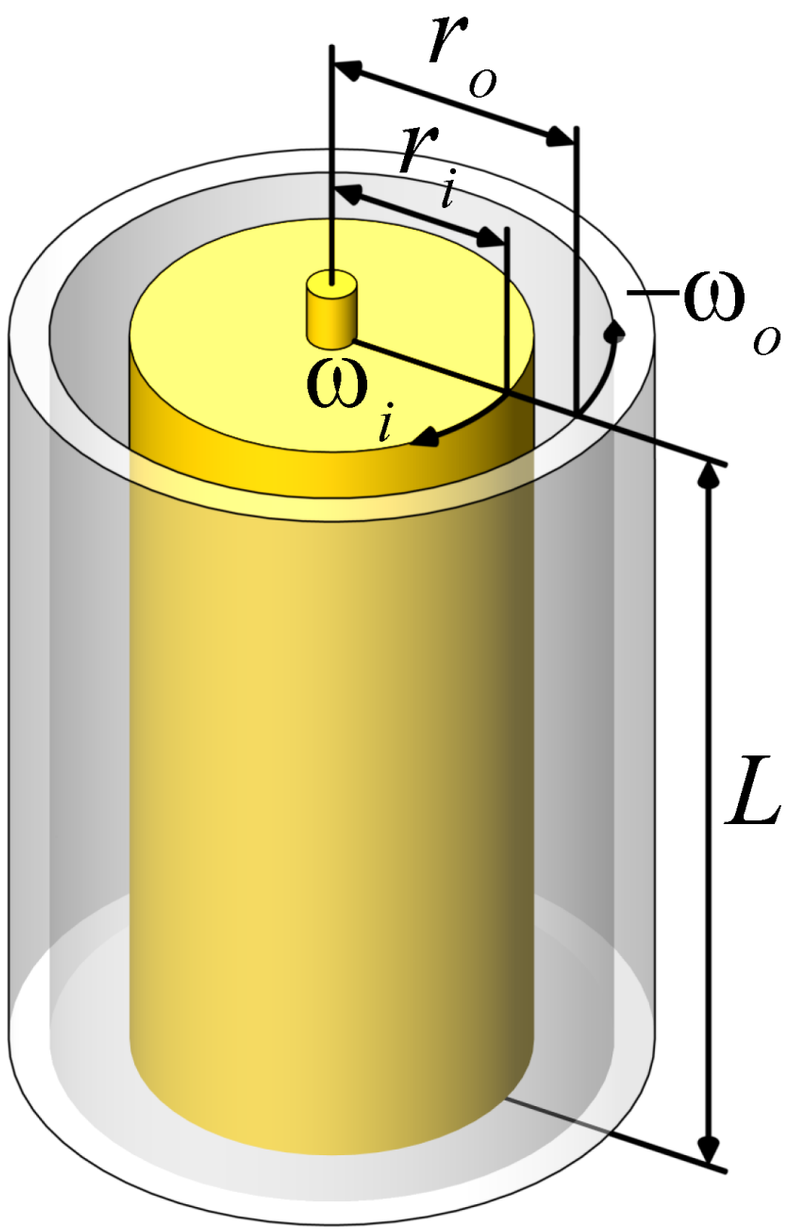
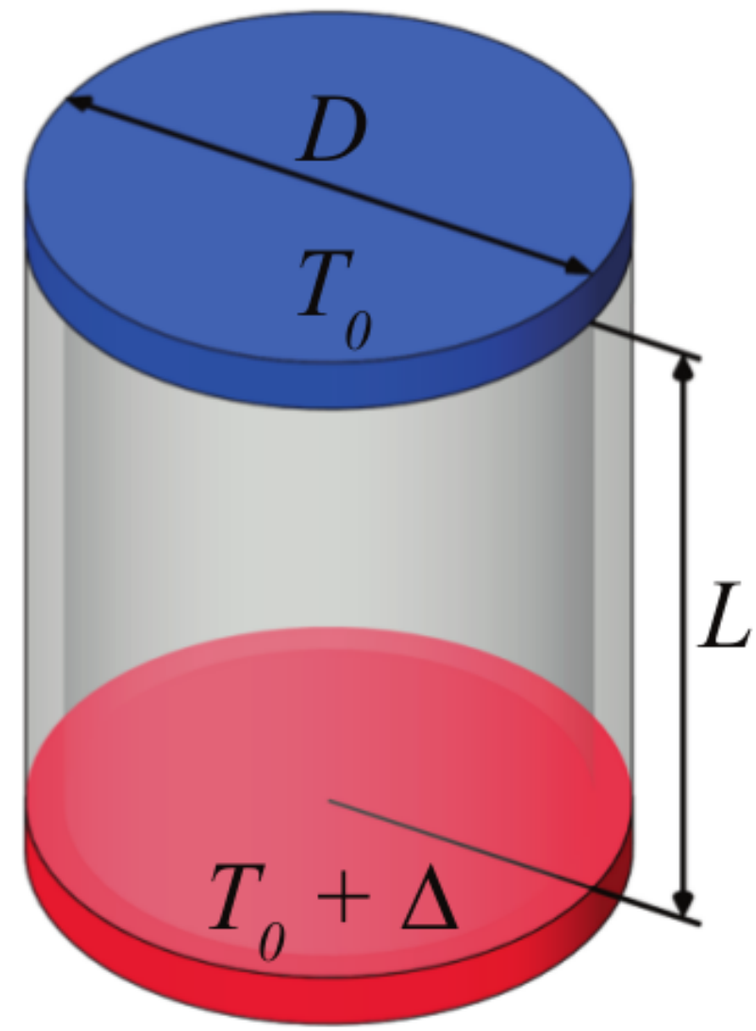
Convection: from small plumes to large coherent turbulent structures

Rodolfo Ostilla Mónico
(and many others)



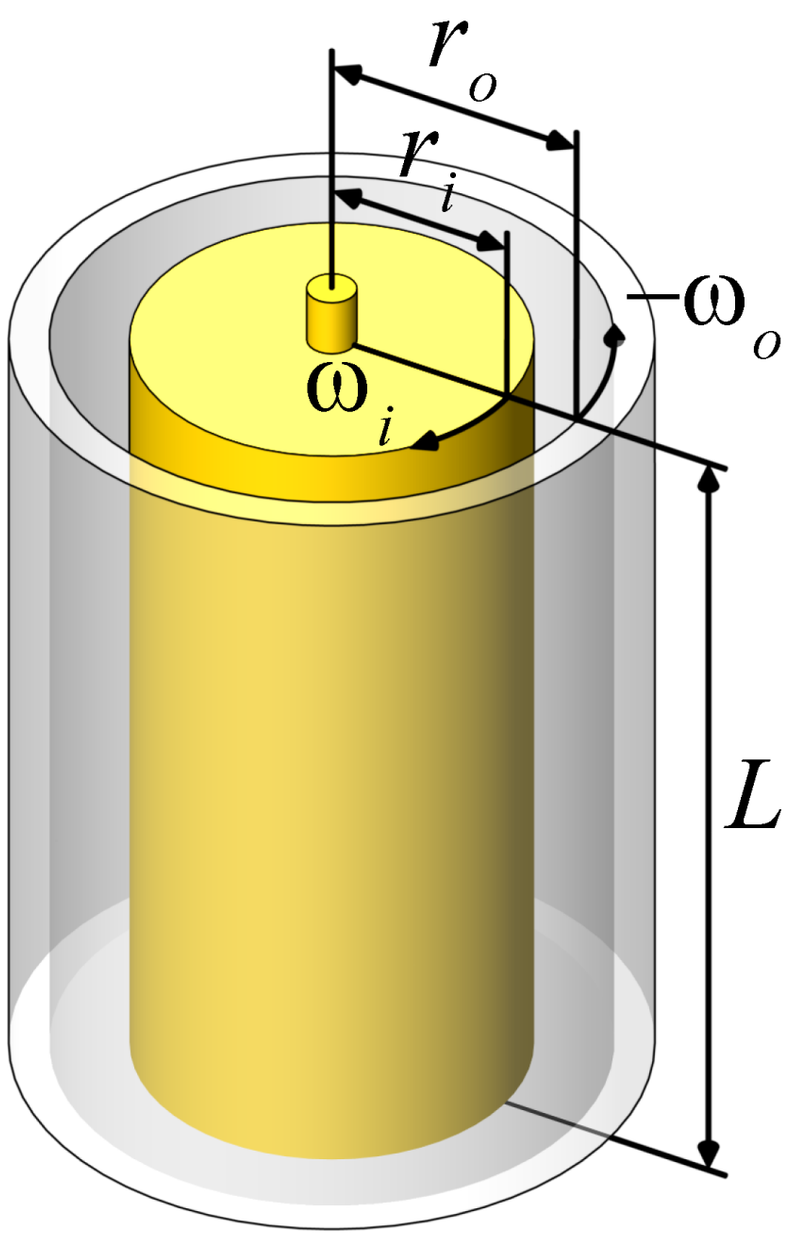
Taylor-Couette flow

Angular momentum convection



Rayleigh-Bénard flow

Heat convection



$$d = r_o - r_i$$

Geometric parameters

$$\eta = \frac{r_i}{r_o} \qquad \Gamma = \frac{L}{d}$$

Driving parameters

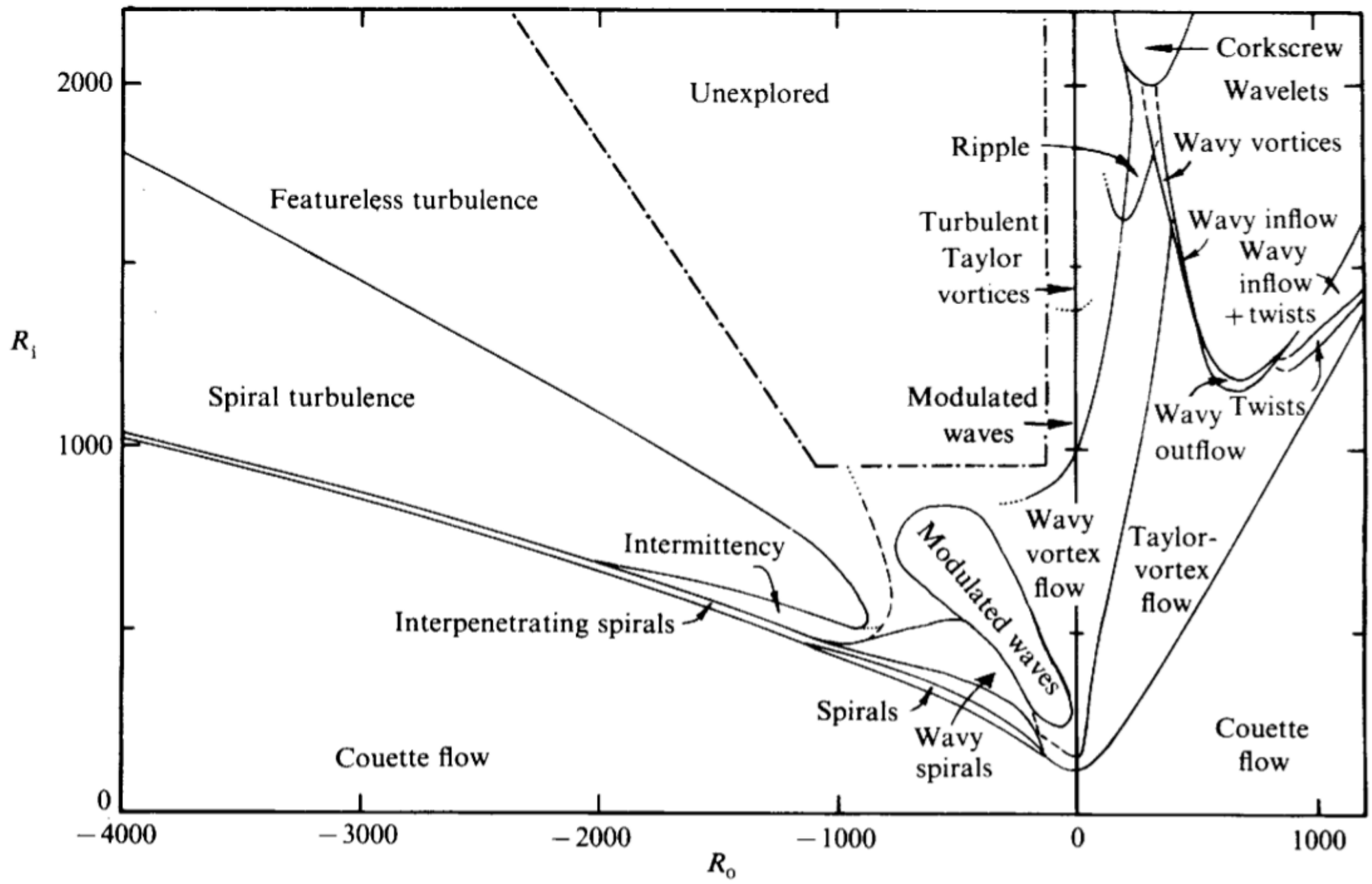
$$Re_i = \frac{r_i \omega_i d}{\nu} \qquad Re_o = \frac{r_o \omega_o d}{\nu}$$

Shear + rotation parameters

$$Re_s = \frac{1+\eta}{2}(Re_i - \eta Re_o)$$

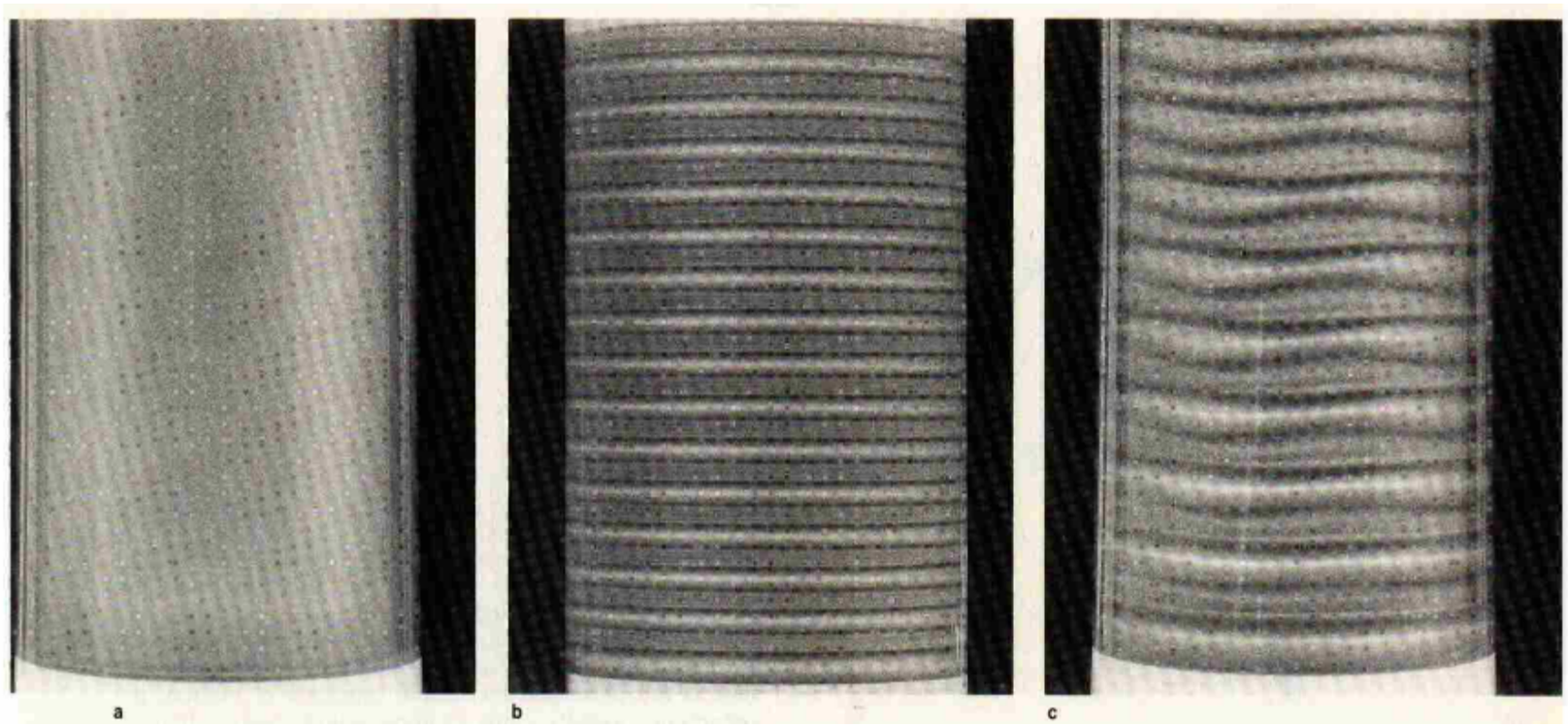
$$\mu = \frac{\omega_o}{\omega_i}$$

Rich flow structures at low Reynolds



Andereck, Liu, Swinney, J. Fluid Mech., 164 (1986): 155-183

The transition to turbulence of TC flow



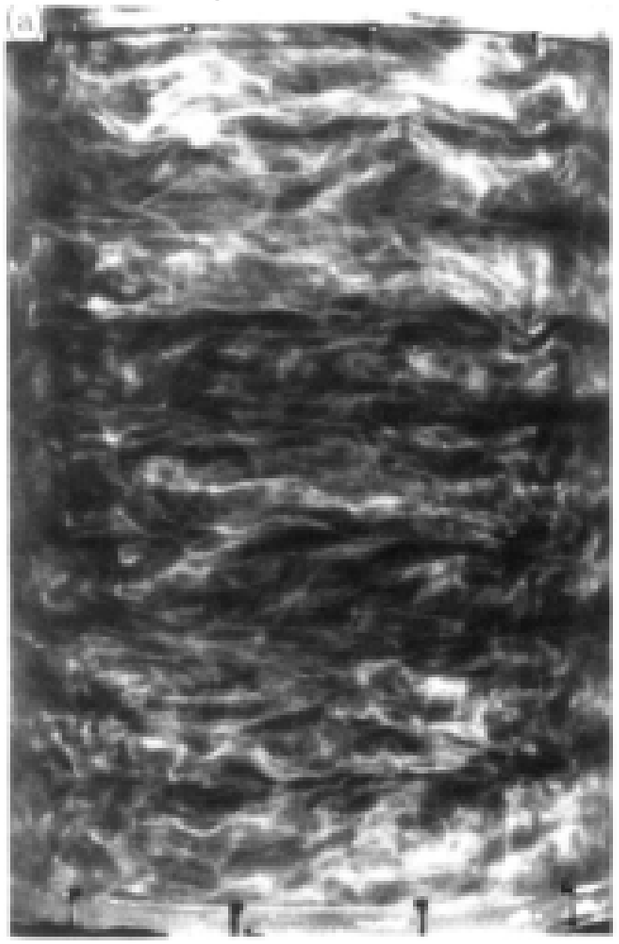
Increasing Reynolds number

R. Donnelly, "Taylor-couette flow: the early days", Physics Today 1991

Do the rolls dissapear at large Re?

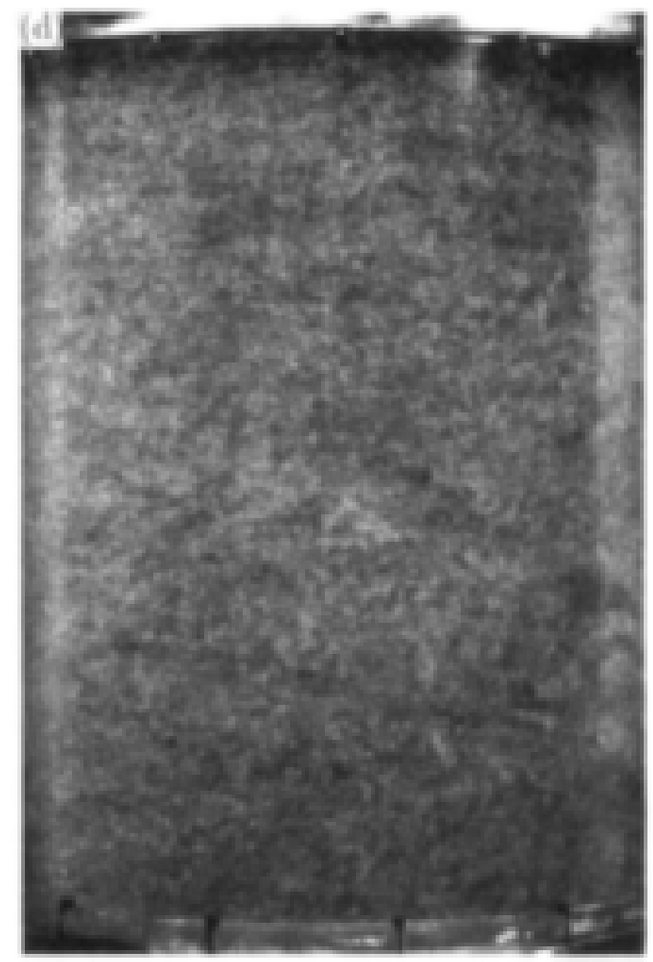
$Re_s \sim 10^4$

(a)



$Re_s \sim 10^5$

(b)

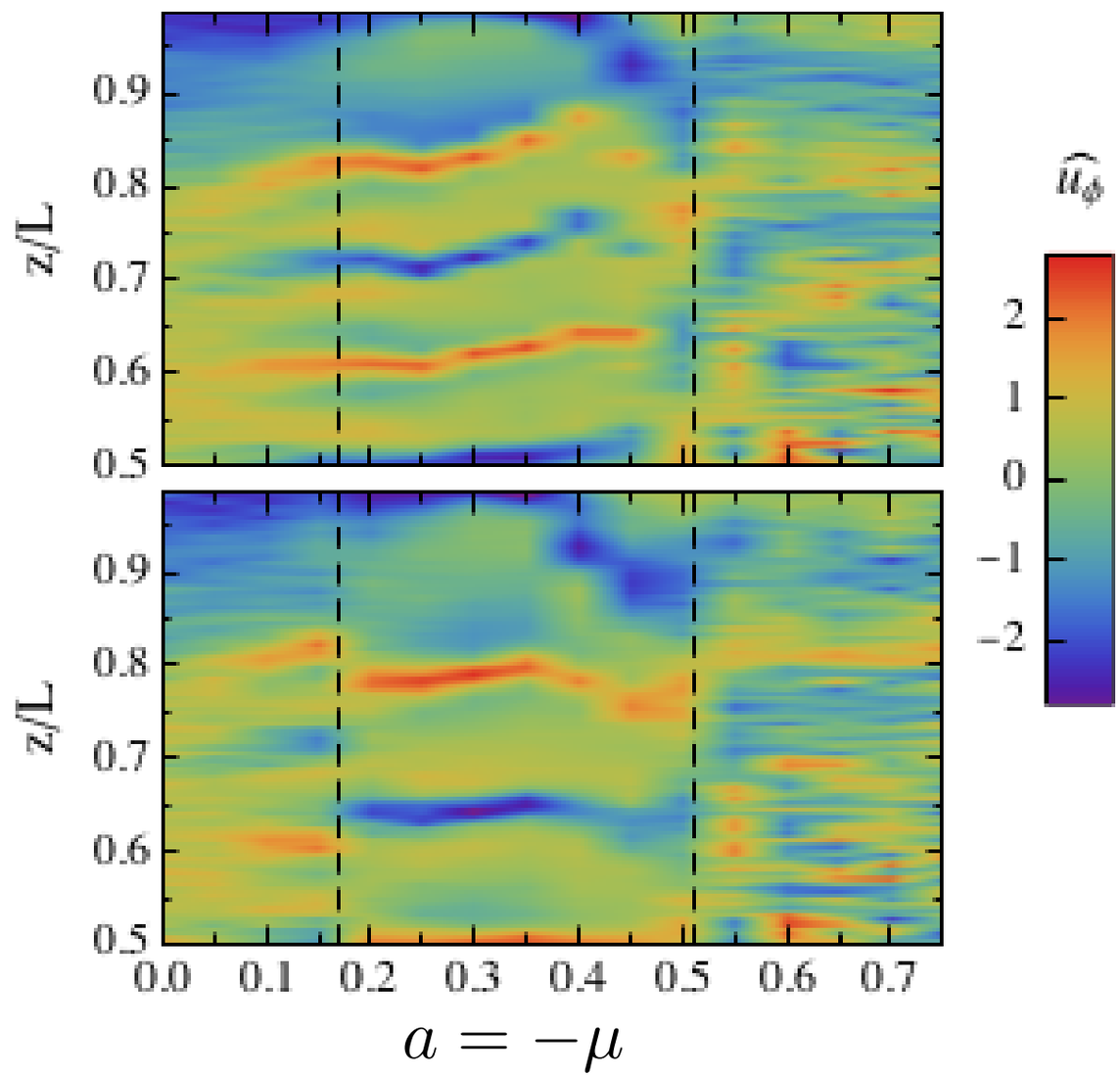


$\eta = 0.714$

Increase Reynolds enough

Lathrop DP, Fineberg J, Swinney HS. 1992. Phys. Rev. A 46:6390–6405

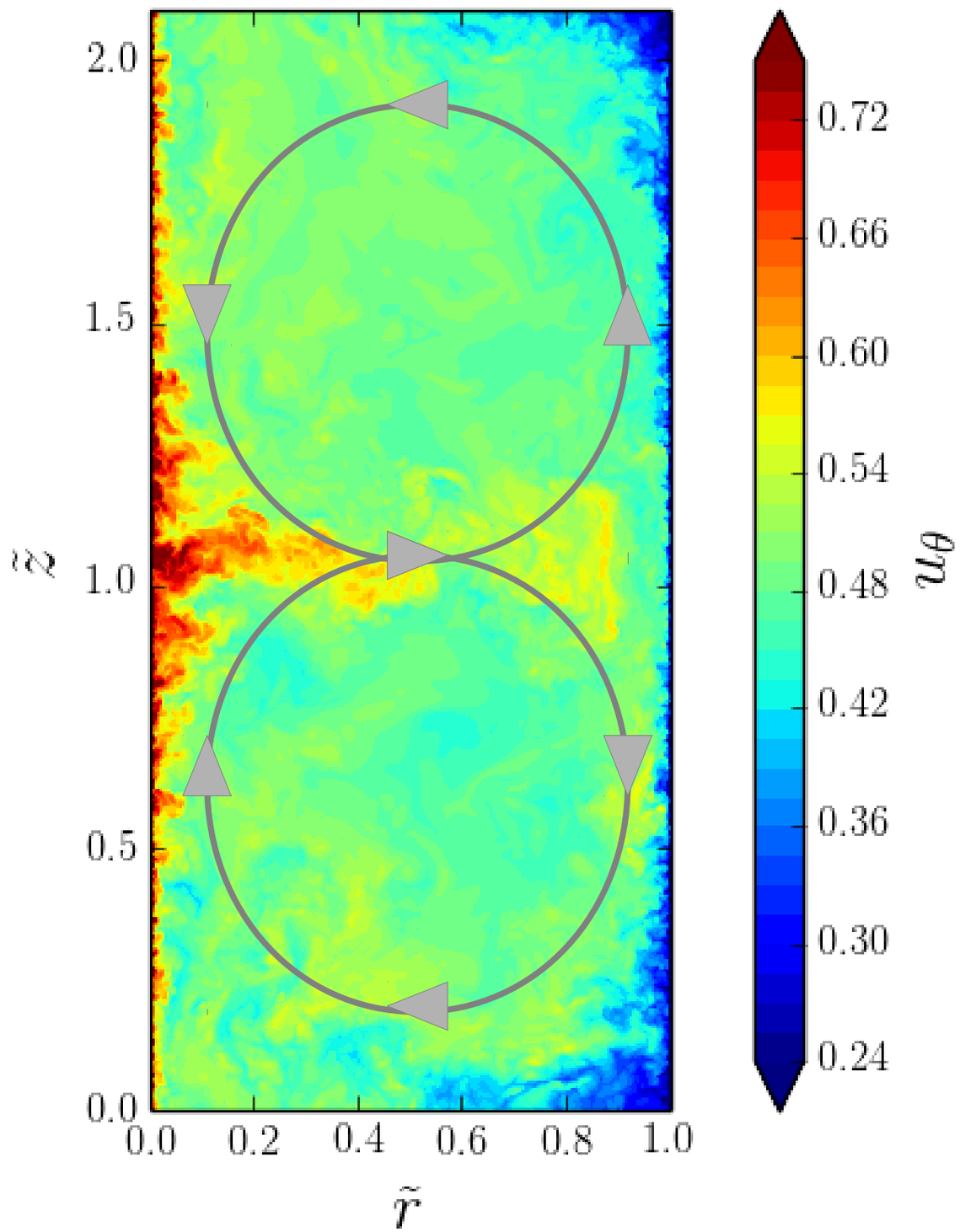
Seen in some experiments at high Reynolds number



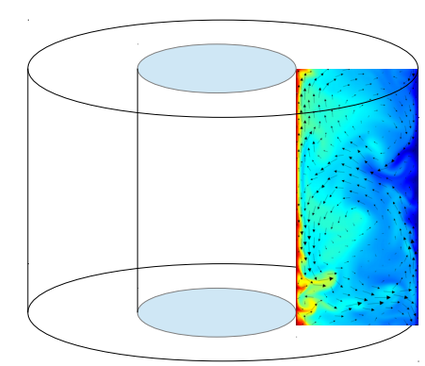
$Re_s \approx 10^6$
$\eta = 0.714$

Huisman SG, van der Veen RCA, Sun C, Lohse D. 2014. Nat. Commun. 5:3820

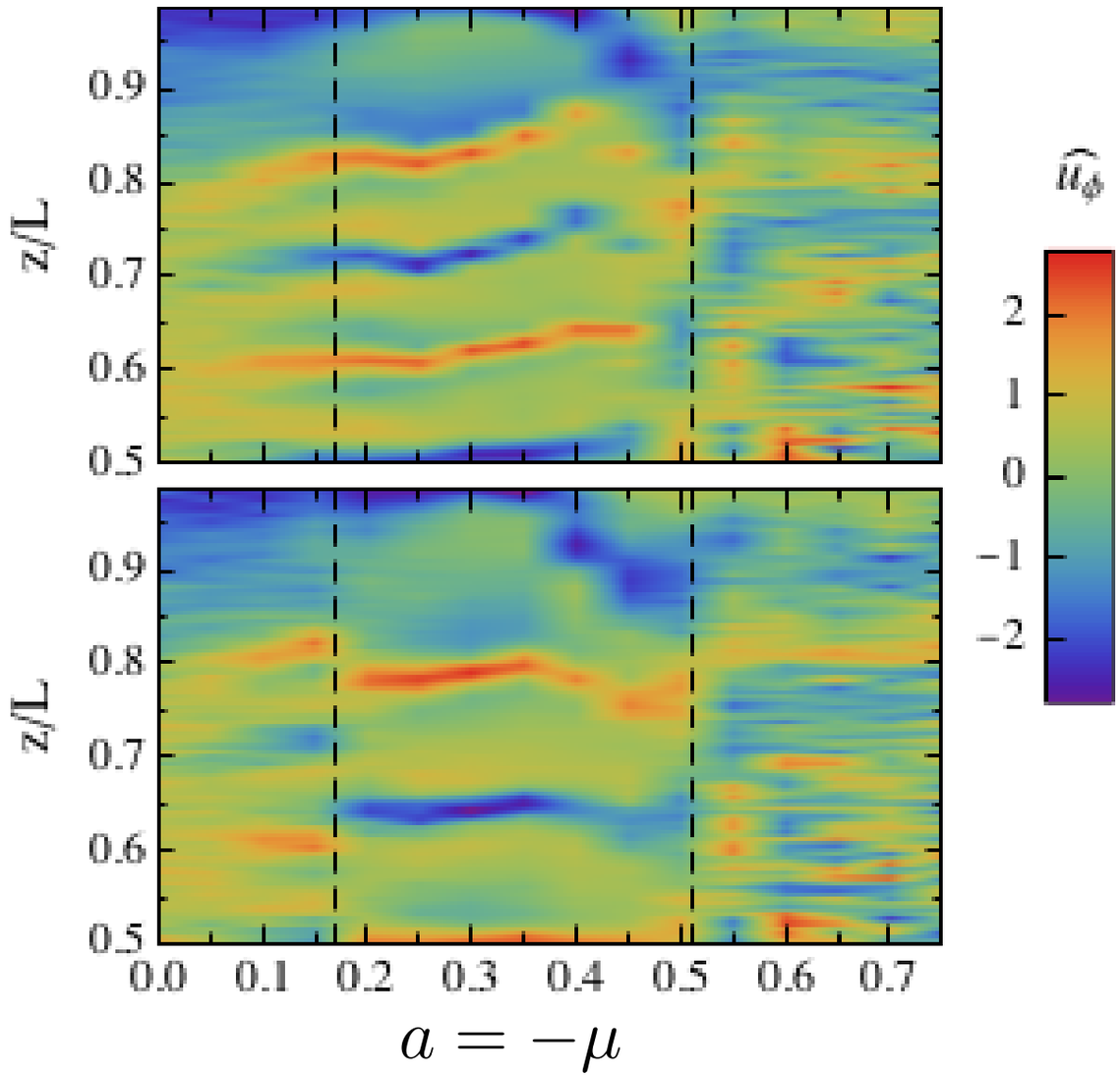
Also in DNS with periodic conditions



$$\eta = 0.909$$
$$Re_s = 10^5$$
$$\Gamma = 2$$
$$\omega_o = 0$$



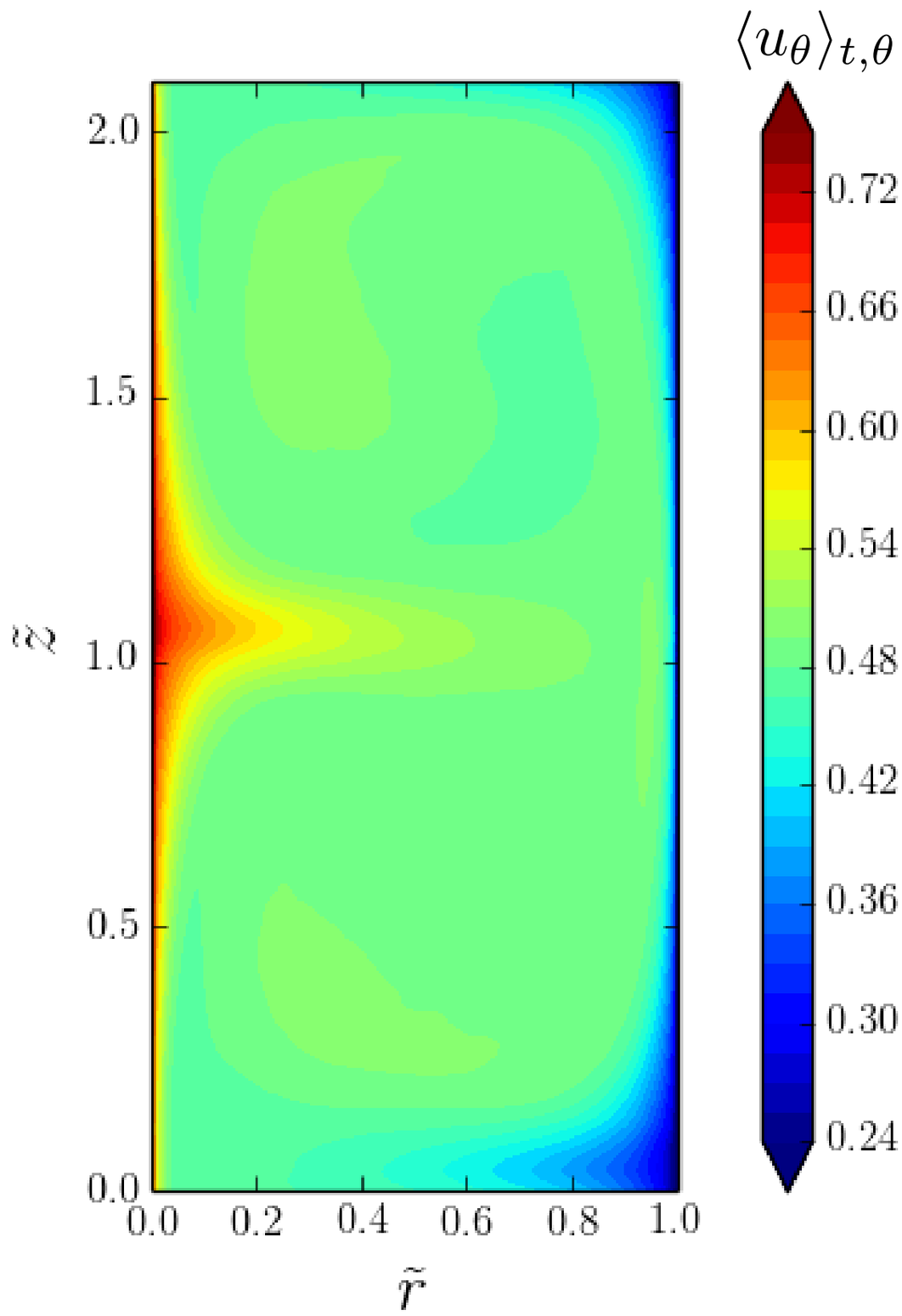
Seen in some experiments at high Reynolds number



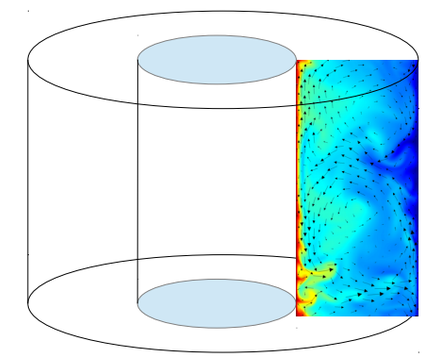
$Re_s \approx 10^6$

Huisman SG, van der Veen RCA, Sun C, Lohse D. 2014. Nat. Commun. 5:3820

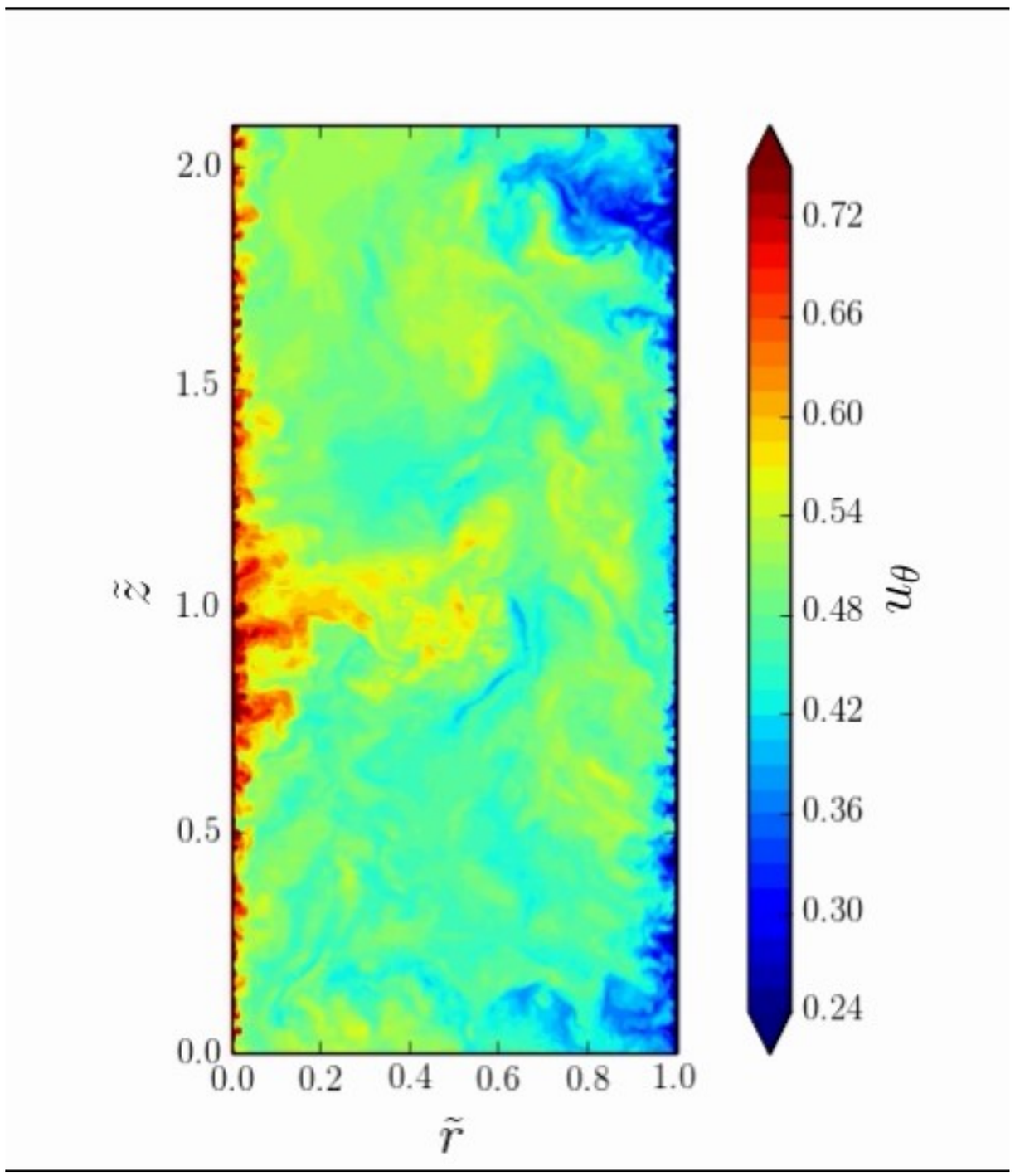
Rolls are persistent in time

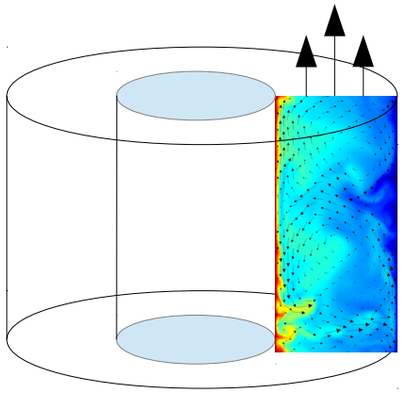


$$\eta = 0.909$$
$$Re_s = 10^5$$
$$\Gamma = 2$$
$$\omega_o = 0$$

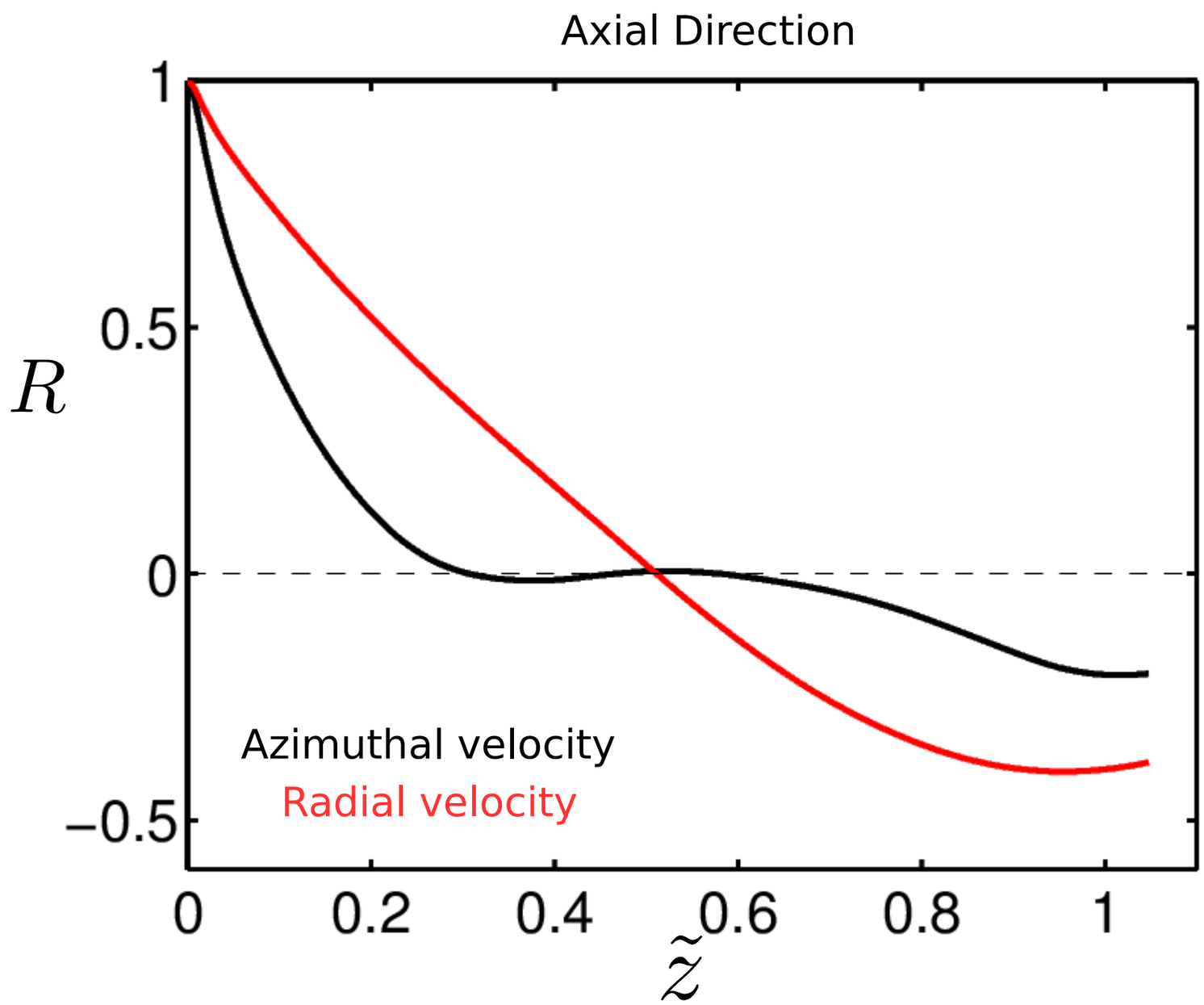


They seem to be resistant to axial flow



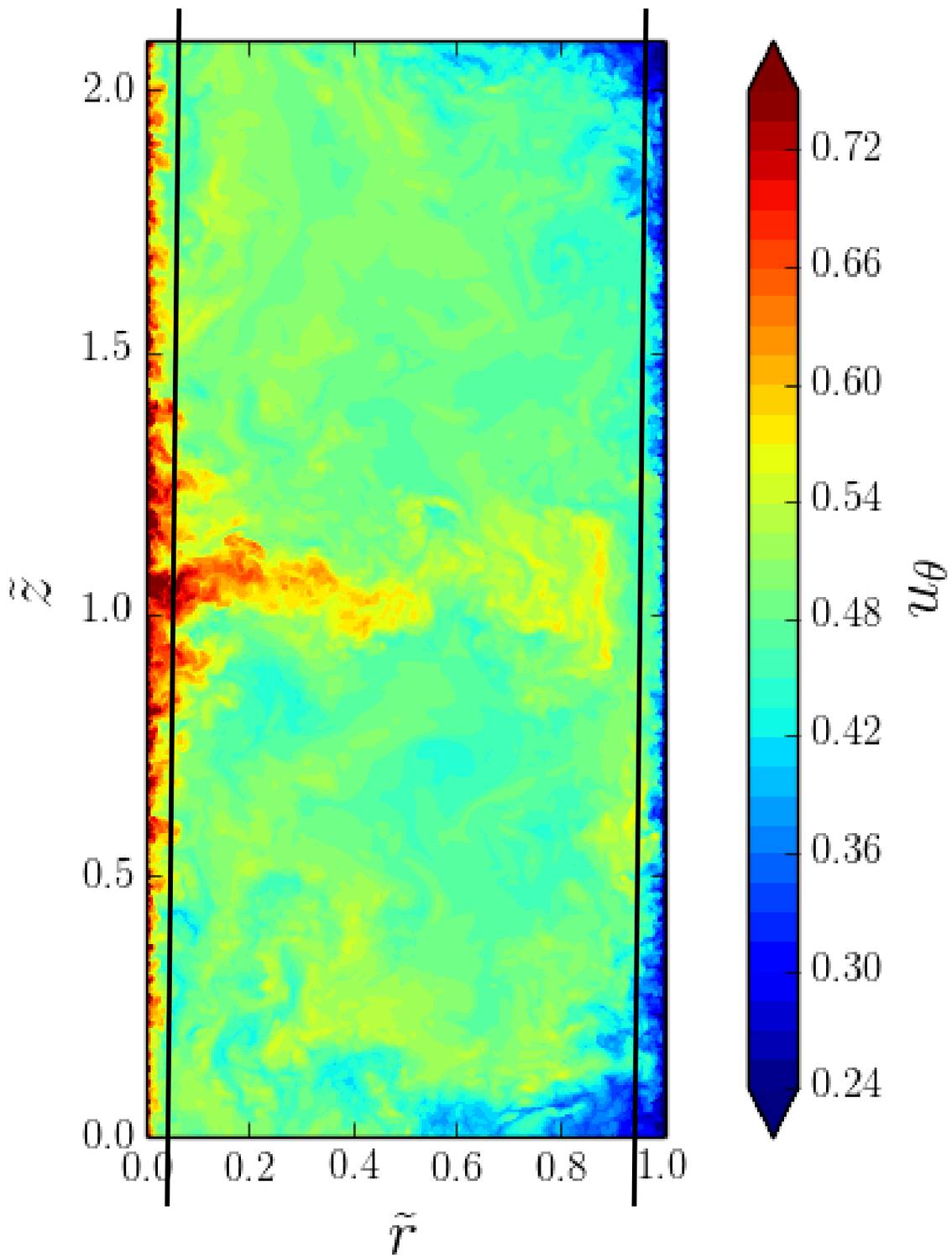


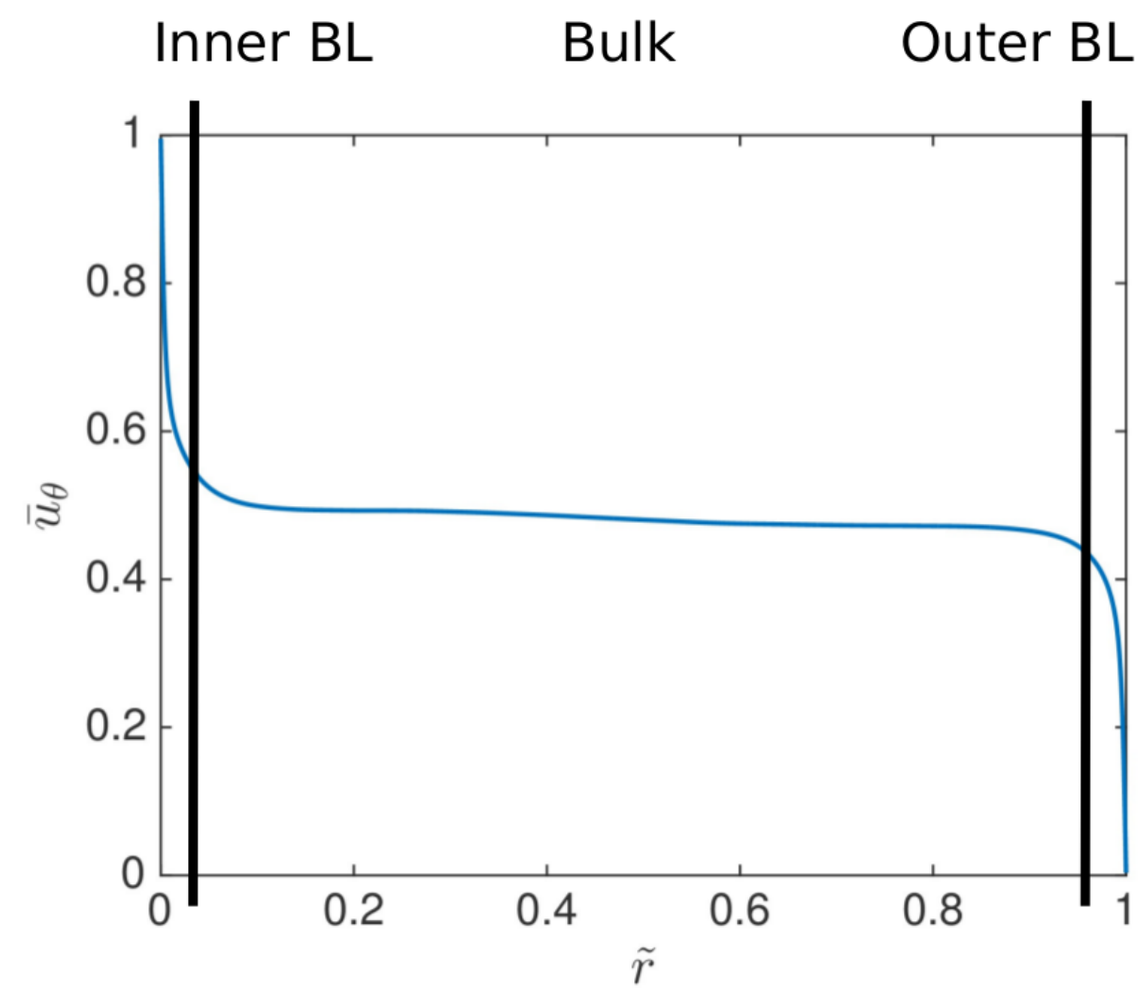
Axial autocorrelations with imposed flow



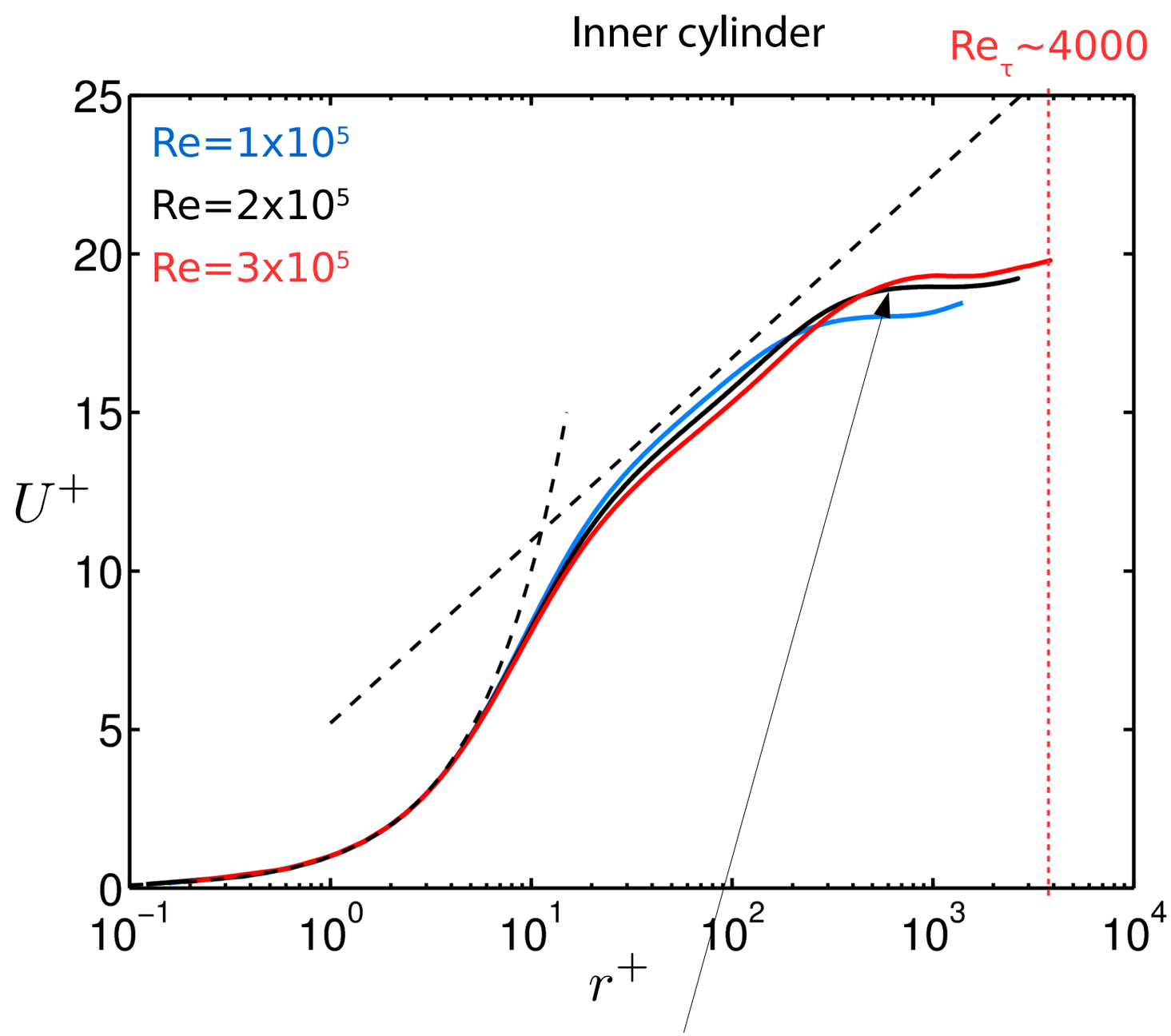
$$\eta = 0.909$$
$$Re_s = 10^5$$
$$\Gamma = 2$$
$$\omega_o = 0$$

Rolls cause a very clear separation of BL & bulk





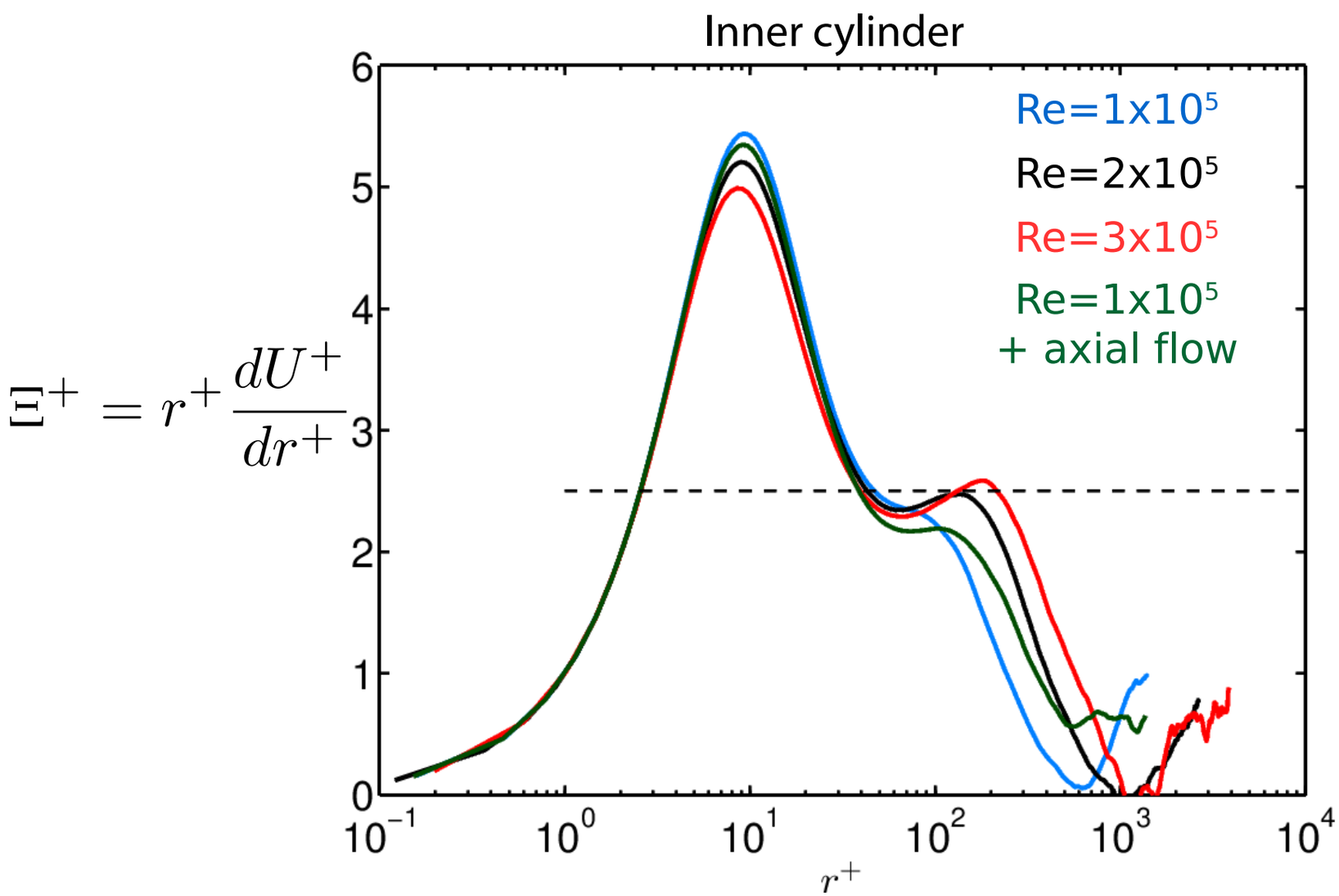
Boundary layers show log-layers



$$\eta = 0.909$$
$$\Gamma = 2$$
$$\omega_o = 0$$

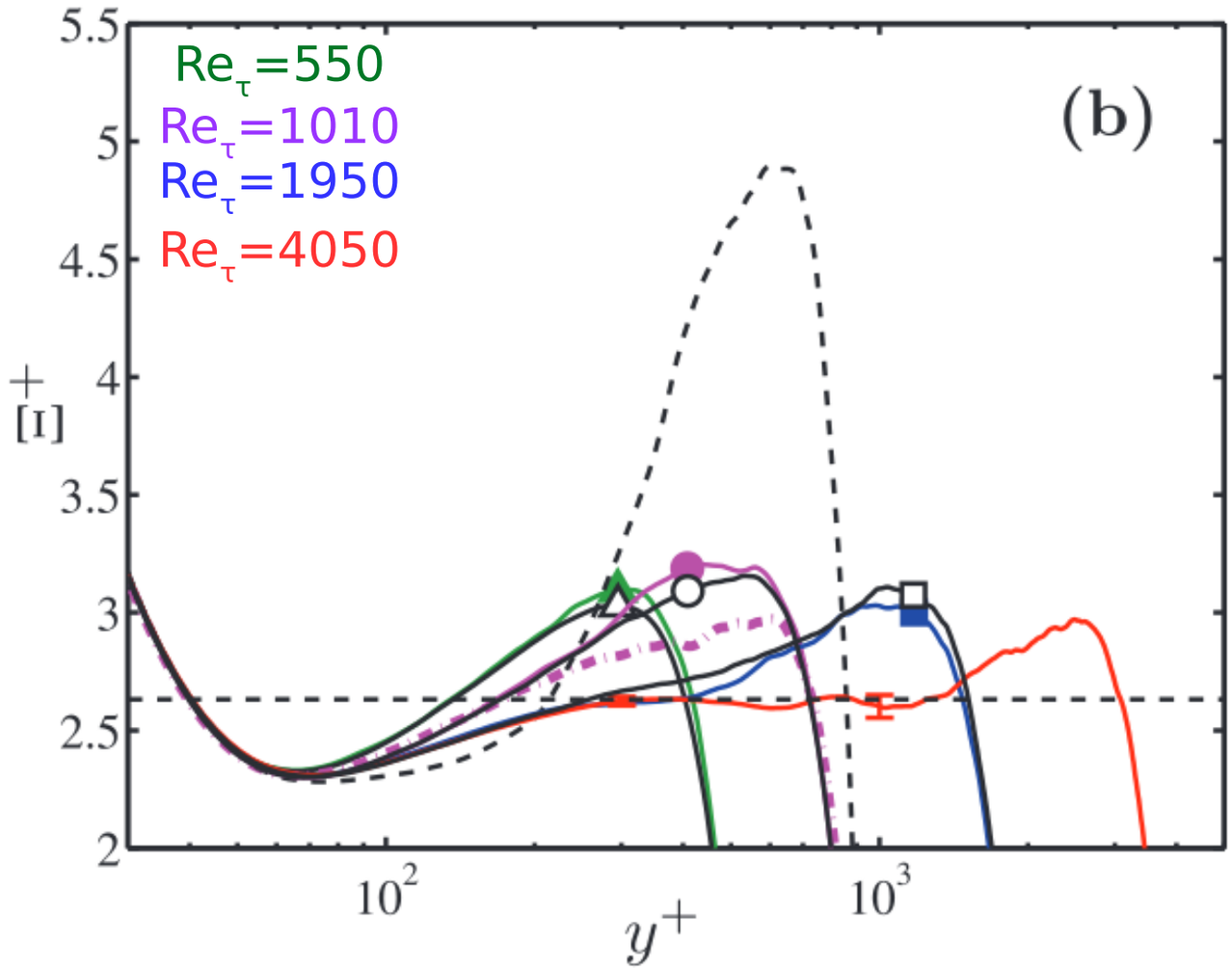
Profiles "bend" around 5% gap width: flat bulk

How logarithmic are the profiles?



$$\eta = 0.909$$
$$\Gamma = 2$$
$$\omega_o = 0$$

S-like behaviour in similar Re_τ channels

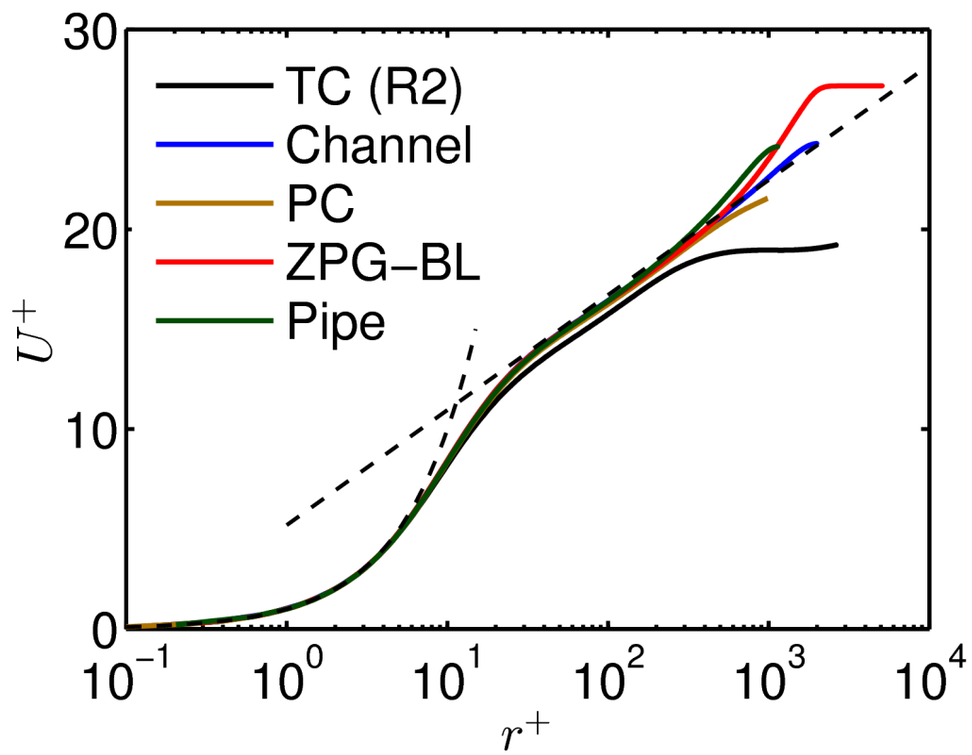


$$\Xi^+ = y^+ \frac{dU^+}{dy^+}$$

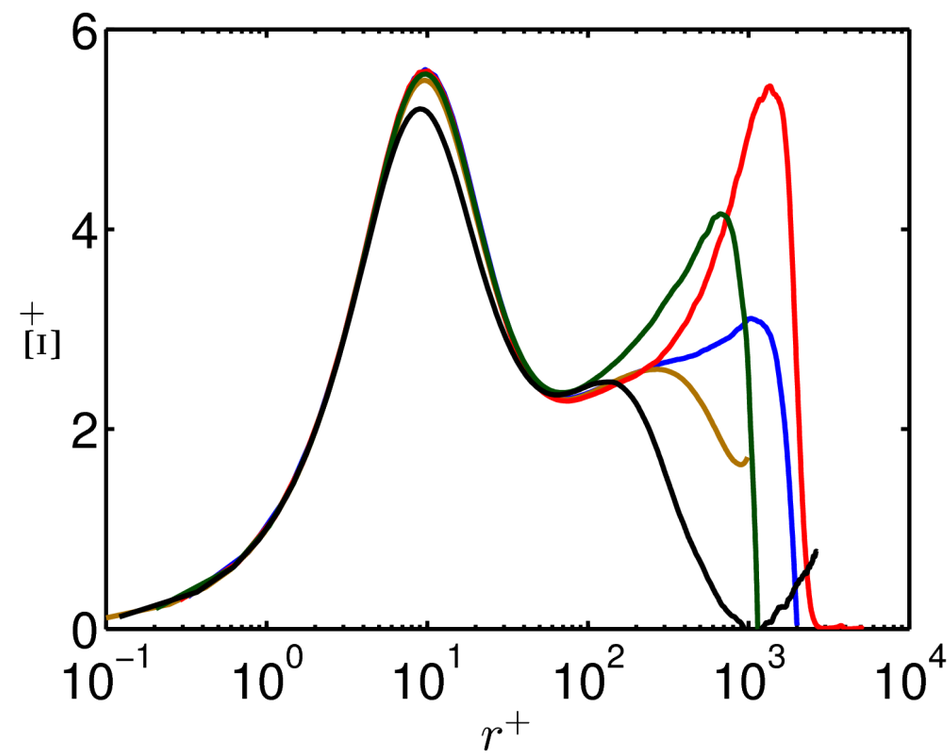
Lozano-Durán, Jiménez, Phys. Fluids , 26 (2014), 011702

Mean profile looks similar to other flows

Streamwise velocity profile for $Re_\tau = 2000$

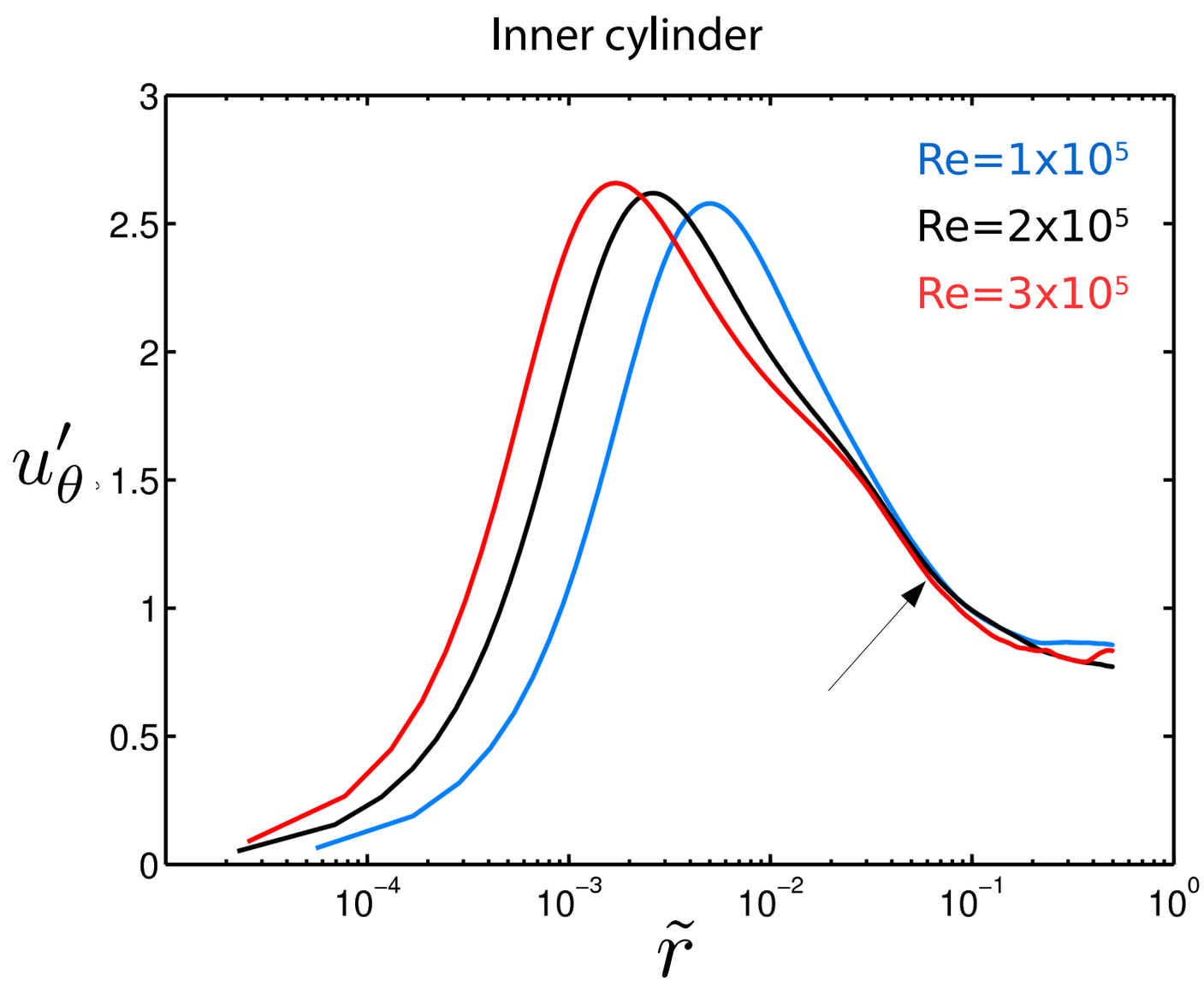


$$\Xi^+ = r^+ \frac{dU^+}{dr^+}$$



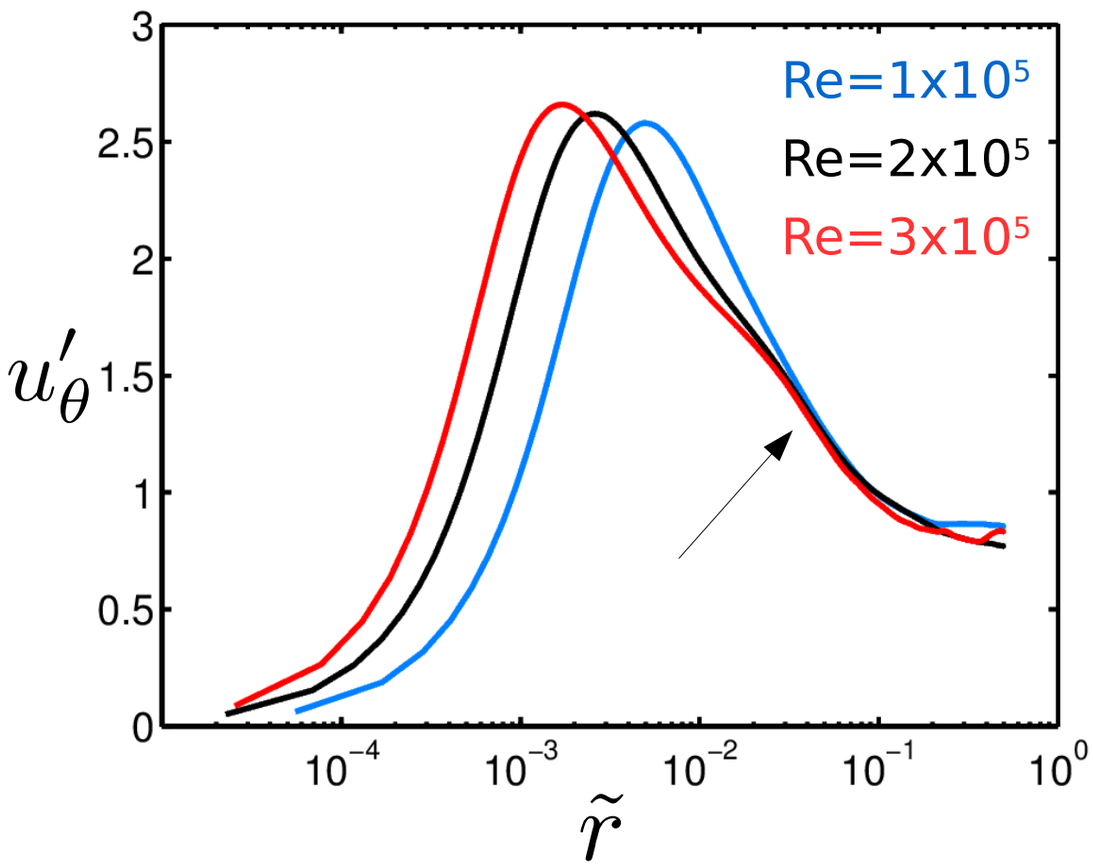
Is this "log-layer" behaviour apparent in other statistics of TC flow?

Overlap layers appear for the fluctuations

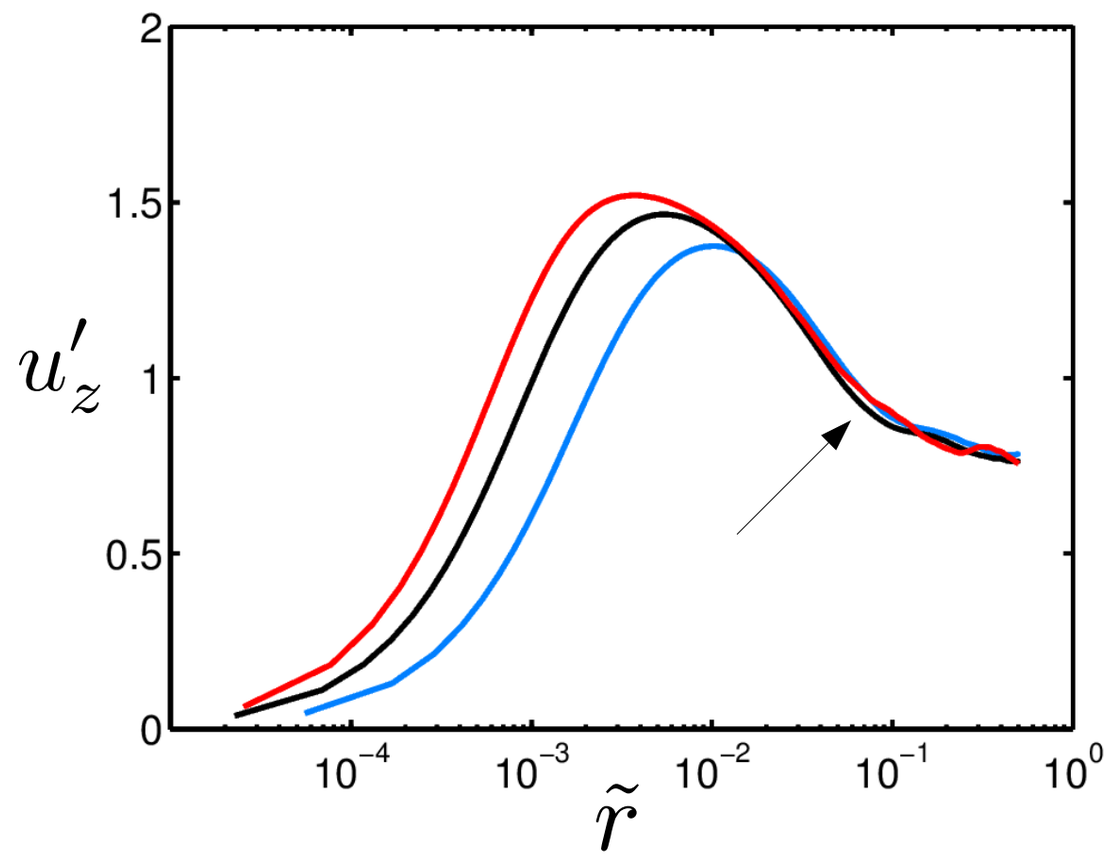


Overlap layer in velocity fluctuations

Streamwise (azimuthal) velocity fluctuations



Spanwise (axial) velocity fluctuations



Re_τ is too small to see overlap in u'_r

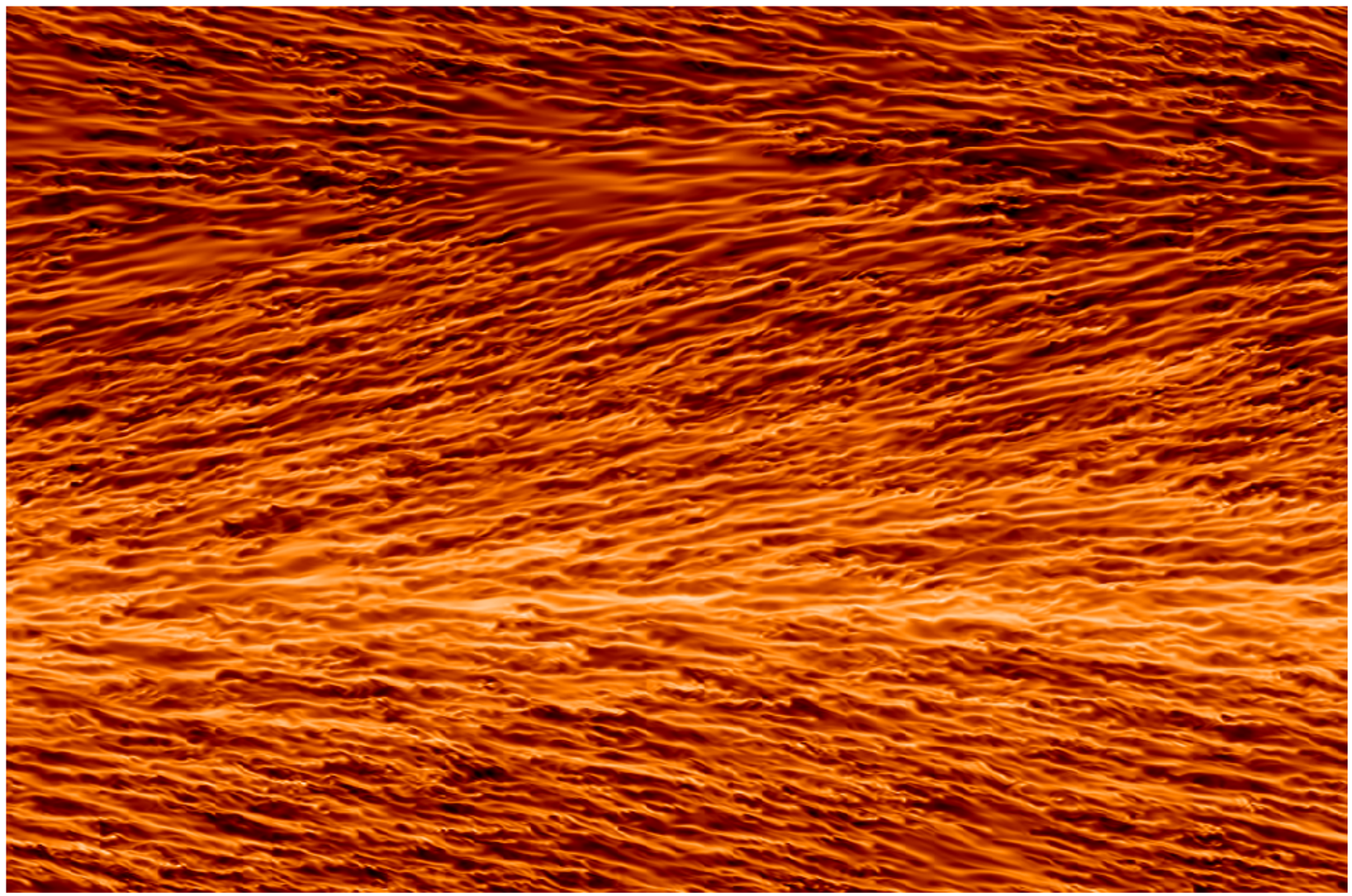
$4(d/2)$

Rolls show up inside the boundary layers

Spanwise length

$y^+ \approx 15$

$4(d/2)$



$\tilde{z}$

$\eta = 0.909$

$Re_s = 10^5$

$\Gamma = 2$

$\omega_o = 0$

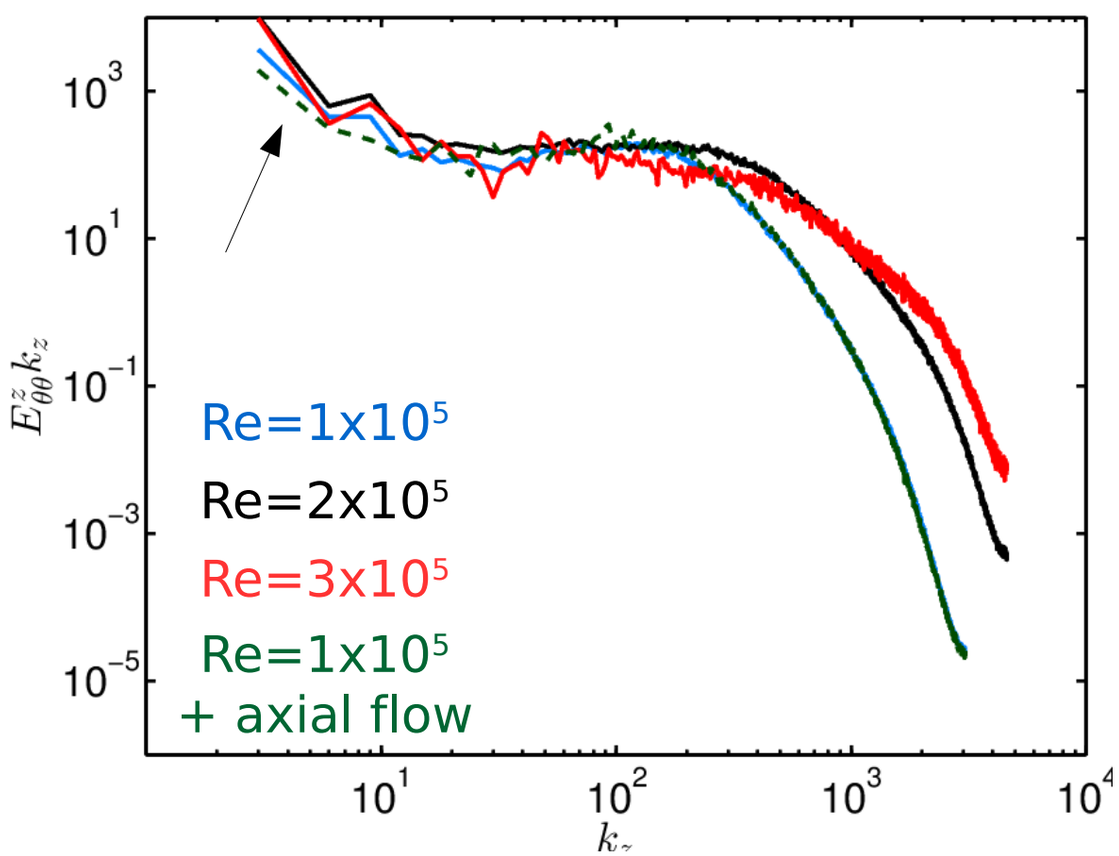
0

$2\pi(d/2)$

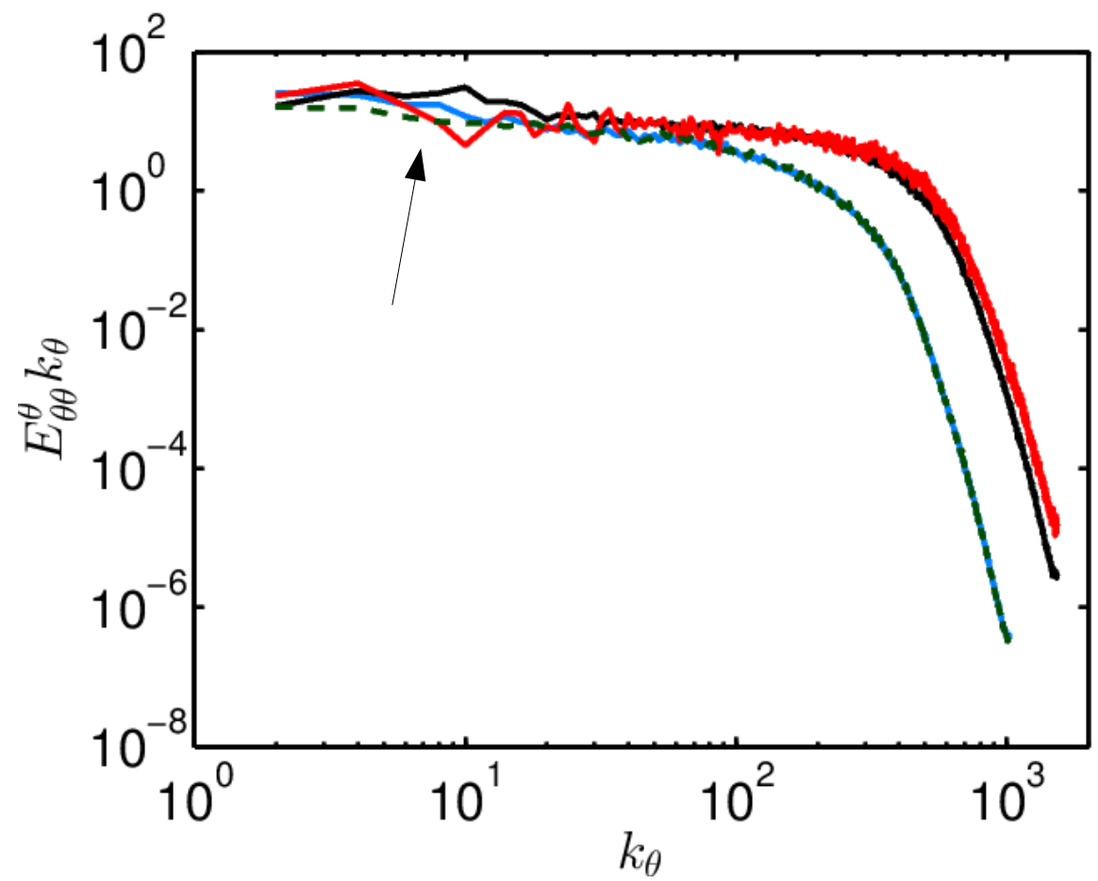
Streamwise length

Look at the azimuthal velocity spectra

Premultiplied azimuthal spectra at $y^+ \approx 15$



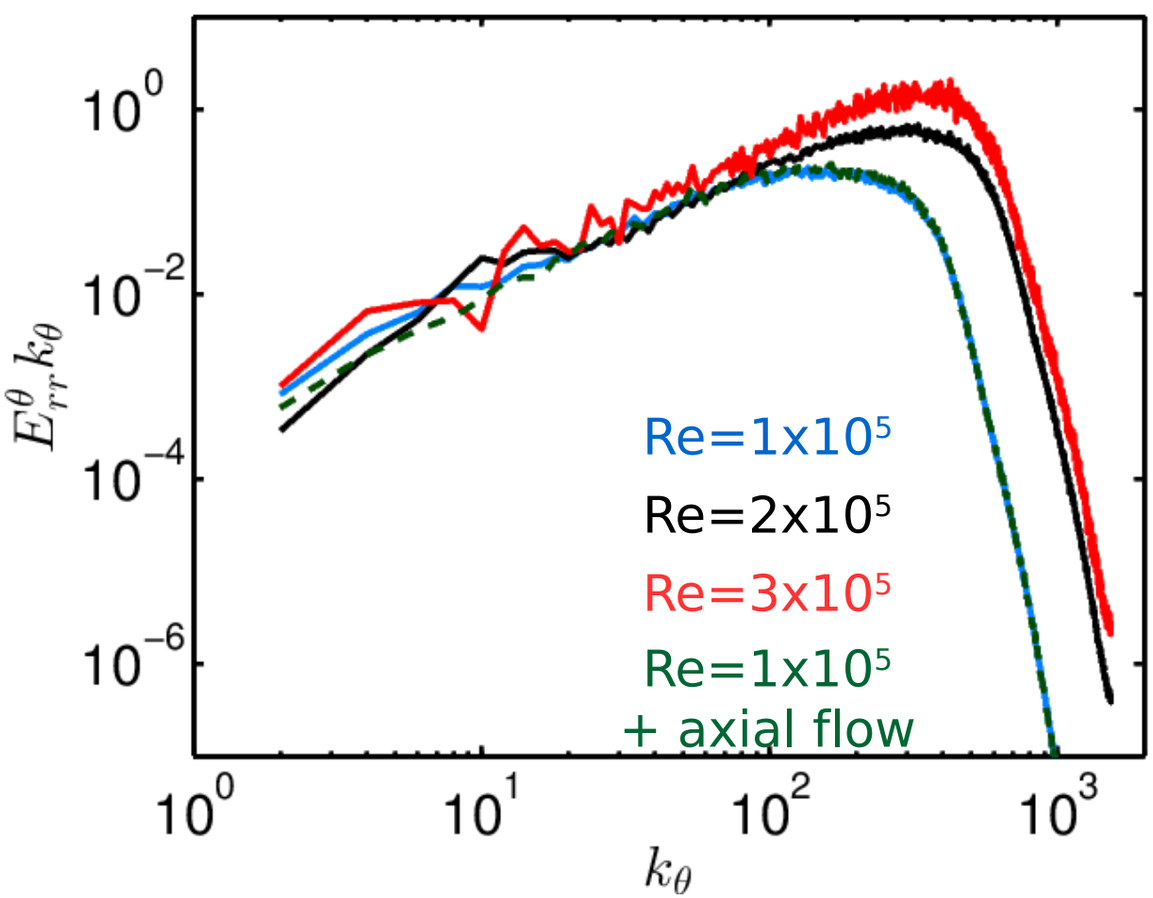
Premultiplied axial spectra at $y^+ \approx 15$



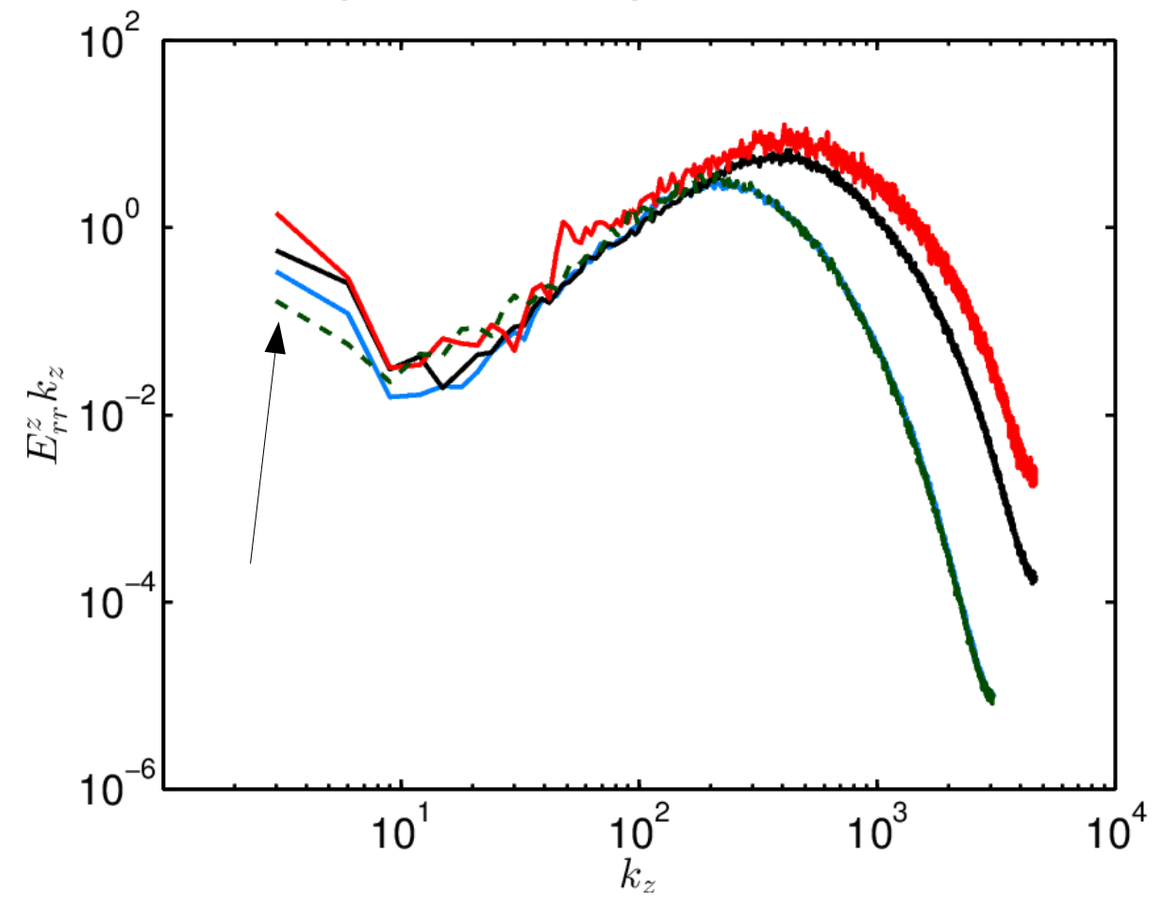
Large-scale rolls are attached to the wall

What about the radial velocity?

Premultiplied azimuthal spectra at $y^+ \approx 15$



Premultiplied axial spectra at $y^+ \approx 15$

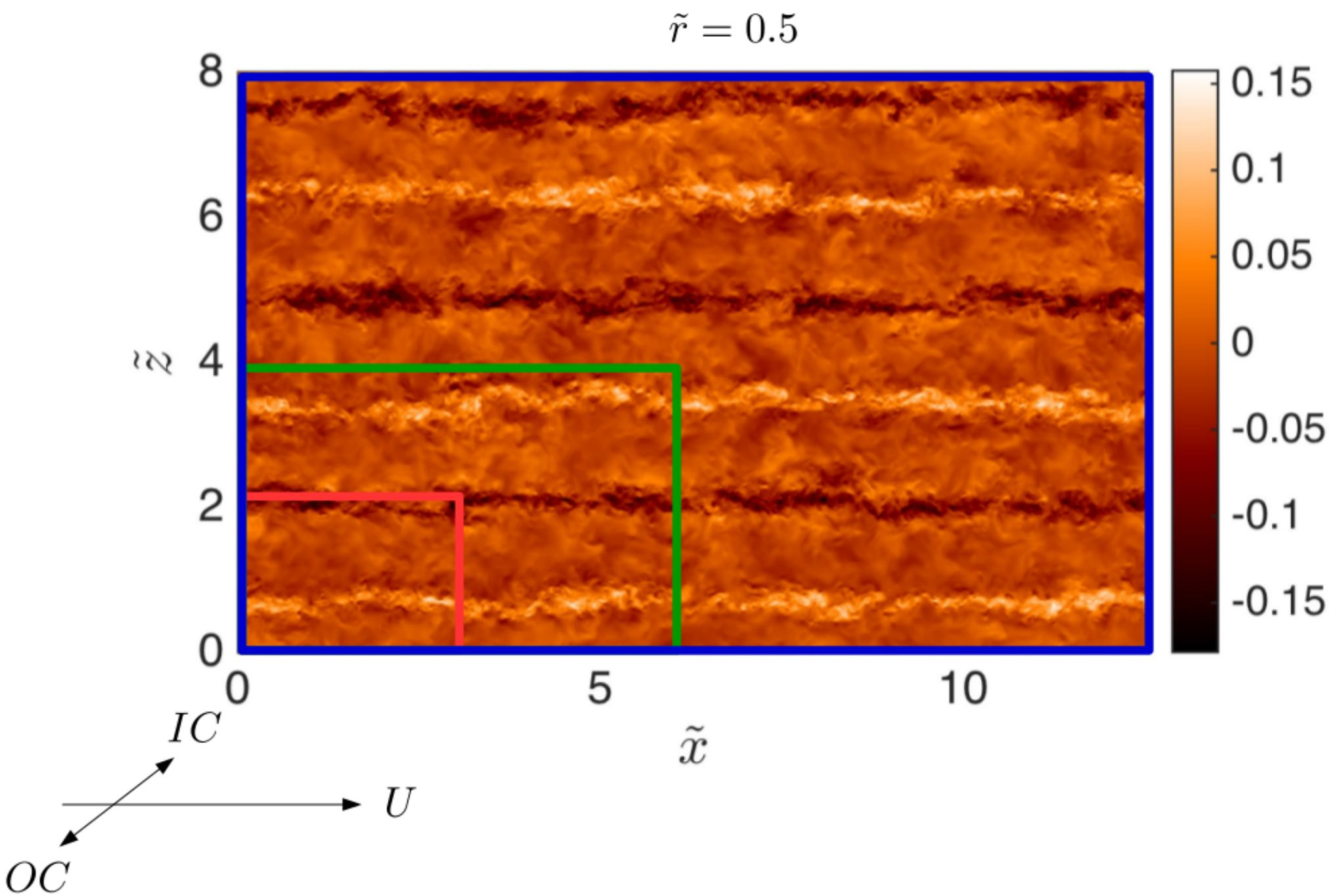


Ostilla-Mónico, Verzicco, Lohse, J. Fluid Mech. (2016)

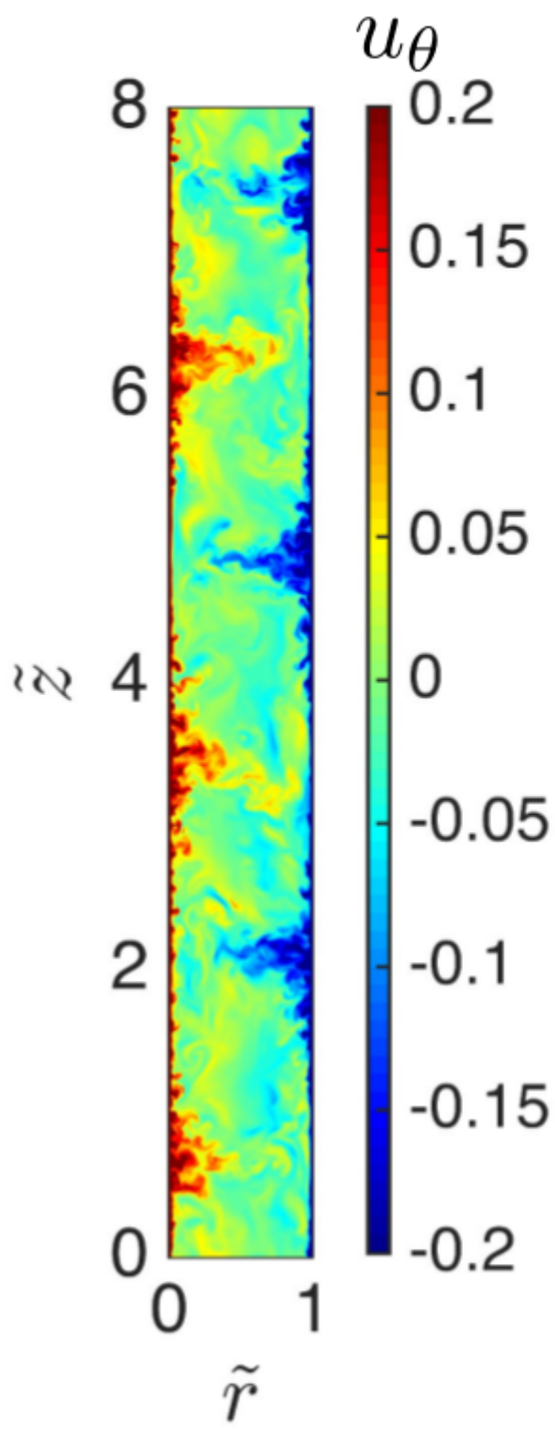
Rolls are active, they transport angular velocity near the wall

There is a maxima in the cospectra for axisymmetric rolls inside the BL

Going to large computational boxes...



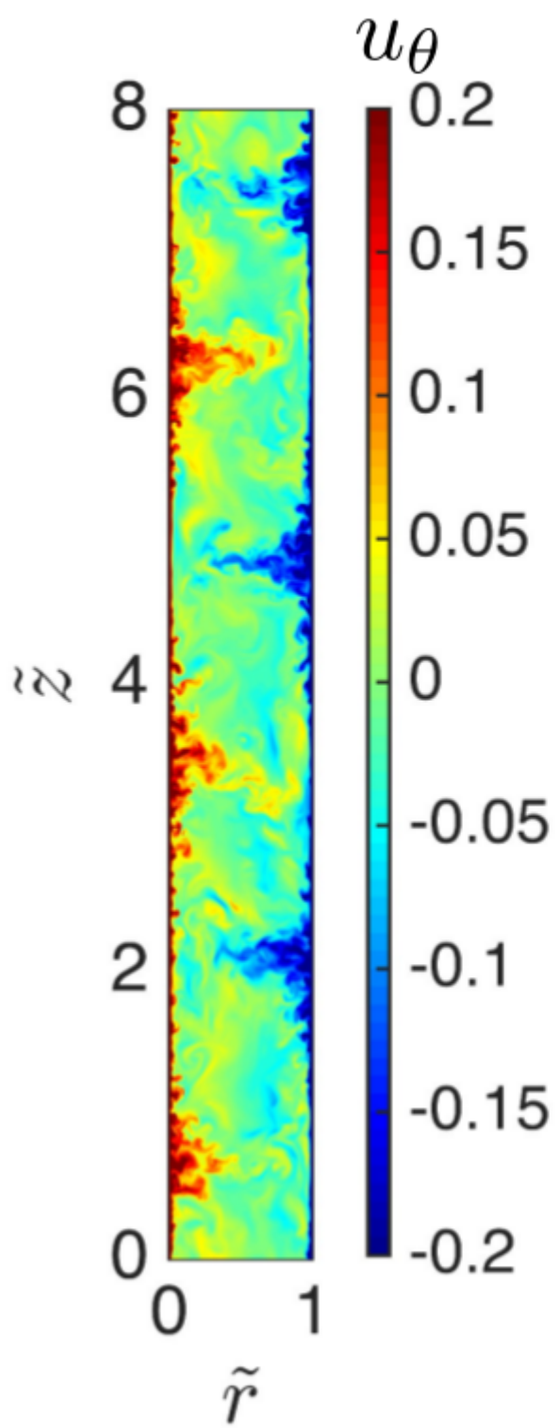
Fixed structures persist in larger boxes

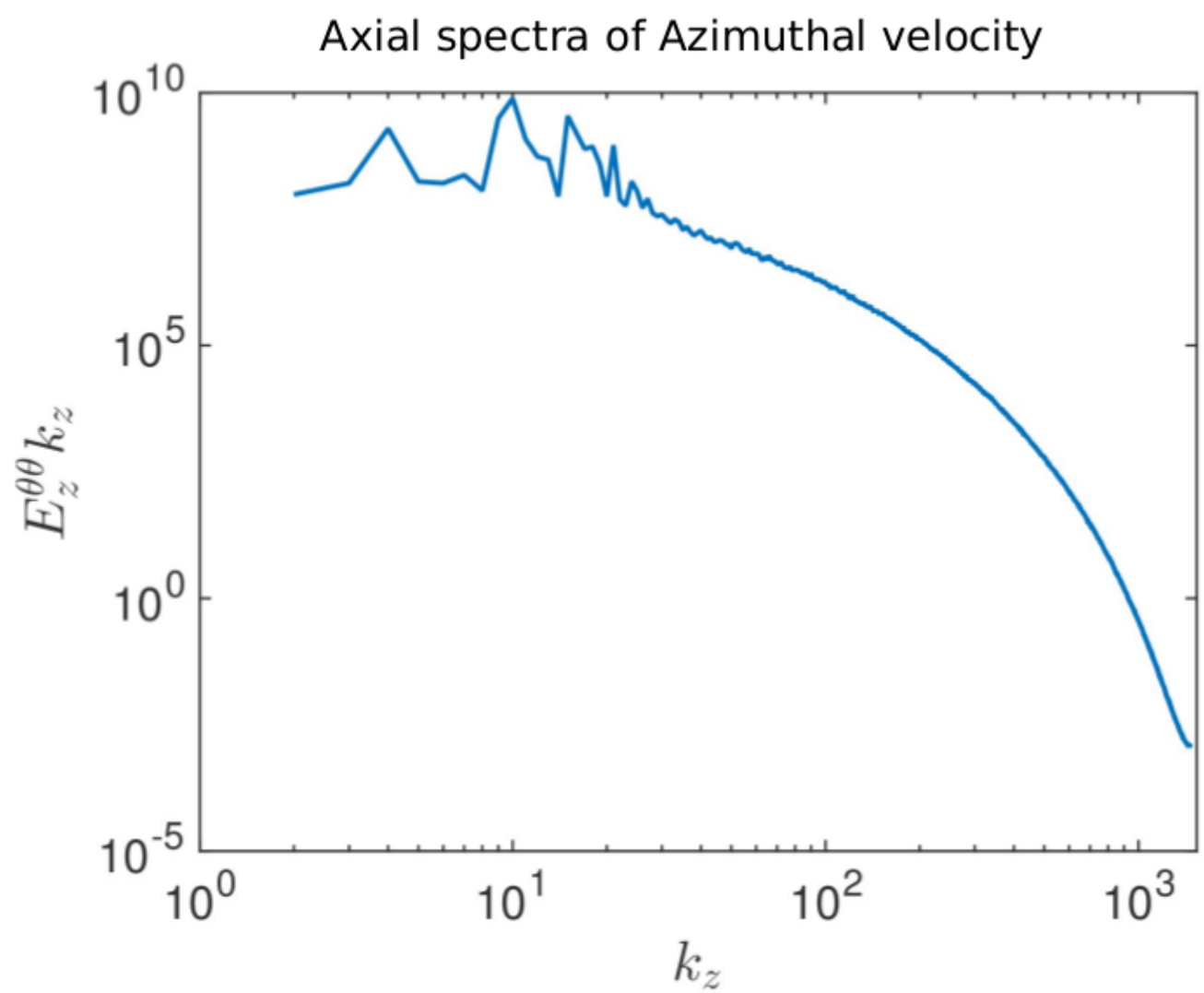


$Re_i = 3.4 \times 10^4$

$\eta = 0.909$

Fixed structures persist in larger boxes





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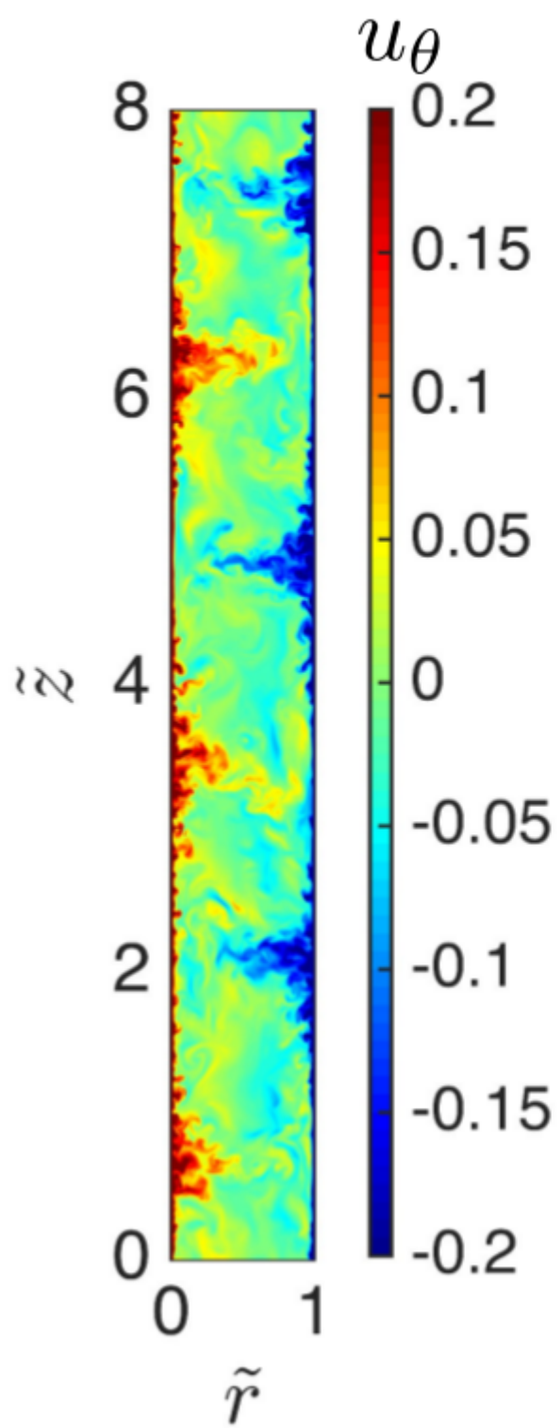
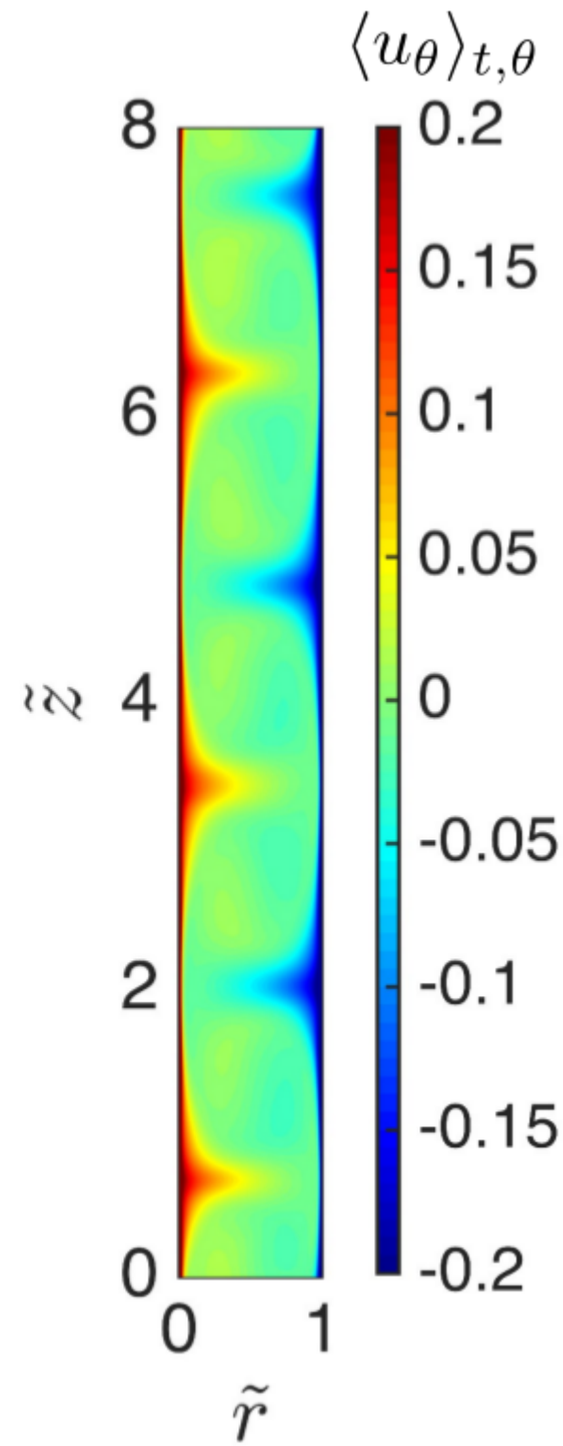
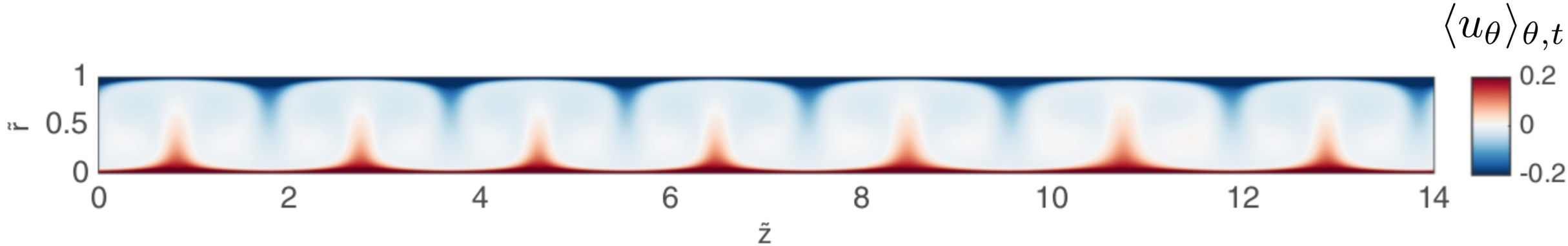




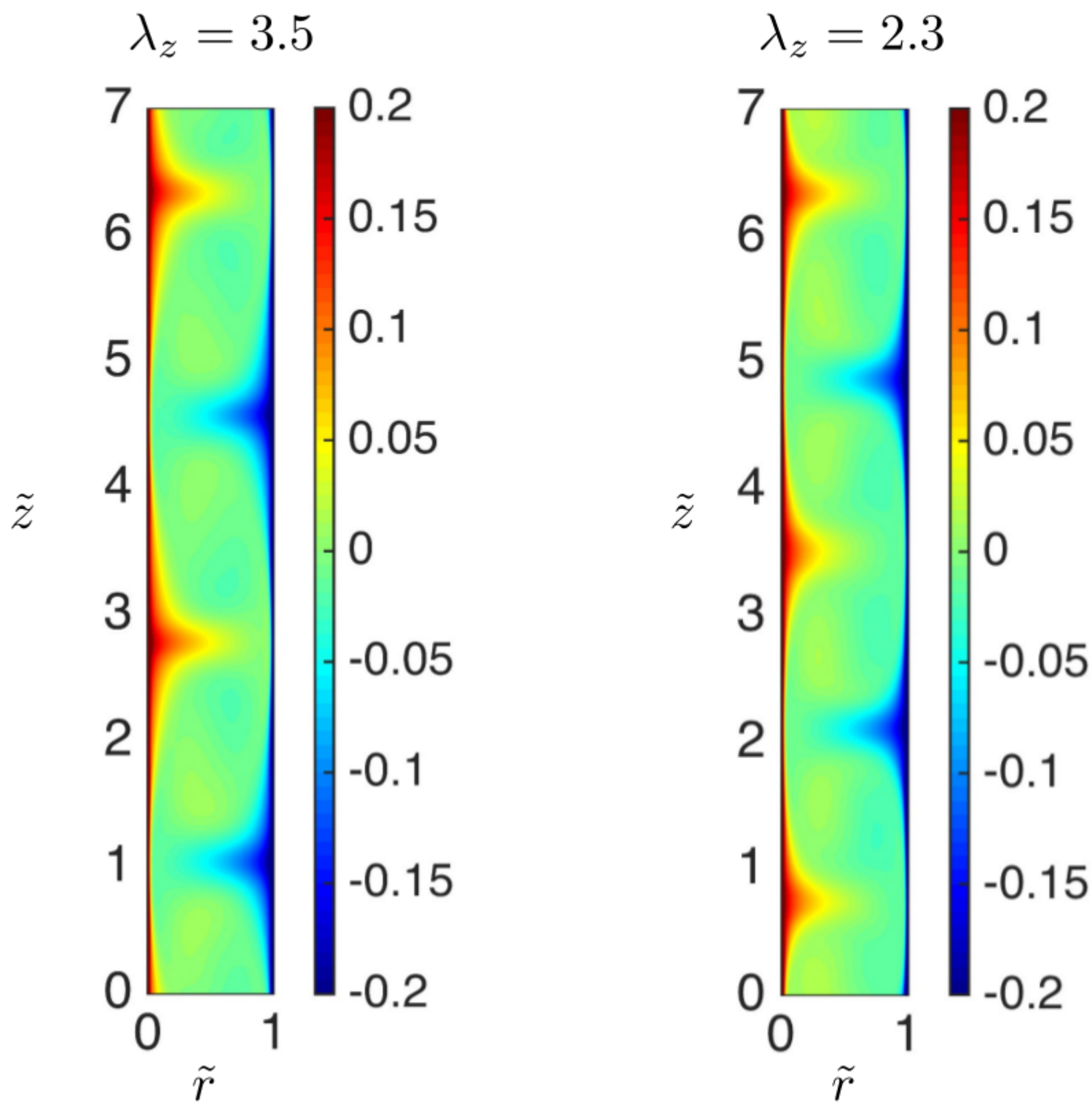
And even larger boxes

$$L_\theta \times L_r \times L_z = 21\pi \times 2 \times 9\pi$$
$$t_s d/(2u_\tau) \approx 29$$



$$Re_i = 1.57 \cdot 10^4$$
$$Re_\tau \approx 240$$
$$\eta = 0.909$$

Different roll states for same box



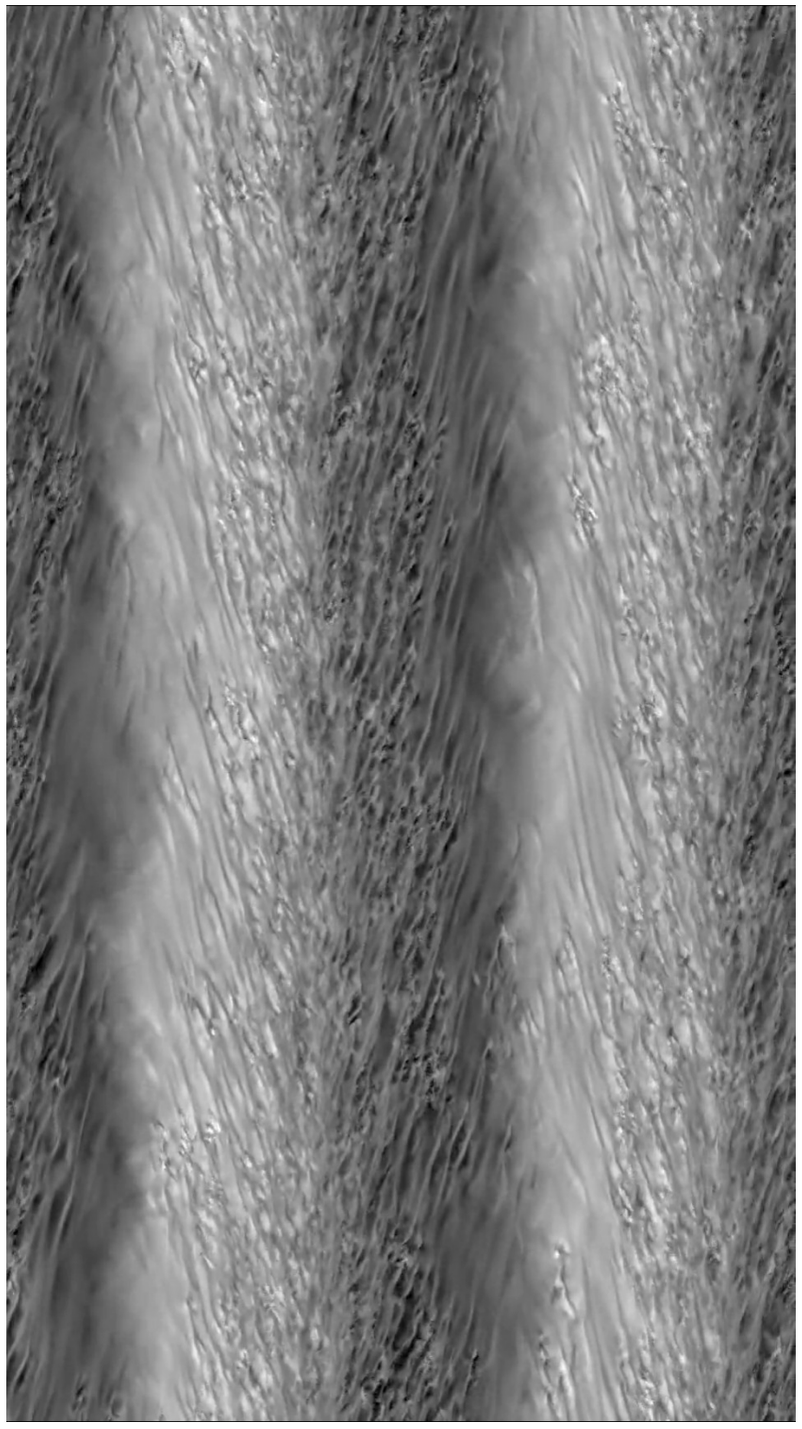
$Re_i = 3.4 \times 10^4$

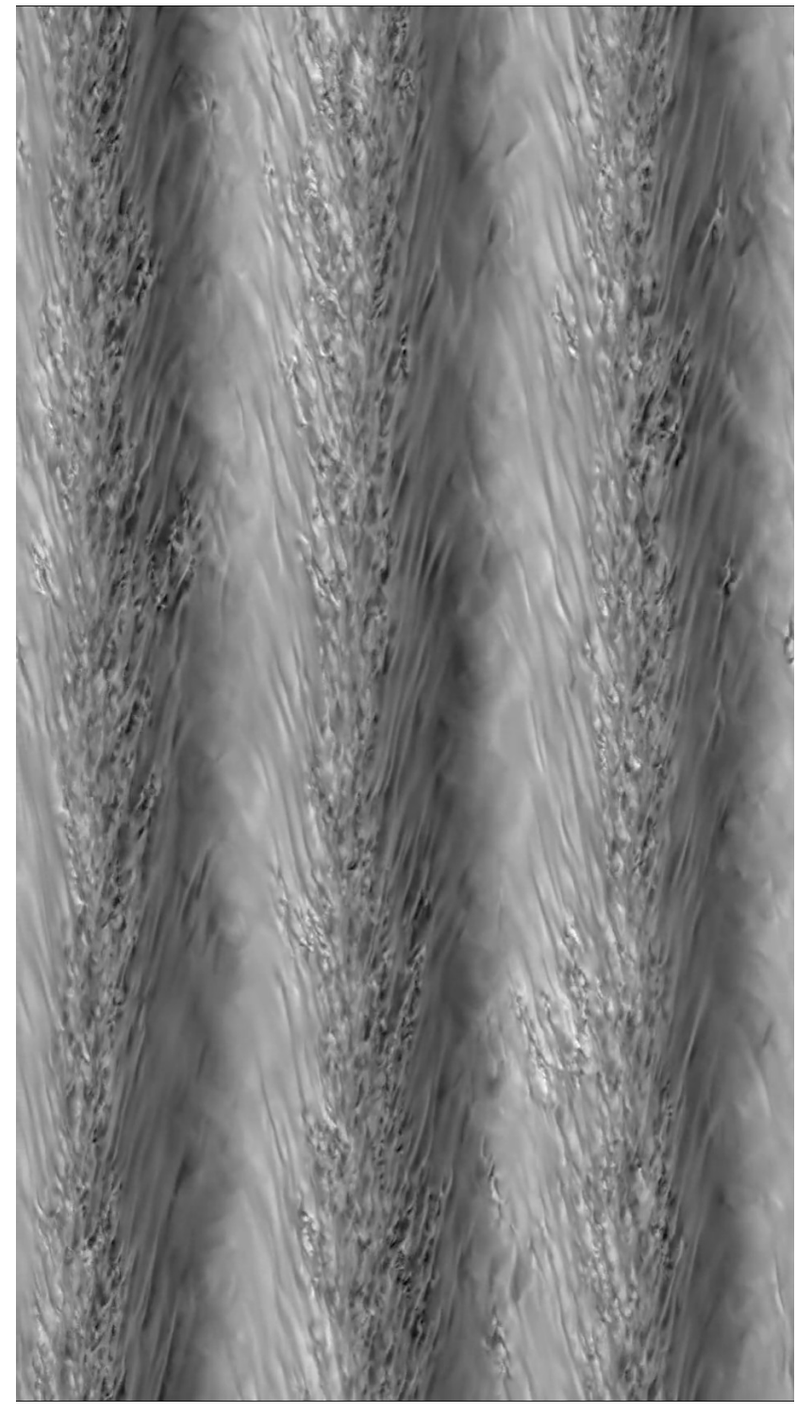
$Re_o = 0$

$\Gamma = 7$

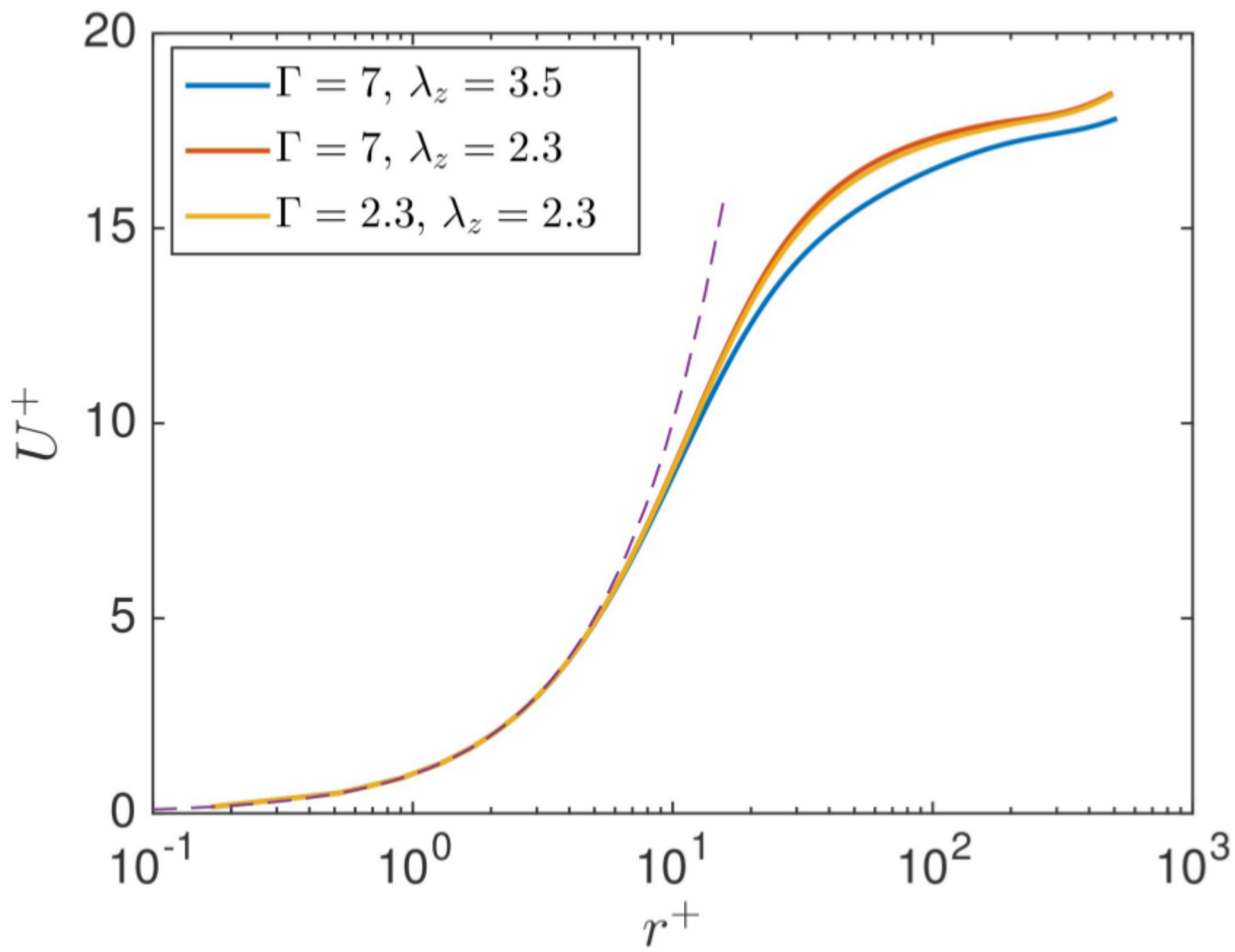
$\eta = 0.909$

Spanwise velocity (streaks) in BL

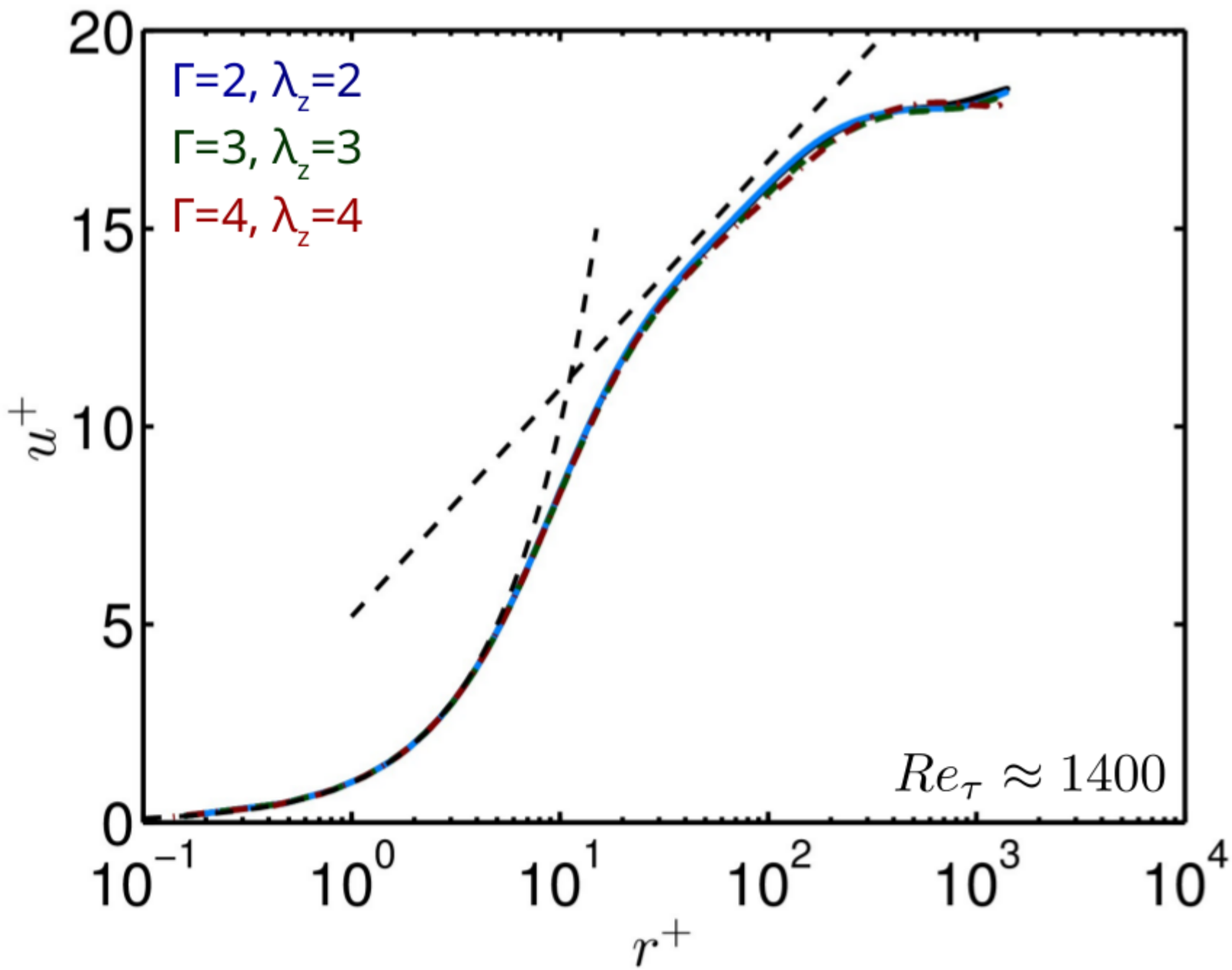




Velocity profiles depend on roll wavelength and not number

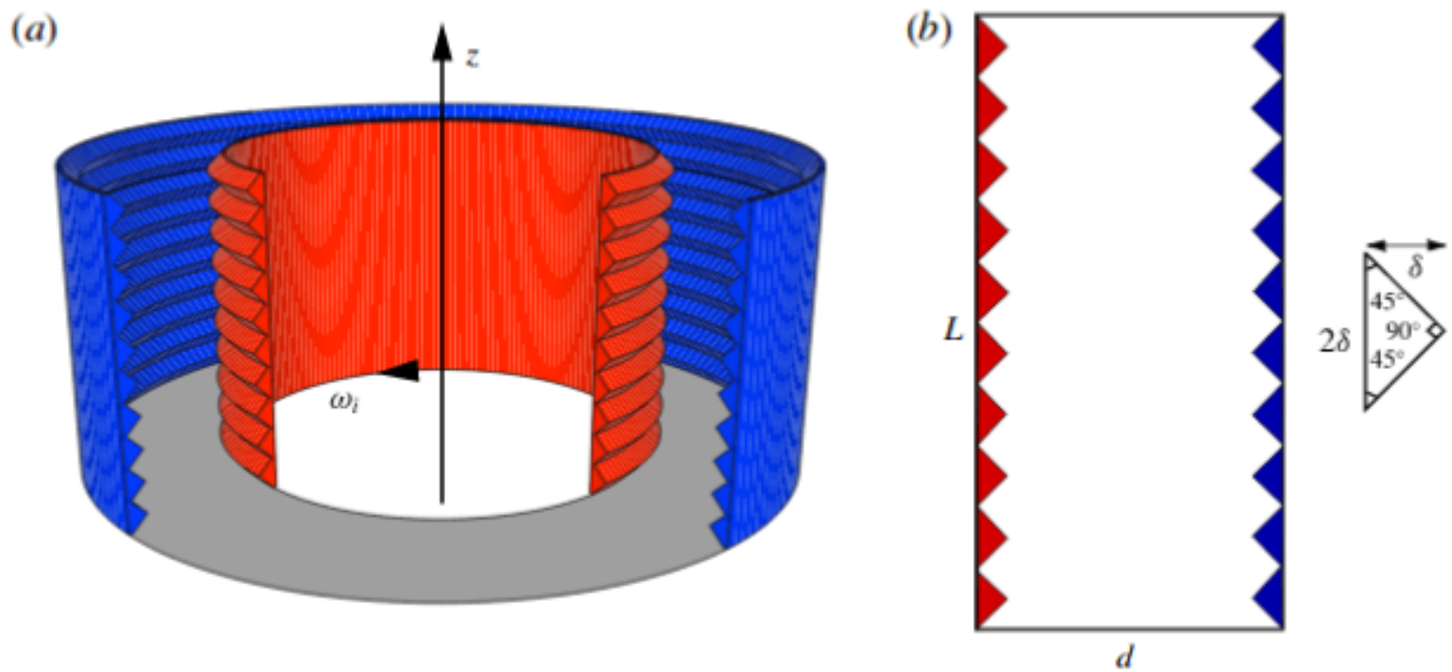


Dependence becomes weaker at higher Re



Ostilla-Mónico, Verzicco, Lohse. Phys. Fluids (2015)

Add "riblets" to regularize streaks:



Zhu, Ostilla-Mónico, Verzicco, Lohse. J Fluid Mech (2016)

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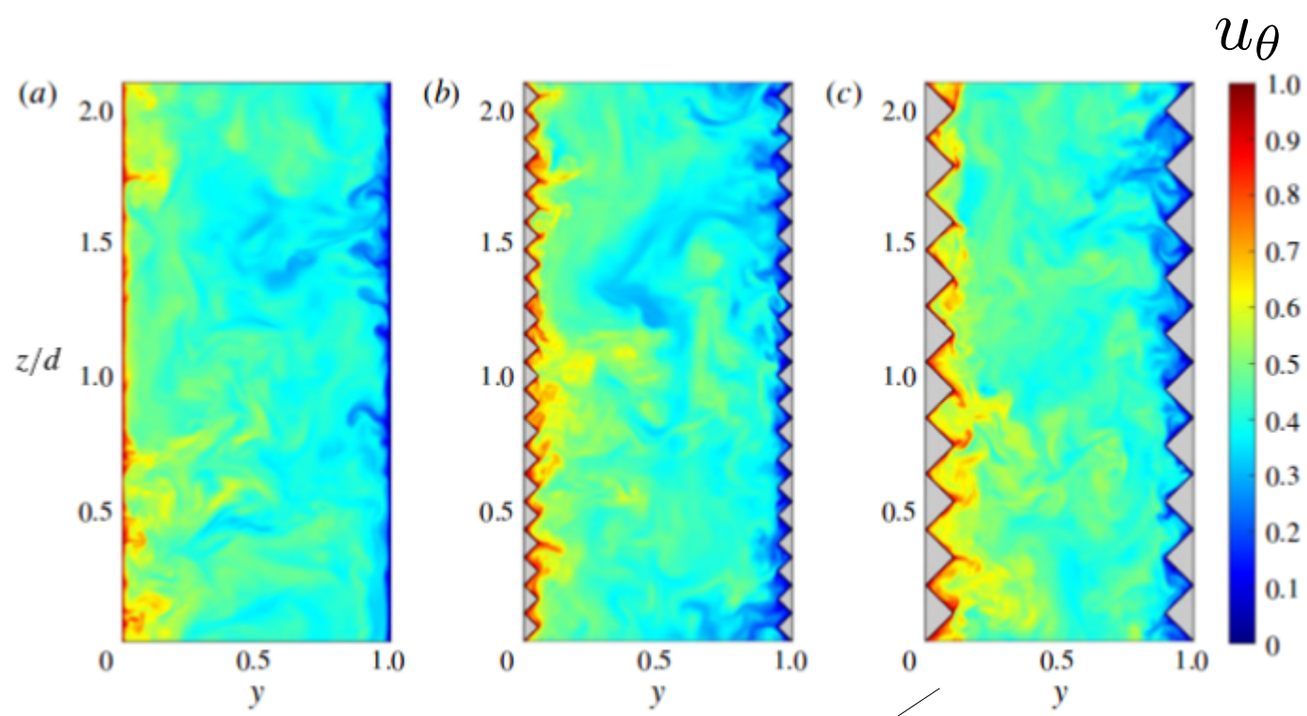
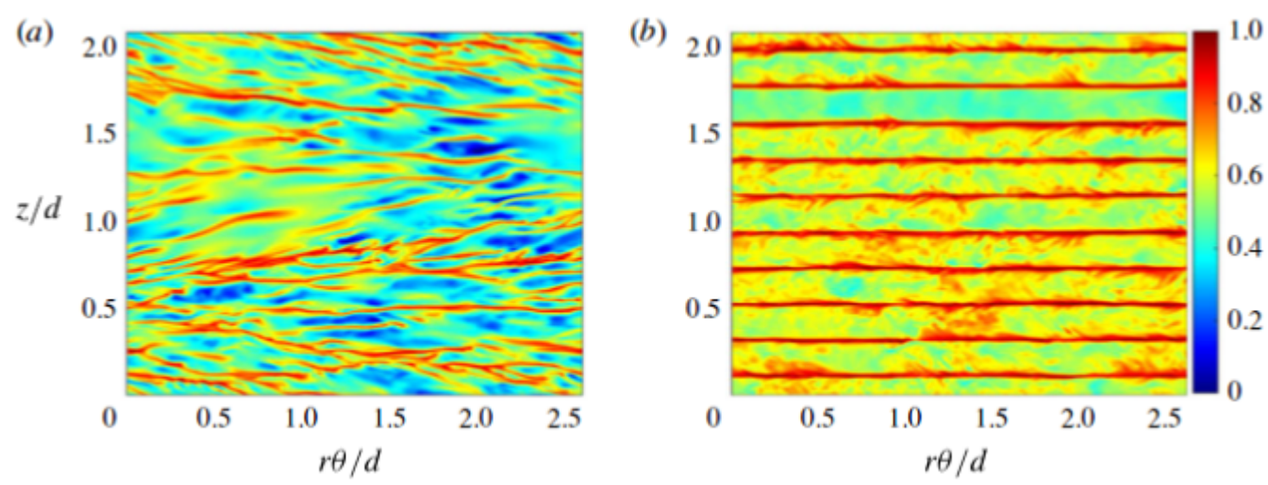


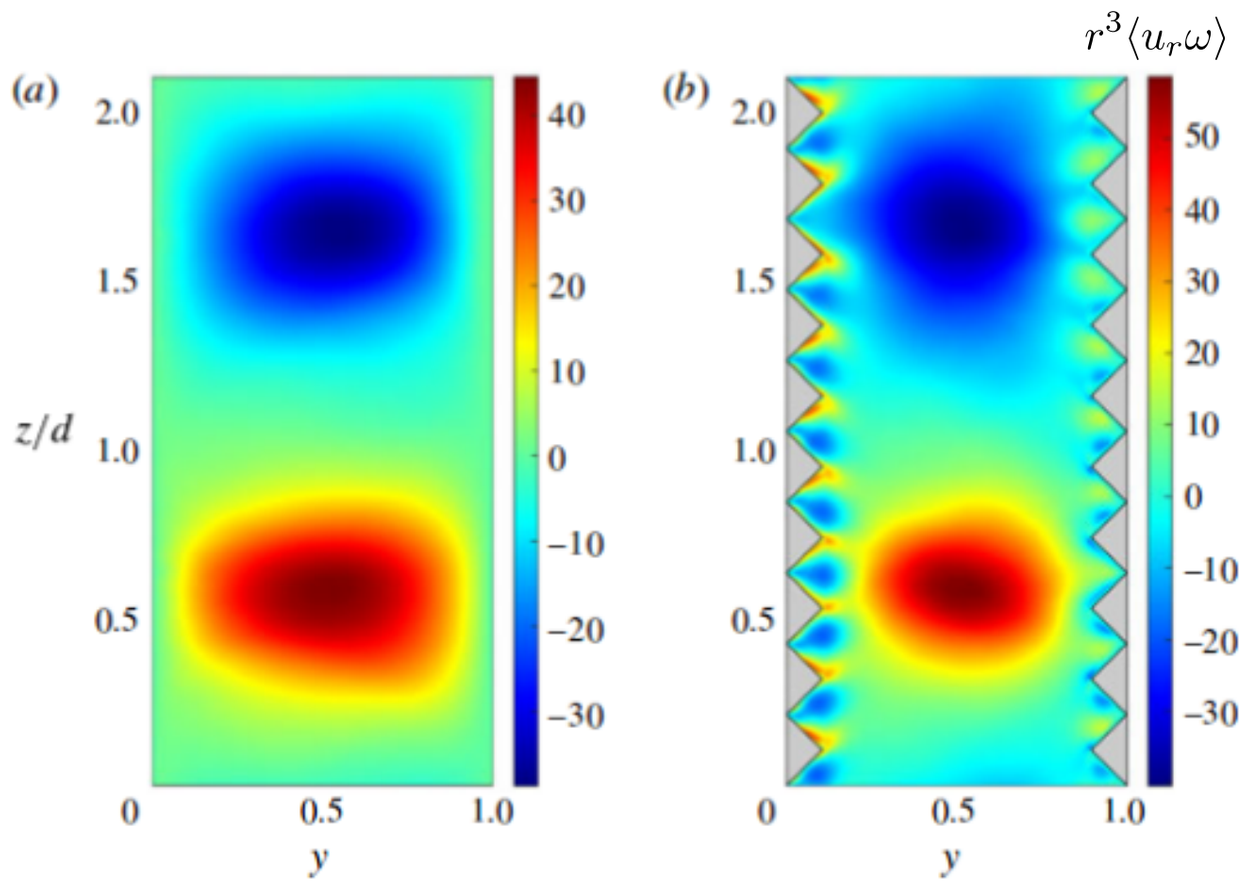


$$\eta = 0.714$$

$$Re_s = 3 \times 10^4$$

Zhu, Ostilla-Mónico, Verzicco, Lohse. J Fluid Mech (2016)

Suddenly make cylinders free-slip: decaying turbulence



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Zhu, Ostilla-Mónico, Verzicco, Lohse. J Fluid Mech (2016)

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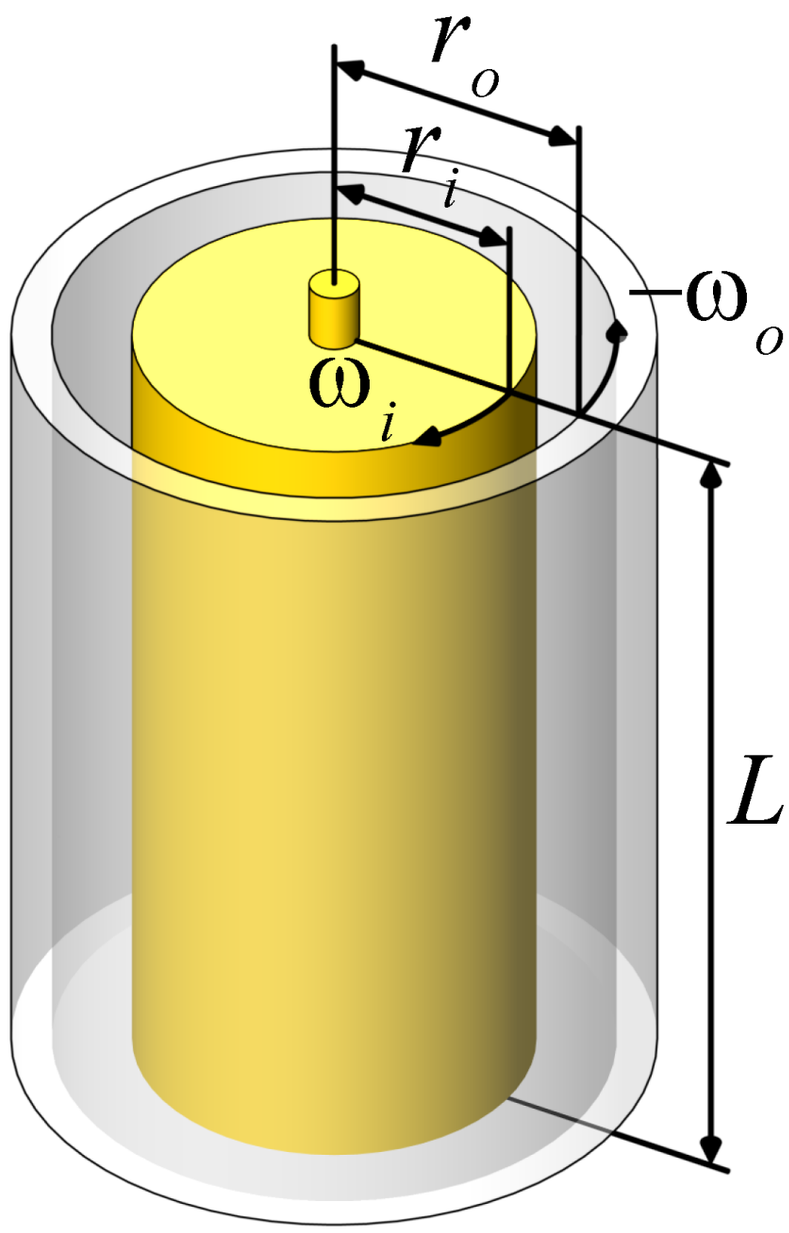
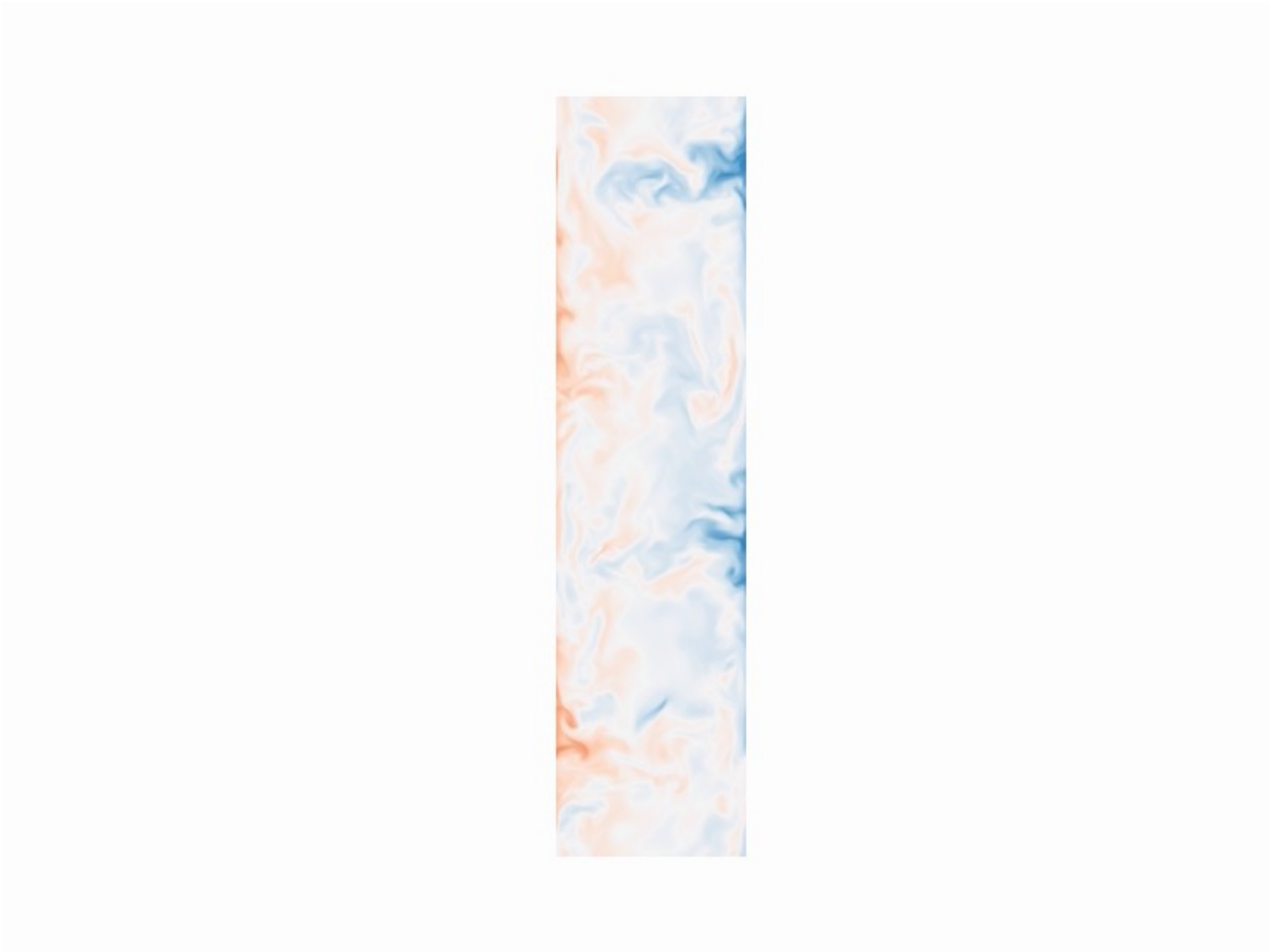
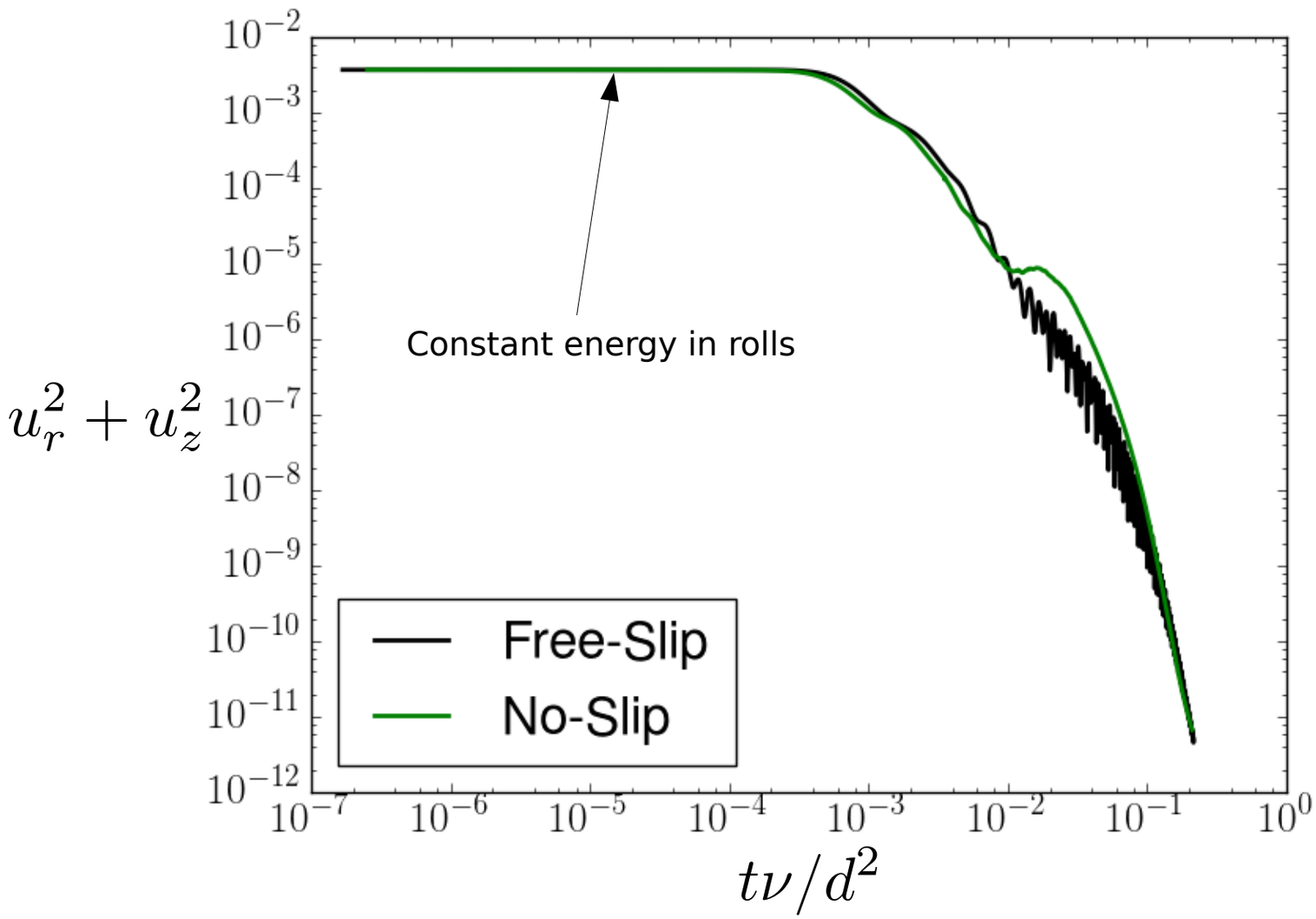


Decaying TC tubulence preserves rolls for some time

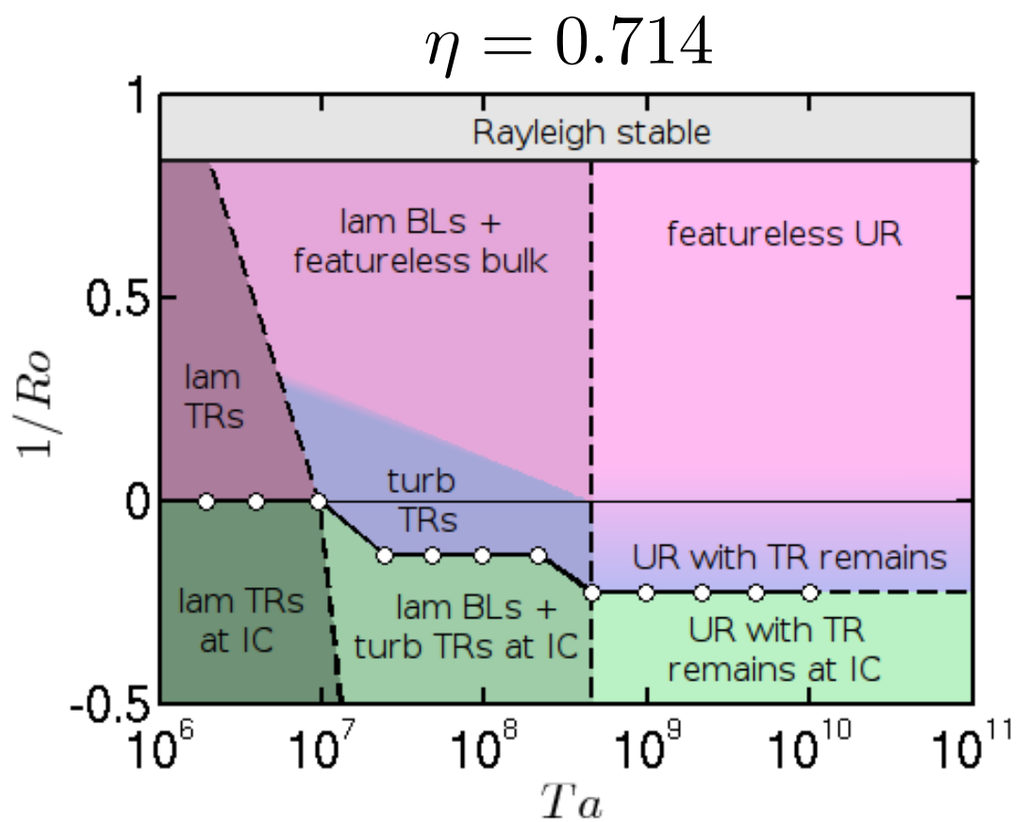


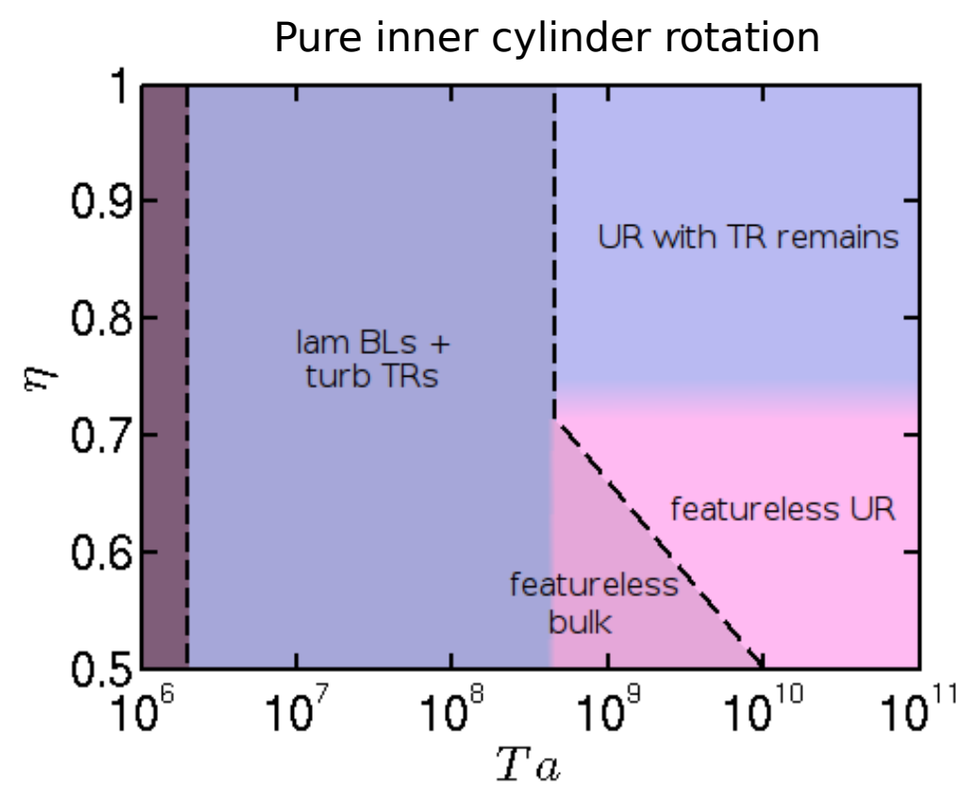
First decay regime: rolls remain active



Ostilla-Mónico & others, in preparation

Large scale rolls exist only in certain regions of parameter space

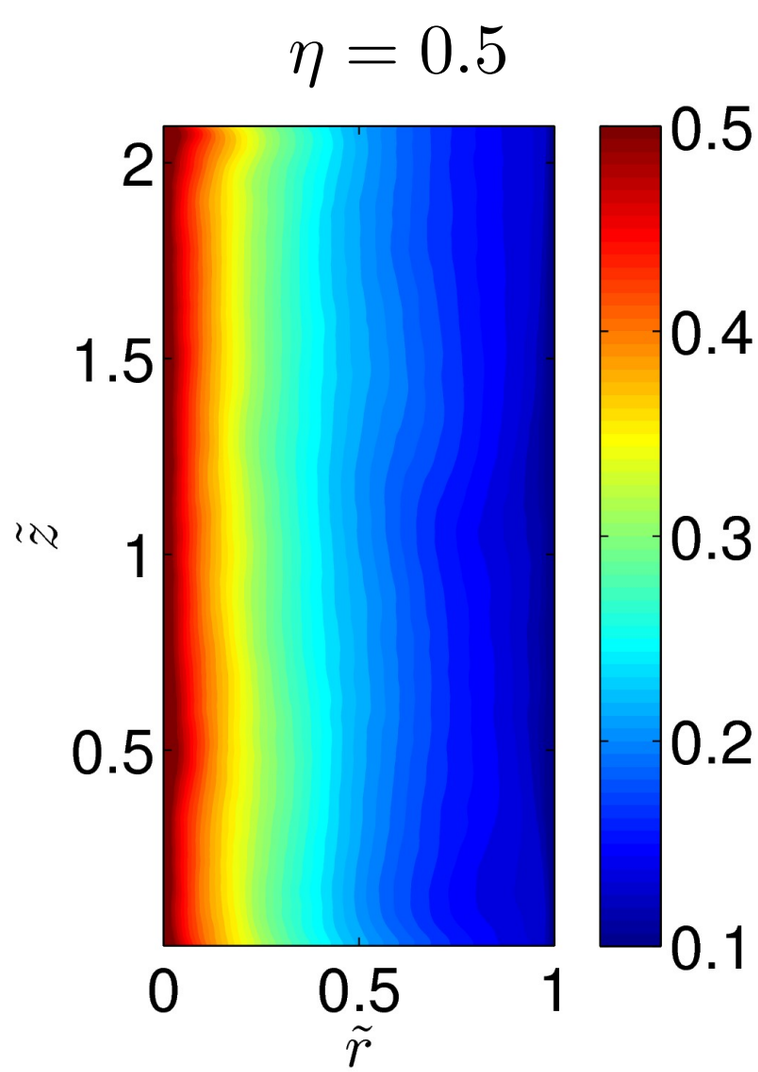


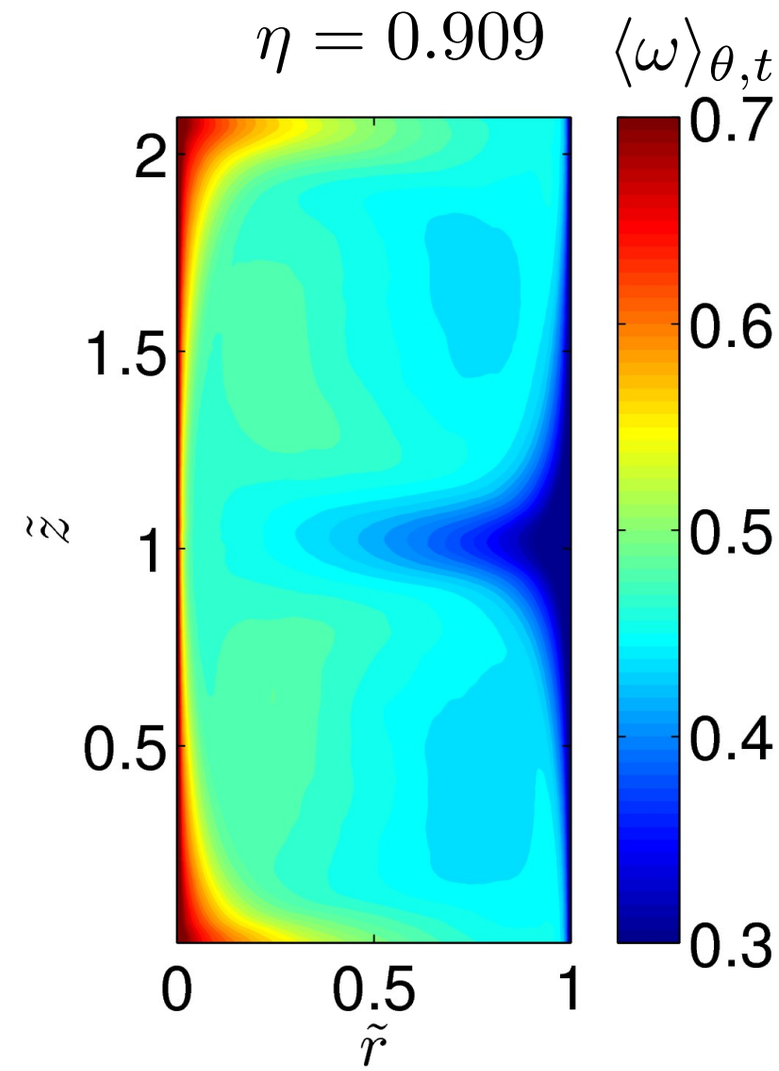


Curvature & mean rotation prevent the existence of rolls

Ostilla-Mónico, van der Poel, Verzicco, Grossmann, Lohse. J. Fluid. Mech, 2015

Local flow organization depends on curvature

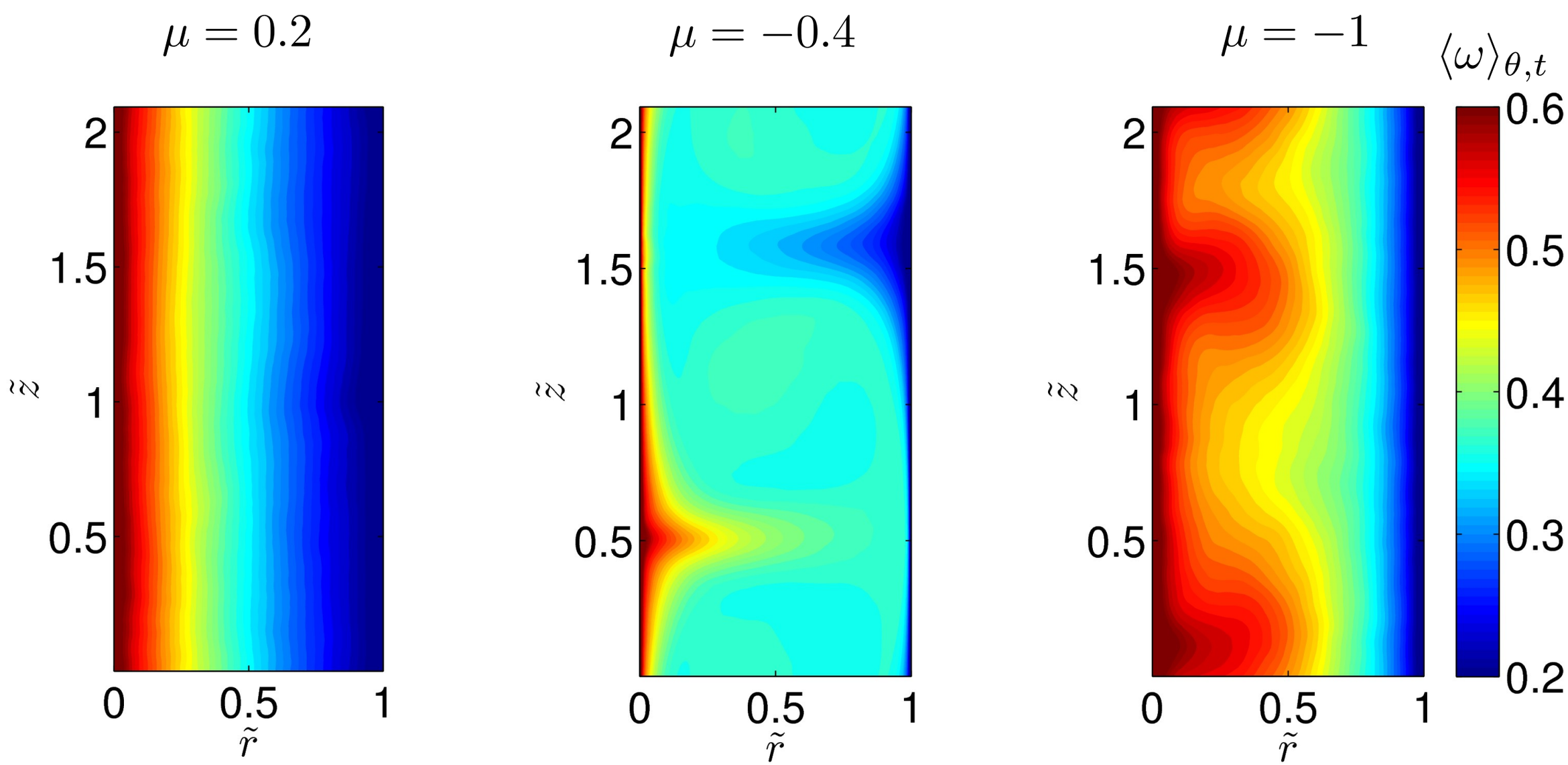




$\eta = 0$ Vanishing inner cylinder

$\eta = 1$ Two plates (plane Couette)

Local flow organization depends on mean rotation

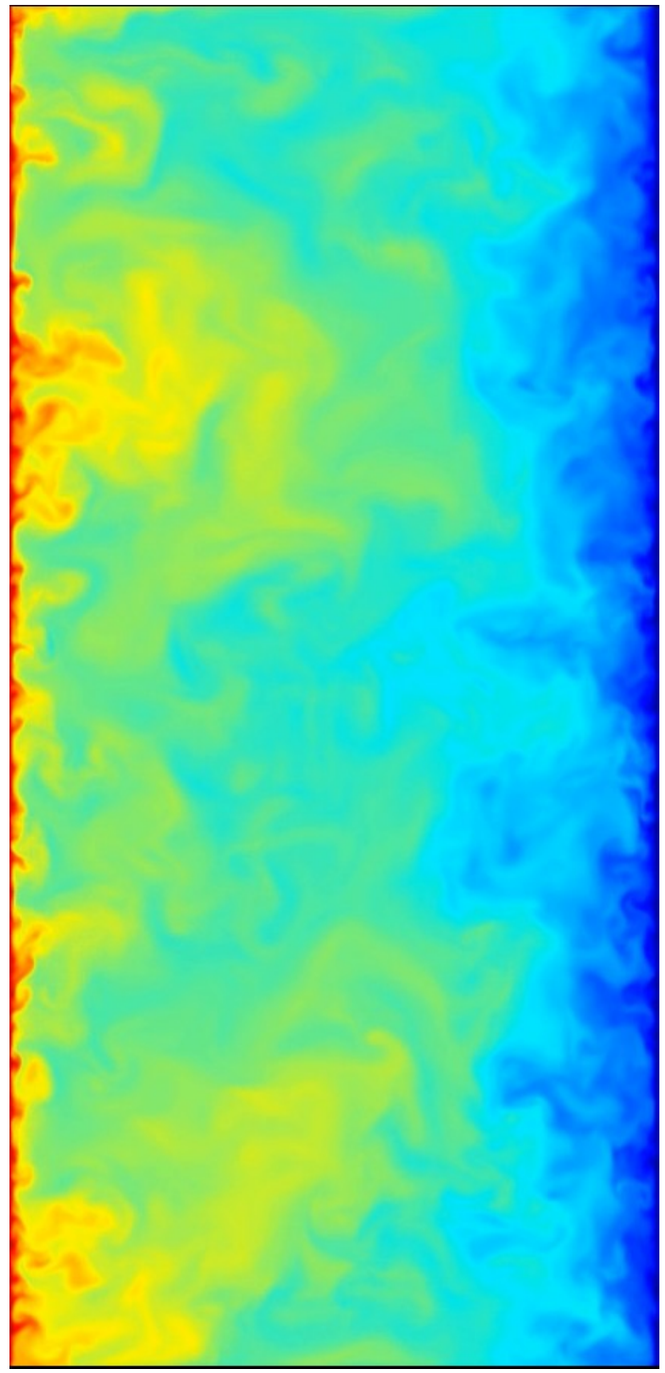


Co-rotation
No large scale rolls

Weak counter-rotation:
Large scale rolls
which fill the entire gap

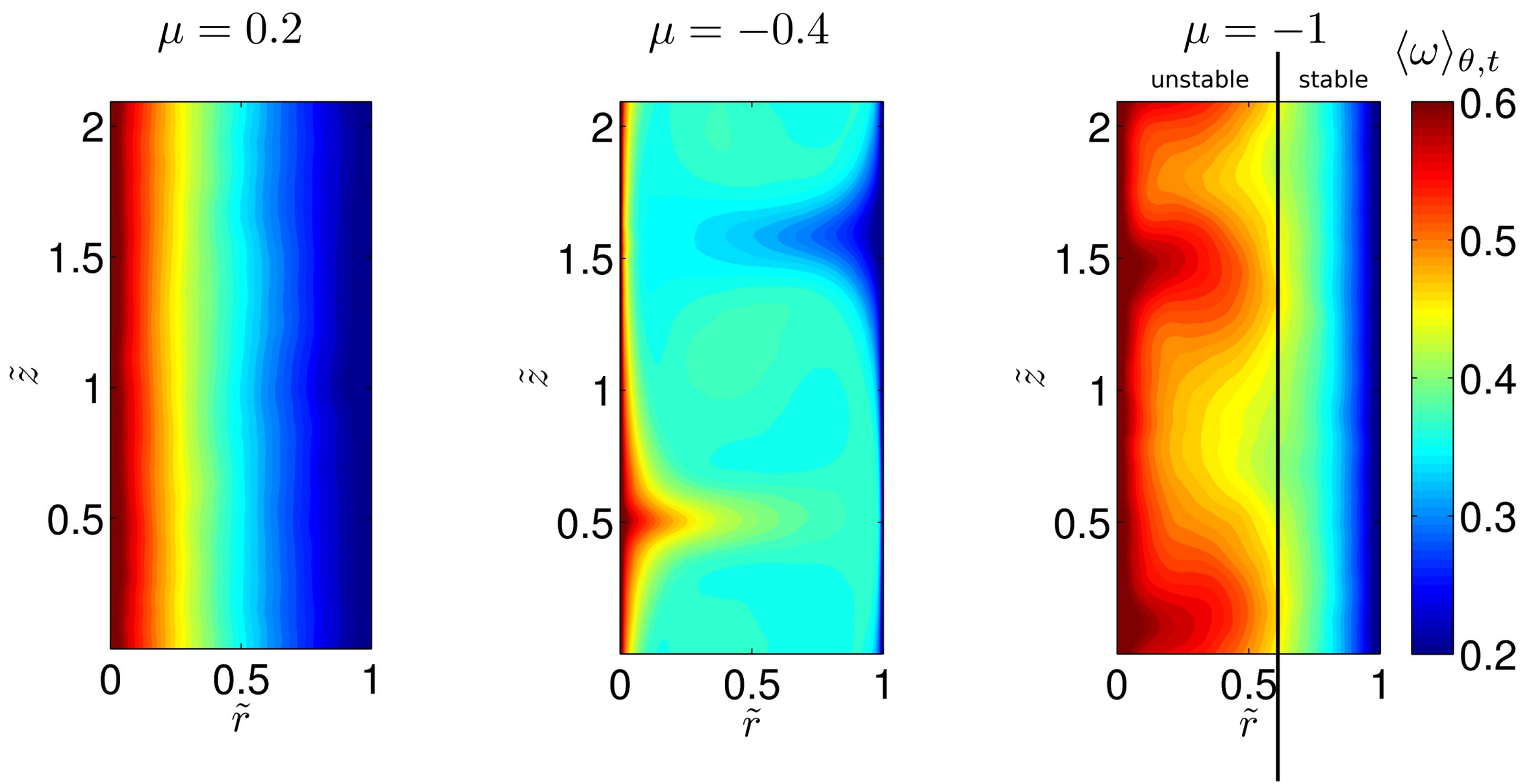
Strong counter-rotation:
Large scale rolls pushed
towards inner cylinder

Counter-rotation: mixed dynamics



$$\mu = -1$$
$$\eta = 0.909$$
$$Re_s = 10^5$$

Local flow organization depends on mean rotation



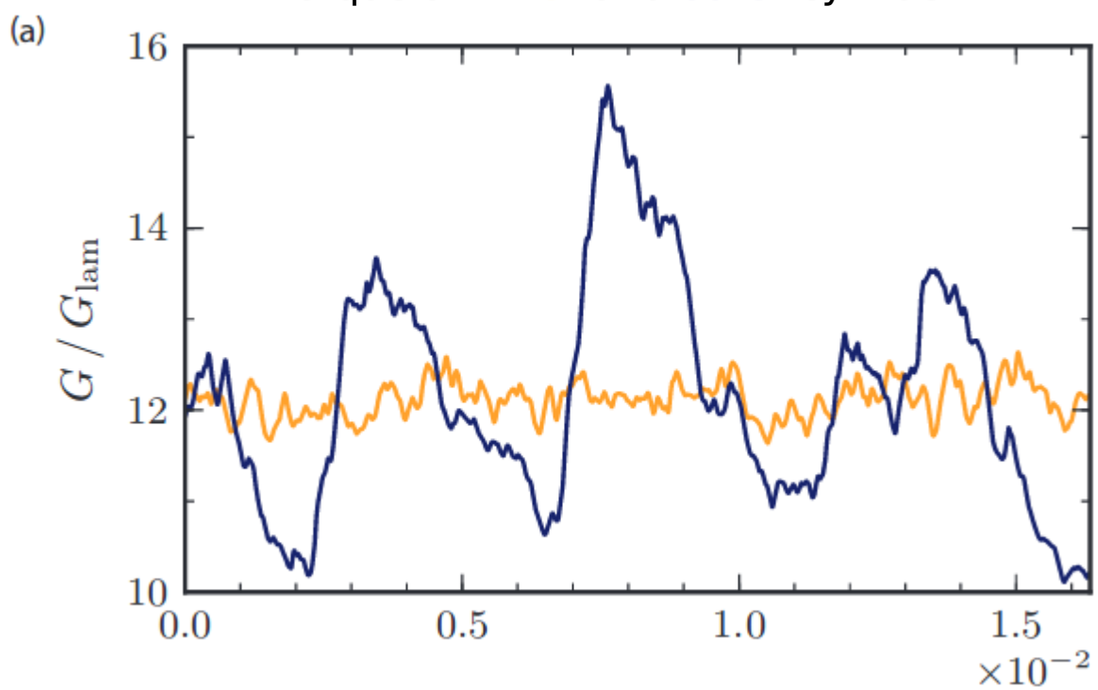
Co-rotation
No large scale rolls

Weak counter-rotation:
Large scale rolls
which fill the entire gap

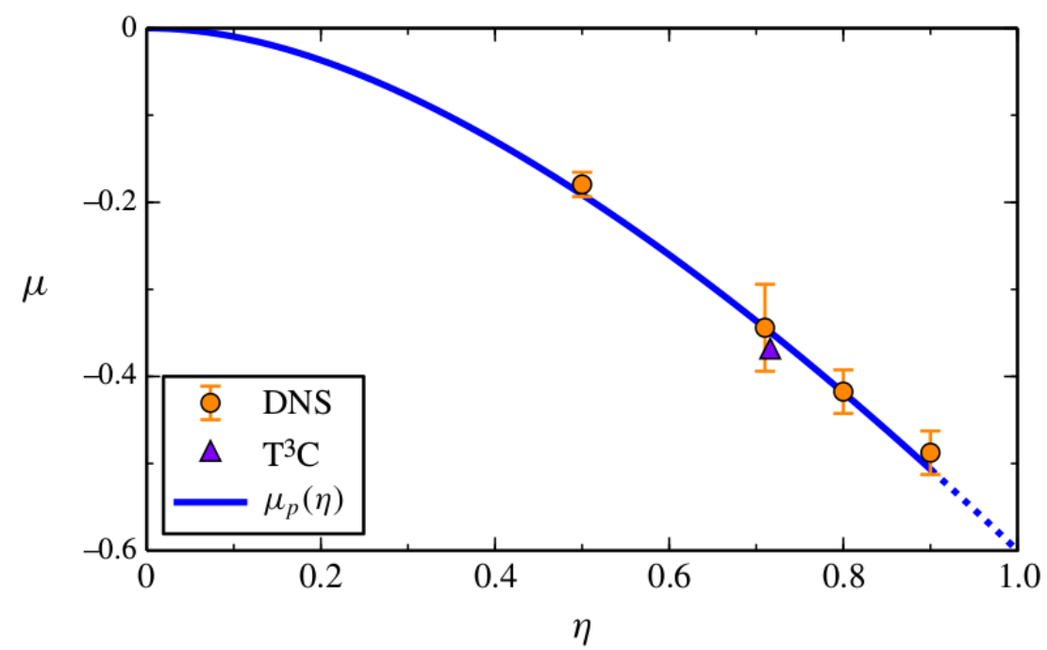
Strong counter-rotation:
Large scale rolls pushed
towards inner cylinder

Different behaviour at both cylinders

Torque at inner and outer cylinder

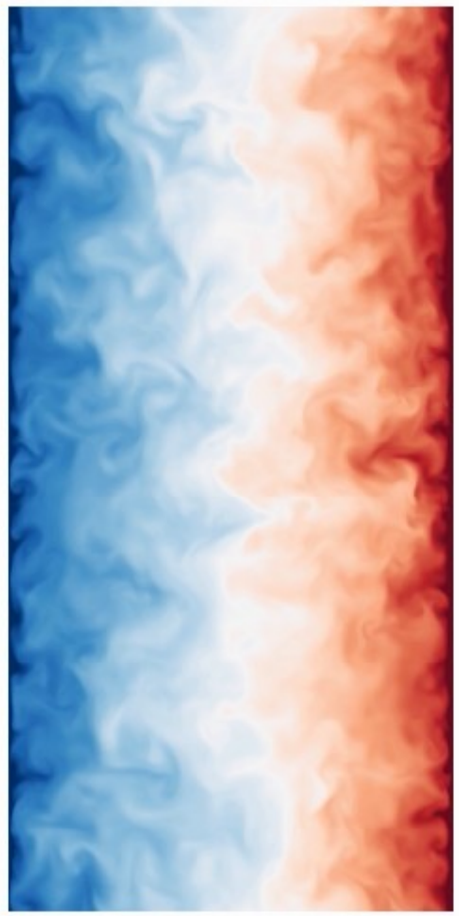


Onset of intermittency



Brauckmann, Eckhardt, Phys. Rev. E (2013)
Brauckmann, Salewski, Eckhardt, J. Fluid Mech (2016)

Pure outer cylinder rotation: linearly stable



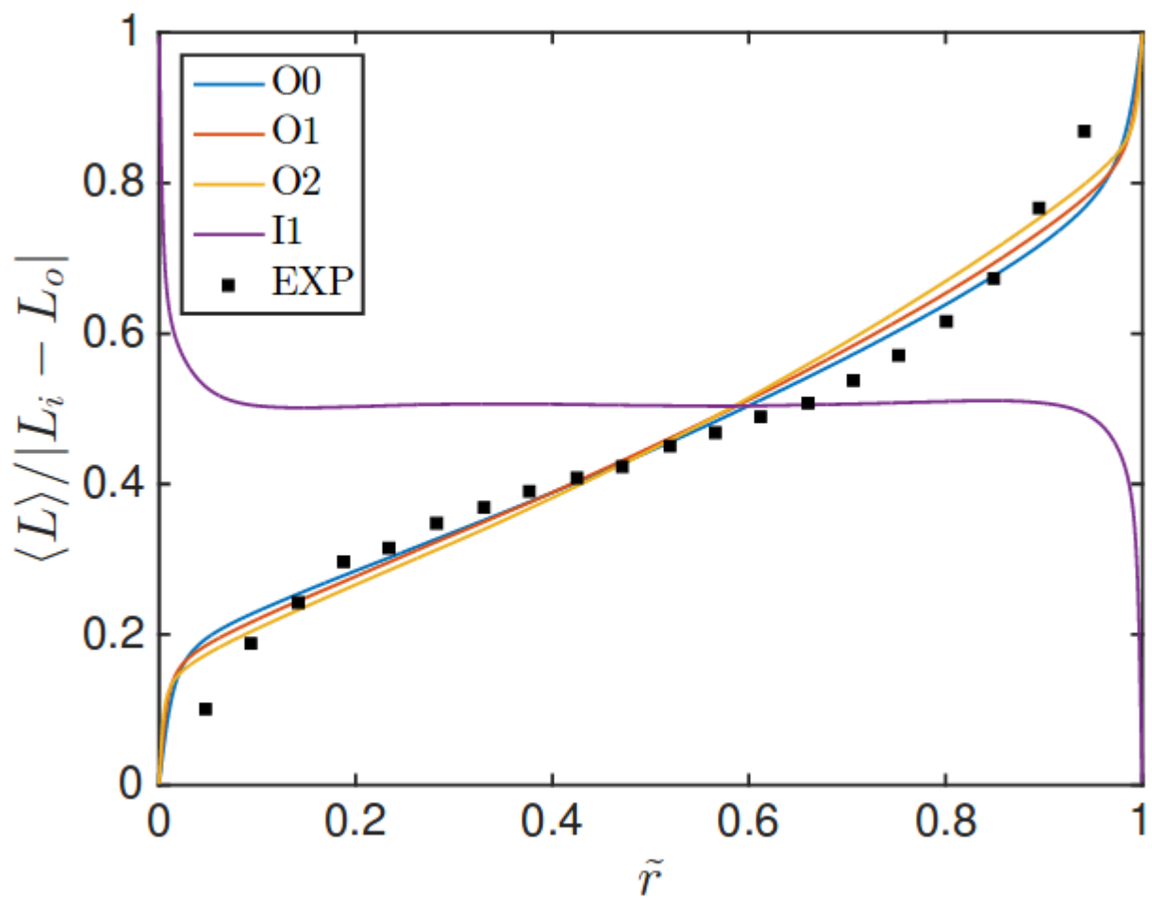
$$\mu \to -\infty$$

$$\eta = 0.909$$

$$Re_o = 2 \cdot 10^5$$

Ostilla-Mónico, Verzicco, Lohse, JFM Rapids (2016)

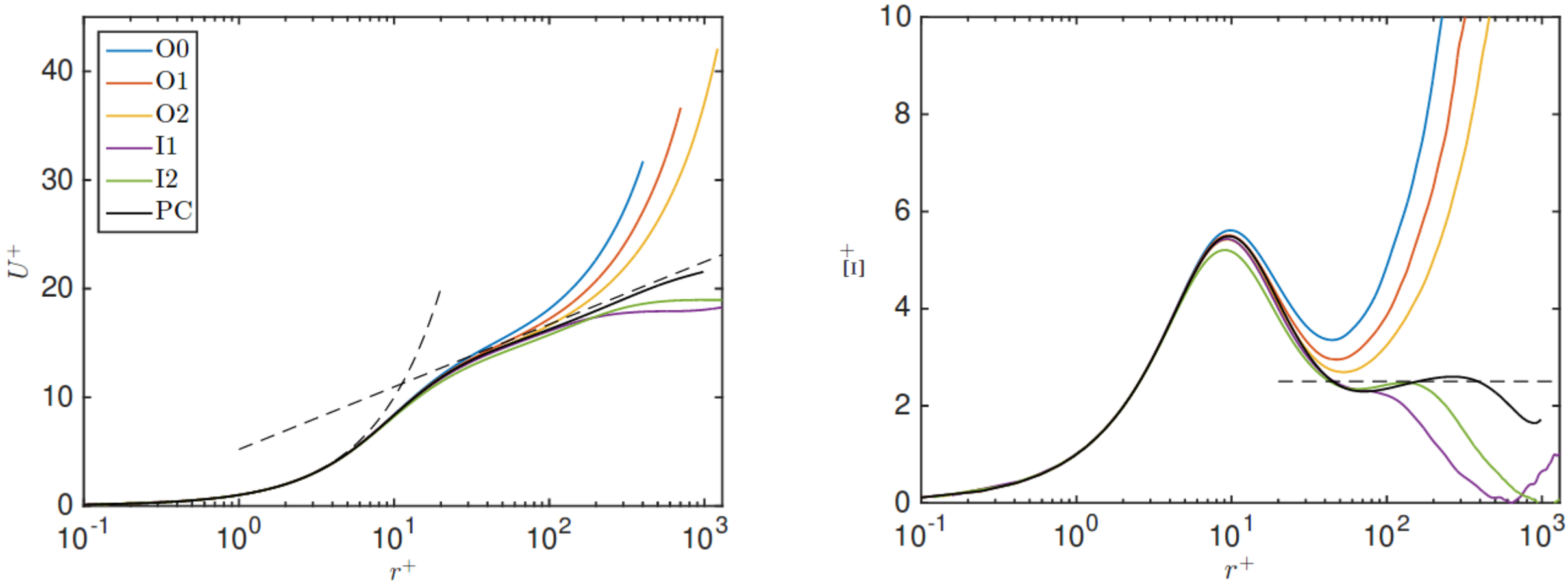
Angular momentum profile looks very different



$$\eta = 0.909$$

Ostilla-Mónico, Verzicco, Lohse, JFM Rapids (2016)

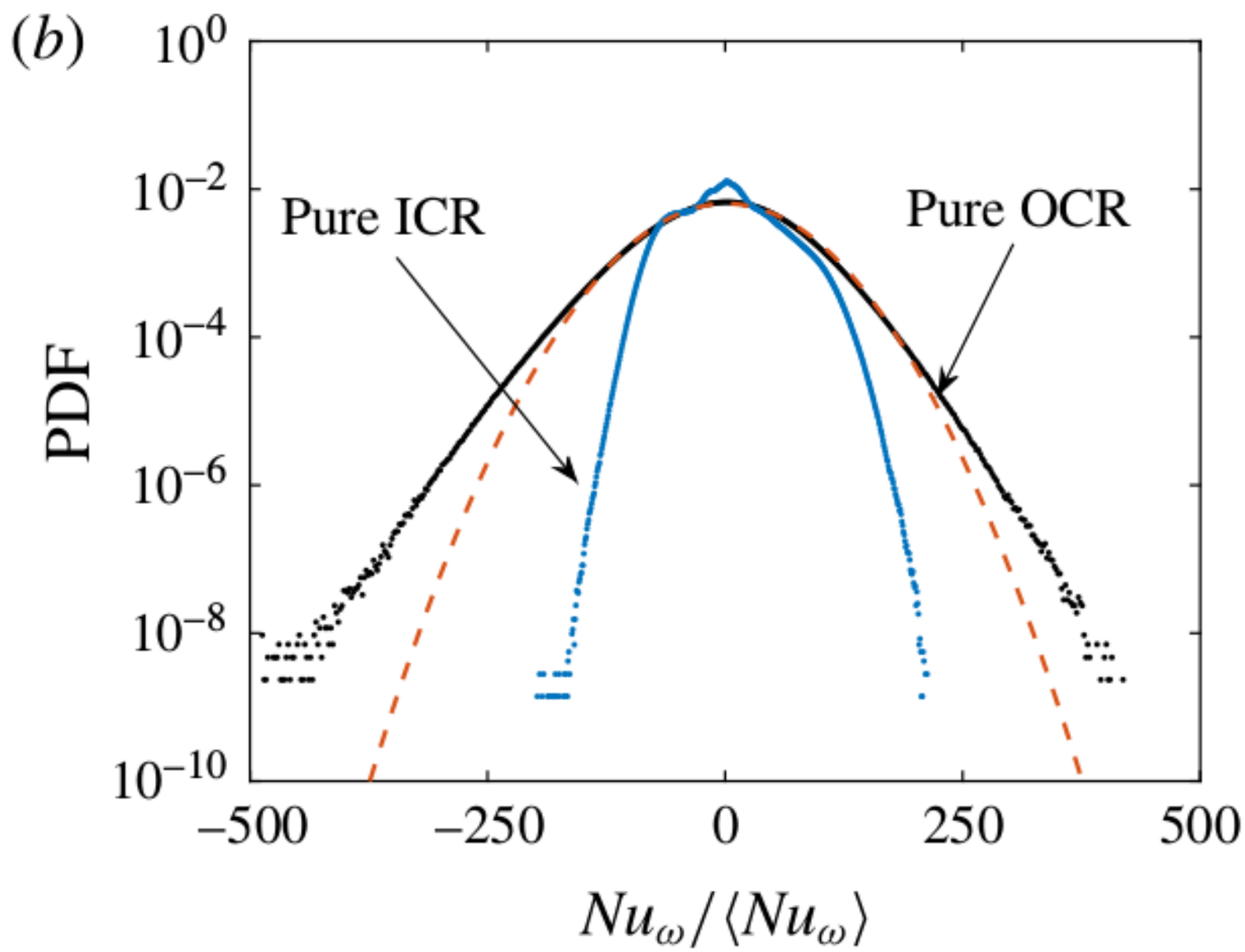
Wall profiles look different in the bulk



$\eta = 0.909$

Ostilla-Mónico, Verzicco, Lohse, JFM Rapids (2016)

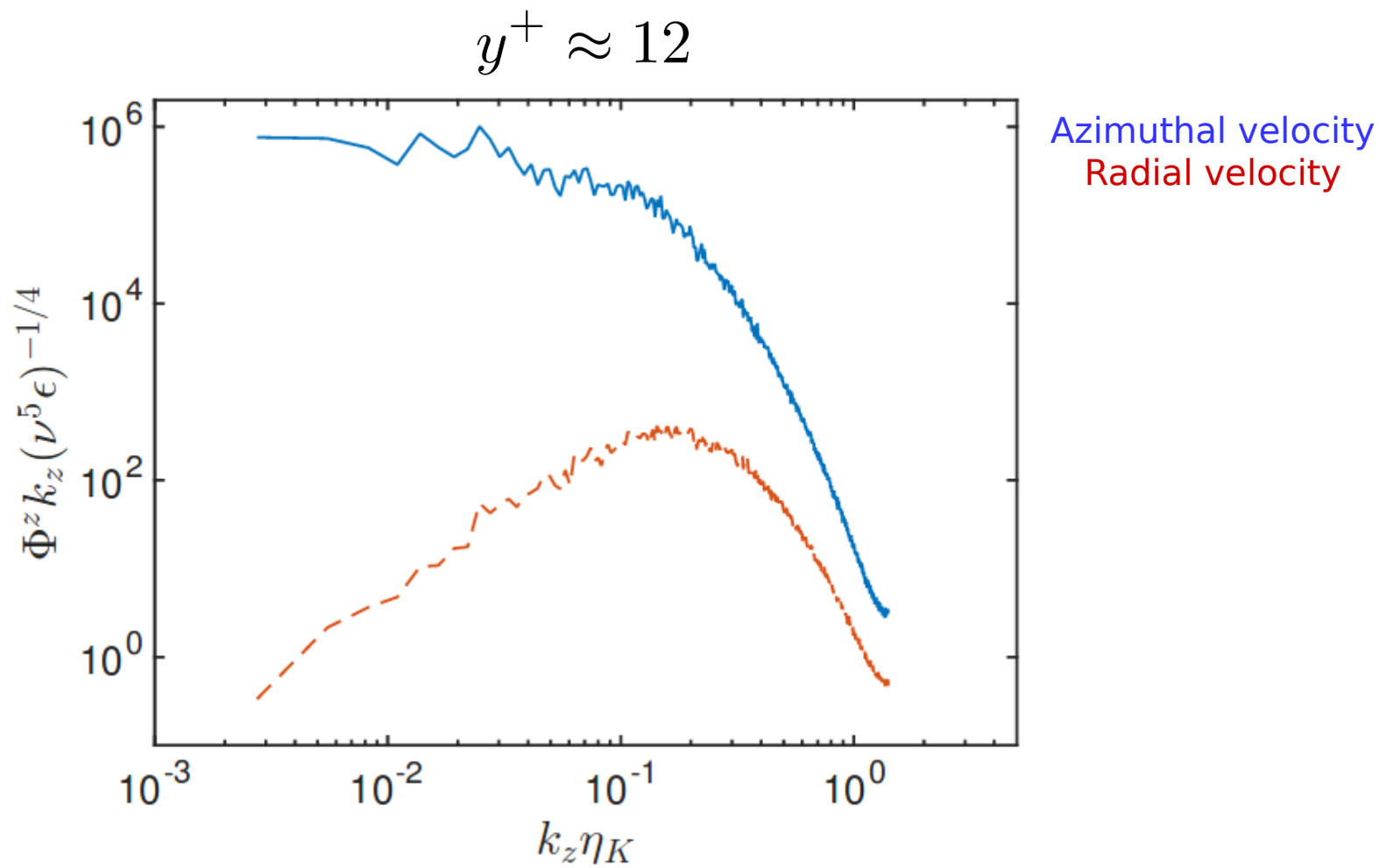
Transport takes a very different character



$$Nu_\omega \sim \langle u_r \omega \rangle$$

$$Re_s = 2 \times 10^5$$

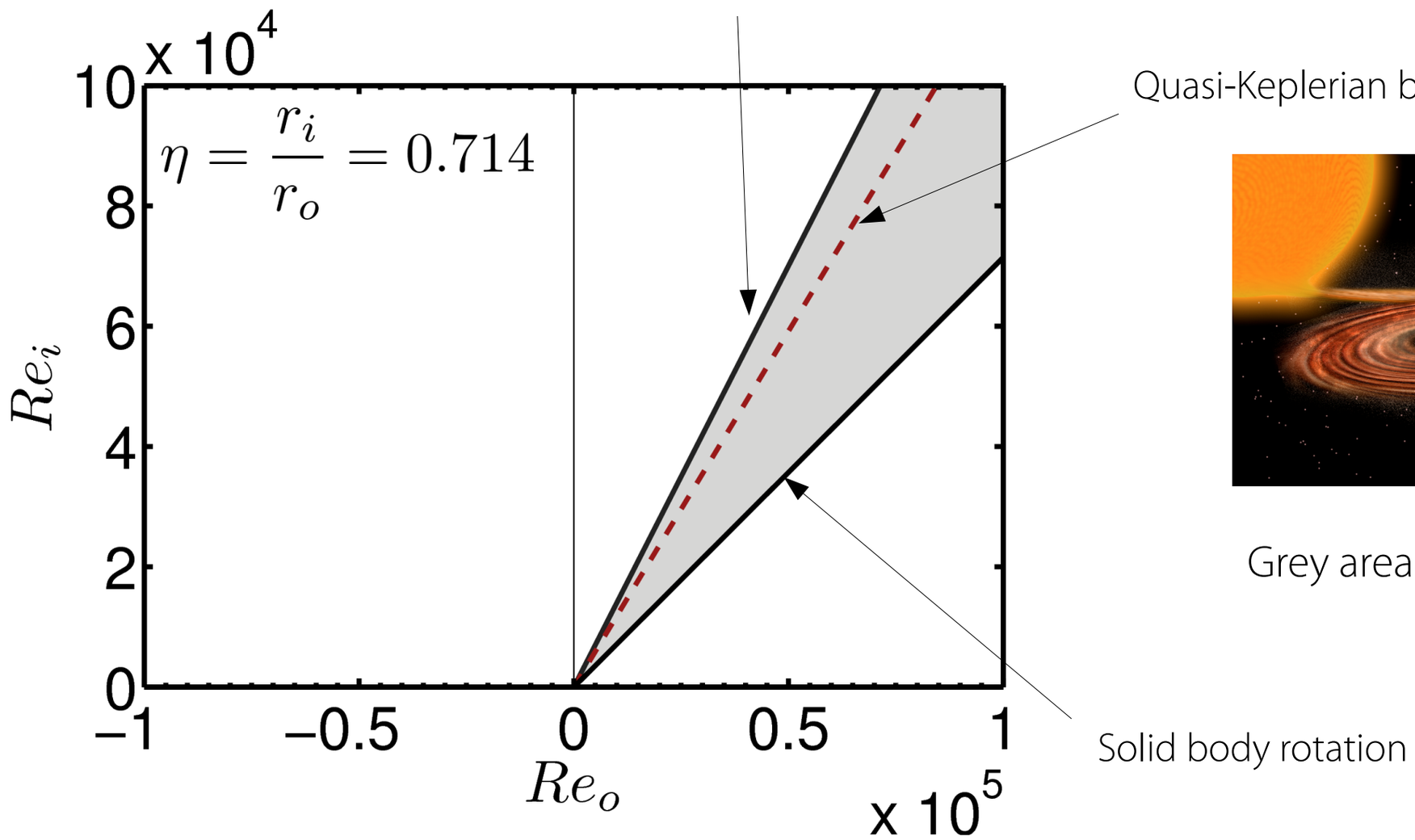
Spectra at wall become similar to other flows

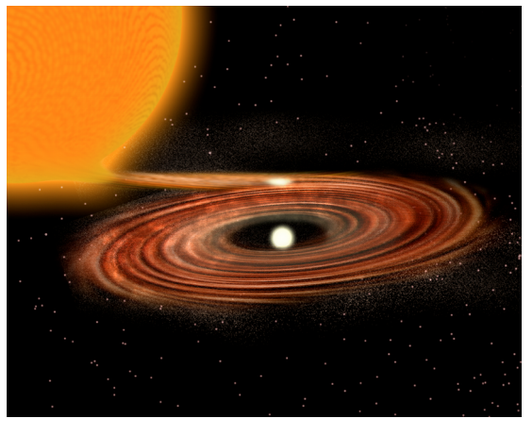


Ostilla-Mónico, Verzicco, Lohse, JFM Rapids (2016)

Quasi-Keplerian regime

TC is linearly stable if: $\dfrac{d(r^2\omega)}{dr} > 0$

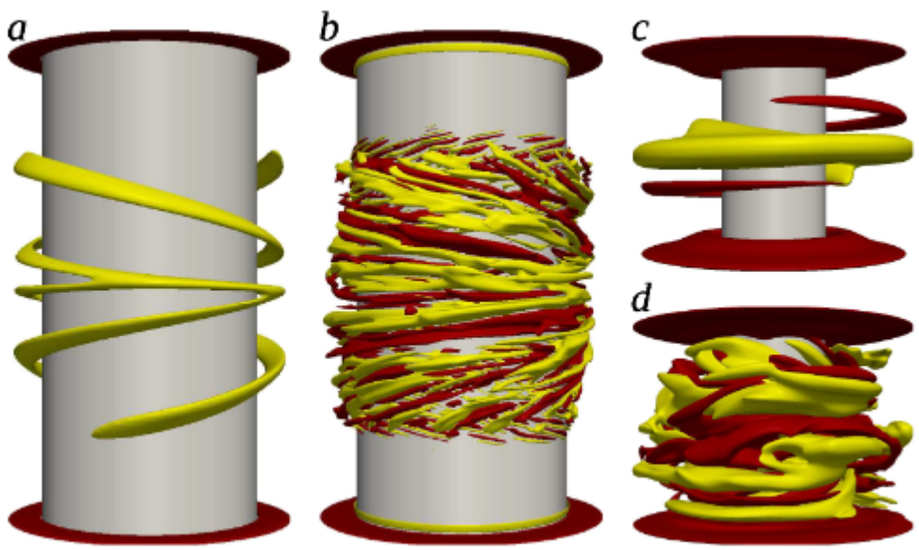




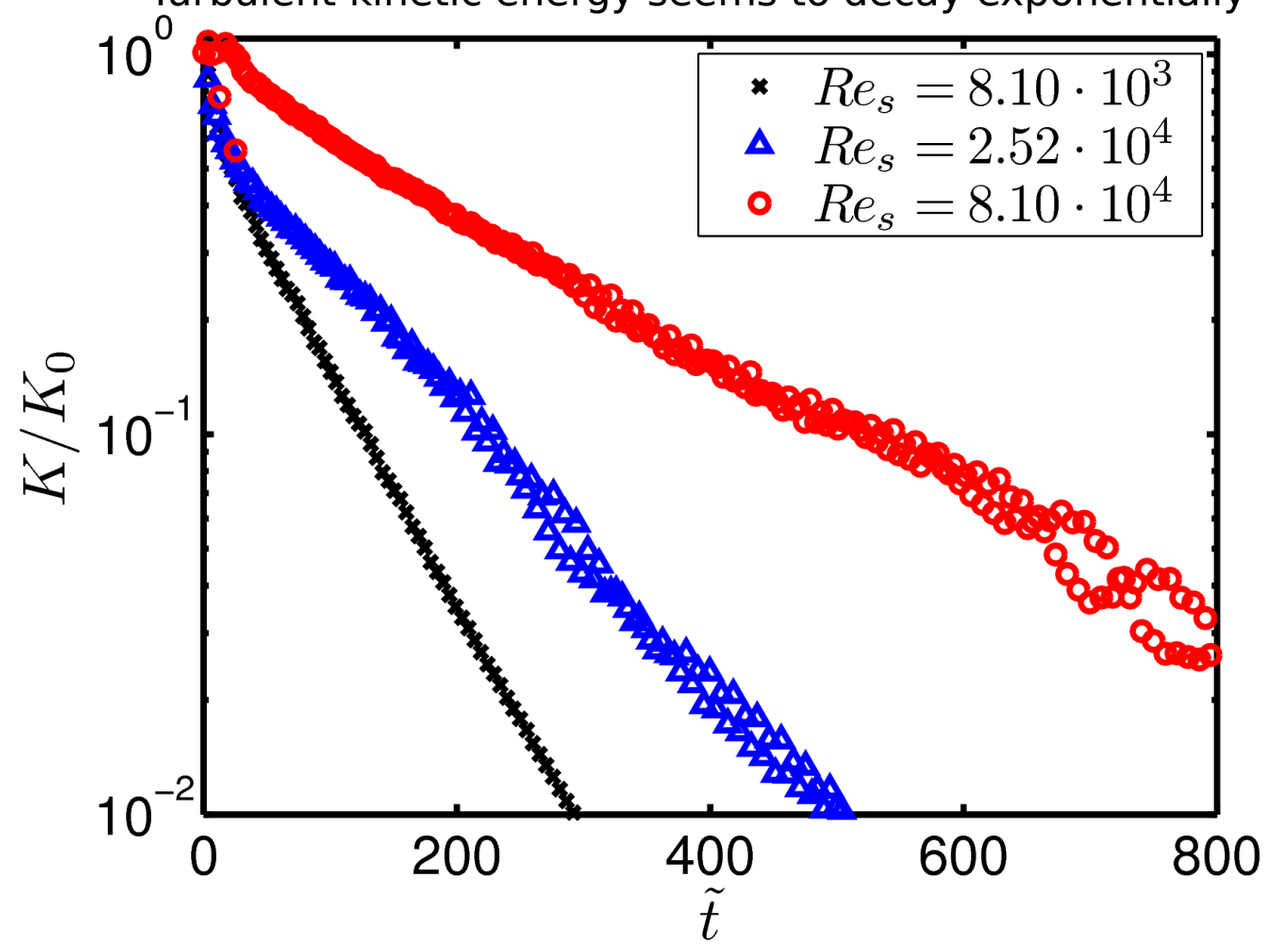
Grey area: $\nabla L < 0$, $\nabla \omega > 0$

No turbulence seen up to Re~10^5

Simulation of experimental setups



Turbulent kinetic energy seems to decay exponentially



Avila, Phys. Rev. Lett (2012)
Ostilla-Mónico, Verzicco, Grossman, Lohse, J. Fluid Mech. Rapids (2014)
Shi, Rampp, Höf, Avila, Comp&Flu (2015)

Summary

- Taylor-Couette parameter space is extremely rich

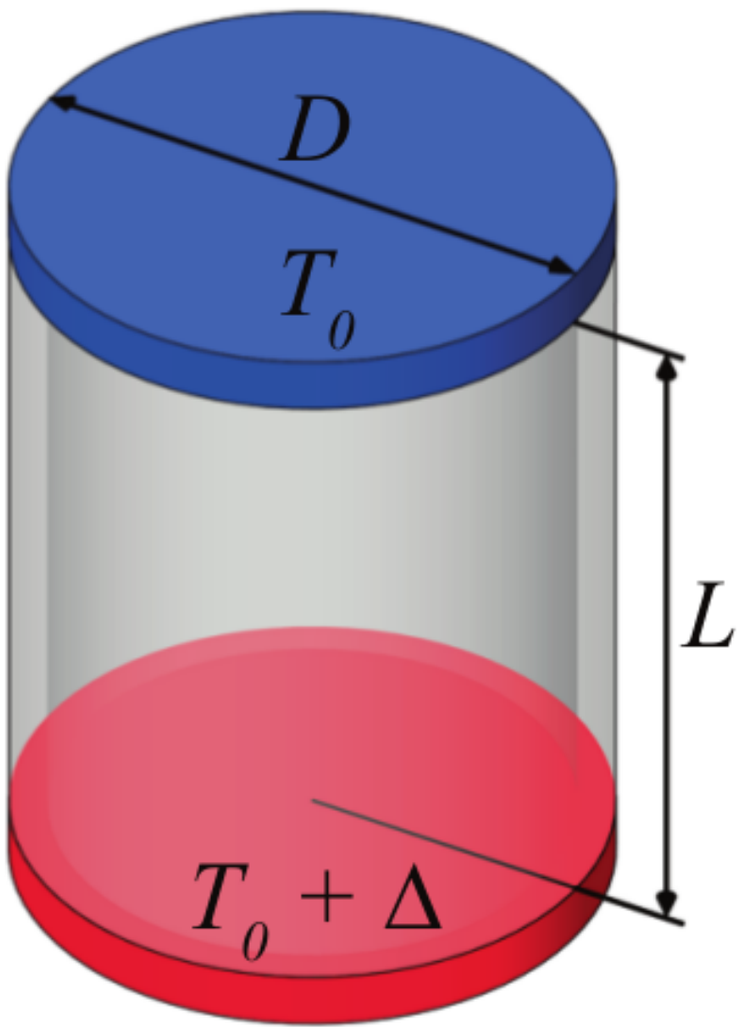
- Three possible scenarios: linearly stable, linearly unstable and mixed

- Linearly unstable TC flow has large-scale rolls which cause very distinct properties.

- Linearly stable lacks the large-scale rolls, but still some properties are different.

- Quasi-Keplerian regime: up to now no conclusive evidence for turbulence up to $Re \sim 10^5$

Small taste of Rayleigh-Bénard



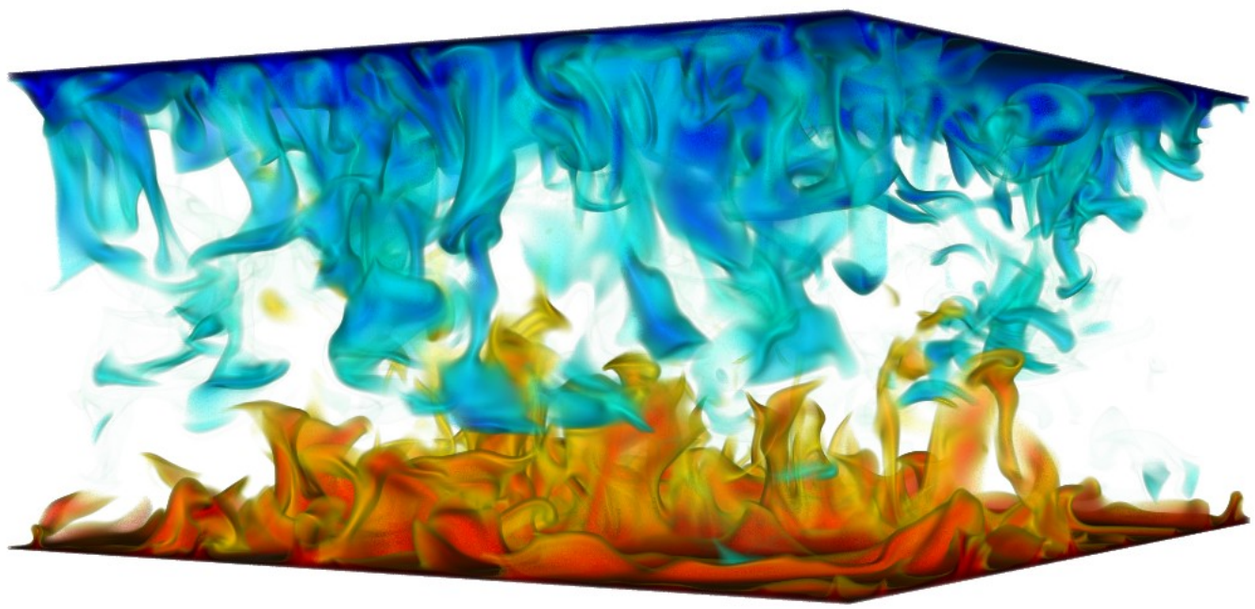
$$Ra = \frac{\beta g \Delta L^3}{\nu \kappa}$$

$$Pr = \frac{\nu}{\kappa}$$

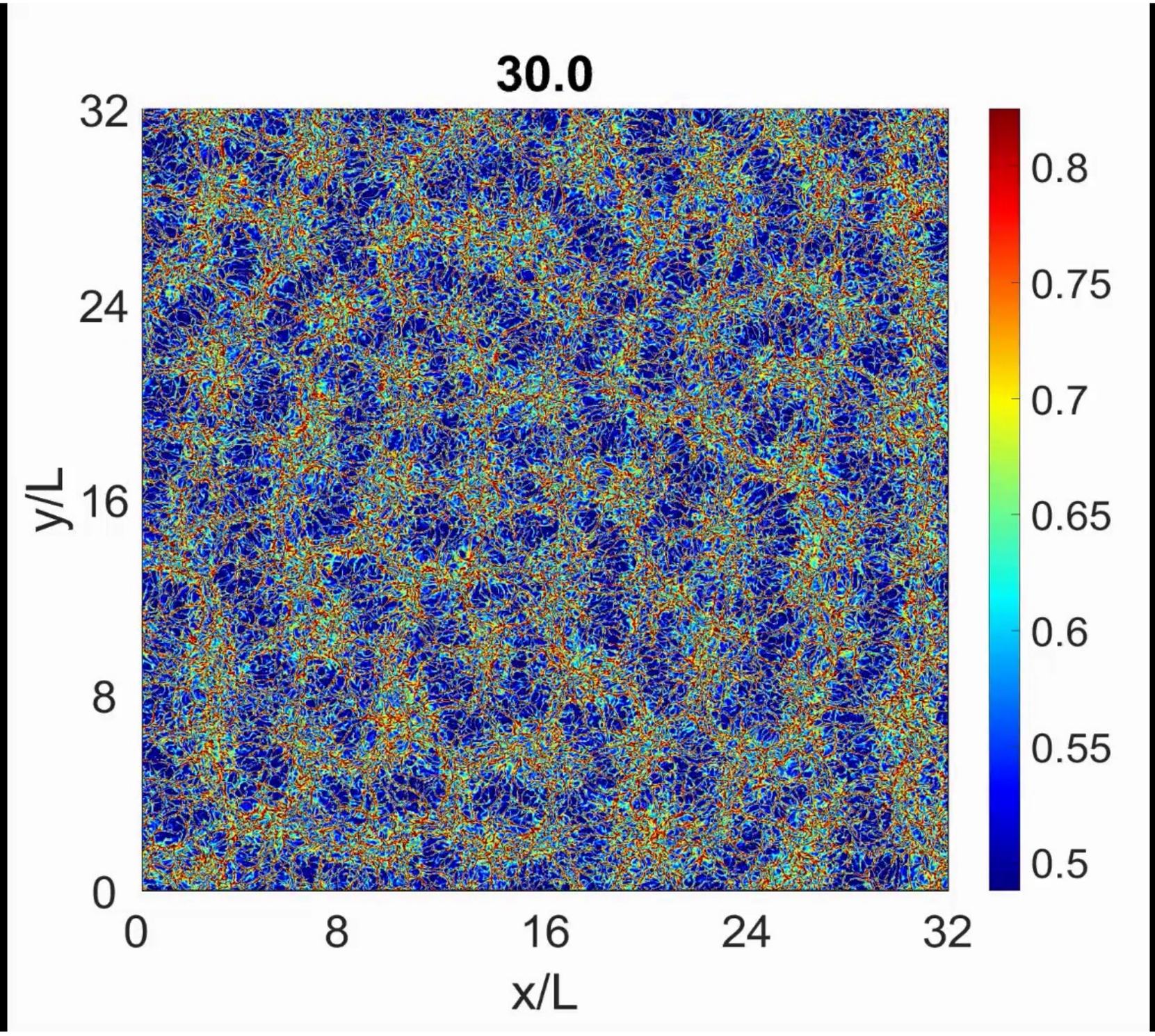
$$Ro = \frac{\sqrt{\beta g \Delta / L}}{2\Omega}$$

$$Nu = \frac{Q}{\kappa \Delta L^{-1}}$$

3D flow: large scale patterns



Inside the Boundary Layer at Ra = 10^8



Video by R. J. A. M. Stevens & A. Blass

2D: Control structure topology with velocity boundary conditions

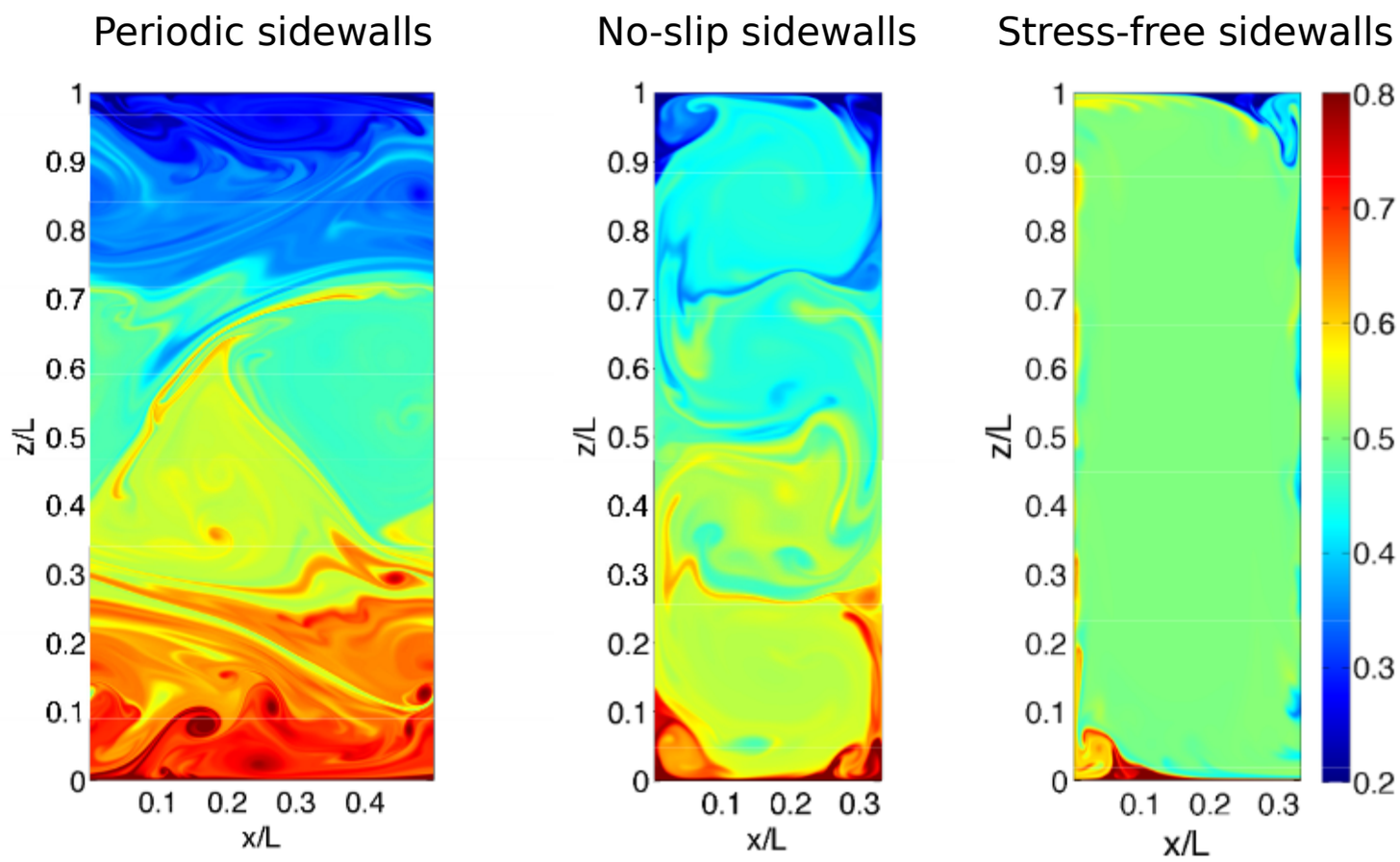


FIG. 1. (Color online) Temperature field snapshots for different simulations. Red and blue indicate hot and cold fluid respectively. The colour varies between $\theta = 0.2$ and $\theta = 0.8$. $\mathrm{Ra} = 10^{11}$ and $\Gamma = 1/2$ for lateral periodicity, showing zonal flow (left panel), $\mathrm{Ra} = 10^{10}$ and $\Gamma = 0.33$ for no-slip sidewalls, showing roll structures (center panel), and $\mathrm{Ra} = 10^{10}$ and $\Gamma = 0.33$ for stress-free sidewalls (right panel). The plates are no-slip in all cases. Movies can be found in the supplementary material.

van der Poel, Ostilla-Mónico, Verzicco, Lohse, Phys. Rev. E (2014)

Bursting with periodic BC in small domains

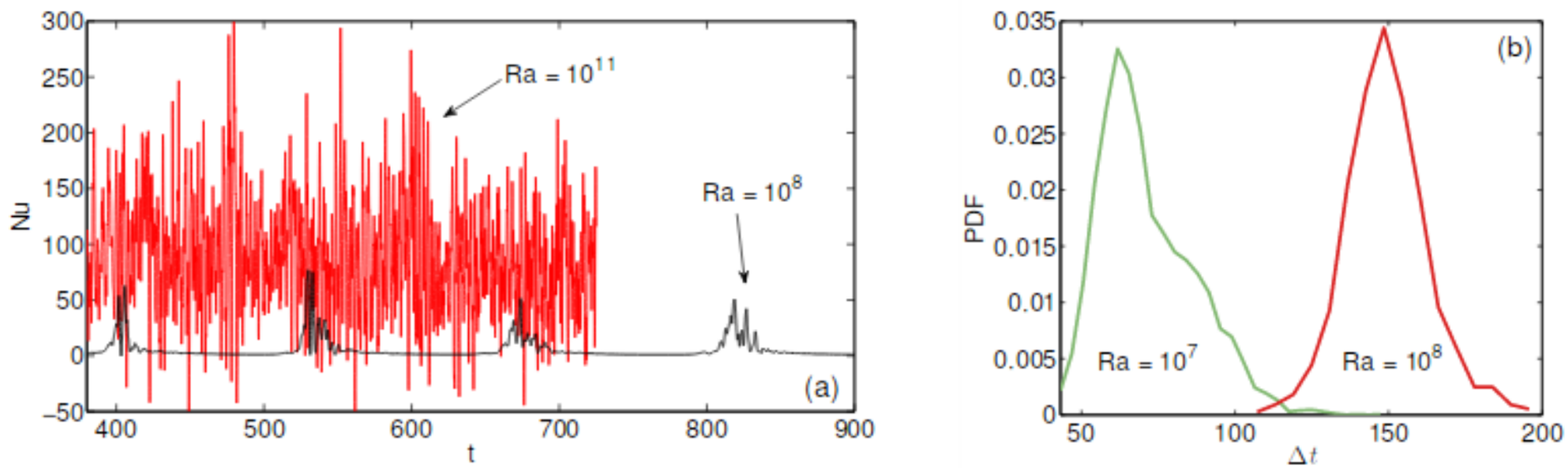


FIG. 7. (Color online) a) Nu as a function of dimensionless time in freefall time units at $\Gamma = 1/2$, for $\mathrm{Ra} = 1 \times 10^8$ and $\mathrm{Ra} = 1 \times 10^{11}$, NS plates and PD sidewalls. At $\mathrm{Ra} = 10^8$ we observe bursting behaviour while at $\mathrm{Ra} = 10^{11}$ we do not: The zonal flow is bursting for lower Ra and sustained for higher Ra. Note the large difference between the time scale of the bursts at $\mathrm{Ra} = 10^8$ and the fluctuations at $\mathrm{Ra} = 10^{11}$. b) Probability density function of the time interval between the bursts Δt for $\mathrm{Ra} = 10^7$ and $\mathrm{Ra} = 10^8$.

van der Poel, Ostilla-Mónico, Verzicco, Lohse, Phys. Rev. E (2014)

Reduced heat transport with small Γ

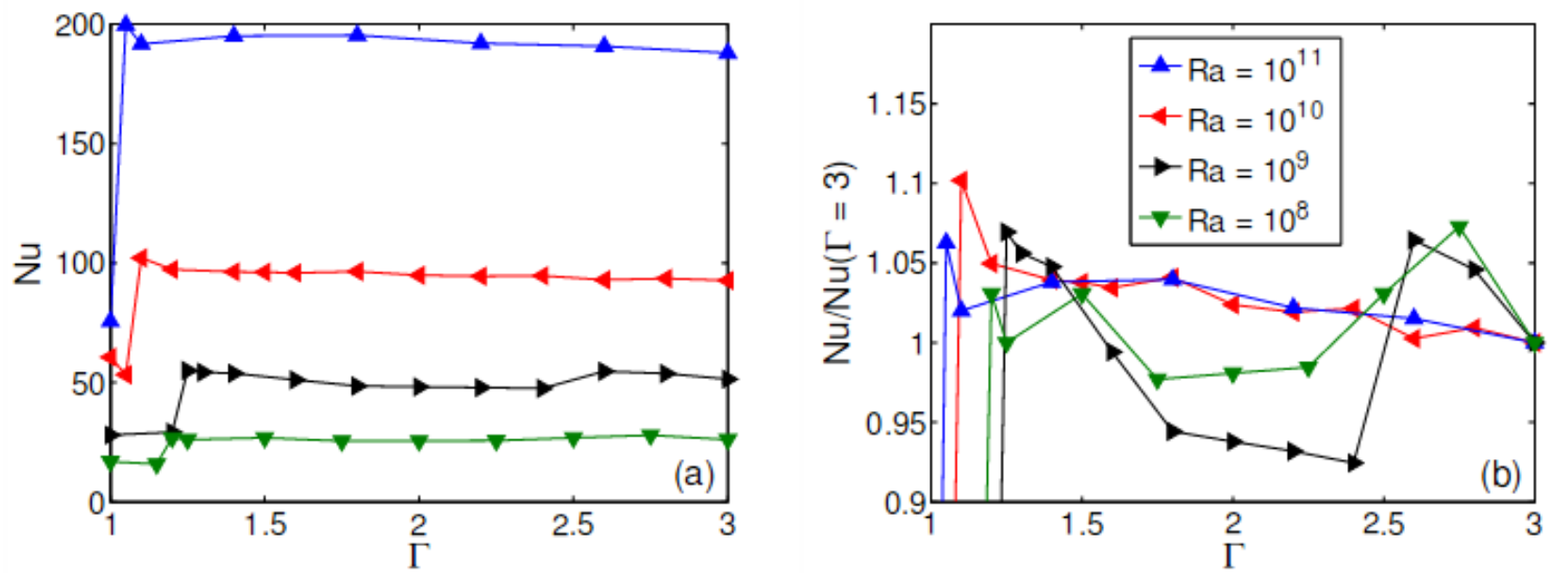


FIG. 4. (Color online) a) Nu vs Γ for periodic sidewalls and no-slip plates for Ra $\in \{10^8, 10^9, 10^{10}, 10^{11}\}$, see legend in b). Jumps in Nu can be seen around $\Gamma \approx 1.25$, $\Gamma \approx 1.25$, $\Gamma \approx 1.10$ and $\Gamma \approx 1.05$ for Ra $= 10^8$, Ra $= 10^9$, Ra $= 10^{10}$ and Ra $= 10^{11}$, respectively. The Nusselt number statistics at lower Γ than the jump are more difficult to converge than the other data points due to the bursting nature of Nu(t) and have a larger error. They are included to indicate the change of flow state. In b) Nu is compensated with Nu($\Gamma = 3$).

van der Poel, Ostilla-Mónico, Verzicco, Lohse, Phys. Rev. E (2014)

Certain BCs cause onset of "zonal" flow

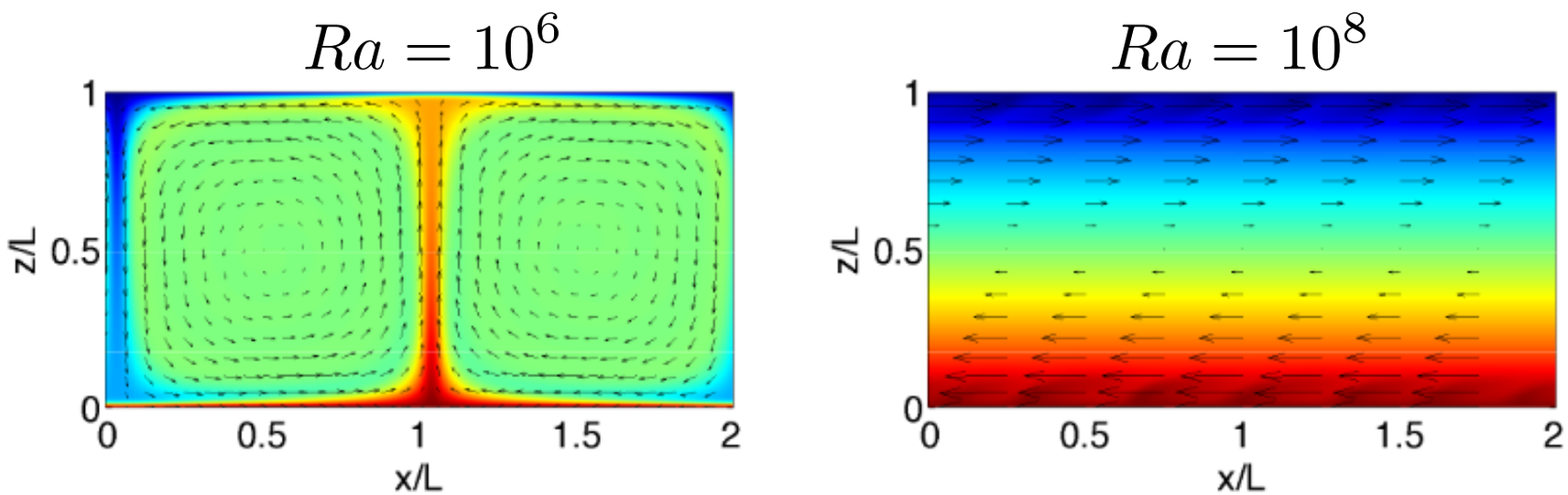
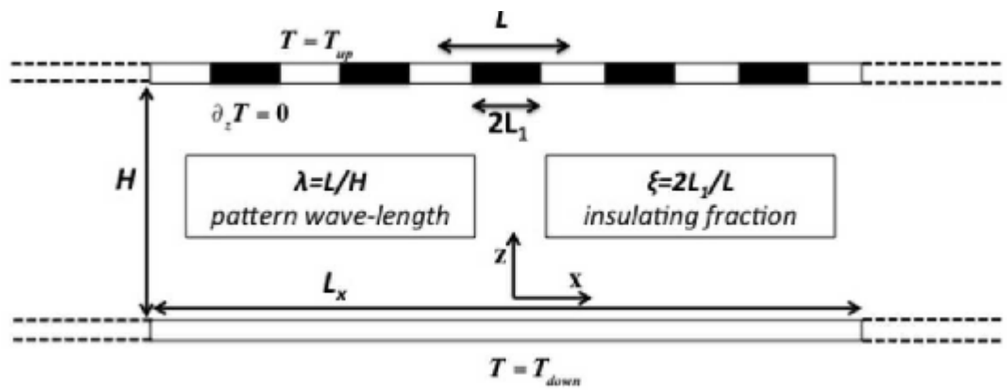


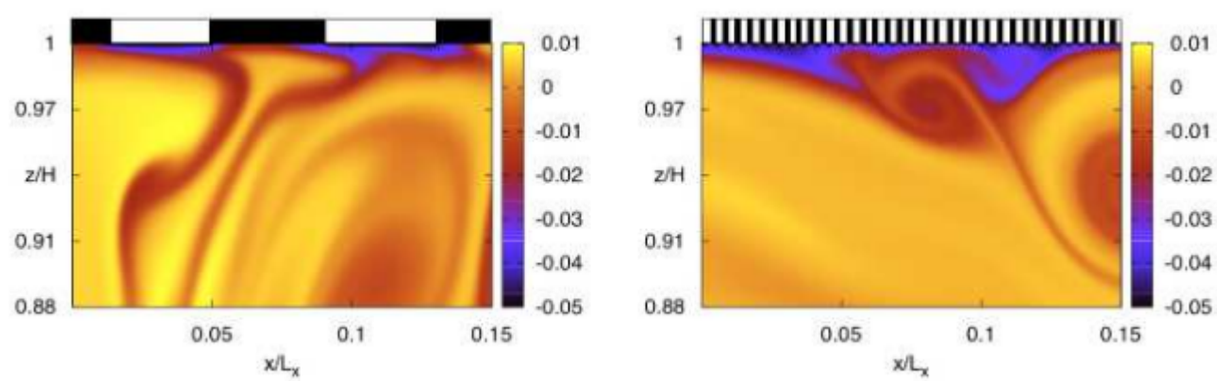
FIG. 6. (Color online) Temperature field snapshots with velocity vectors superimposed for stress-free plates and periodic sidewalls for Pr = 1 and at Ra = 10^6 (left panel) and Ra = 10^8 (right panel) in a $\Gamma = 2$ cell. The temperature ranges from $0 \leq \theta \leq 1$. The size of the arrows is proportional to the absolute velocity. The flow is roll-like in a) and zonal in b). Corresponding movies can be found in the supplemental material.

Heat transport significantly reduced at high Ra

van der Poel, Ostilla-Mónico, Verzicco, Lohse, Phys. Rev. E (2014)

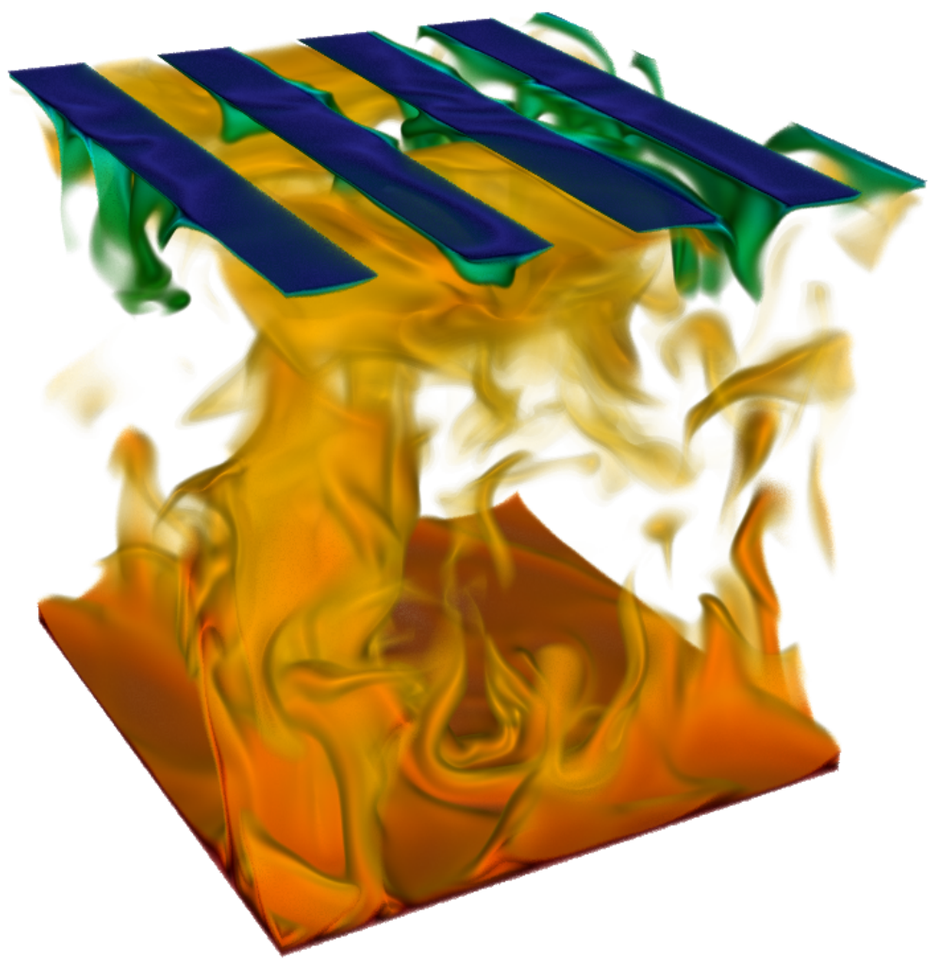
Mixed temperature BCs have little effect

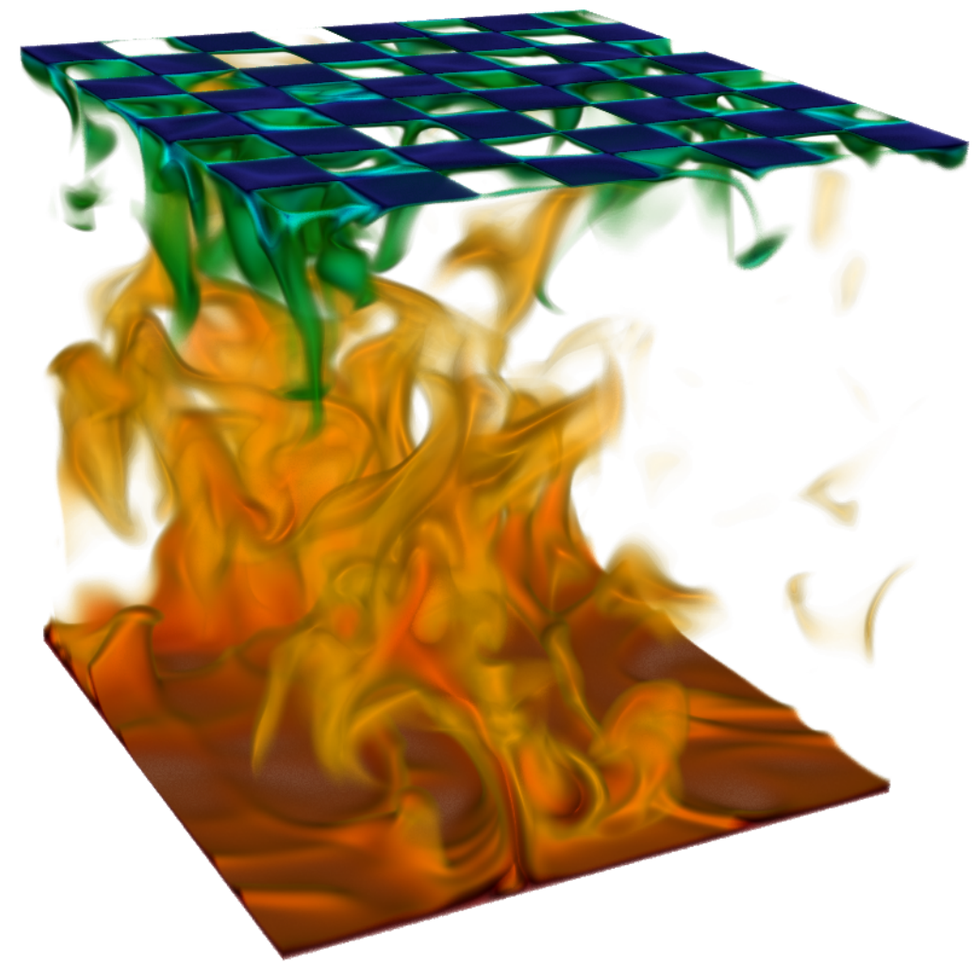




Ripesi, Biferale, Sbragaglia, Wirth, J. Fluid Mech (2014)

Mixed temperature BC in 3D: little effect





Bakhuis, Ostilla-Mónico, van der Poel, Verzicco, Lohse, in preparation

Rapid rotation causes different organization

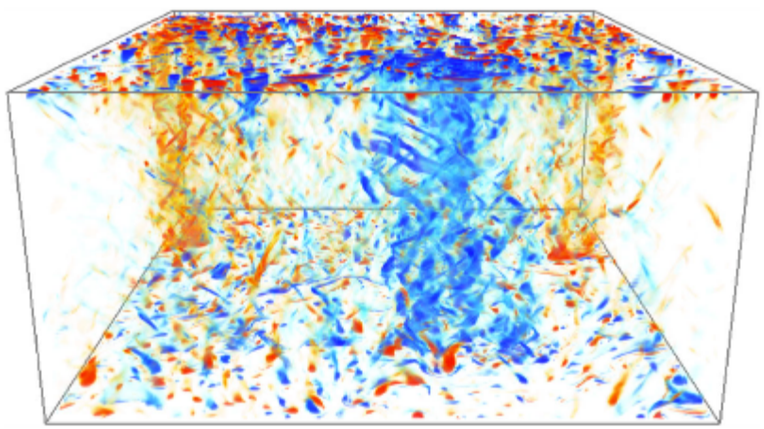


FIG. 1 (color online). Volume rendering of vertical vorticity ζ in geostrophic turbulence showing the development of a large scale dipole and the organization of small-scale convective eddies for $\mathrm{Ra}E^{4/3} = 100$ and $\sigma = 1$ at $t = 100$.

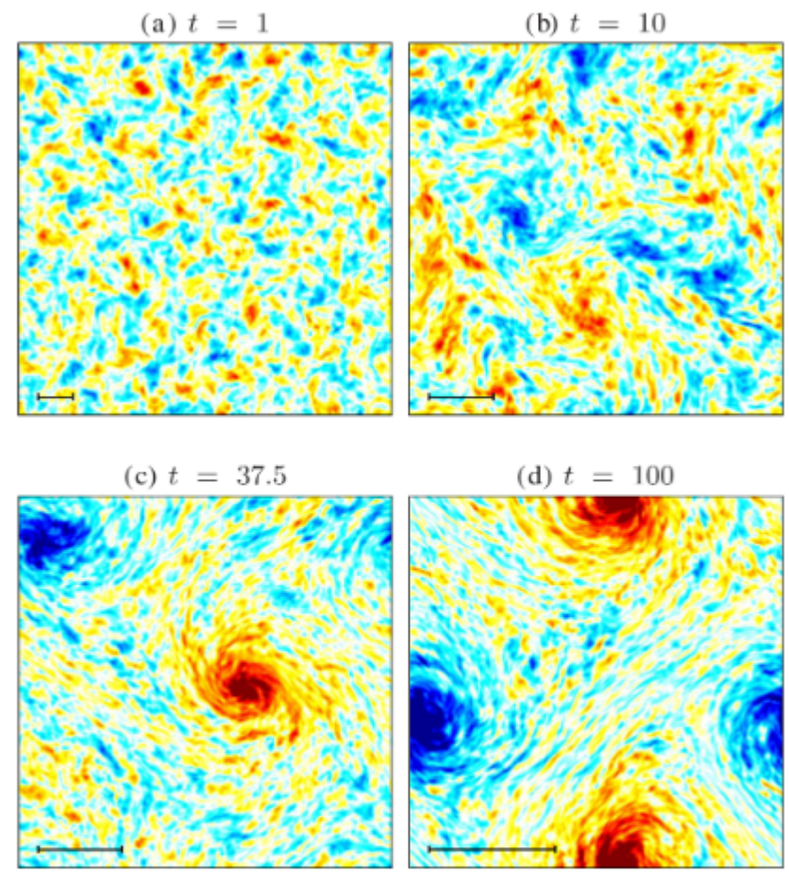


FIG. 2 (color online). Barotropic vertical vorticity at $t = 1$, 10, 37.5, and 100, respectively, showing the organization of the flow into structures at progressively larger scales. The black lines indicate one-half wavelength of the dynamically-evolving baroclinic forcing scale $1/k_f$ defined in the text.

Rubio, Julien, Knobloch, Weiss, Phys. Rev. Lett, 112 (2014), 144501

Depends on No-slip or Free-slip BCs

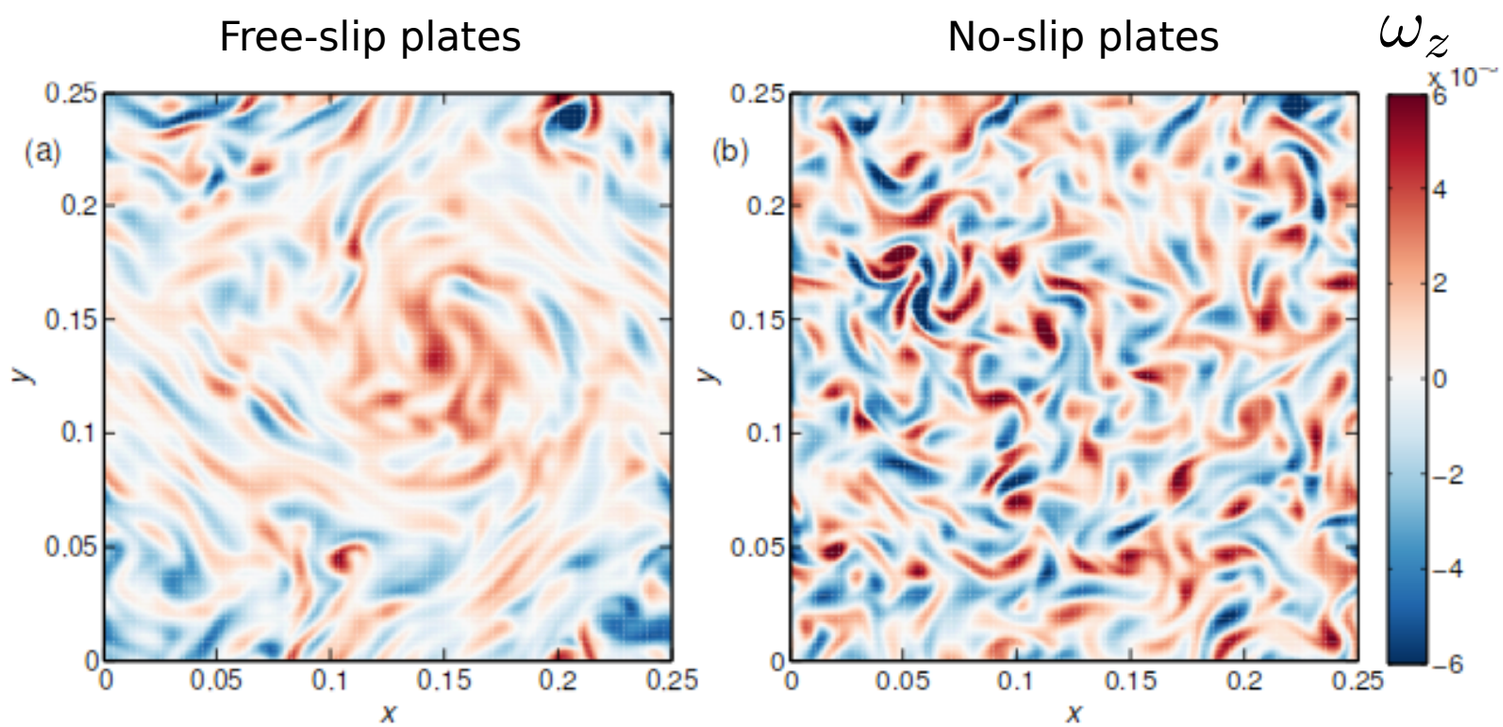


FIGURE 6. Snapshot at midheight ($z = 0.5$) of the vertical vorticity from runs at $Ra = 5 \times 10^{10}$ and $Ek = 1.34 \times 10^{-7}$. (a) SF plates. (b) NS plates. Red is positive (cyclonic) vorticity, while blue is negative (anticyclonic) vorticity. Both plots have the same colour scale.

Kunnen, Ostilla-Mónico, van der Poel, Verzicco, Lohse, J. Fluid Mech (2016)

Rayleigh-Bénard asymptotics:

Numerical simulation of an asymptotically reduced system for rotationally constrained convection

By MICHAEL SPRAGUE[1,2], KEITH JULIEN[1], EDGAR KNOBLOCH[3] AND JOSEPH WERNE[4]

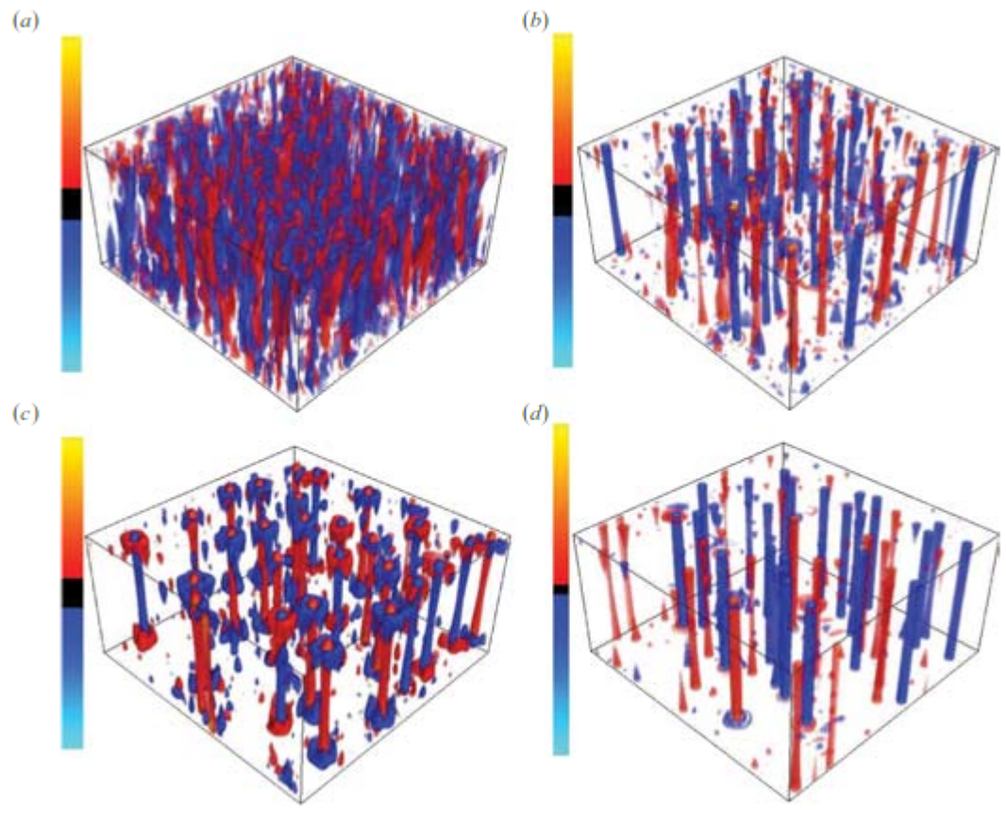


FIGURE 3. As figure 2 but for $\widetilde{Ra} = 40$ and different Prandtl numbers Pr. (a) $Pr = 1$; (b) 3; (c) 7; (d) $\rightarrow \infty$.

Rayleigh-Bénard asymptotics:

$Pr >> 1$

On upper bounds for infinite Prandtl number convection with or without rotation

Charles R. Doering
Department of Mathematics, University of Michigan, Ann Arbor, Michigan 48109-1109

Peter Constantin[a)]
Department of Mathematics, University of Chicago, Chicago, Illinois 60637

$$\frac{1}{\mathrm{Pr}}\left(\frac{\partial \mathbf{u}}{\partial t}+\mathbf{u}\cdot\nabla\mathbf{u}\right)+\sqrt{\mathrm{Ta}}\,\mathbf{k}\times\mathbf{u}+\nabla p=\Delta\mathbf{u}+\mathrm{Ra}\,\mathbf{k}T,$$

$$\nabla\cdot\mathbf{u}=0,$$

$$\frac{\partial T}{\partial t}+\mathbf{u}\cdot\nabla T=\Delta T,$$

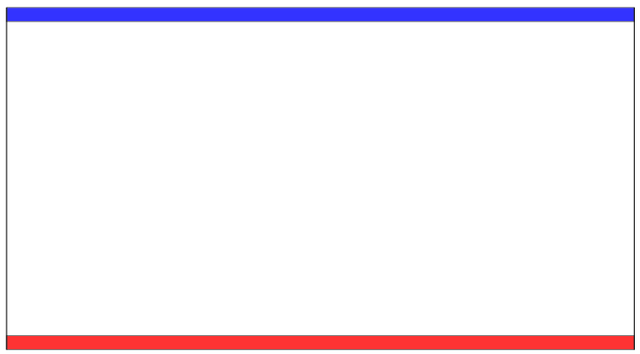
$Pr << 1$

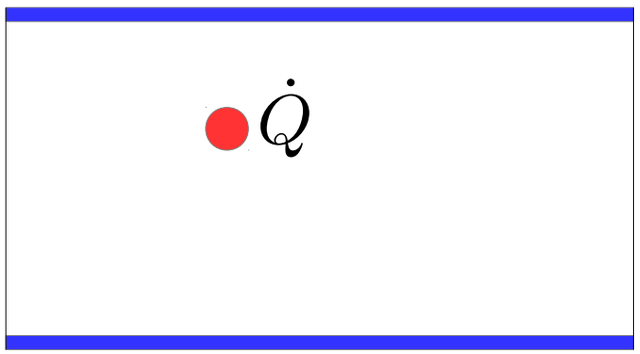
Zero-Prandtl-number convection

By OLIVIER THUAL†
National Center for Atmospheric Research, PO Box 3000, Boulder, CO 80307, USA

$$\left.\begin{aligned}\boldsymbol{v}_t+\boldsymbol{v}\cdot\nabla\boldsymbol{v}&=-\nabla p+\nabla^2\boldsymbol{v}+R\theta\,\boldsymbol{e}_3,\\ \nabla\cdot\boldsymbol{v}&=0,\\ 0&=w+\nabla^2\theta.\end{aligned}\right\}$$

Rayleigh-Bénard vs Internal Heating





$$Nu = \frac{Q}{\kappa \Delta L^{-1}}$$

$$Re = \frac{U_w L}{\nu}$$

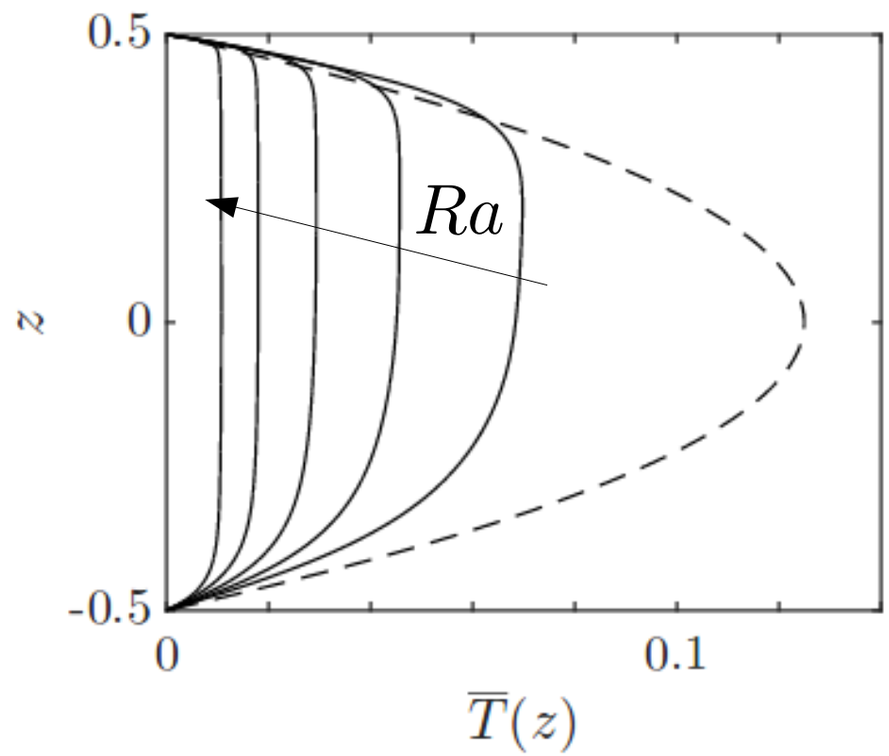
Everywhere unstable

$$\langle T \rangle$$

$$\mathcal{F}_B = \frac{Q_b}{Q}$$

Stable/unstable

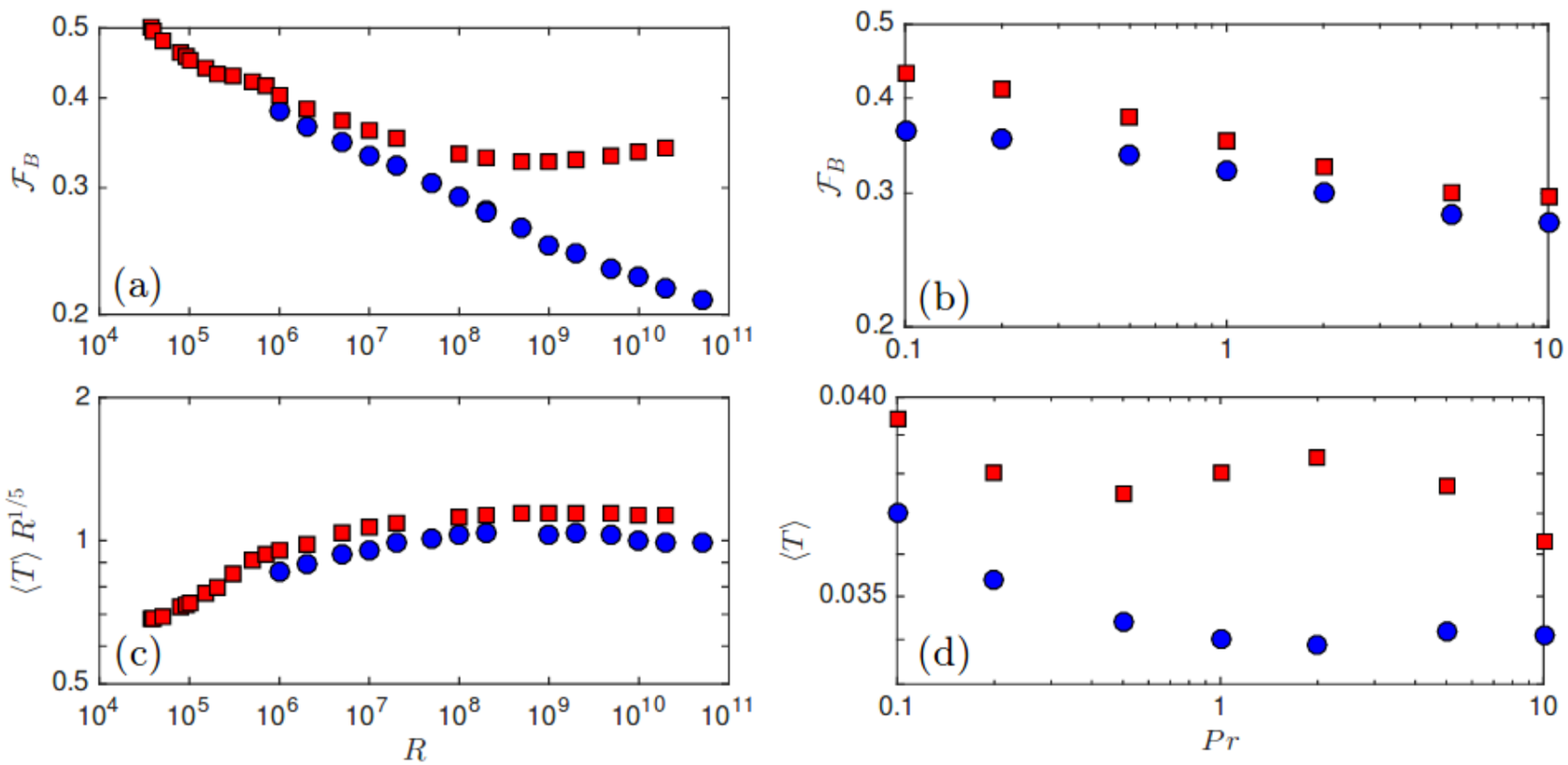
Little exploration of this problem



Goluskin, van der Poel, J. Fluid Mech Rapids (2016)

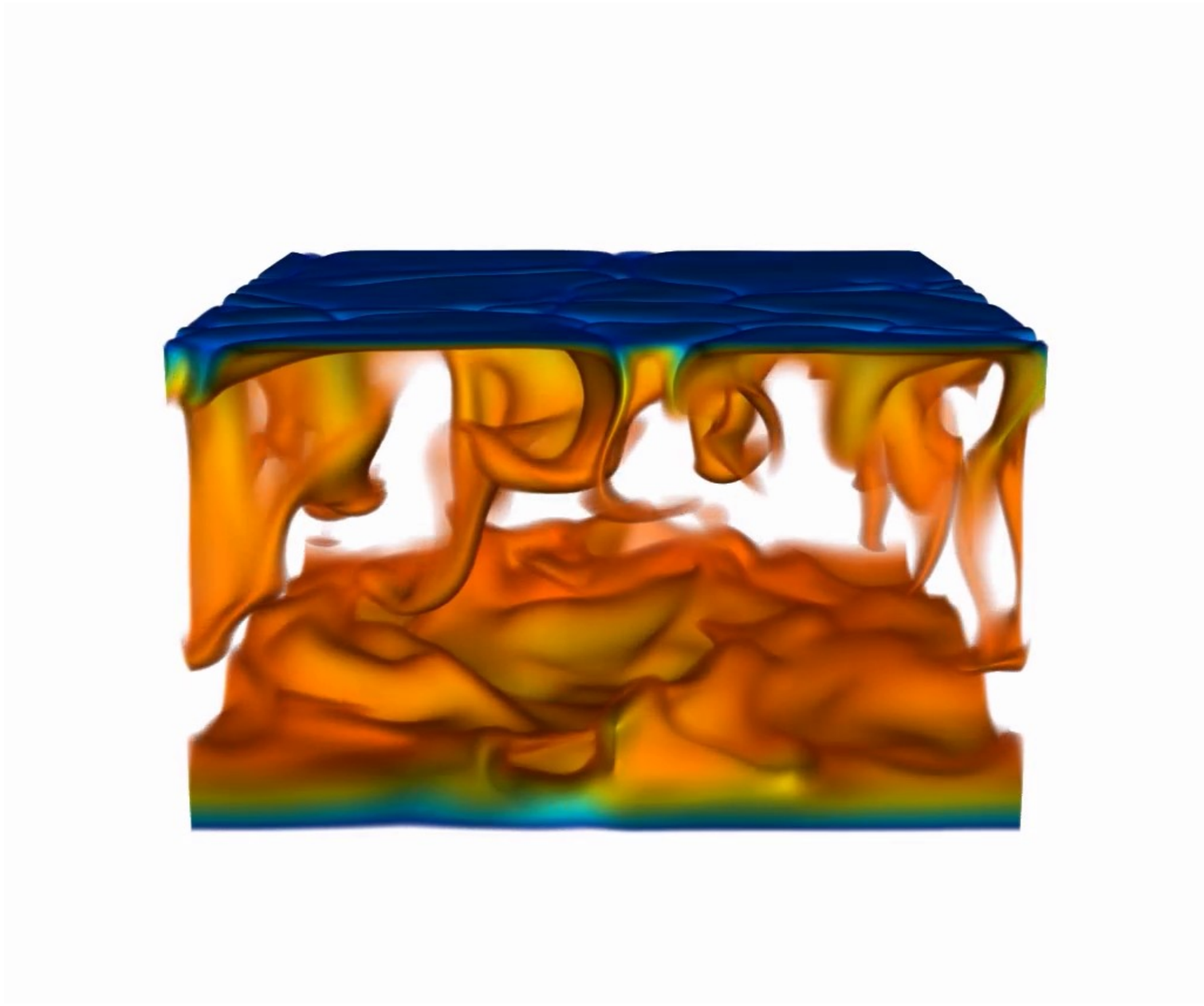
Different physics in 2D and 3D

2D vs 3D phenomena



Goluskin, van der Poel, J. Fluid Mech Rapids (2016)

Internal Heating



Goluskin, van der Poel, J. Fluid Mech Rapids (2016)