The Mixing Length Theory

Past, Present & Future

Probes of Transport in Stars
KITP
29 November, 2021

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- please argue with me

Past

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1979: Mihalas 1978, Kurucz 1979 – study of radiative transfer models of stellar atmospheres in connection to sub-surface convection zones characterized by MLT

Mixing Length Theory (MLT) Formalism

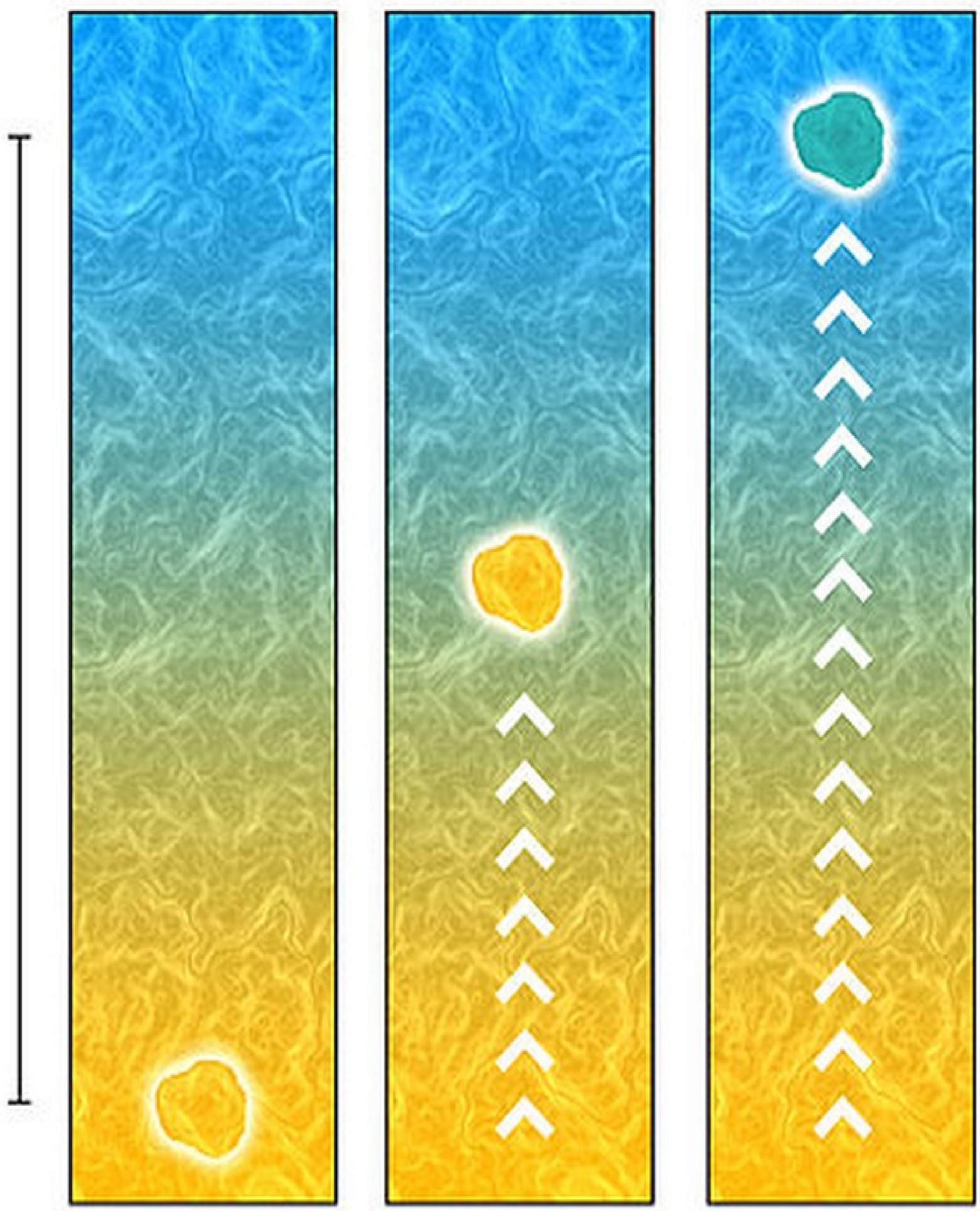

Fig: Wikipedia commons public domain

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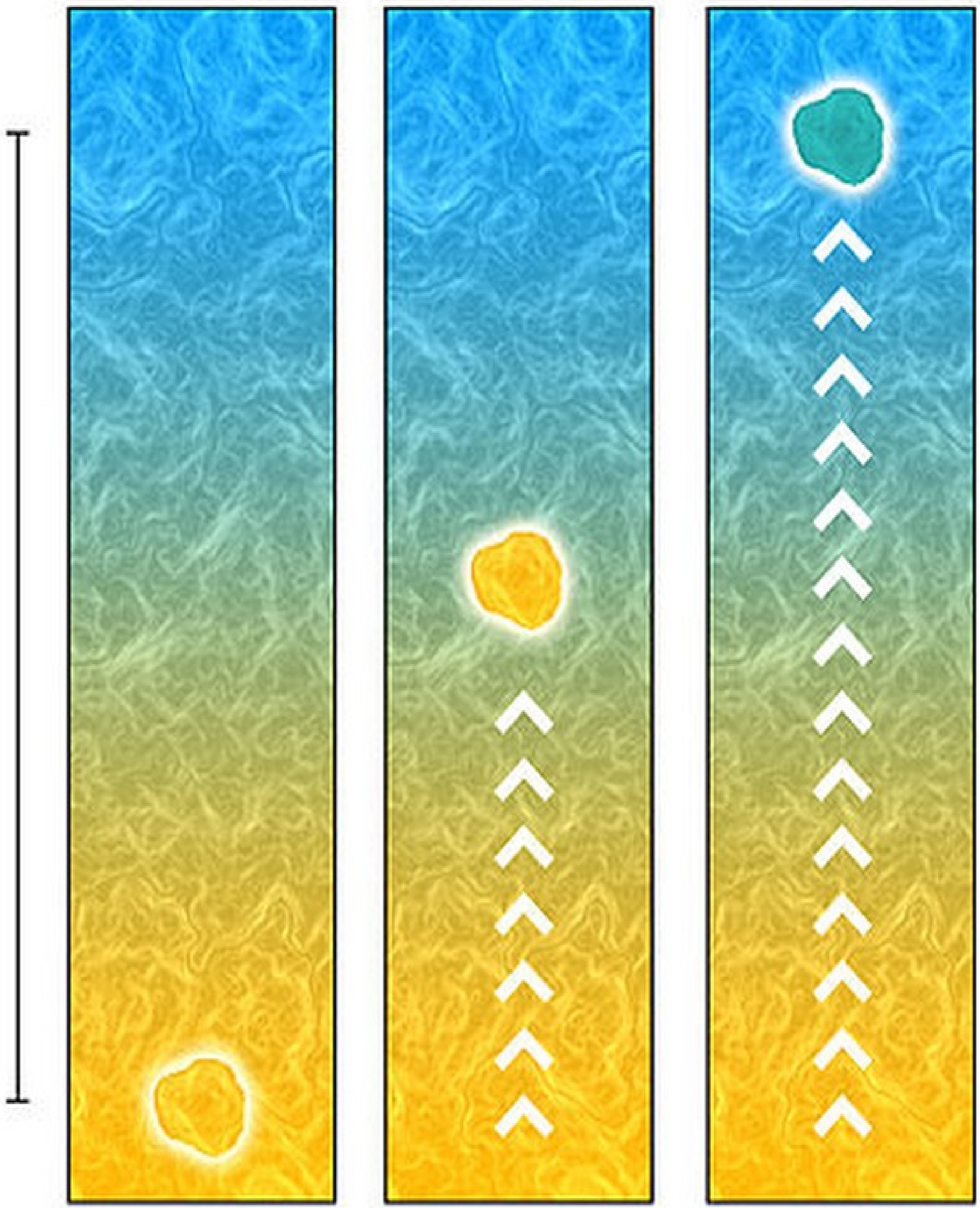

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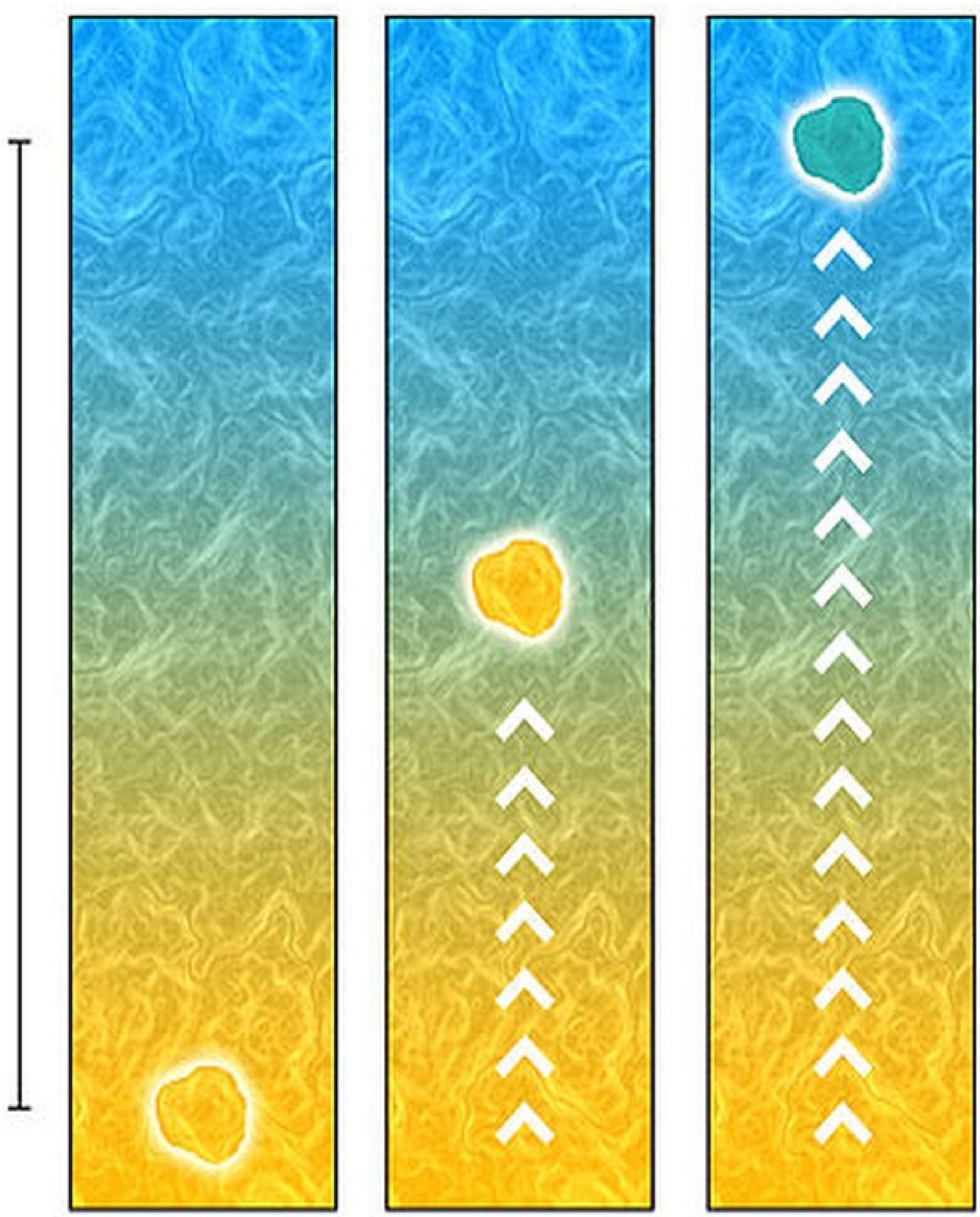

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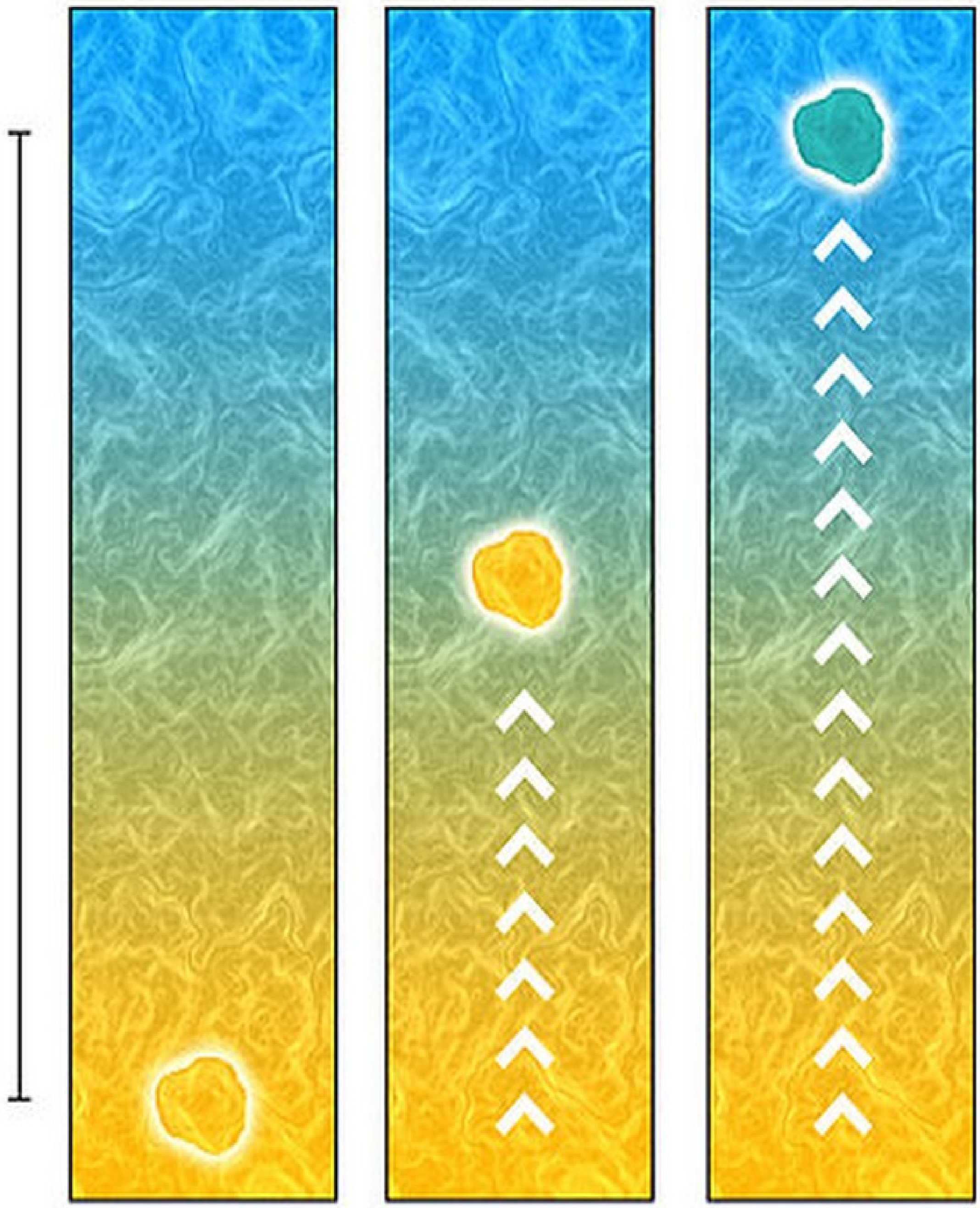

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$$F_{\mathrm{conv}} = \frac{1}{2}\rho v c_p T \frac{\lambda}{\mathrm{H_P}}(\nabla_T - \nabla_{\mathrm{ad}}).$$

Let's break this down...

$$F_{\text{conv}} = \frac{1}{2}\rho v c_p T \frac{\lambda}{H_P}\left(\nabla_T - \nabla_{\text{ad}}\right).$$

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Reminder: Stellar evolution definitions:

$$\nabla_T = \left(\frac{d\ln T}{d\ln P}\right)$$ General temperature gradient

$$\nabla_{\text{rad}} = \left(\frac{d\ln T}{d\ln P}\right)_{\text{rad}} = \frac{3}{16\pi acG}\frac{\kappa l P}{mT^4}$$ Radiative temperature gradient

$$\nabla_{\text{ad}} = \left(\frac{d\ln T}{d\ln P}\right)_{\text{ad}} = \frac{P\delta}{T\rho c_p}$$ Adiabatic temperature gradient

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$$F_{\rm conv} = \frac{1}{2}\rho v c_p T \frac{\lambda}{H_P}(\nabla_T - \nabla_{\rm ad}).$$

~ Local flux of convective energy (local pressure, specific heat, temperature, velocity)

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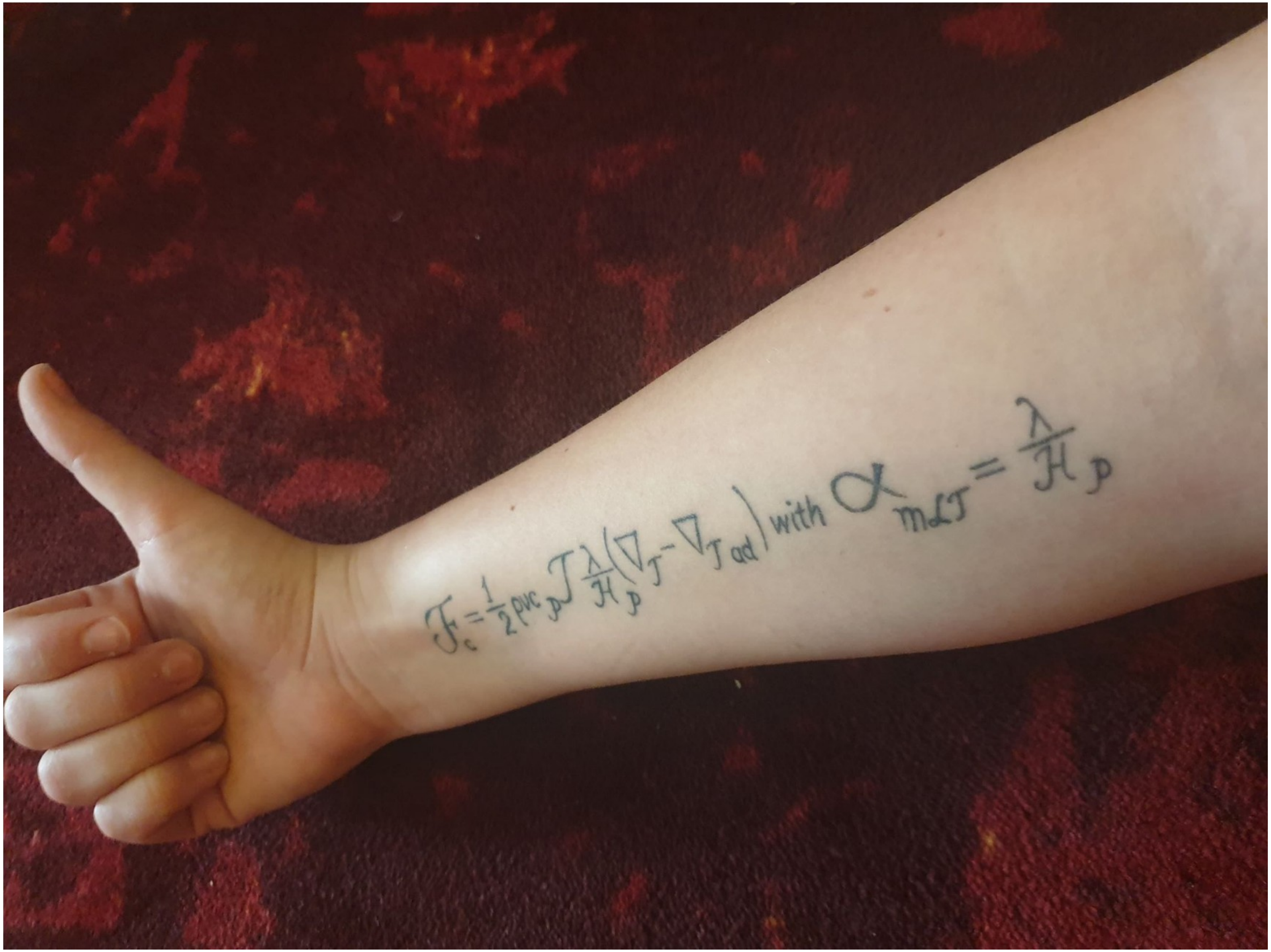
Mean free path λ in terms of the pressure scale height


$F_c = \frac{1}{2} \rho v c_p T \frac{\lambda}{H_p} (\nabla_T - \nabla_{T\,ad})$ with $\alpha_{MLT} = \frac{\lambda}{H_p}$

Quick review: convective stability criteria

- The Ledoux criterion for dynamical stability is given by

$\nabla_{rad} < \nabla_{ad} + [\varphi/\delta] \nabla_{\mu}$

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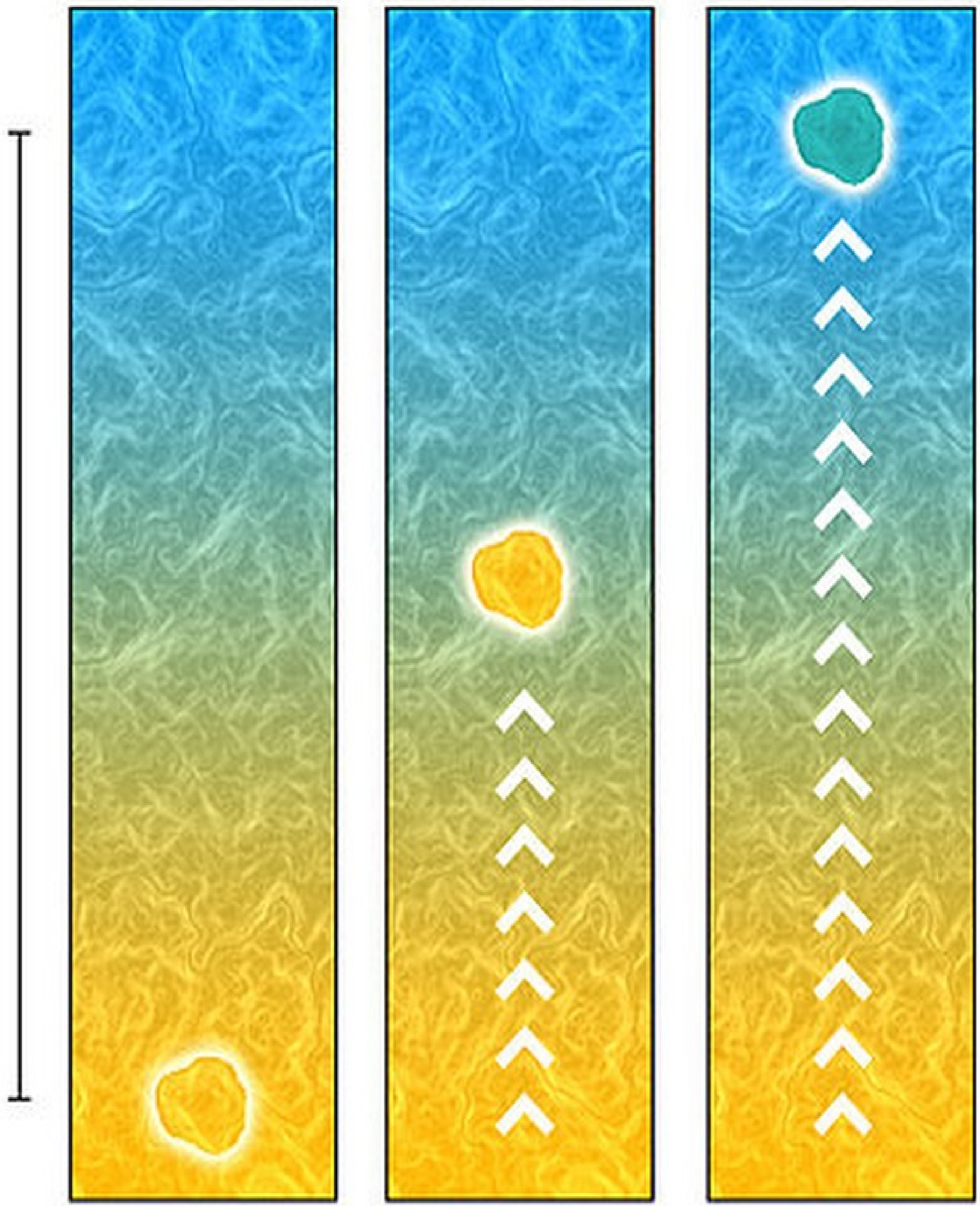
- Dynamically stable regions do not produce convective motions, and the energy flux is carried out exclusively by radiation (or conduction).

- If the relevant condition is violated, convection will activate and share in the transport of flux

Summary: Mixing Length Theory



$$F_{\rm conv} = \frac{1}{2}\rho v c_p T \frac{\lambda}{H_P}(\nabla_T - \nabla_{\rm ad}).$$

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How is MLT inadequate?

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Let us count the ways...

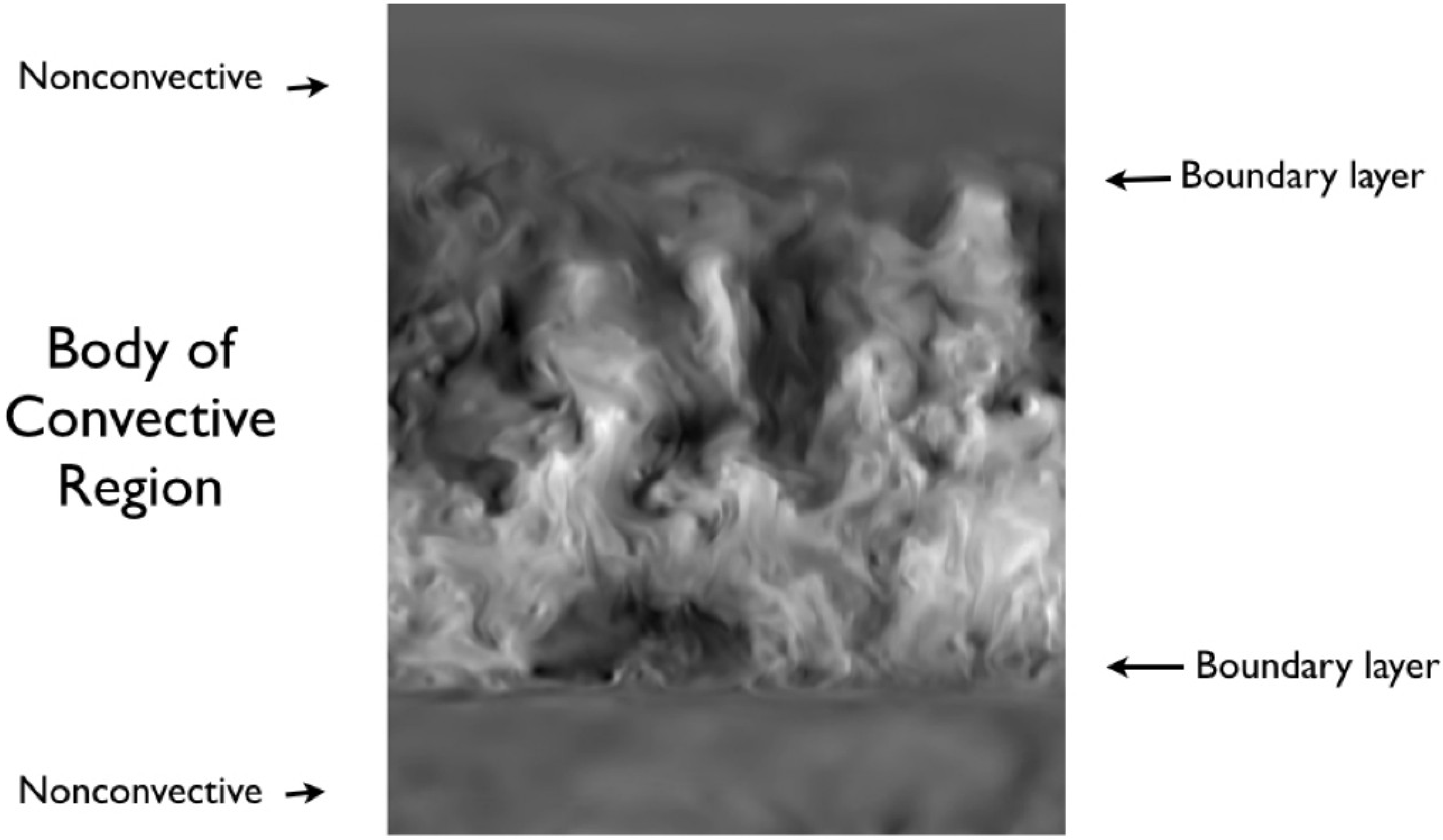


Figure: Maxime Viallet. MUSIC

(1) Convective boundaries are not rigid; local mixing occurs. MLT cannot account for this

(2) There is no symmetry between upflows and downflows, which creates **negative kinetic flux**. MLT cannot account for this.

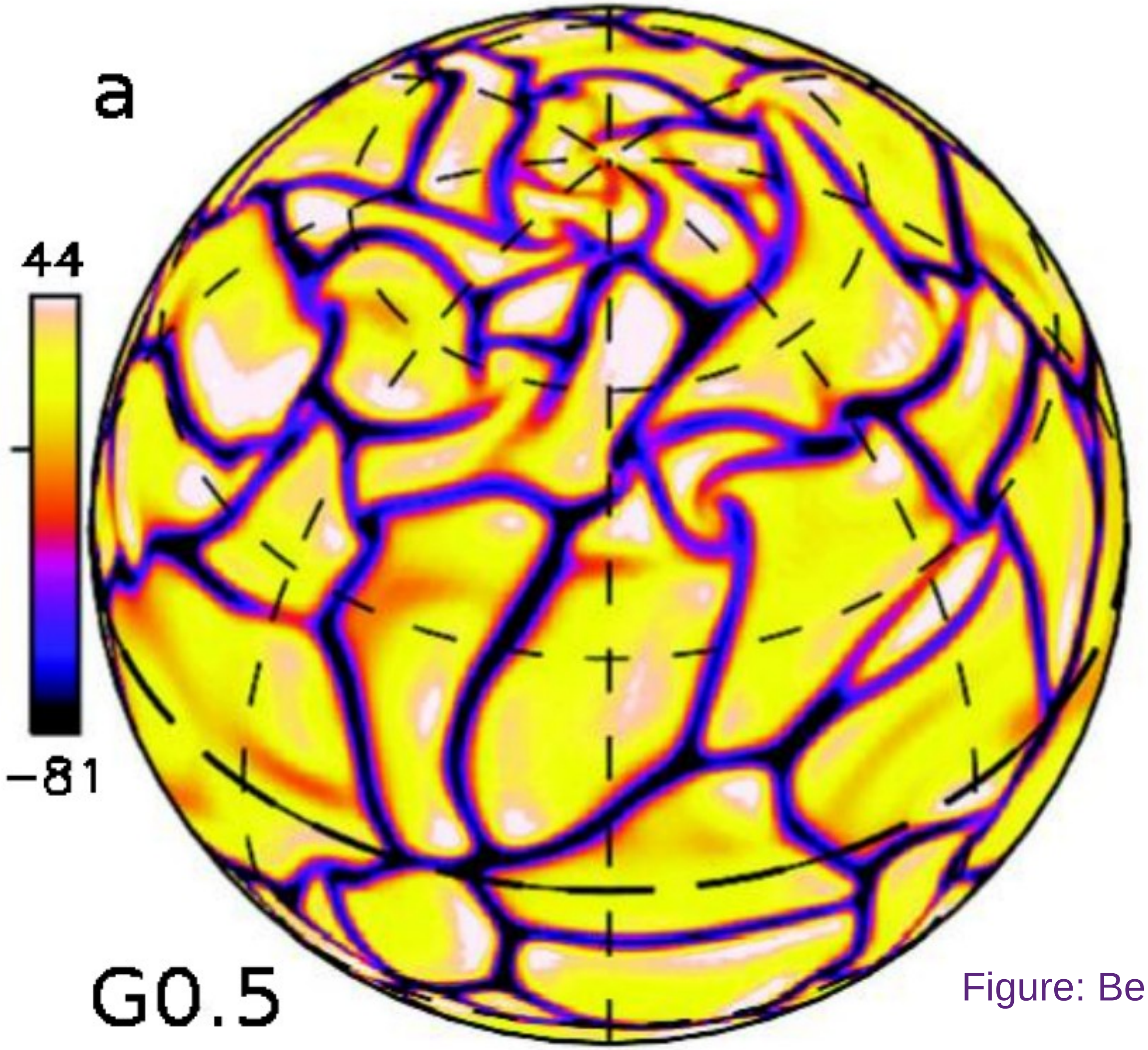
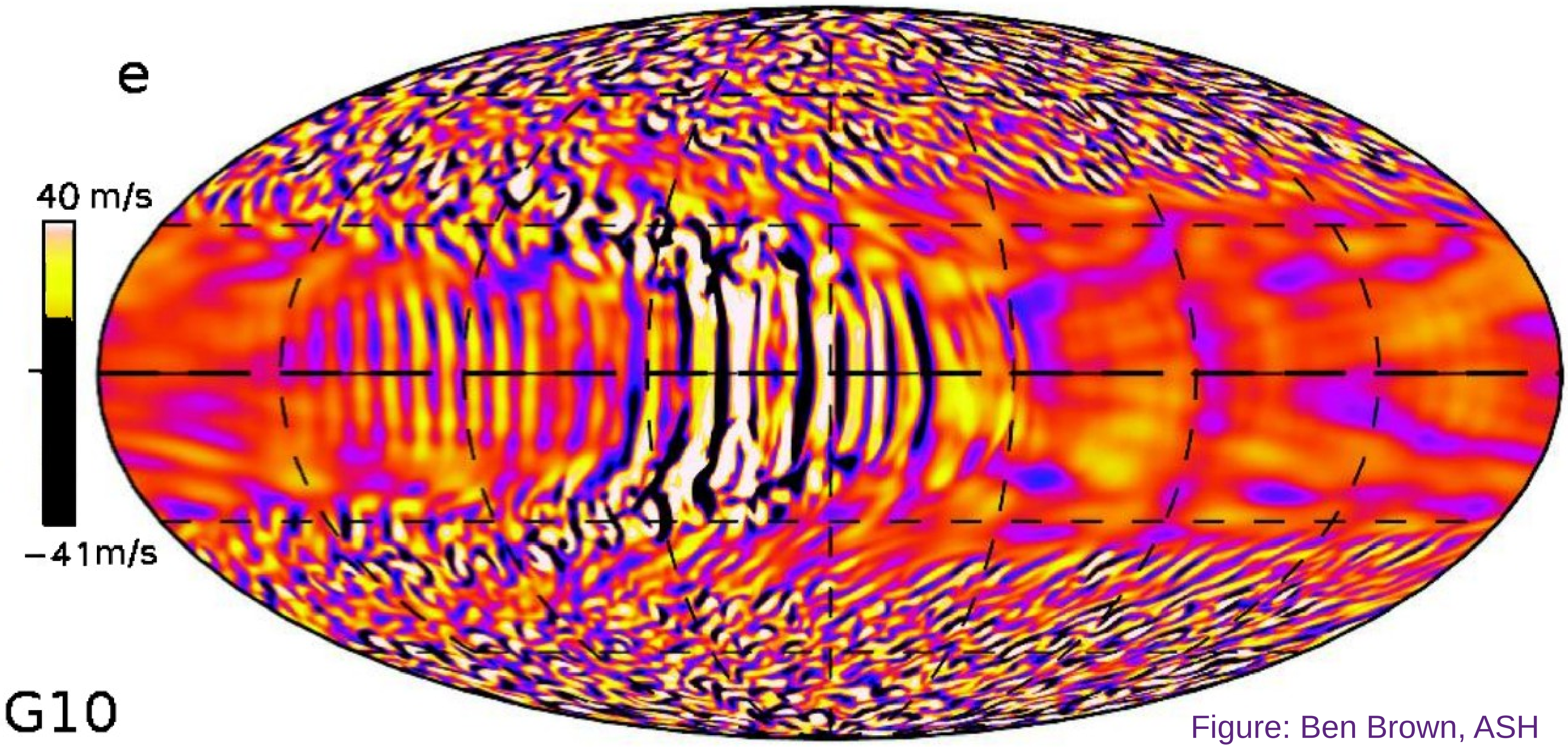


Figure: Ben Brown, ASH

(3) Real convective cells undergo shearing and fragmentation. MLT cannot account for this.



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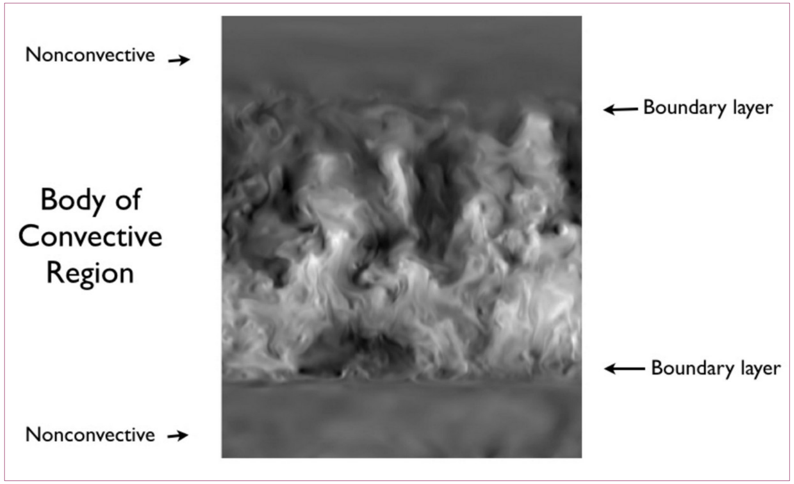

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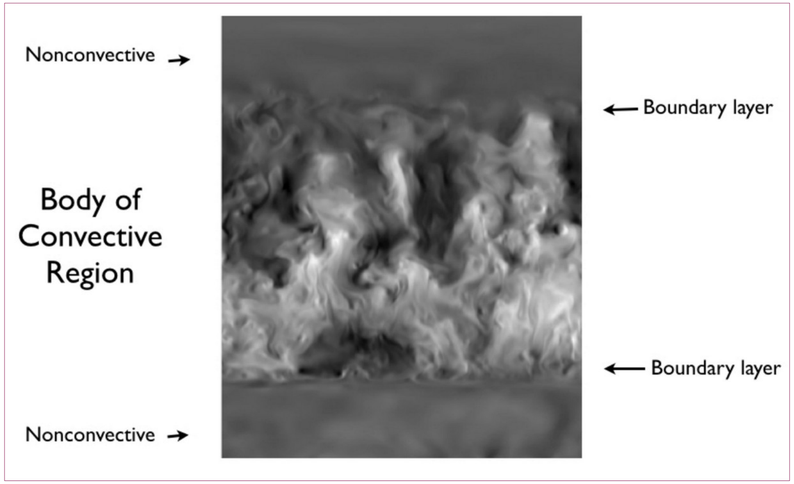

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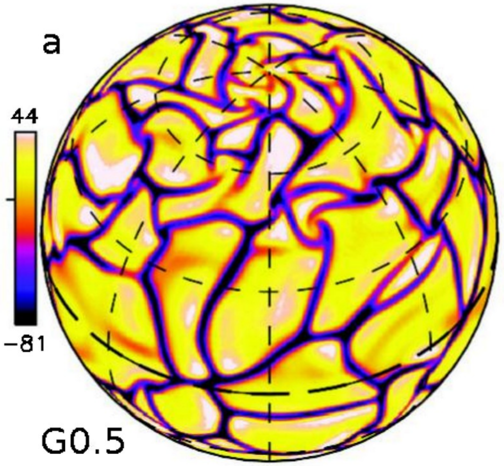

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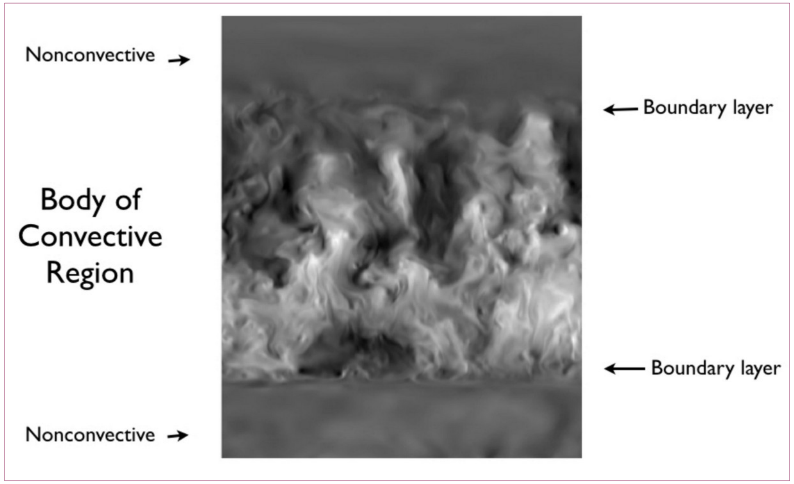

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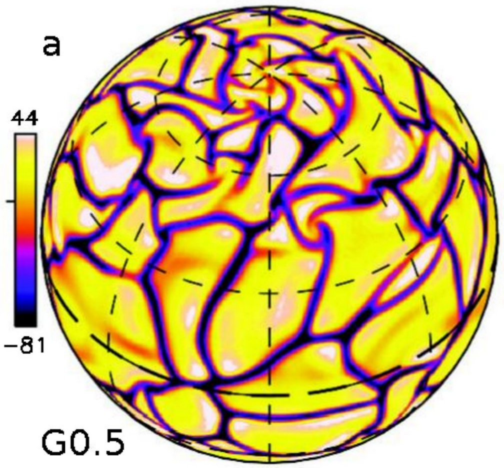

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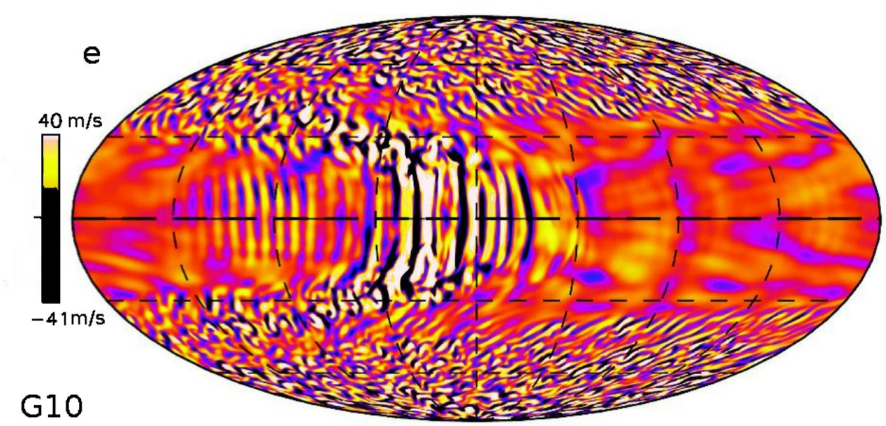

(3) No concept of shearing, fragmentation, nor any 2D or 3D effects

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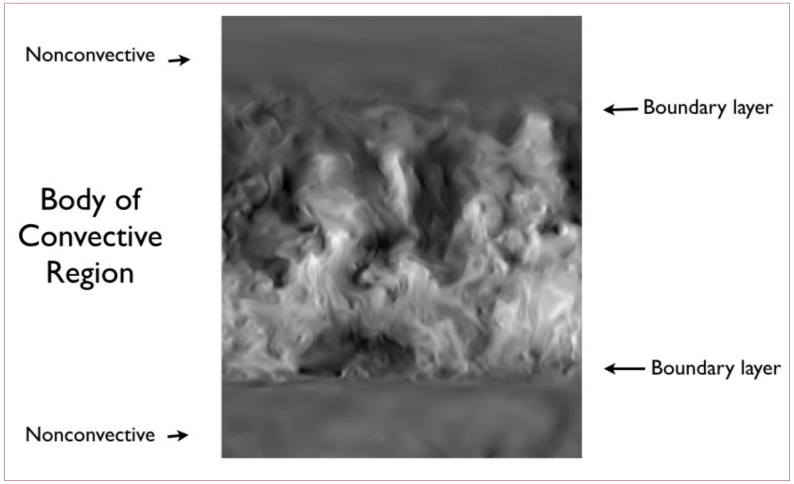

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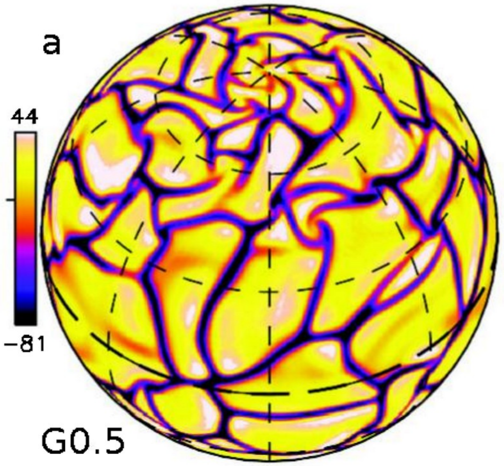

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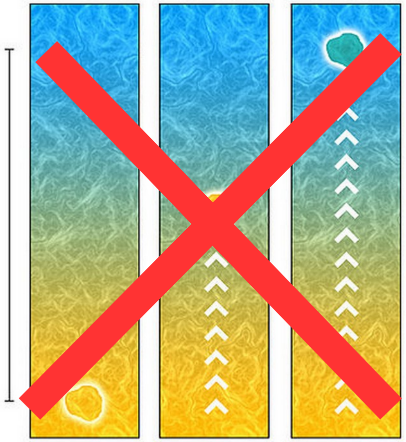
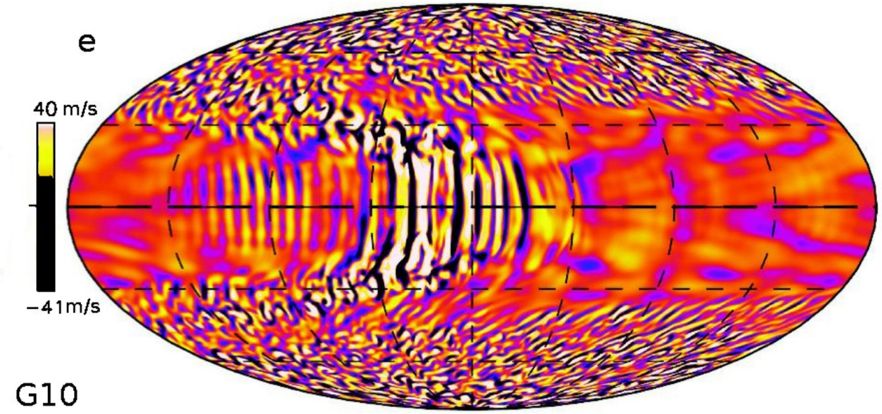


(4) BONUS: there is no such thing as a quantized unit of convection, so what does a “parcel” even mean?



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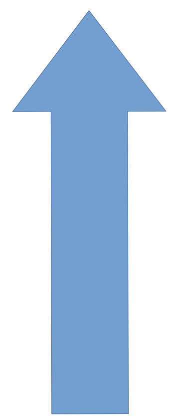
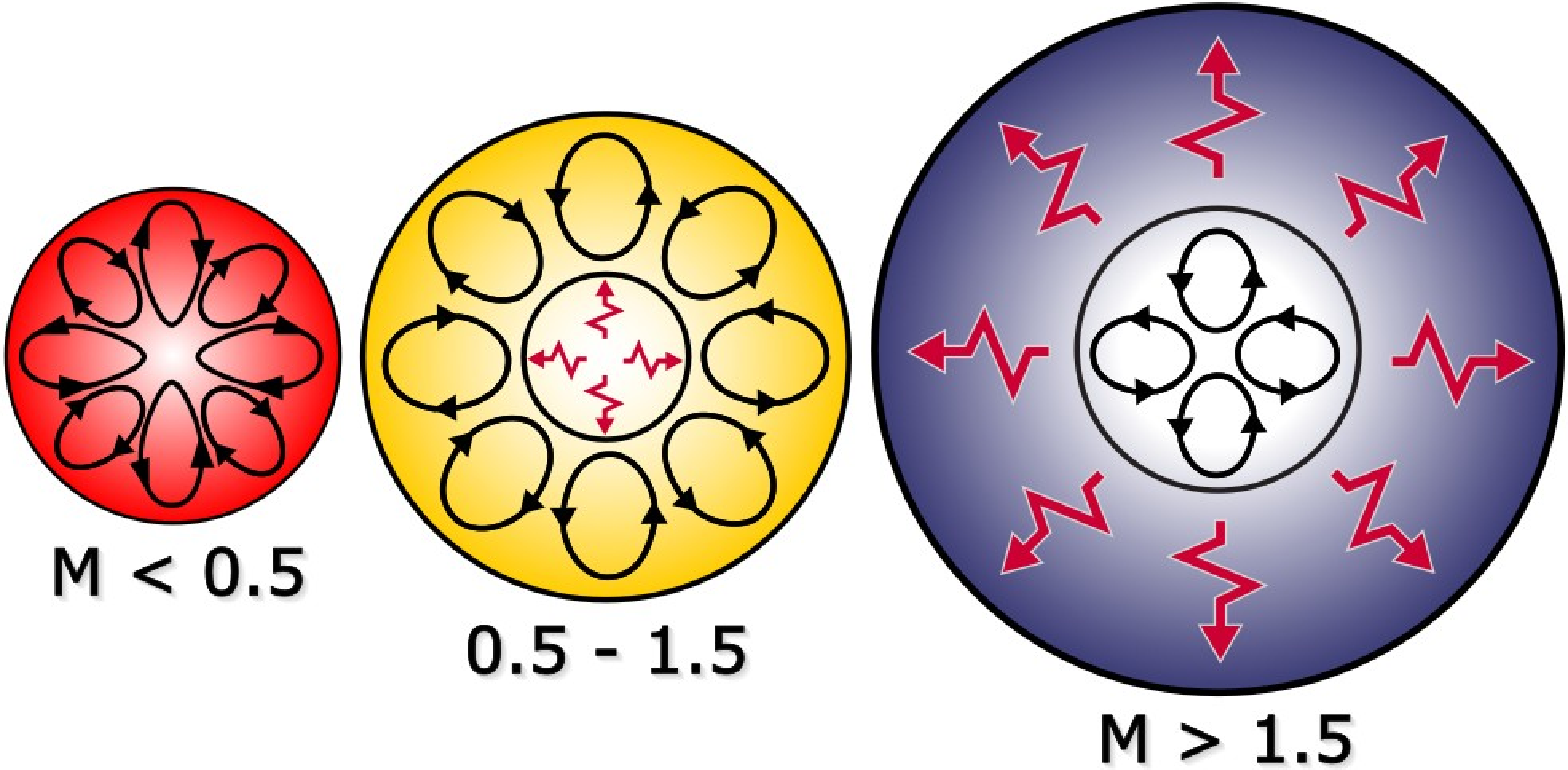
Nevertheless, we persist…

Present



Let’s talk about 1D “stars”

Where does MLT matter?



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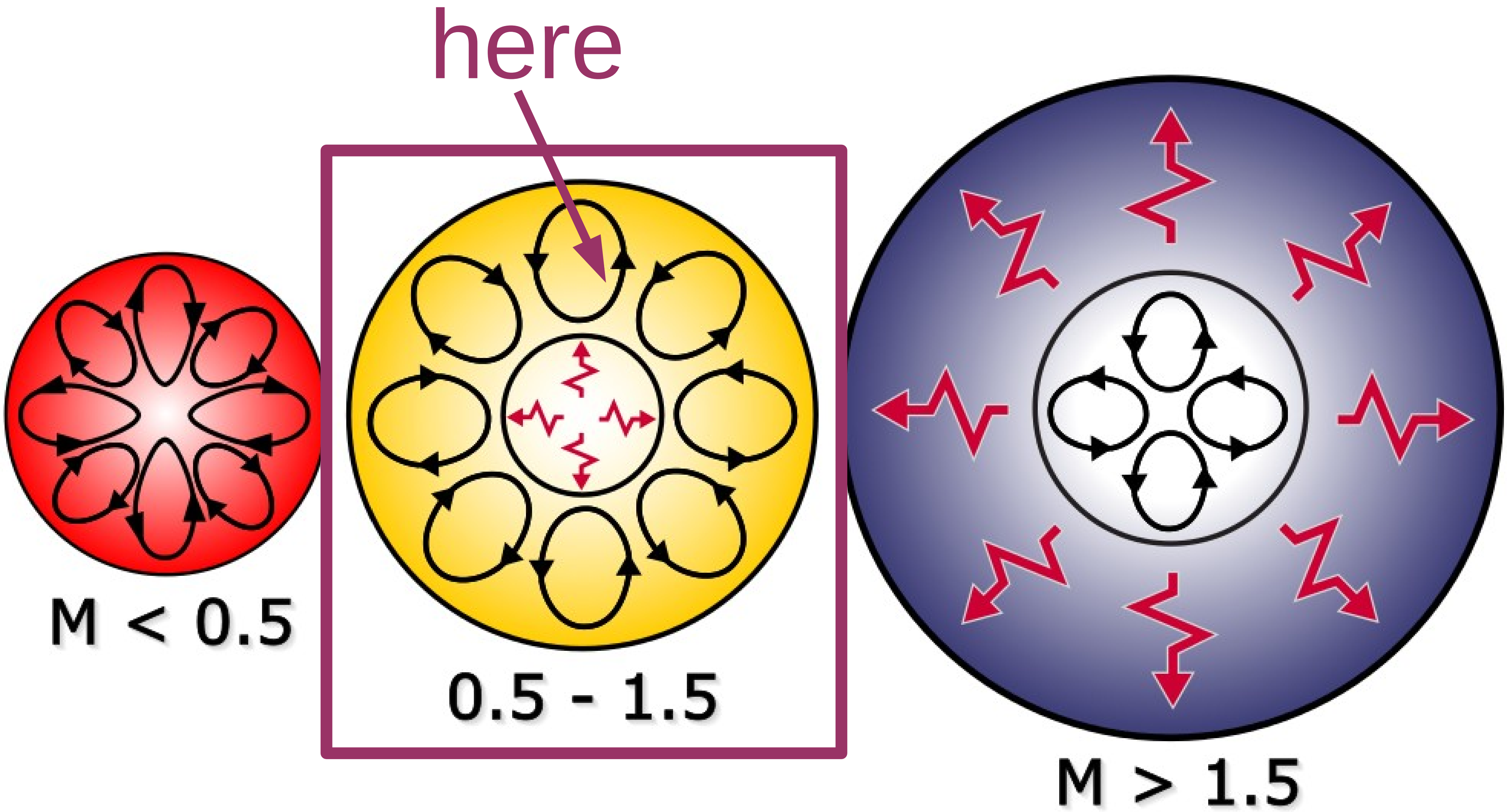

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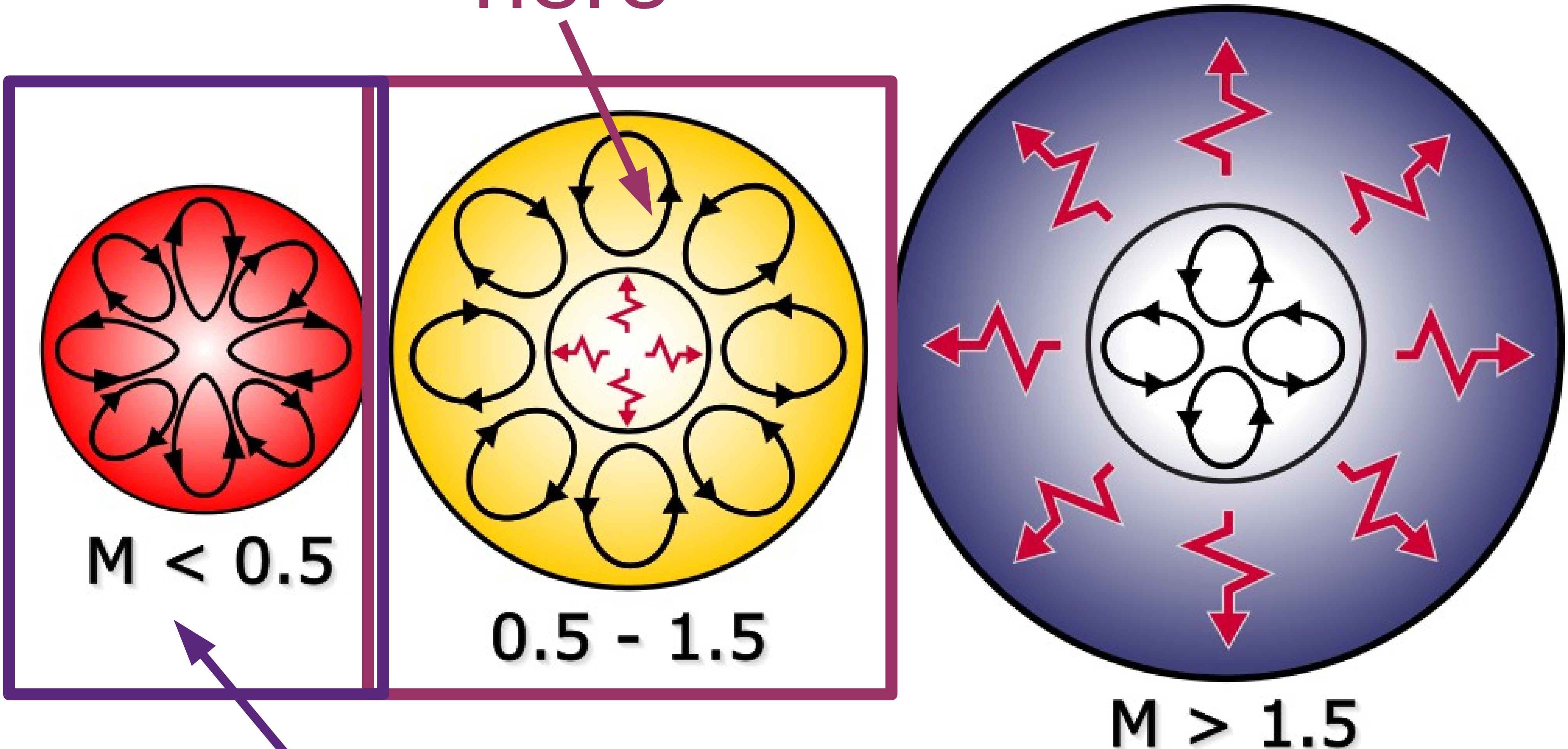

also here (less intuitive)

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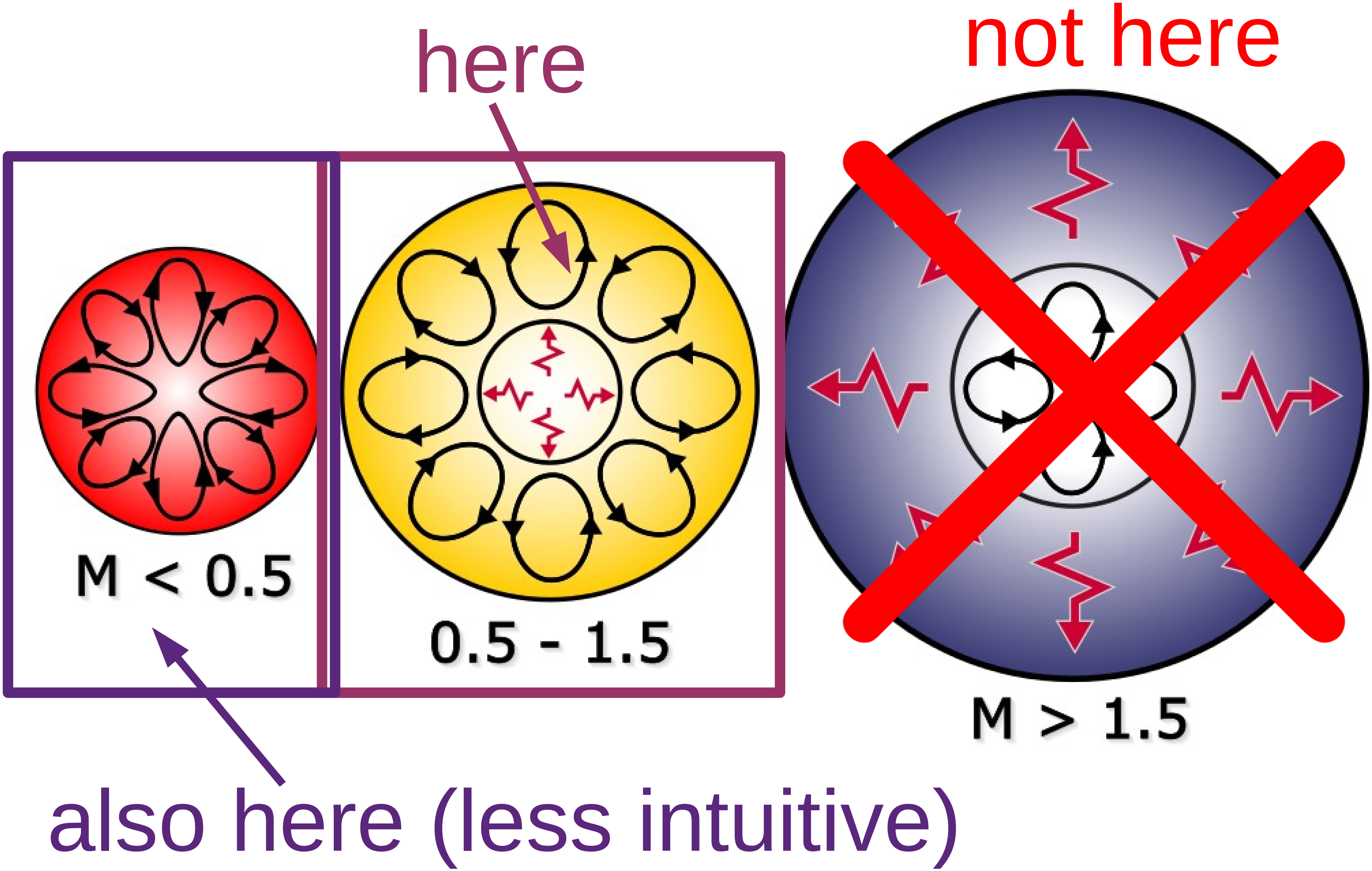

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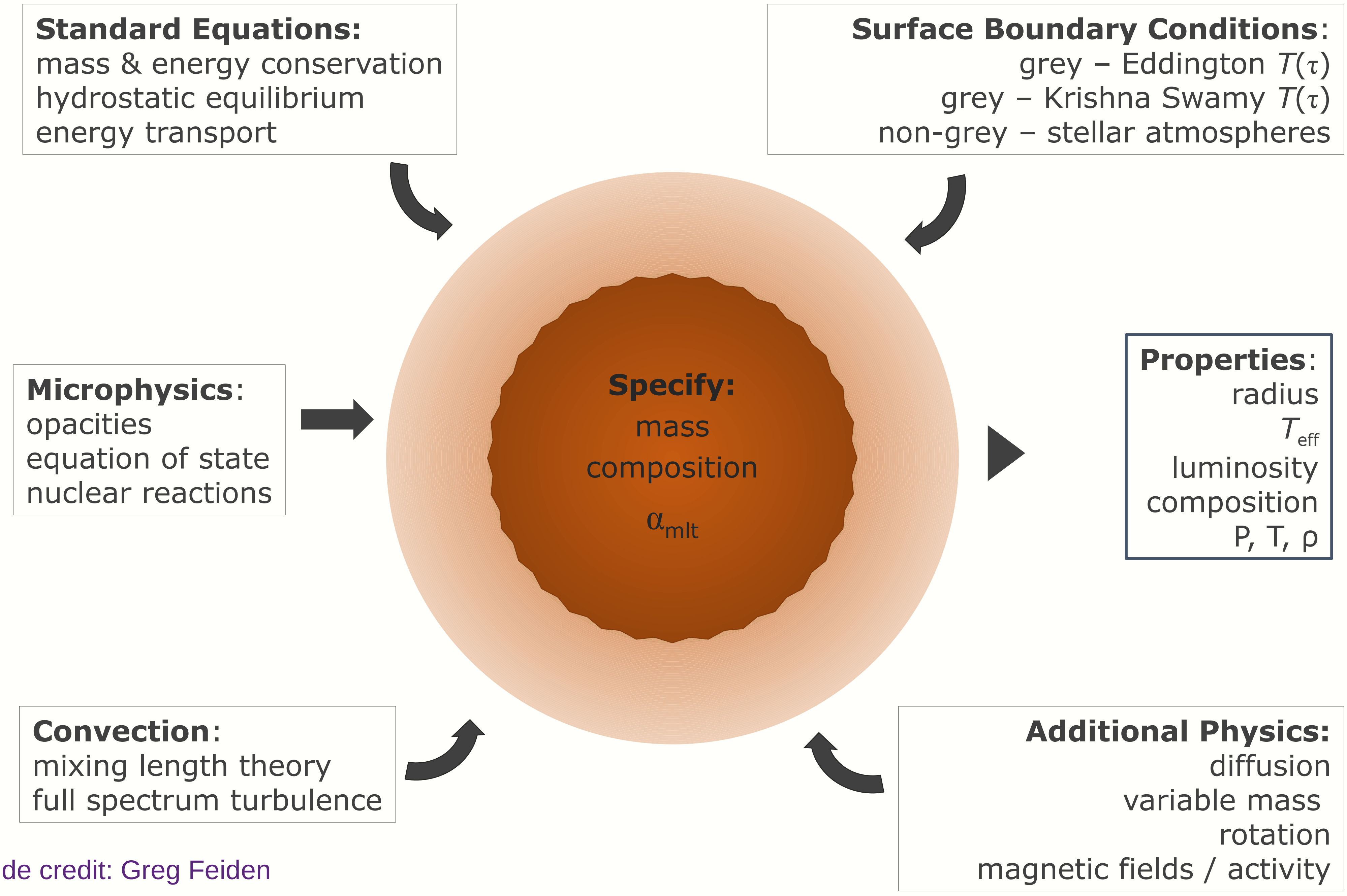
- the entropy jump implies the depth of the convective envelope

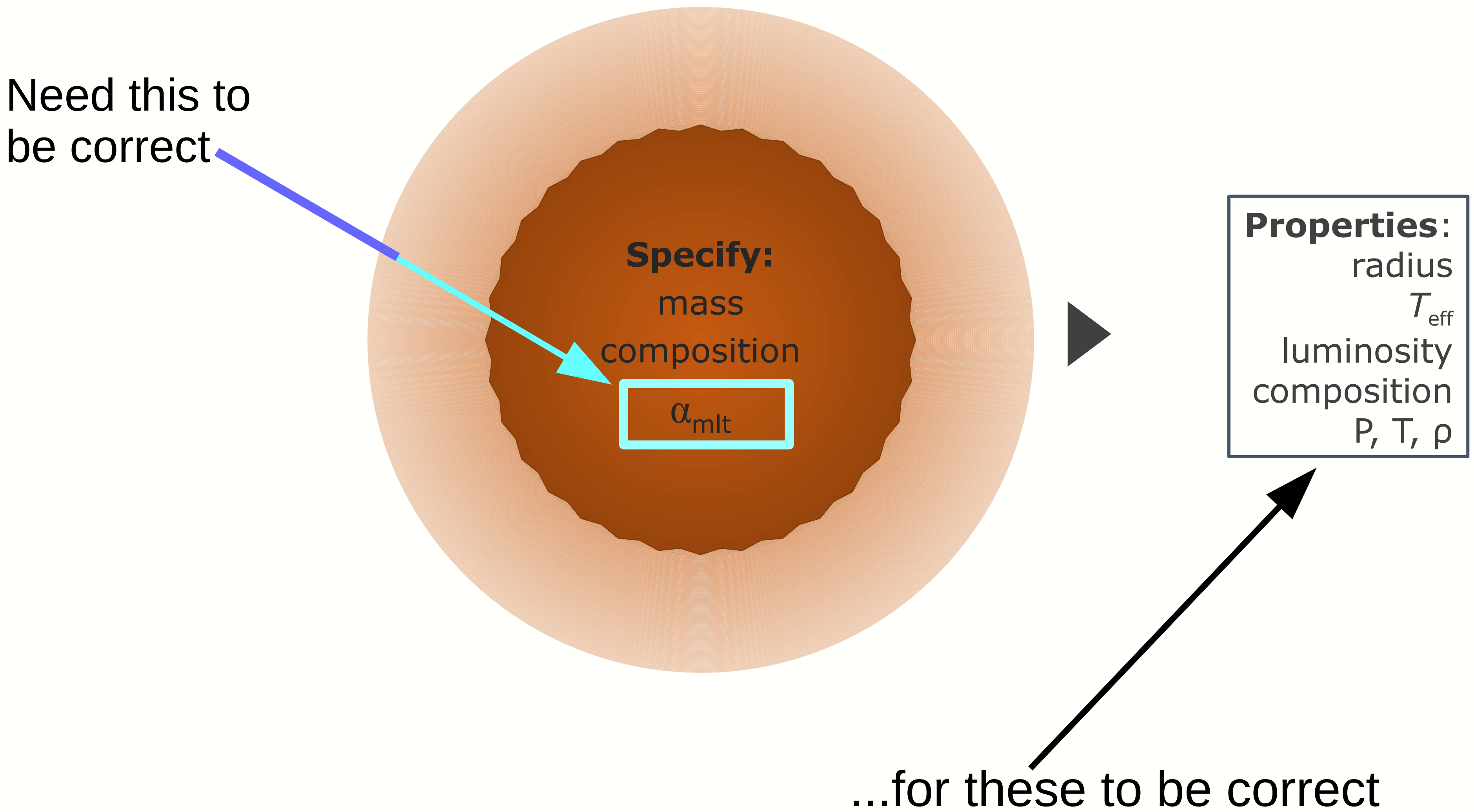
Adapted from Joyce PhD thesis

Components of a Stellar Structure & Evolution Program

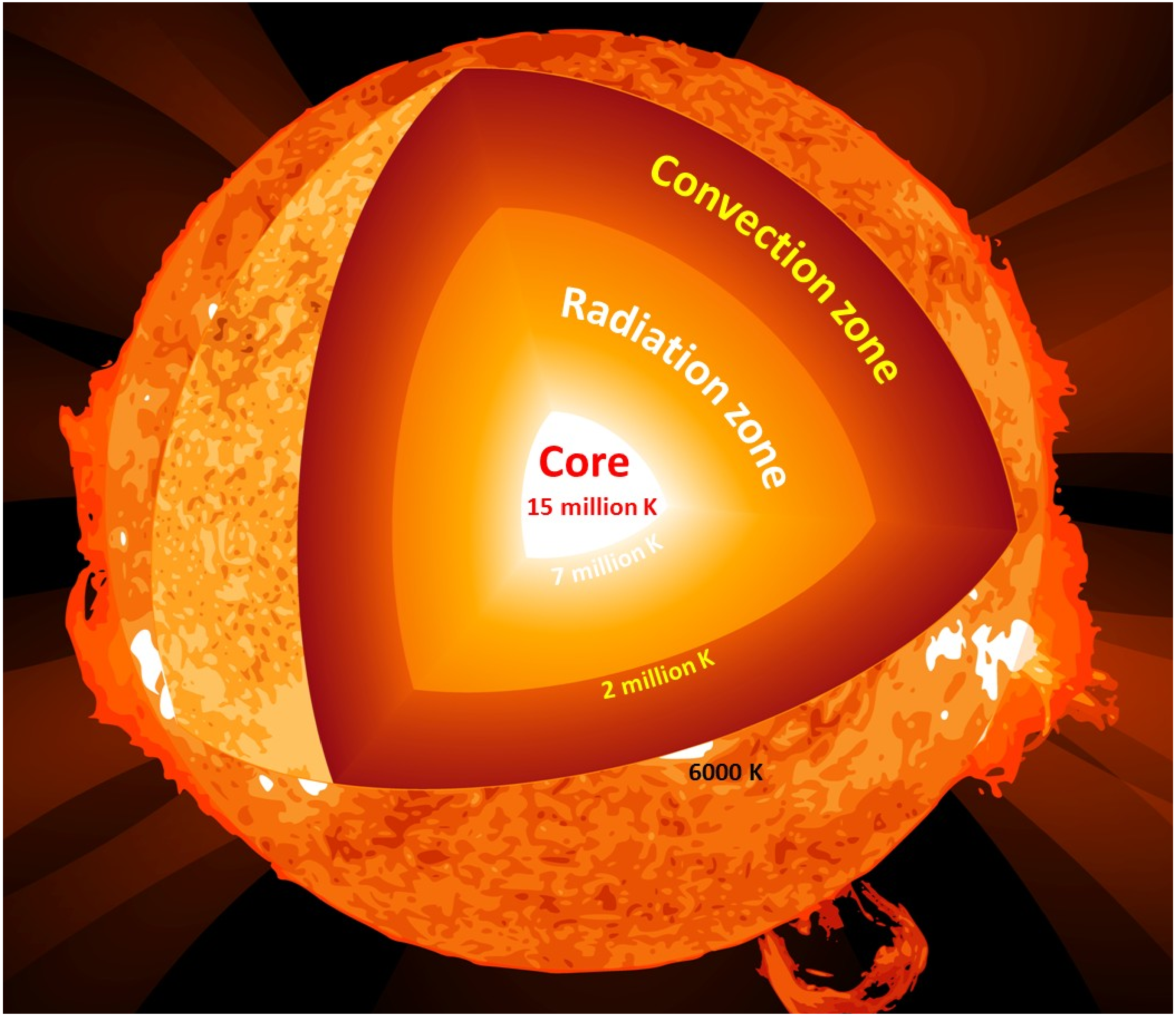


Slide credit: Greg Feiden

Components of a Stellar Structure & Evolution Program



MLT in practice: The solar calibration



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—so yes, of course it is wrong...but is it still good enough?

A path forward: calibrating the mixing length to stars other than the Sun

Two separate science questions:

(1) How does α_{MLT} vary among stars with different global properties?

(2) How does α_{MLT} change within a single star's evolution?

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> Tayar et al., 2017
> Joyce & Chaboyer, 2018a, 2018b
> Viani et al., 2018

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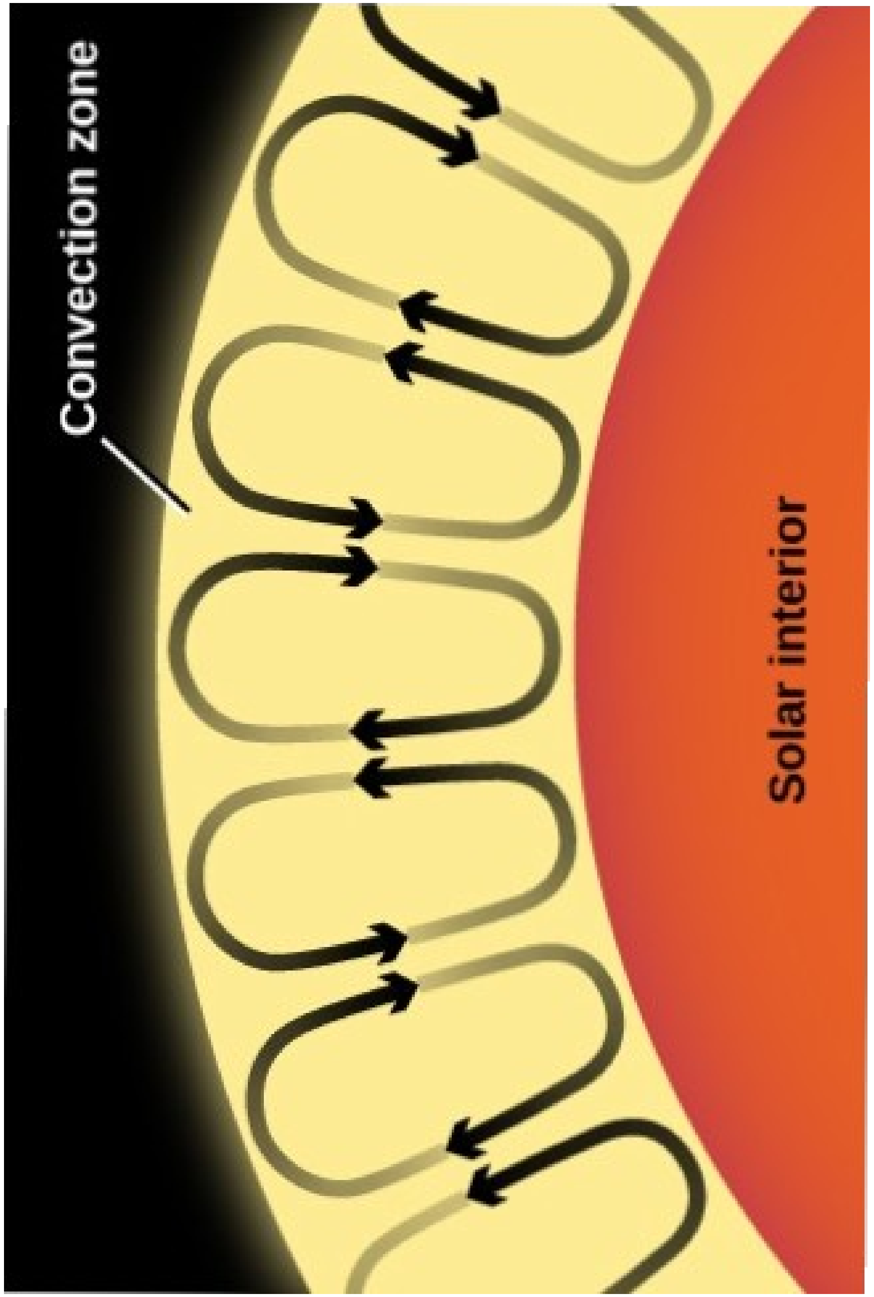
(2) How should α_{MLT} change within a single star's evolution?

> Trampedach et al., 2014
> Spada et al., 2021

What we need:



Calibrate here!

- low mass stars (0.5 – 1.2 Ms)
- Stars with sub-surface convective envelopes (preferably thick)
- main sequence, subgiant, or early RGB stars
- Star should ideally have at most one critical deviation from the Sun at a time
- As many independent measurements of the candidate’s fundamental properties as possible

alpha Centauri A & B- the perfect lab

The high number of independent observational measurements removes degrees of freedom and **allows the mixing length parameter to be isolated** in the stellar evolution models

Mass – kinematics

Radius – interferometry

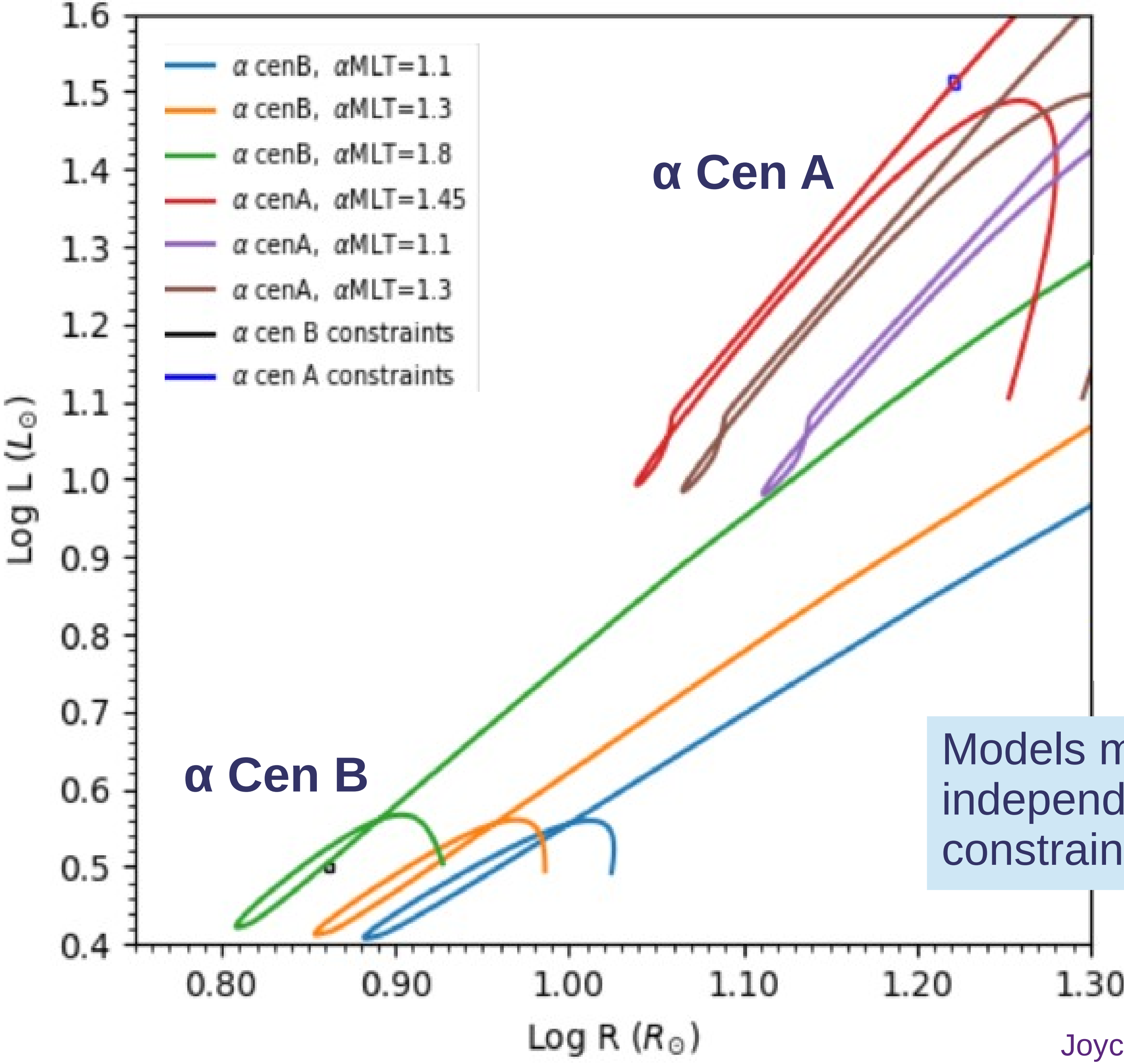
Luminosity – photometry

Surface abundance – high resolution spectroscopy

Stellar interior constraints **from which surface effects can be removed** – seismology

Because the candidate is a binary with all classical measurements satisfied in both components, we get a prior-independent age constraint for free

Mixing length calibrations to alpha Centauri A & B



(1) Mass A
(2) Mass B
(3) Radius A
(4) Radius B
(5) Luminosity A
(6) Luminosity B
(7) Common surface abundance Z/X
(8) r02 A
(9) r02 B

Models must satisfy 7 independent observational constraints at a common age

Joyce & Chaboyer, 2018b

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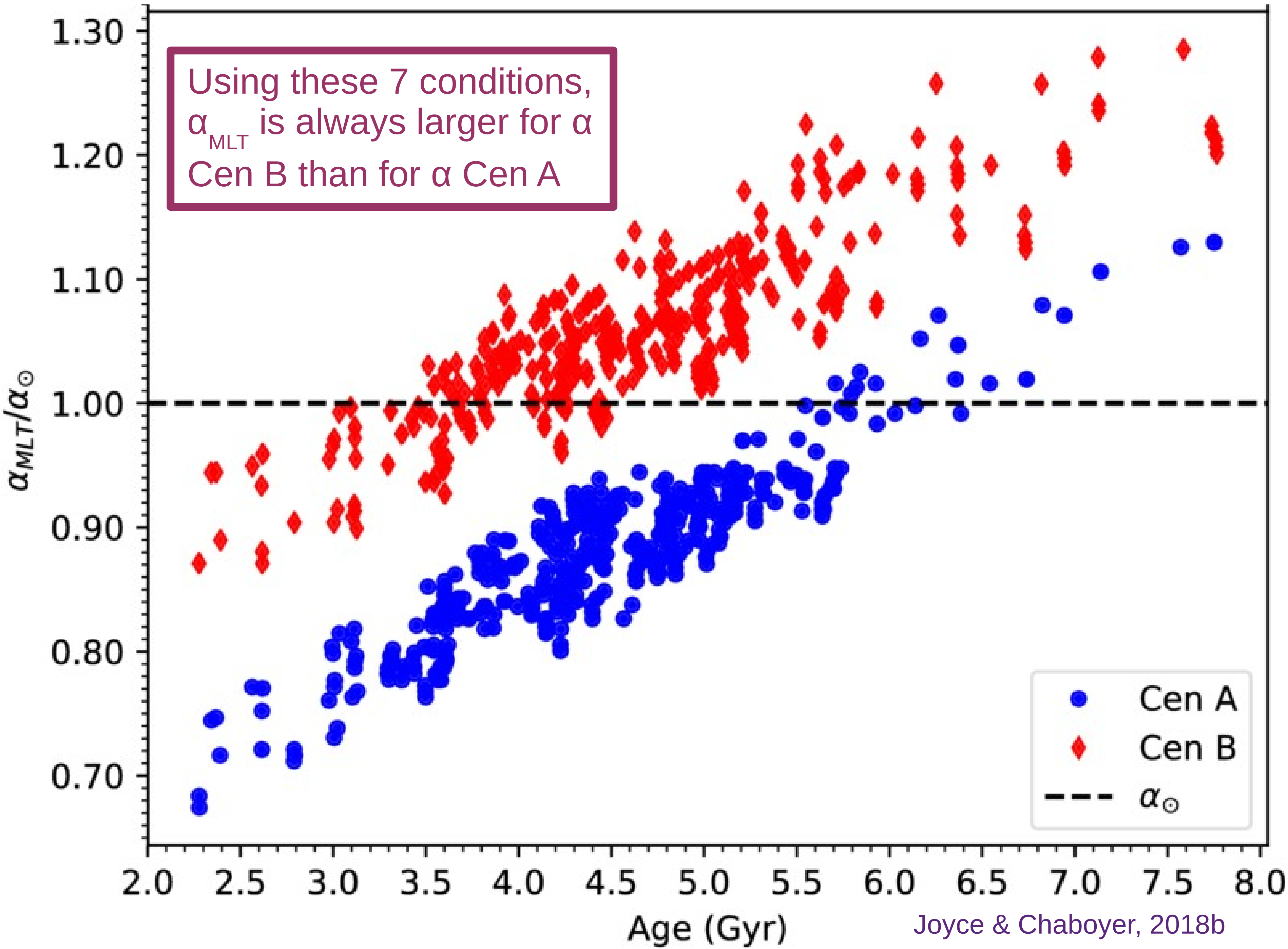
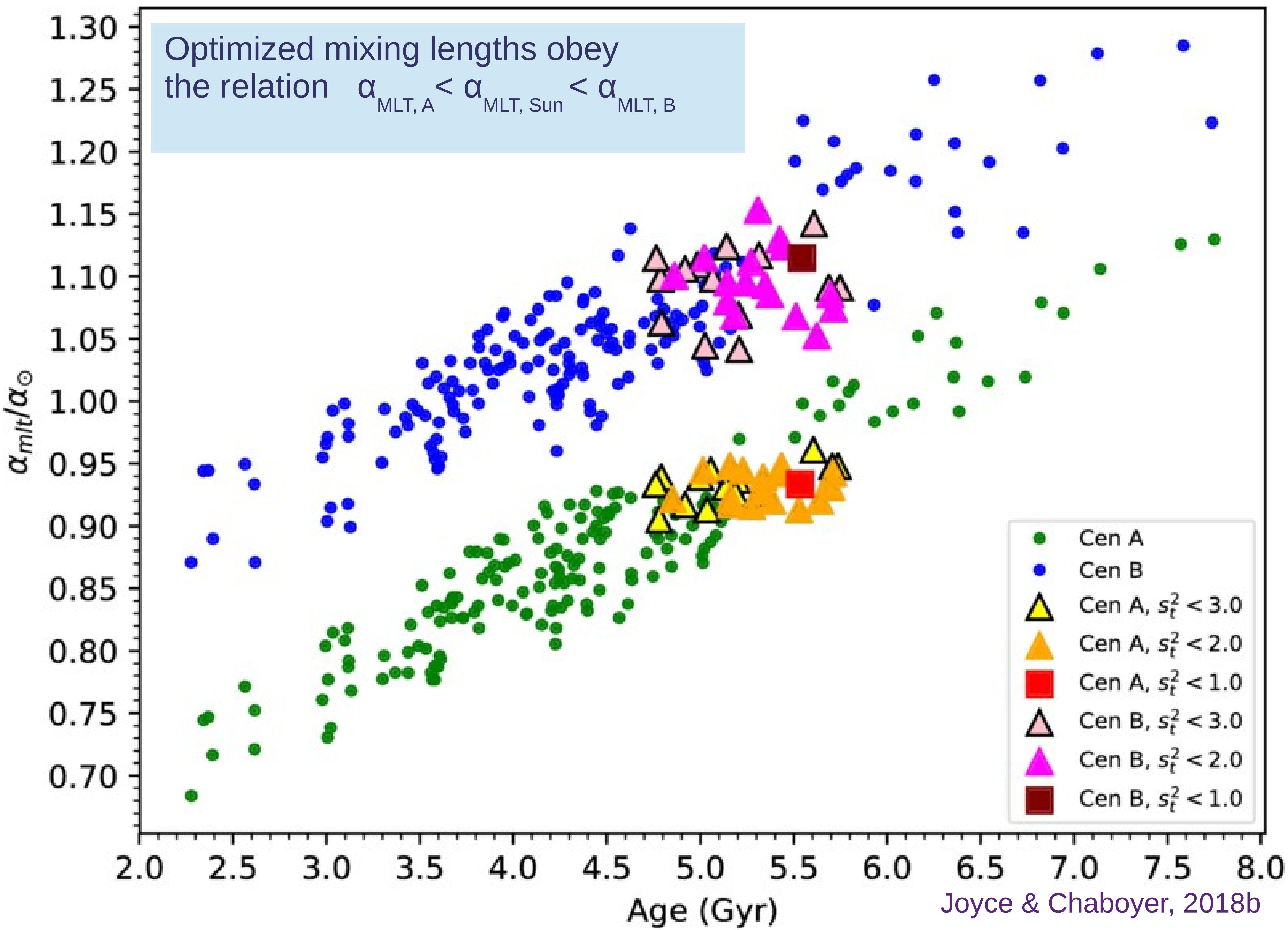
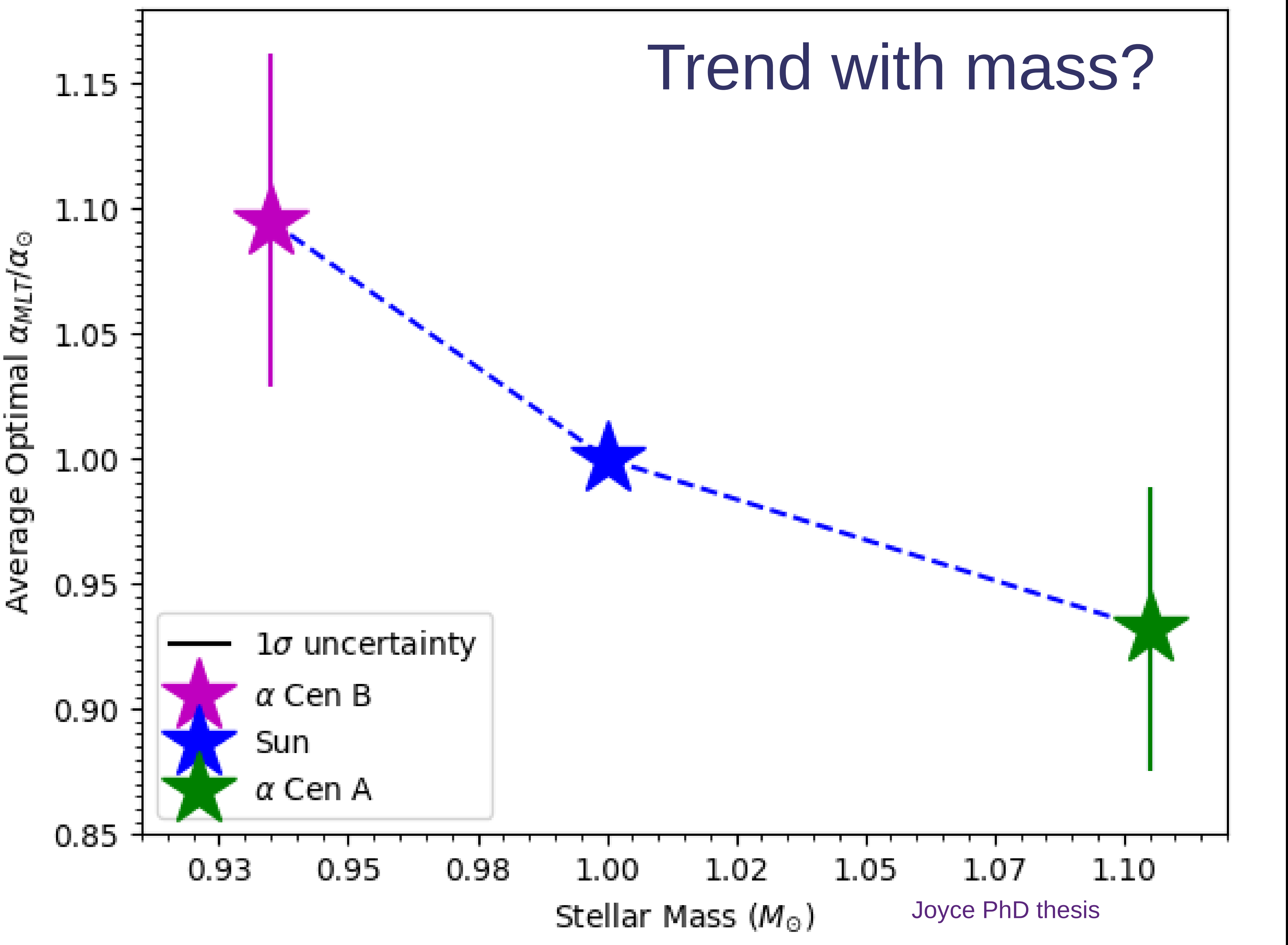


Joyce & Chaboyer, 2018b

When seismic constraints are added to the original 7 constraints, the models converge to precise values

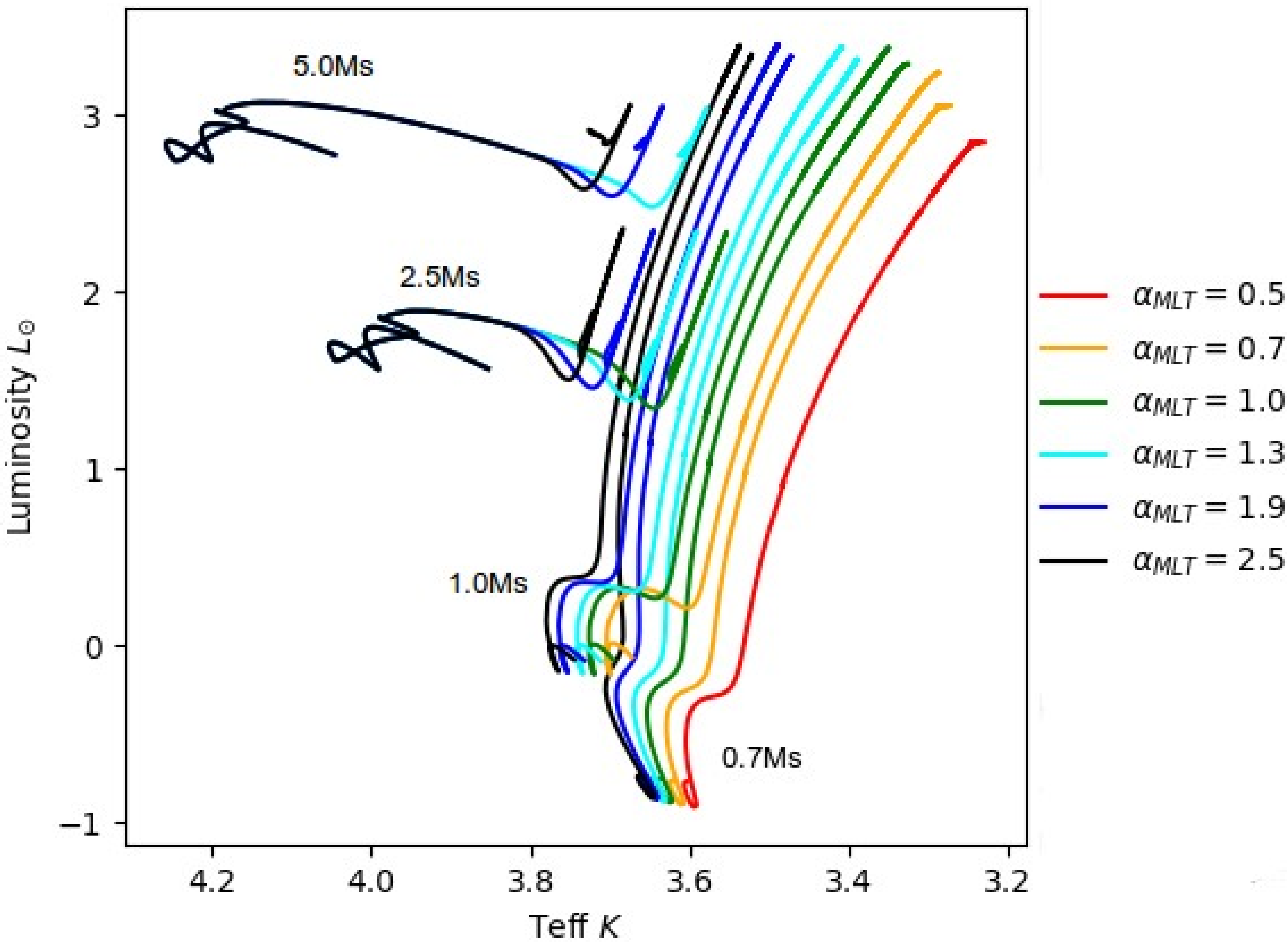


Joyce & Chaboyer, 2018b


Trend with mass?
Average Optimal $\alpha_{MLT}/\alpha_{\odot}$
1.15
1.10
1.05
1.00
0.95
0.90
0.85
0.93
0.95
0.98
1.00
1.02
1.05
1.07
1.10
Stellar Mass ($M_{\odot}$)
1σ uncertainty
α Cen B
Sun
α Cen A
Joyce PhD thesis

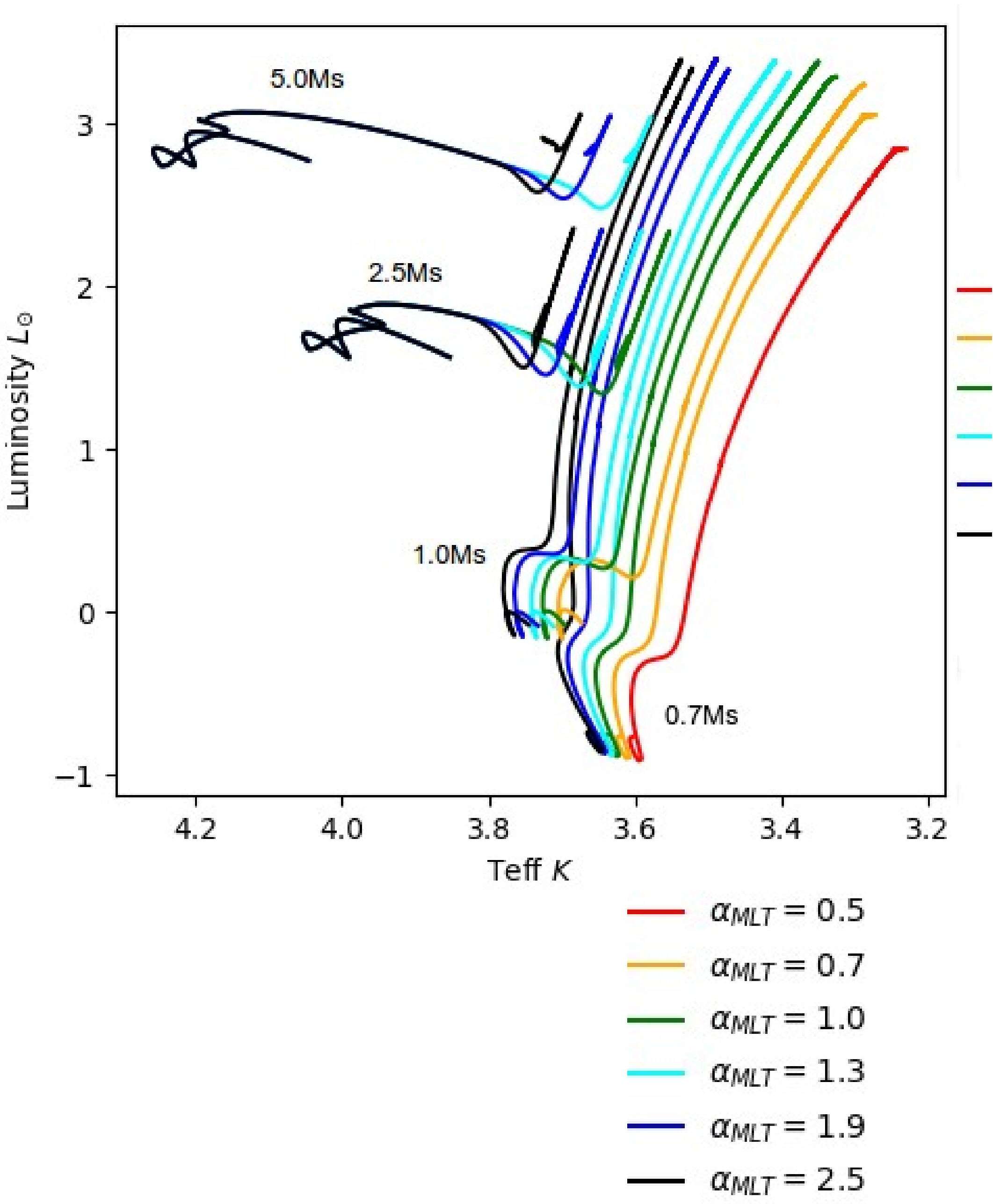
To appreciate the importance of calibrations, we must understand how changes to α_{MLT} impact stellar models, and how much

DSEP (Dartmouth Stellar Evolution Program) tracks



Building intuition for general trends:

- Low-mass stars are impacted by changes to alpha_MLT on the main sequence because they have convective envelopes



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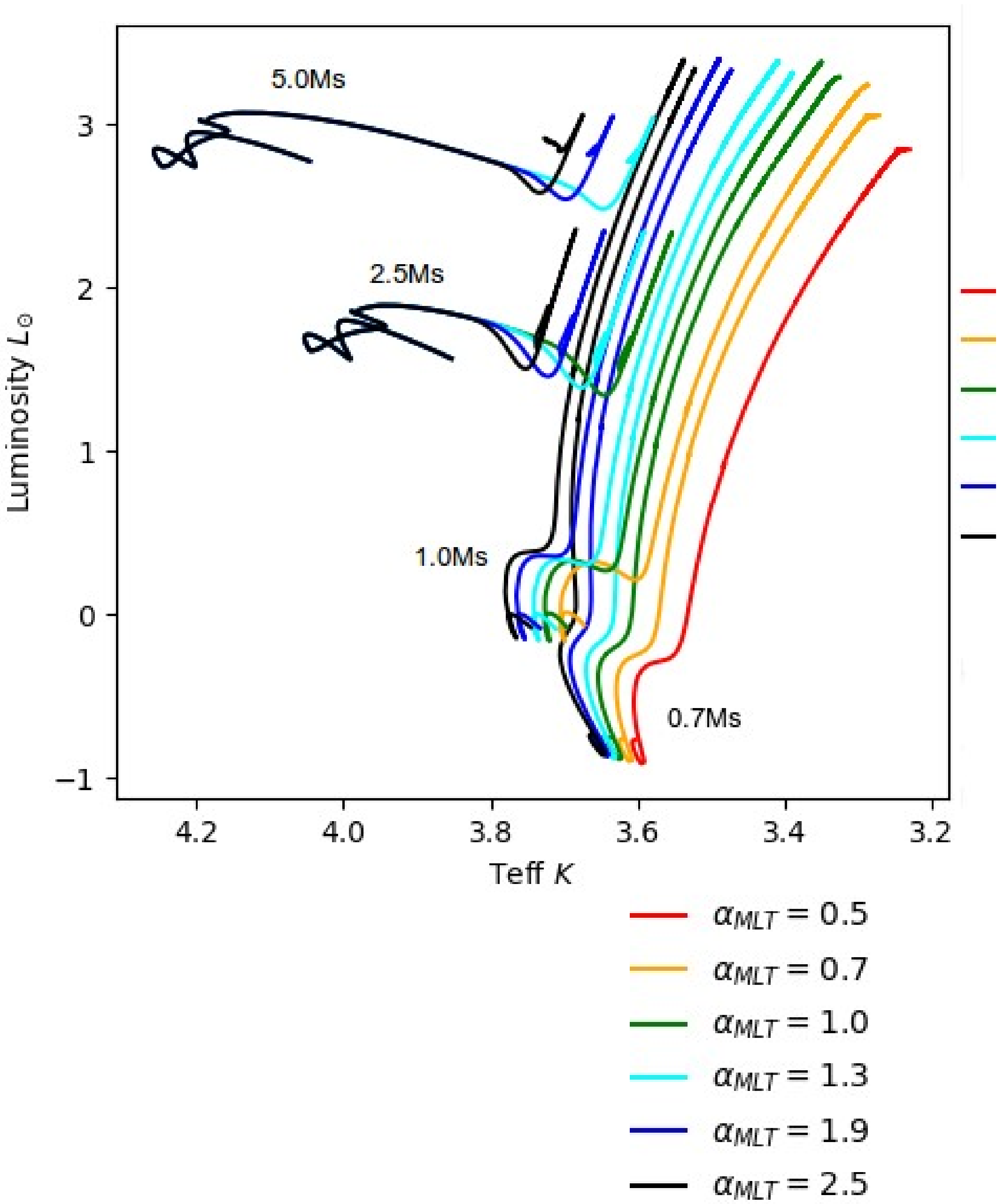

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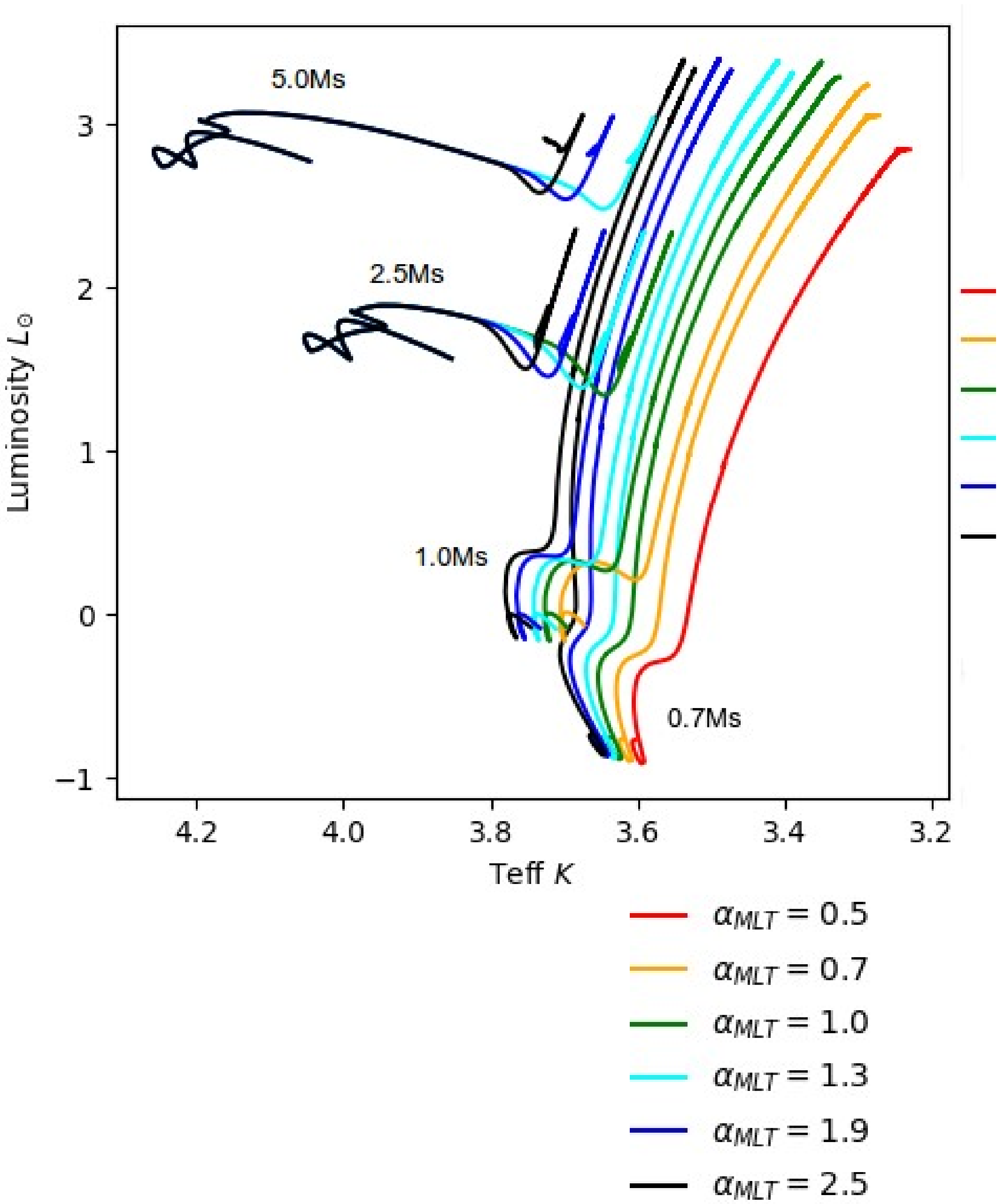

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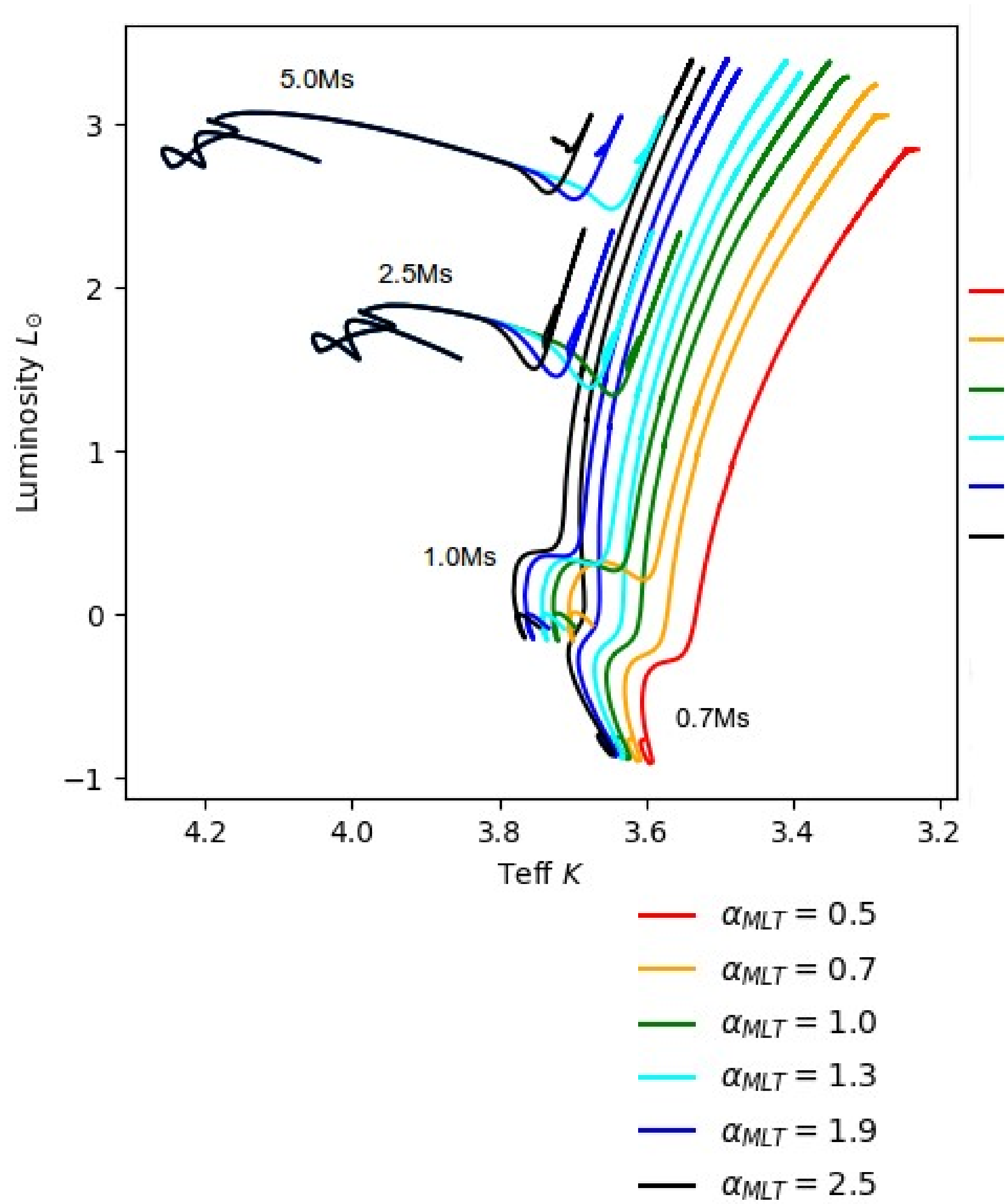

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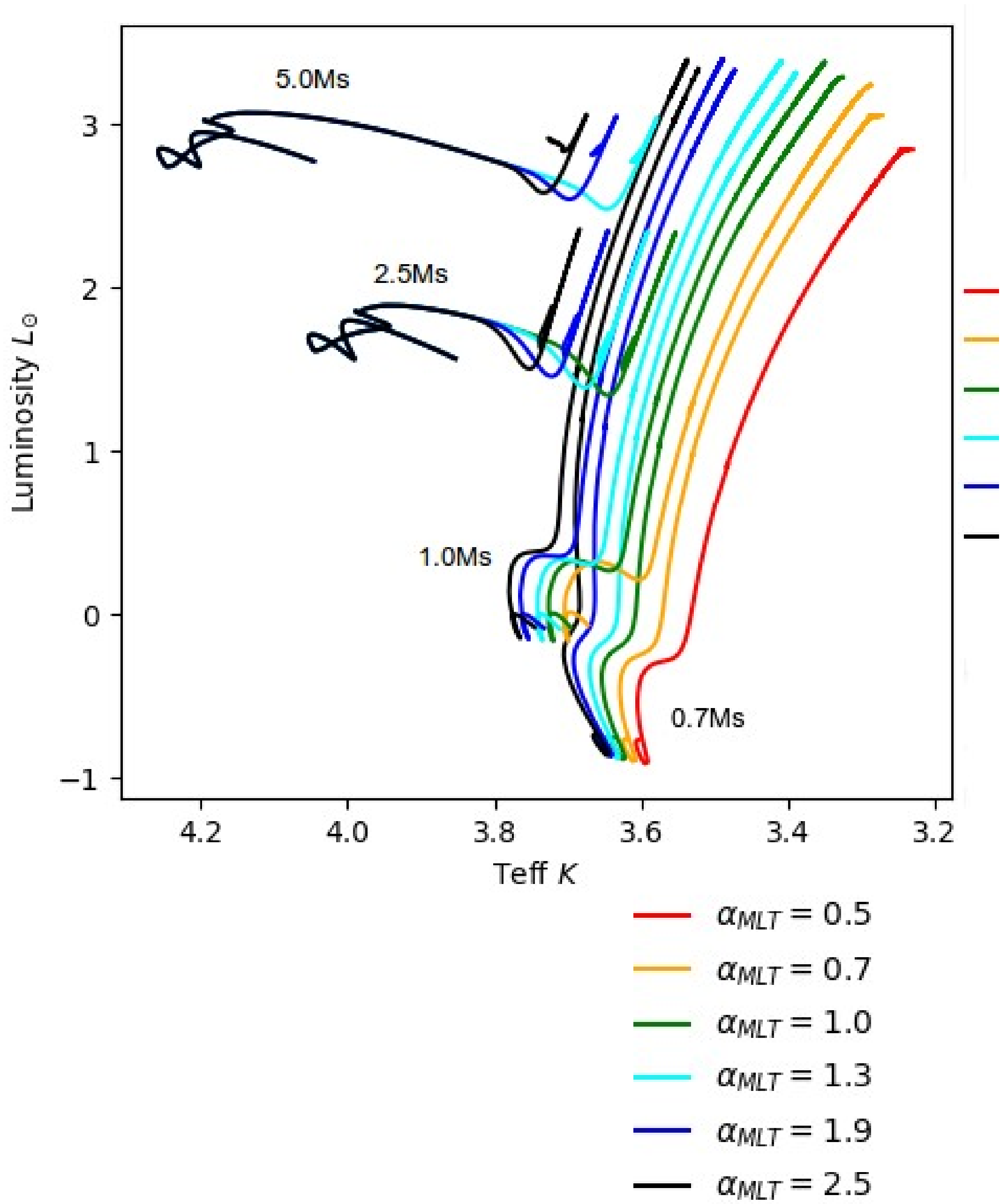

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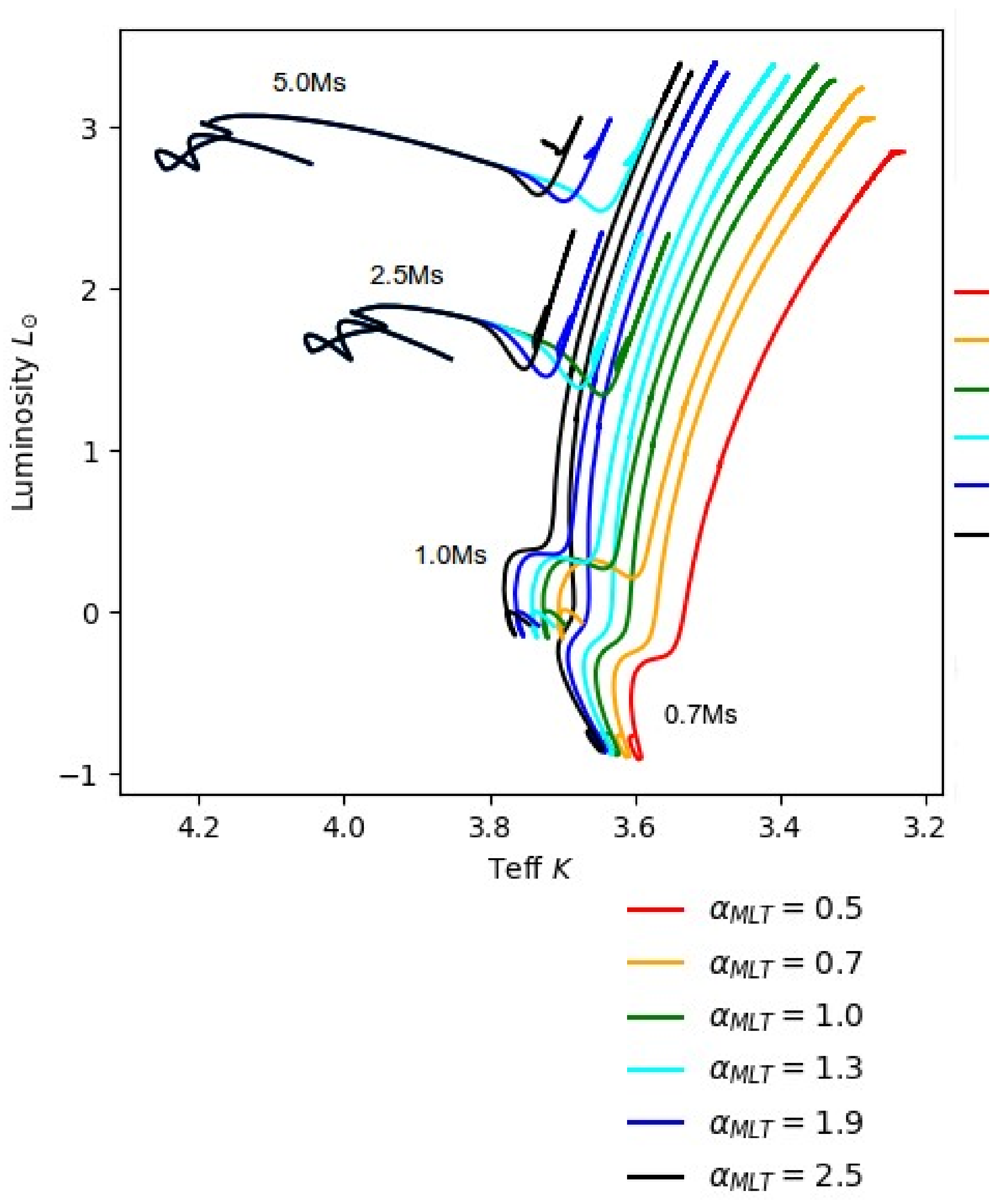
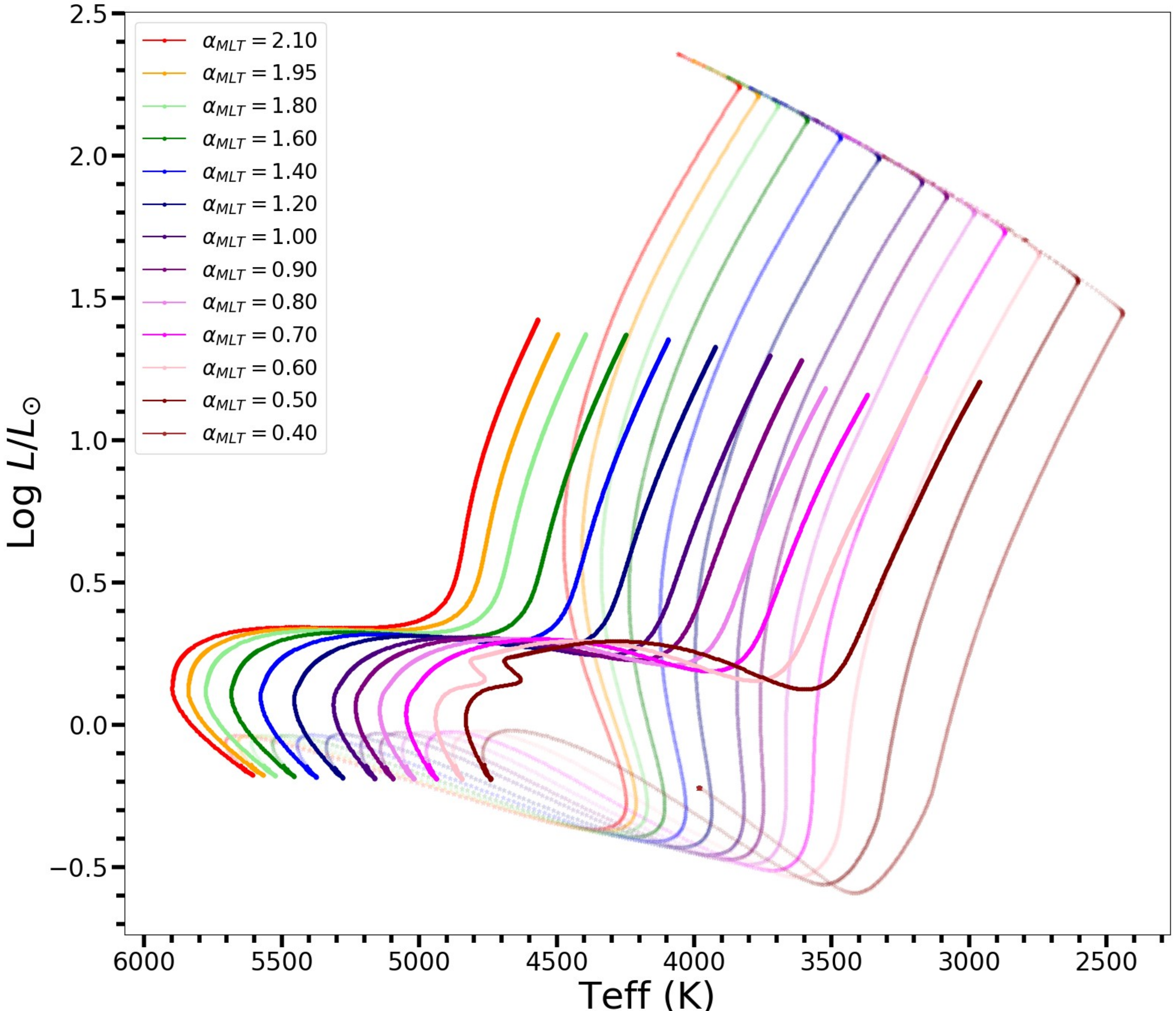
- Efficient transporters of energy should be hotter



Solar-like tracks, MESA v15140+



Joyce et al., this workshop

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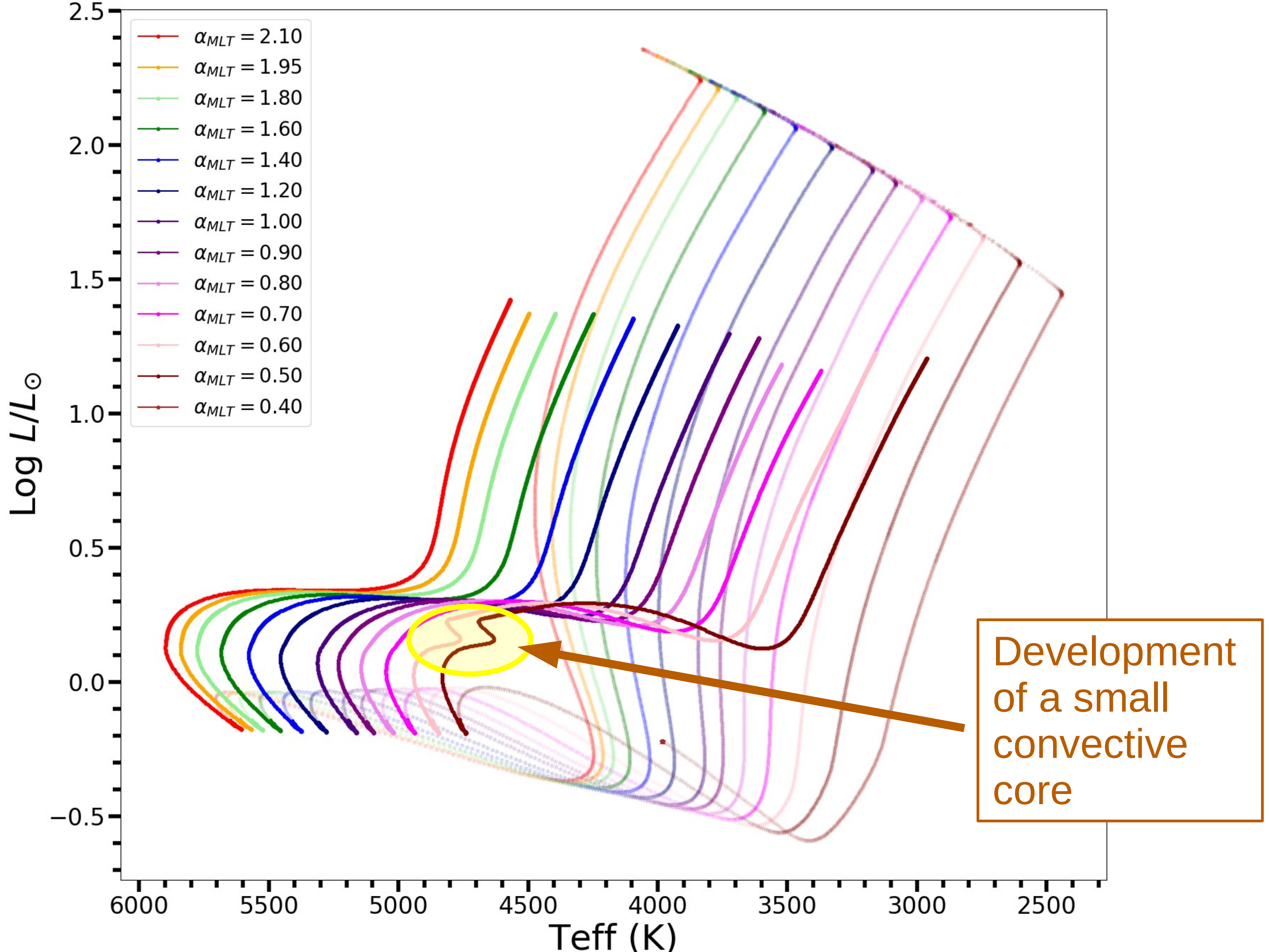


Joyce et al., this workshop

Custom DSEP isochrones with non-solar α_{MLT} and fits to M92

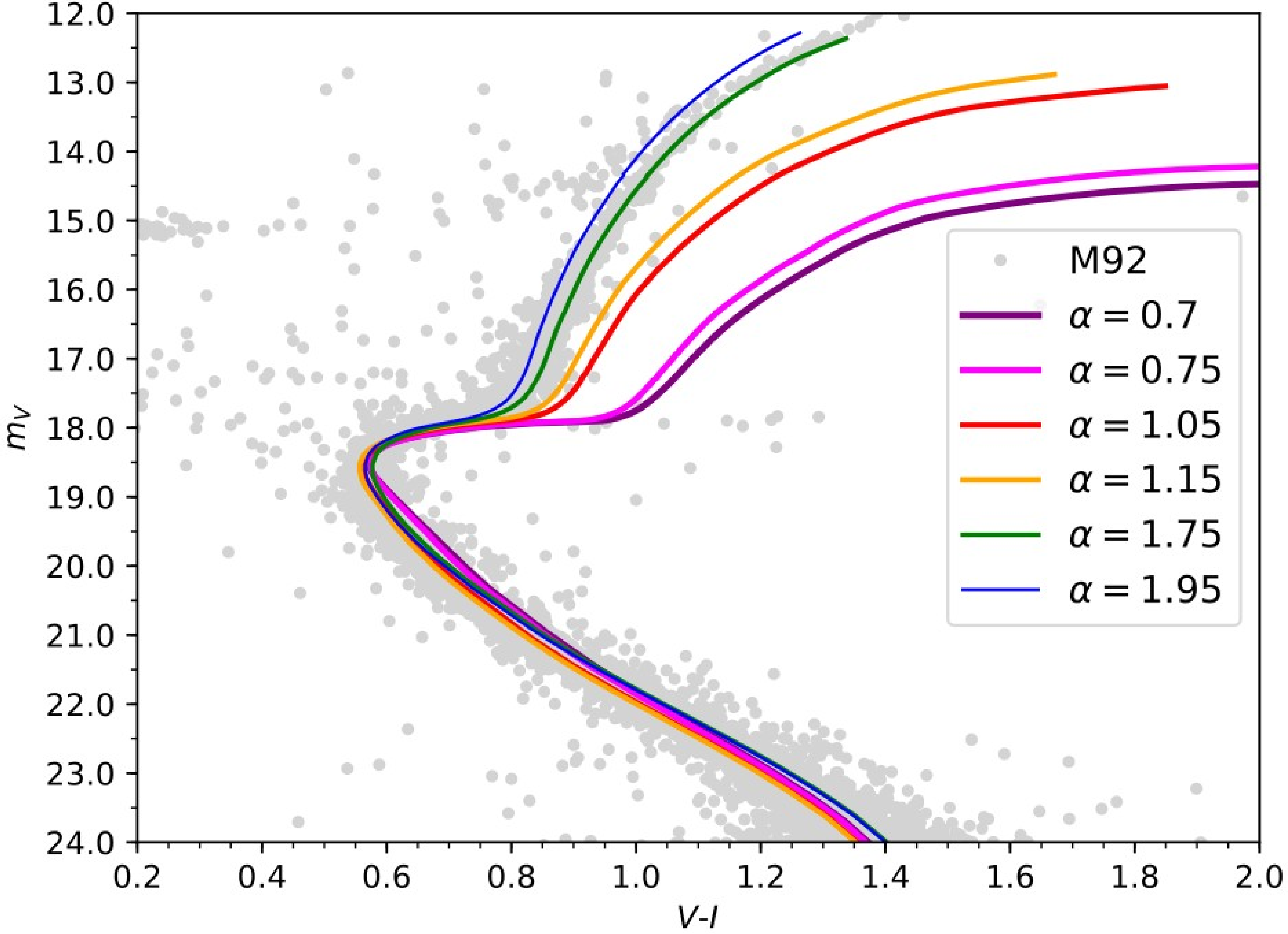
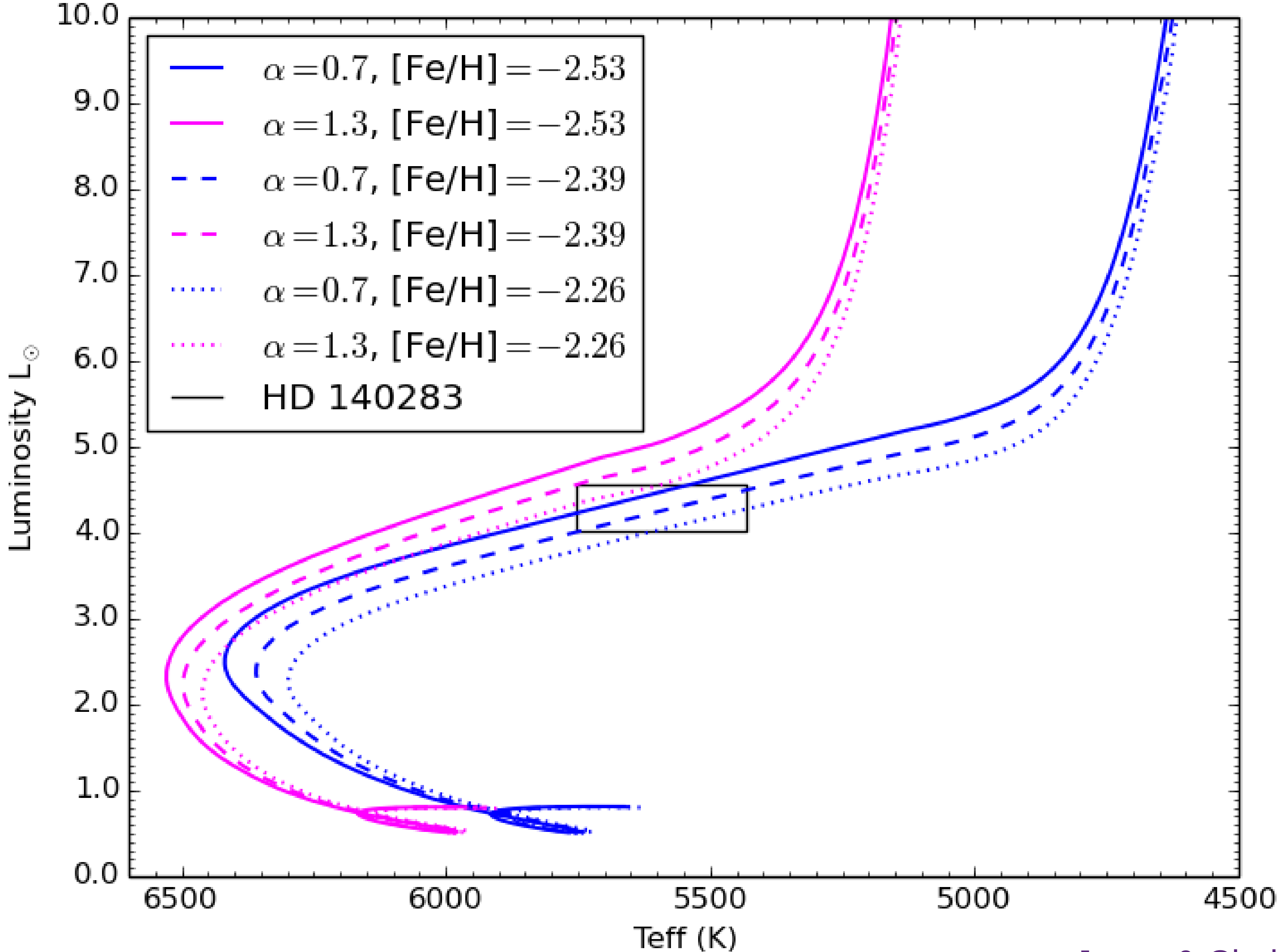


Figure 5. Six isochrones, each of age 13 Gyr, generated with different mixing lengths and shown against M92 for reference.

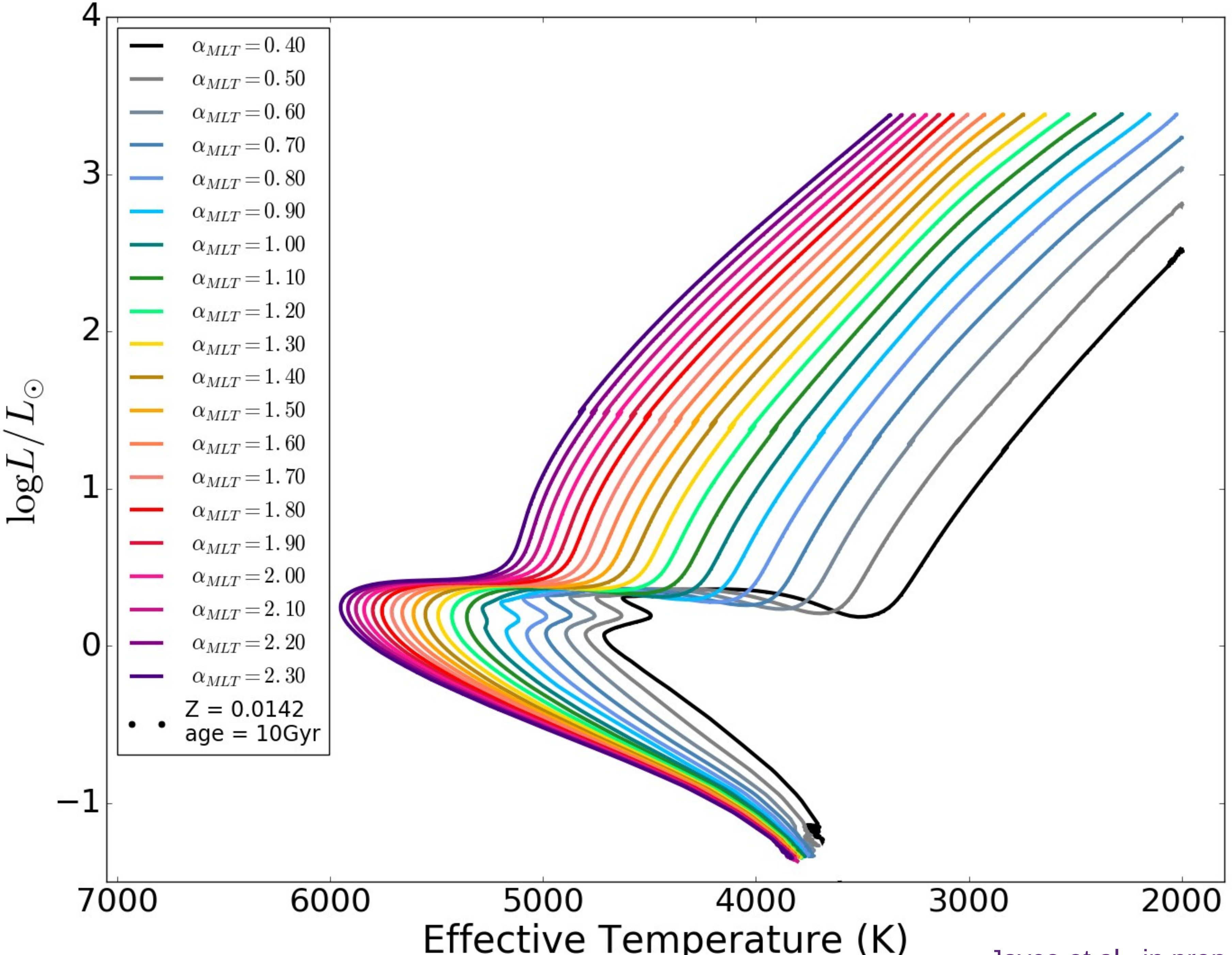
Joyce & Chaboyer, 2018a

In stellar tracks, changes to the mixing length are indistinguishable from changes in chemical abundance (an observed quantity) below a certain precision



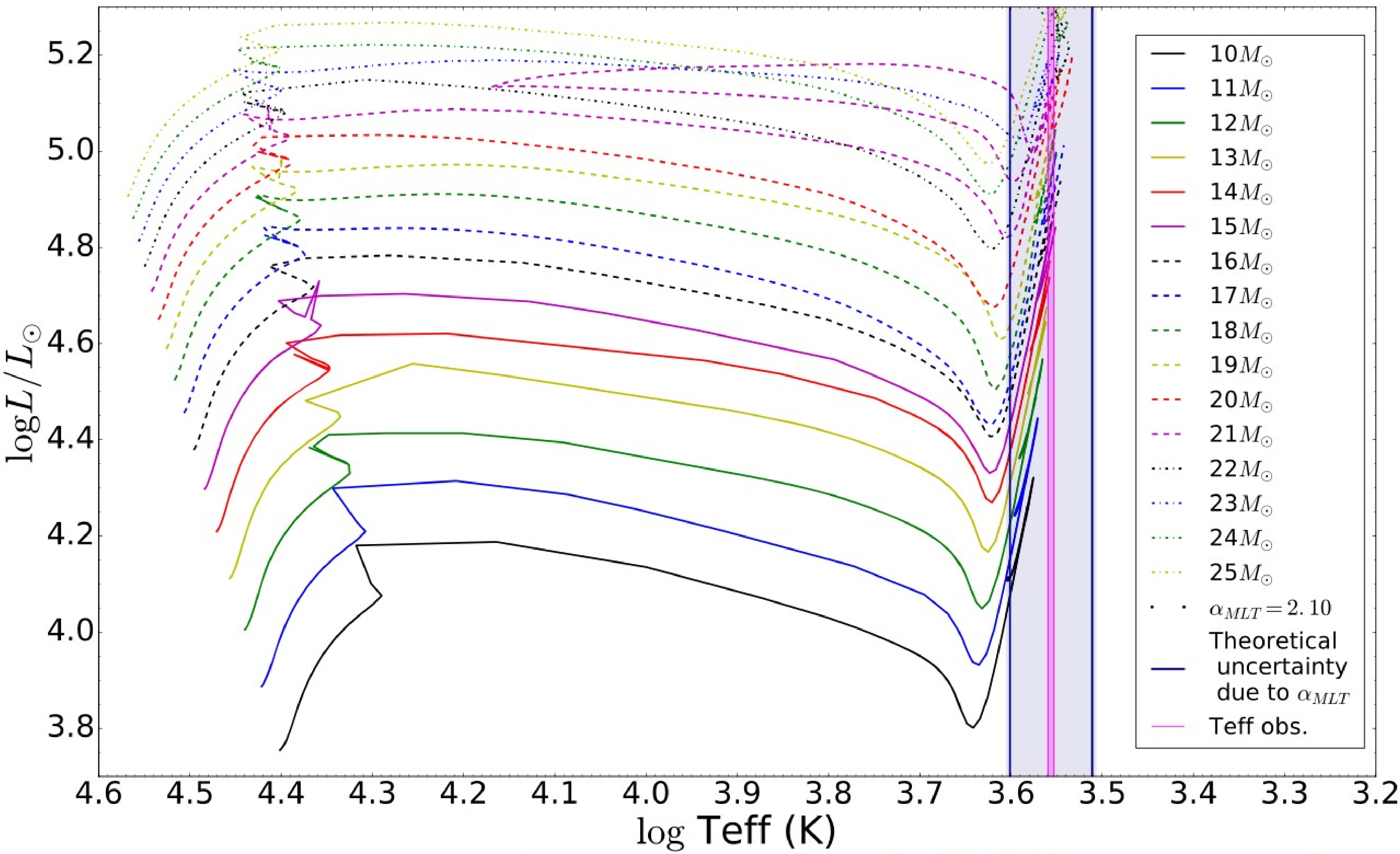
Joyce & Chaboyer, 2018a

Custom isochrones based on MESA tracks



Joyce et al., in prep

Observational vs modeling uncertainties: Betelgeuse



Joyce et al., 2020

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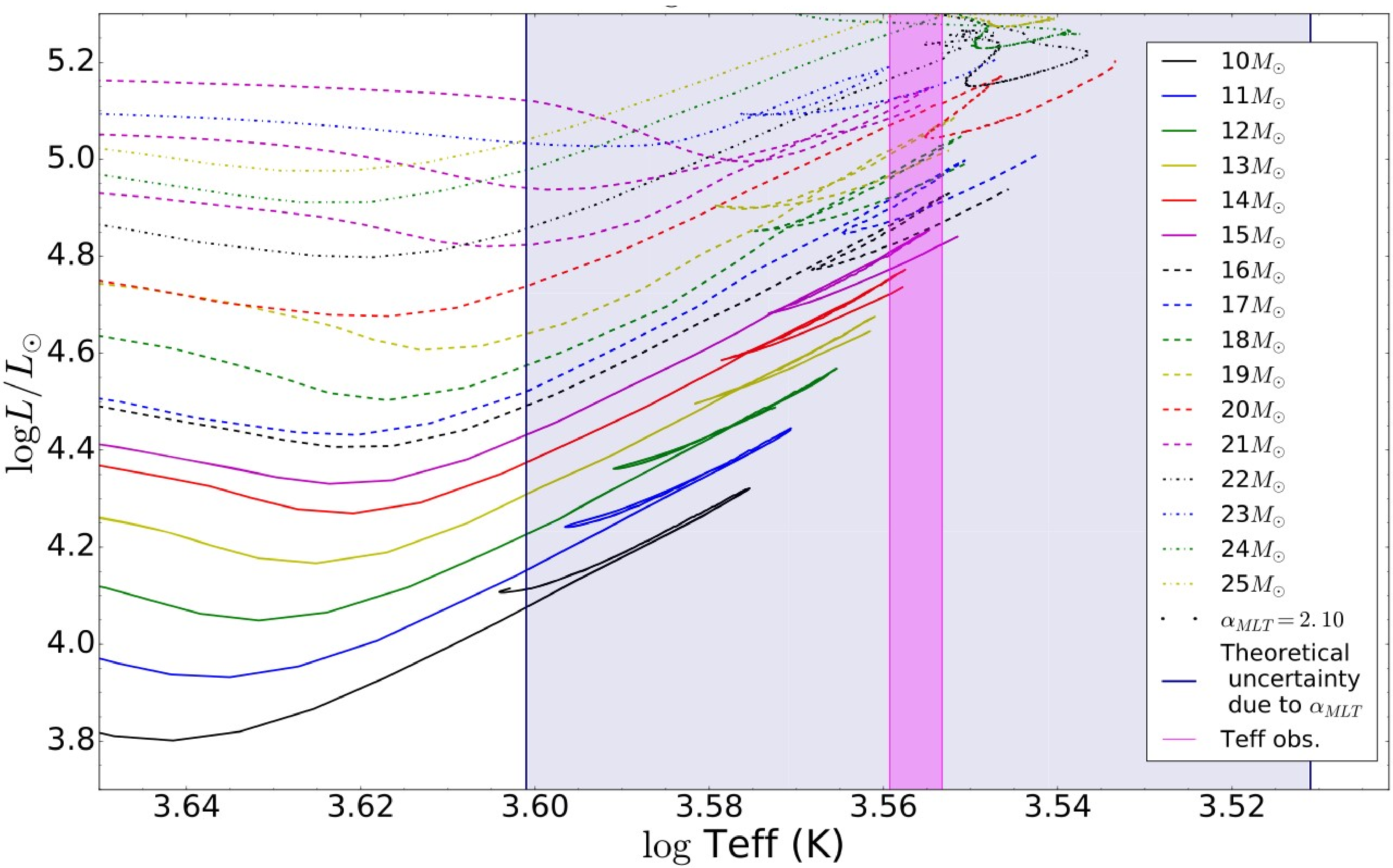
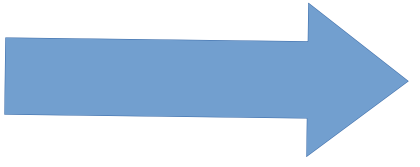


Joyce et al., 2020

Future



The future begins with the past: Summary of modern attempts to improve MLT

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(5) direct calibrations of the mixing length parameter against hydrodynamic simulations (ex. Trampedach et al., 2014)

A deeper dive into three of these modern improvements

(2) Full spectrum turbulence models (Canuto et al., 1991)

Statement of problem:

1. *The MLT treats the energy spectrum of the turbulent eddies as if it consisted of only one large eddy.*

(2) Full spectrum turbulence models (Canuto et al., 1991)

Solution (without math):

In conclusion:

1. The new model describes turbulence in a more realistic way than the MLT, for it includes the large spectrum of eddies that characterize a nearly inviscid medium such as a stellar interiors.
2. The new model can easily be implemented in existing stellar codes.
3. The new model requires $\alpha < 1$, which is a welcome feature on its own right.
4. The new model gives rise to several differences with respect to the MLT; on the basis of items 1–3 above, they are expected to be in the right direction.

When all the above considerations are taken together, they suggest that the new expression for the convective flux, equation (32), even with the $\Lambda = \alpha H_p$ expression, is to be preferred to the standard MLT.

(2) Full spectrum turbulence models (Canuto et al., 1991)

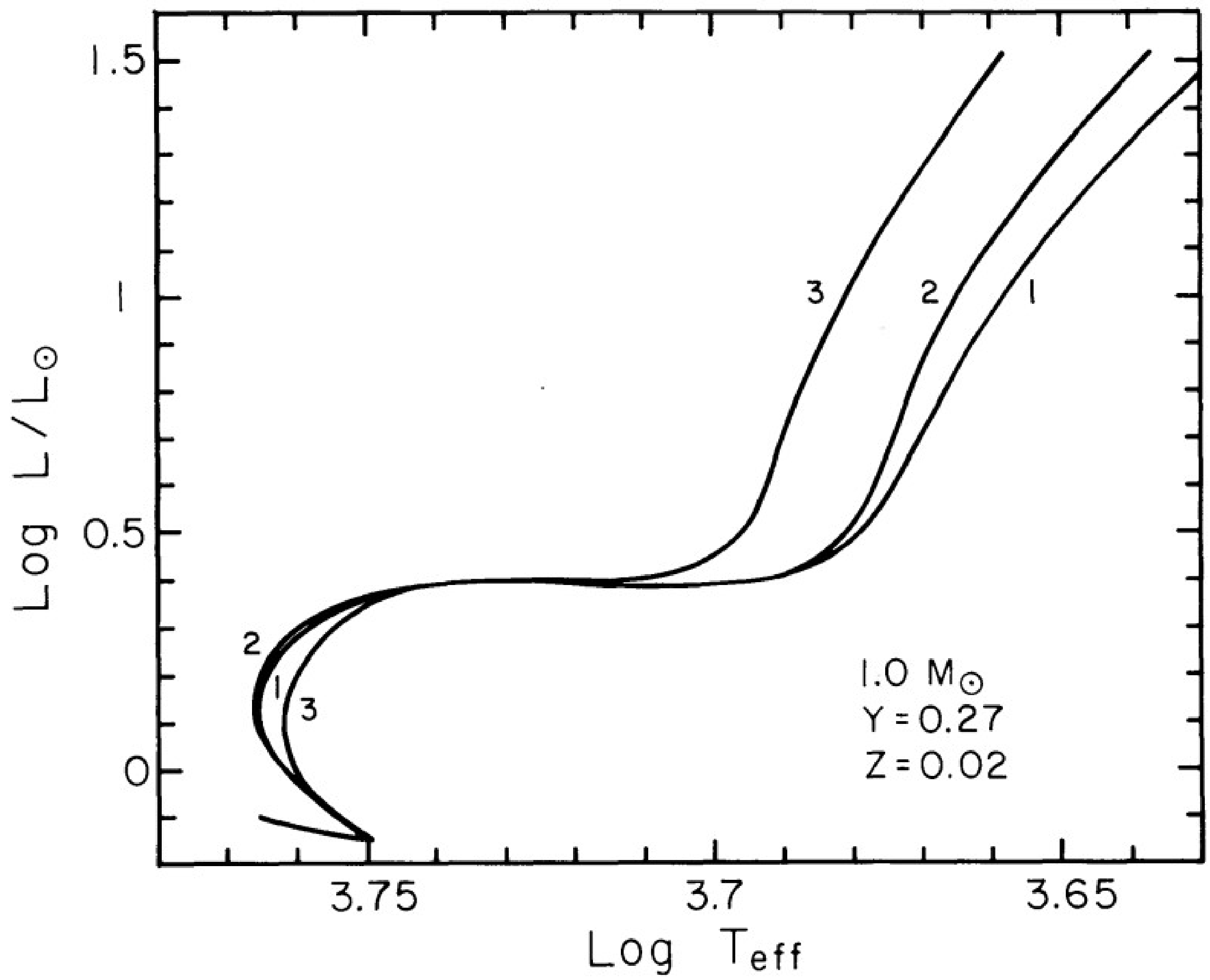

FIG. 6.—Evolutionary tracks in the H-R diagram for a 1 $M_\odot$ star, with $Y = 0.27$, $Z = 0.02$, for the three cases (1) MLT, $\Lambda = 1.4H_p$; (2) new theory, $\Lambda = 0.7H_p$, no turbulent pressure; (3) new theory, $\Lambda = z$, with turbulent pressure.

(2) Full spectrum turbulence models (Canuto et al., 1991)

1. *The MLT treats the energy spectrum of the turbulent eddies as if it consisted of only one large eddy.*

We thus conclude that the one-eddy model adopted by the MLT may be considered a reasonable approximation for viscous flows, but is a poor approximation for nearly inviscid stellar interiors which are characterized by a wide spectrum of eddies of all sizes.

(4) Using 3D radiative hydrodynamics to inform the choice of α_{MLT} (Ludwig et al., 1999)

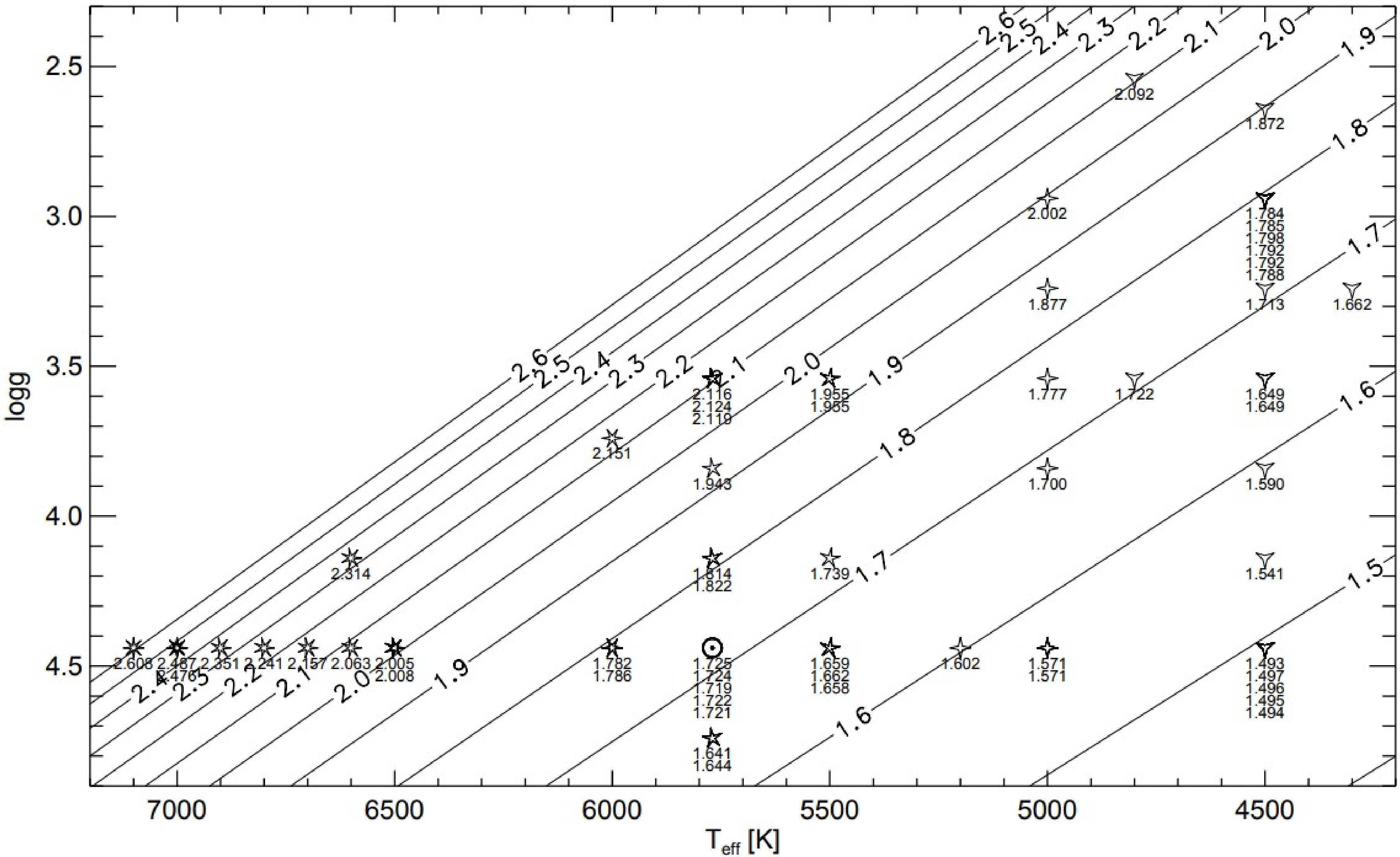

Fig. 3. s_{env} in units of $s_0 = 10^9$ erg/g/K from the grid of RHD models for Y = 0.28 and solar metallicity. Symbols indicate RHD models. Attached to the symbols the actual s_{env} values are given; the contour lines present a fit to them. The fitting function is given in Appendix C.

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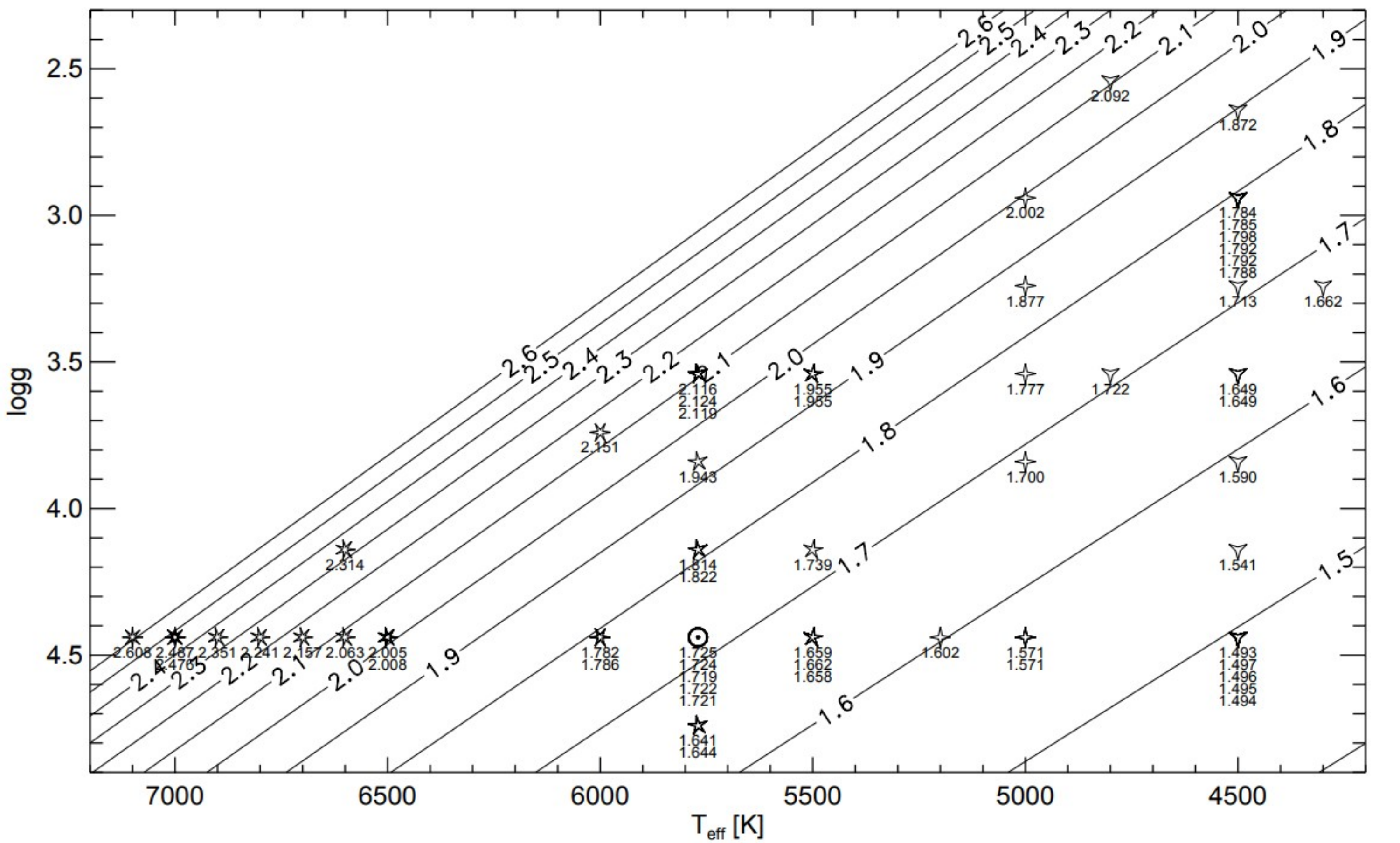
H.-G.Ludwig et al.: A calibration of the mixing-length for solar-type stars. I 117



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- trends are qualitatively the same as those shown by 1D sims
- in their own words: “do not extrapolate outside of this graph!”

(5) Direct calibration against 3D hydrodynamics (Trampedach et al., 2014)

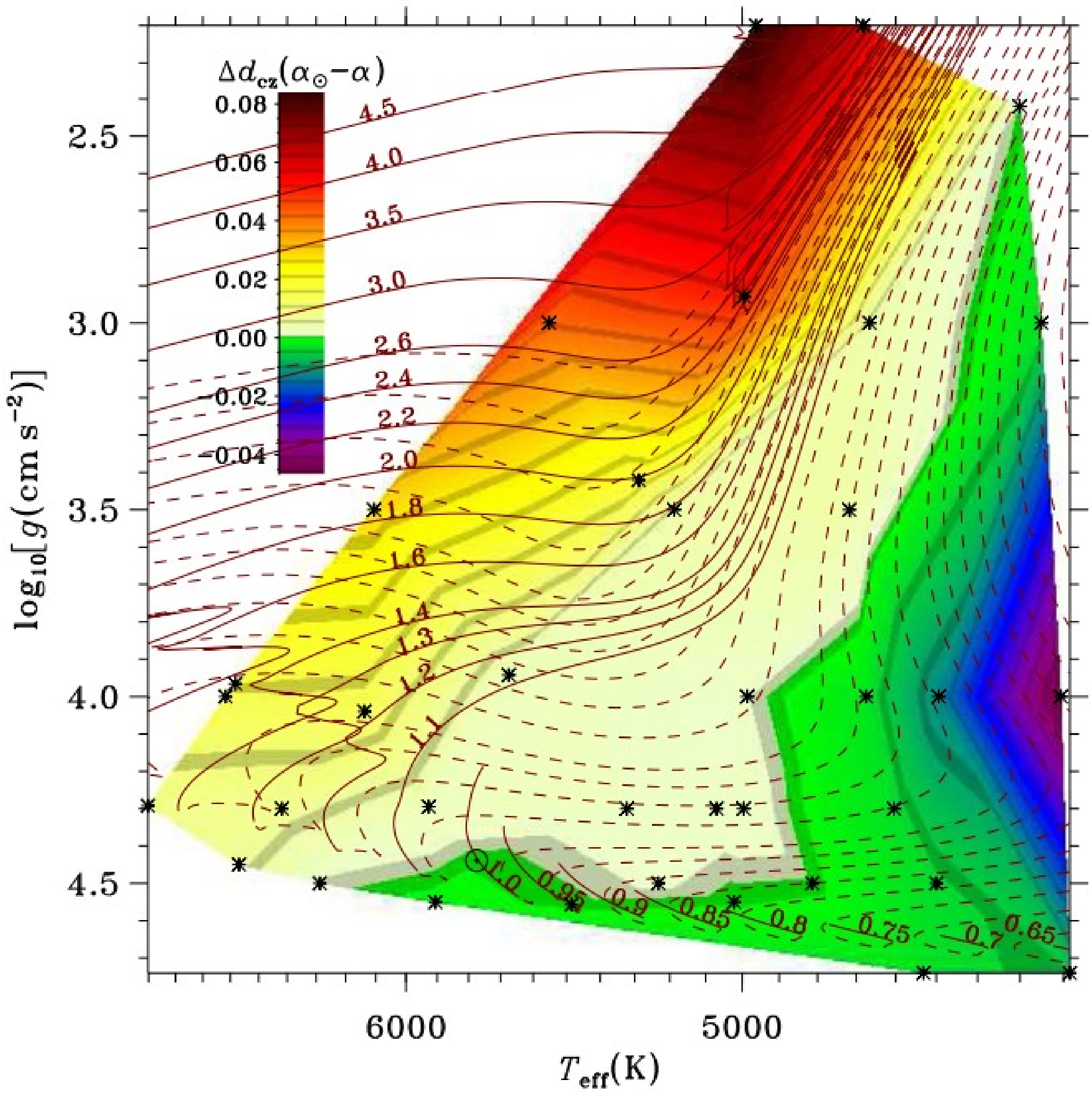

Figure 11. As Fig. 1, but showing differences in the depth of the convective envelope between models using solar-calibrated and individually calibrated values of α, in the sense that warm giants with $\alpha_\odot$ have deeper convective envelopes.

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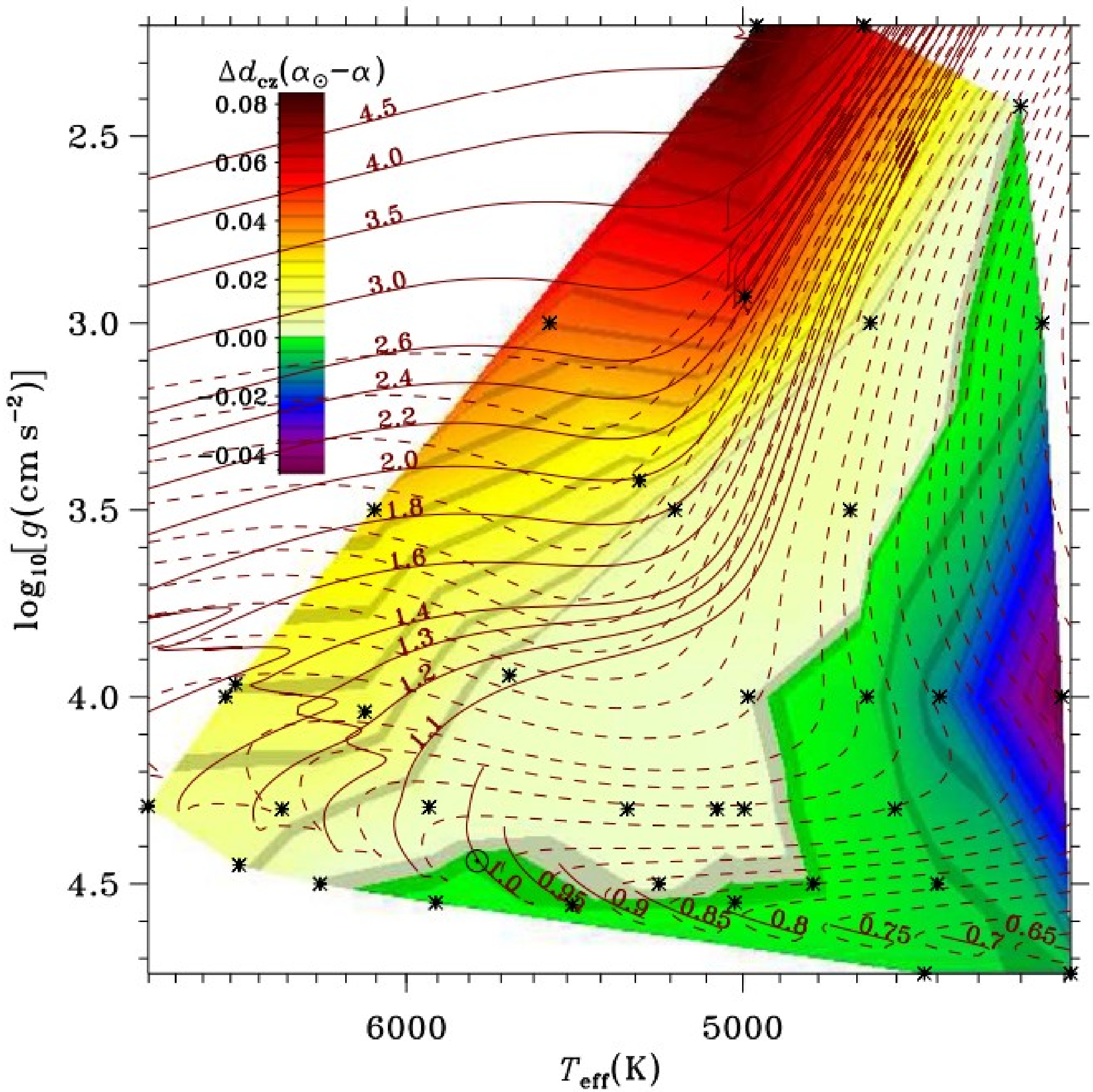
Trend: lower values of alpha (less efficient convection) are required at higher masses and later evolutionary stages



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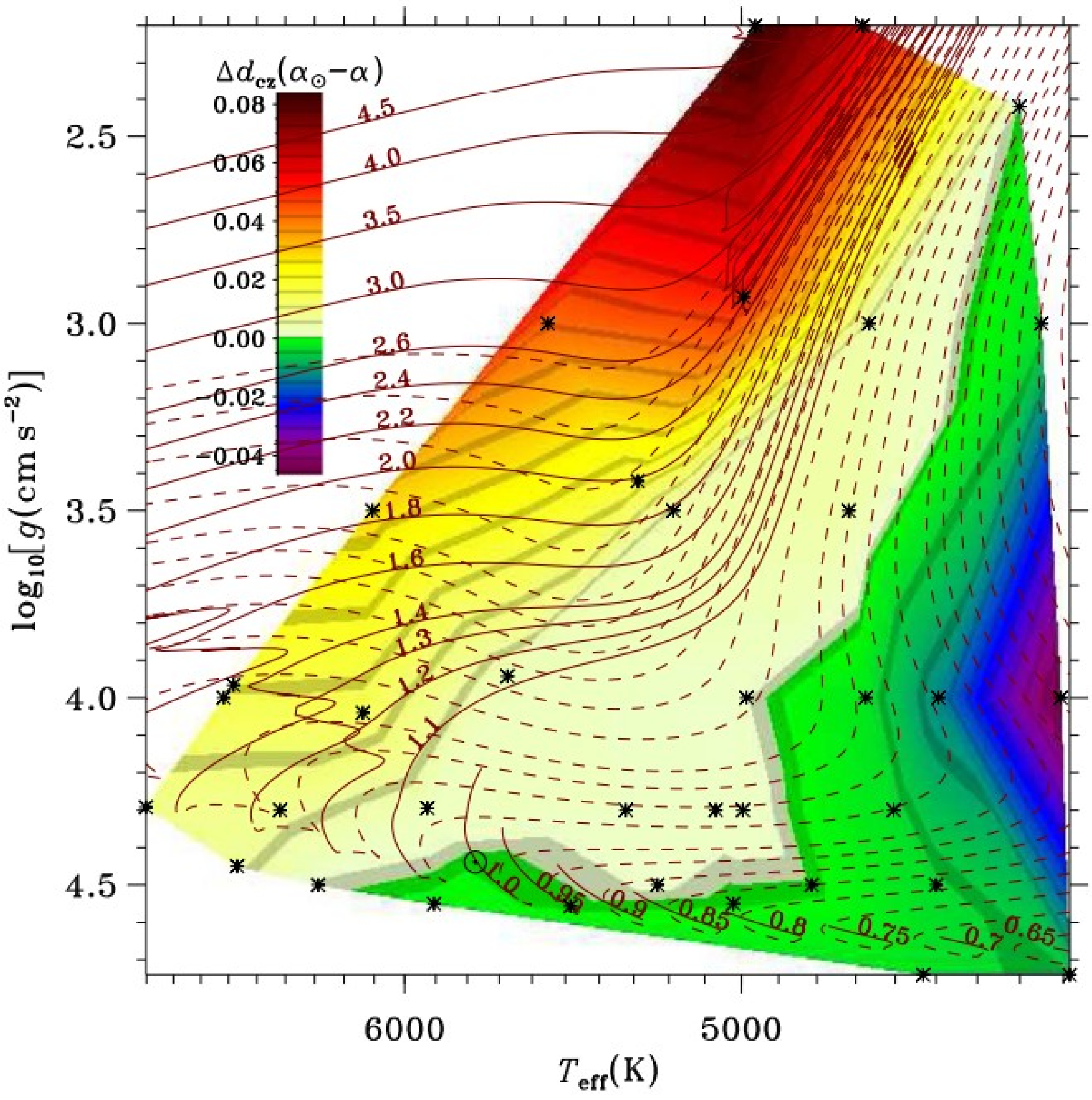

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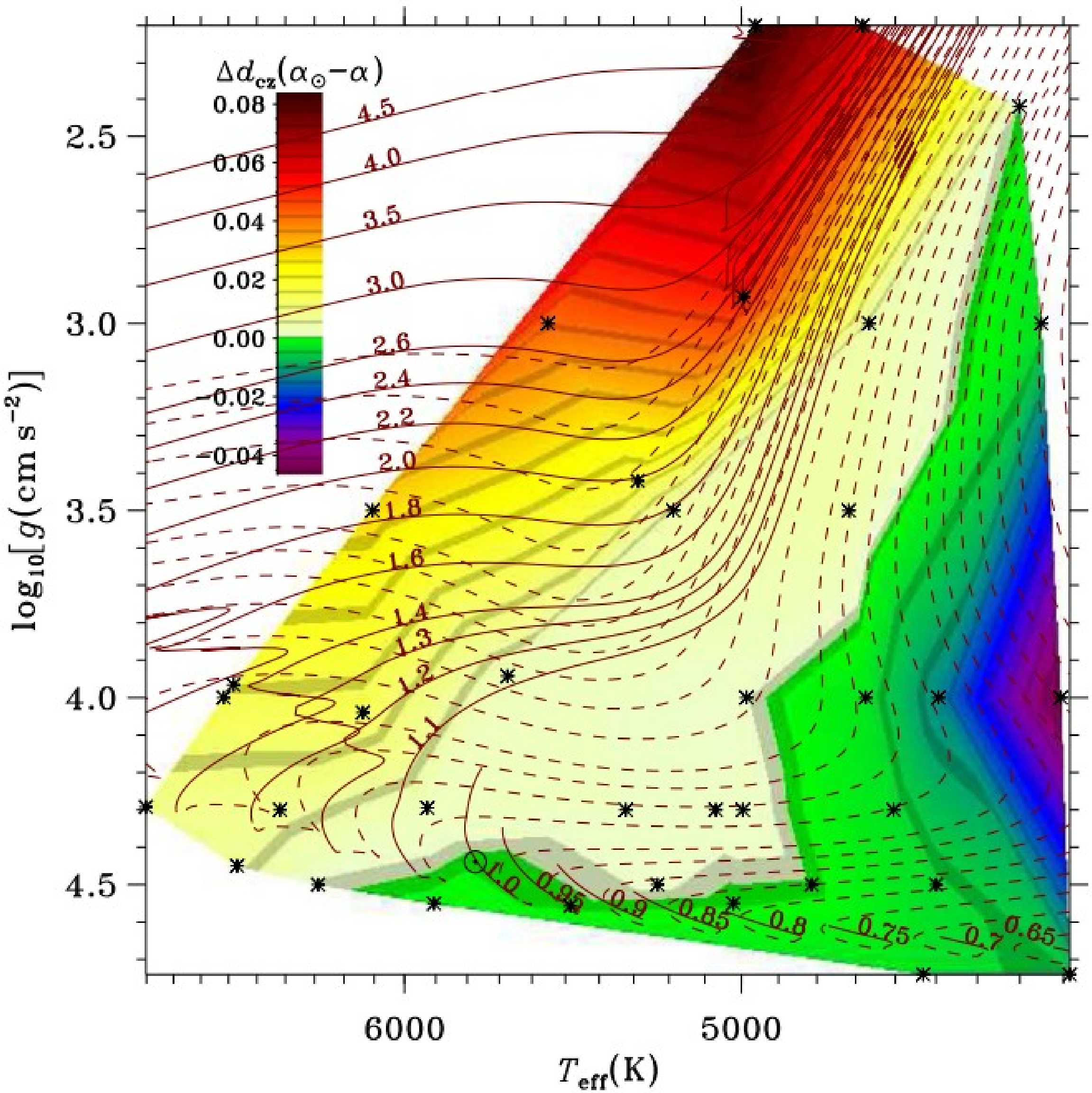
Question: how is this compatible with other work suggesting fully convective M-dwarfs require ultra-low alpha_MLT values?
(See Mann et al., 2015, and references therein)



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Where do we go from here?

Q: Is mixing length theory going away?

Q: Is this formalism going to get better?

Q: What role does 3D hydro play?

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*and it most likely is not, unless you are me or one of a handful of other people intentionally probing this regime. Your choices for, e.g., atmospheric boundary conditions or color corrections should be examined first

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How?

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Why- because MLT is a calibration-based formalism inherently, and stellar evolution codes are designed around this treatment

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As more targets become available for precision 1D calibrations, we must continue to validate these results against 3D simulations with the goal of building a calibration library that is **informed in equal parts by observations and the truth of fluid dynamics**


Fin