Self-sustaining attached eddies in wall-bounded turbulence: Pressure, skin friction & invariant solutions

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\[ U^+ (y^+) = y^+ \]

\[ U^+ = \frac{1}{\kappa} \ln y^+ + B \]

Viscous sublayer | Buffer layer | Log region (Overlap layer) | Wake region

\[ \delta_v (= \nu / u_\tau) \]

\[ y \]

\[ \delta \]
Evidence: Linear spanwise length scale growth

\[ k_z E_{uu}(\lambda_z; y) \]

\[ y \sim 0.1\lambda_z \]

\[ \lambda_z^+ \approx 100 \quad \lambda_z \approx 1.5h \]

Hoyas & Jimenez (2006, PoF)

DNS (channel) at \( Re_x = 2003 \)
Attached eddies do exist and sustain themselves


Hwang, 2015, *J. Fluid Mech.* **767** p254

Hwang & Bengana, 2016, *J. Fluid Mech.* **795** p708
Isolating the motions at a given $\lambda_{z,0}$ ($100\delta_y < \lambda_{z,0} < 1.5h$)

$\lambda_{z,0}$

![Diagram with annotations](image)

**Narrow spanwise domain + additional removal of quasi 2D motion**

$L_z = \lambda_{z,0}$

Isolating the motions at a given $\lambda_{z,0}$ ($100\delta_v < \lambda_{z,0} < 1.5h$)

Hoyas & Jimenez (2006, PoF)

LES with an excessively large eddy viscosity (artificial elevation of $C_s$)

Example: isolating the motions at $\lambda_{z,0}^+ \approx 1000 \ (\text{Re}_\tau \approx 2000)$

$\lambda_{z,0}^+ \approx 1000$

$k_z^+ E_{uu}$

$\lambda_z^+ \approx 100$

$y^+ \sim 0.1 \lambda_z$

$C_s = 0.05$

$\lambda_z^+ > 1000 \quad \Rightarrow \quad $Filtering

$\lambda_z^+ < 1000 \quad \Rightarrow \quad $Over-damped LES
Statistics of self-sustaining Townsend’s attached eddies

\[ \frac{\lambda_x}{\lambda_z} \approx 3 \]

\[ \frac{y}{\lambda_{z,0}} \]

\[ k_x^+ E_{uu} \]

\[ k_x^+ E_{vv} \]

\[ \lambda_x^+ = 572 \]

\[ \lambda_x^+ = 871 \]

\[ \lambda_x^+ = 1517 \]
Bursting of self-sustaining attached eddies

\[ E_u = \int_V u^2 \, dV \]

\[ E_v = \int_V v^2 \, dV \]

Streaks

Vortices

Flow

Vortices

Streaks
Causality of the three elements

**Streaks**

\[ E_u \equiv \int_V (u^+)^2 dV \]

**Vortices**

\[ E_v^+ \equiv \int_V (v^+)^2 dV \]

**Wave (streak instability)**

\[ E_1 \equiv \int_V \left| \hat{u} \right|^2 + \left| \hat{v} \right|^2 + \left| \hat{w} \right|^2 dV \quad \text{for} \quad k_x = 2\pi/L_x \quad \text{and} \quad k_z = 2\pi/L_z \]

\[
C_{uv}(\tau) = \frac{\langle E_u(t+\tau)E_v(t) \rangle}{\langle E_u(t) \rangle^{1/2} \langle E_v(t) \rangle^{1/2}},
\]

\[
C_{vw}(\tau) = \frac{\langle E_v(t+\tau)E_w(t) \rangle}{\langle E_v(t) \rangle^{1/2} \langle E_w(t) \rangle^{1/2}},
\]

\[
C_{lv}(\tau) = \frac{\langle E_1(t+\tau)E_v(t) \rangle}{\langle E_1(t) \rangle^{1/2} \langle E_v(t) \rangle^{1/2}},
\]
Self-sustaining process of attached eddies

Streamwise vortices

Lift-up

\[ \frac{D\omega_y}{Dt} \sim \frac{\partial U}{\partial y} \omega_x \]

Streamwise wavy structures

Streak instability or transient growth

Nonlinear processes

Hamilton, Kim & Waleffe (JFM, 1995)
Pressure fluctuation is generated by nonlinear feeding processes of vortices

Minjeong Cho (SNU, Korea)
Haecheon Choi (SNU, Korea)
Classical description on pressure in a turbulent flow

\[ \nabla^2 p = - \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i} \]

\[ u_i = U_i + u'_i \]

\[ \nabla^2 p_{\text{rapid}} = -2 \frac{dU}{dy} \frac{\partial v'}{\partial x} \]

\[ \nabla^2 p_{\text{slow}} = - \frac{\partial u'_i}{\partial x_j} \frac{\partial u'_j}{\partial x_i} \]

Rapid (linear) pressure
: rapidly responding pressure under direct effect of mean shear

Slow (nonlinear) pressure
: slow responding pressure by the following nonlinear interaction

See e.g. Kim (1989, JFM)
Self-similar pressure of self-sustaining attached eddies

\[ \lambda_x \approx 3\lambda_{z,0} \]

Pressure shows strong correlation with vortices (not surprisingly!).

Slow pressure is about twice larger than rapid pressure.

Rapid pressure extends to the wall due to mean shear:

\[ \nabla^2 p_{\text{rapid}} = -2 \frac{dU}{dy} \frac{\partial v'}{\partial x} \]
Causality of pressure in self-sustaining process

\[
C_{ij}(\tau) = \frac{\langle E_i(t+\tau)E_j(t) \rangle}{\sqrt{\langle E_i^2(t) \rangle} \sqrt{\langle E_j^2(t) \rangle}} \quad \left( \langle \cdot \rangle: \text{average in time} \right)
\]

\[
\begin{align*}
C_{up_r} & \quad C_{up_s} \\
C_{1p_r} & \quad C_{1p_s} \\
C_{vp_r} & \quad C_{vp_s}
\end{align*}
\]

\[
\tau u_r/\lambda_{z,0}
\]

Streak \quad \rightarrow \quad \text{Streamwise wavy motions} \quad \rightarrow \quad \text{Vortices}

Rapid & slow pressures
Self-sustaining process and pressure

Lift-up (interaction with mean shear)

\[ \frac{D\omega_y}{Dt} \sim \frac{\partial U}{\partial y} \omega_x \]

Rapid pressure should decay in time in the linear process

Streamwise vortices

\[ \nabla^2 p_{\text{rapid}} = -2 \frac{\partial U}{\partial y} \frac{\partial v'}{\partial x} \]

Streak

Nonlinear processes

Streamwise wavy structures

Rapid pressure is the mediator of the lift-up effect

Streak instability or transient growth

\[ \frac{du_s}{dz} \]
The mechanisms of slow pressure generation

\[ \nabla^2 p_{\text{slow}} = -\frac{\partial u'_i}{\partial x_j} \frac{\partial u'_j}{\partial x_i} \]

The leading nonlinear vortex dynamics is found with

\[ \frac{\partial \omega'_x}{\partial t} \sim -v' \frac{\partial \omega'_x}{\partial y} + \omega'_x \frac{\partial u'}{\partial x} \]

Nonlinear wall-normal advection

Vortex stretching by streamwise wavy streak

Skin friction at high Re turbulence is dominated by log-layer attached eddies

de Giovanetti, Hwang & Choi, 2016, J. Fluid Mech. 808 p51

Matteo de Giovanetti (Imperial)
Haecheon Choi (SNU, South Korea)
How do they contribute to turbulent skin friction at high Re?
A fundamental issue of skin-friction control

Performance of flow control for near-wall structures decays with Reynolds number.
Assessment I – FIK identity based approach

FIK (Fukagata-Iwamoto-Kasagi) identity

\[
C_f (\zeta_{z,t}) = \frac{122}{Re_{nm}} + 12 \int_0^h 22 \left( 1 - \frac{\lambda}{h} \right) \left( \frac{u'w'(y;\lambda)}{4U_m^2} \right) dy
\]

\[-k_z E_{uv} (\lambda_z; y)\]

\[
\int_0^\infty E_{uv} (k) \left( \frac{k}{\lambda_{z,t}} \right) dk_z
\]

Hoyas & Jimenez (2006, PoF)
DNS (channel) at Re_τ = 2003

Fukagata et al. (2002, Phys. Fluids)
Assessment II – confined spanwise domain

Restrict the computational box + removal of 2D uniform motion

\[ L_z = \lambda_{z,t} \]

\[ -k_z E_{uv}(\lambda_z ; y) \]

Hoyas & Jimenez (2006, PoF) DNS (channel) at \( \text{Re}_t = 2003 \)

Assessment III – artificial damping of large structures

Damp out the motions at $\lambda_z > \lambda_{z,t}$

$$\frac{\partial u}{\partial t} + (u \cdot \nabla)u = -\nabla p + \frac{1}{\text{Re}} \nabla^2 u + f$$

where $\hat{f} = \mu(\lambda_z)\hat{u}$

$$- k_z E_{uv}(\lambda_z; y)$$

$\lambda_z / h$

$\mu(\lambda_z) = \begin{cases} 
0 & \text{for } \lambda_z \leq \lambda_{z,t} \\
\mu_0 & \text{for } \lambda_z > \lambda_{z,t}
\end{cases}$

Hoyas & Jimenez (2006, PoF)

DNS (channel) at $\text{Re}_\tau = 2003$
Log-layer attached eddies dominate skin-friction generation

Three different assessments at $Re_\tau \approx 2000$

\[ \lambda_+^z = 100 \quad \lambda_z = 1.5h \]
Scale interaction is not trivial and important

$-k_z E_{uv}(\lambda_z; y)$

**Shaded** (full simulation)
**Solid** (box confinement)
**Dashed** (spectral damping)

Generation of new Reynolds stress in the absence of large motions

Modified actual size of inner scale

Wall-resolved LES at $Re_x = 2003$
Lift-up effect is a skin-friction generation process

\[ \frac{D \omega_y}{Dt} \sim \frac{\partial U}{\partial y} \omega_x \]

Streamwise vortices

Streak

Streak instability or transient growth

\[ \frac{du_s}{dz} \]

Streamwise wavy structures

Nonlinear processes

\[ \frac{\partial \omega_x'}{\partial t} \sim -v' \frac{\partial \omega_x'}{\partial y} + \omega_x' \frac{\partial u'}{\partial x} \]

Suppression of lift-up effect leads to drag reduction

Full LES

LES without lift-up a
\[ \lambda_z = 1.5h \]

8% of drag reduction !!!

Hwang & Bengana, 2016, *J. Fluid Mech.* **795** p708
Invariant solutions of attached eddies

Hwang, Willis & Cossu, 2016, *J. Fluid Mech.* 802 R1

Oliver Yang (Warwick)
Ashley Willis (Sheffield)
Carlo Cossu (IMFT)
SSP reflects the existence of invariant solutions

The terribly difficult puzzle to complete

\[ k_z E_{uu}(\lambda_z; y) \]

- \( \lambda_z^+ \approx 100 \)
- \( \lambda_z^+ \approx 1000 \)
- \( \lambda_z^+ \approx 10000 \)

- \( y^+ \approx 10 \)
- \( y^+ \approx 100 \)
- \( y^+ \approx 1000 \)

- \( \lambda_z / h \)
- \( 10^0 \)
- \( 10^1 \)
- \( 10^2 \)
- \( 10^3 \)

- \( Re_\tau \approx 200 \)
- \( Re_\tau \approx 1000 \)
- \( Re_\tau \approx 1800 \)

Hoyas & Jimenez (2006, PoF)

Flow

Wallefe (2001)

Hwang, Willis & Cossu (2016)

Jimenez & Simens (2001)

Jimenez et al. (2005)

Yang, Willis & Hwang (2017, ongoing)
Traveling wave solutions of large-scale structures at $\text{Re}_\tau \approx 1000$

Bifurcation with $C_s$ ($\lambda_z = 1.5h$)

Hwang, Willis & Cossu, 2016, *J. Fluid Mech.* 802 R1
The UB solution conceptually represents large-scale structures

\[ E_{\text{streak}} = \frac{1}{2V} \int_V (u')^2 \, dV \]
\[ E_{\text{vor}} = \frac{1}{2V} \int_V (v')^2 + (w')^2 \, dV \]
A glimpse of log-layer traveling wave solution at $\text{Re}_\tau \approx 1800$

Any connection with the one in Gibson & Brand (2014)?

Yang, Willis & Hwang, 2017, Ongoing work
Conclusions

Townsend’s attached eddies do exist and sustain with SSP.

Pressure fluctuations (both rapid and slow) are generated by nonlinear feeding processes of vortices.

Skin friction at high Re is dominated by log-layer eddies and the lift-up effect is an important mechanism of momentum transfer to the wall.

There exists traveling wave form of invariant solutions for attached eddies, but their computation is very challenging.