Physics and modeling of laminar-turbulent interfaces in pipe flow

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Why pipe flow?

Channel flow

Pipe flow

Boundary layer flow

TURBULENT FLOWS, by S.B. Pope
Transitional pipe flow

Reynolds *PTRSL* 1883

Wygnanski & Champagne *JFM* 1973: obstacle at inlet

Re=2400 Spatio-temporal intermittency  Re=4200 Slug sequence
Laminar-turbulent fronts

Nishi et al., JFM 2008: controlled (pulsed) disturbance
DNS: weak slug

Re=2600, weak downstream front

Song et al., JFM (in press)
DNS: strong slug

Re=5000, strong downstream front

upstream
downstream

bulk

Song et al., JFM (in press)
Front speeds

Duguet et al. 2010
Nishi et al. 2008
Durst et al. 2006
Wygnanski & Champagne 1973
Lindgren 1957
The rise of fully turbulent flow

\[ u_t + uu_x = \varepsilon g(q, u) \]
\[ q_t + (u - \zeta)q_x = f(q, u) + Dq_{xx} \]

Barkley et al., *Nature* 2015; Barkley *JFM* 2016
Front definitions

$\sqrt{\int \int (u_r^2 + u_\theta^2) r dr d\theta}$

Song et al., JFM (in press)
Front structure in \((u,q)\)-space

**DNS**

- DNS data for different Re numbers:
  - Re=2000
  - Re=2600
  - Re=2800
  - Re=3200
  - Re=4250
  - Re=5500

**Model**

- Strong slug
- Weak slug
- Puff

Song et al., JFM (in press)
Front switching

Song et al., JFM (in press)
Turbulent kinetic energy equation

- Turbulent kinetic energy:  \( k = \frac{1}{2} u' \cdot u' \)

\[
\frac{\overline{Dk}}{Dt} + \nabla \cdot T = P - \epsilon
\]

- Co-moving with the front:

\[
\frac{\partial k}{\partial t} = P - \epsilon - (\overline{u} - U_F) \cdot \nabla k - \nabla \cdot T = 0
\]  

Song et al., JFM (in press)
Kinetic-energy budget at Re=2600

Re=2600, UF

Re=2600, DF

Pipe axis, z(D)

Energy loss              Energy gain

P

-\epsilon

-\nabla \cdot T

-(\vec{u} - \vec{U}_F) \cdot \nabla k

Advection

Song et al., JFM (in press)
Kinetic-energy budget at Re=5000

Re=5000, UF

energy loss  energy gain

pipe axis, z(D)

Re=5000, DF

energy loss  energy gain

pipe axis, z(D)

UF

advection

Song et al., JFM (in press)
Mean turbulent advection speed

\[ c = - \frac{\langle \partial_z u(r, \theta, z, t) \cdot f(u(r, \theta, z, t), p, t) \rangle}{\| \partial_z u(r, \theta, z, t) \|^2} \]

Kreilos, Zammert & Eckhardt, JFM 2014

As assumed in Dwight’s model!

Song et al., JFM (in press)
Crossover weak to strong fronts

![Graph showing the crossover of weak to strong fronts. The graph plots speed (U) against Reynolds number. The data points are labeled with "DNS upstream," "DNS downstream," and "advection speed." The graph also includes lines for speed asymptotics for both weak and strong fronts.]
Crossover weak to strong fronts

$q_{\text{max}}$ vs $Re$

UF, DF, bulk
Fronts and ECS: localized RPO

\[ \left( \frac{u - u_{\text{lam}}}{u_{\text{lam, max}}} \right) \]

RPO\* → torus → chaos → transient chaos → spatio-temporal chaos

* Sub-harmonic bifurcation from TW: Chantry et al., PRL 2014

Avila et al., PRL 2013
Ritter et al., NJP 2016
Exponential localization

$\mathcal{L}$

$m=2$

$m=3$

Ritter et al., in preparation
Localization: Re-dependence

$m=2$

$m=3$

Ritter et al., in preparation
Model vs. ECS

\[ u = \tilde{u}(r) \exp \left[ i m \theta + \frac{\mu}{Re} (z - ct) \right] \]
\[ p = \tilde{p}(r) \exp \left[ i m \theta + \frac{\mu}{Re} (z - ct) \right] \]

Quadratic eigenvalue problem

Ritter et al., in preparation
$m=2$

upstream

$\mathbf{u}_r$

$\mathbf{u}_\theta$

$\mathbf{u}_z$

downstream
Modeling fronts with linear model

pipe axis (D)

Downstream profile

Upstream profile
Conclusions

• 1D-modeling (ARD) describes dynamics of interfaces

• Weak/strong fronts are physically distinct:
  – Weak: relaminarization at near (local) equilibrium
  – Strong: peak in production with delayed dissipation
  – Key role of mean advection speed of turbulence
  Song, Barkley, Hof & Avila, *JFM* 2017 (in press)

• Fronts of ECS:
  Ritter, Zammert, Eckhardt & Avila, in preparation

Further reading: Holzner, Song, Avila & Hof, *JFM* 2013:
“Lagrangian approach to laminar–turbulent interfaces in transitional pipe flow”