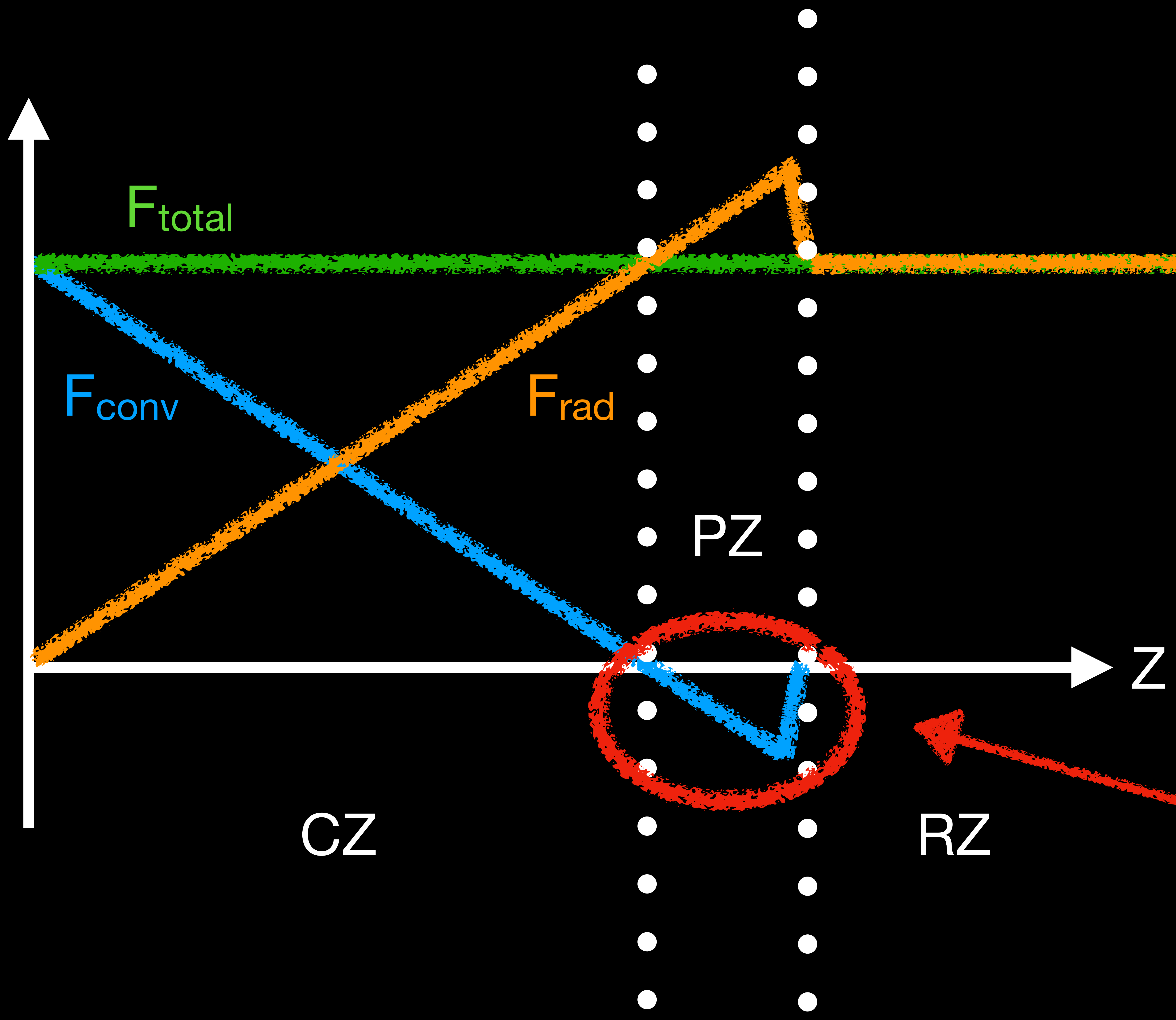
$$\frac{\partial \mathcal{K}}{\partial t} + \nabla \cdot \left(\overrightarrow{\mathcal{F}} \right) = \mathcal{B} - \Phi$$

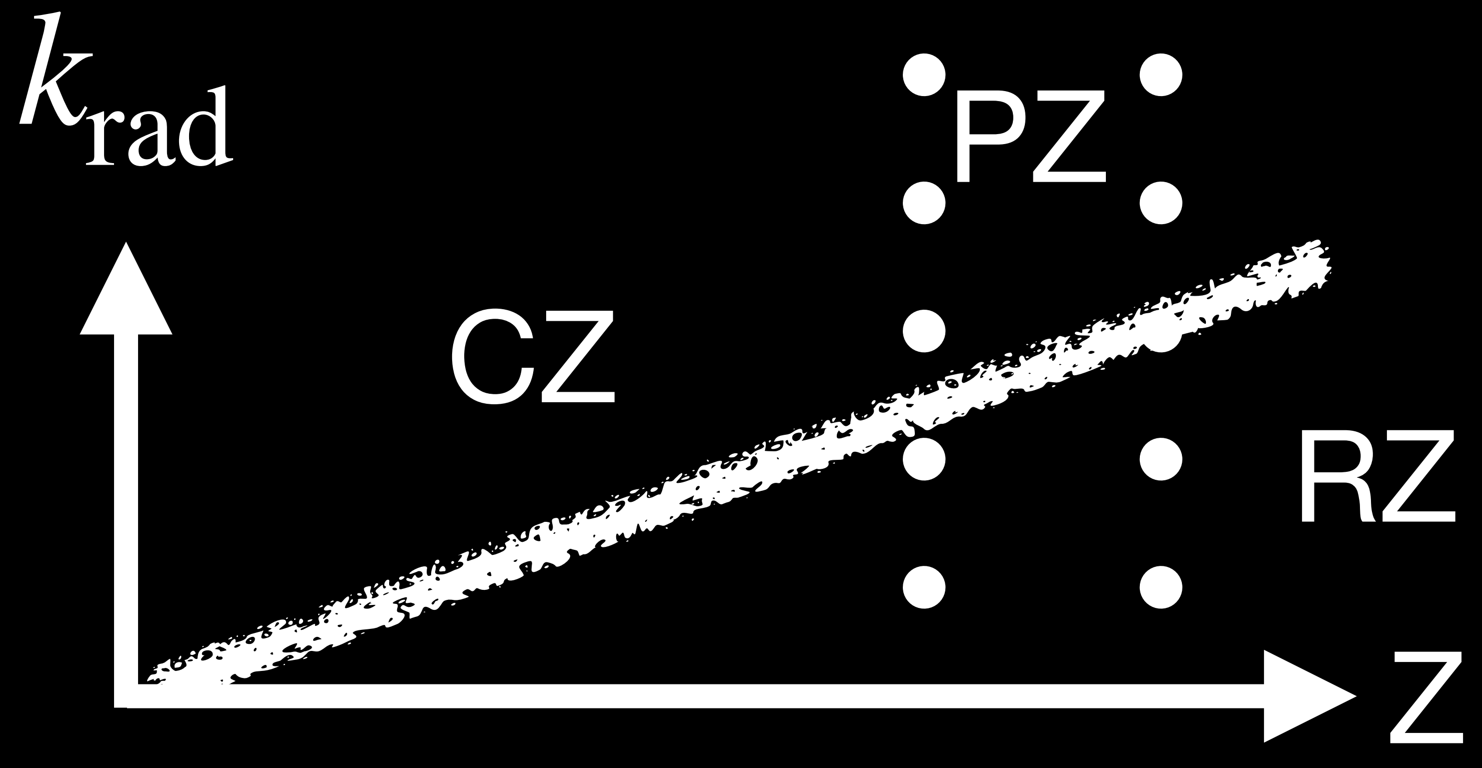
$$\iiint \mathcal{B}\, dV = \iiint \Phi\, dV$$

Buoyant work balances viscous losses.

[Roxburgh 1989]

A toy model of fluxes in *convective penetration*

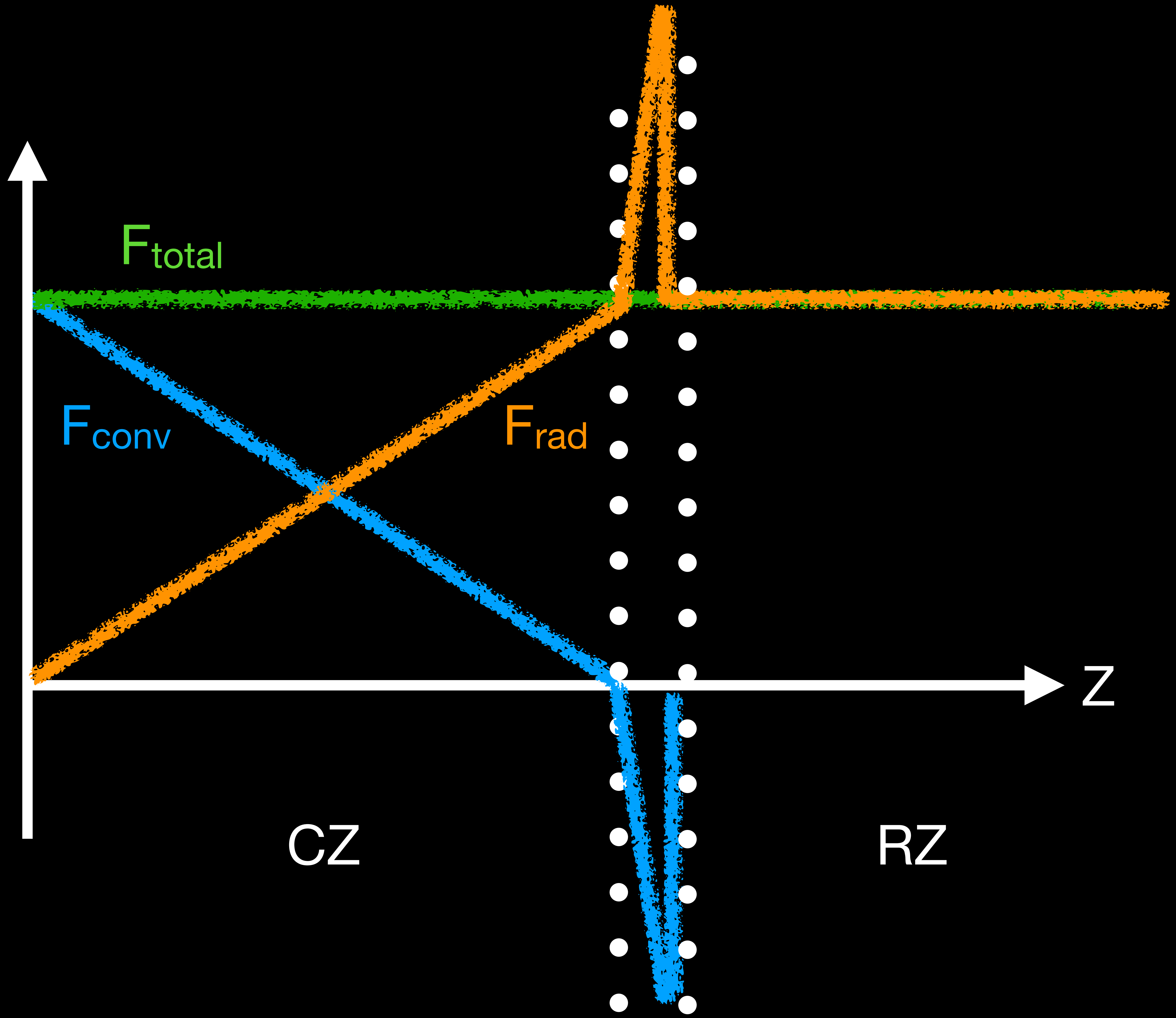


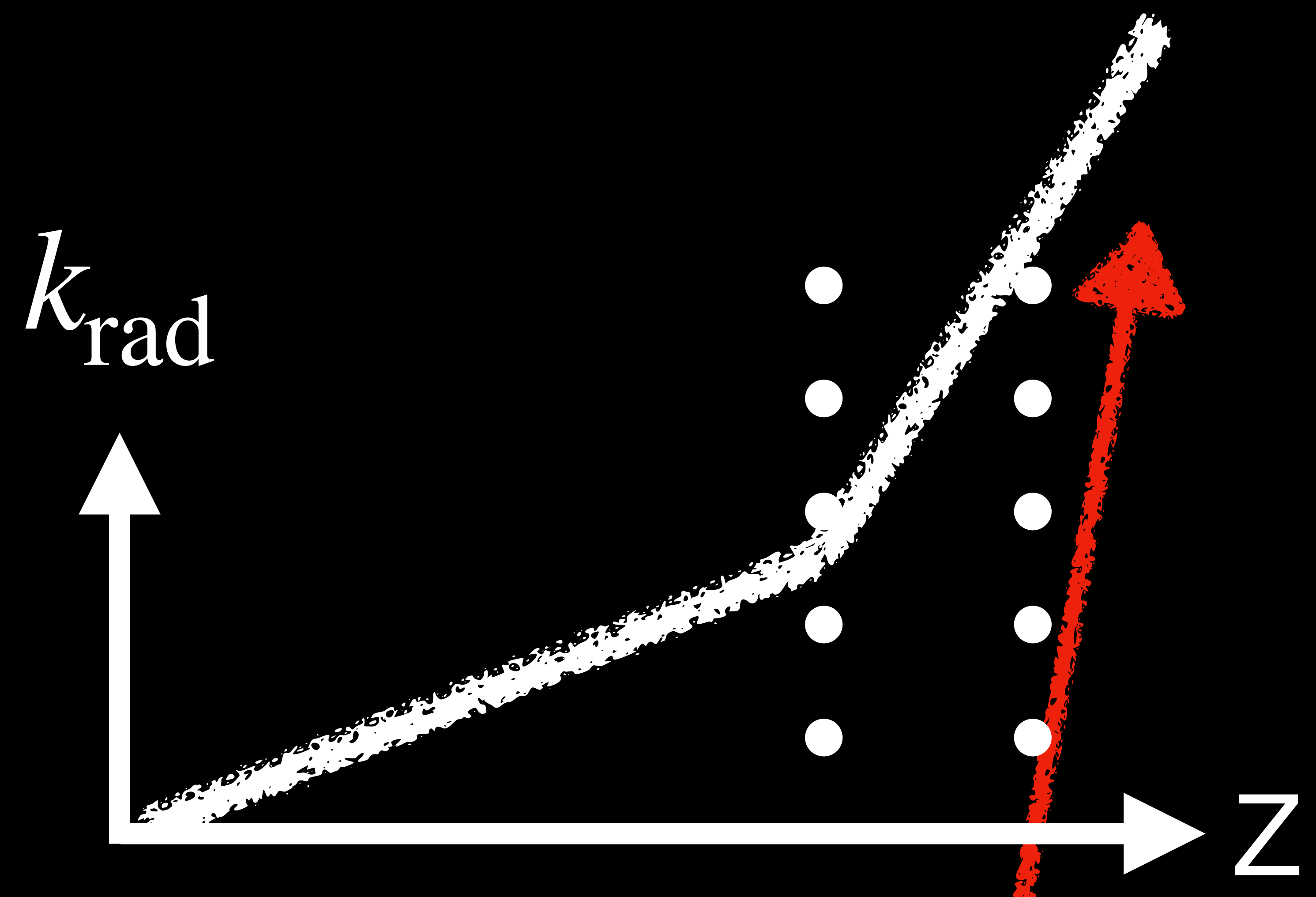


The ‘negativeness’ here is set by the radiative conductivity profile!

[Zahn 1991]

A toy model of fluxes in *convective penetration*

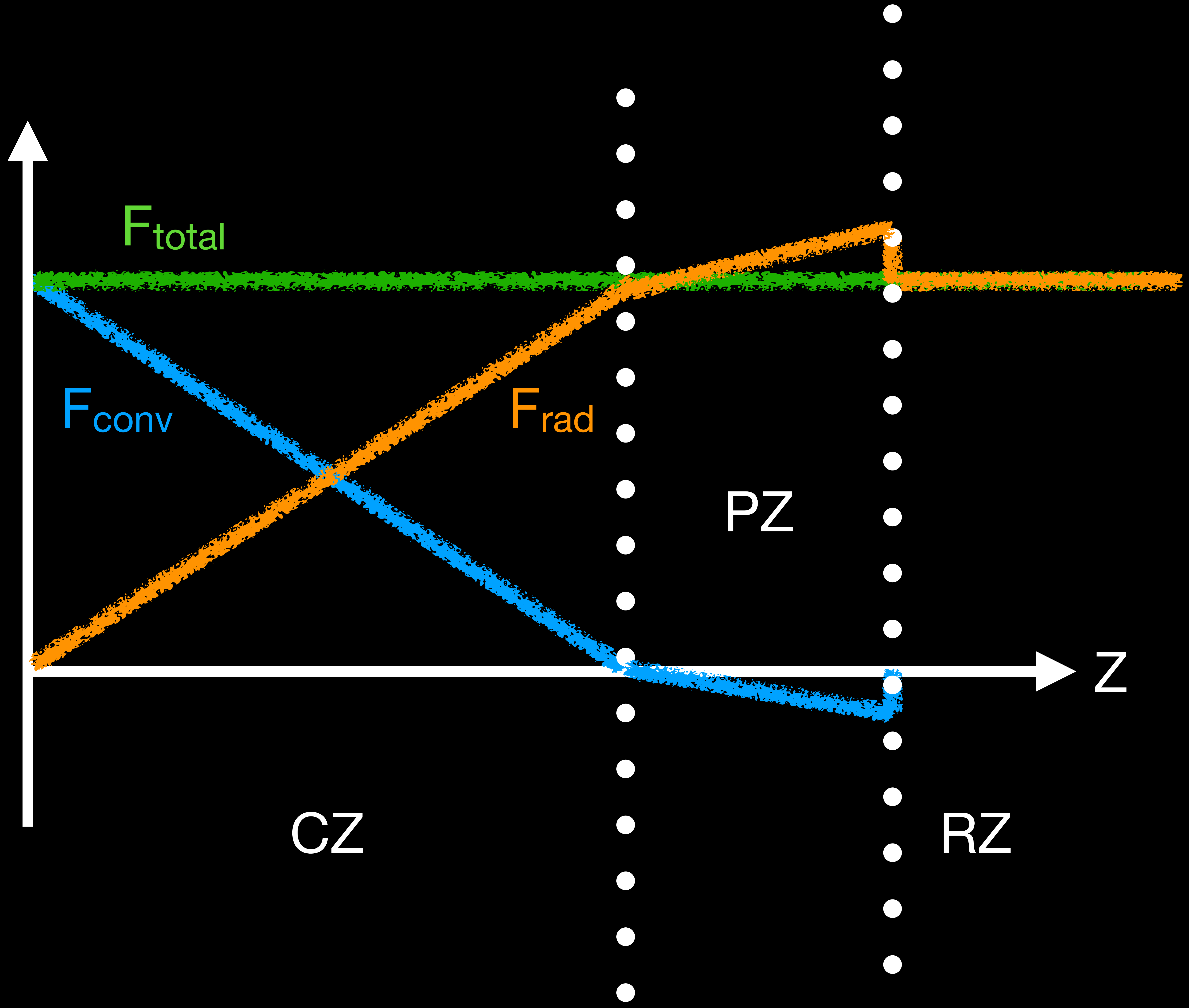


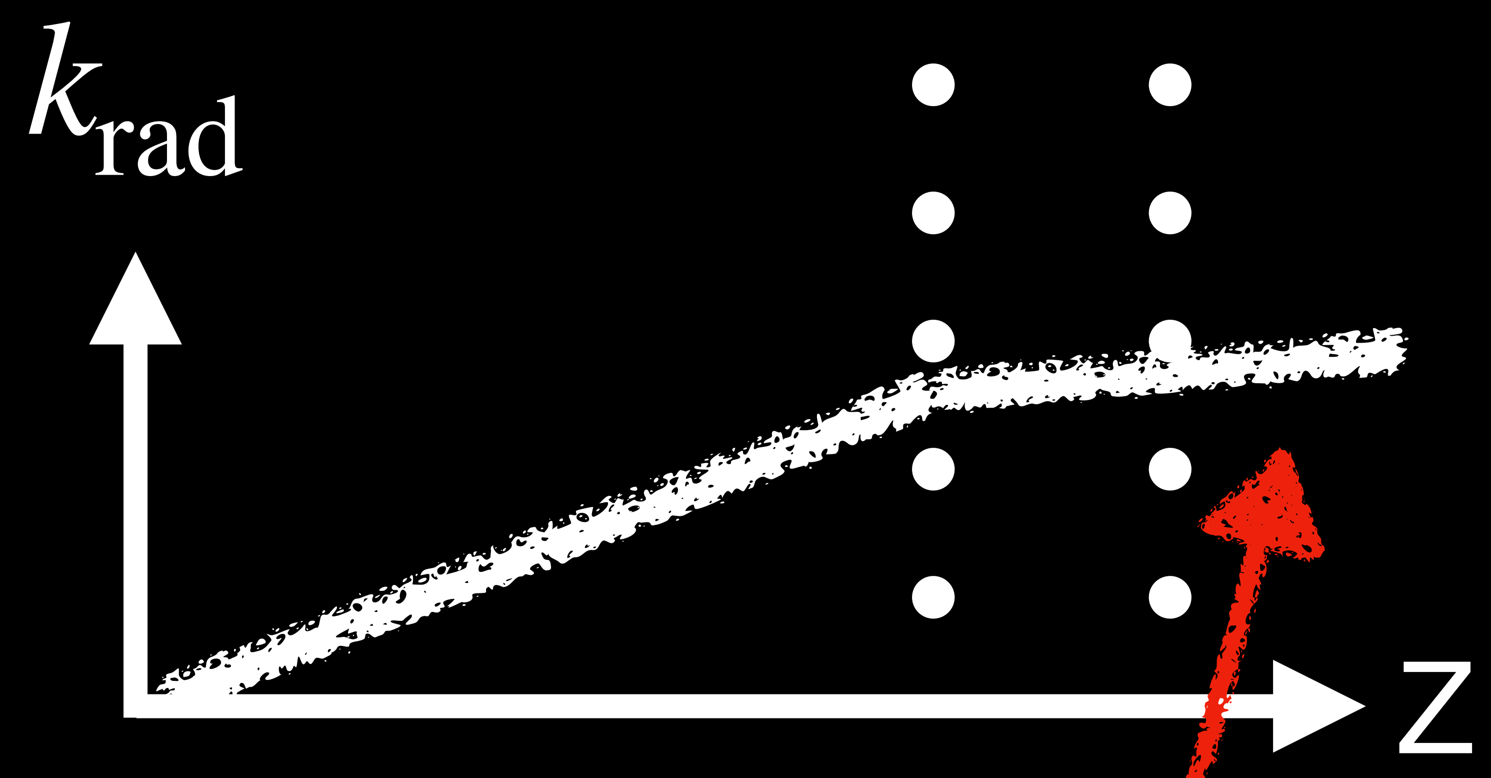


Steep profile here leads to smaller PZ

[Zahn 1991]

A toy model of fluxes in *convective penetration*

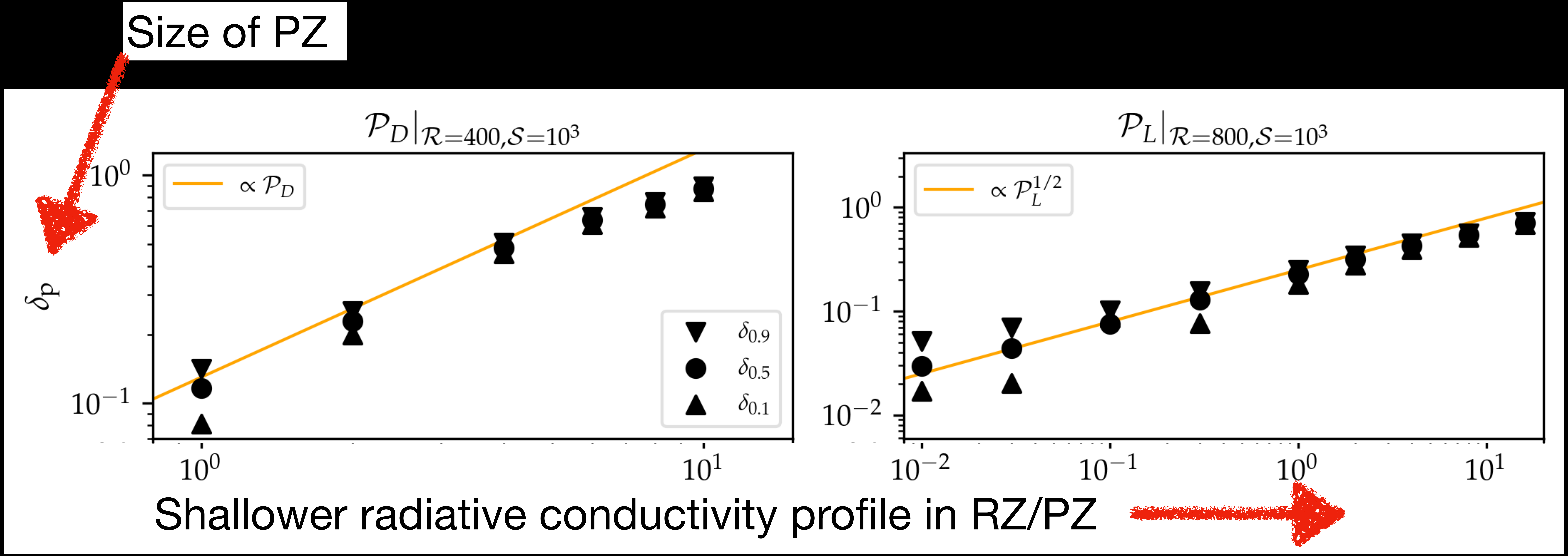




Shallow profile here leads to larger PZ

[Zahn 1991]

Strong PZ dependence on P in simulations



[fig 7, Anders et al 2021 / arxiv: 2110.11356]

From 3D to MESA

We derived a closed form equation from energy arguments that relates the radiative conductivity to the extent of the PZ. See eqn 44 in Anders+2021.

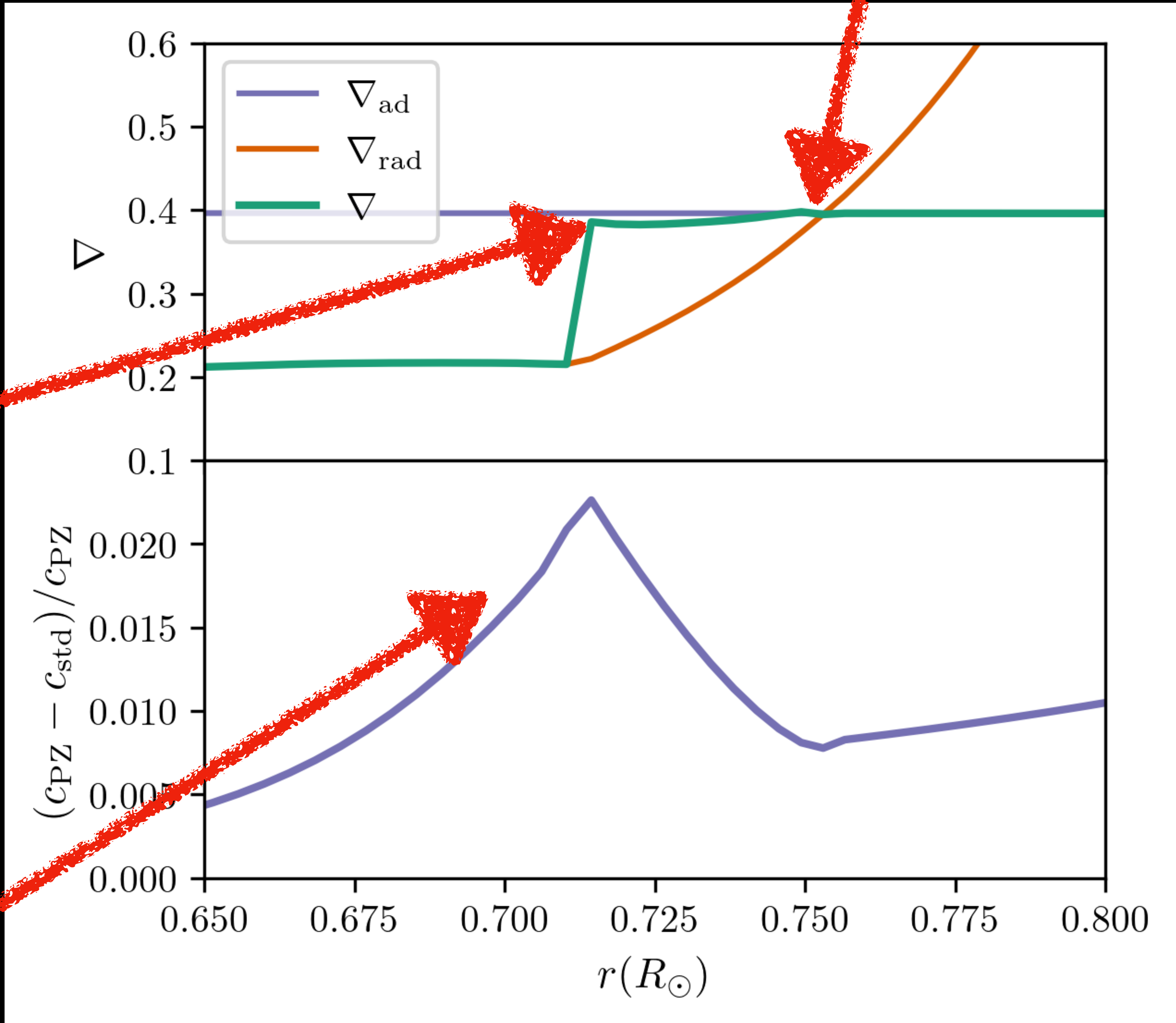
From 3D to MESA

Simple MESA model of 1 $M_\odot$ star with penetration. (not a solar model)

Bottom of penetration zone

Produces measurable acoustic glitch

Schwarzschild boundary

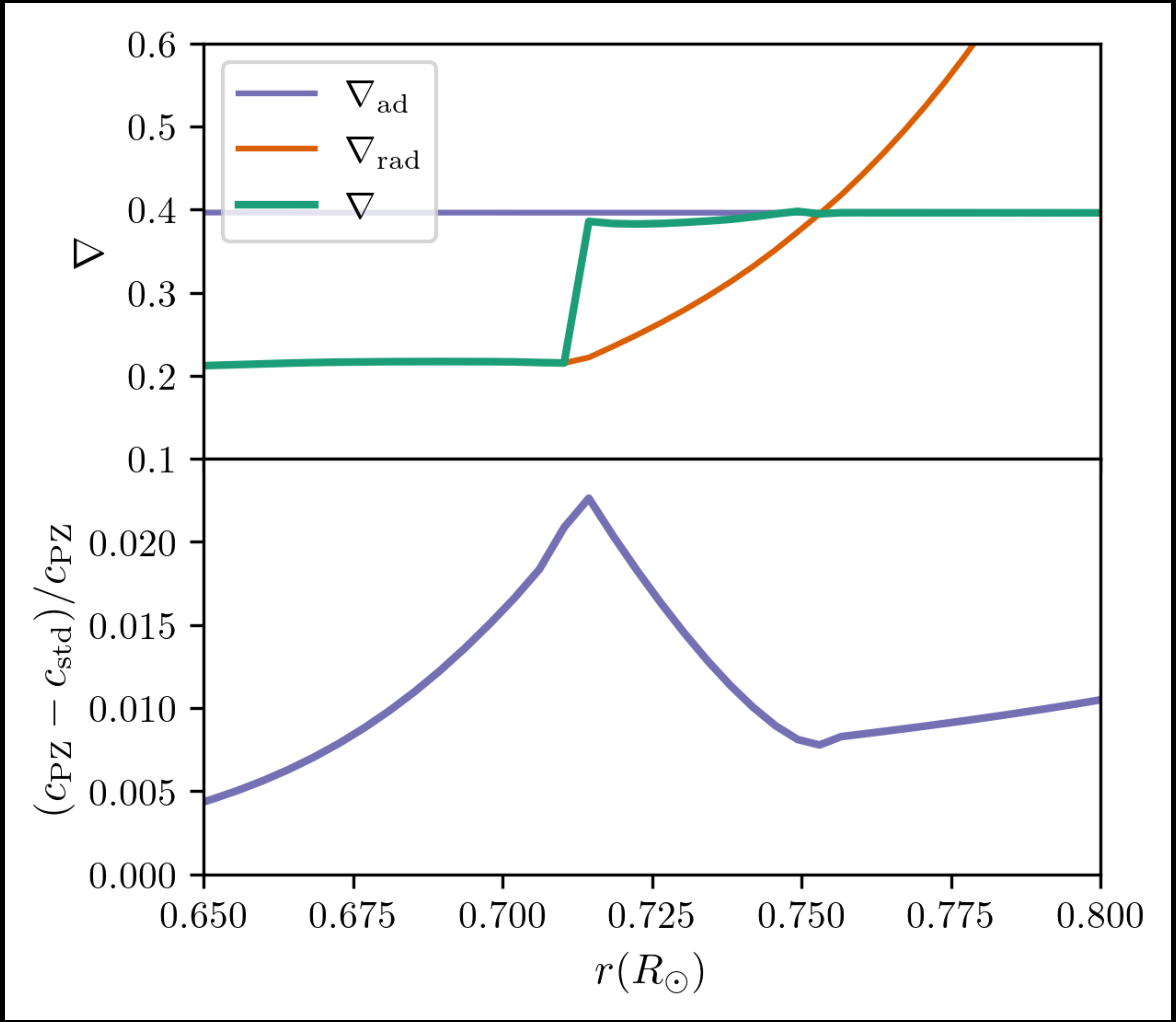


[fig 10, Anders et al 2021 / arxiv: 2110.11356]

From 3D to MESA

There are obvious problems with this model:

- Boussinesq theory (unsurprising it over predicts for the very stratified 'solar' CZ)
  - Not concerning. The logical process applies to the full equations.
- Some kinks to work out in making an adiabatic PZ in MESA



[fig 10, Anders et al 2021 / arxiv: 2110.11356]

Wrap up

We think we understand convective penetration, generally.
We think it's parameterizeable.
We need to expand the model & nail down parameter values.
Questions?

[Anders et al 2021 / arxiv: 2110.11356]