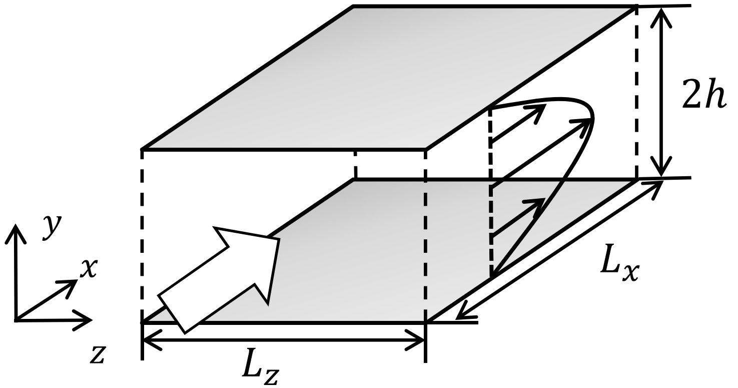
Periodic solutions representing the origin of turbulent bands in channel flow

Takahiro Kanazawa   Masaki Shimizu

Genta Kawahara

Graduate School of Engineering Science
Osaka University

Plane channel flow

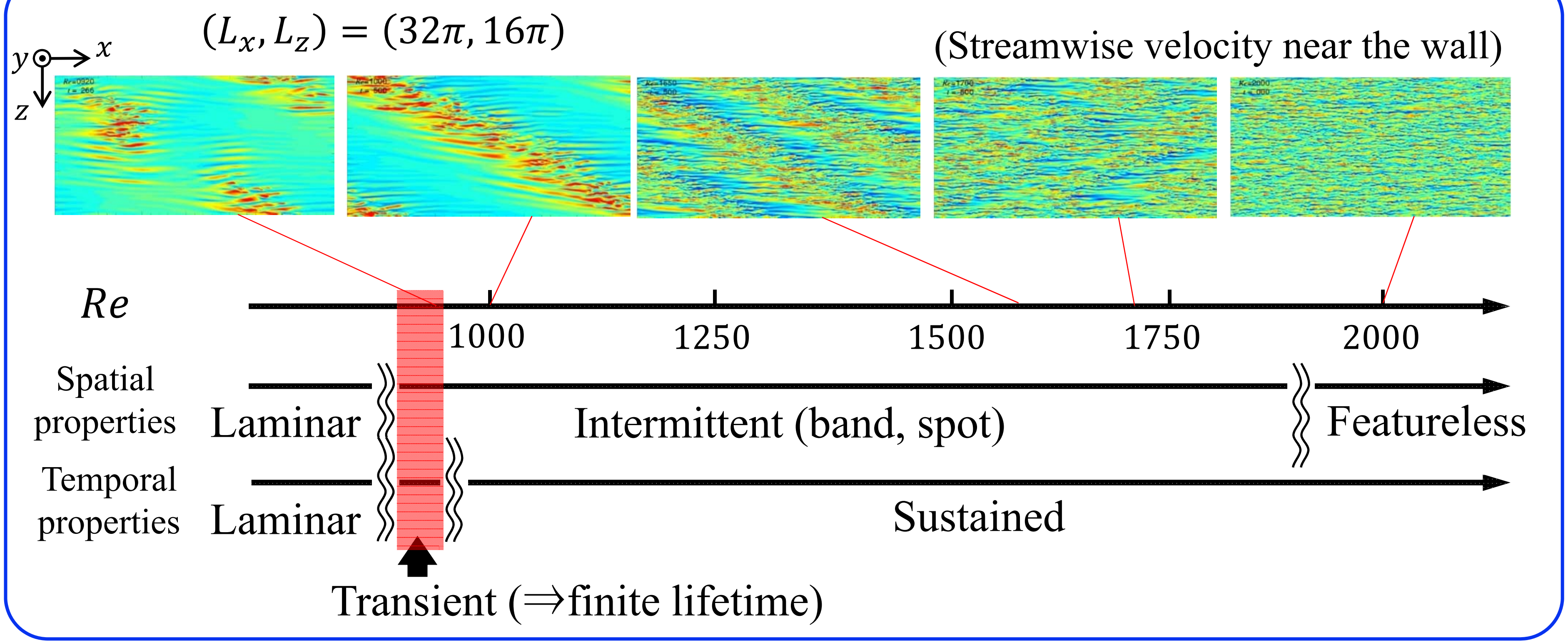


Reynolds number : $Re = \frac{U_c h}{\nu}$

U_c : centerline velocity

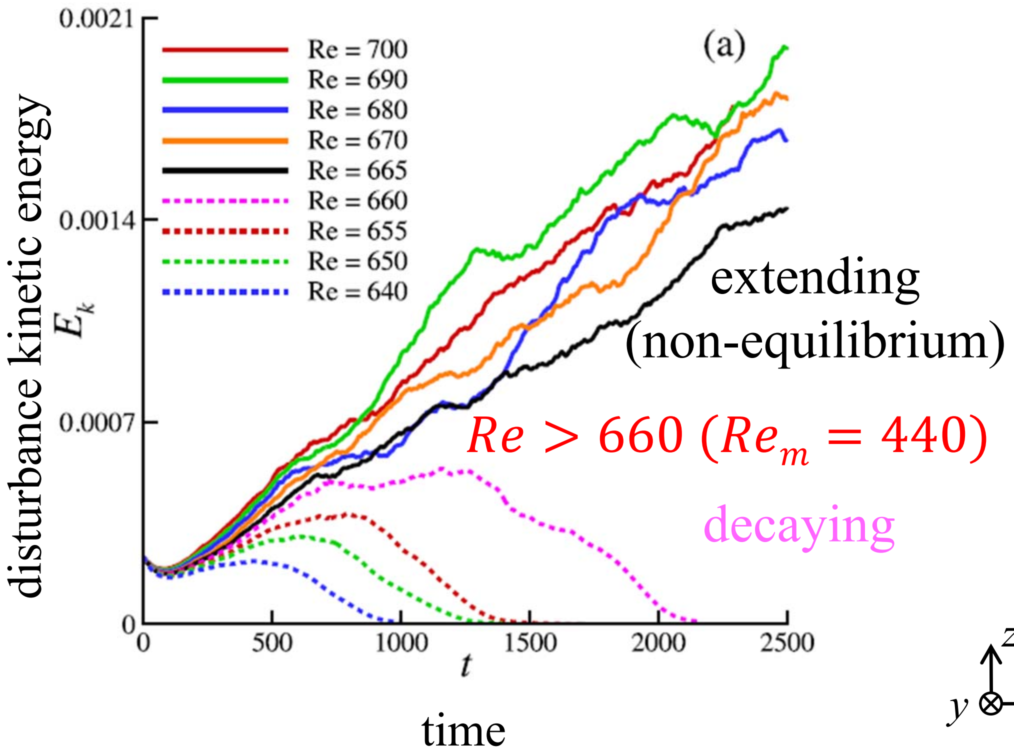
- Laminar flow becomes unstable at $Re = Re_c^{[1]}$
- Turbulence is observed at $Re < Re_c$

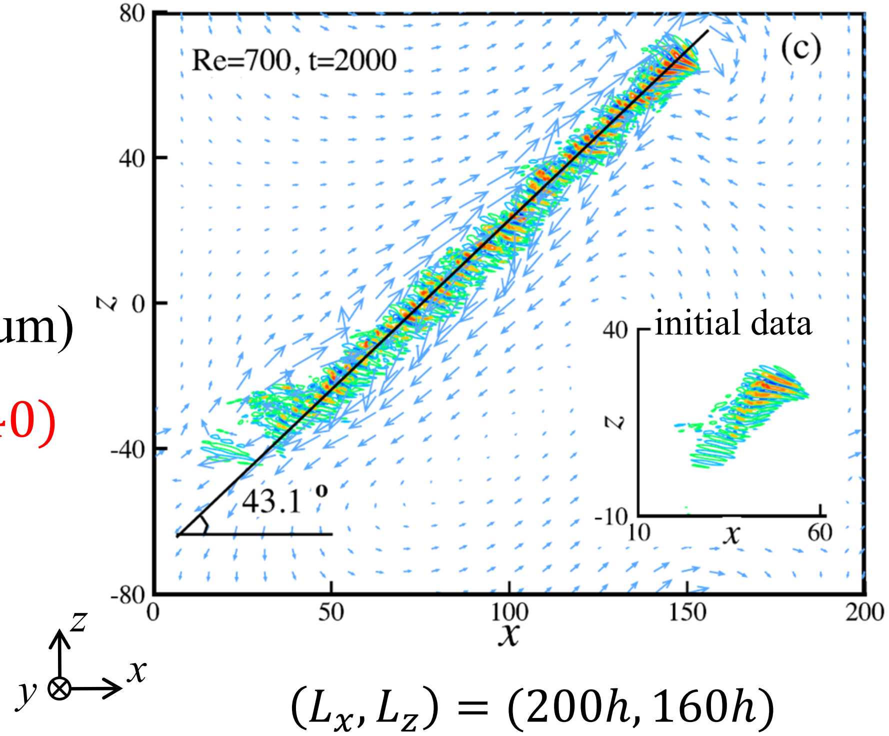
[1] $Re_c = 5772$, Orszag, JFM (1971)



Isolated turbulent band

Isolated extending turbulent band in large computational domain





Xiong, Tao, Chen and Brandt, Phys. Fluids (2015)

System & Parameters

- Governing equation

  Incompressible Navier-Stokes equation

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\frac{1}{\rho} \nabla p + \nu \Delta \boldsymbol{u}$$

$$\nabla \cdot \boldsymbol{u} = \boldsymbol{0}$$

- Dimensionless parameter

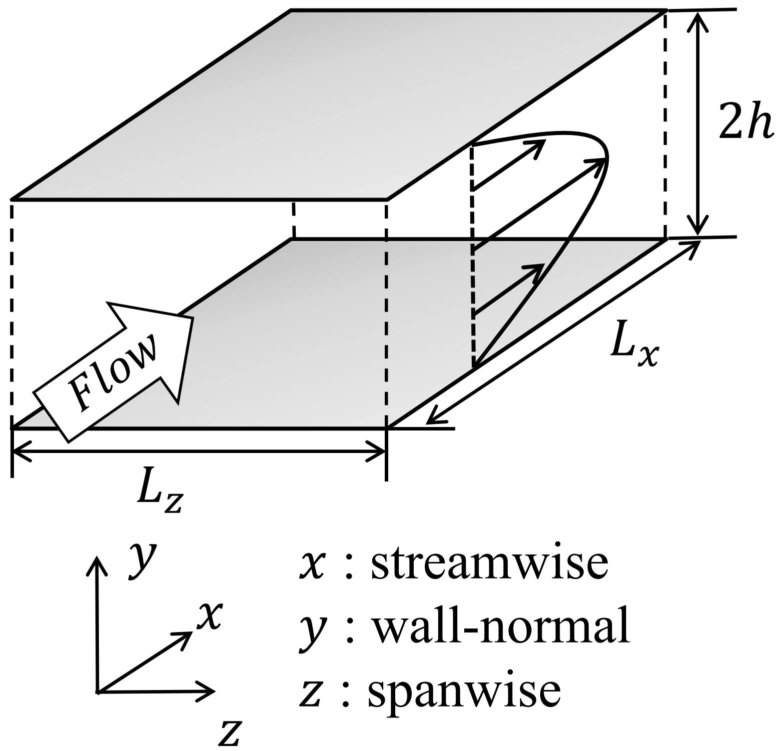
  Reynolds number : $Re_m = \dfrac{Uh}{\nu}$

  (U : constant bulk mean velocity)

- Boundary conditions

  streamwise & spanwise ⇒ periodic

  wall-normal ⇒ no-slip impermeable



x : streamwise

y : wall-normal

z : spanwise

$x \in [0, L_x]$

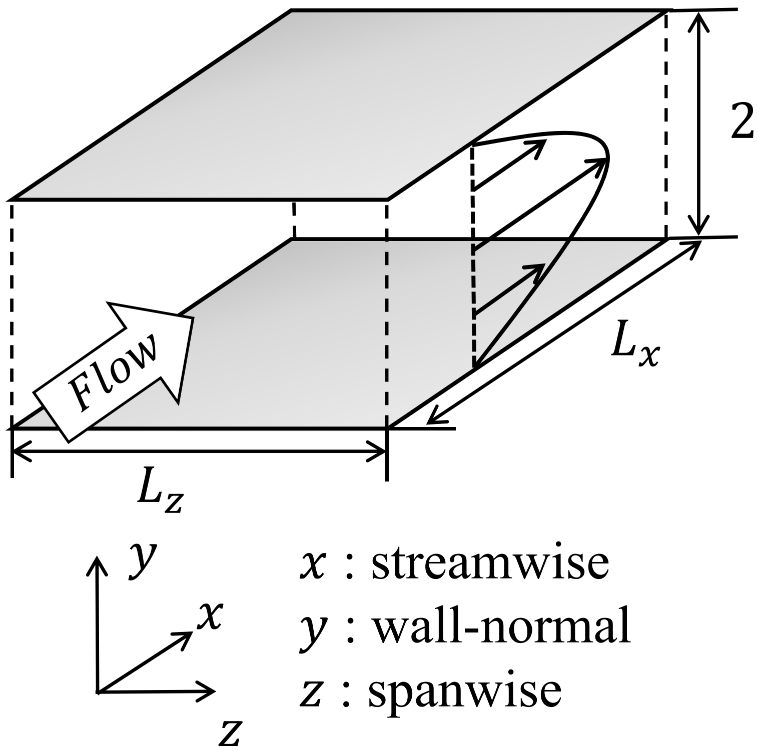
$y \in [-h, h]$

$z \in [0, L_z]$

System & Parameters

- Numerical domain & Grid numbers

(L_x, L_z)	(N_x, N_y, N_z)
$(500, 250)$	$(3840, 49, 1920)$
$(200, 200)$	$(768, 49, 768)$
$(100, 100)$	$(384, 49, 384)$

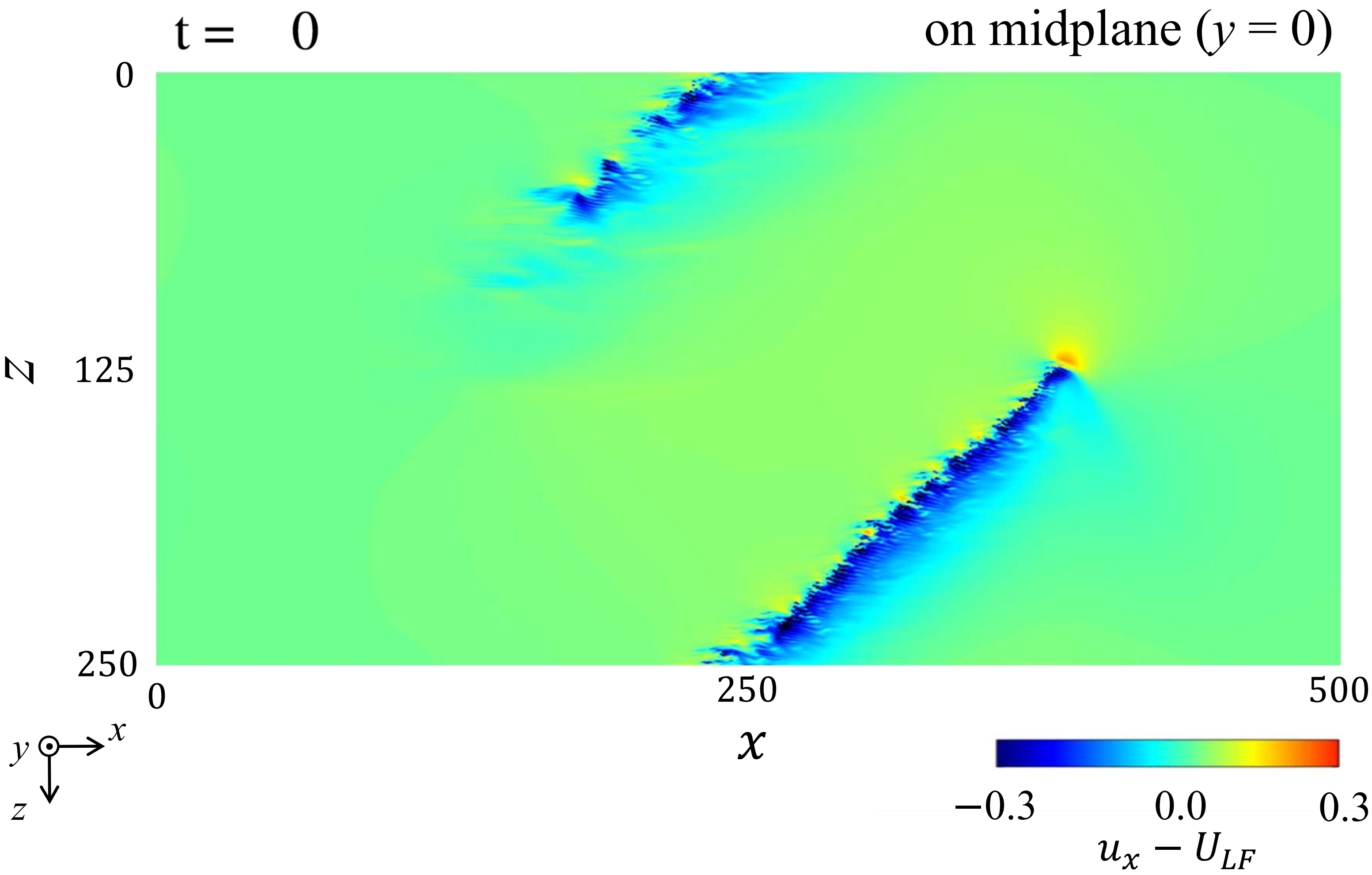


x : streamwise
y : wall-normal
z : spanwise

$$x \in [0, L_x]$$
$$y \in [-1, 1]$$
$$z \in [0, L_z]$$

Example

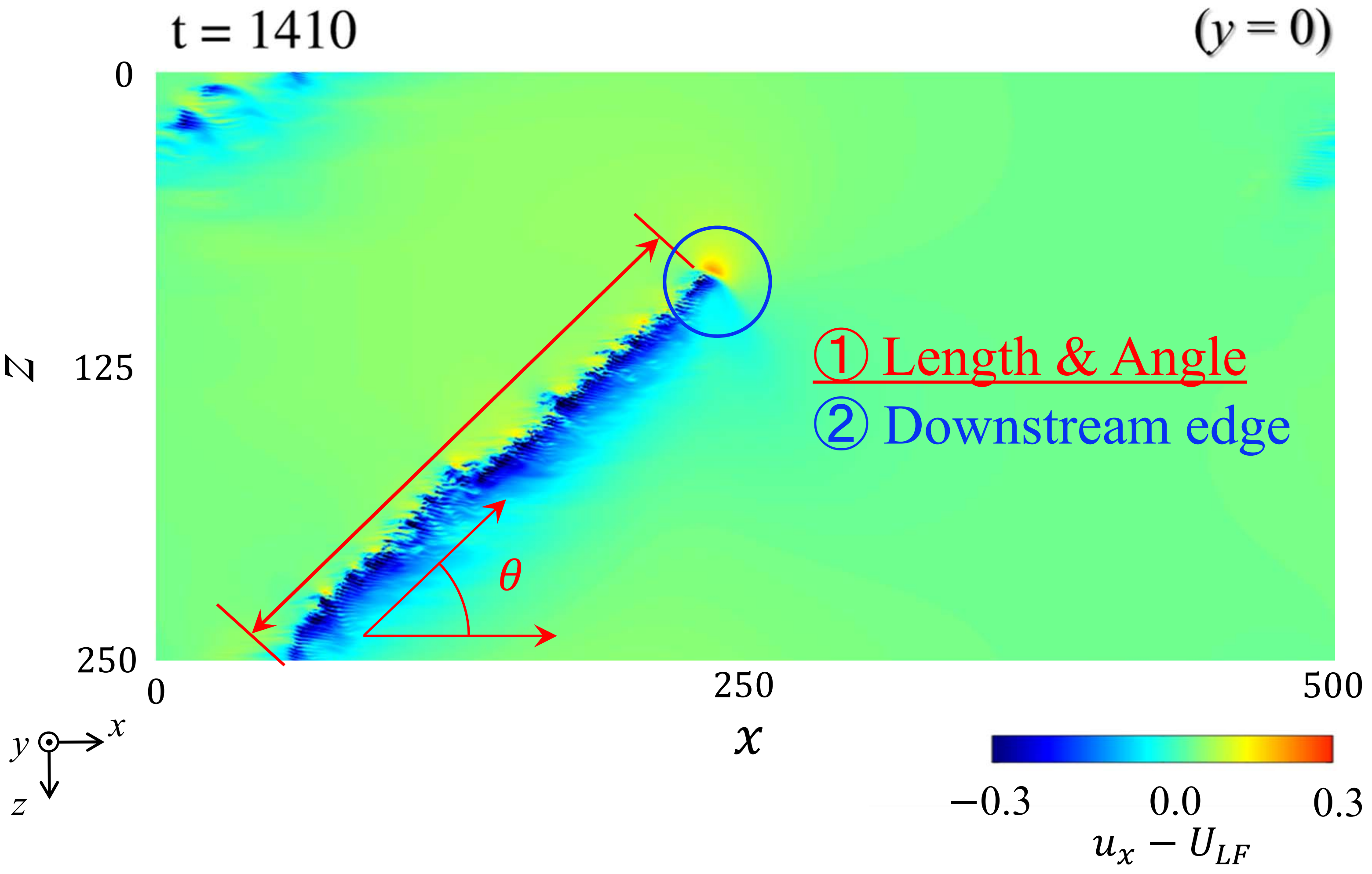
$(L_x, L_z) = (500, 250),\ Re_m = 440$



U_{LF} : Laminar flow velocity

Example

$(L_x, L_z) = (500, 250),\ Re_m = 440$

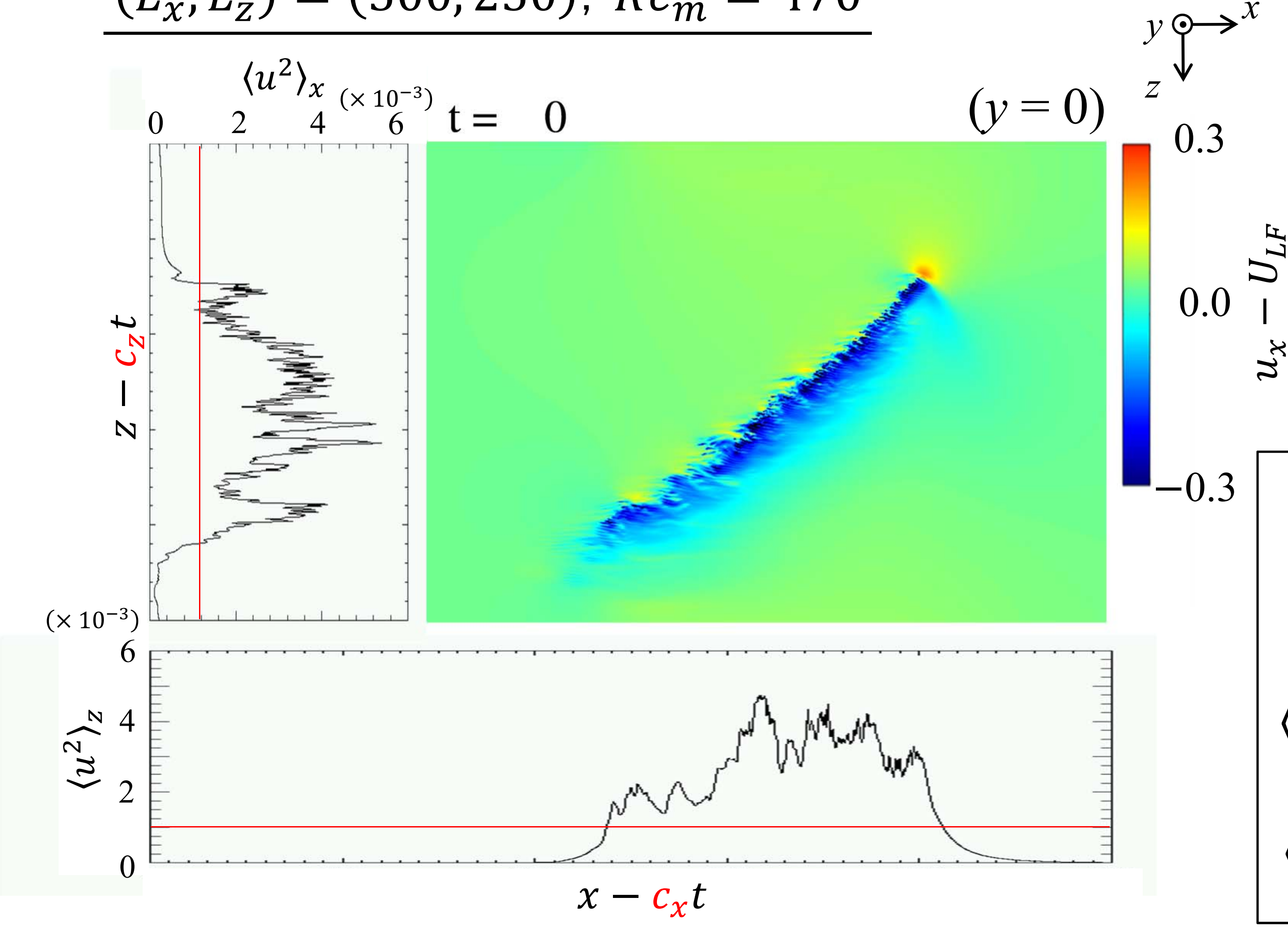


U_{LF} : Laminar flow velocity

Length & Angle

Measurement method

$(L_x, L_z) = (500, 250),\ Re_m = 470$



$c_x \simeq 1.23$

$c_z \simeq -0.14$

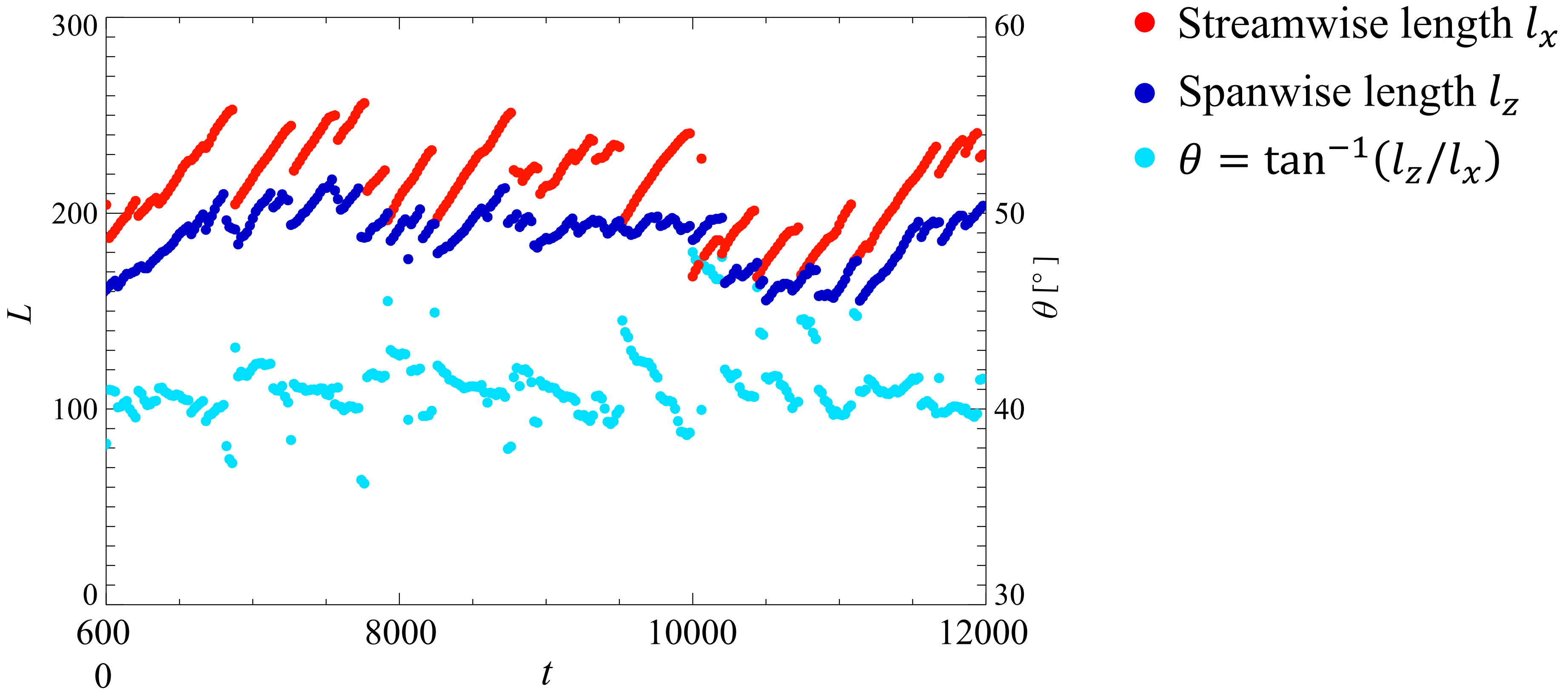
$$u^2 = (u_x - U_{LF})^2$$

$$\langle \cdot \rangle_x = \frac{1}{L_x} \int_0^{L_x} \cdot \, dx$$

$$\langle \cdot \rangle_z = \frac{1}{L_z} \int_0^{L_z} \cdot \, dz$$

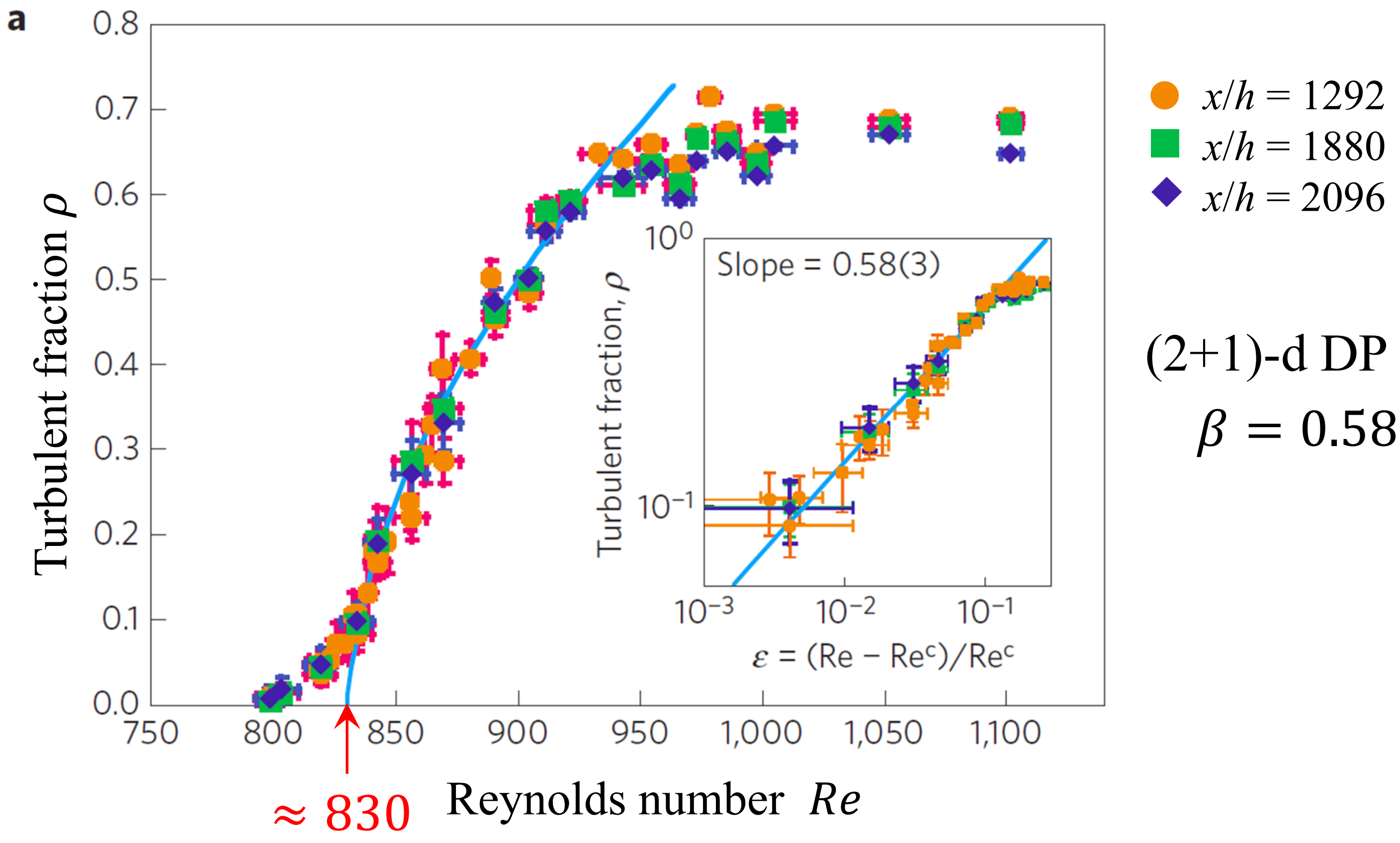
Length & Angle

$(L_x, L_z) = (500, 250)$, $Re_m = 440$ $(Re = 3Re_m/2 = 660)$



Length & Angle

$(L_x, L_z) = (500, 250),\ Re_m = 440$ $\quad (Re = 3Re_m/2 = 660)$



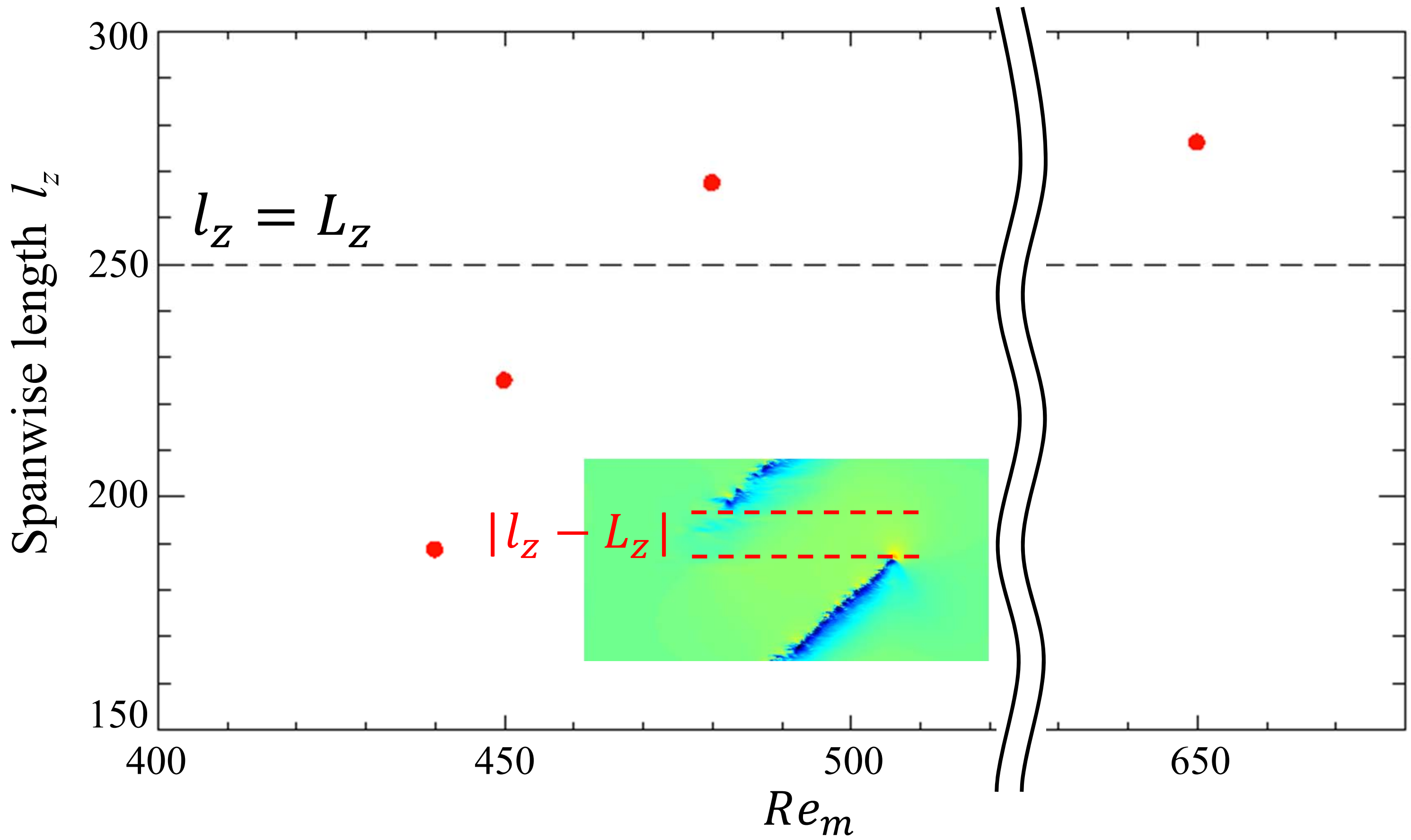
(2+1)-d DP

$\beta = 0.583$

Sano and Tamai, Nature Phys. (2016)

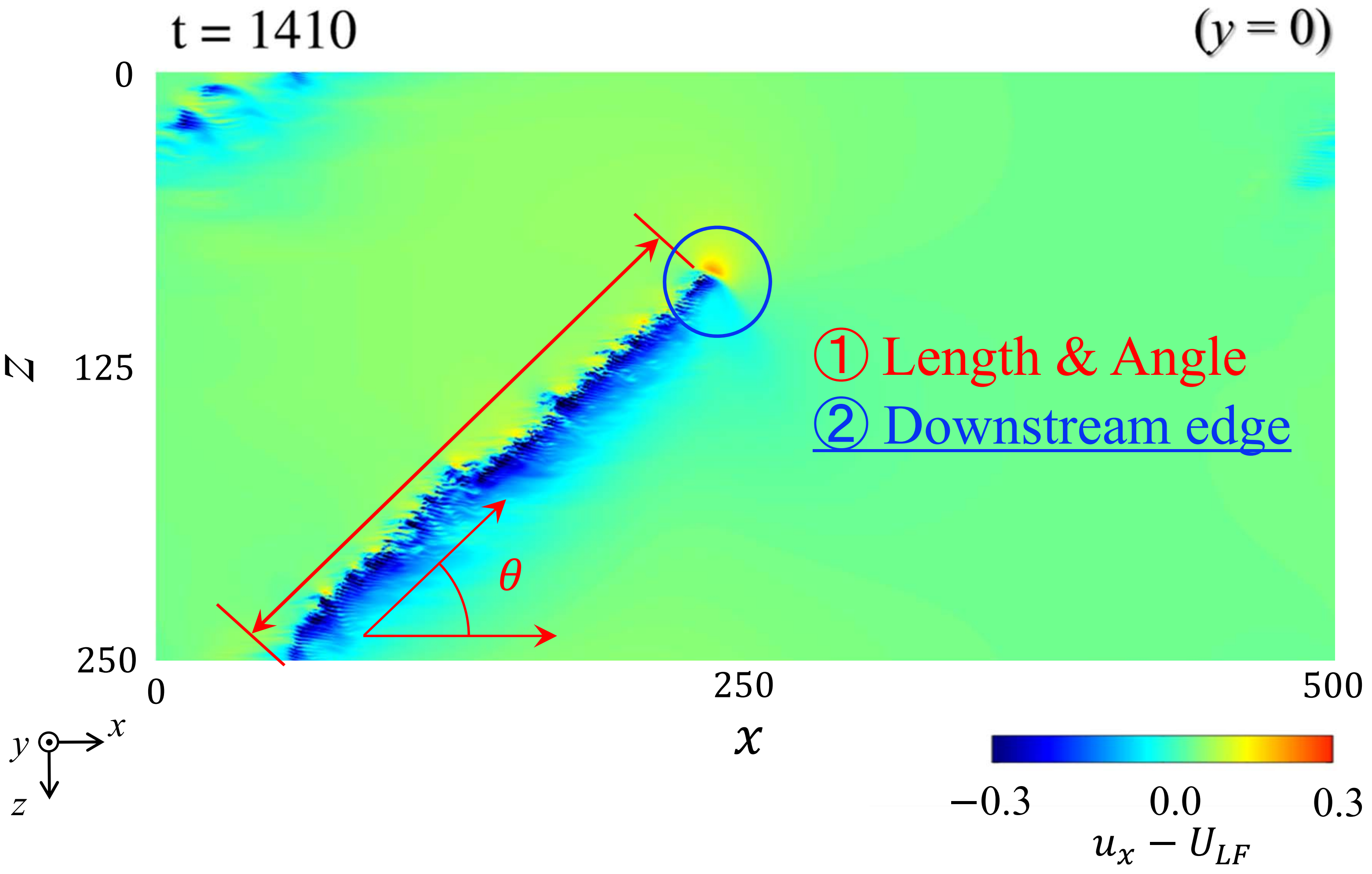
Effects of spatial periodicity

$(L_x, L_z) = (500, 250)$



Example

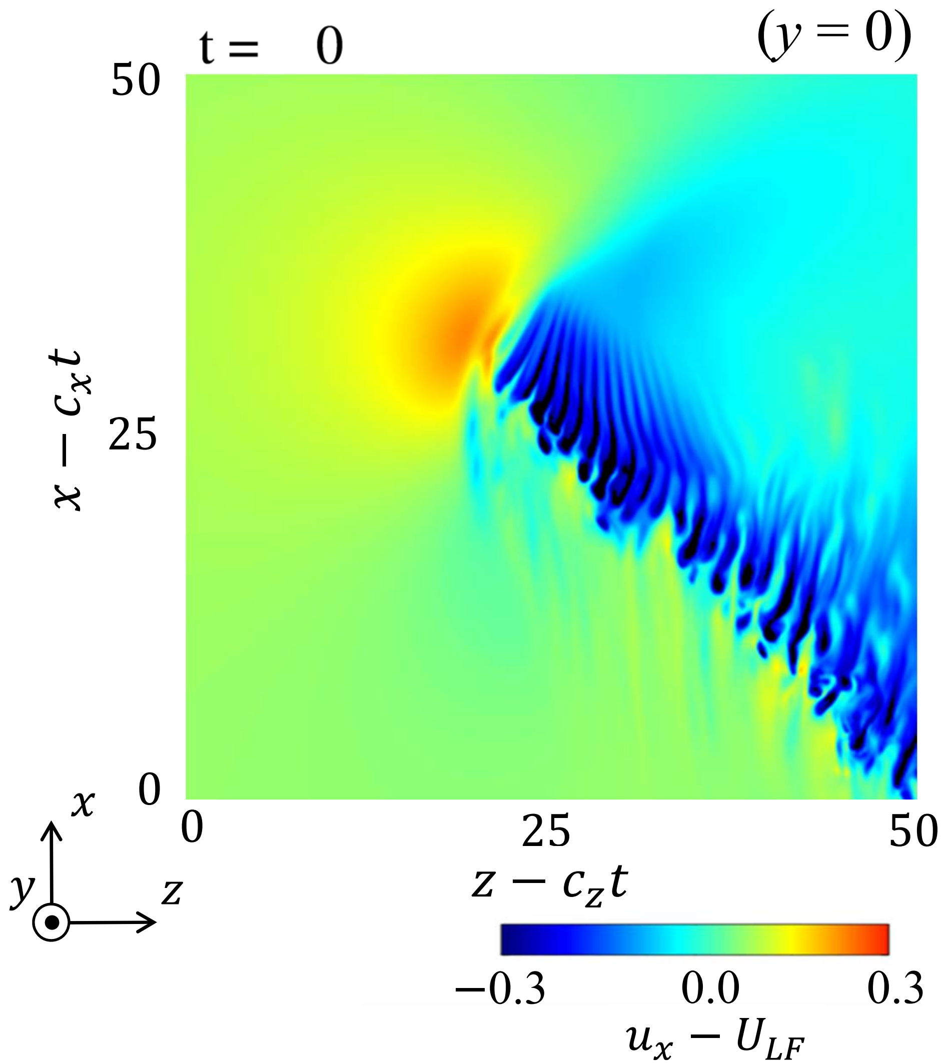
$(L_x, L_z) = (500, 250)$, $Re_m = 440$



U_{LF} : Laminar flow velocity

Downstream edge

$(L_x, L_z) = (500, 250),\ Re_m = 450$

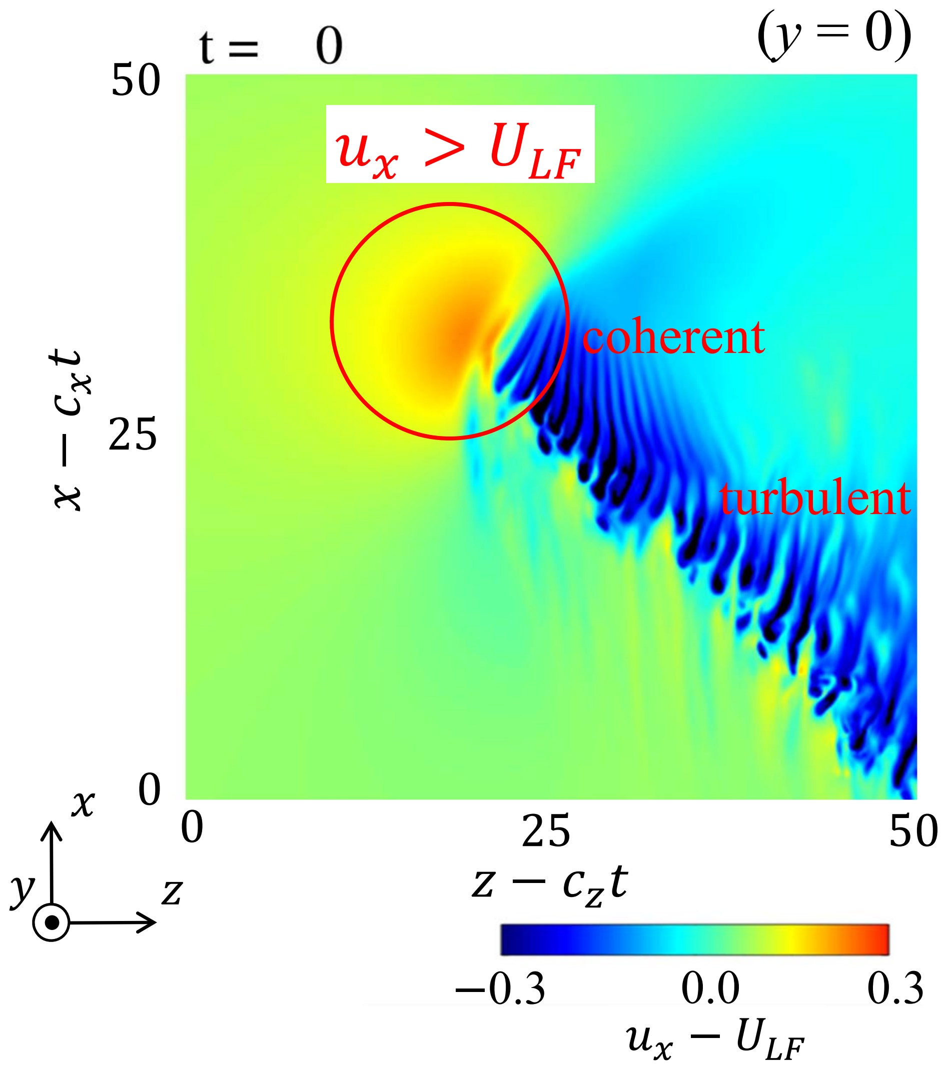


$c_x \simeq 1.29$

$c_z \simeq -0.14$

Downstream edge

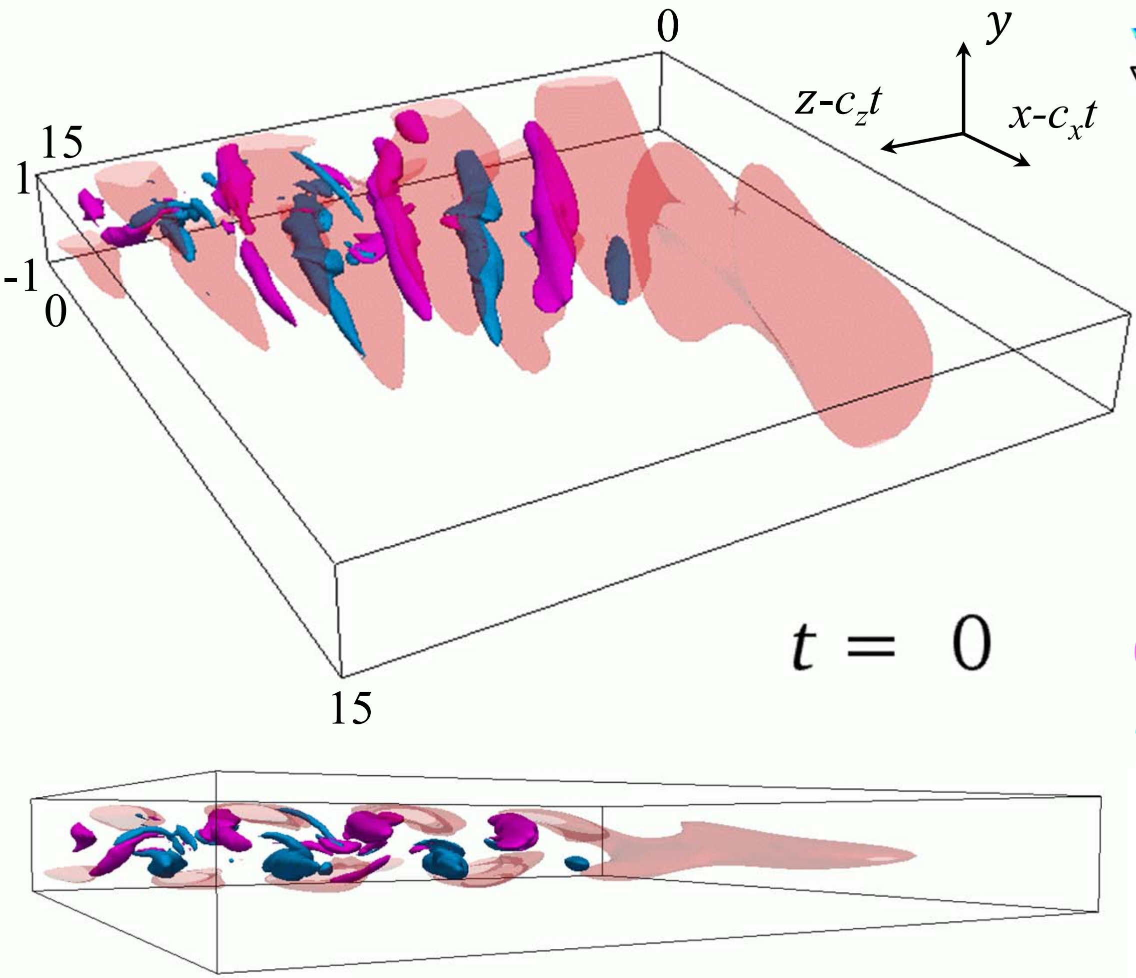
$(L_x, L_z) = (500, 250)$, $Re_m = 450$

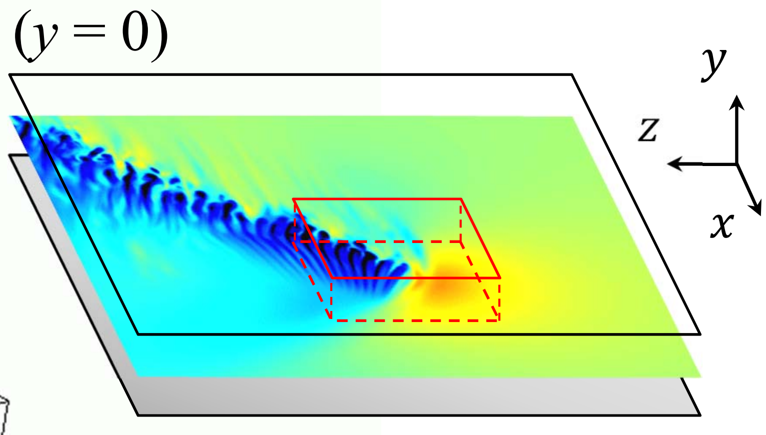


$c_x \simeq 1.29$
$c_z \simeq -0.14$

Downstream edge

$(L_x, L_z) = (500, 250),\ Re_m = 450$





$c_x \simeq 1.29$

$c_z \simeq -0.14$

$u_x - U_{LF} = +0.1$

$Q = 0.075, \omega_x > 0$

$Q = 0.075, \omega_x < 0$

Q : second invariant of velocity gradient tensor

Isolation of downstream edge

⇒ Add the spatially localized damping force

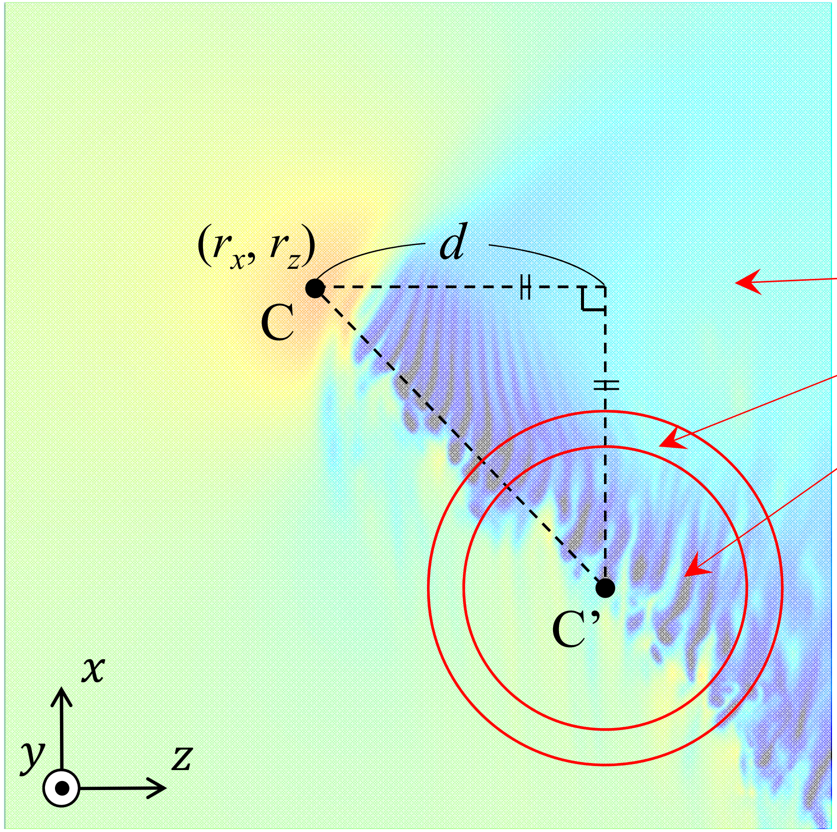
$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\nabla p + \frac{1}{Re} \Delta \boldsymbol{u} - F(x, z, t)(\boldsymbol{u} - U_{LF} \mathbf{e}_x)$$

$$\nabla \cdot \boldsymbol{u} = \mathbf{0}$$

$$F(x, z, t) = \alpha f(x, z, t)$$

α : damping intensity

$f(x, z, t)$: spatial distribution function



$f = 0$

$0 < f < 1$ hyperbolic-tangent type

$f = 1$

C : local maximum point of u_x (y=0) (r_x, r_z)

C': $(r_x - d, r_z + d)$

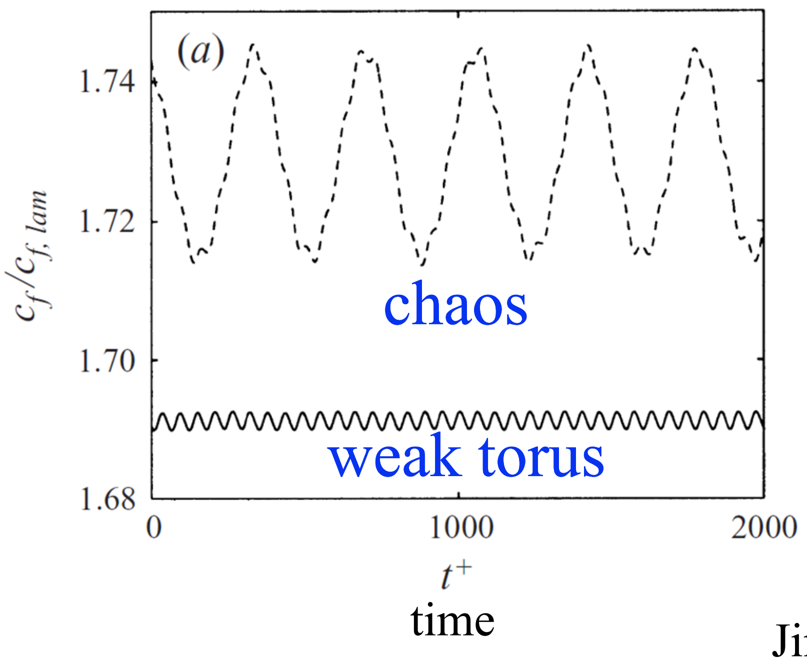
$d \simeq 19$

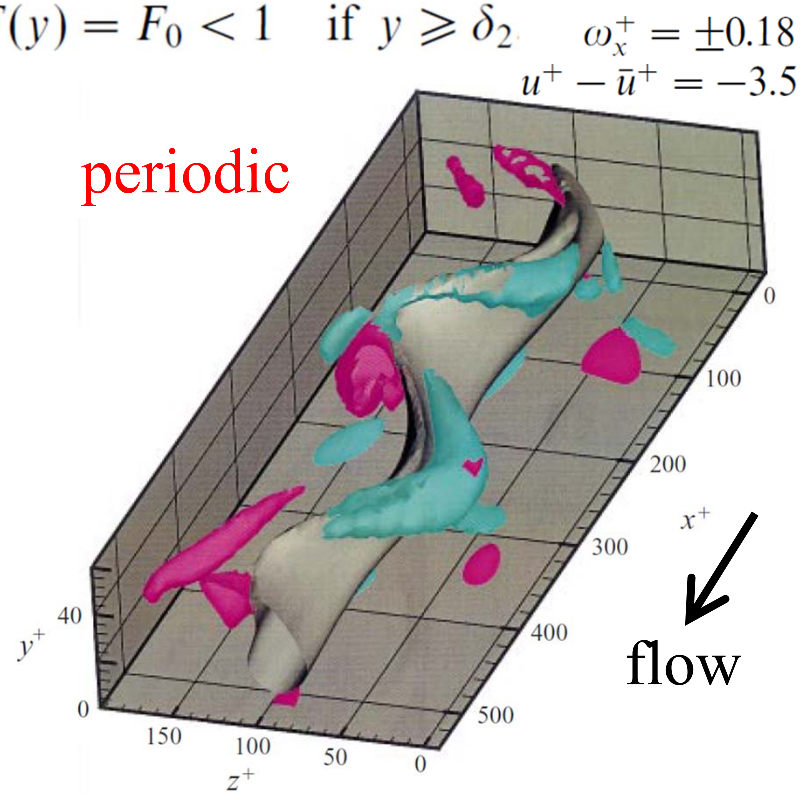
Isolation of downstream edge

Damping mask in minimal turbulent channel

$$\frac{\partial \boldsymbol{u}}{\partial t} = F \times (\text{RHS of Navier—Stokes})$$

$$F(y) = 1 \quad \text{if } y \leqslant \delta_1, \qquad F(y) = F_0 < 1 \quad \text{if } y \geqslant \delta_2$$





Jiménez and Simens, J. Fluid Mech. (2001)

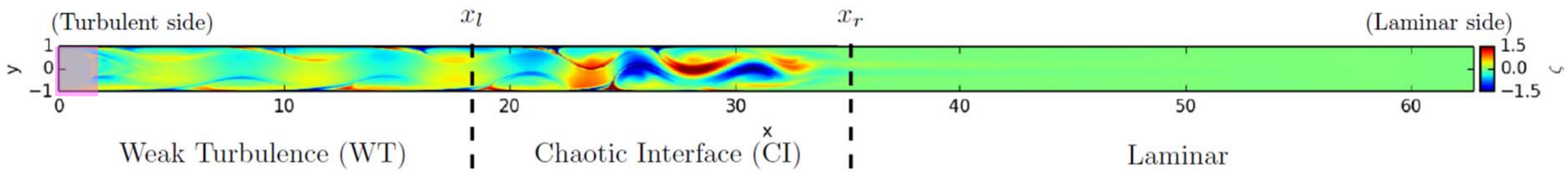
Isolation of downstream edge

Damping force in high-*Re* 2-d turbulent channel

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} = -\nabla p + \frac{1}{\mathrm{Re}}\nabla^2\boldsymbol{u} - H_{\sigma^2,\Omega}(x)(\boldsymbol{u}-\boldsymbol{U}_L),$$

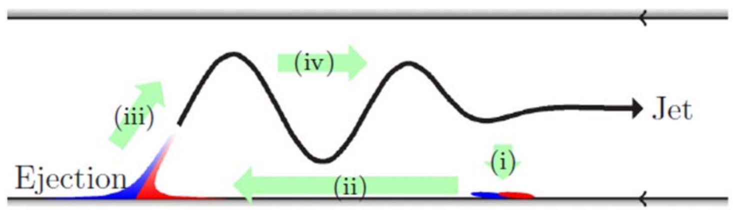
$$H_{\sigma^2,\Omega}(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\int_\Omega dx' \exp\left(\frac{(x-x')^2}{2\sigma^2}\right)$$

turbulence $Re = 8000$



→ flow

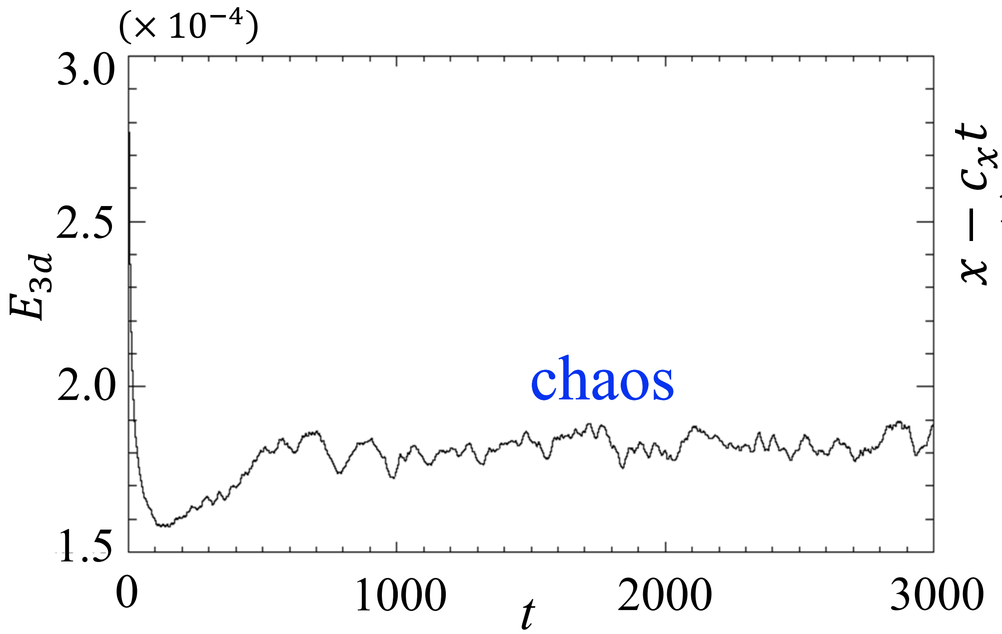
ejection-jet cycle



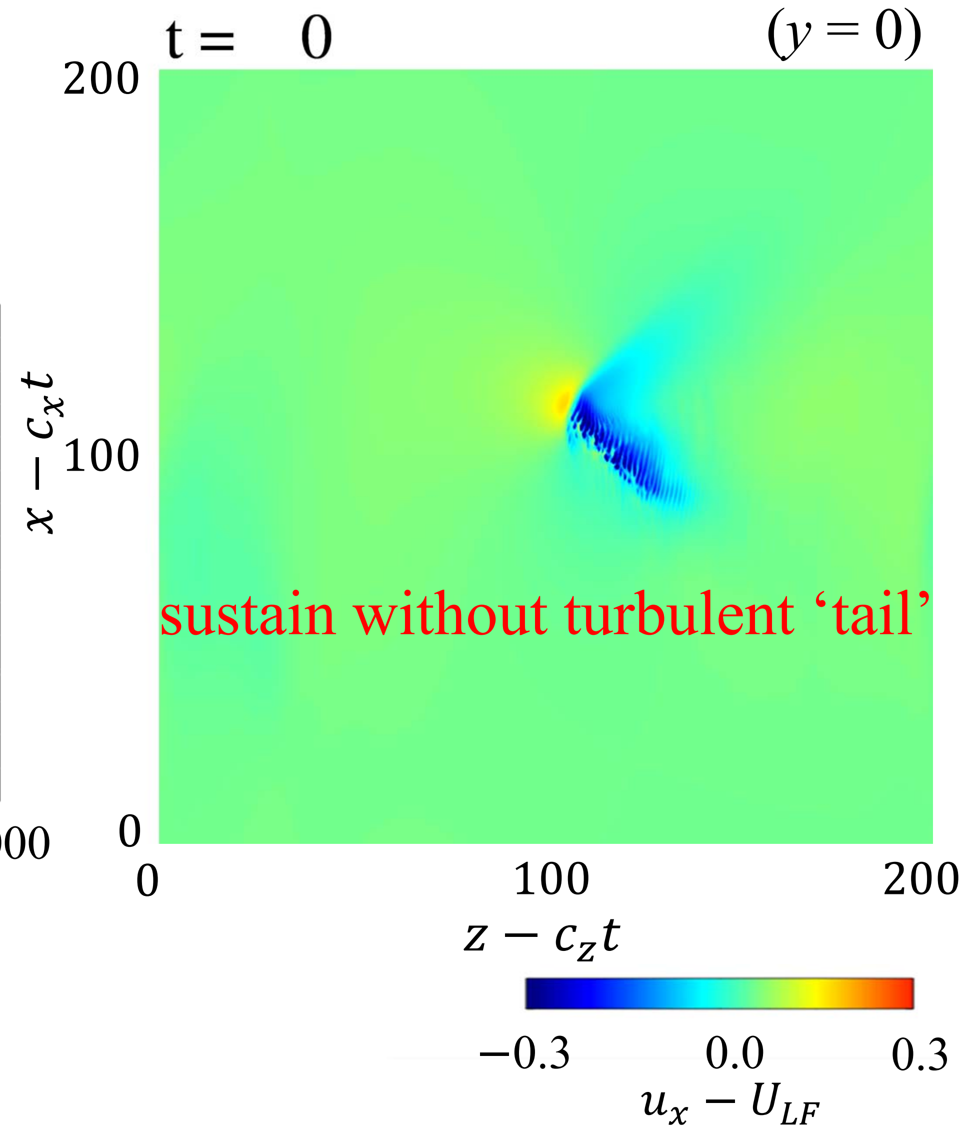
Teramura and Toh, Phys. Rev. E (2016)

Isolated downstream edge

$(L_x, L_z) = (200, 200),\ Re_m = 470, \alpha = 0.1$

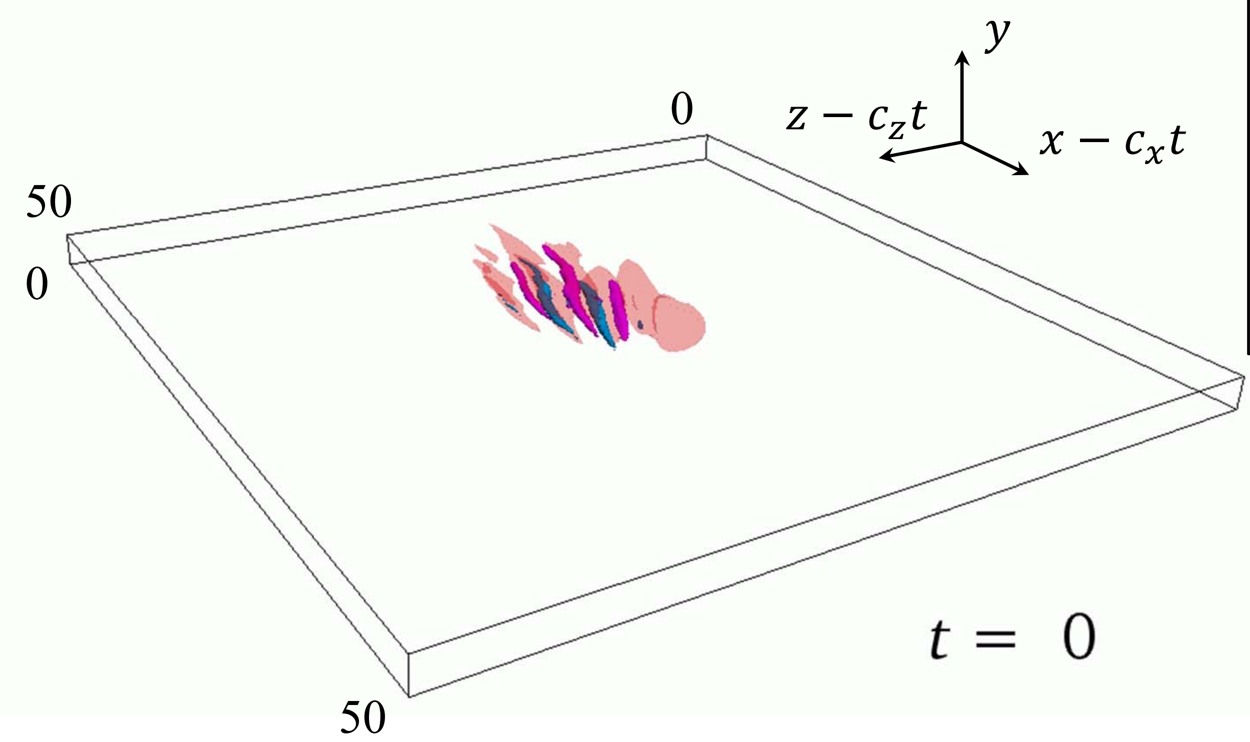


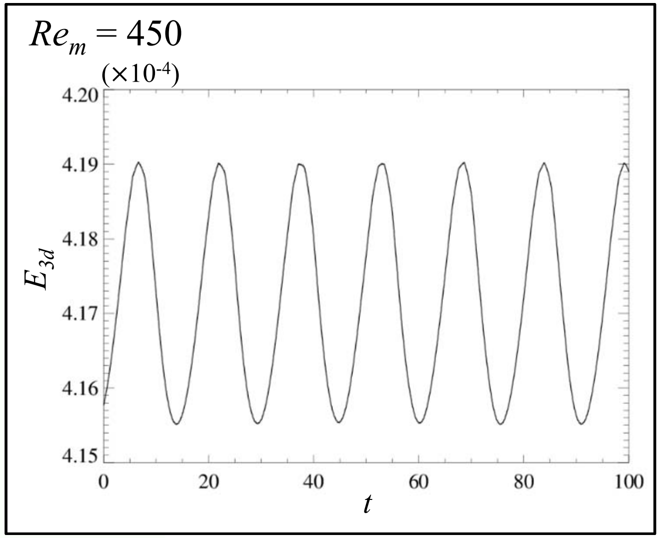
$$E_{3d} = \frac{1}{2V}\int_V (\boldsymbol{u} - U_{LF}\mathbf{e}_x)^2\, \mathrm{d}V$$



Periodic solution extracted using damping force

$(L_x, L_z) = (100, 100),\ Re_m = 450, \alpha = 0.1$





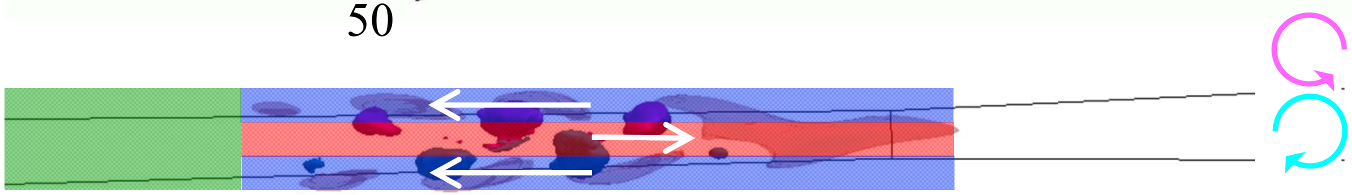
sustaining edge

$\Rightarrow$ invariant solution

$u_x - U_{LF} = +0.1$

$Q = 0.075, \omega_x > 0$

$Q = 0.075, \omega_x < 0$



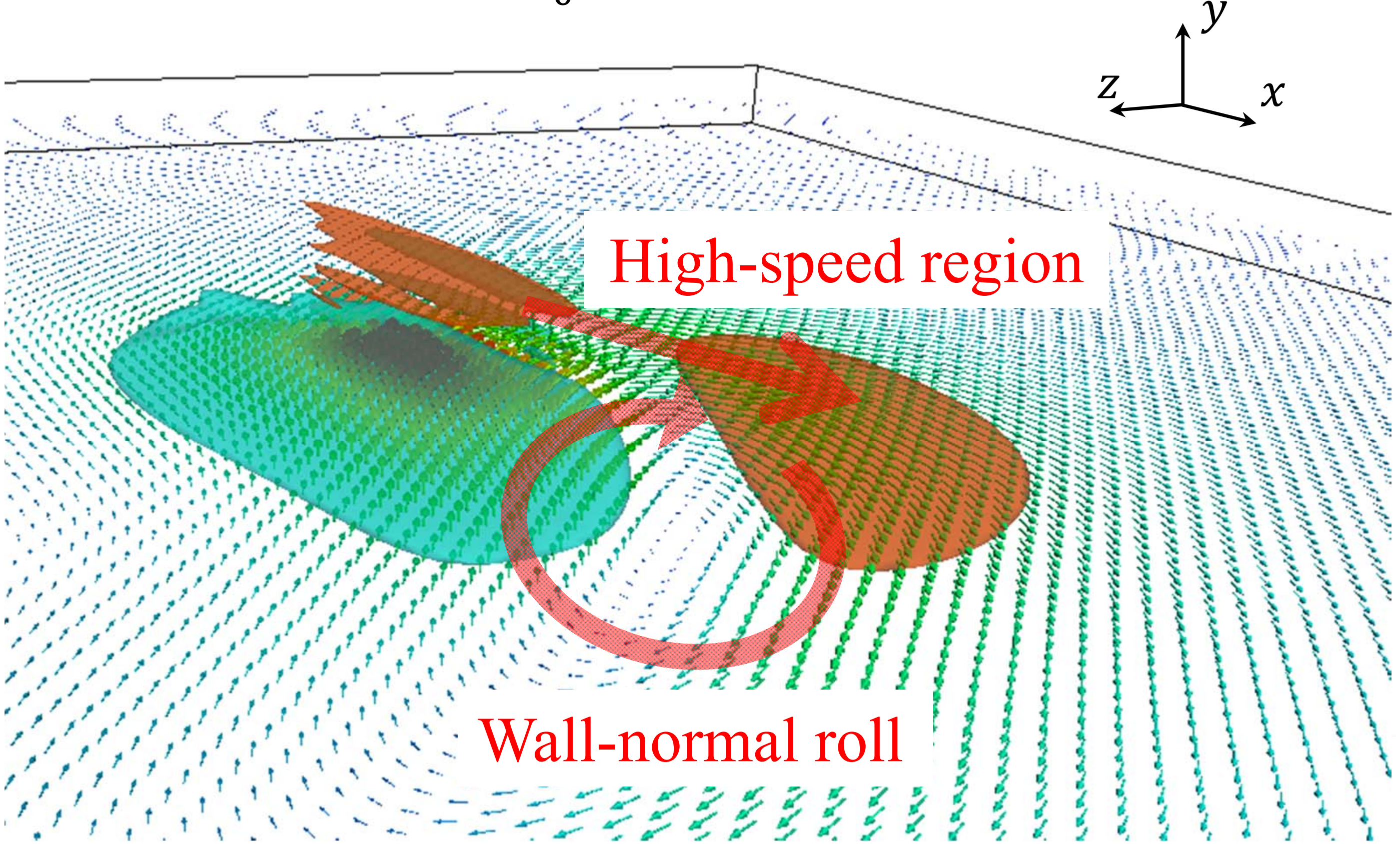
‘shear-layer’ sinuous instability $\Rightarrow$ staggered vortices $\Rightarrow$ chaos

Sustaining mechanism of downstream edge

$(L_x, L_z) = (100, 100),\ Re_m = 460, \alpha = 0.1$

Mean flow field for periodic solution

$$\langle \boldsymbol{u} - U_{LF}\mathbf{e}_x \rangle_t = \frac{1}{T}\int_0^T (\boldsymbol{u} - U_{LF}\mathbf{e}_x)\,\mathrm{d}t$$



$\overline{u_x} - U_{LF} = 0.1$

$\overline{u_x} - U_{LF} = -0.1$

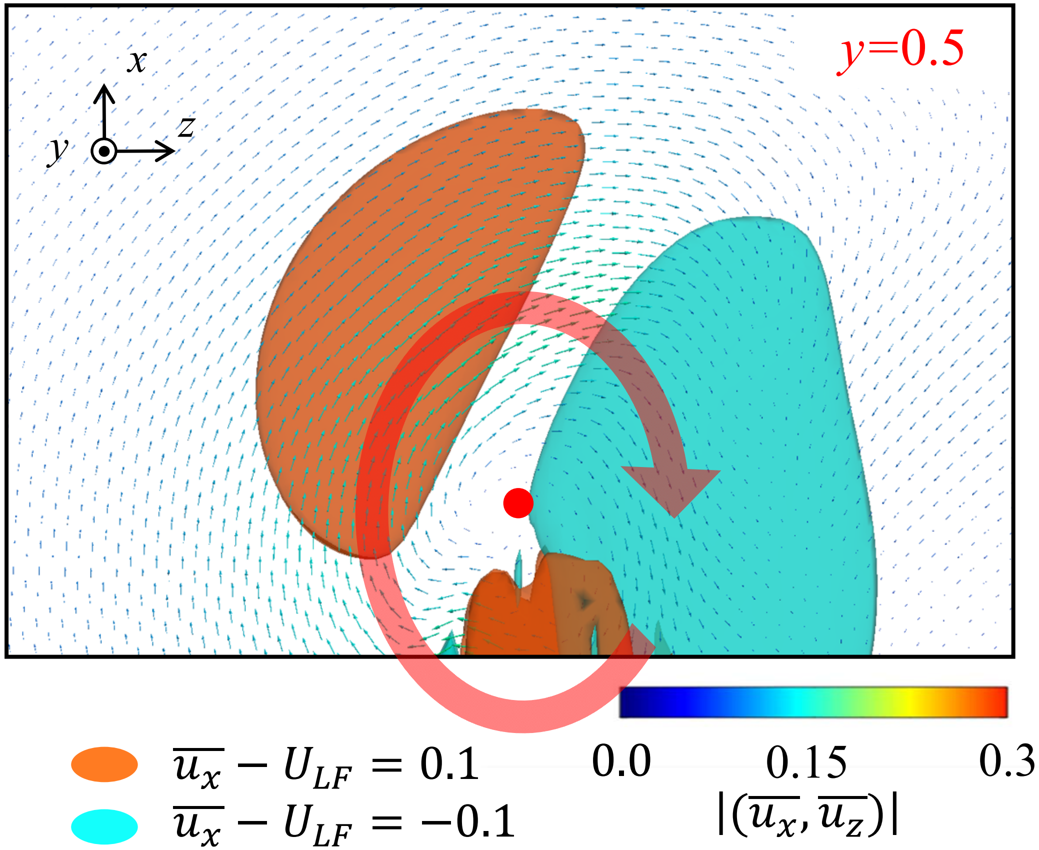
velocity vectors
on y=0

$|(\overline{u_x}, 0, \overline{u_z})|$

Sustaining mechanism of downstream edge

Mean flow field for periodic solution

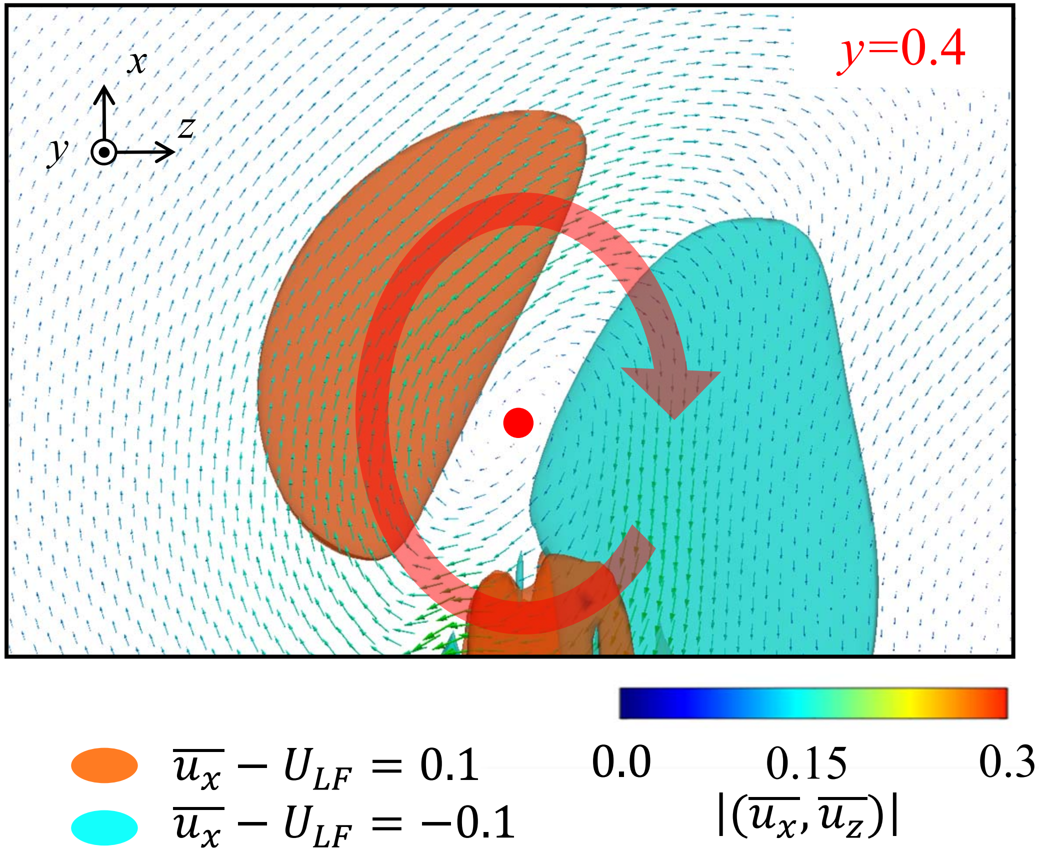
$$\langle \boldsymbol{u} - U_{LF}\mathbf{e}_x \rangle_t = \frac{1}{T}\int_0^T (\boldsymbol{u} - U_{LF}\mathbf{e}_x)\,\mathrm{d}t$$



Sustaining mechanism of downstream edge

Mean flow field for periodic solution

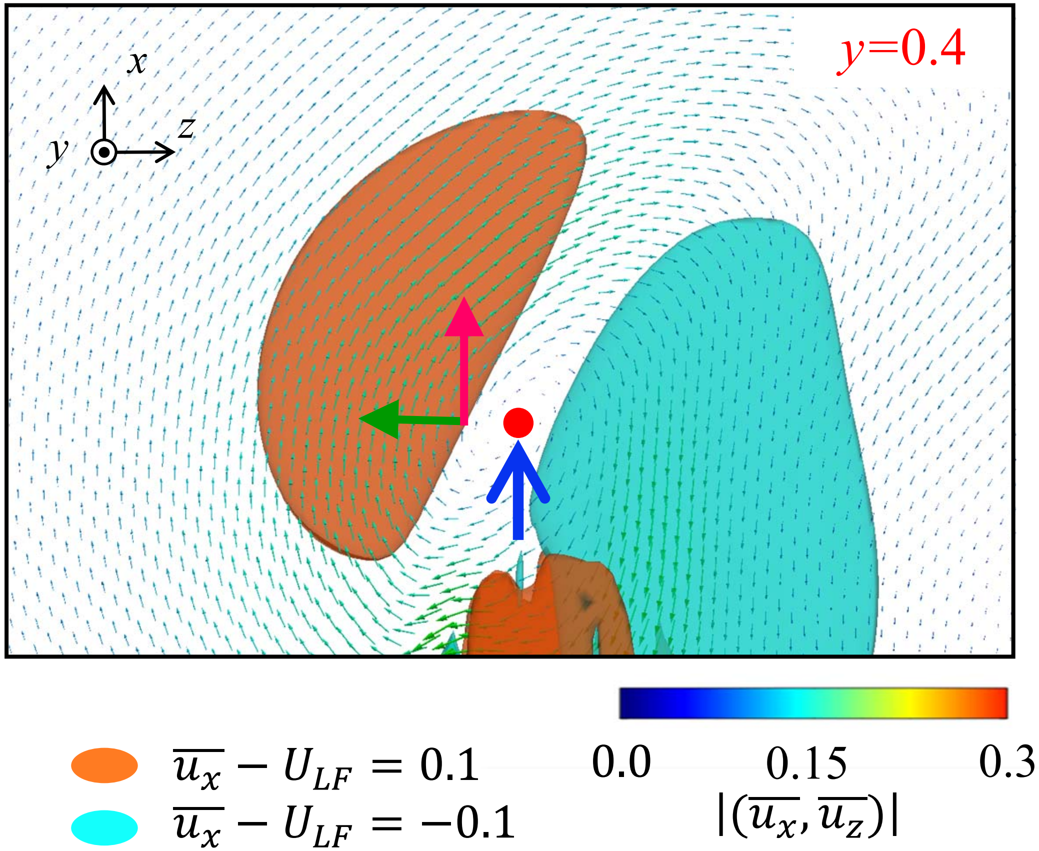
$$\langle \boldsymbol{u} - U_{LF}\mathbf{e}_x \rangle_t = \frac{1}{T}\int_0^T (\boldsymbol{u} - U_{LF}\mathbf{e}_x)\,\mathrm{d}t$$

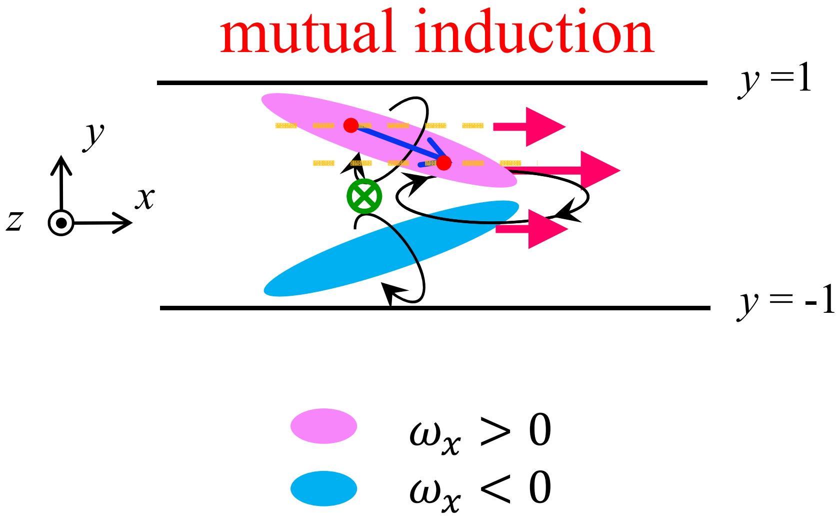


Sustaining mechanism of downstream edge

Mean flow field for periodic solution

$$\langle \boldsymbol{u} - U_{LF}\mathbf{e}_x \rangle_t = \frac{1}{T}\int_0^T (\boldsymbol{u} - U_{LF}\mathbf{e}_x)\,\mathrm{d}t$$

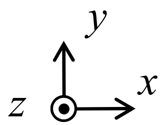


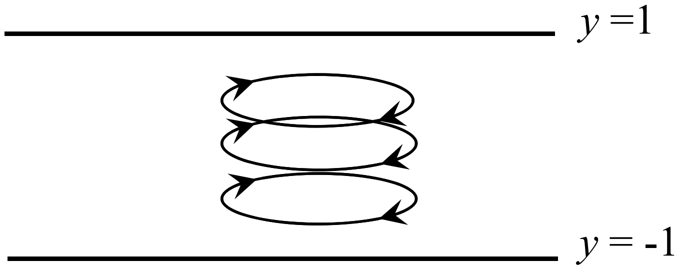


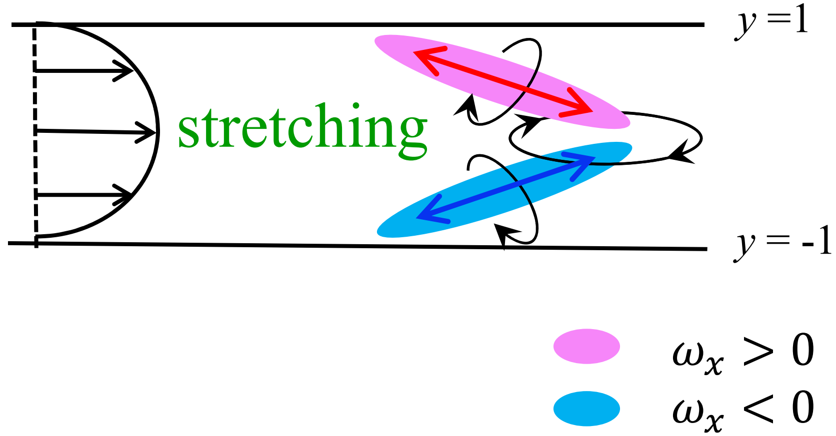
Sustaining mechanism of downstream edge

If the roll were purely wall-normal

Real inclined rolls





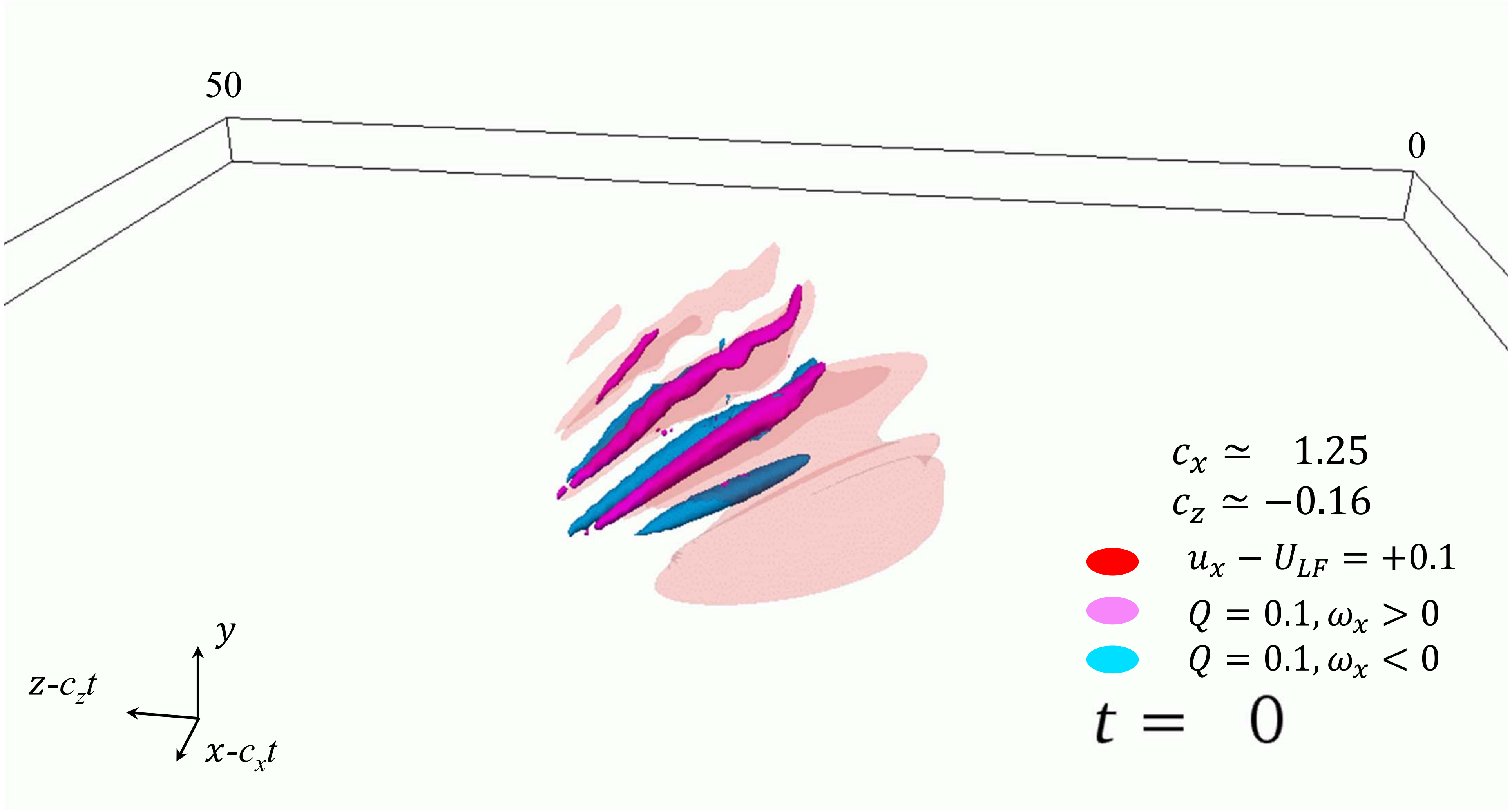


would decay

can sustain!

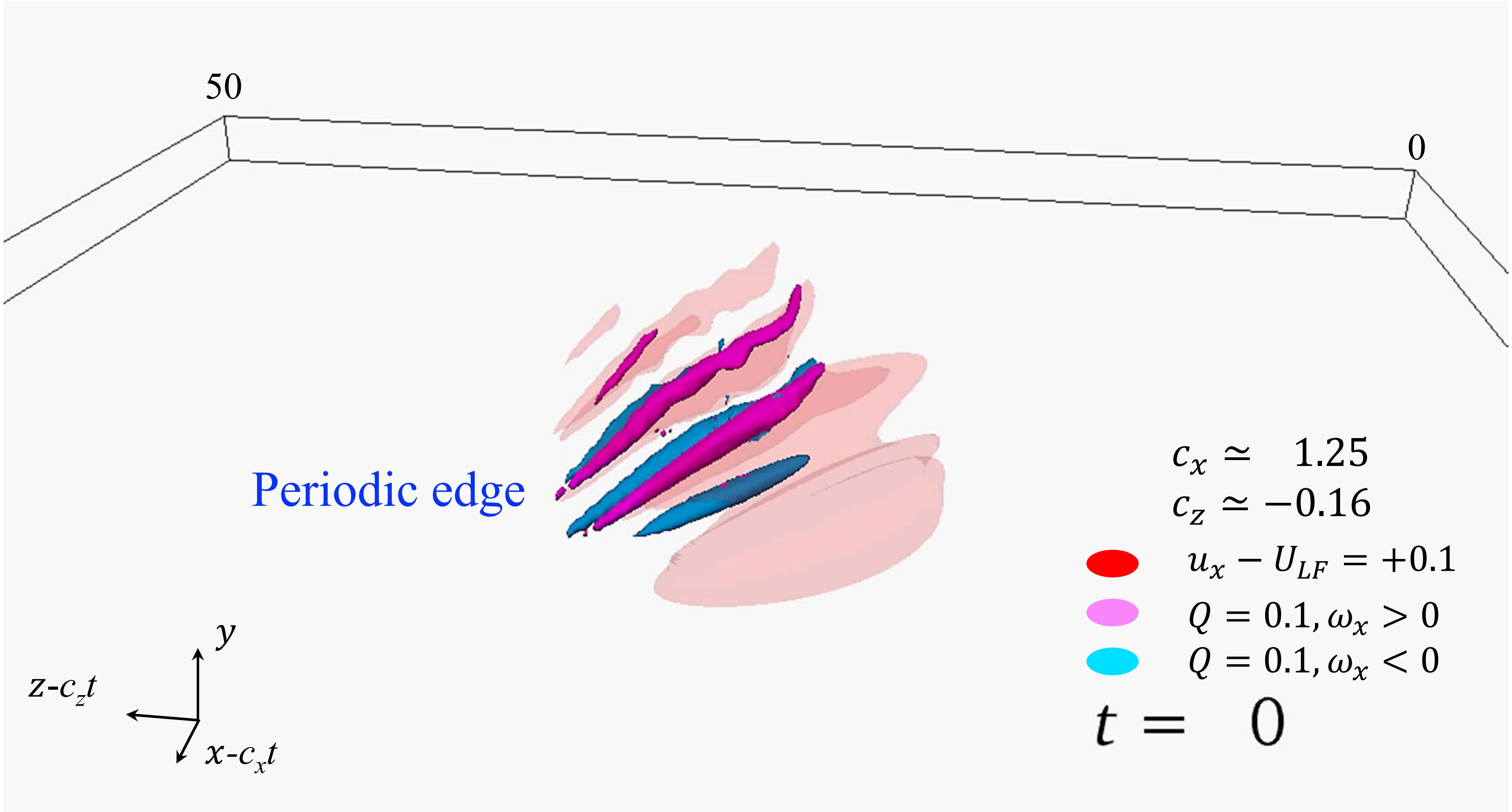
Extension to turbulent band

$(L_x, L_z) = (100, 100), Re_m = 460, \alpha = 0$



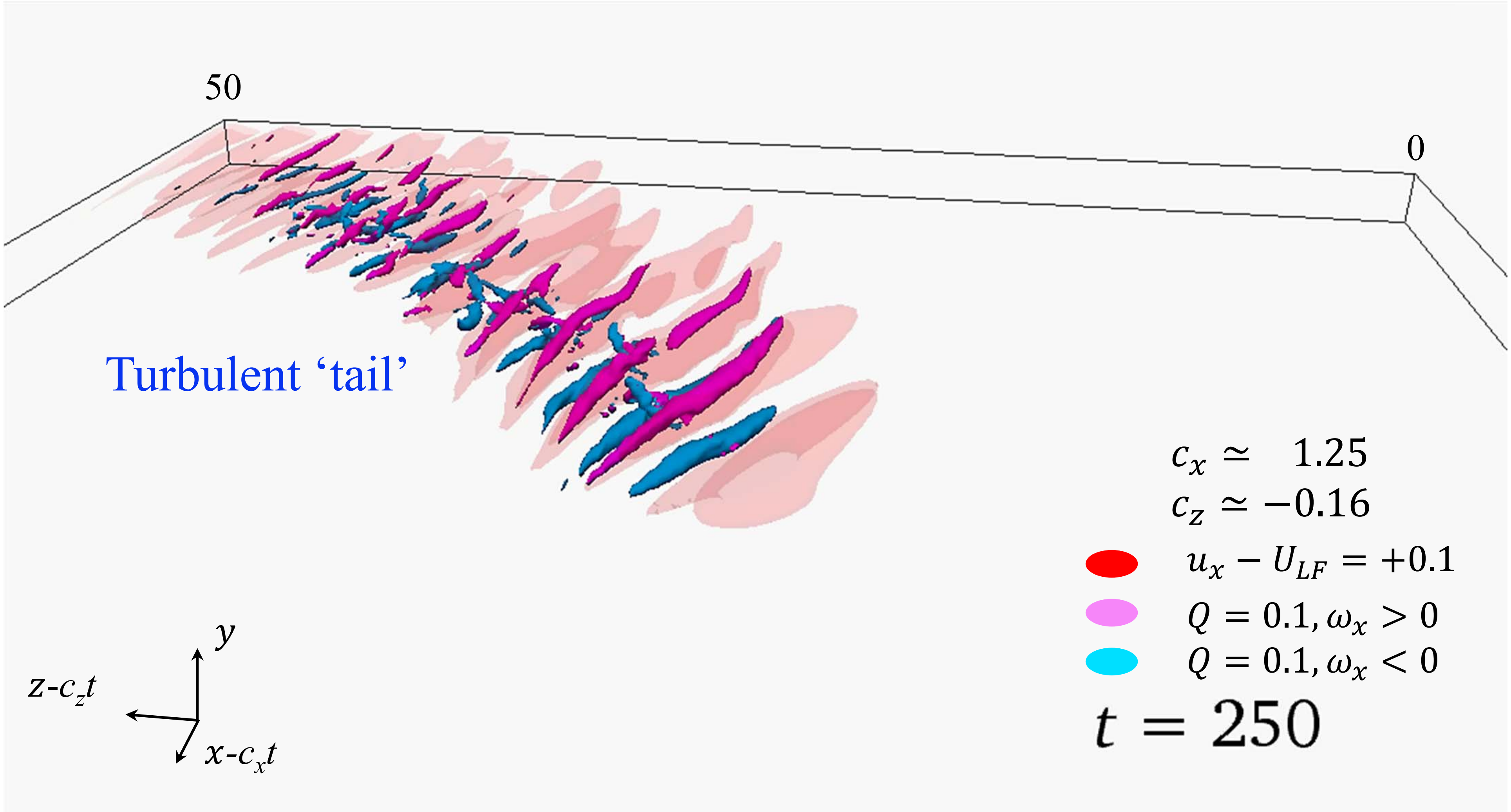
Extension to turbulent band

$(L_x, L_z) = (100, 100), Re_m = 460, \alpha = 0$



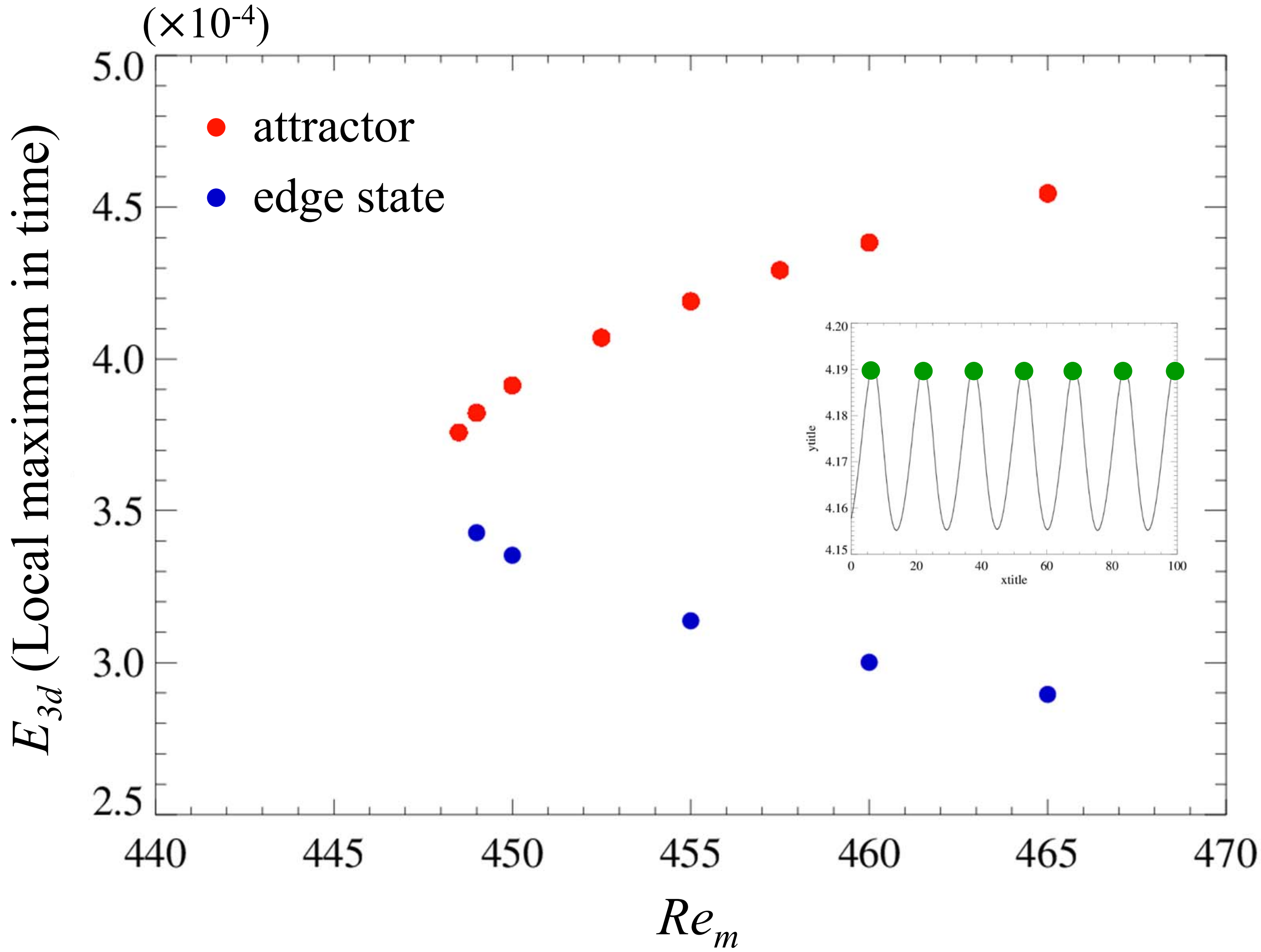
Extension to turbulent band

$(L_x, L_z) = (100, 100), Re_m = 460, \alpha = 0$



Bifurcation diagram of periodic solution

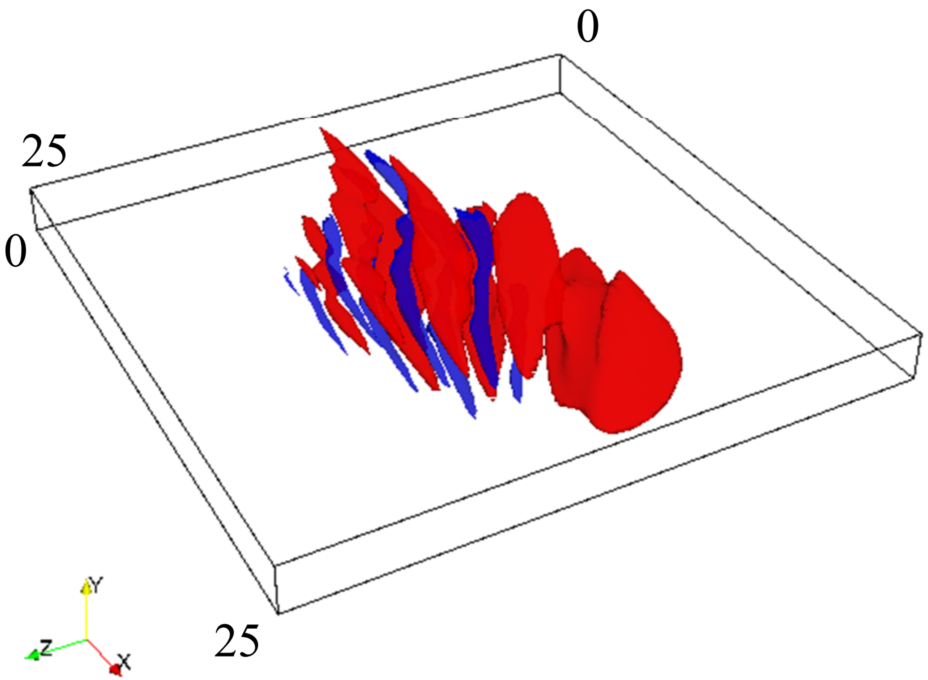
$(L_x, L_z) = (100, 100), \alpha = 0.1$



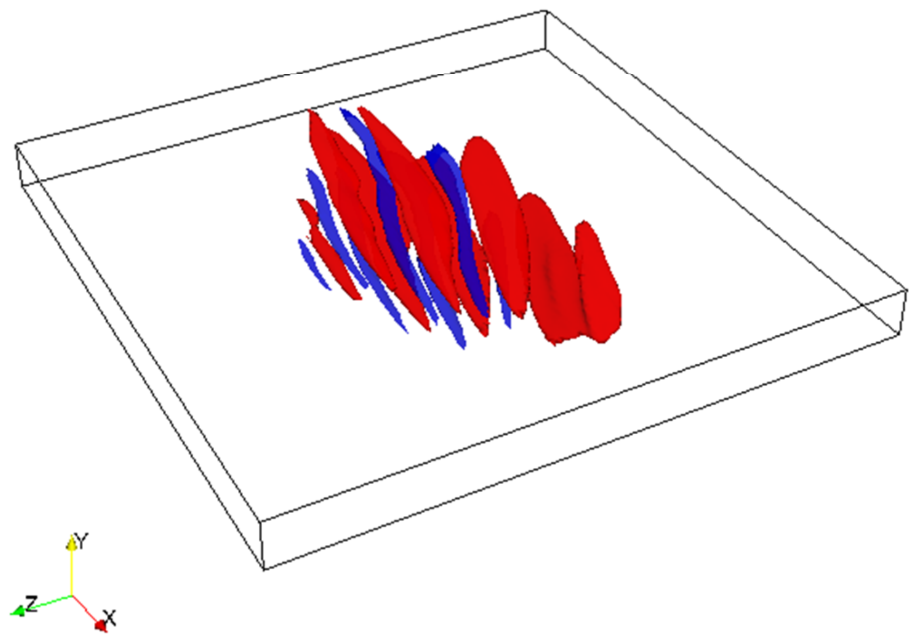
Snapshot

$(L_x, L_z) = (100, 100), Re_m = 460, \alpha = 0.1$

Upper branch



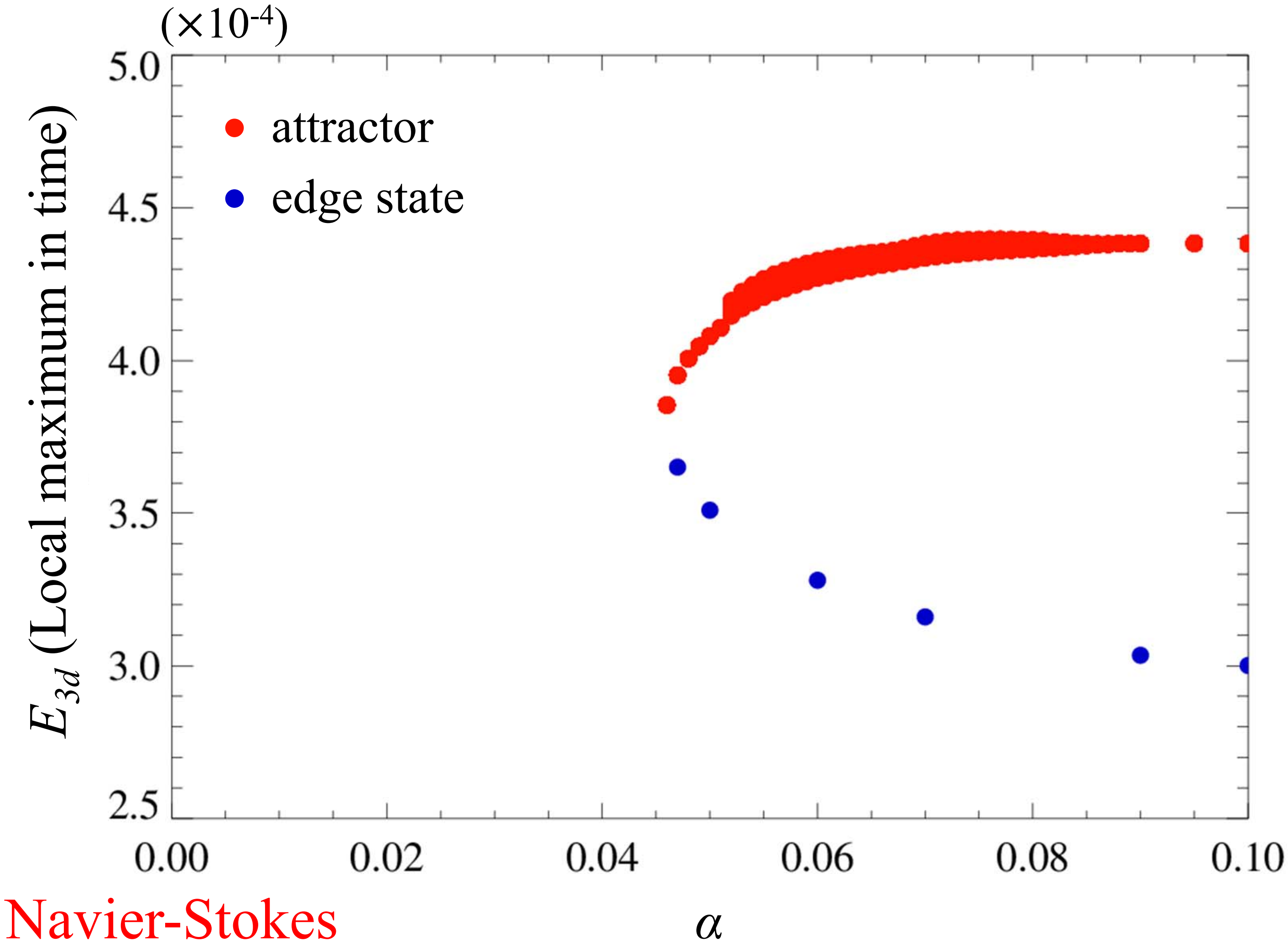
Lower branch



$u_x - U_{LF} = \pm 0.1$

Bifurcation diagram of periodic solution

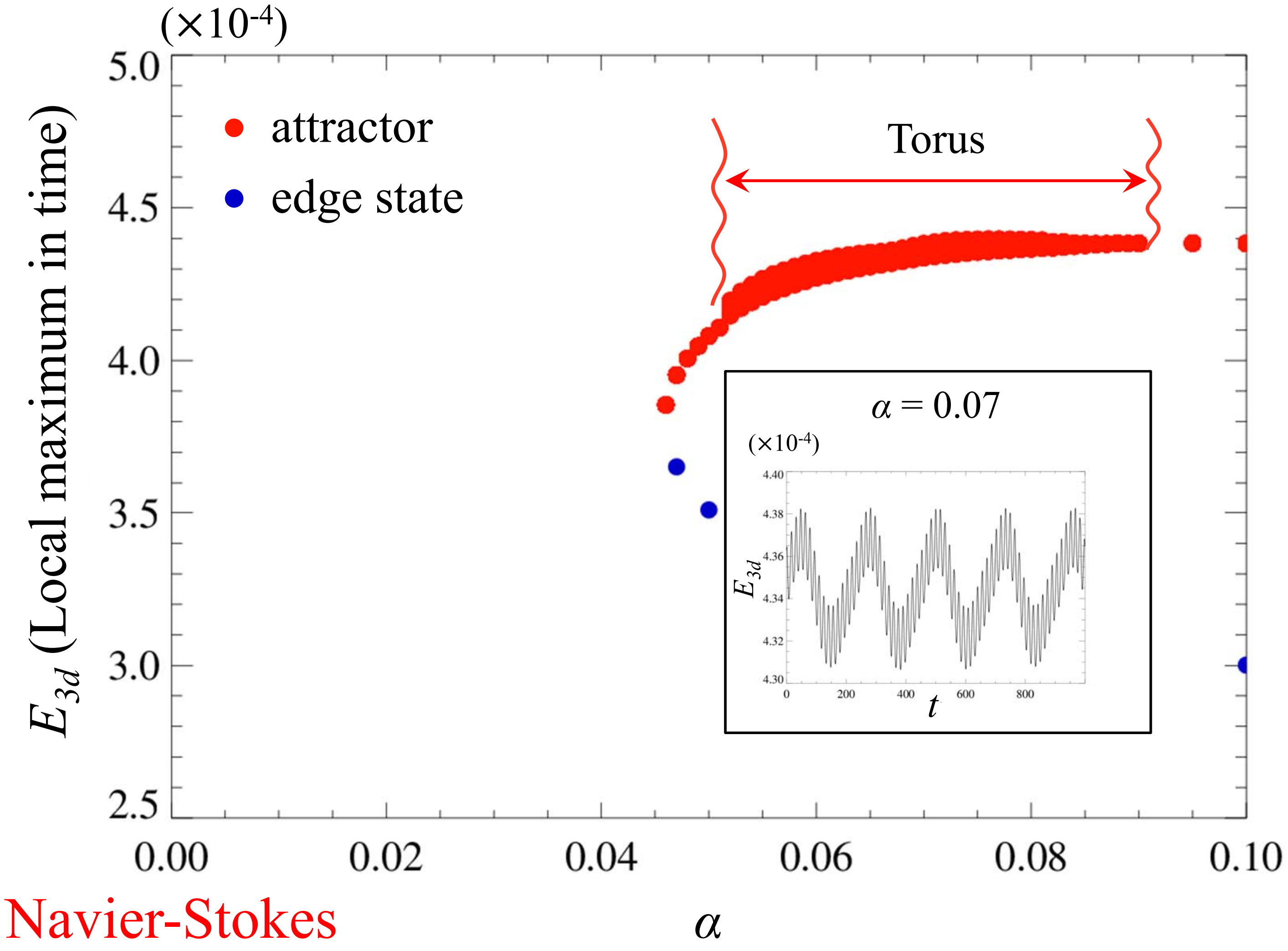
$(L_x, L_z) = (100, 100),\ Re_m = 460$



Navier-Stokes

Bifurcation diagram of periodic solution

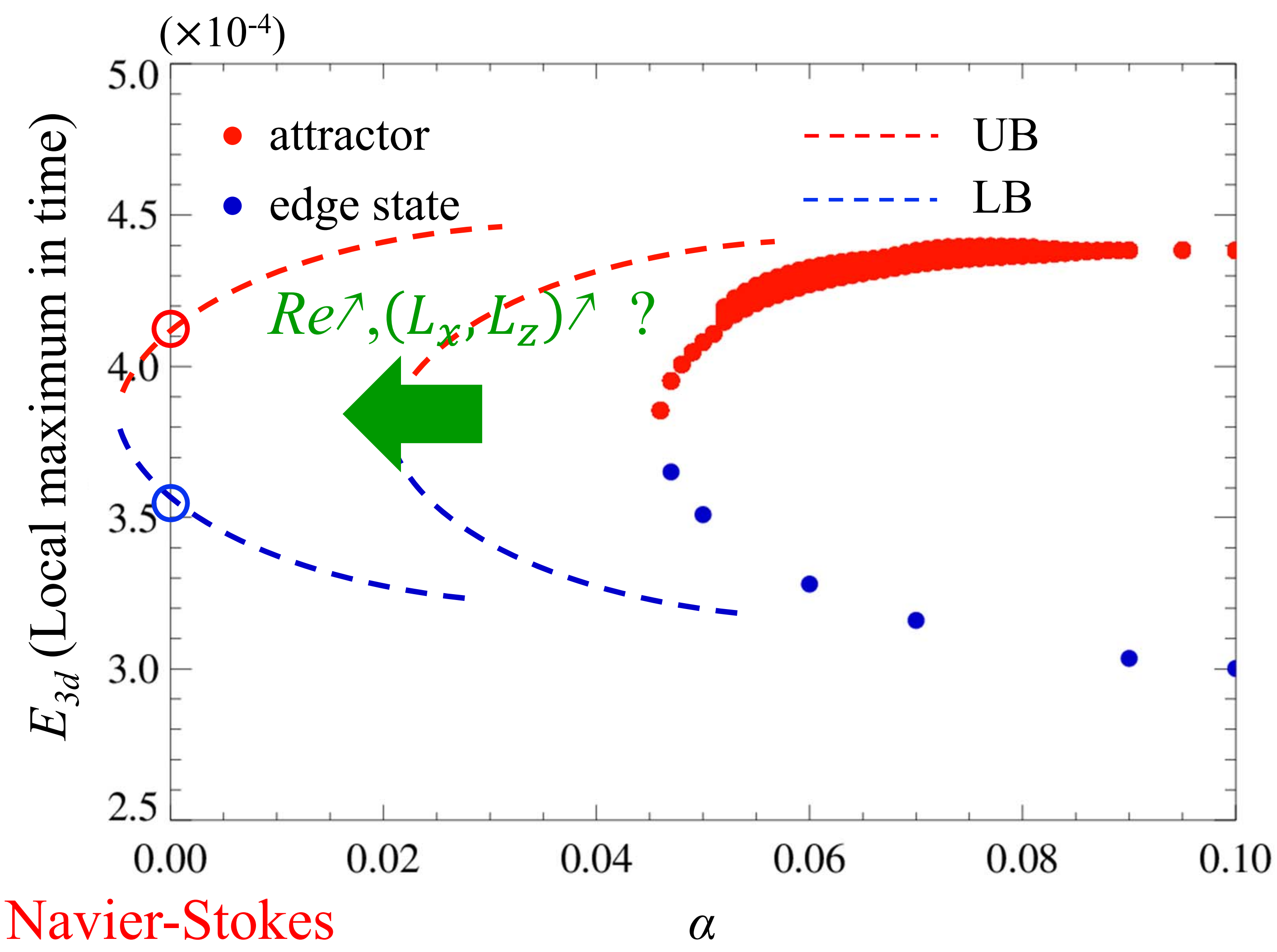
$(L_x, L_z) = (100, 100),\ Re_m = 460$



Navier-Stokes

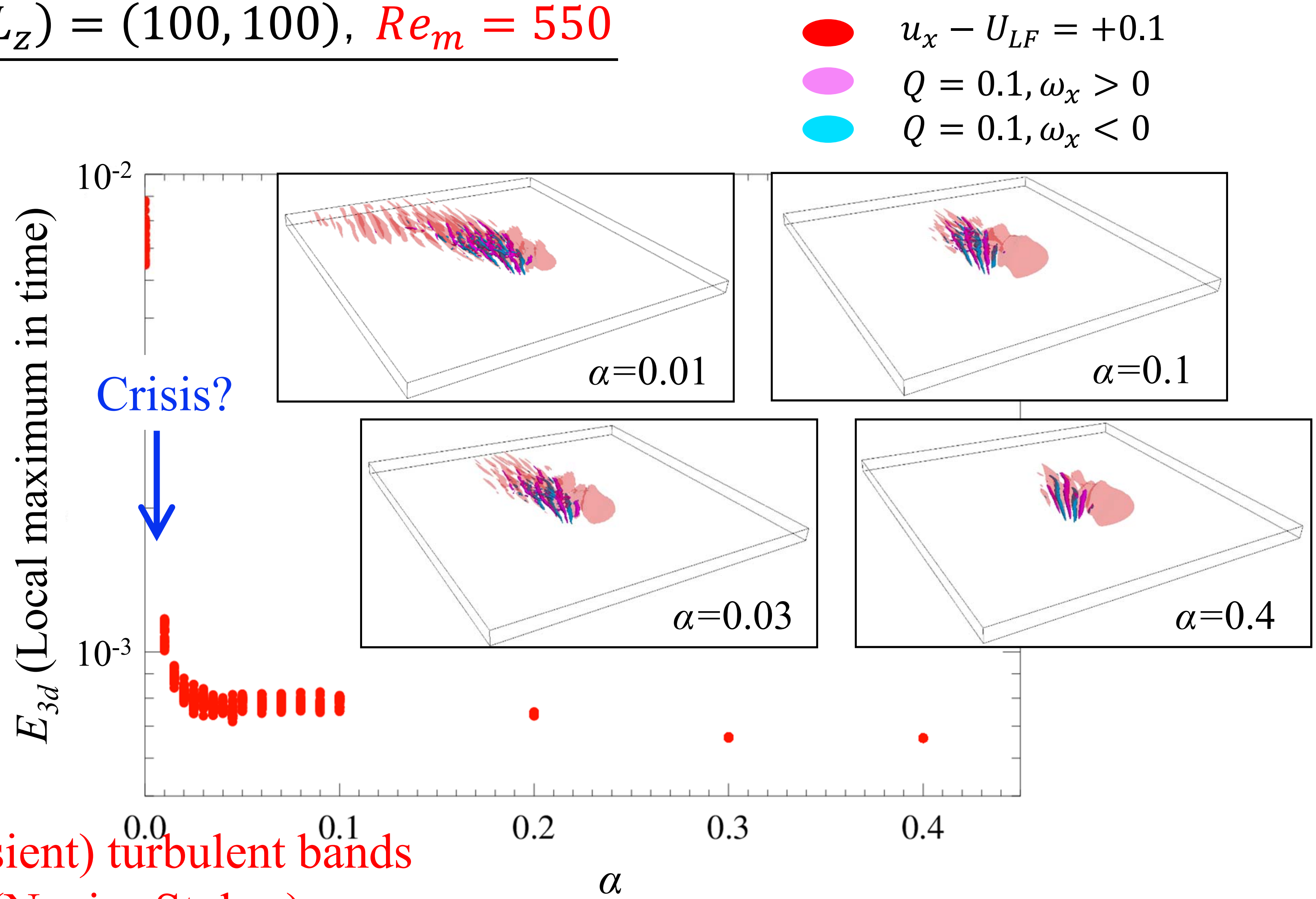
Relevance to full Navier-Stokes system

$(L_x, L_z) = (100, 100),\ Re_m = 460$



Relevance to full Navier-Stokes system

$(L_x, L_z) = (100, 100)$, $Re_m = 550$



(Transient) turbulent bands
(Navier-Stokes)

Concluding remarks

- Turbulent bands of equilibrium length have been observed in large numerical domain.
- Turbulent bands can be sustained up to around $Re_m = 440$.
- Relative periodic orbits have been discovered in spatially-localized damping-forced Navier-Stokes system.
- Periodic solutions mathematically provide self-sustaining mechanism of downstream edge (physically, inclined and thus stretched wall-normal rolls).
- If damping force is reduced, upper-branch solution loses its stability and eventually chaotic solution appears to represent turbulent bands of longer array of complex vortices.
- Periodic solutions representing turbulent bands might be connected to full Navier-Stokes system (cf. Hof et al.'s invariant solutions).