Information flow and causality of streak-roll interactions in wall-bounded turbulence

Adrián Lozano-Durán

Center for Turbulence Research, Stanford University

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Introduction and Motivation

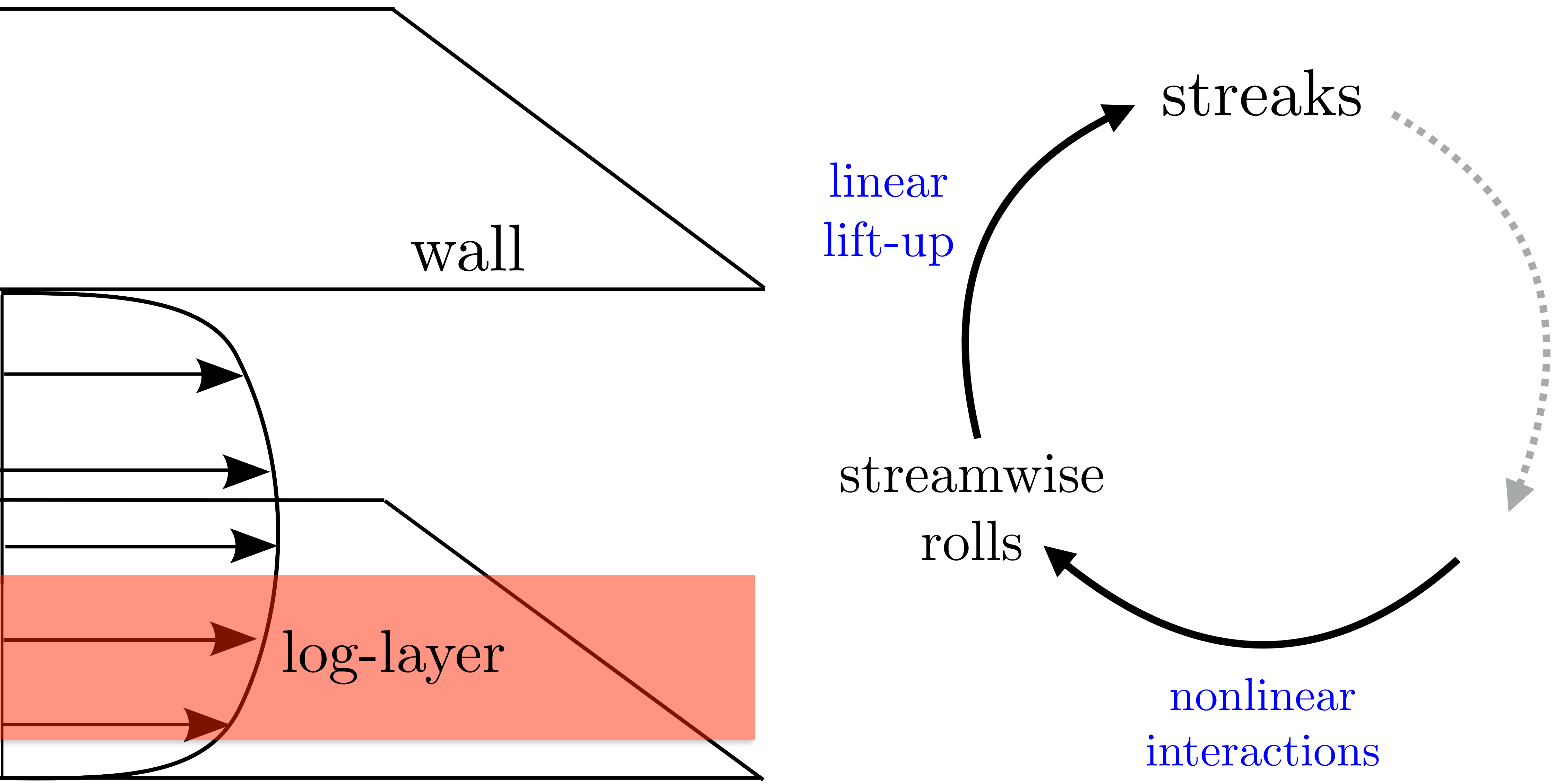
Collaborators:

- Javier Jiménez (Universidad Politécnica de Madrid)
- Gilles Tissot (Institut de Mathématiques de Toulouse)
- Laurent Cordier (Institut PPRIME, Poitiers)
- X. Sang Liang (Nanjing Institute of Meteorology)

Funded by:

- First Multiflow Summer School, Madrid 2014
- CTR Summer Program, Stanford 2016

Self-sustaining process in log-layer



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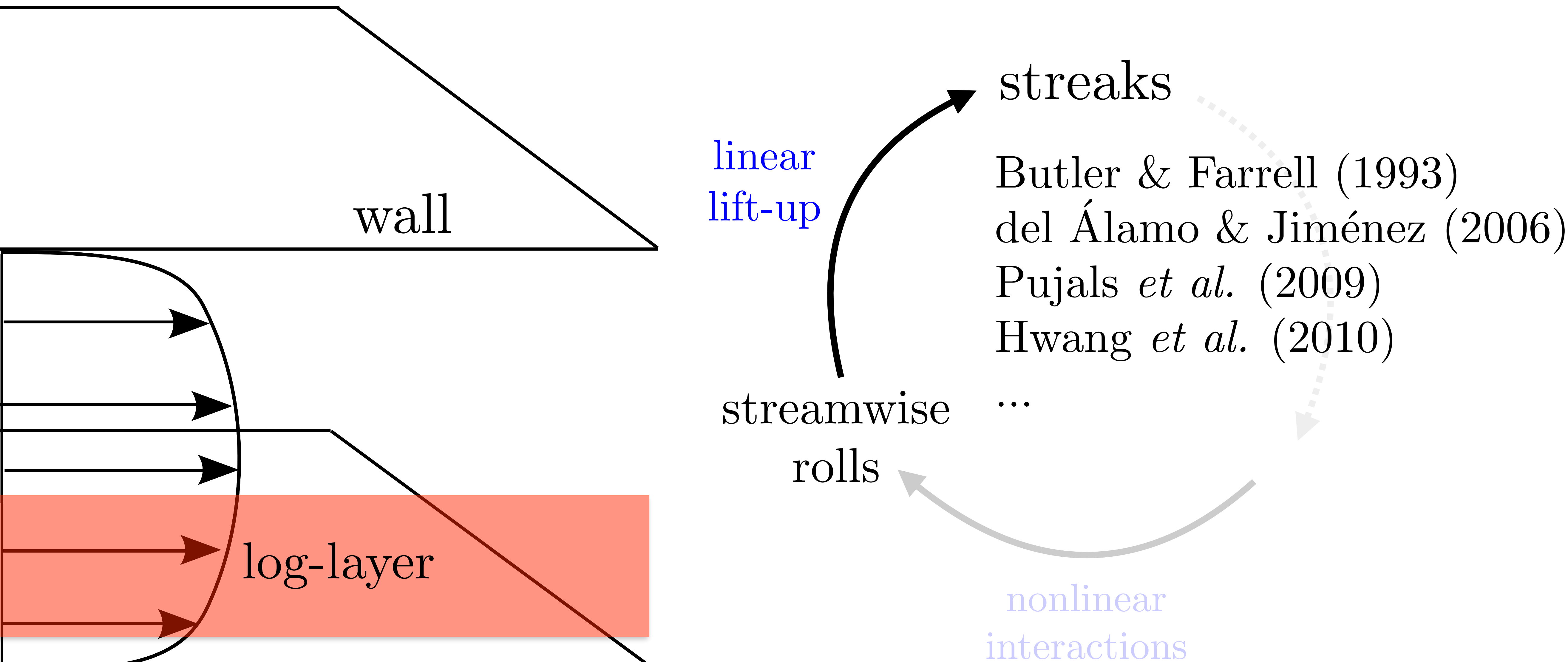

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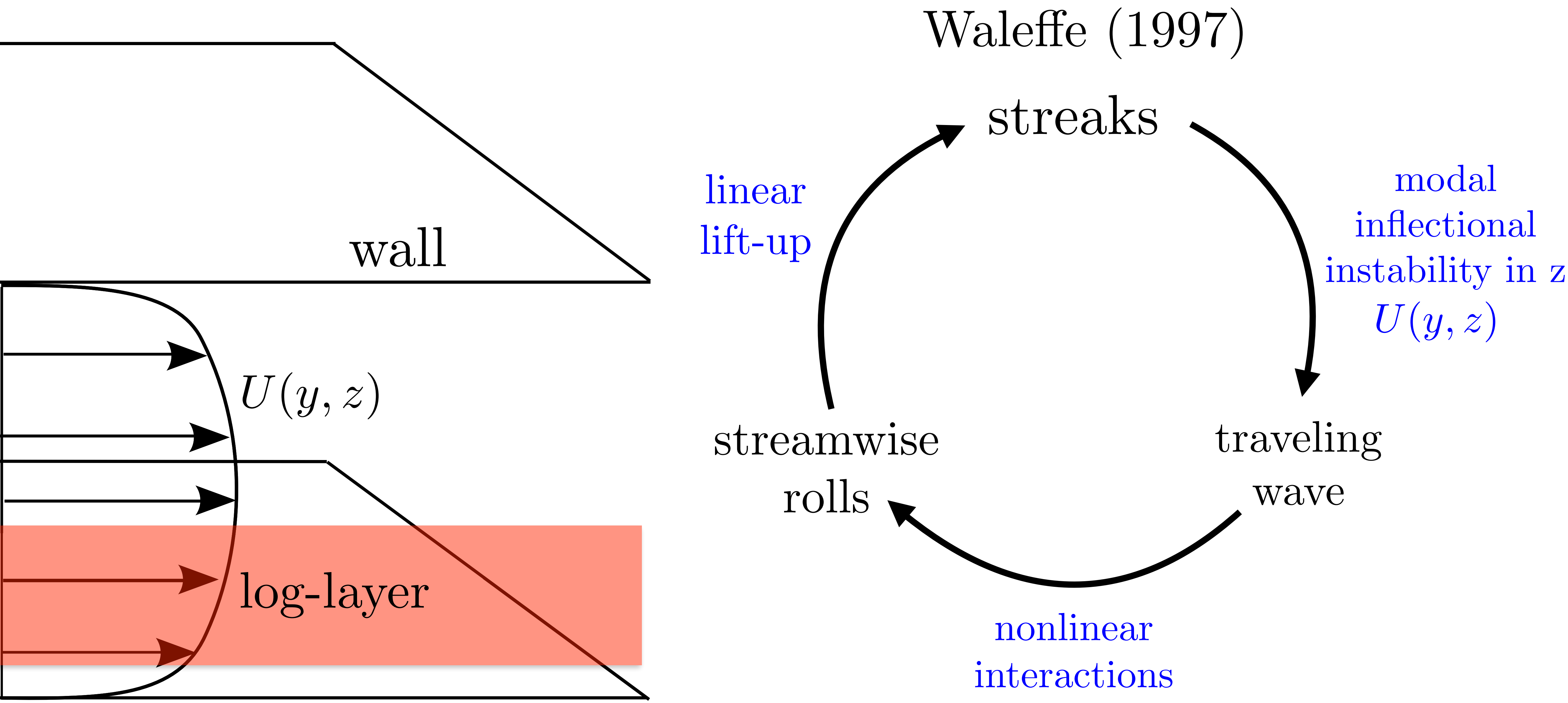

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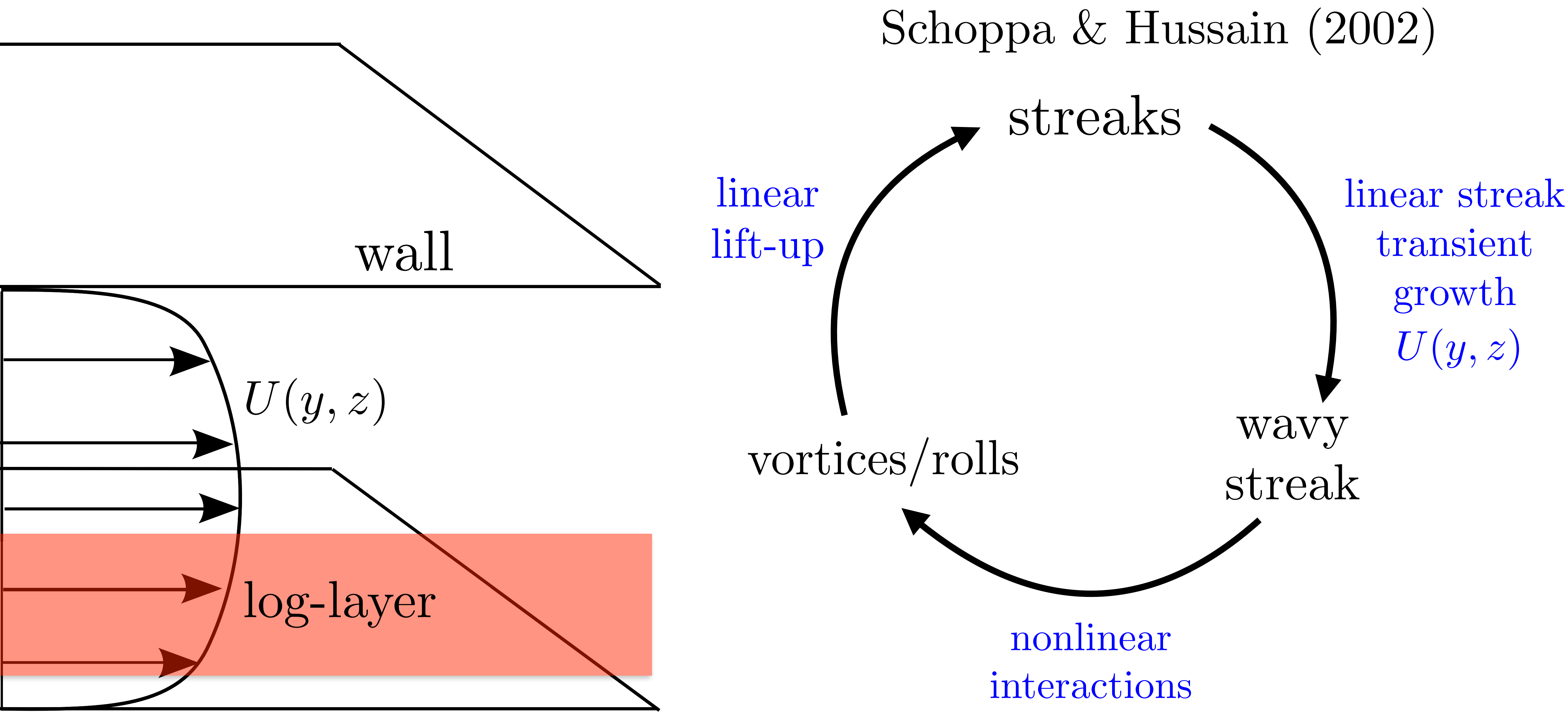

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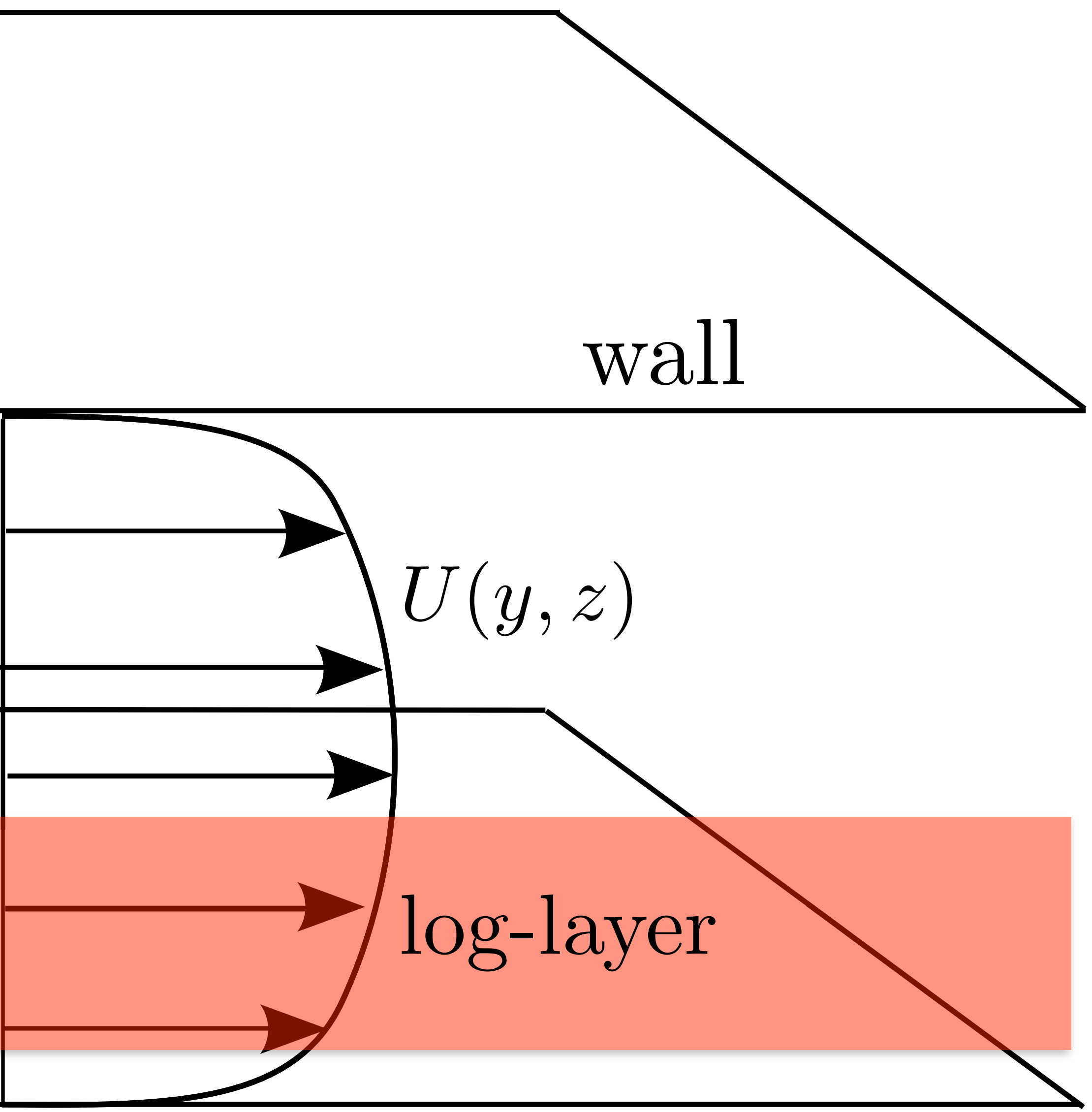
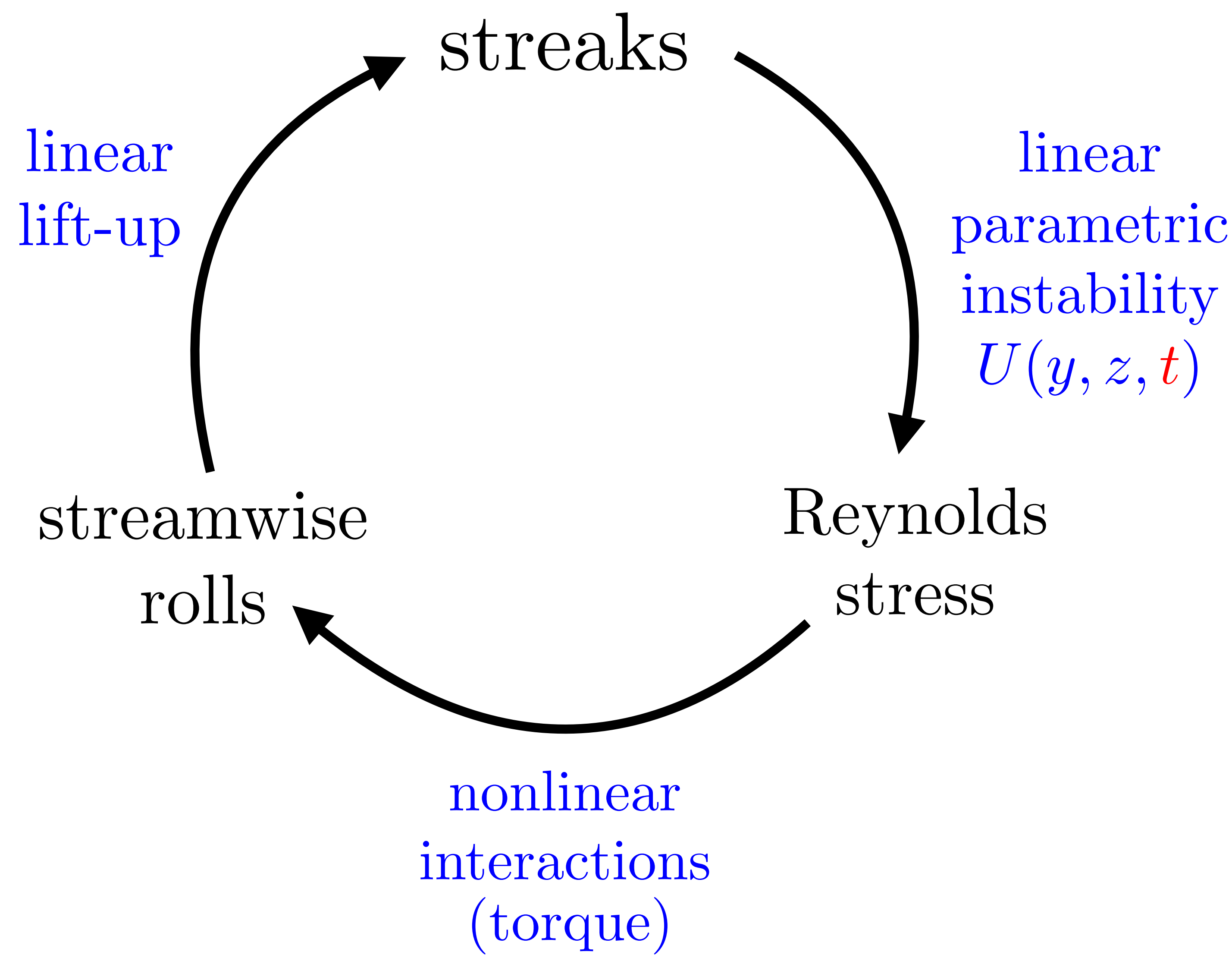


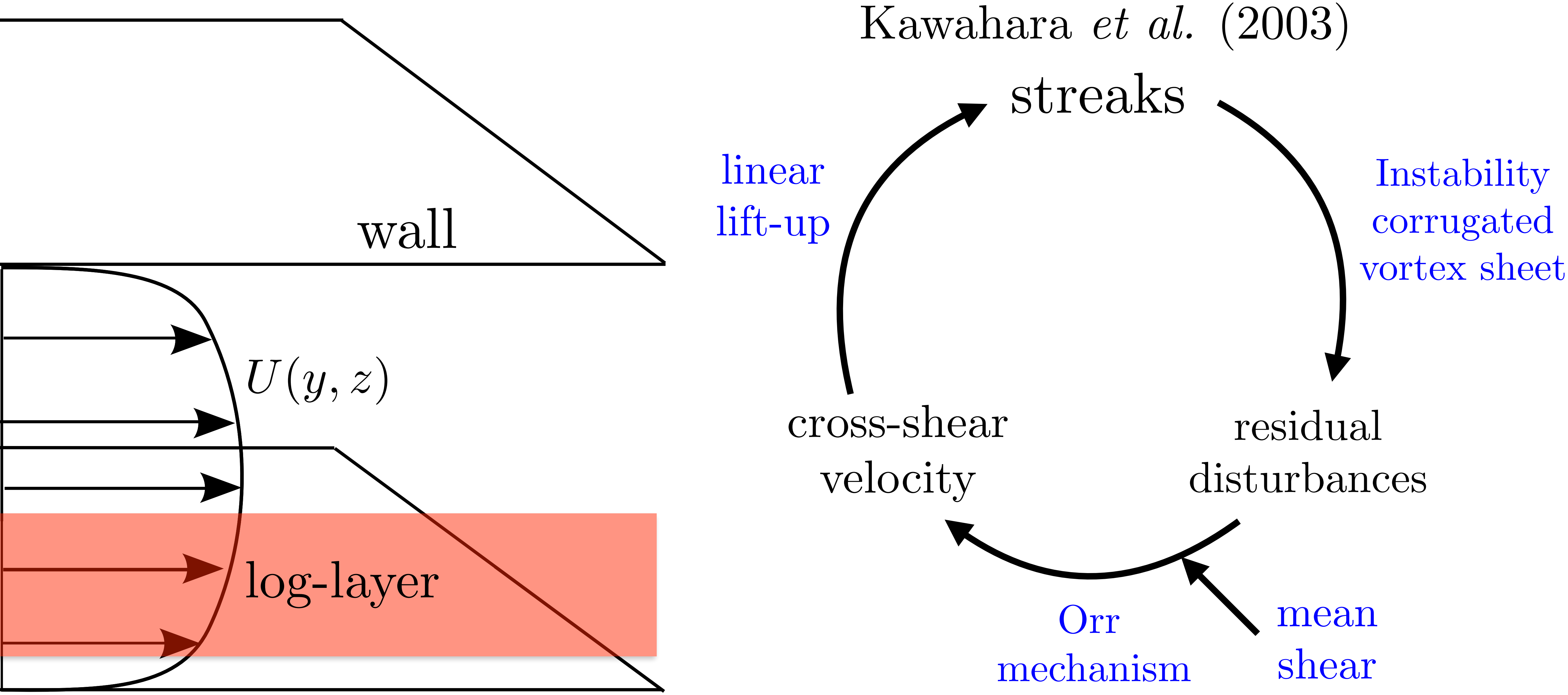
Farrell & Ioannou (2012)

Farrell *et al.* (2016)



Self-sustaining process in log-layer

Jiménez (2013, 2015)
Kawahara *et al.* (2003)



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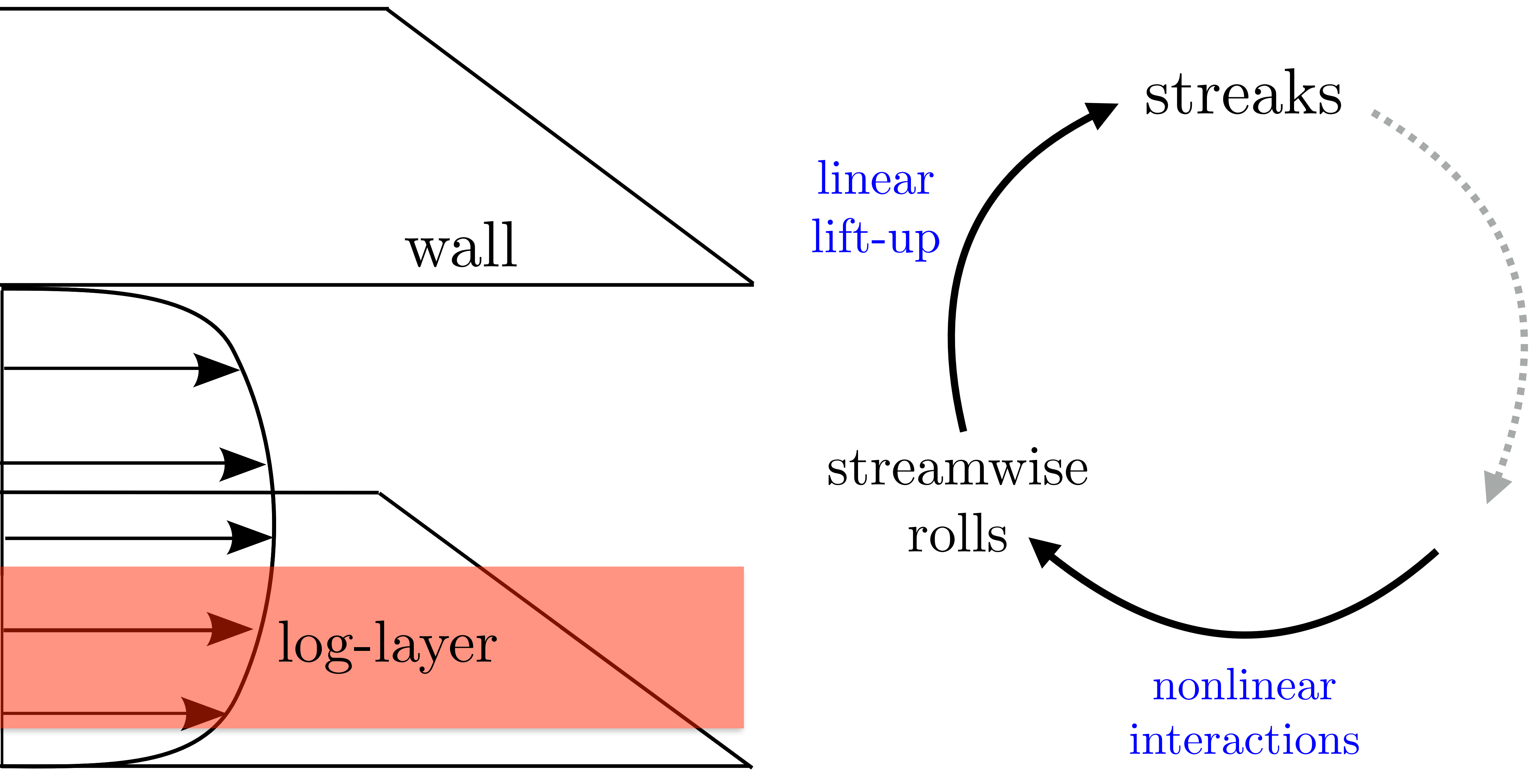

How to study self-sustaining process?

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Tools in self-sustaining turbulence:

- Exact coherent structures
- Energy budget
- Correlations
- Linear stability analysis
- Reduced-order models
- DNS data interrogation
- ...

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For a complete understanding:

cause $\rightarrow$ effect

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For a complete understanding:

cause → effect

Goal: Present a tool to quantify causality and application to interaction between streaks and rolls in the log-layer of a channel flow

Outline

- Introduction and motivation
- Causality as information transfer
- Numerical experiment
- Identification of streaks and rolls
- Causality flow between streaks and rolls
- Conclusions

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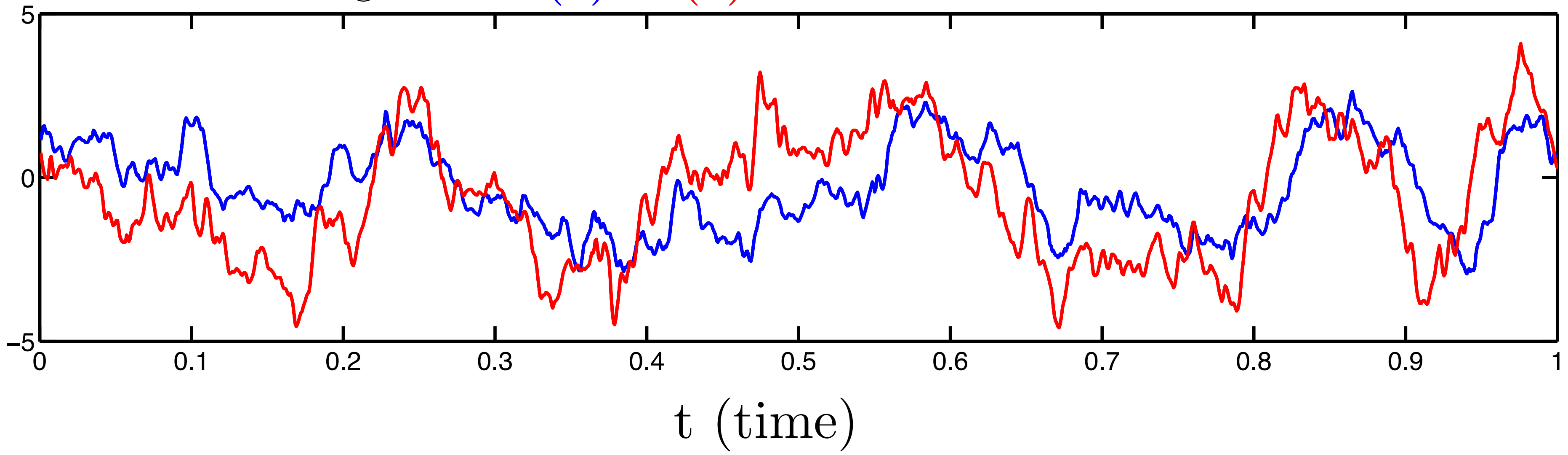
Causality as information transfer

Consider two signals: $u(t)$ $v(t)$



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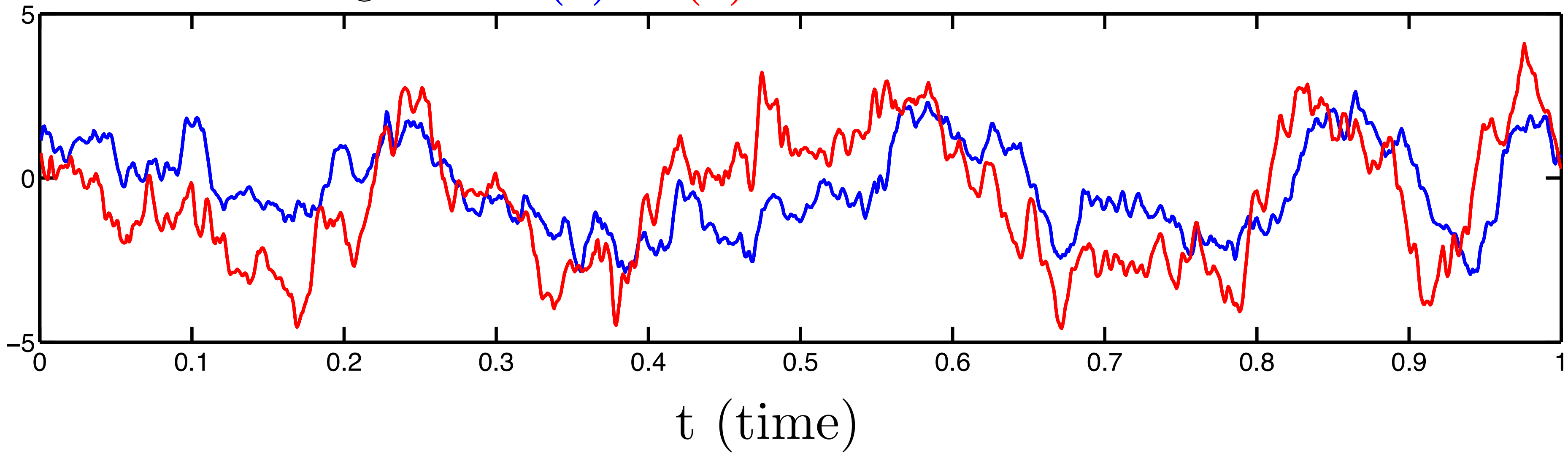

How to measure the causality from v to u?

Methods to infer causality:

- Time-correlation → → Jiménez (2013)
- Granger causality → Granger (1969) → Tissot *et al.* (2014)
- Information flow → Information Theory → Liang and Lozano-Durán (2017)

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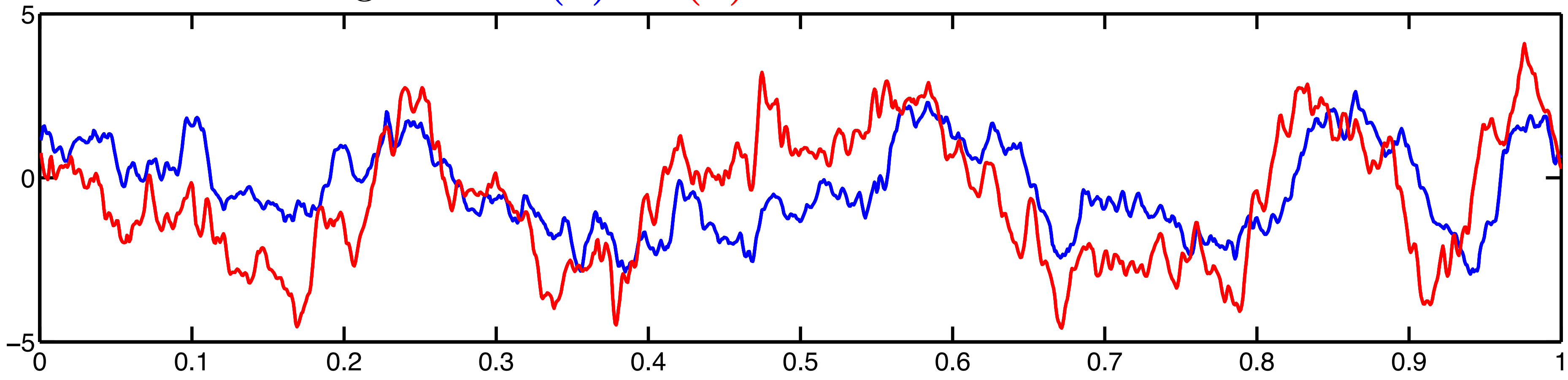

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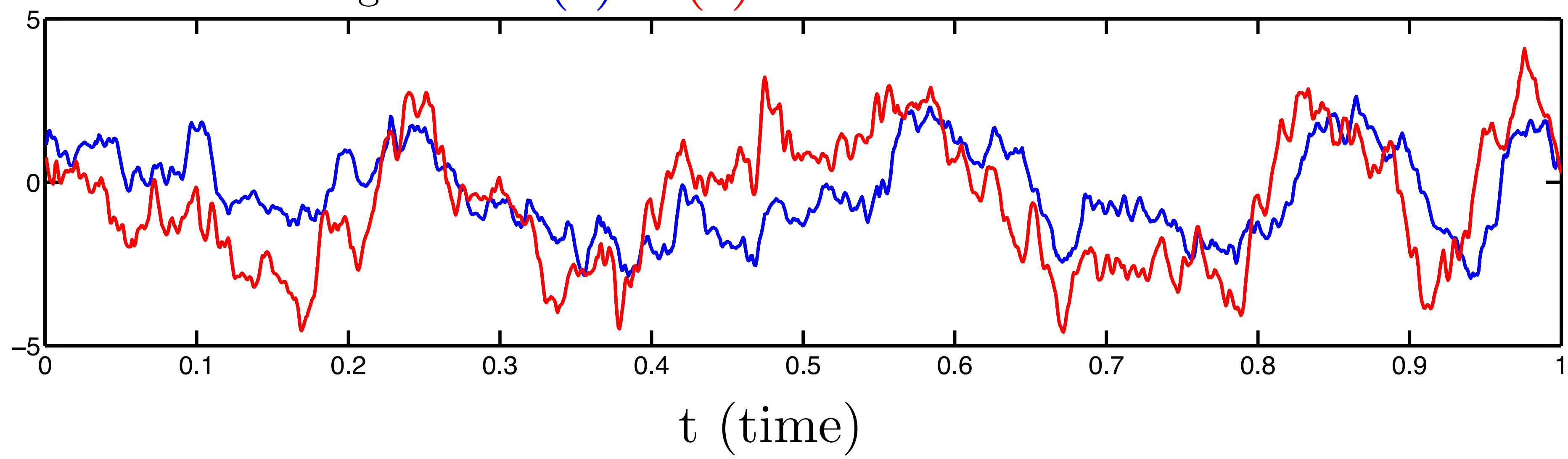

Causality from v to u := Information flow from v to u

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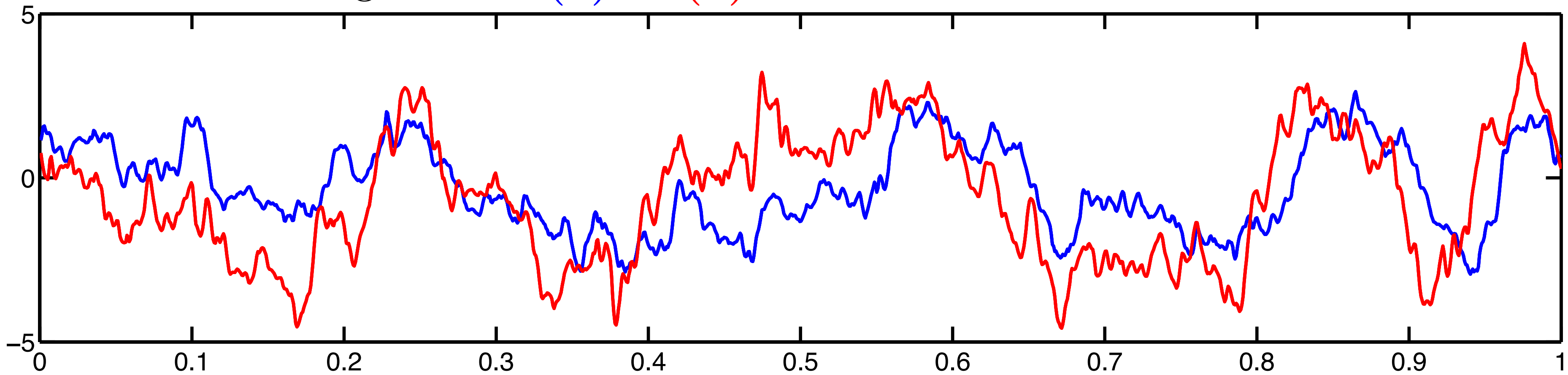

t (time)

Causality from v to u := Information flow from v to u

(Shannon, 1948)

$$I(u) = -\log(p(u))$$

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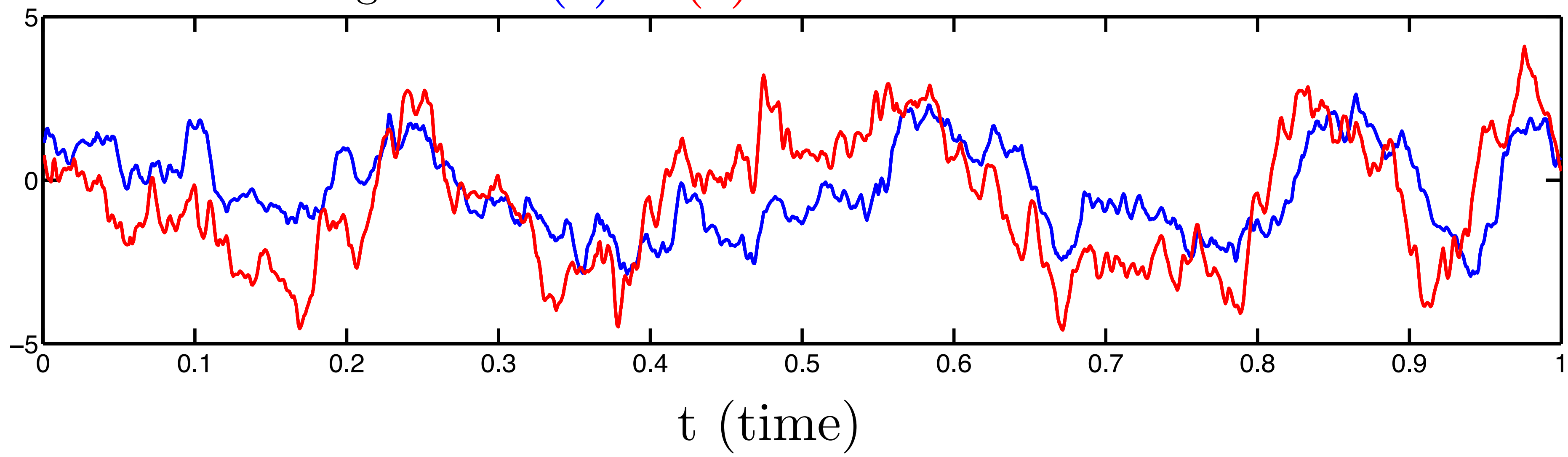

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Probability density function of u

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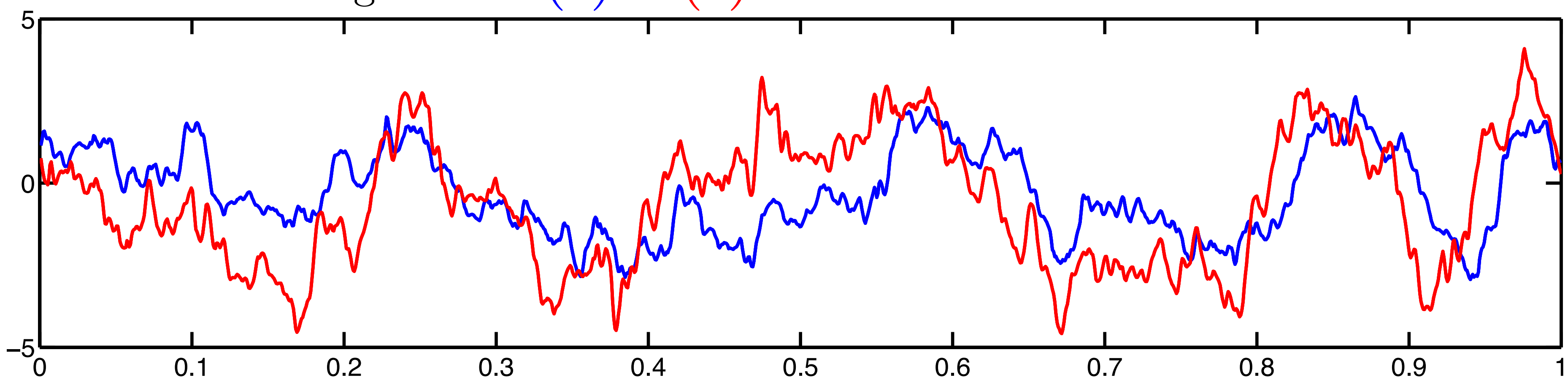

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Example: coin flipping

$$p(\text{heads}) = 0.5$$

$$p(\text{tails}) = 0.5$$

$$\Downarrow$$

$$I(\text{heads}) = -\log_2(0.5) = 1 \text{ bit}$$

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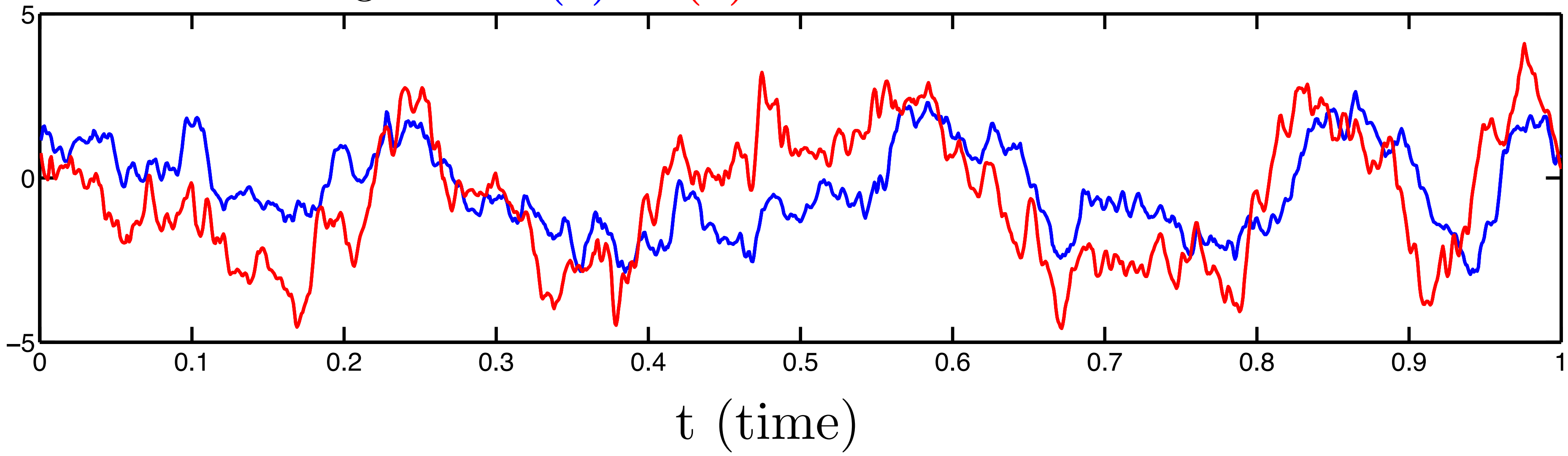

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$$I(u) = -\log(p(u))$$

$I(u)$ ↓ Information in u

$p(u)$ ↓ Probability density function of u

Example: coin flipping n times

$$p(\text{heads}) = 0.5$$

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$$\Downarrow$$

$$I(\text{head}, \text{tails}, ...) = -\log_2(0.5^n) = n \text{ bits}$$

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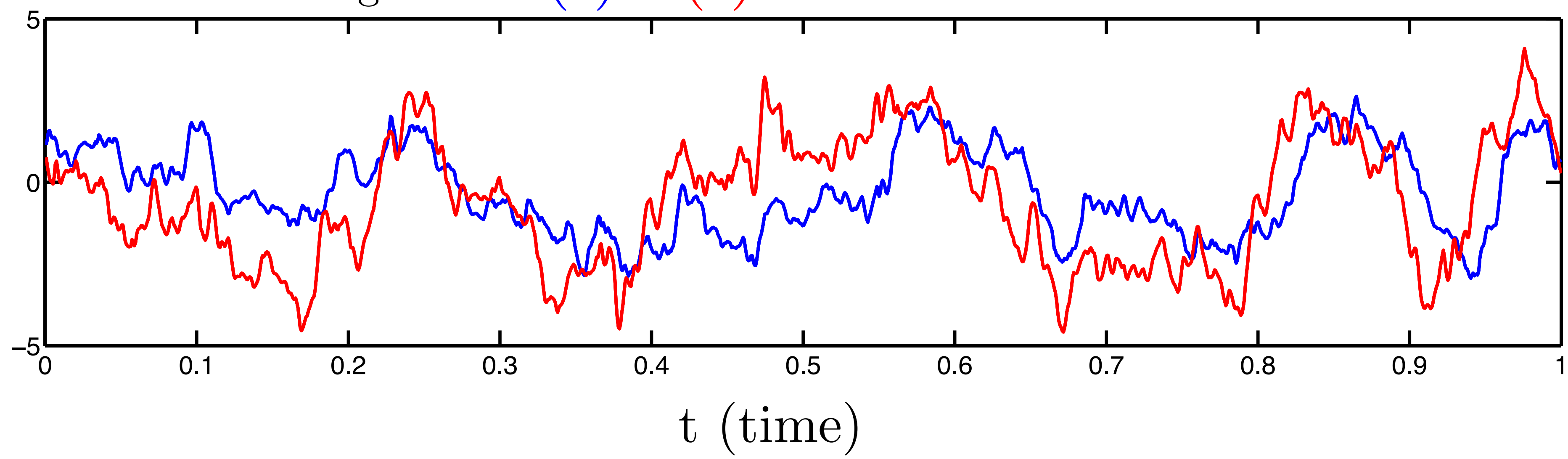

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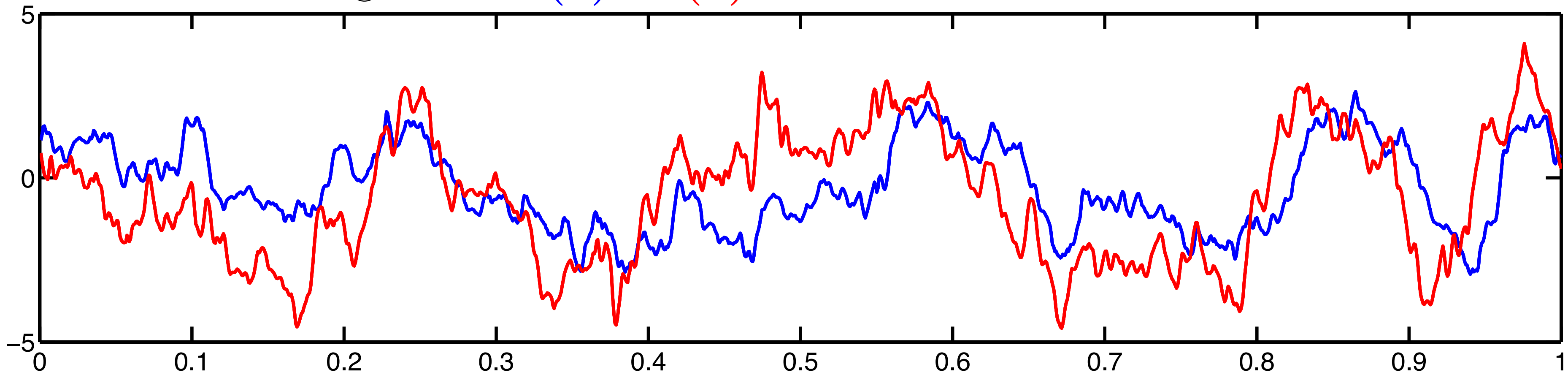

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- $I(u)$: Information in u
- $p(u)$: Probability density function of u
- $\langle I(u) \rangle$: Information entropy

Causality as information transfer

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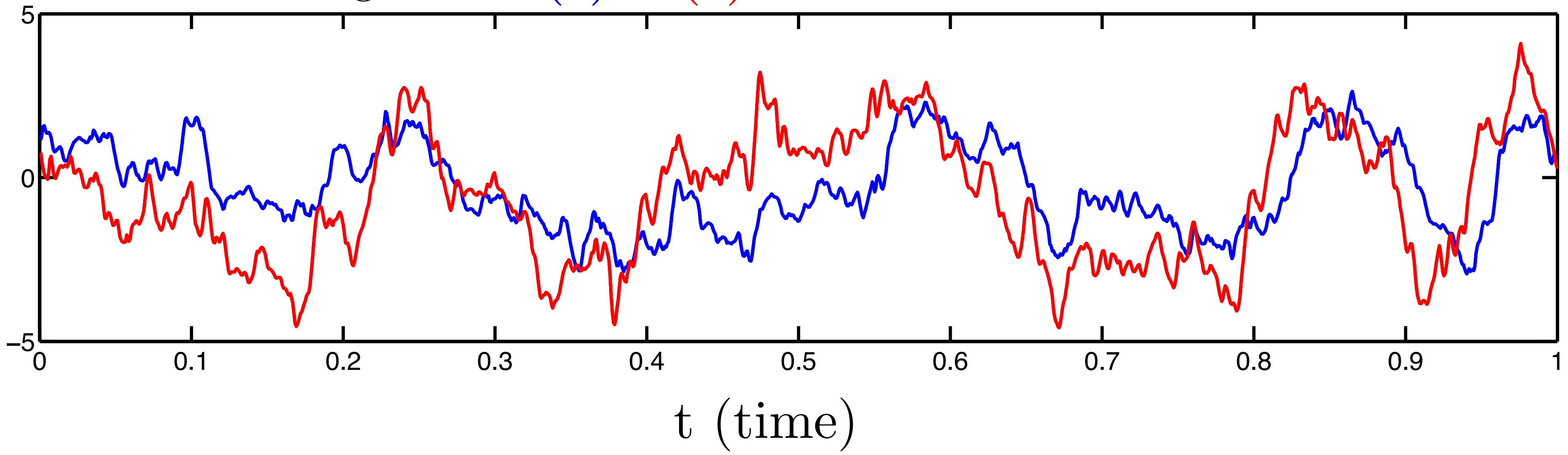

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$$\frac{\mathrm{d}\langle \mathrm{I(u)} \rangle}{\mathrm{d}t} = ?$$

Causality as information transfer

Information equation (Liang, 2016)

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} u \\ v \end{pmatrix} = \begin{bmatrix} F_u(u,v) \\ F_v(u,v) \end{bmatrix} \quad + \quad \text{Liouville equation for } P(u,v)$$

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$$\frac{\mathrm{d}\langle \mathrm{I(u)} \rangle}{\mathrm{d}t} = \int P(u,v)\frac{\partial F_u(u,v)}{\partial u}\mathrm{d}u\mathrm{d}v - \int P(v|u)\frac{\partial (p(u)F_u(u,v))}{\partial u}\mathrm{d}u\mathrm{d}v$$

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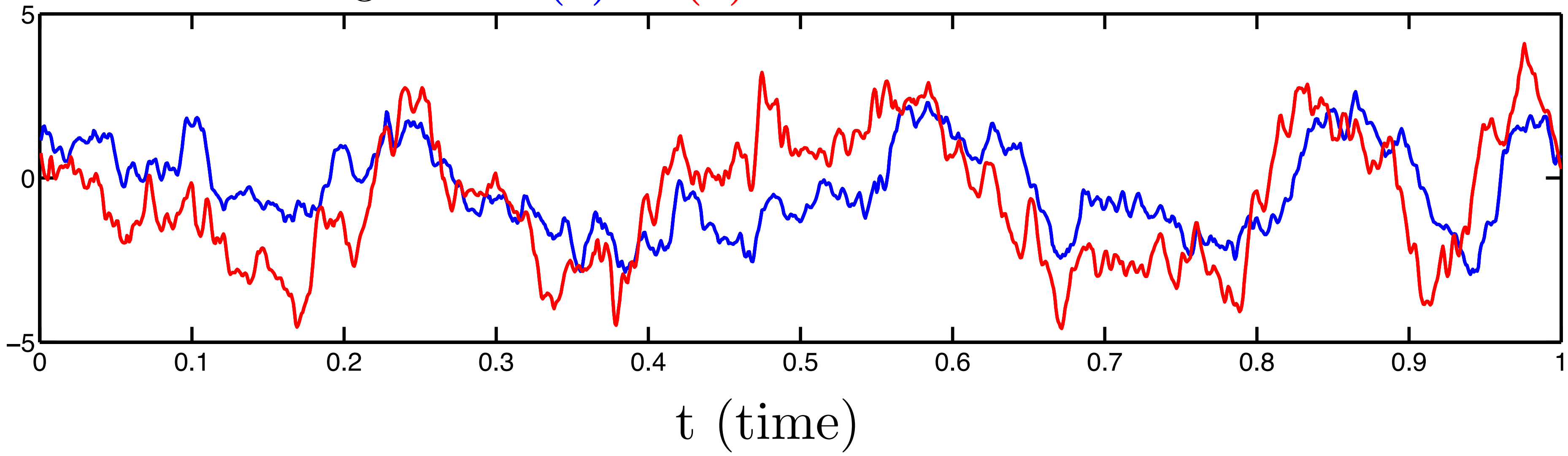

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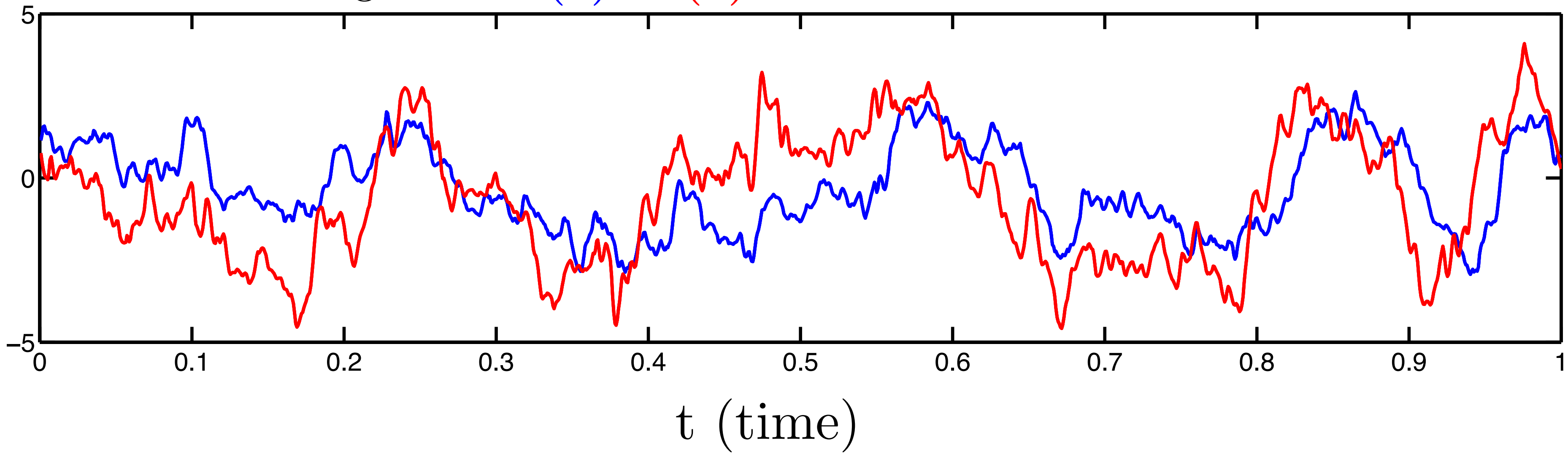

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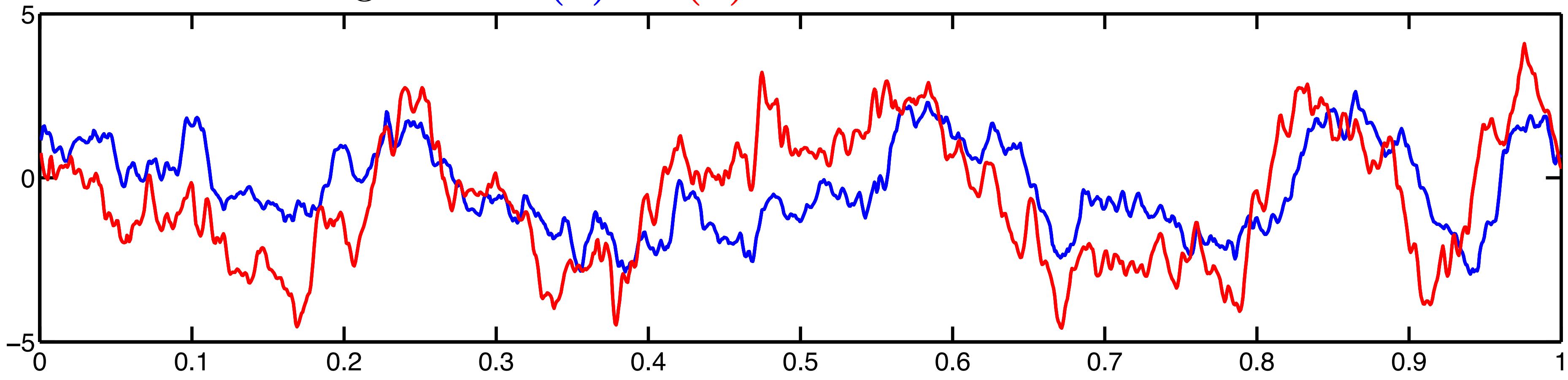

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- $\frac{\mathrm{d}\langle I(u)\rangle}{\mathrm{d}t}$ → Change of information in u
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- $\ldots$ → Flow from remaining variables

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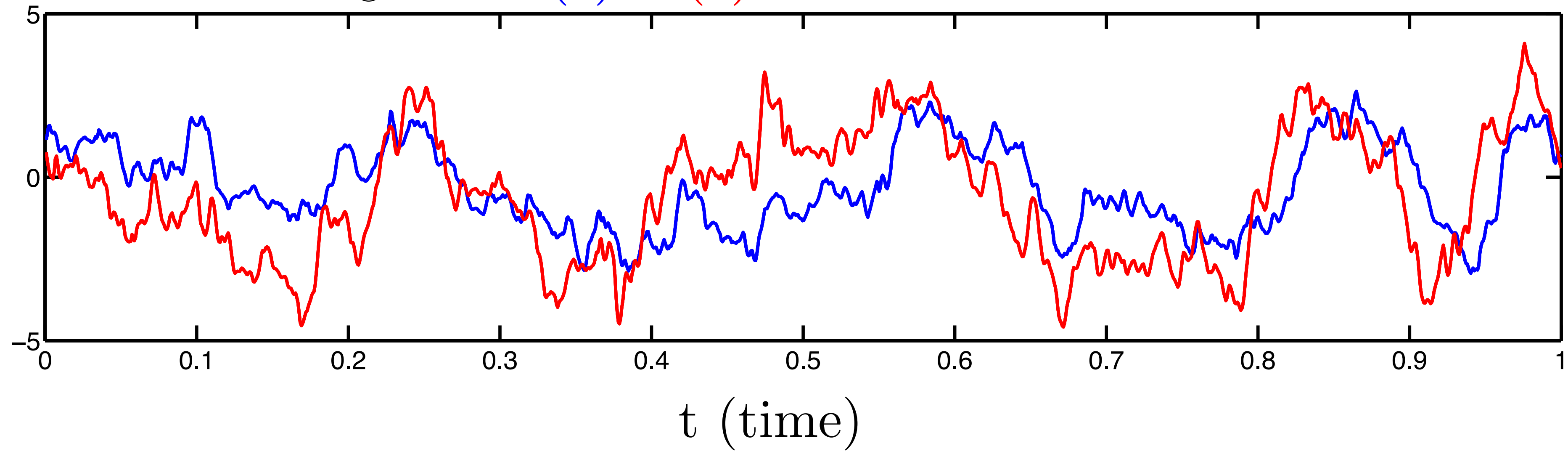

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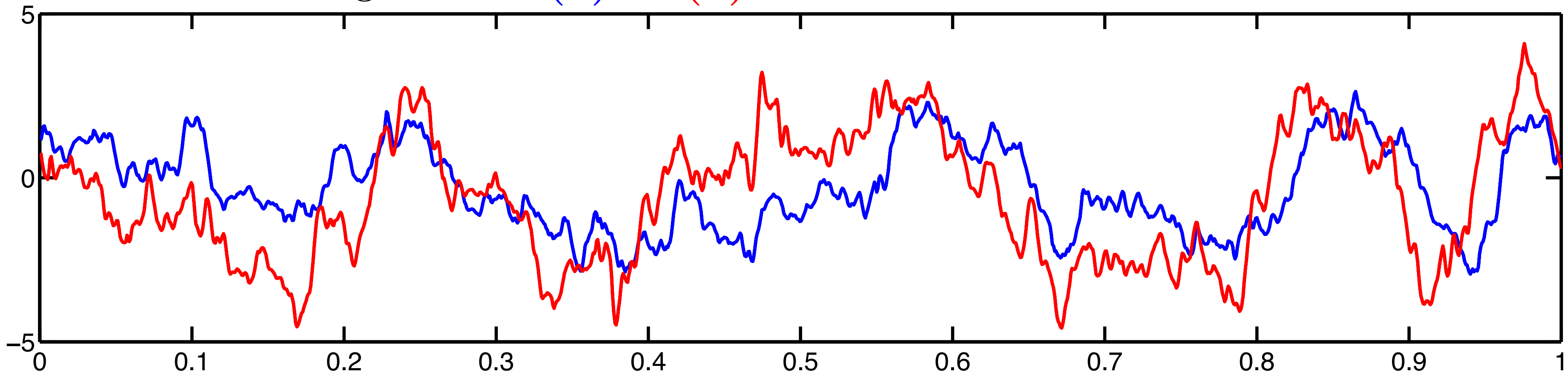


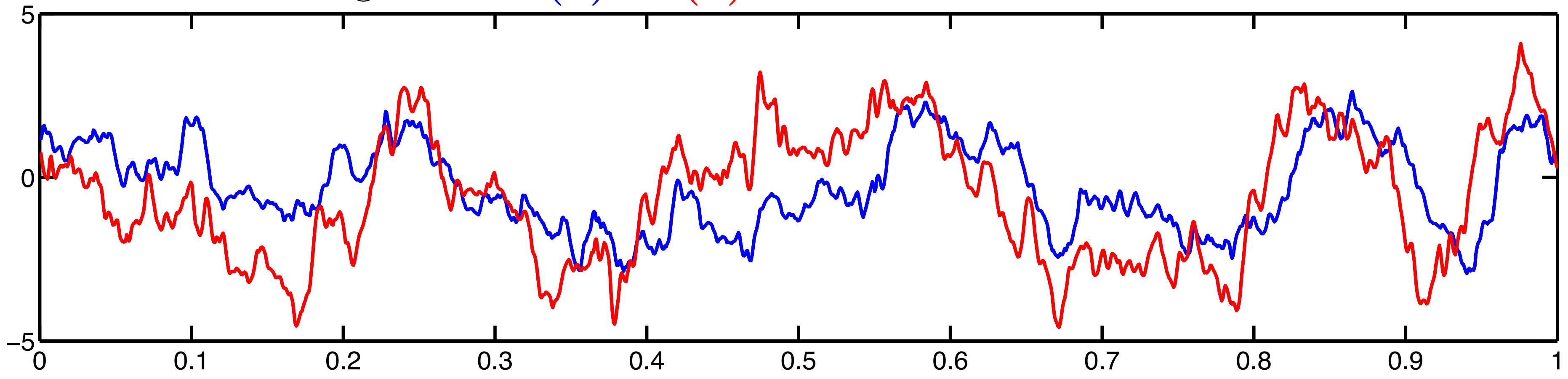
$$\text{Normalized Causality from } v \text{ to } u = \frac{|\tau_{v\to u}|}{\left|\frac{\mathrm{d}\langle I_{self}\rangle}{\mathrm{d}t}\right| + |\tau_{v\to u}| + |\ldots|} \in [0,1]$$

0 if v is fully non-causal to u
1 if v is fully causal to u

(Liang, 2016)

Causality as information transfer

Consider two signals: $u(t)$ $v(t)$



t (time)

$$\text{Normalized Causality from } v \text{ to } u = \frac{\left|\tau_{v\to u}\right|}{\left|\frac{\mathrm{d}\langle I_{self}\rangle}{\mathrm{d}t}\right| + \left|\tau_{v\to u}\right| + \left|\ldots\right|} \in [0,1]$$

$$\text{Example: } \frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} u \\ v \end{pmatrix} = \begin{bmatrix} F_u(u,\cancel{v}) \\ F_v(u,v) \end{bmatrix} \longrightarrow \tau_{v\to u} = 0$$

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Analytic expression for all terms but... extremely expensive to compute

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Estimator:

- Only $u(t)$, $v(t)$, ... are known
- Ergodicity
- Linear dynamic model
- Gaussian distribution of the variables

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$$\tau_{v\to u} = \frac{C_{uu}C_{uv}C_{v\dot{u}} - C_{uv}^2 C_{u\dot{u}}}{C_{uu}^2 C_{vv} - C_{uu}C_{uv}^2}$$

$C_{ij} \to$ correlation i and j

Liang (2014)

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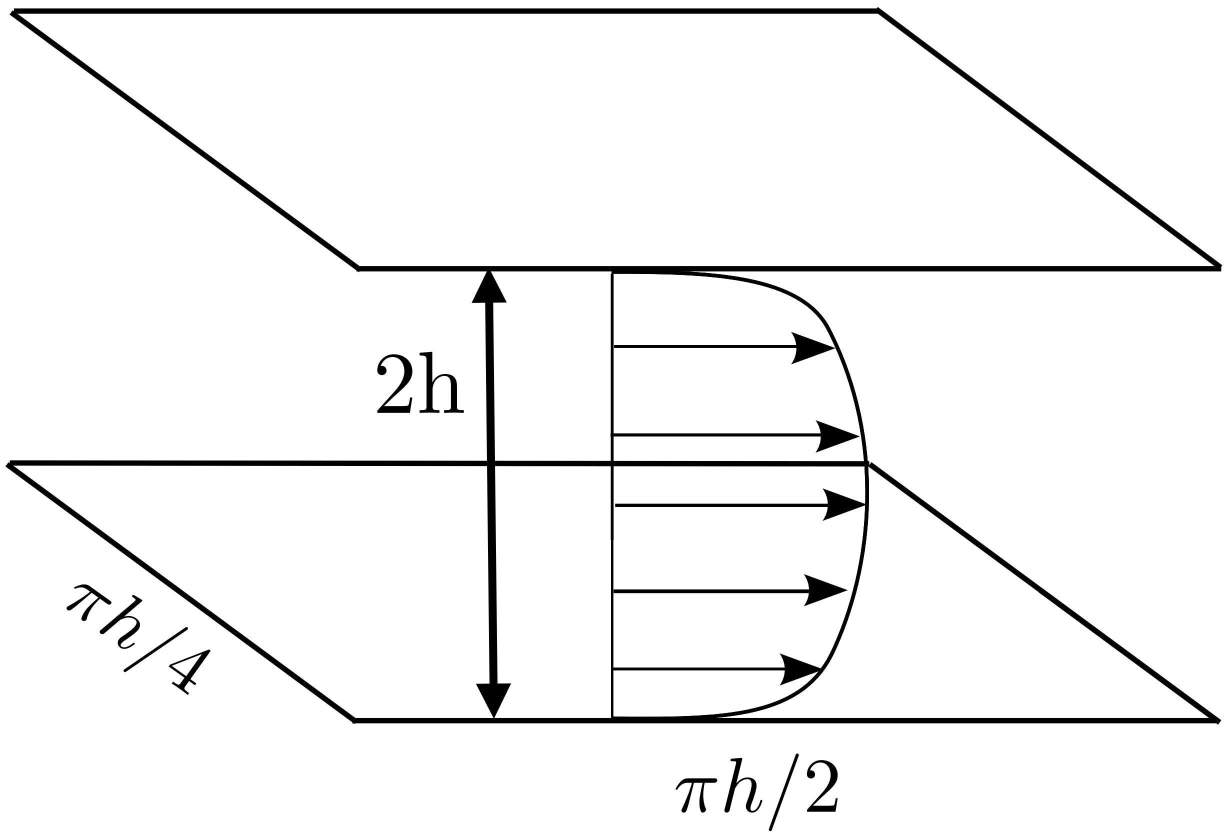
Numerical experiment

DNS channel flow $Re_\tau = 1000$



Minimal channel for the log-layer

(Flores and Jiménez, 2010)

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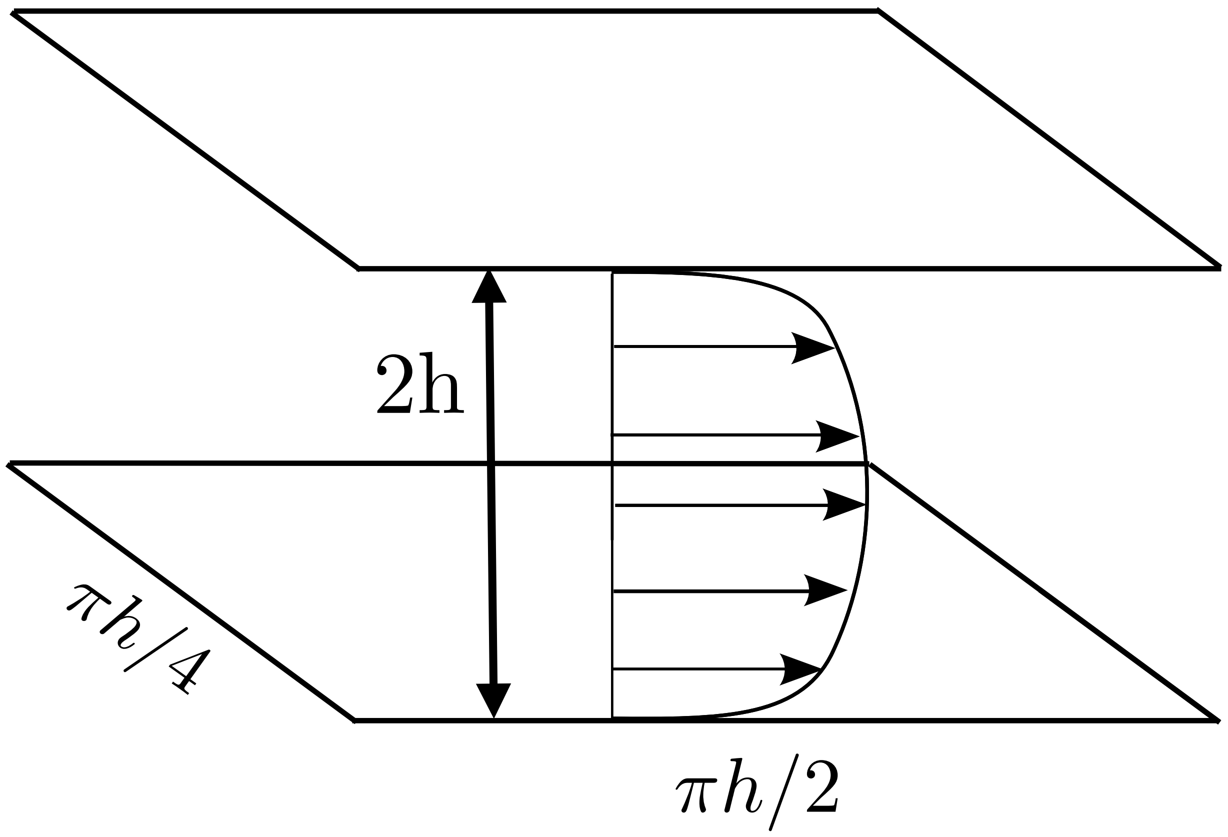

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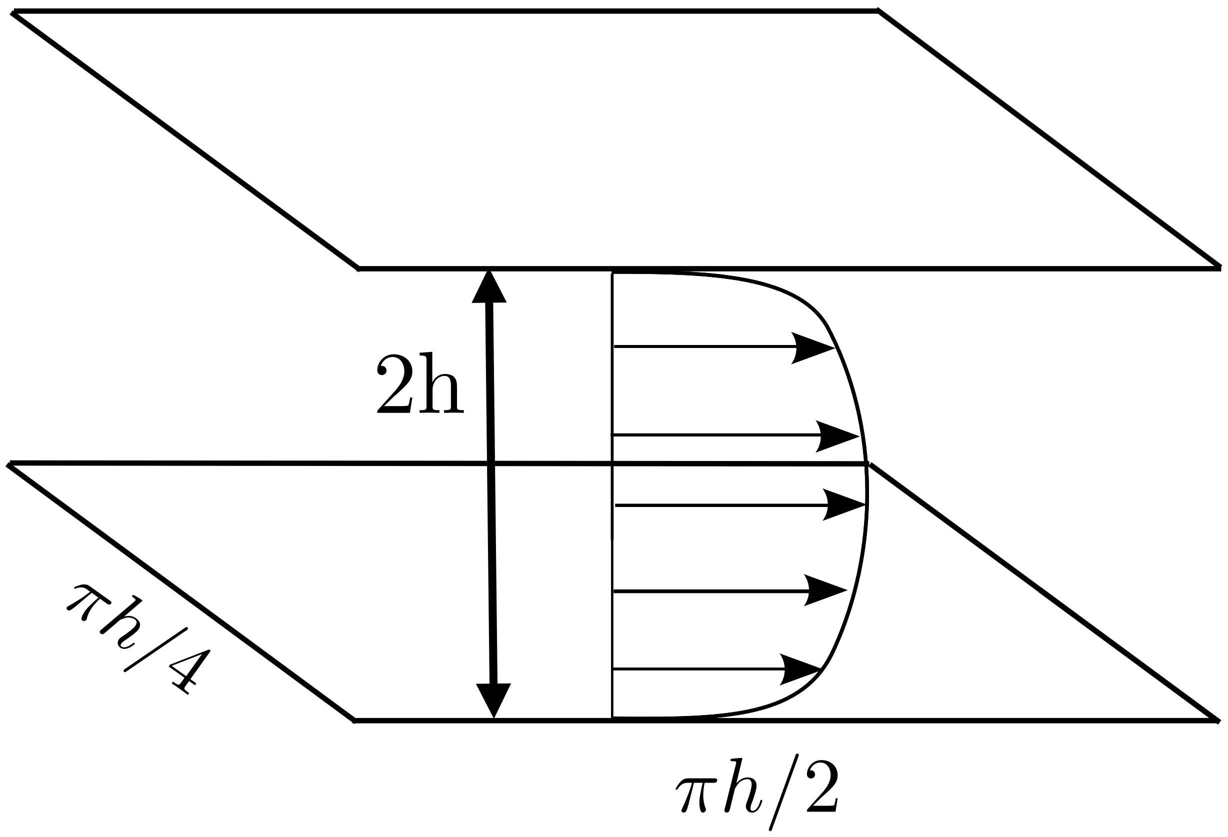

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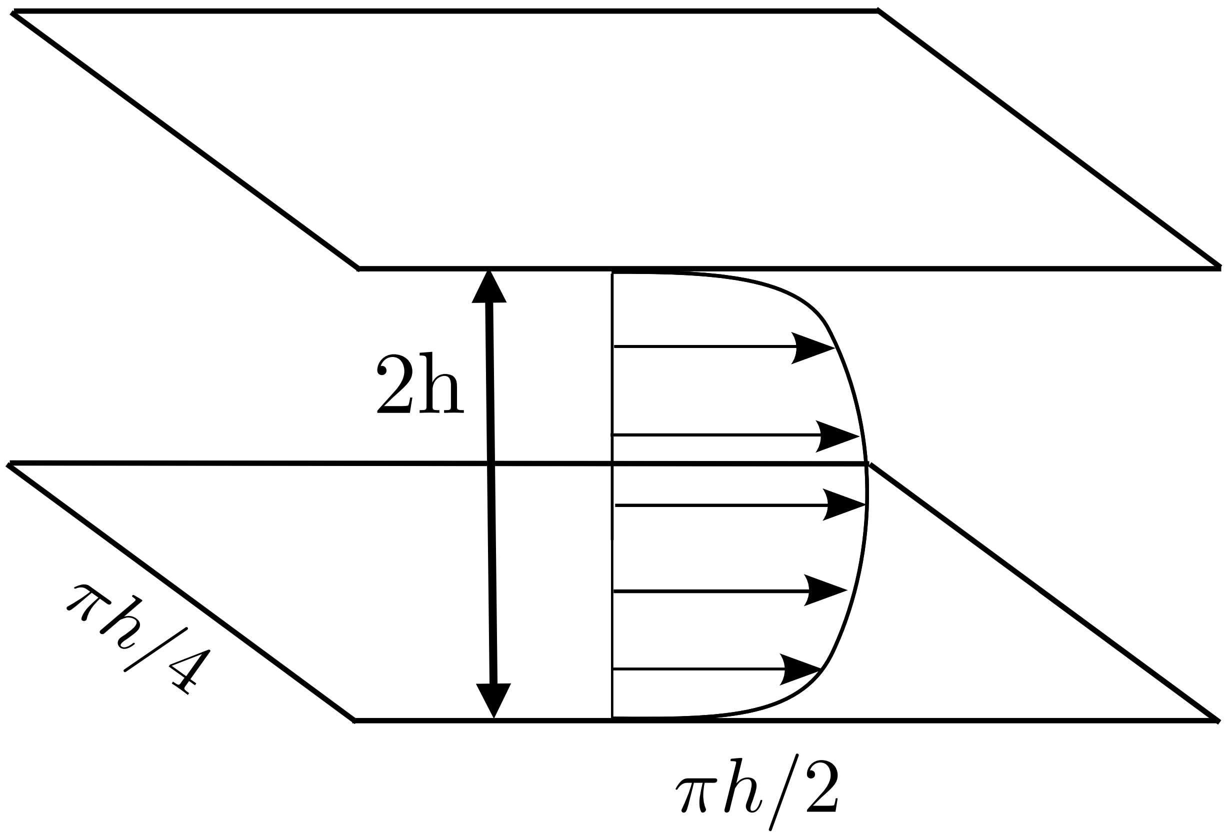

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- Total time simulated: 140 eddy turnovers
- Time between snapshots: 25 wall units

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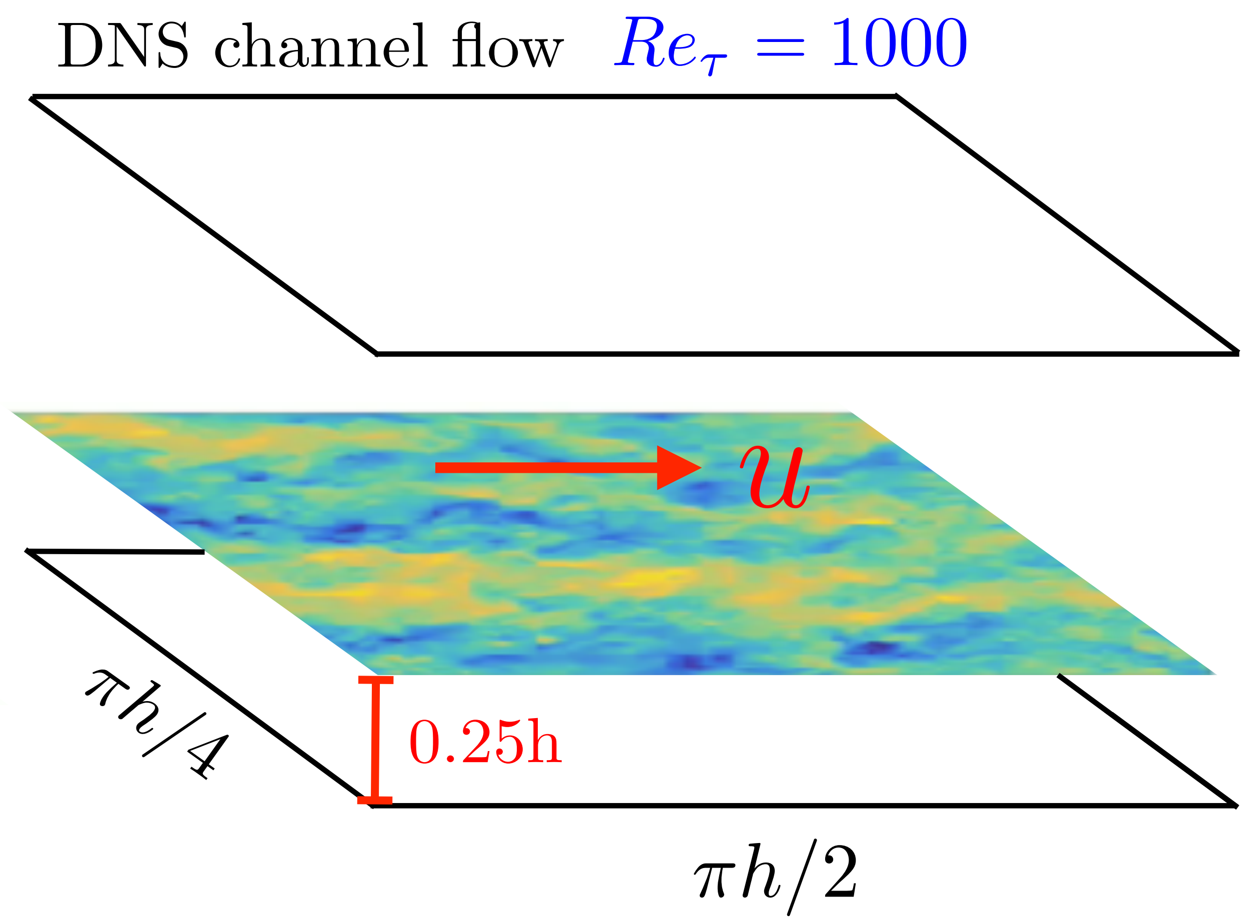

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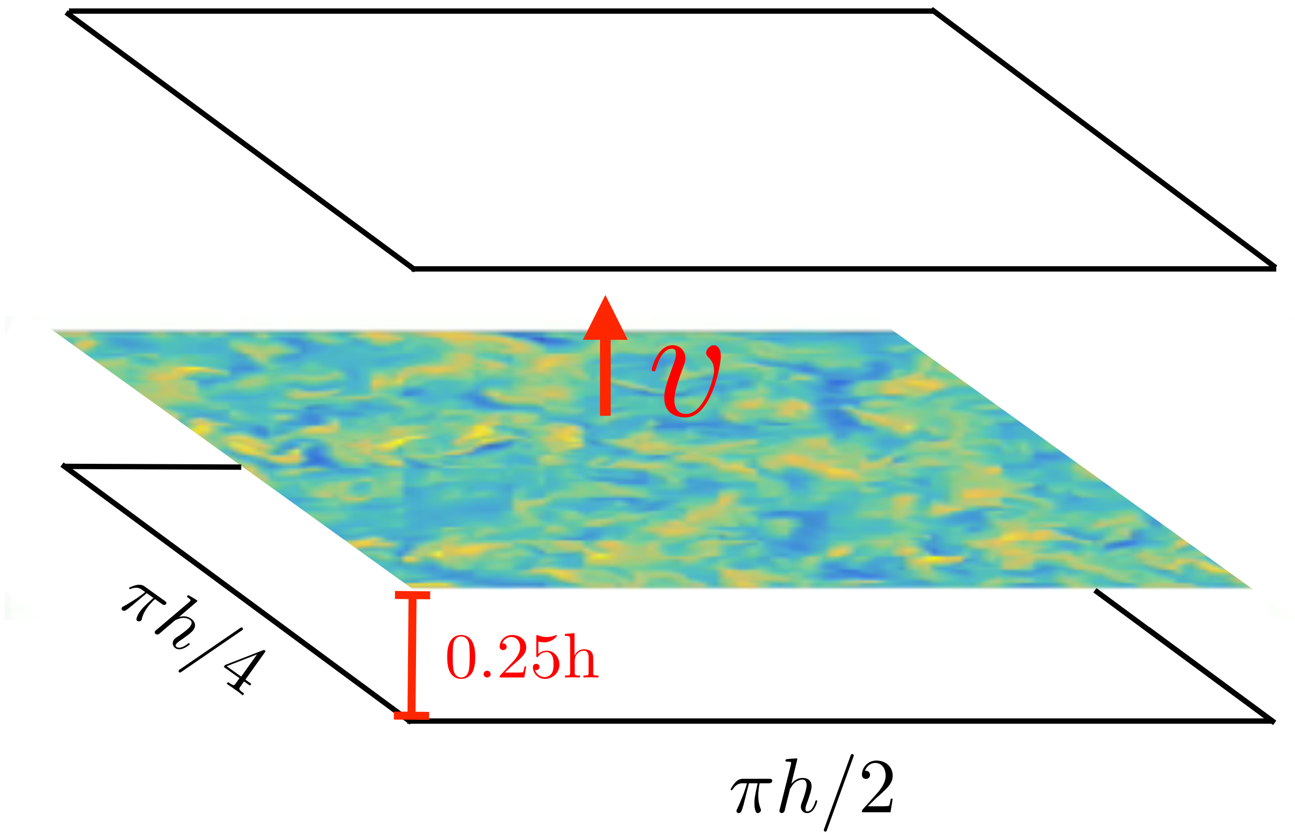

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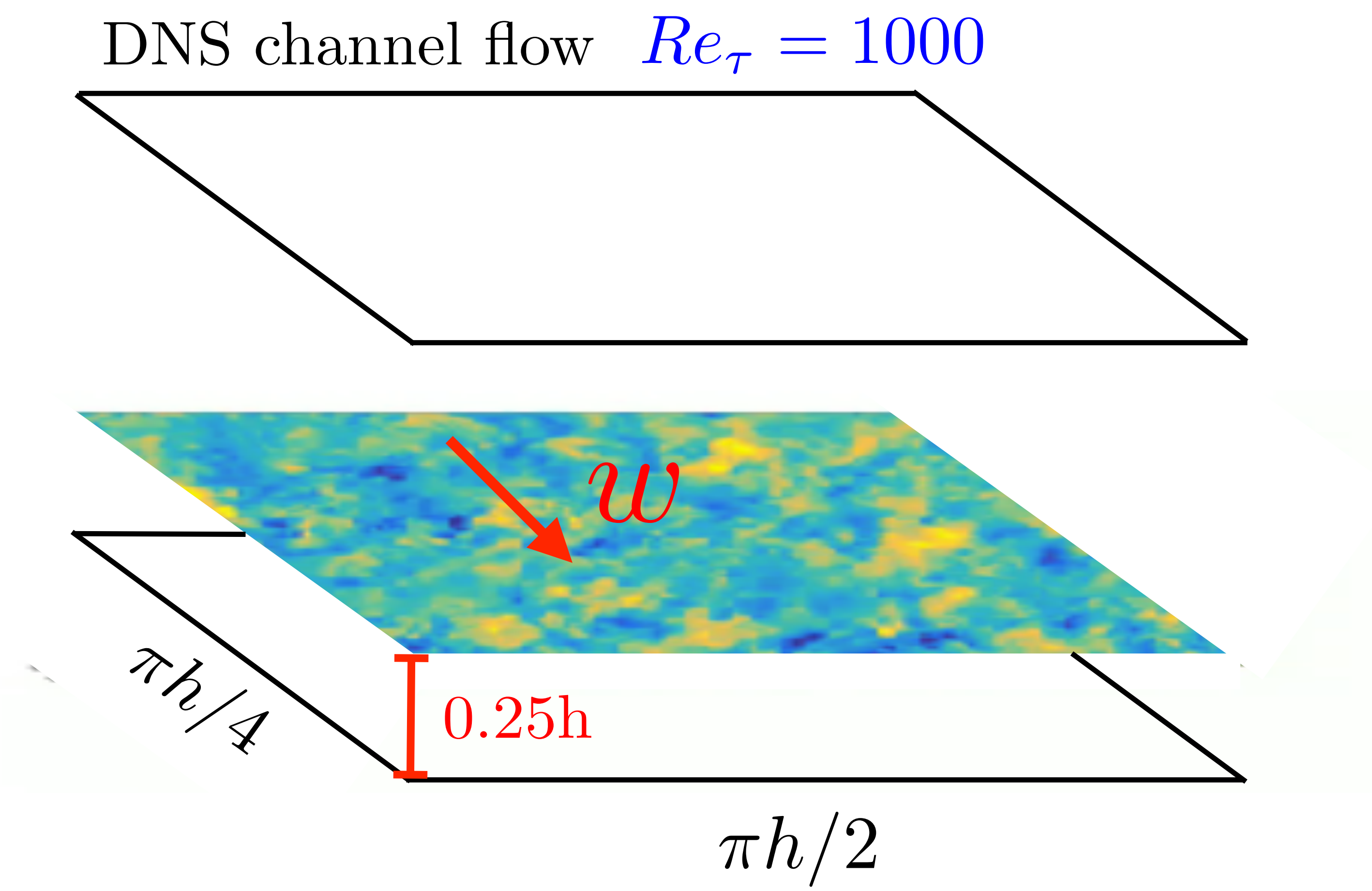

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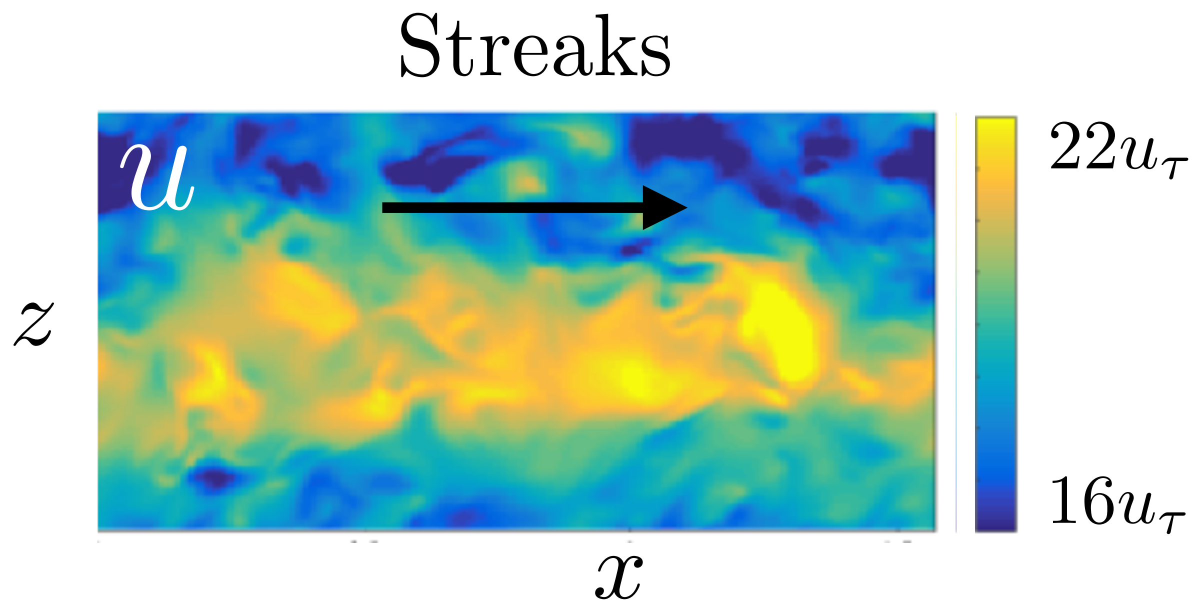
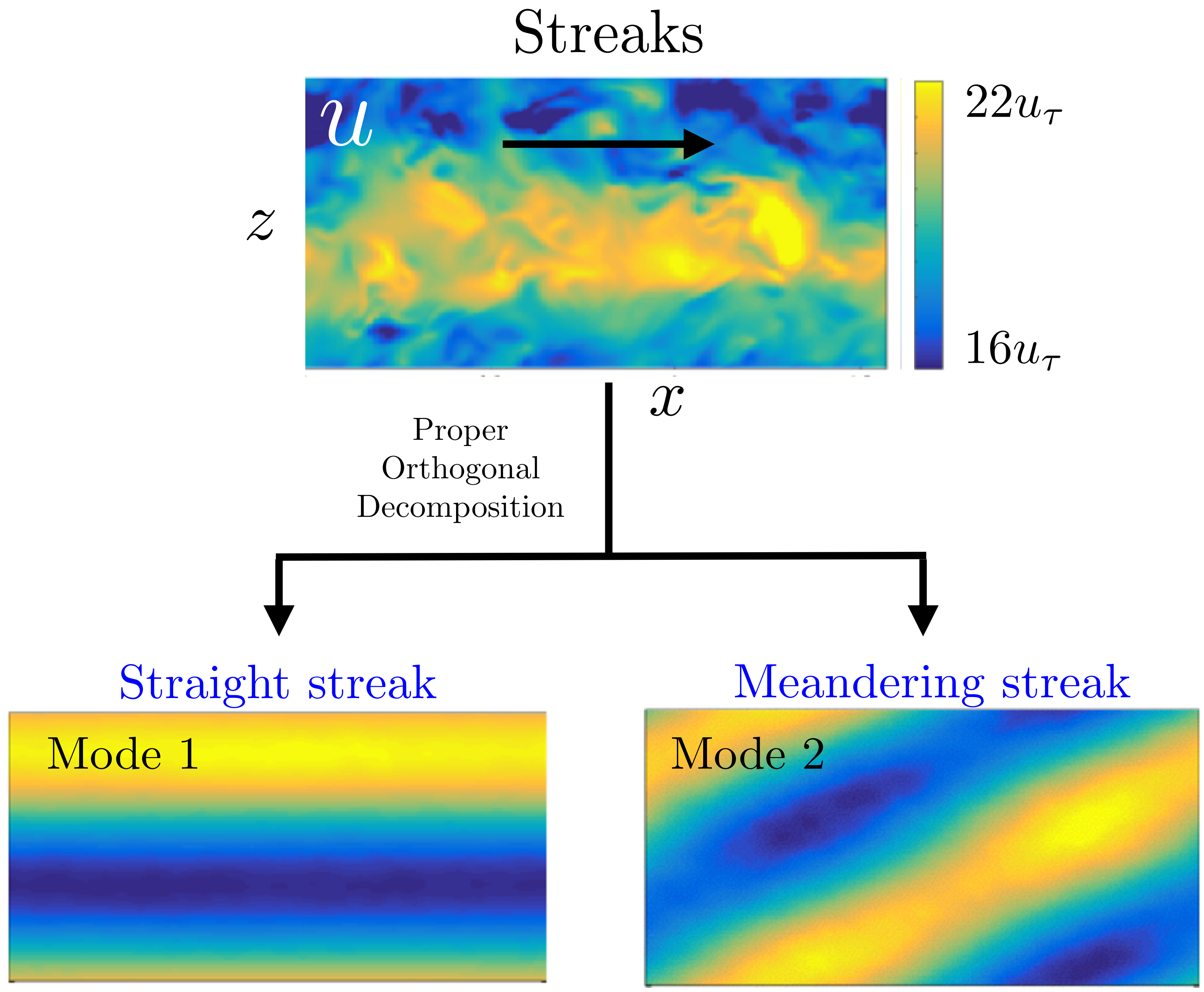
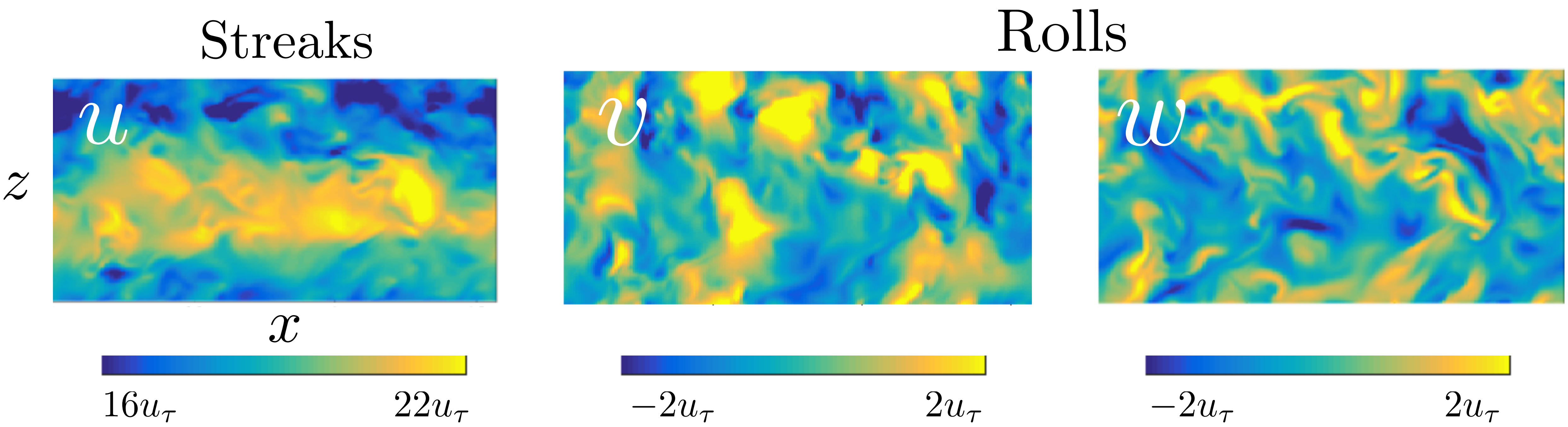
Definition of rolls and streaks in the log-layer



Definition of rolls and streaks in the log-layer



Definition of rolls and streaks in the log-layer



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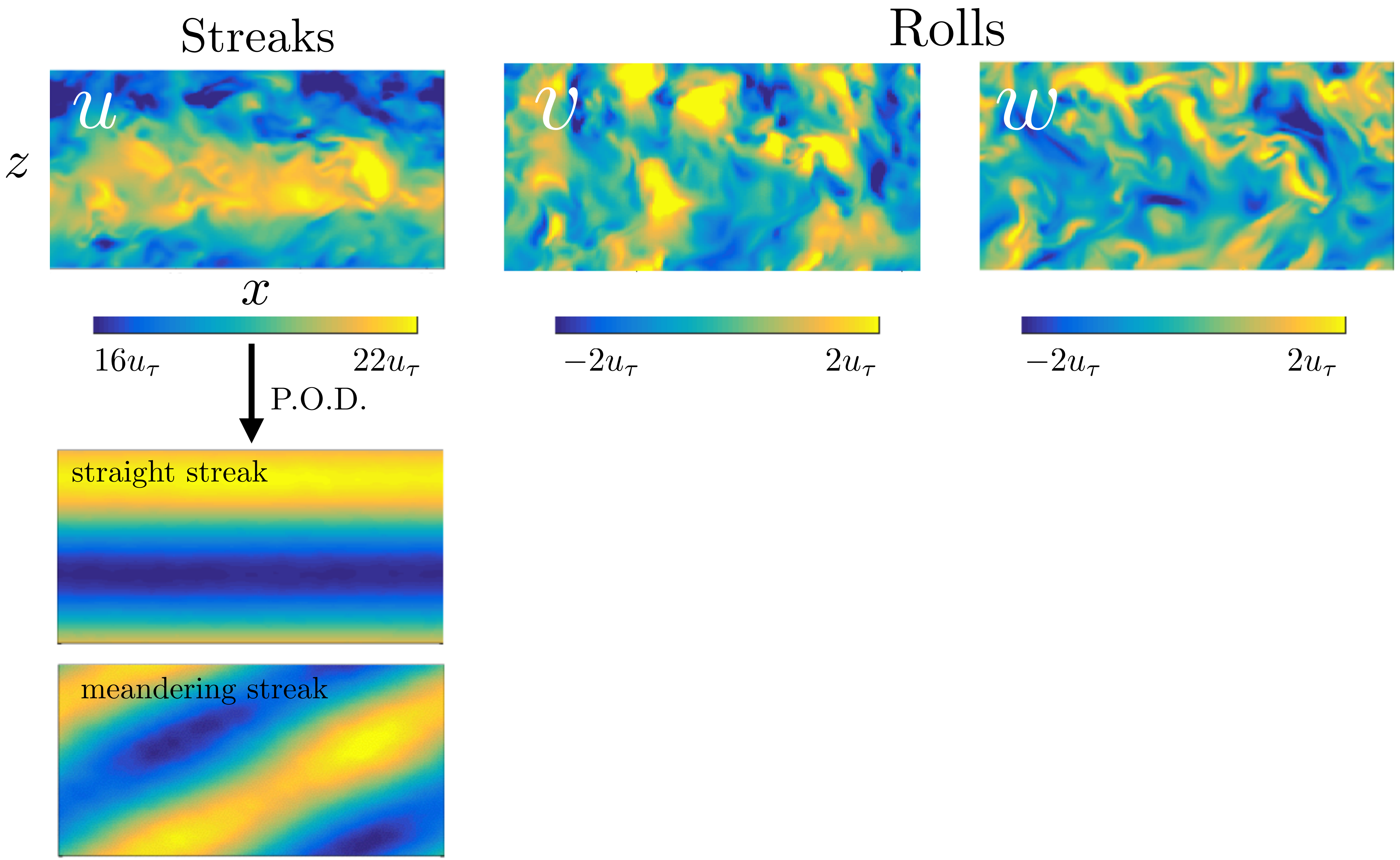

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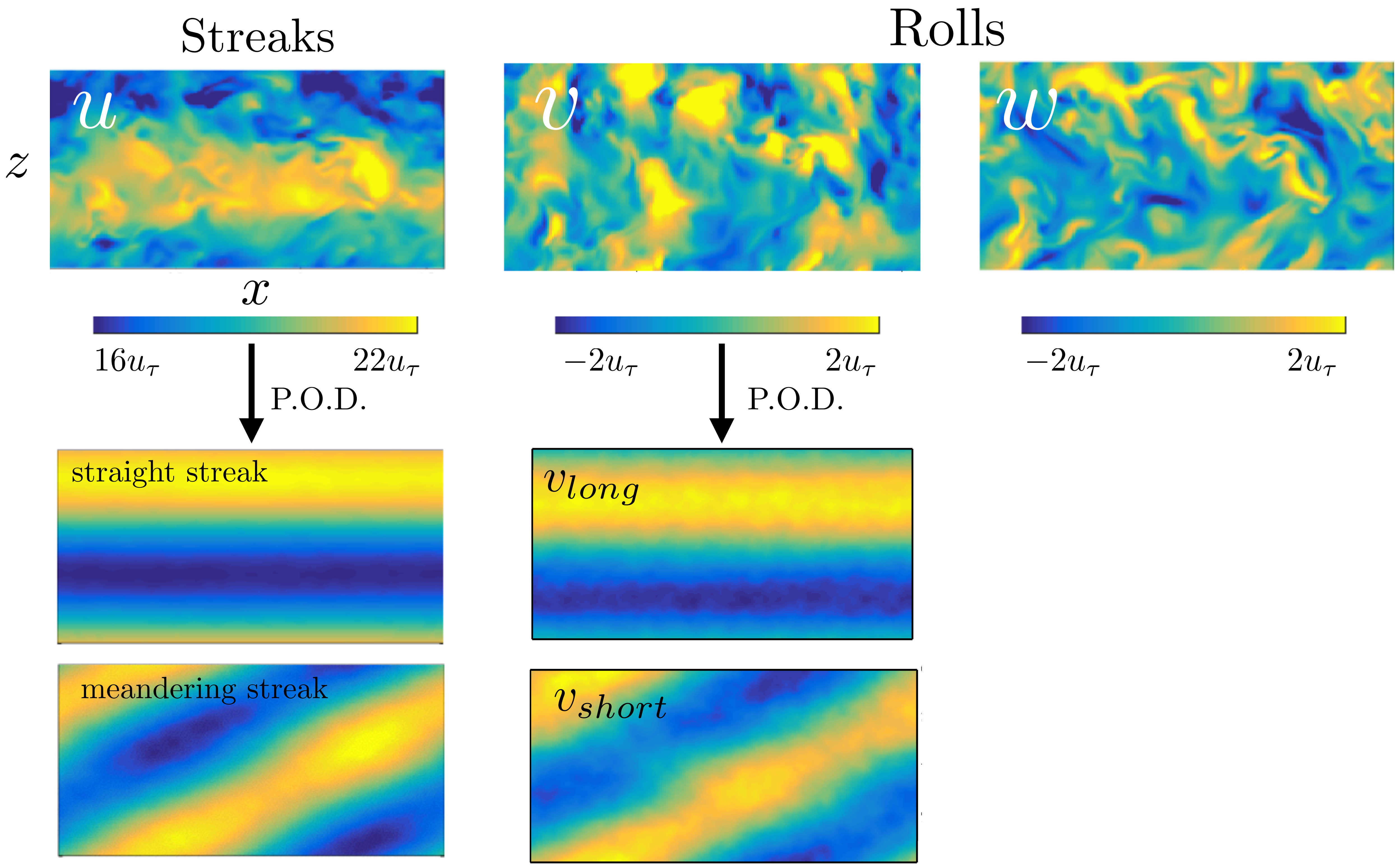

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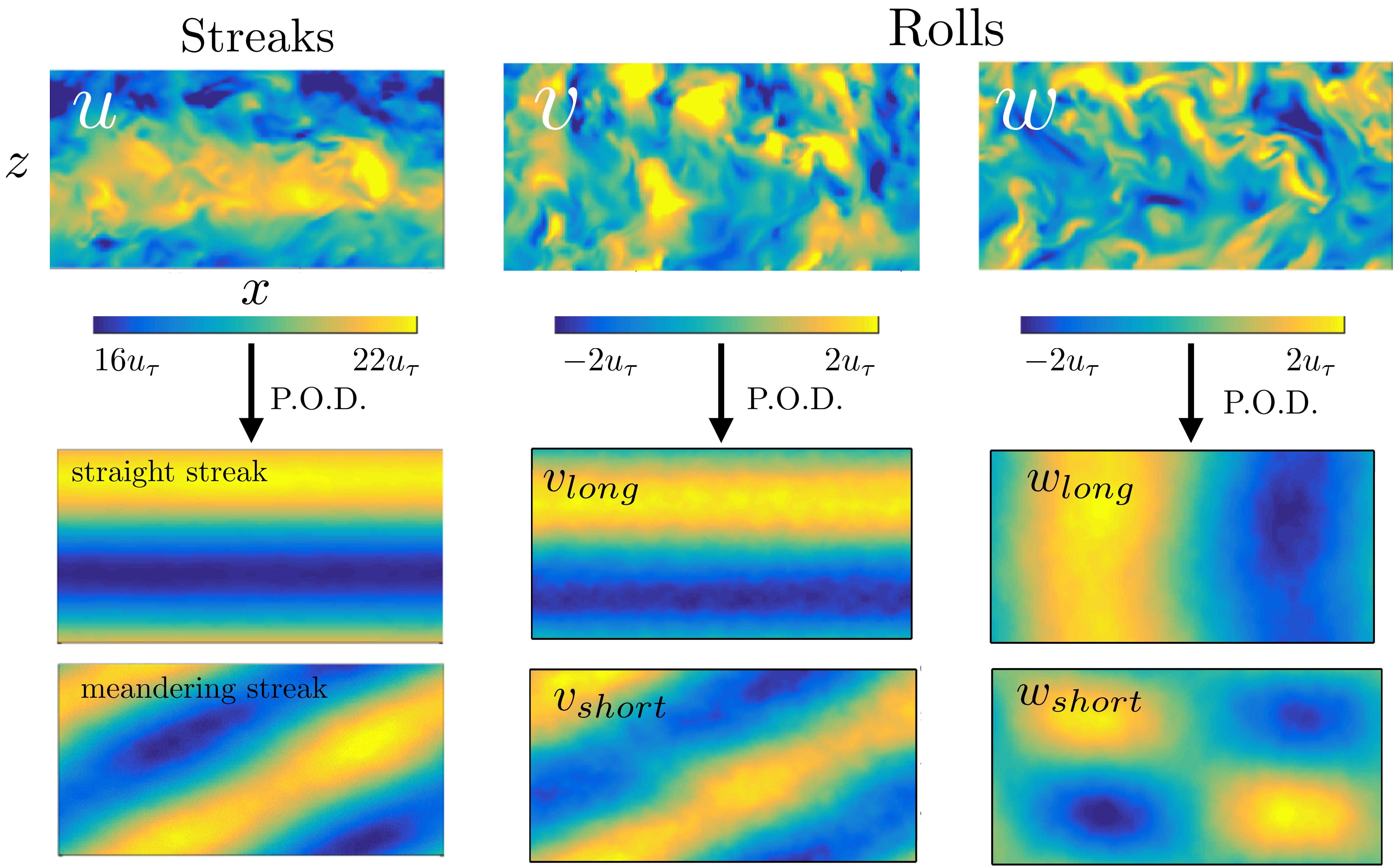


Outline

- Introduction and motivation
- Causality as information transfer
- Numerical experiment
- Identification of streaks and rolls
- Causality flow between streaks and rolls
- Conclusions

Causality flow between rolls and streaks

straight streak

v_{long} v_{short}

w_{long} w_{short}

meandering streak

strong causality

weak causality

Causality flow between rolls and streaks

straight streak

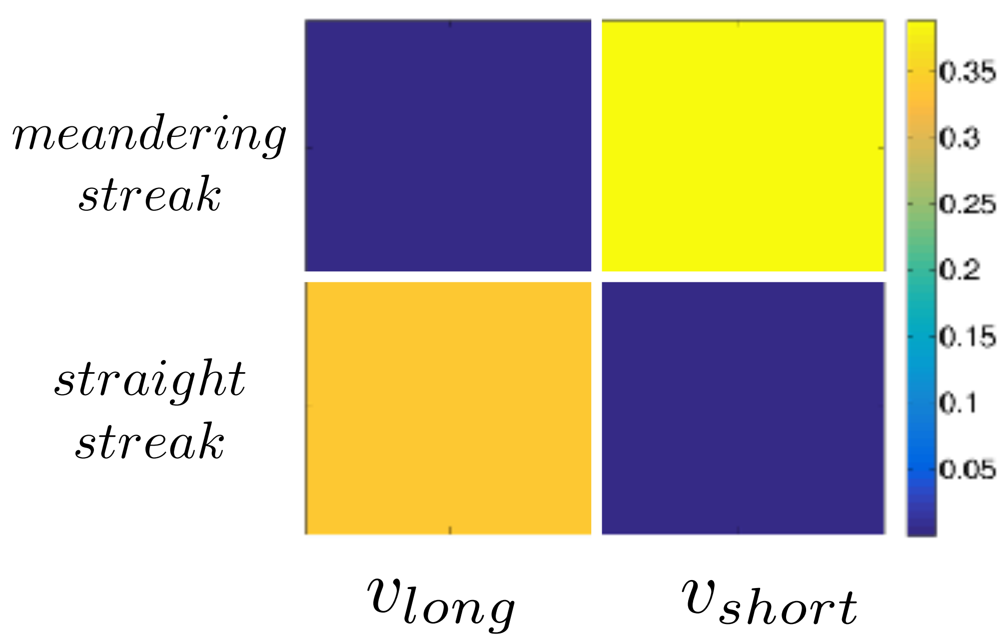
v_{long} v_{short}

w_{long} w_{short}

meandering streak

strong causality
weak causality

Causality $streaks \rightarrow v$



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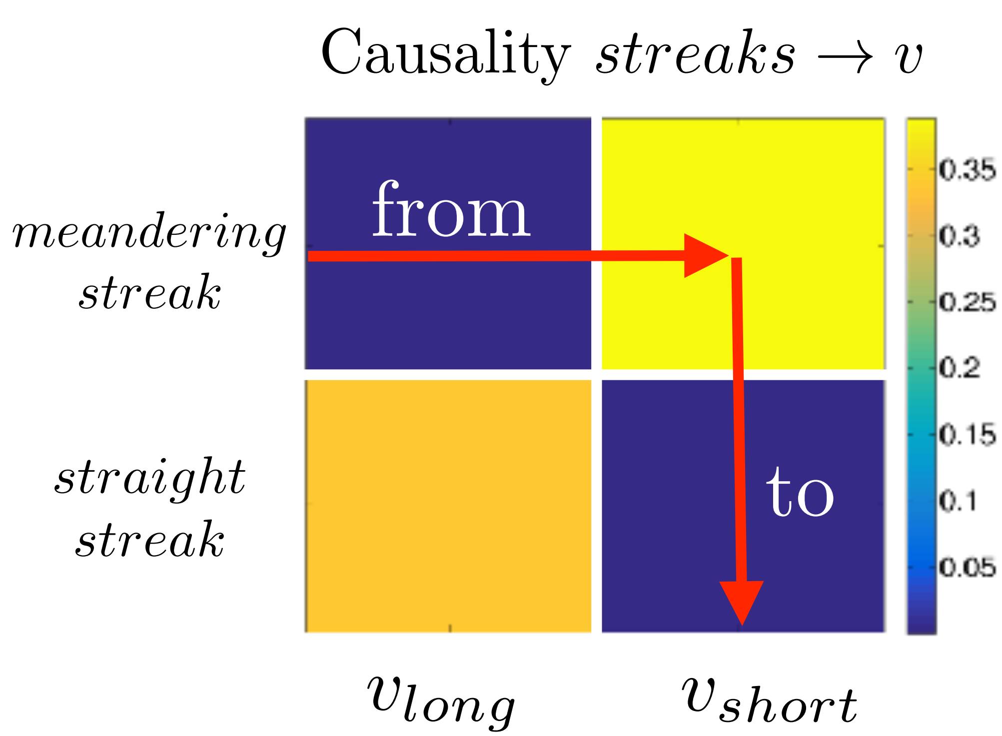
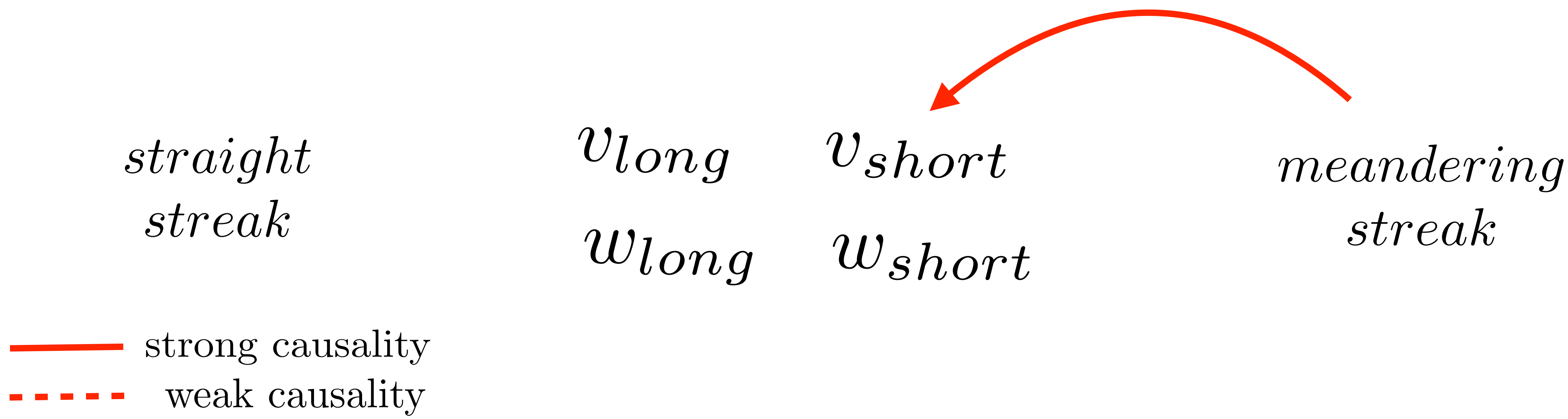
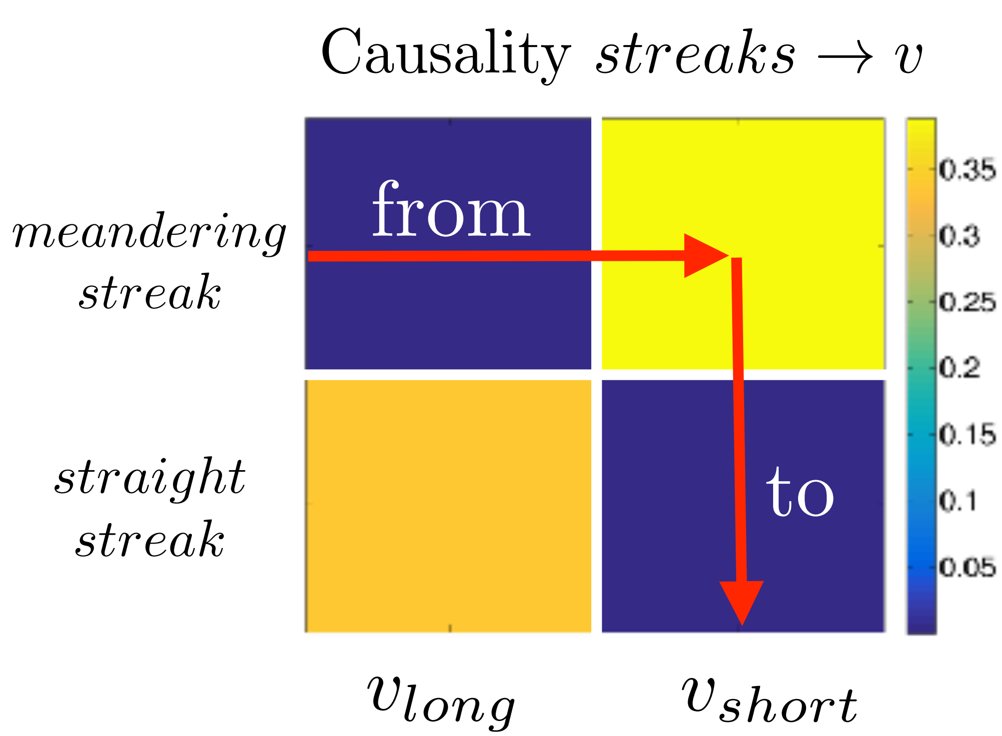


Causality flow between rolls and streaks





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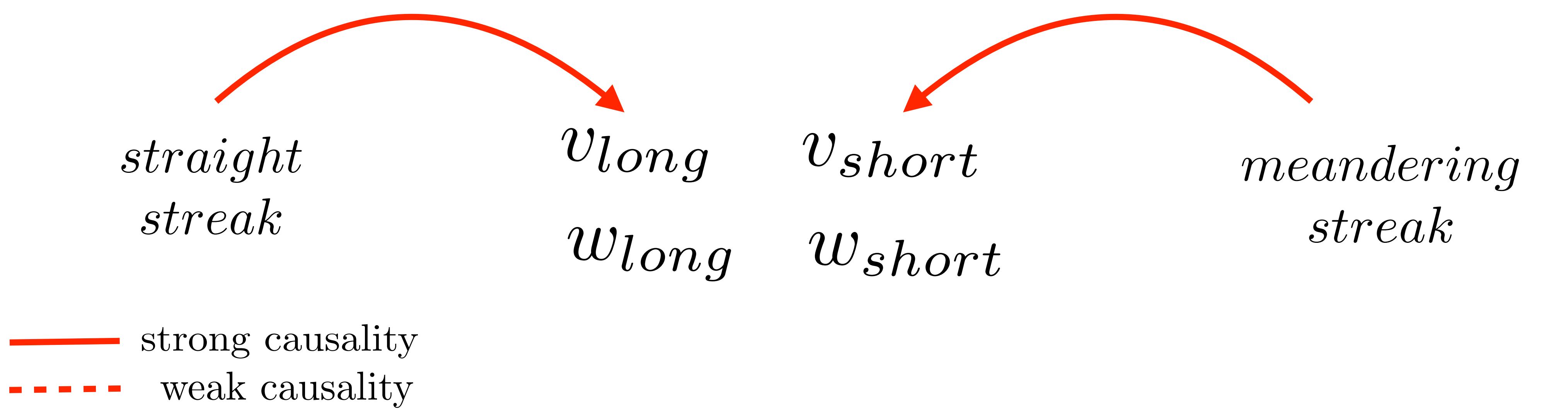
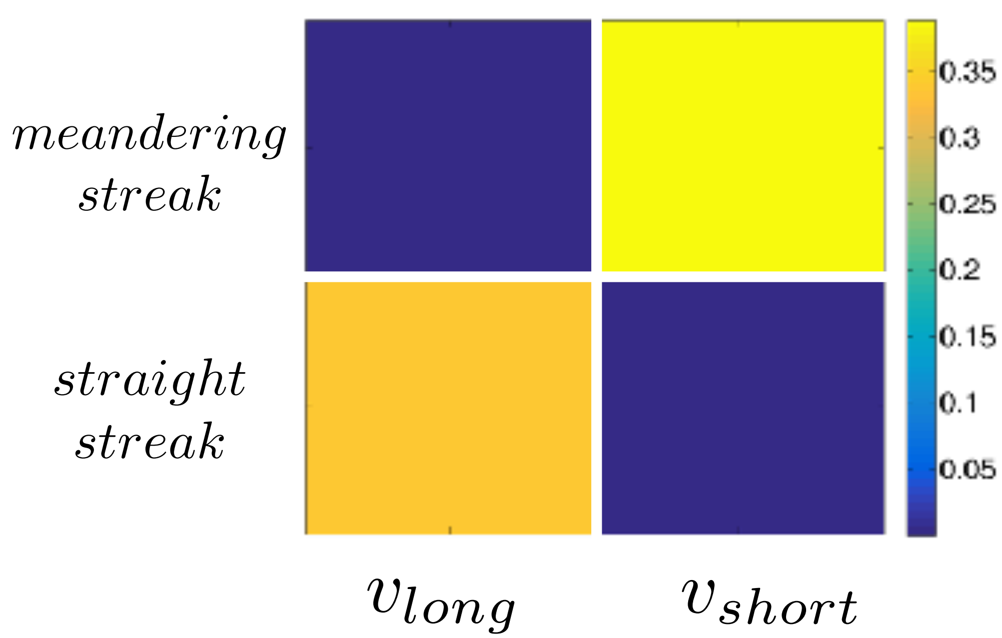


Causality *streaks* $\rightarrow v$



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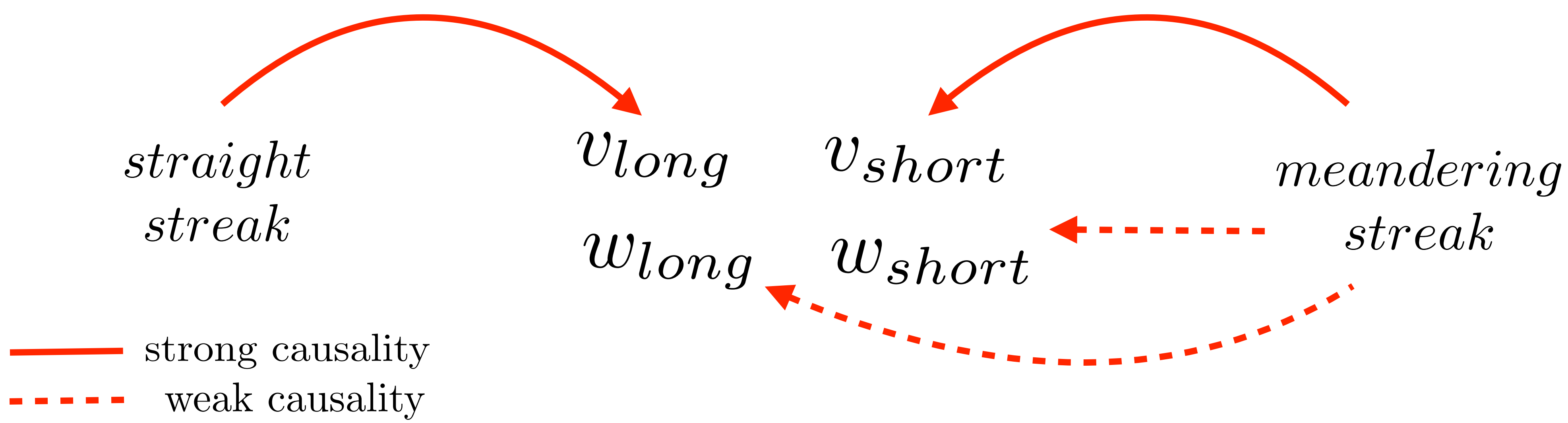
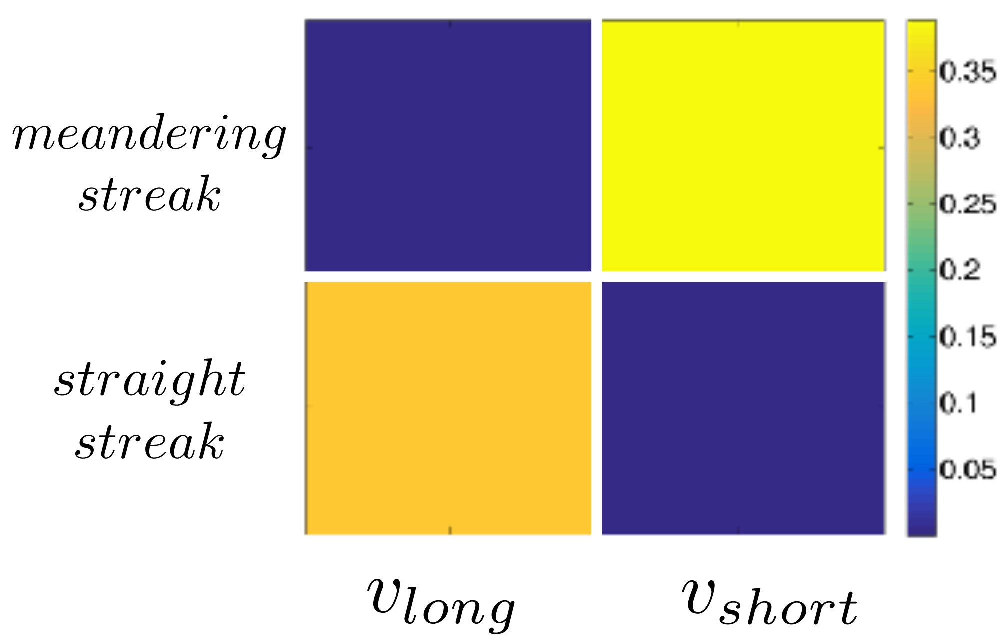
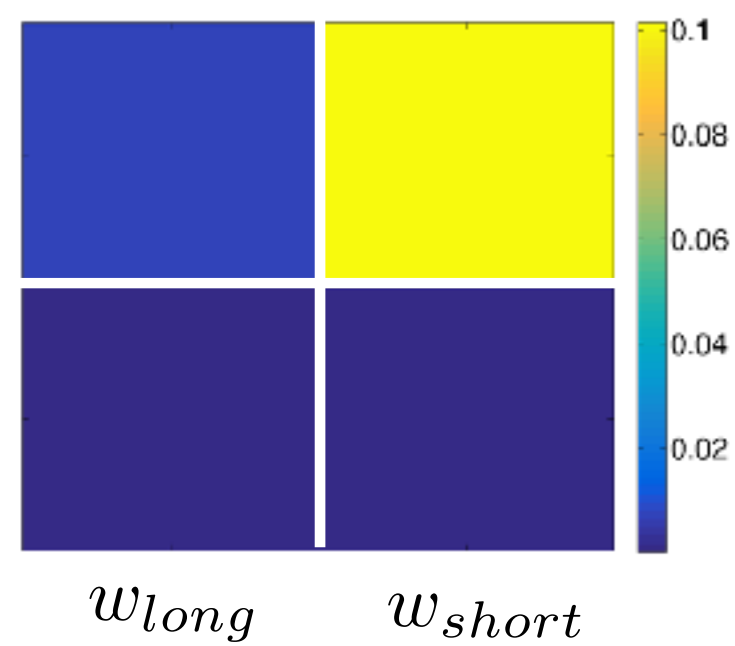


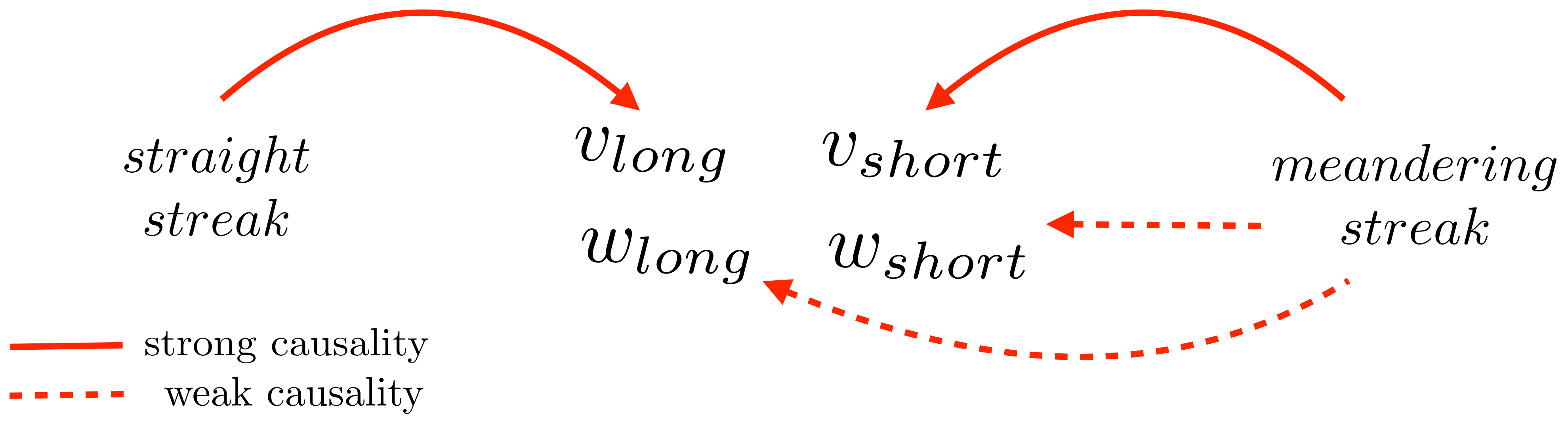
Causality *streaks* → v

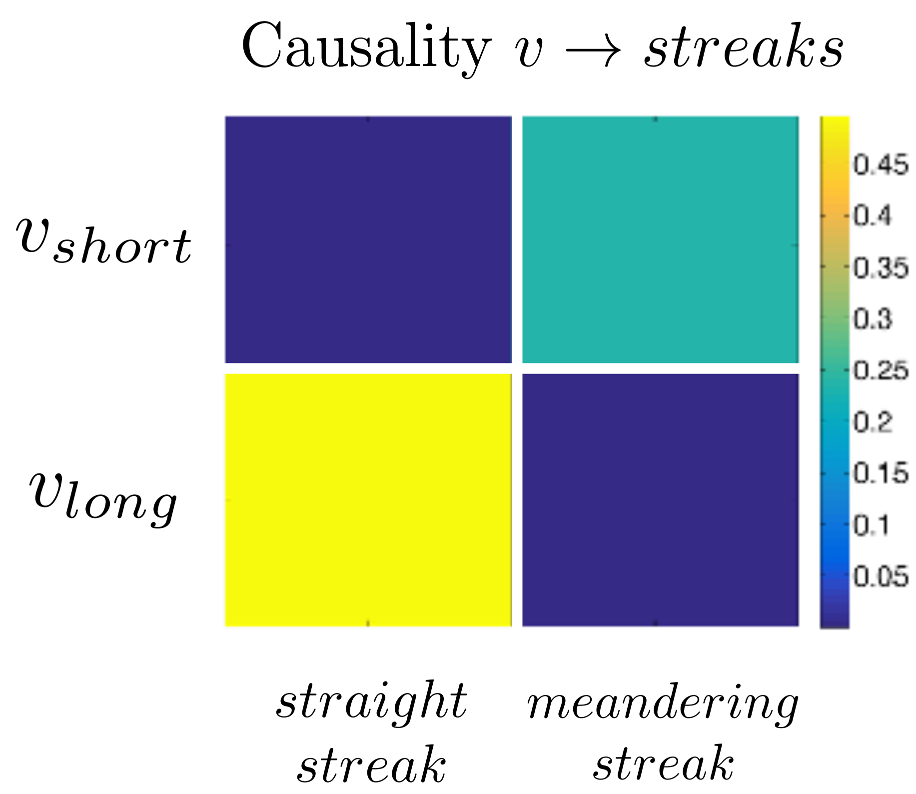


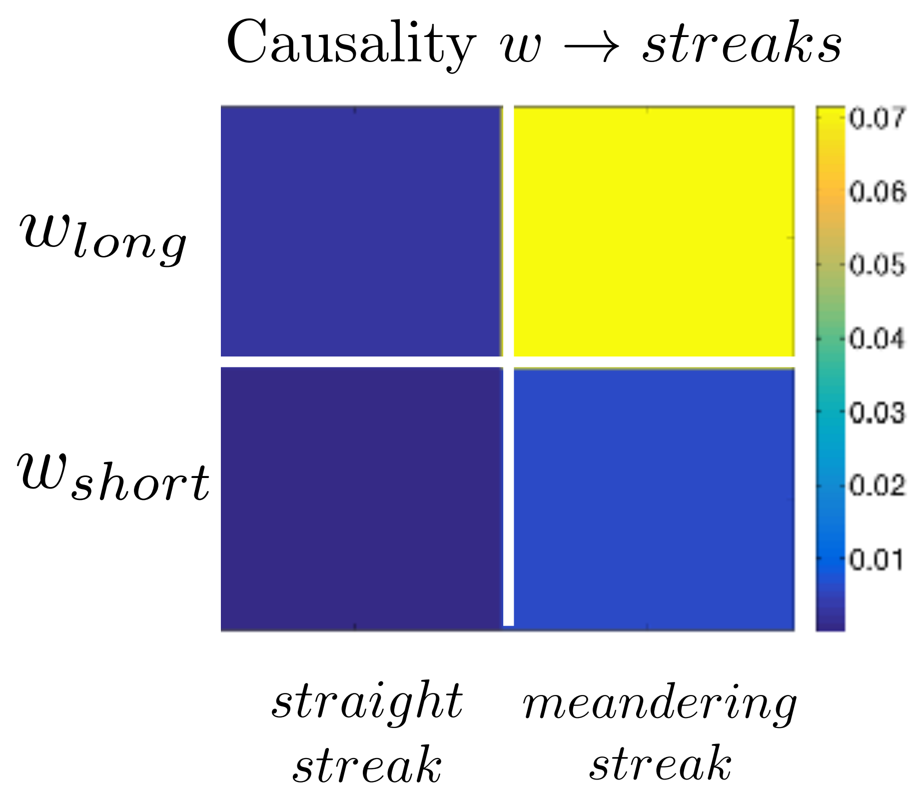
Causality *streaks* → w



Causality flow between rolls and streaks







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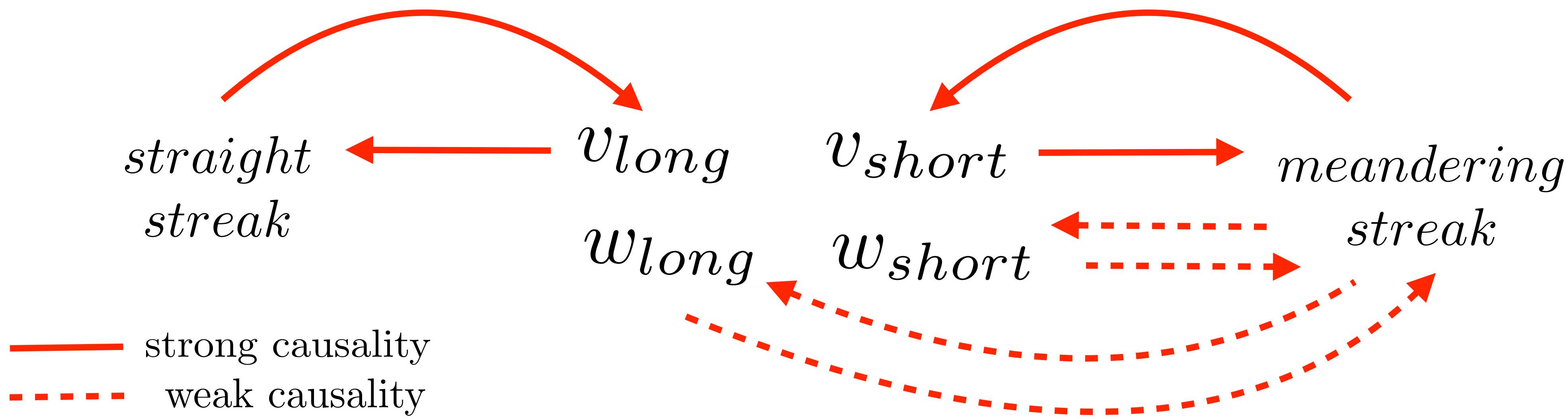

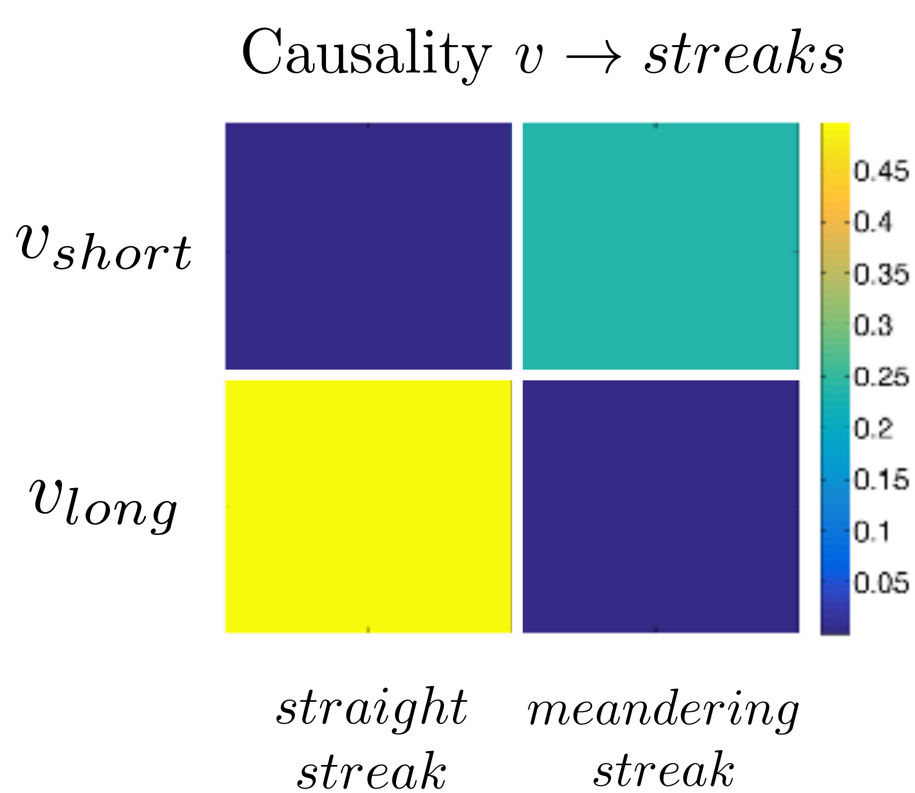

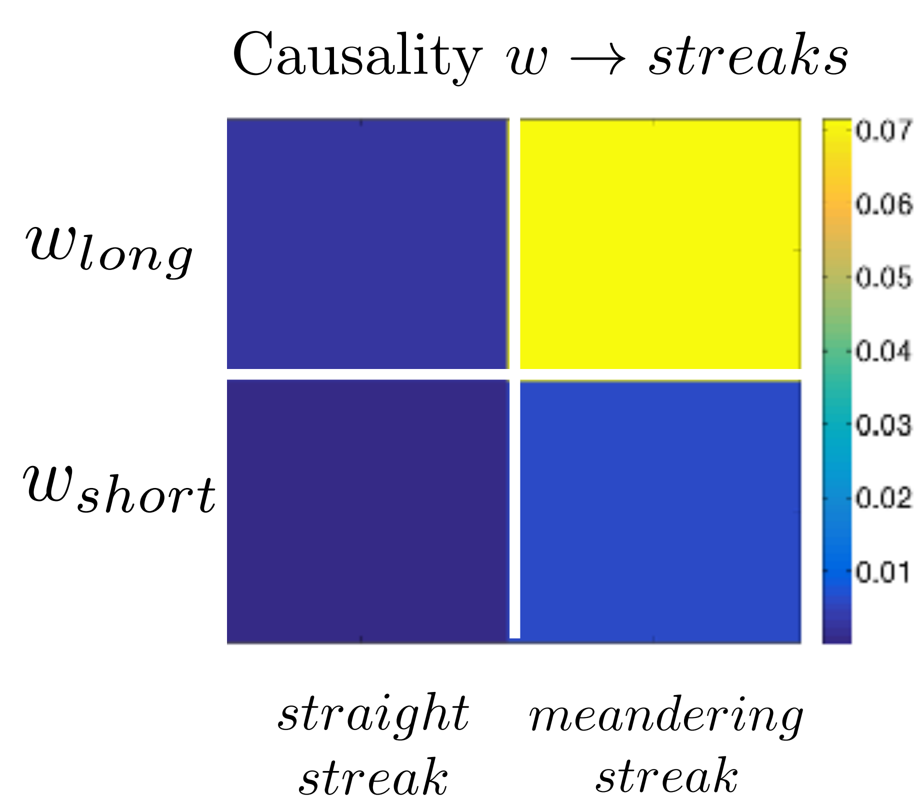

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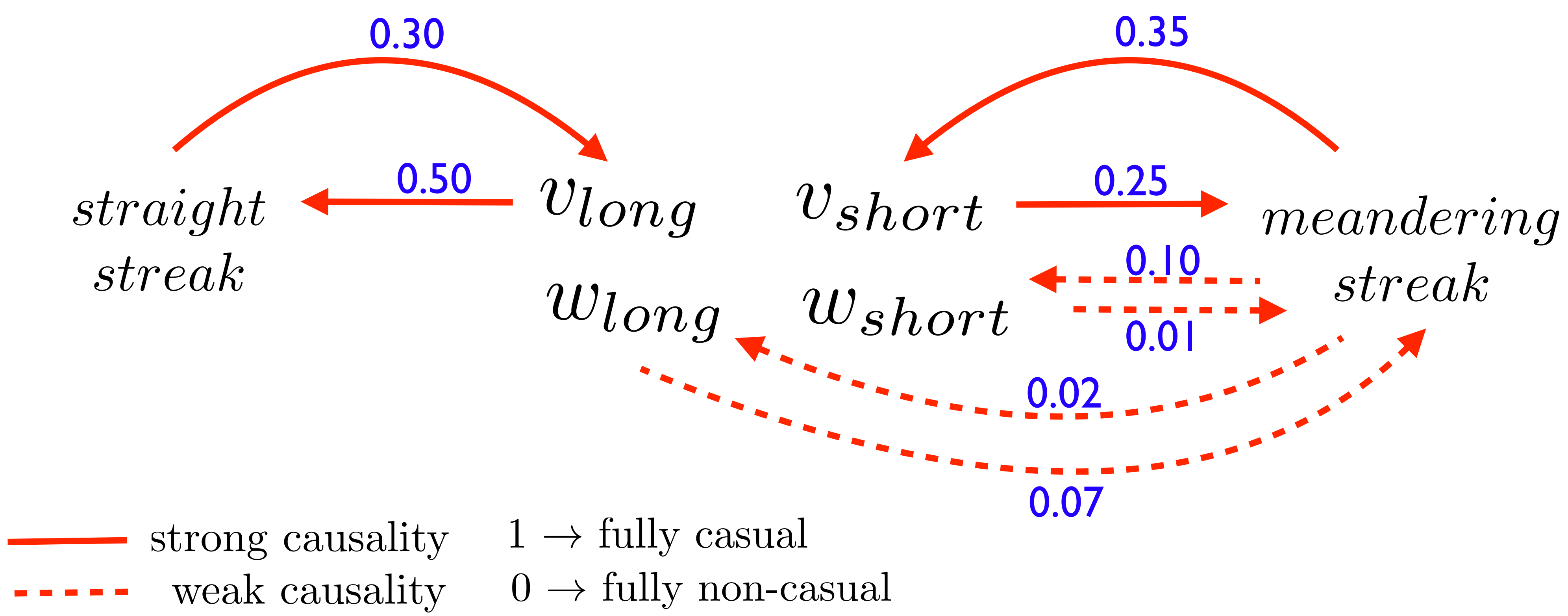

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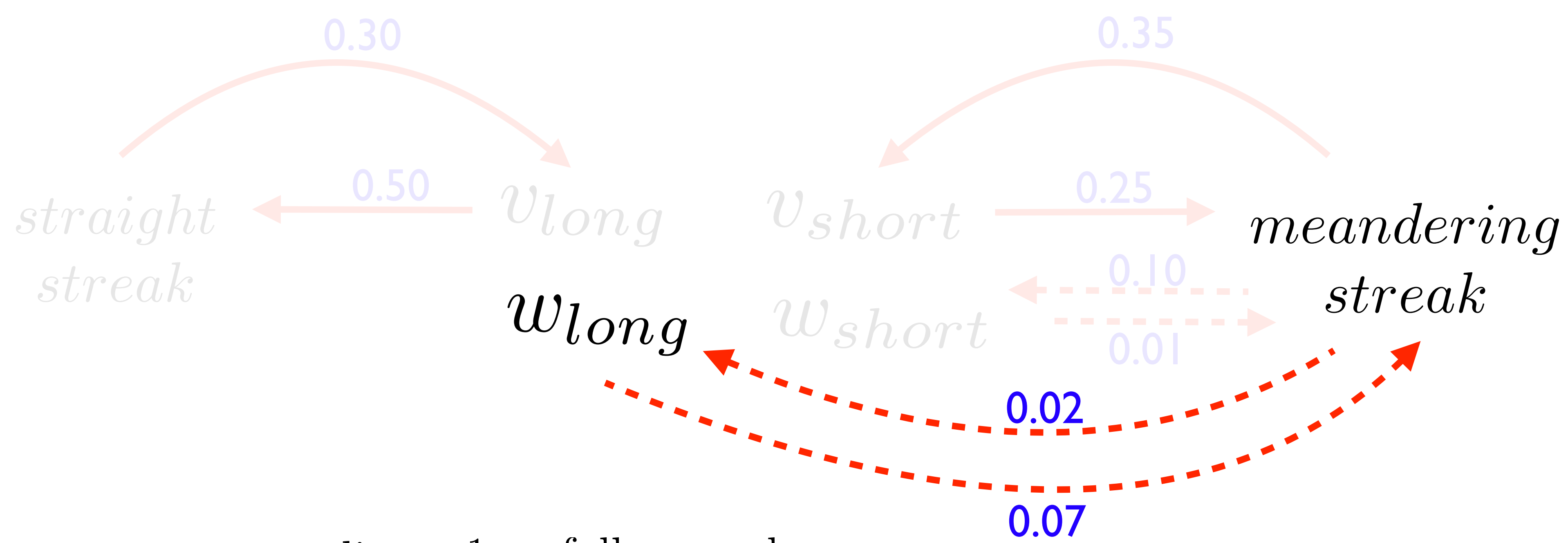
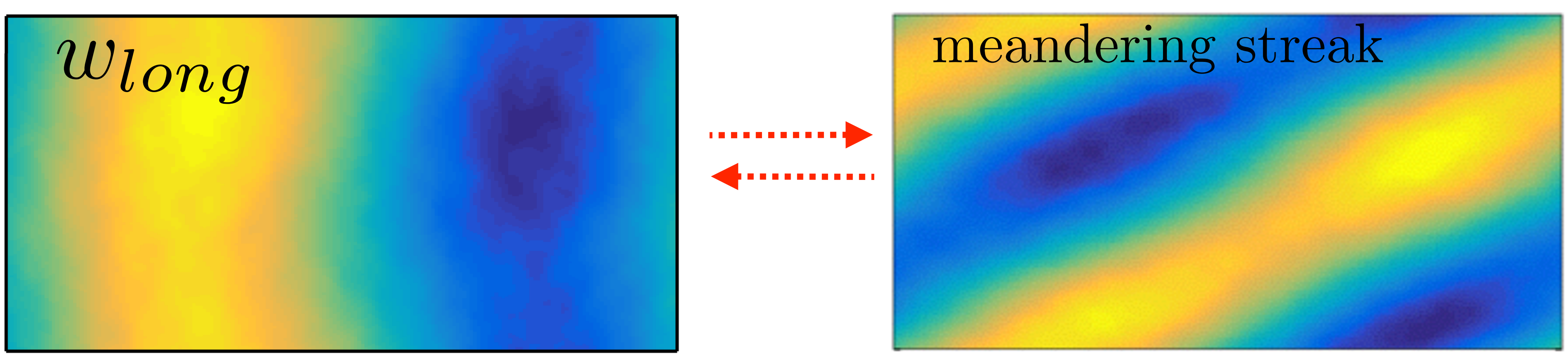


strong causality — $1 \rightarrow$ fully casual

weak causality — $0 \rightarrow$ fully non-casual



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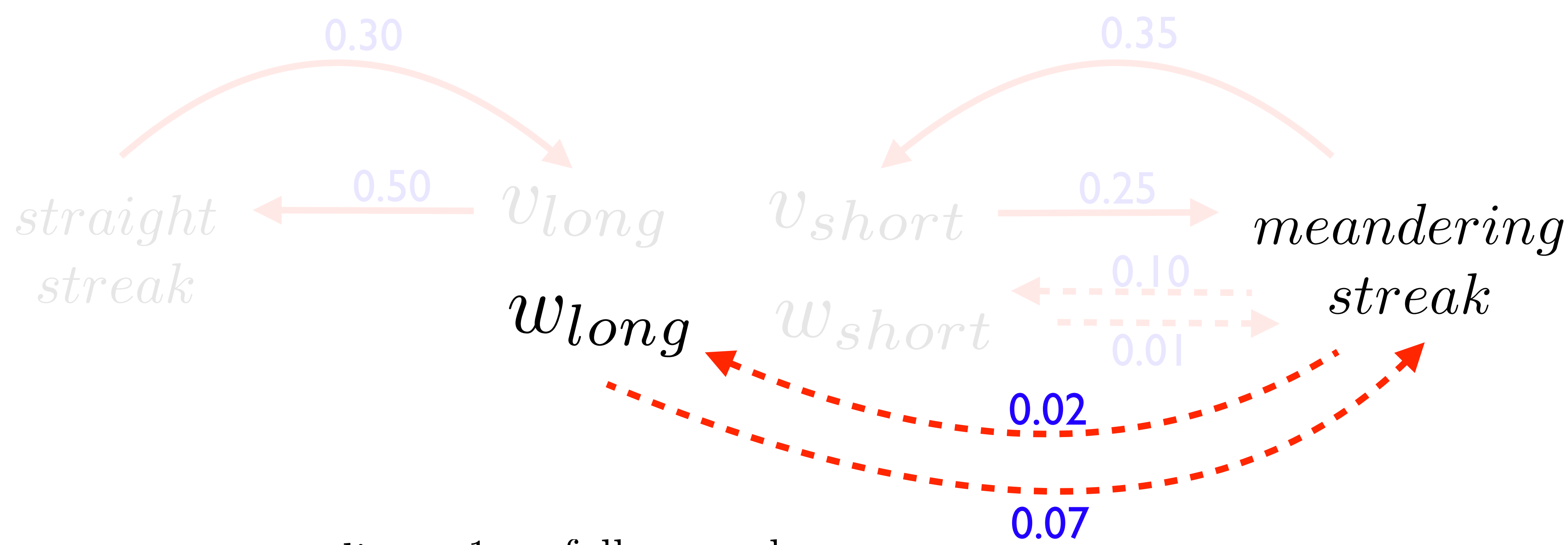

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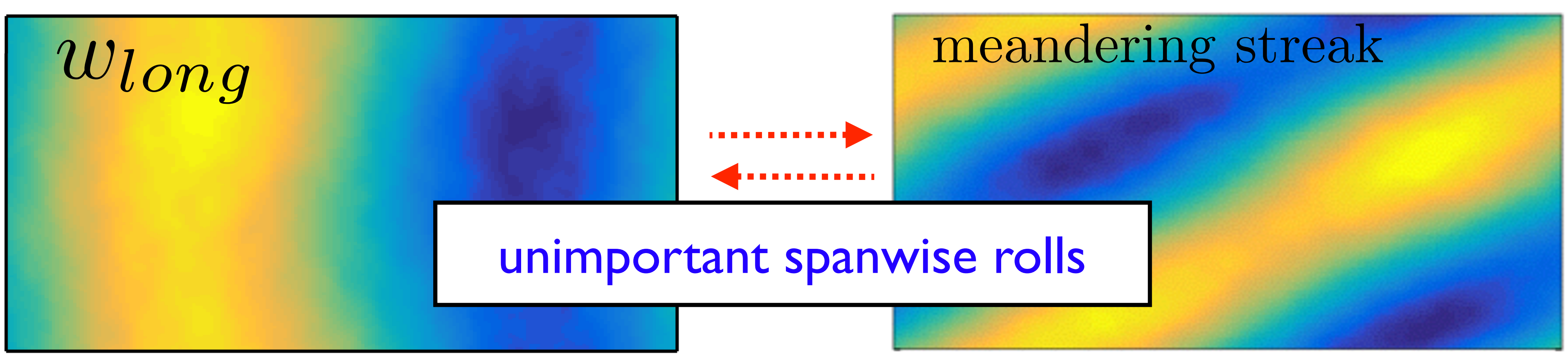

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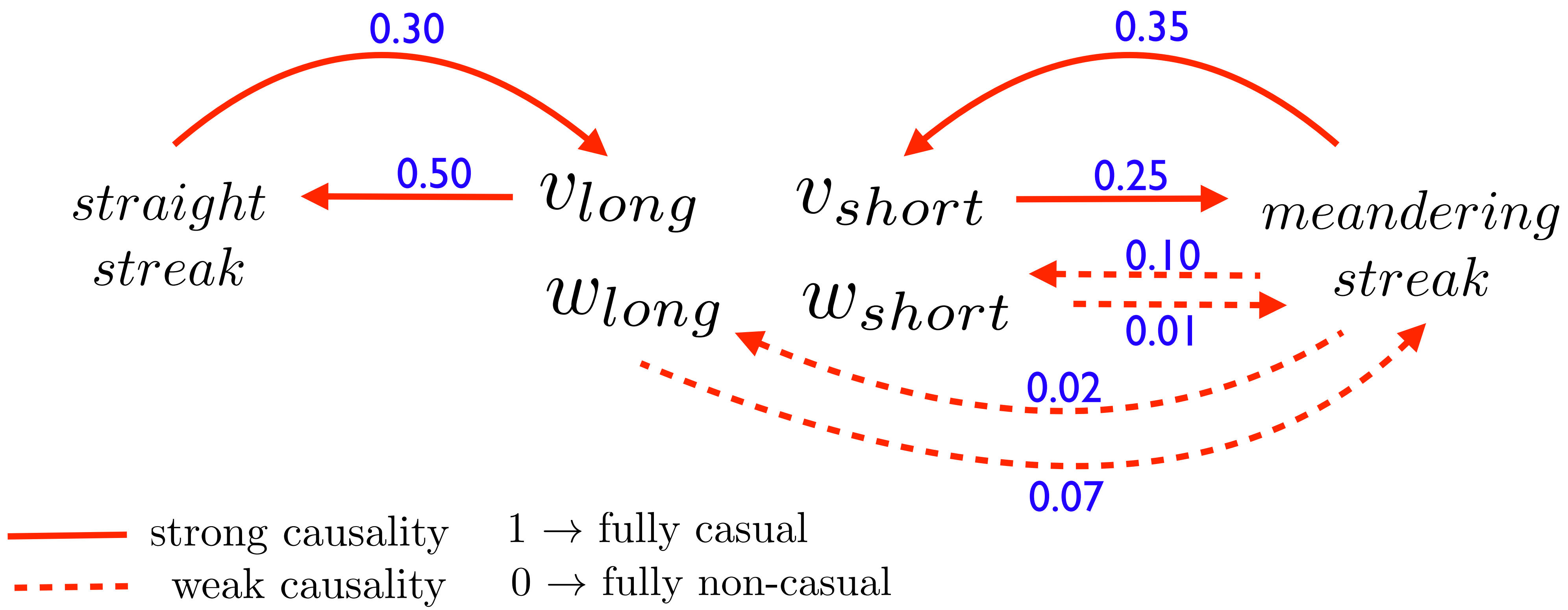

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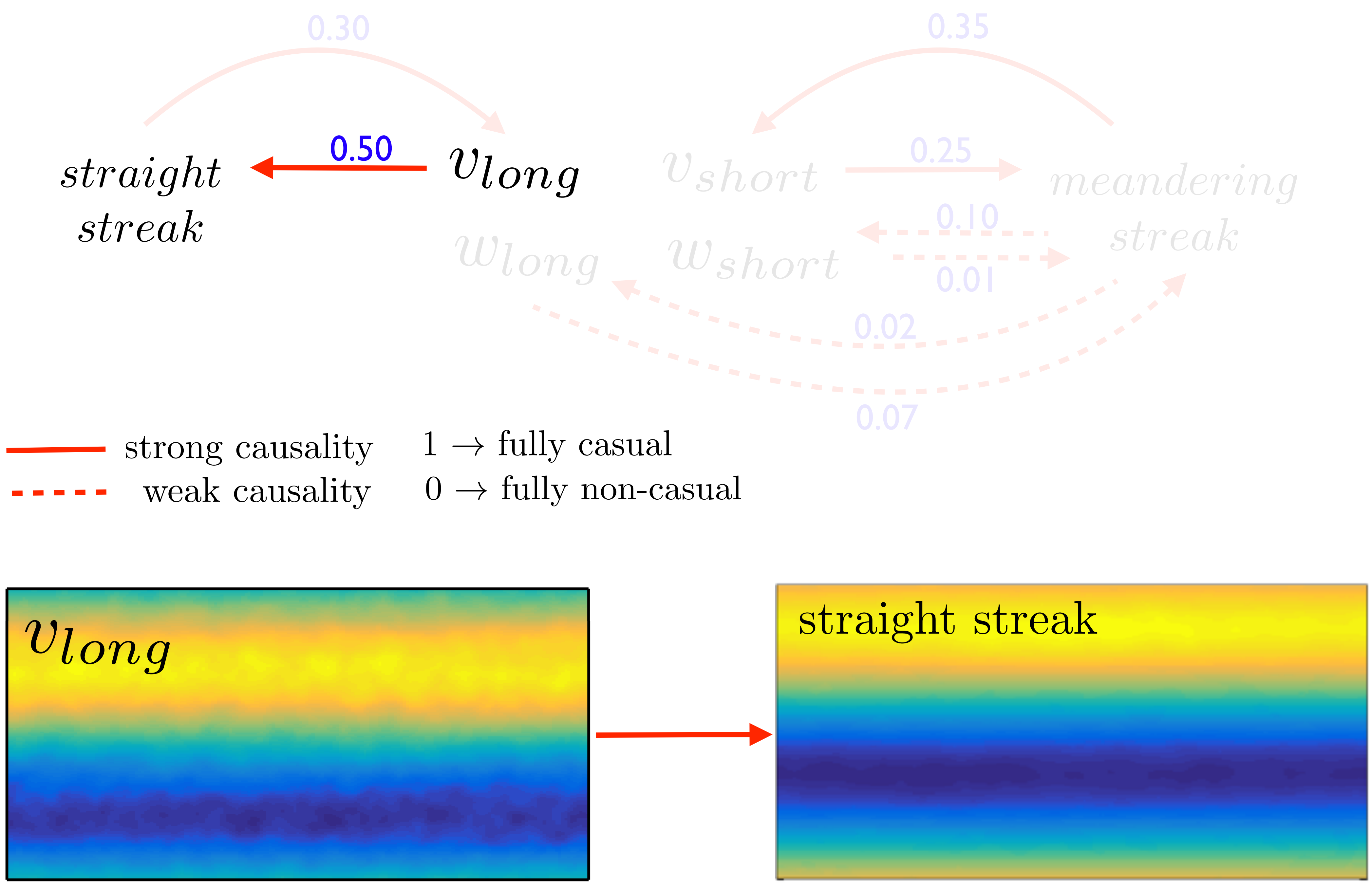

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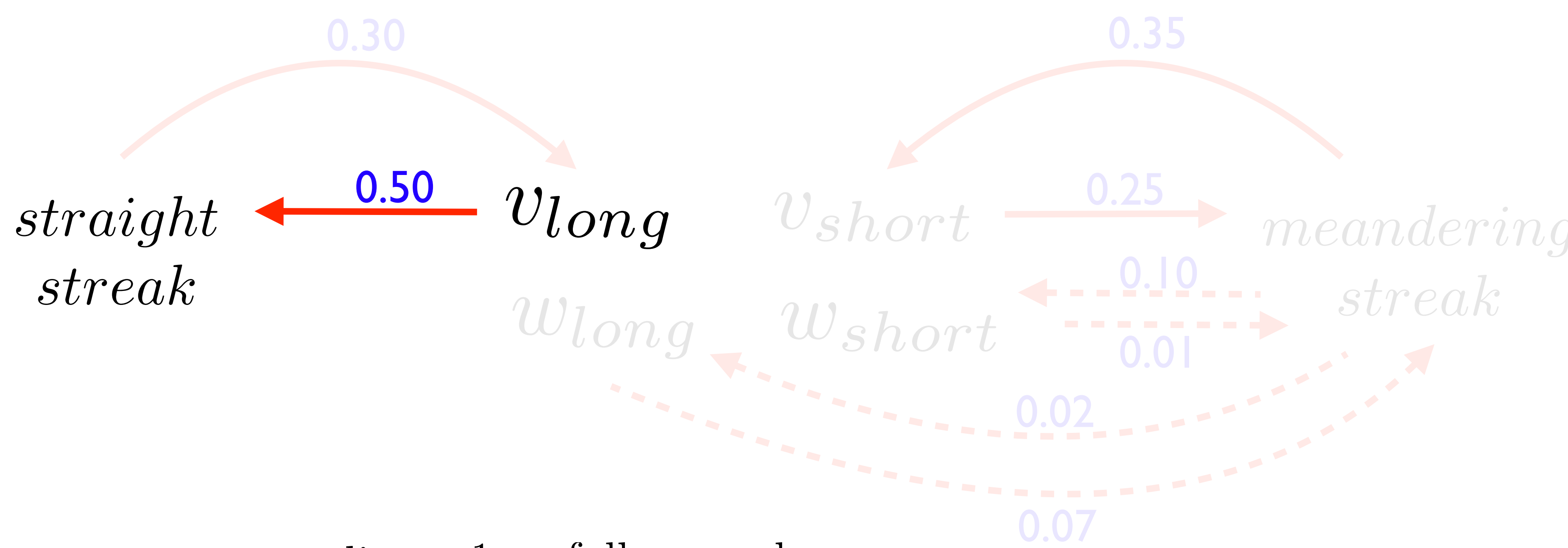

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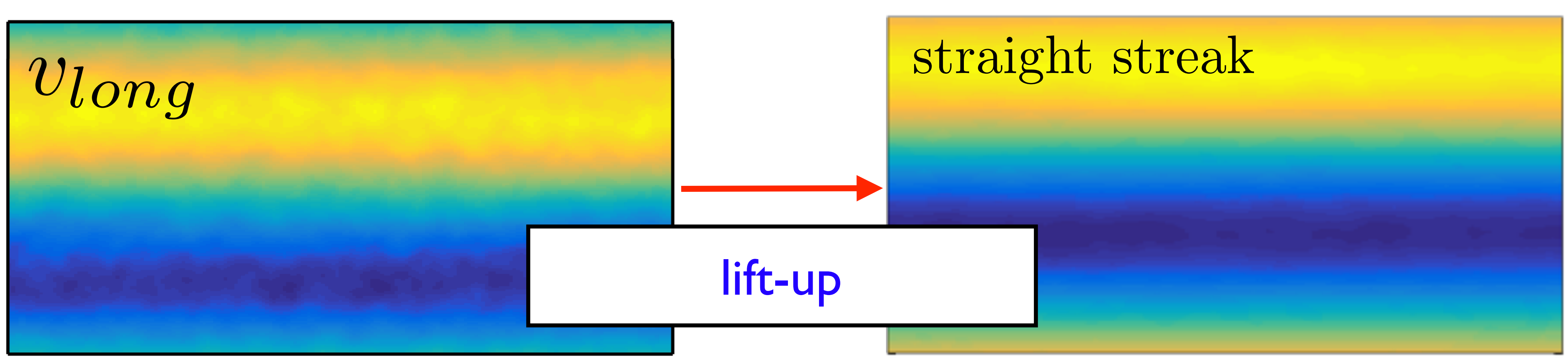

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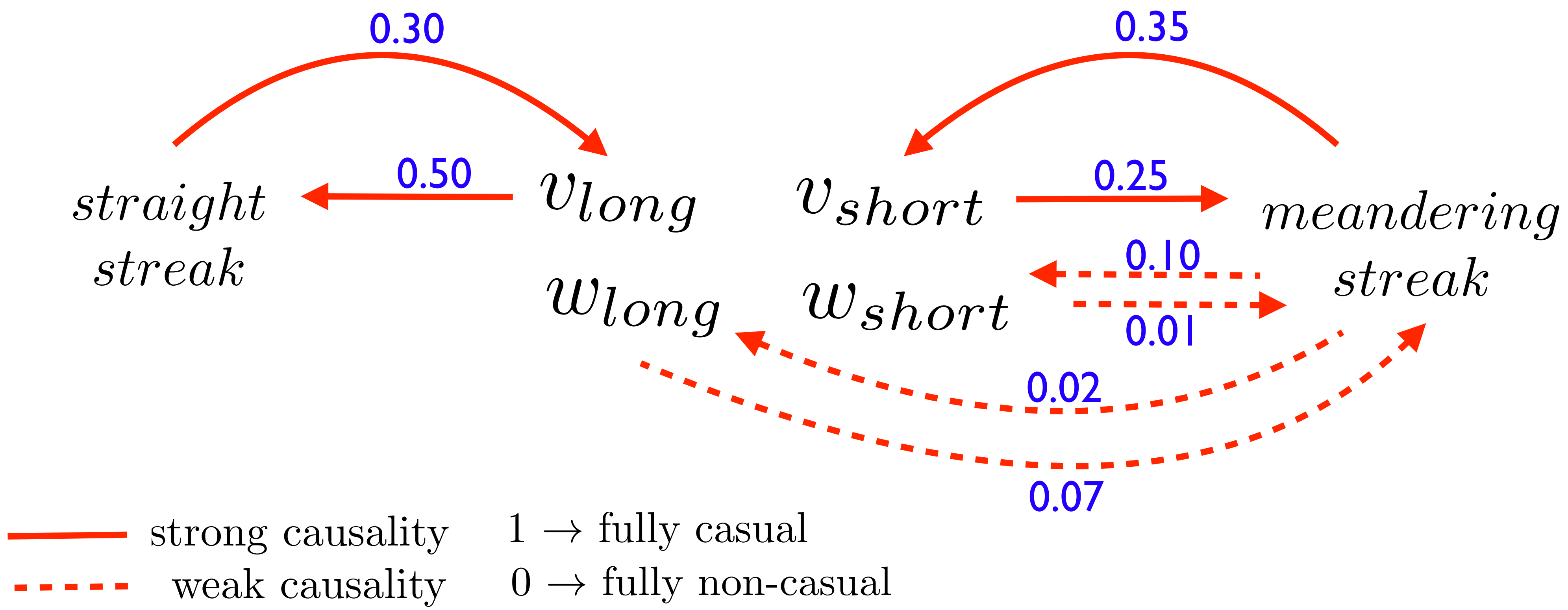

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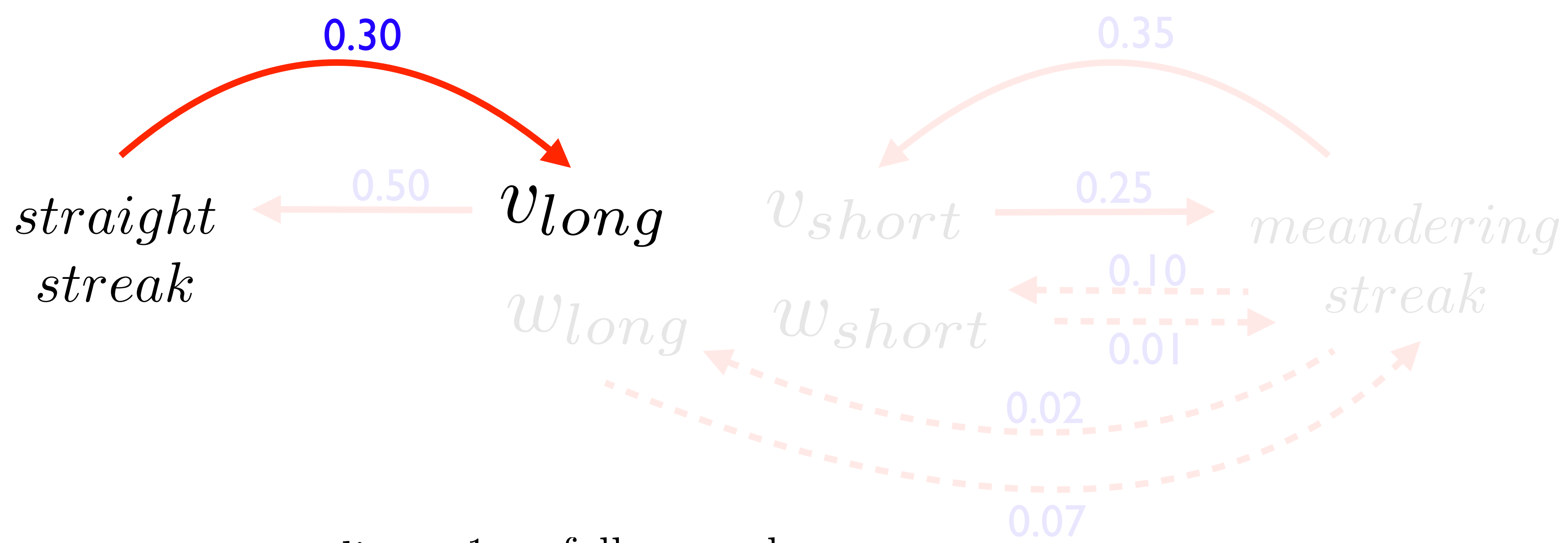

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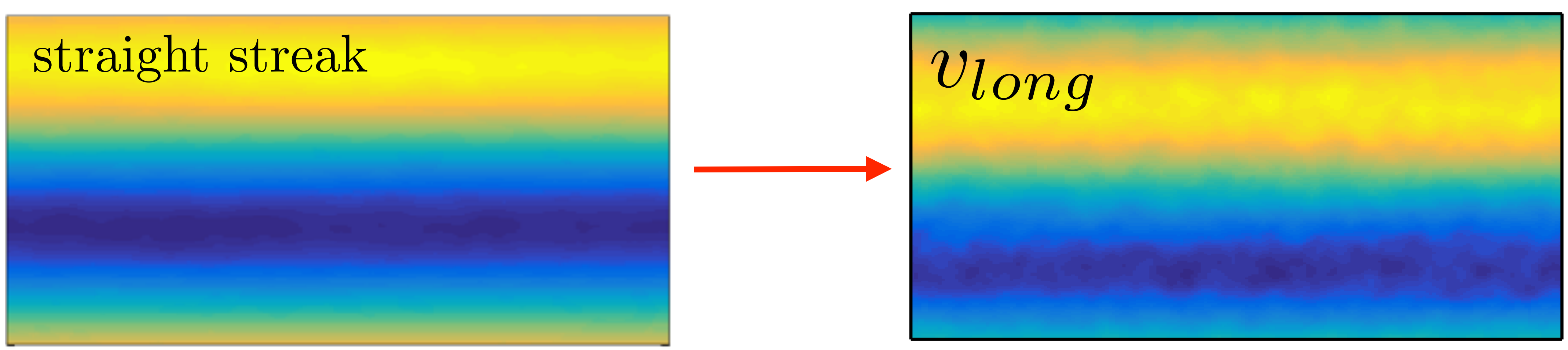

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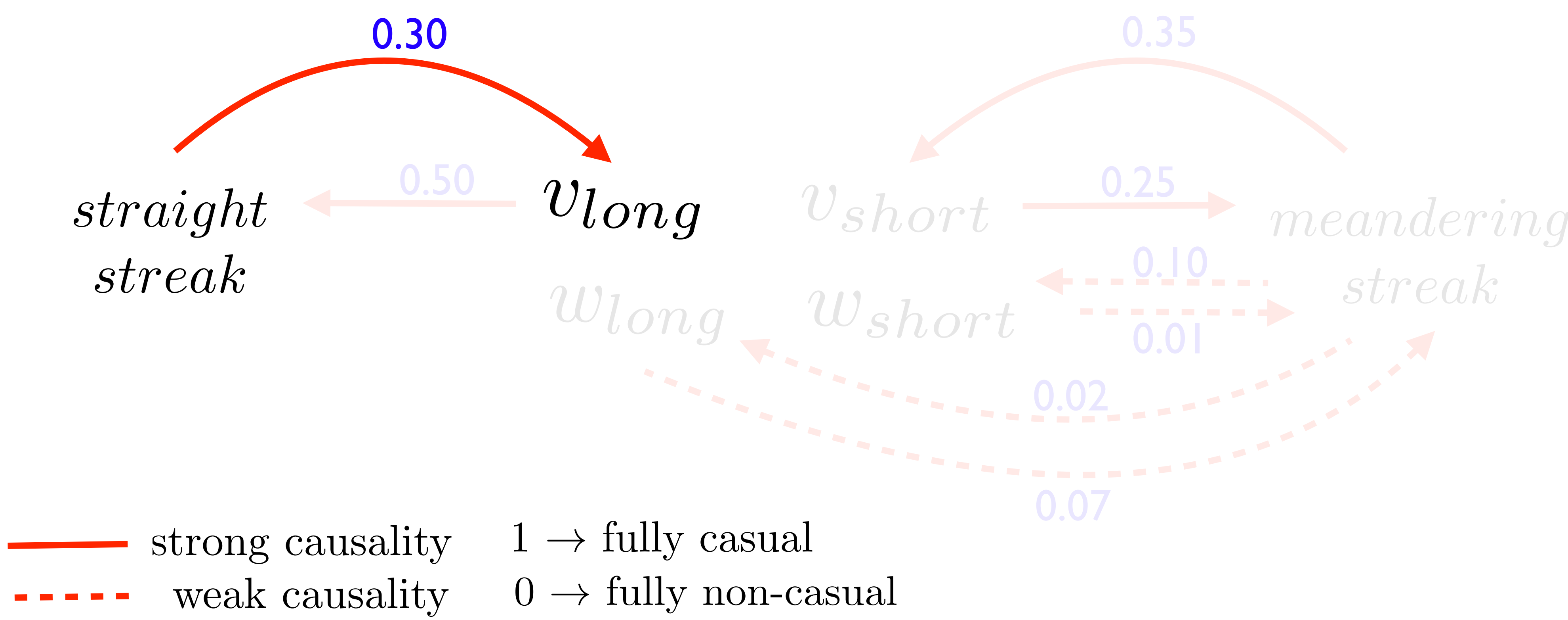

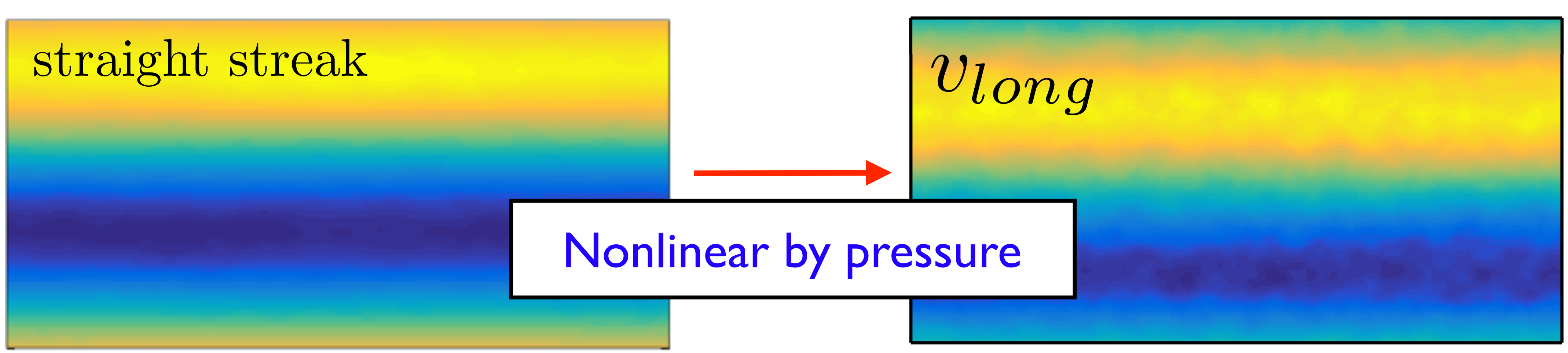

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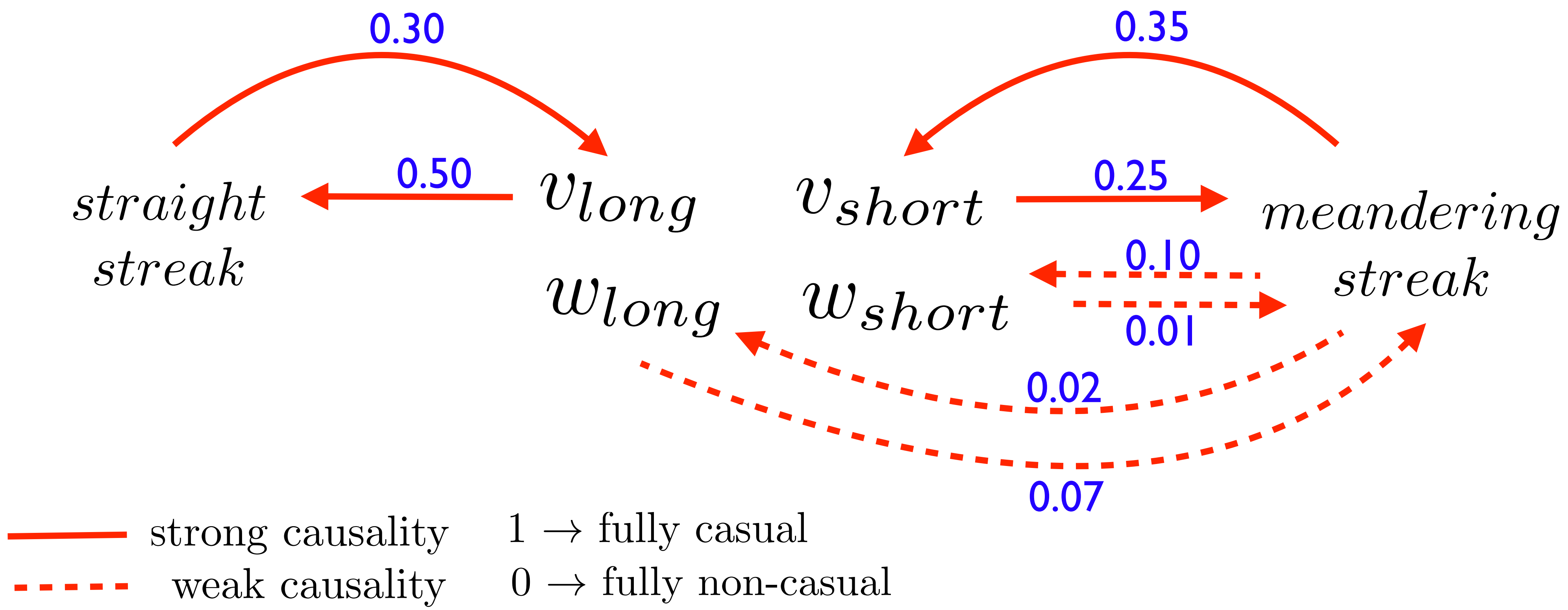

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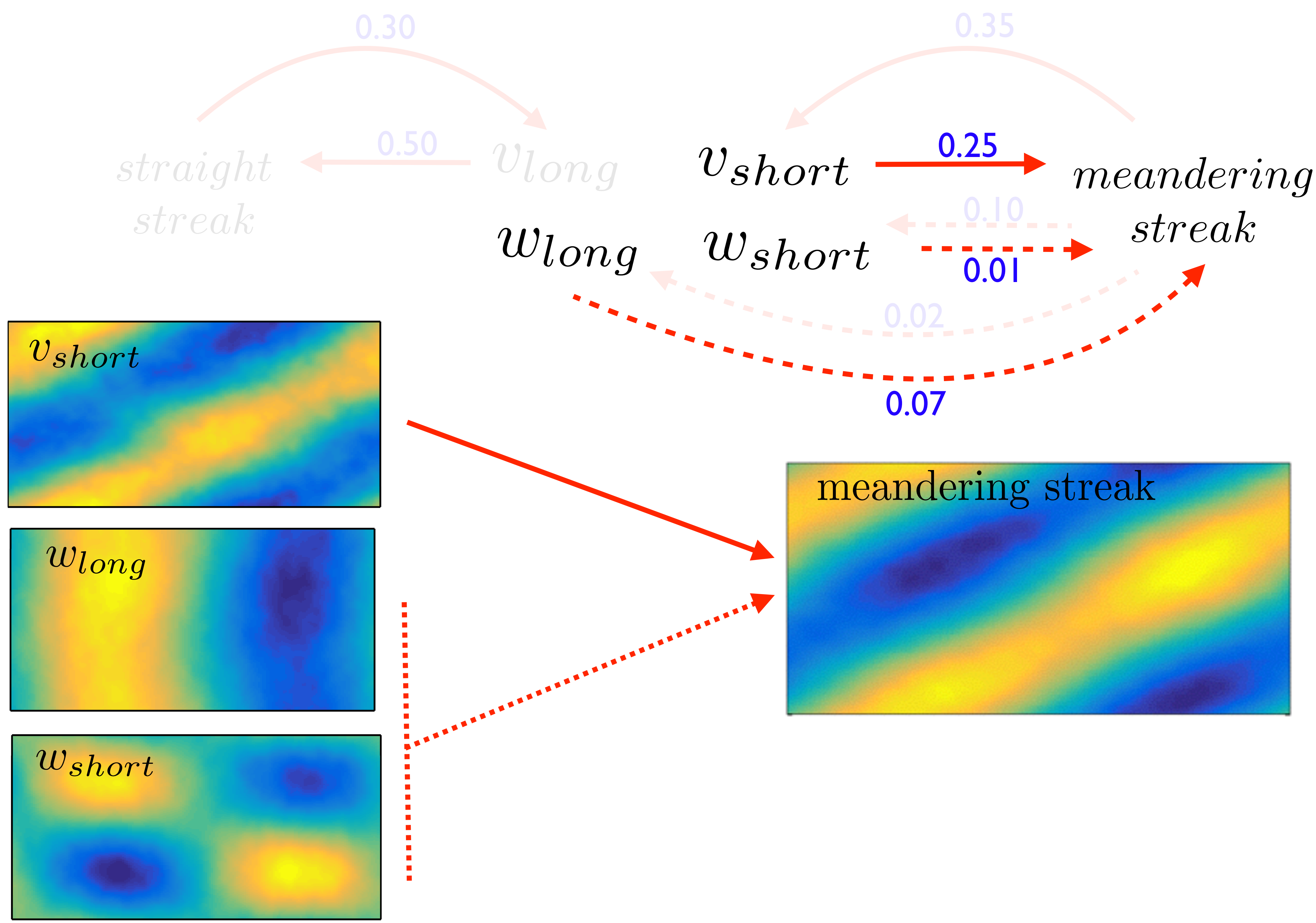

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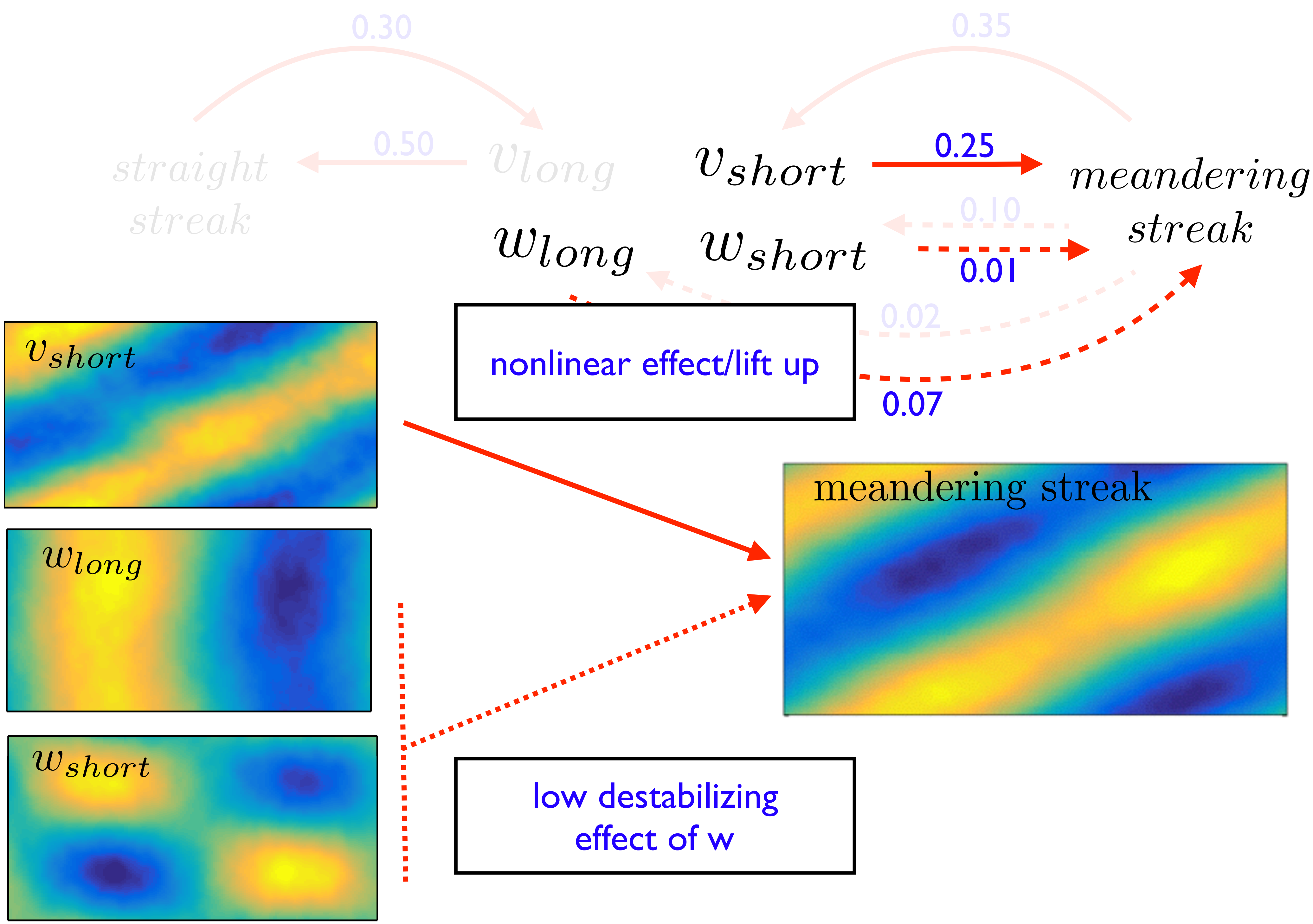

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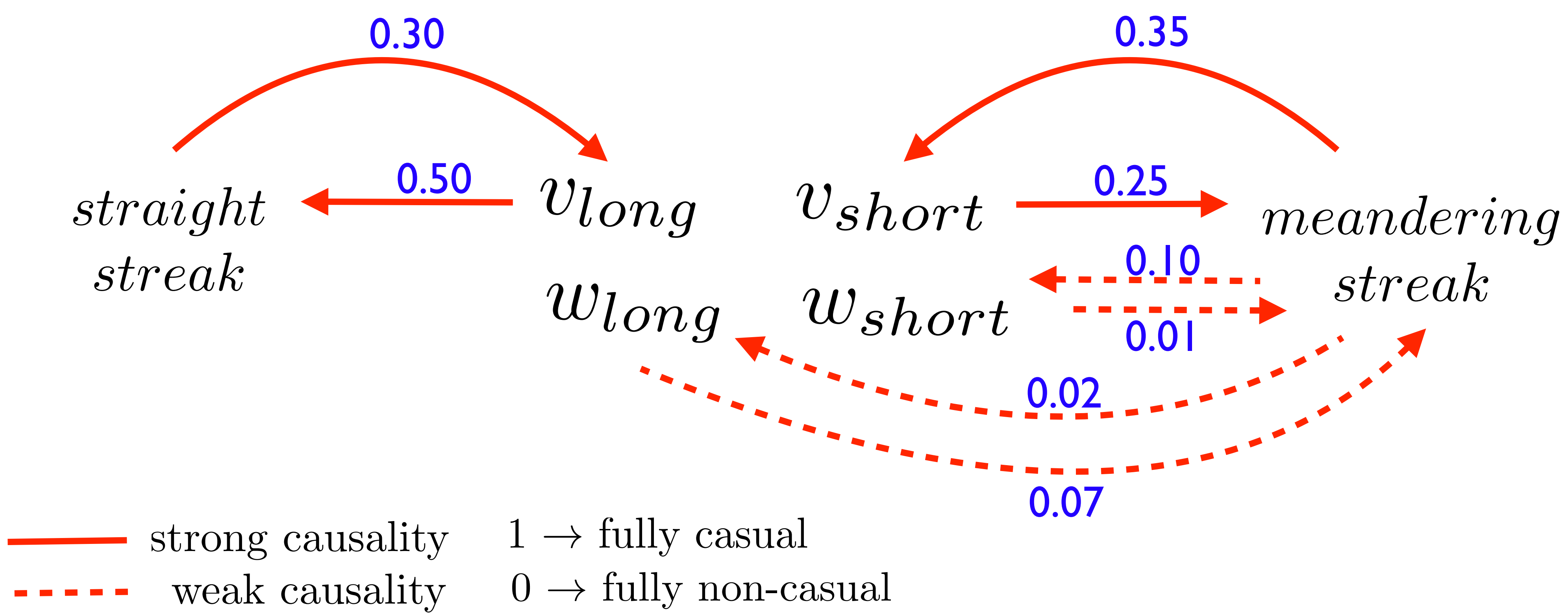

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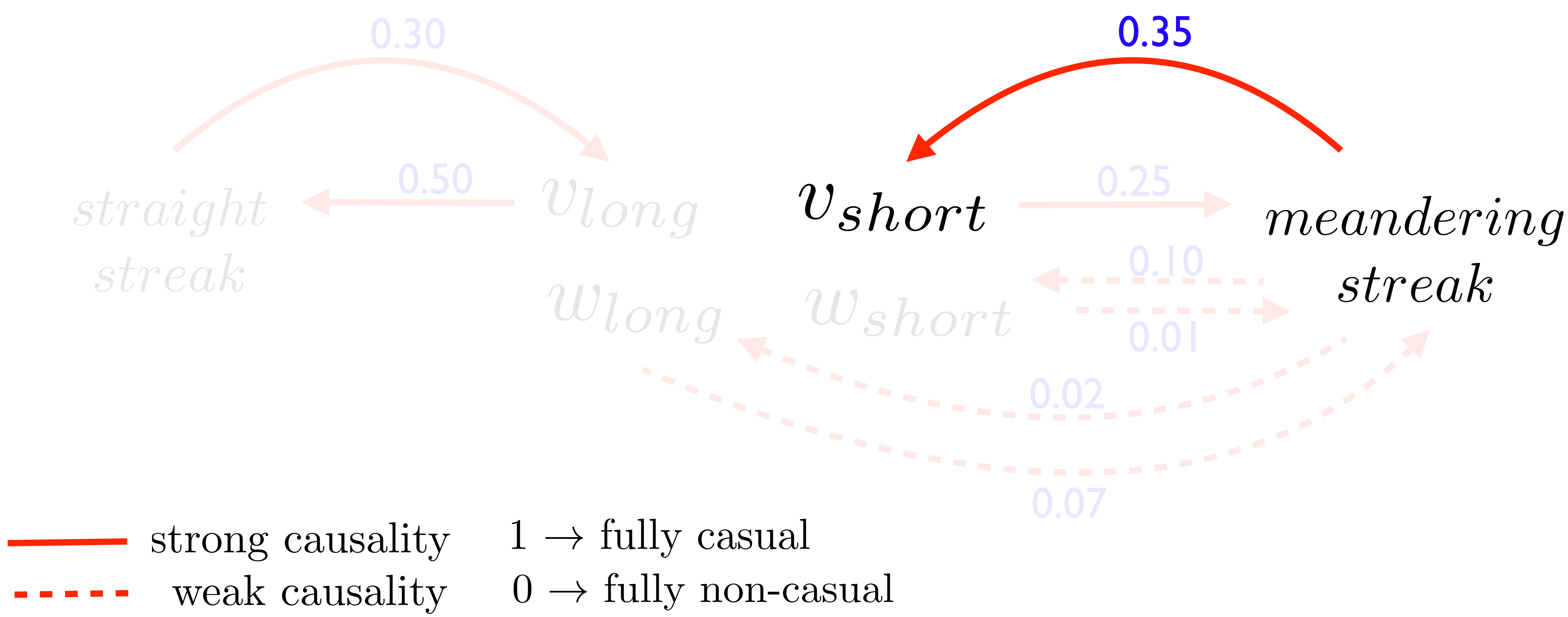

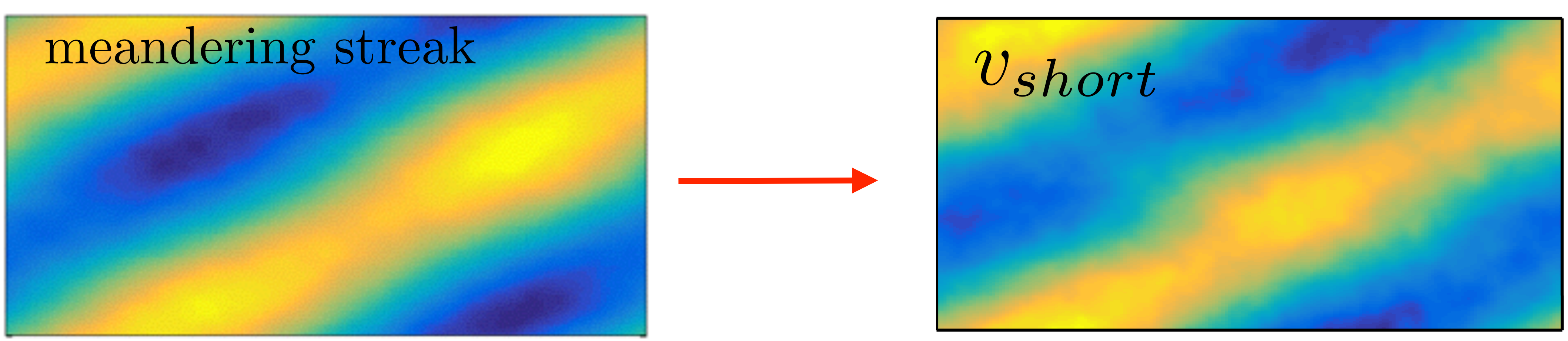

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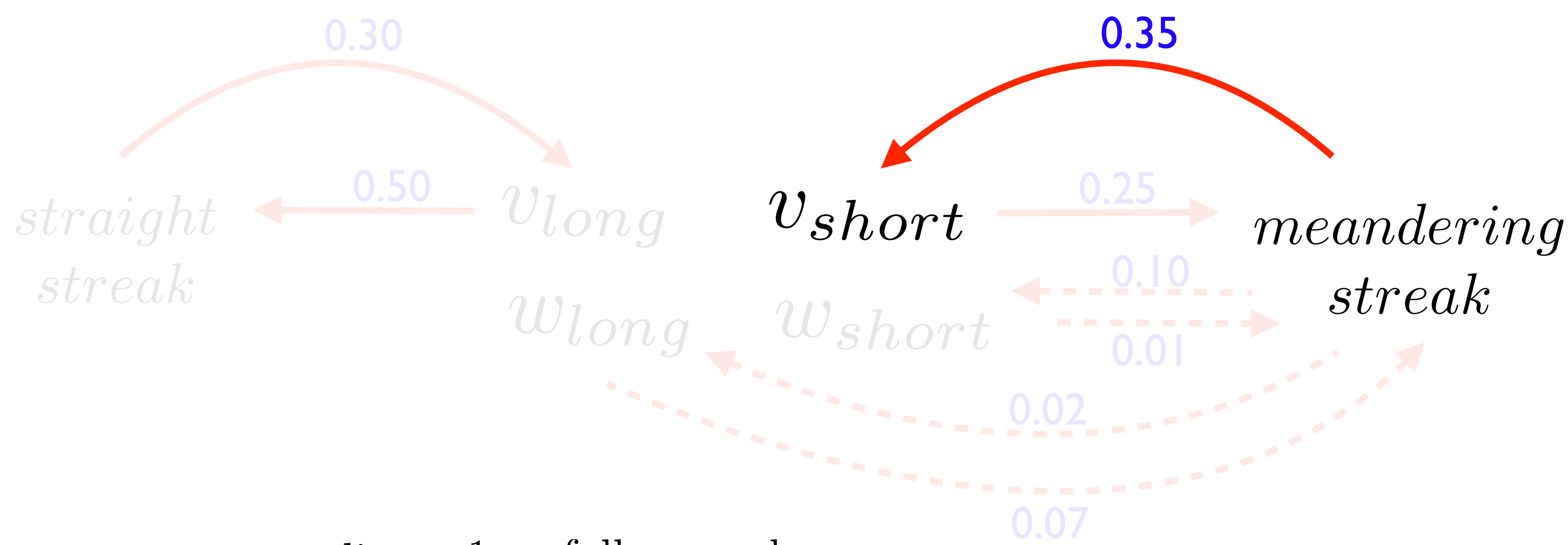

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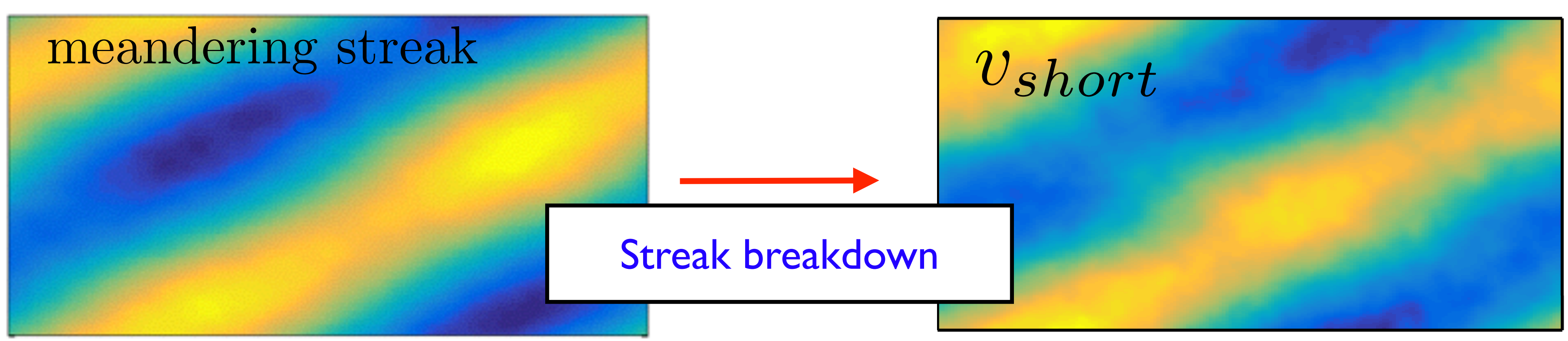

Conclusions

- Causal inference is not straight forward for most common tools used in turbulence research
- Proposed causality analysis based on information flow as a new tool for investigating turbulence dynamics
- Data-driven method with low computational cost and applicable to a wide variety of scenarios
- Application to self-sustaining process in the log-layer of wall-bounded turbulence
- Rolls and straight/meandering streaks identified by POD
- Lift-up, streak breakdown and nonlinear effects quantified