An overview of stellar convection

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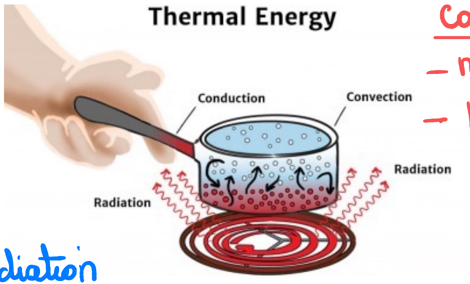
Convection & Convective Boundaries
in 1D stellar evolution codes

A. Thoul - Transport in Stellar Interiors - KITP - 17 november 2021

Three mechanisms for heat transfer
All three are active in stars

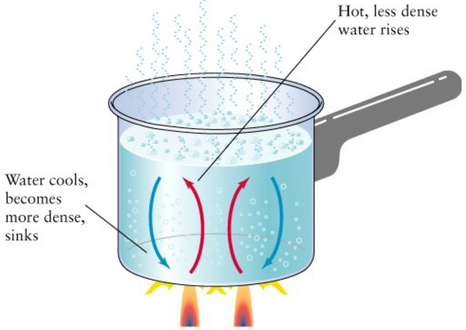
Thermal diffusion
- well understood
- well modeled



Radiation
- well understood
- reasonably well modeled
  (opacity κ)

convection
- not well understood
- poorly modeled

↳ Hydrodynamical instability in a stratified fluid with a "sufficiently large" ∇T



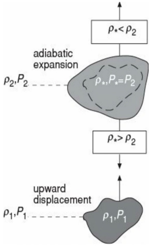
upward motion of hot matter
downward motion of cold matter

⇒ transfer of heat
+ mixing

Convection : Hydrodynamical instability in a stratified fluid with a "sufficiently large" ∇T

How large does ∇T have to be ?

When does a region become unstable to convection?



Convectively unstable if

$$\boxed{\nabla_{rad} = \frac{3 \kappa P L}{16 \pi a c G m T^4} > \nabla_{ad}}$$

Schwarzschild criterion

large L/m
ex: at center
(nuclear reactions)

⇒ convective cores

large opacity κ
ex.: zones of partial ionization near surface

⇒ convective envelopes
and convective shells

In the presence of a composition gradient:

$$\nabla_{rad} > \nabla_{ad} + \nabla_{\mu}$$

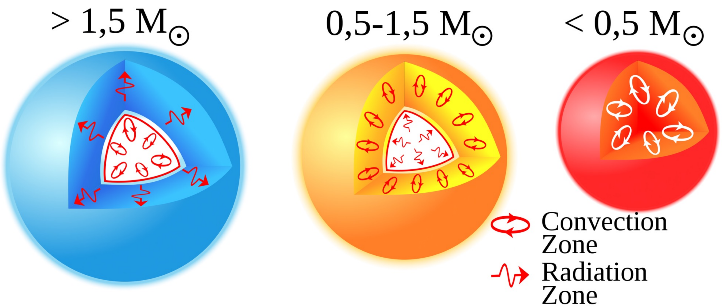
Ledoux criterion

High-mass stars

H is ionized in the envelope

⇒ low opacity

⇒ radiative envelope



Low mass stars

H not ionized in envelope

⇒ radiation easily absorbed

⇒ high opacity

⇒ convective envelopes

In cores : nuclear reactions

⇒ energy production strongly sensitive to temperature

⇒ large heat flux

CNO cycle $E \sim T^{20}$

Radiation cannot handle heat flux

⇒ convective cores

p-p chain $E \sim T^{4}$

radiation can handle heat flux

⇒ radiative cores

Whatever the mass of the star, it has convective regions ...

Convection happens in all stars.

Questions to answer:

- Where exactly? → The problem of the boundaries between radiative and convective regions
- How to model these regions (~~turbulent~~, ~~intrinsically 3D~~, ~~complicated~~!)
  1D ⇒ mixing-length model
  local
- Resulting mixing, energy flux
- (Interactions with magnetic fields, pulsations, nuclear burning, etc...)

Stellar evolution codes are 1D (mostly)

⇒ need a 1-D model for convection

The mixing-length model

very old : Biermann 1932 (in German!)

Böhm-Vitense 1958 : mixing-length model most used in stellar evolution codes

- 2D and 3D simulations : not ready for 1D codes
- MLT generally gives very good results

⇒ MLT Still widely used in stellar structure and evolution codes

The mixing-length approach

A mass element travels a distance l_c (mixing length) adiabatically at velocity v_c until reaches pressure equilibrium and releases heat

$$\delta T = \left[\left(\frac{dT}{dr}\right)_{rad} - \left(\frac{dT}{dr}\right)_{ad} \right] l_c$$

$$= T\,(\nabla_{rad} - \nabla_{ad}) \boxed{\frac{1}{P}\frac{dP}{dr}}\, l_c$$

$$P/\frac{dP}{dr} = \text{pressure scale-height} = l_P$$

Mixing-length α parameter :

$$\boxed{\alpha = \frac{l_c}{l_p} = \frac{l_c}{P(-dP/dr)} = \frac{l_c}{P/\rho g}}$$

$\alpha \sim 1-2$

The mixing-length approach

$$v_c = \sqrt{g' l_c} = \sqrt{\alpha \frac{P}{\rho} \left| \frac{\delta T}{T} \right|}$$

Convective flux = heat released x velocity

$$\Rightarrow \qquad F_c = \rho C_p T \sqrt{P/\rho} \left(\nabla_{rad} - \nabla_{ad} \right)^{3/2} \alpha^2$$

α cannot be determined from first principles

$\Rightarrow$ Free parameter

Convective Boundaries

= where the convective velocity vanishes.

Within MLT : equivalent to $\nabla_{rad} = \nabla_{ad}$

⚠ v_c only defined in convective region

$\Rightarrow$ must have $\nabla_{rad} = \nabla_{ad}$ on the convective side of the boundary!

As an important consequence, using the Ledoux criterion ($\nabla_L = \nabla_{ad} + B$, $B \propto \nabla\mu$) should not affect the boundary location! (IF convective region well mixed)

Very simple criterion !

but often applied incorrectly in stellar evolution codes ...

- when composition is discontinuous at the boundary
- during core Helium burning

In those cases,

using "find the zero of $(\nabla_{rad} - \nabla_{ad})$ or $(\nabla_{rad} - \nabla_{ad} - \nabla_{\mu})$" method does not correctly position the convective boundary !

A few examples :

Low mass star ($M = 1.5\, M_\odot$) growing convective core on MS (Main Sequence)

High mass star ($M = 16\, M_\odot$) retreating core on MS

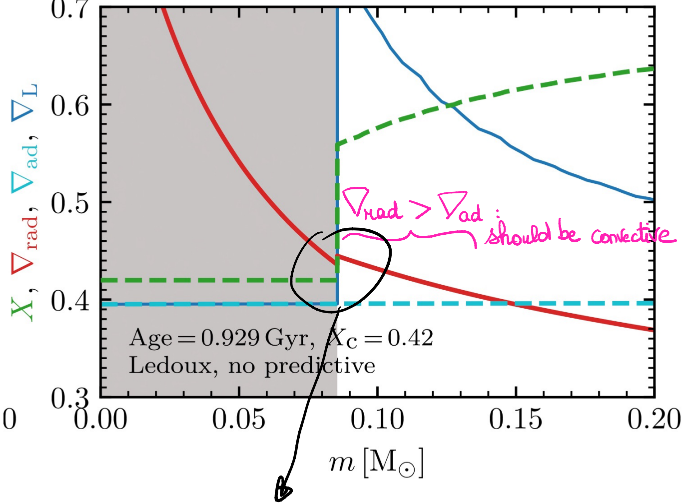
Growing convective core during CHeB (Core Helium Burning)

Growing convective core on MS

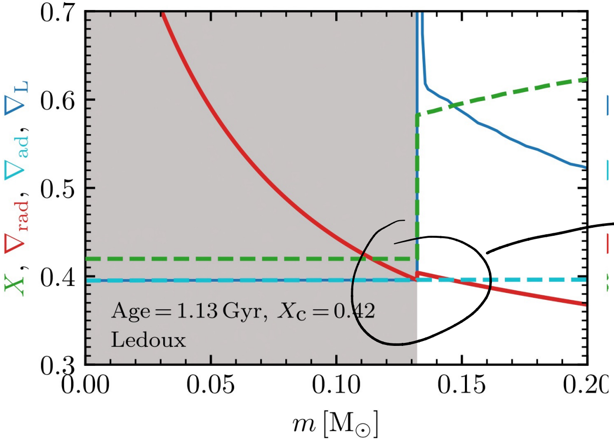
$M = 1.5 M_\odot$

" Find the zero" method

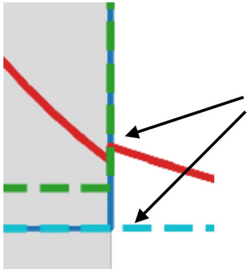
Paxton et al 2018 "



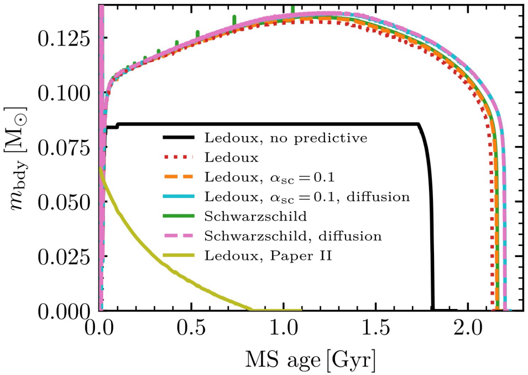
$\nabla_{rad} > \nabla_{ad}$: should be convective



$\nabla_{rad} = \nabla_{ad}$ on convective side of boundary

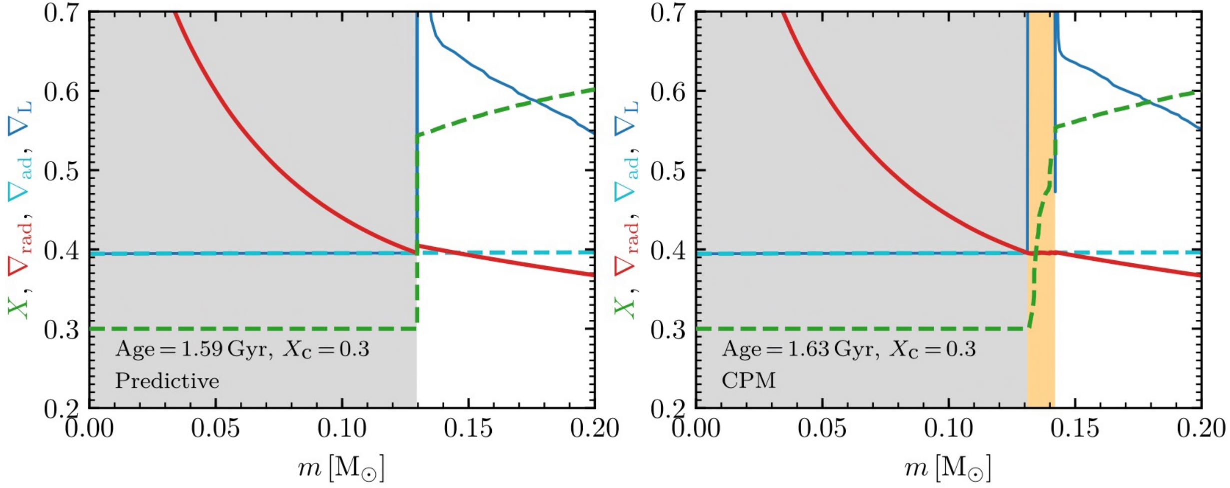


∇rad > ∇ad on convective side of the boundary = BAD



Core grows correctly

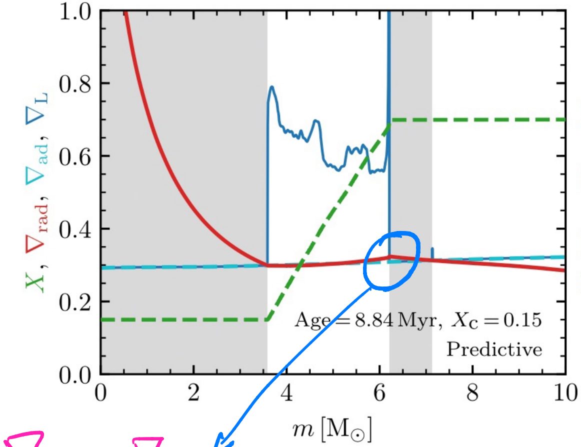
semi-convective region

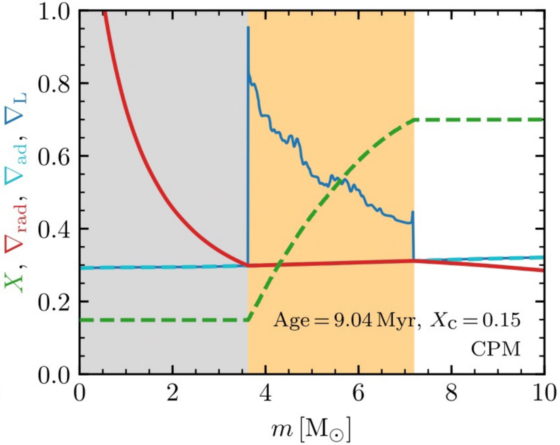


Retreating convective core on MS

$M = 16\,M_\odot$

Paxton et al. 2019

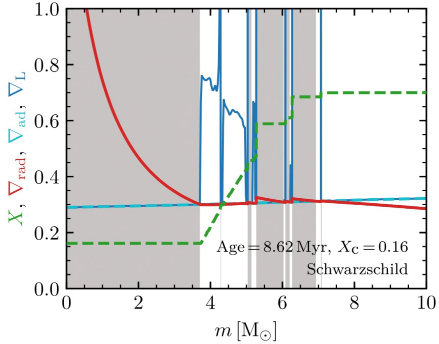


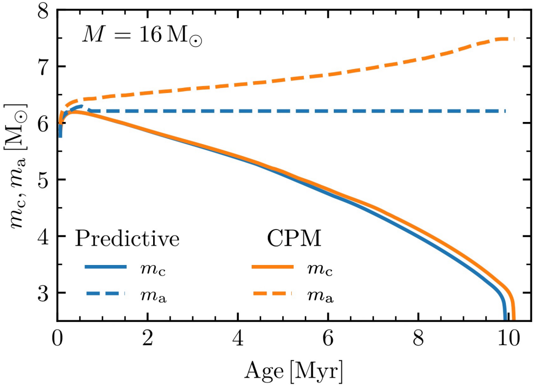


If done correctly, convective shells disappear replaced by a "semi-convective" region where $\nabla_{rad} = \nabla_{ad}$ (yellow)

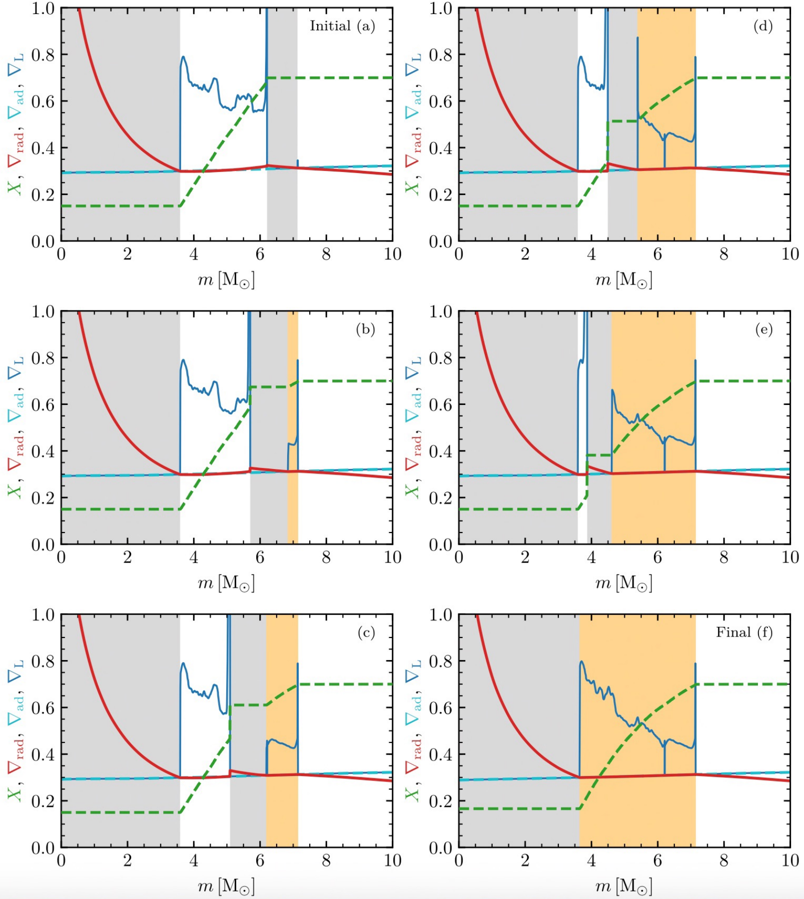
$\nabla_{rad} \neq \nabla_{ad}$

$\Rightarrow$ not ok

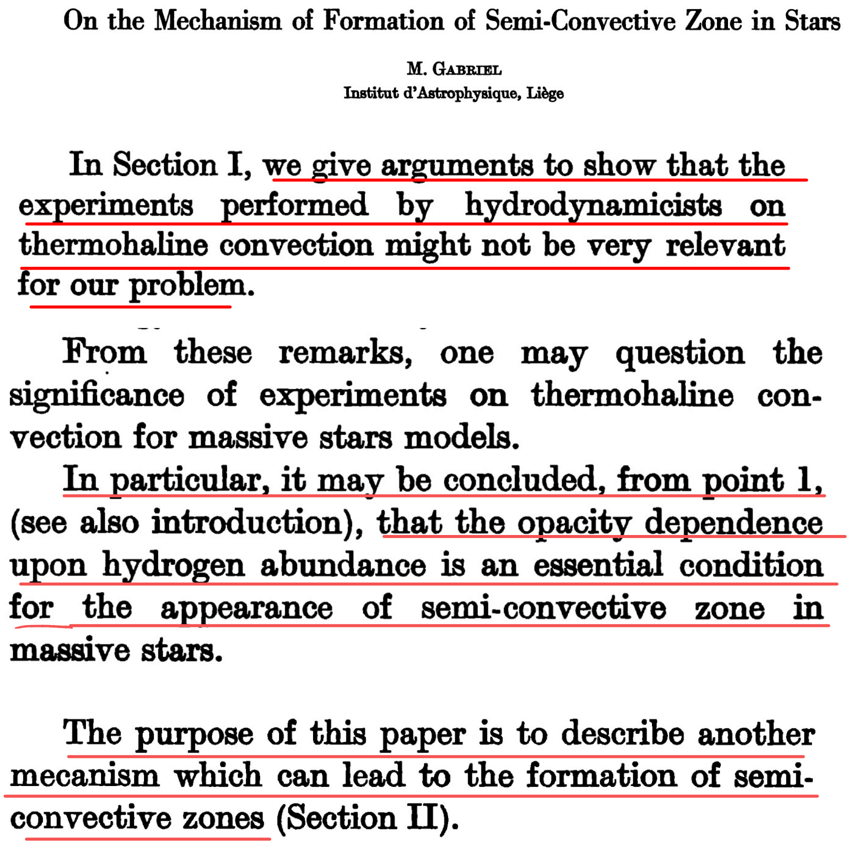




Different composition profile above the convective core


Initial (a)
(b)
(c)
(d)
(e)
Final (f)
X, $\nabla_{\rm rad}$, $\nabla_{\rm ad}$, $\nabla_{\rm L}$
$m\,[{\rm M}_\odot]$
0.0
0.2
0.4
0.6
0.8
1.0
0
2
4
6
8
10

This result had been predicted by Gabriel (1970) !



On the Mechanism of Formation of Semi-Convective Zone in Stars

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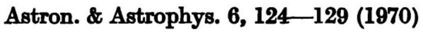
In Section I, we give arguments to show that the experiments performed by hydrodynamicists on thermohaline convection might not be very relevant for our problem.

From these remarks, one may question the significance of experiments on thermohaline convection for massive stars models.

In particular, it may be concluded, from point 1, (see also introduction), that the opacity dependence upon hydrogen abundance is an essential condition for the appearance of semi-convective zone in massive stars.

The purpose of this paper is to describe another mecanism which can lead to the formation of semi-convective zones (Section II).

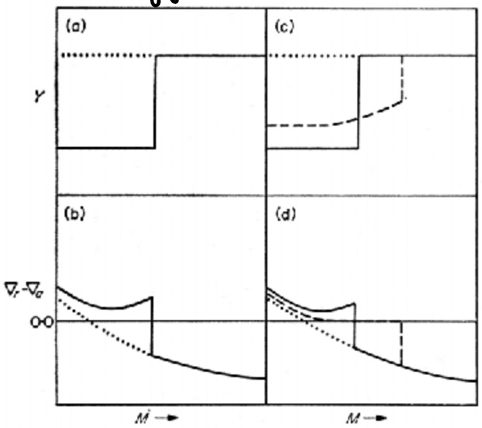


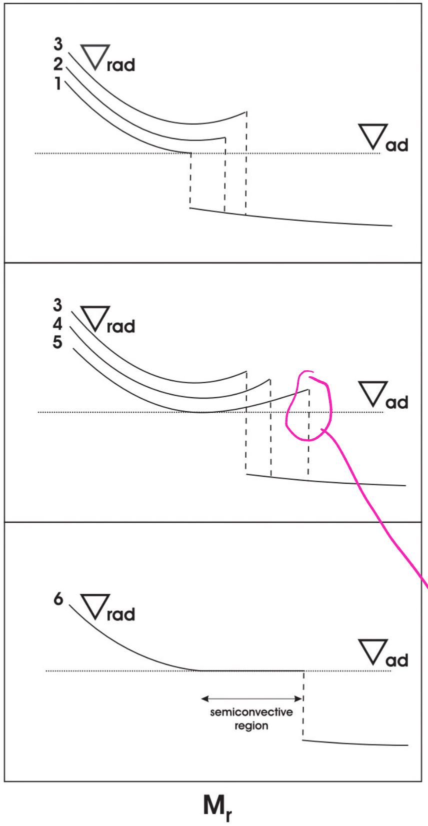
Astron. & Astrophys. 6, 124—129 (1970)

→ This is what we see in MESA (with CPM)

Core Helium Burning stars (CHEB) : Very difficult , Still a challenge

Eggleton 1972





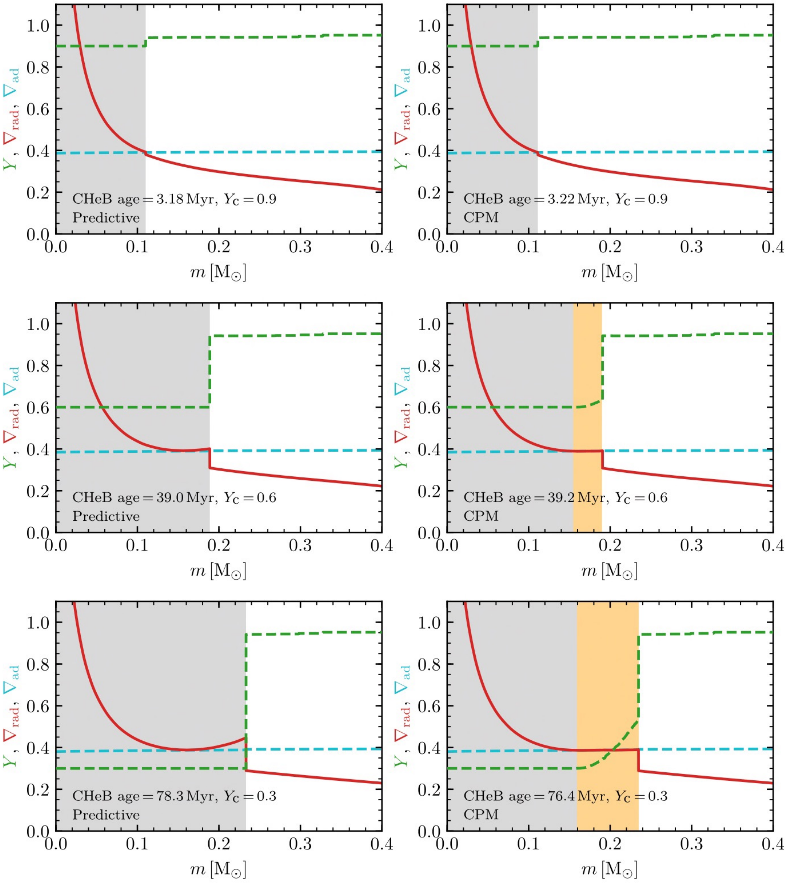
Early stages : Combustion of He → C increases κ ⇒ $\nabla_{rad} \nearrow$ ⇒ core boundary moves outward

A min. in ∇_{rad} develops. More mixing ⇒ increase of He and decrease in C ⇒ $\nabla_{rad} \searrow$

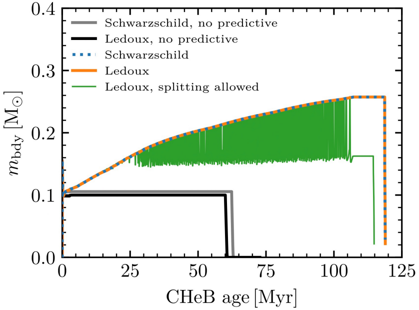
At stage 5 , $\nabla_{rad} = \nabla_{ad}$ at the min.
⇒ the core splits and convective core stops growing

The "convective" shell above the core does not obey Schw. or Ldx criterion
⇒ a semi-convective region must develop.

Core Helium Burning

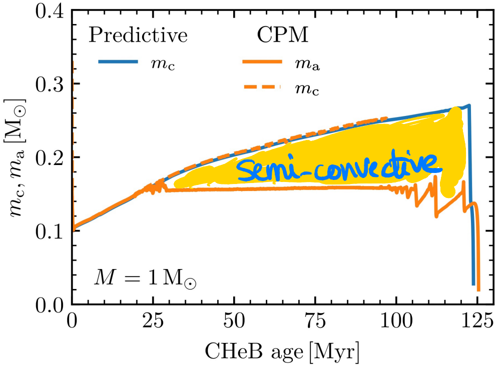


Paxton et al. 2019



Core allowed to split

⇒ huge error on age on CHEB



2 schemes

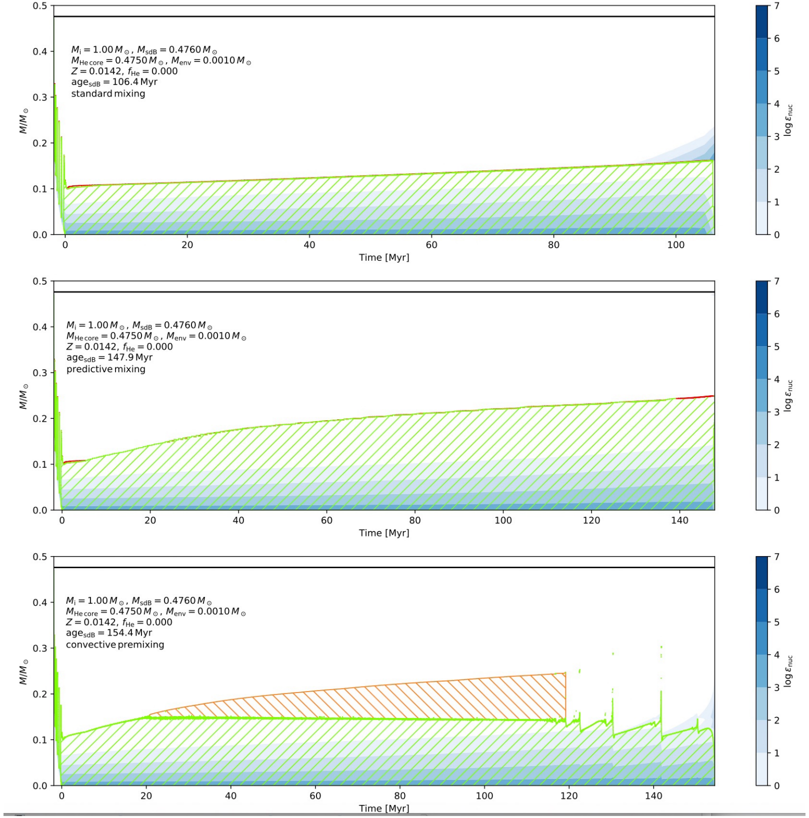
same total mixed region

different abundance profiles

Core Helium Burning

sdB star

Ostrowski et al. MNRAS 503,2021



"Find the zero of $\nabla_{rad} - \nabla_{ad}$" method
→ convective core too small
→ lifetime on CHeB too small

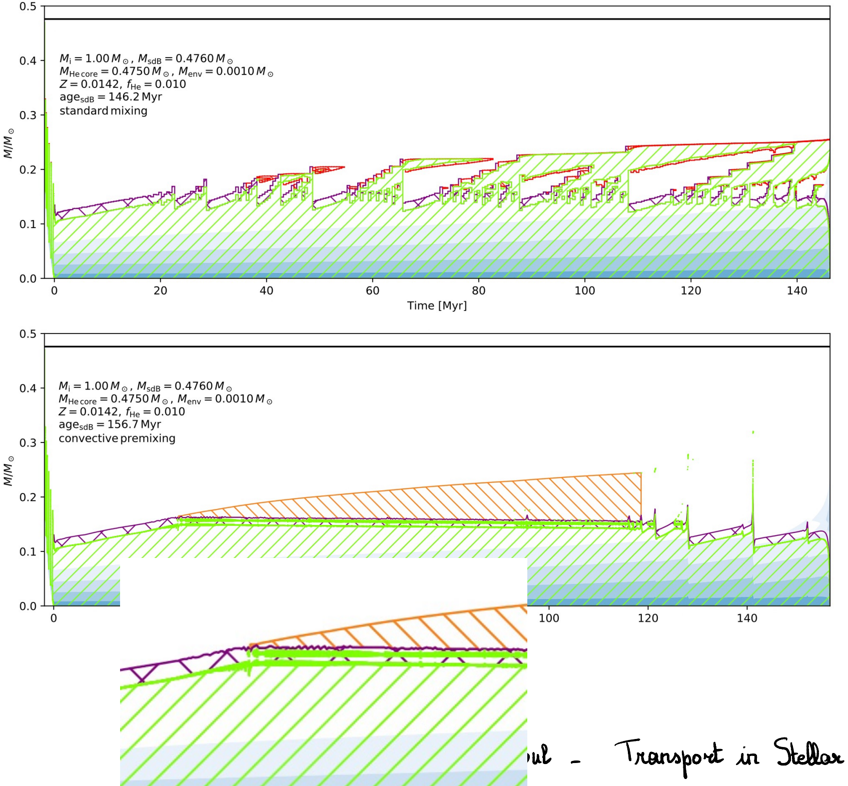
"Predictive" scheme in MESA
→ similar to "Maximum Overshoot" in Monash code

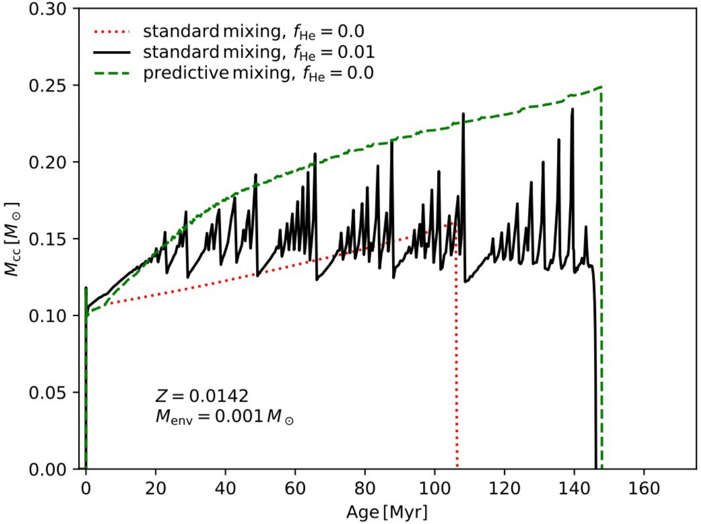
"Convective pre-mixing" scheme in MESA
→ Physically motivated
→ semi-convective (partially mixed) region
→ same total mixed zone
→ problem with "breathing pulses"

Max. extent of convective core on CHeB

+ "overshooting" in MESA (purple)

Ostrowski et al. MNRAS 503,2021





"Overshooting" does not make the convective core bigger

Breathing Pulses

At end of core helium burning ($Y \lesssim 0.1$), dominant reaction is no longer 3α but $^{12}C + {}^{4}He \rightarrow {}^{16}O + \gamma$

$\Rightarrow$ fast increase in ^{16}O which has a higher opacity than C

small ingestion of helium in convective core

$\Rightarrow$ huge production of energy, $L \nearrow$, $\kappa \nearrow$, $\nabla_{rad} \nearrow$, $M_{cc} \nearrow$

$\Rightarrow$ spikes of luminosity ("breathing pulses")

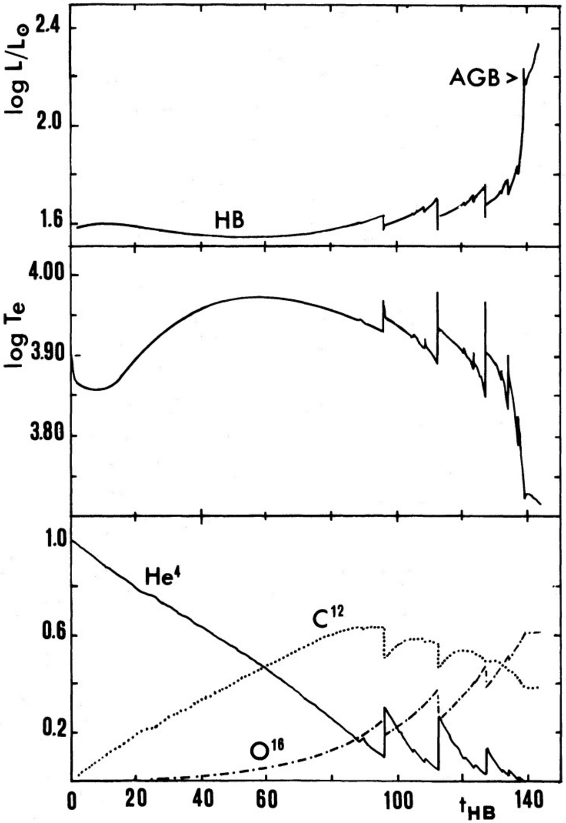
and spikes in M_{cc}

When He is burnt, star readjusts its structure, $M_{cc} \searrow$

Probably a numerical artefact

Important consequences on internal structure of WDs!

Castellani et al. ApJ 296 (1985)

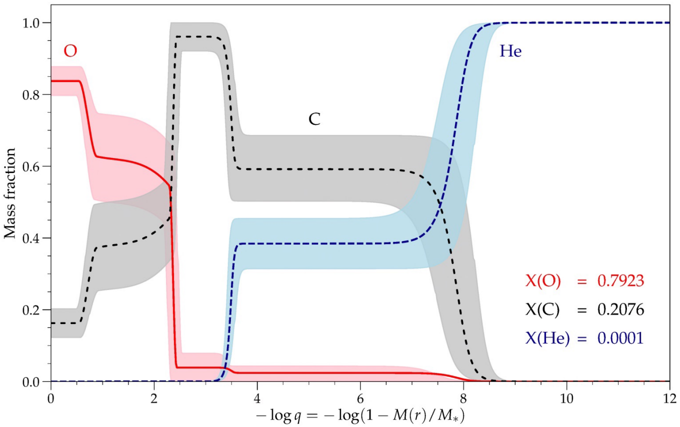


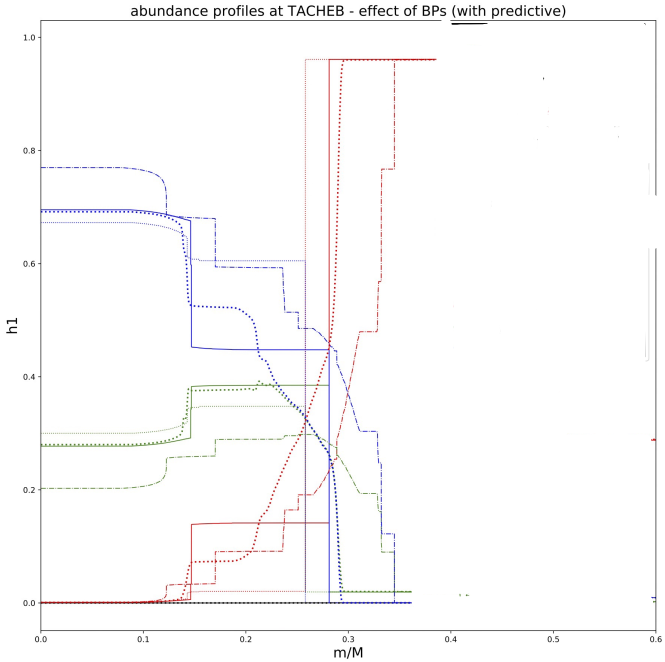
Asteroseismology of White Dwarfs

profiles depend on treatment of convective boundaries
on CHeB

c/o ratio and core size are fixed at the TACHEB !

Charpinet et al. AA 628 (2019)

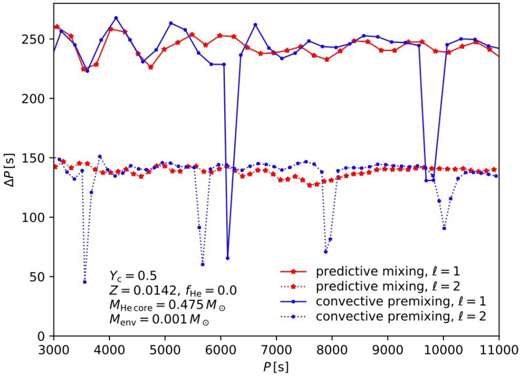




F Timmes

Pulsations in sdB stars

Ostrowski et al. MNRAS 503,2021

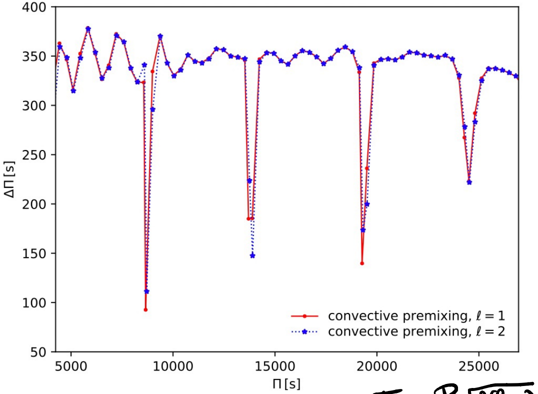


CPM (blue) : semi convective zone

⇒ trapped modes

Predictive mixing (red) : no semi convective zone

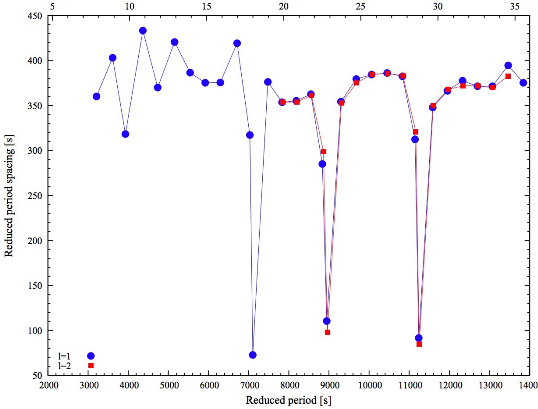
⇒ no trapped modes



$\Pi = P\sqrt{\ell(\ell+1)}$

GYRE

trapped modes in KIC 10001893, pulsating sdB star

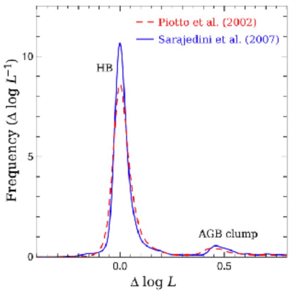


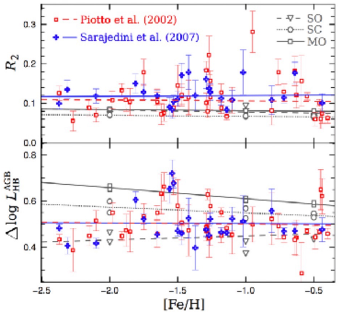
Uzundag et al. MNRAS 472 (2017)

Core Helium Burning – obs. vs models

Constantino et al. MNRAS (2018)

Cluster star count



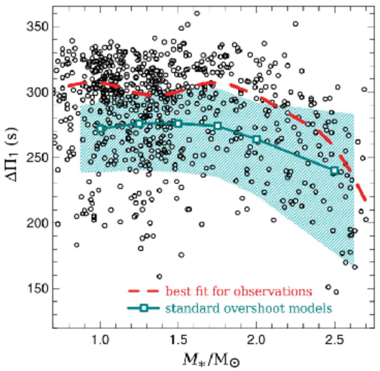


Ratio of CHeB stars to AGB stars

Luminosity of AGB clump vs luminosity of HB

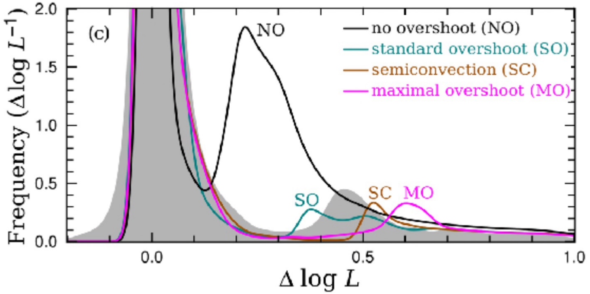
Constantino et al. MNRAS (2015)

Asteroseismology



observed period spacings in HB stars
≠ calculated period spacings

Constantino et al. MNRAS (2018)



↳ strong dependence on treatment of convective boundaries on CHeB

Position of the convective boundaries is still a challenge

Especially true during the CHeB

⚠ The opacity κ plays an important role in the evolution of the convective boundaries. It changes when composition changes ...

⚠ How to deal with ingestion of small amounts of "fresh fuel" in burning cores (cf. breathing pulses)

↳ Help from 3D ?