Exact Coherent Structures in Stratified Plane Couette Flow

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Recurrence, Self-Organization, a

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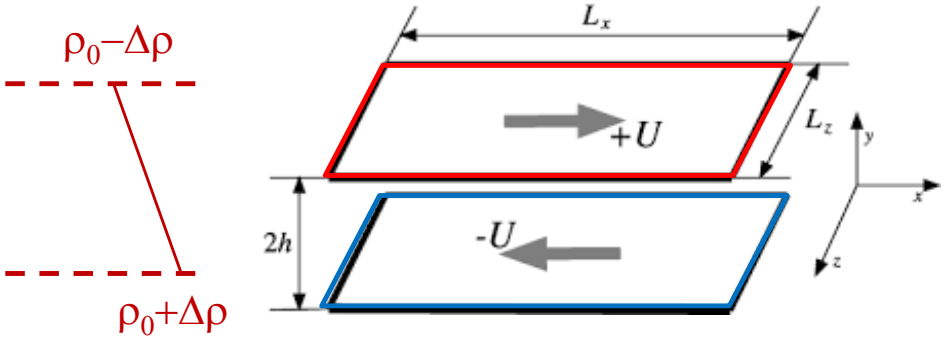
Turbulence remains one of physics' longest-standing and gnarliest problems.
Stokes equations are complex beyond closed-form expression. In recent yea
of steady states, traveling waves, and periodic orbits, in a wide variety of ca
structure the dynamics of transitionally turbulent flows in closed domains, a

The goal of this conference is to convene researchers with a wide variety of
directions of future work. Of particular interest are furthering the developme

...or How to Make a 'Gnarly' plane Couette flow *Gnarlier*

Stratified plane Couette flow



$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla p - Ri_b\,\rho\,\hat{\mathbf{y}} + \frac{1}{Re}\nabla^2\mathbf{u}$$

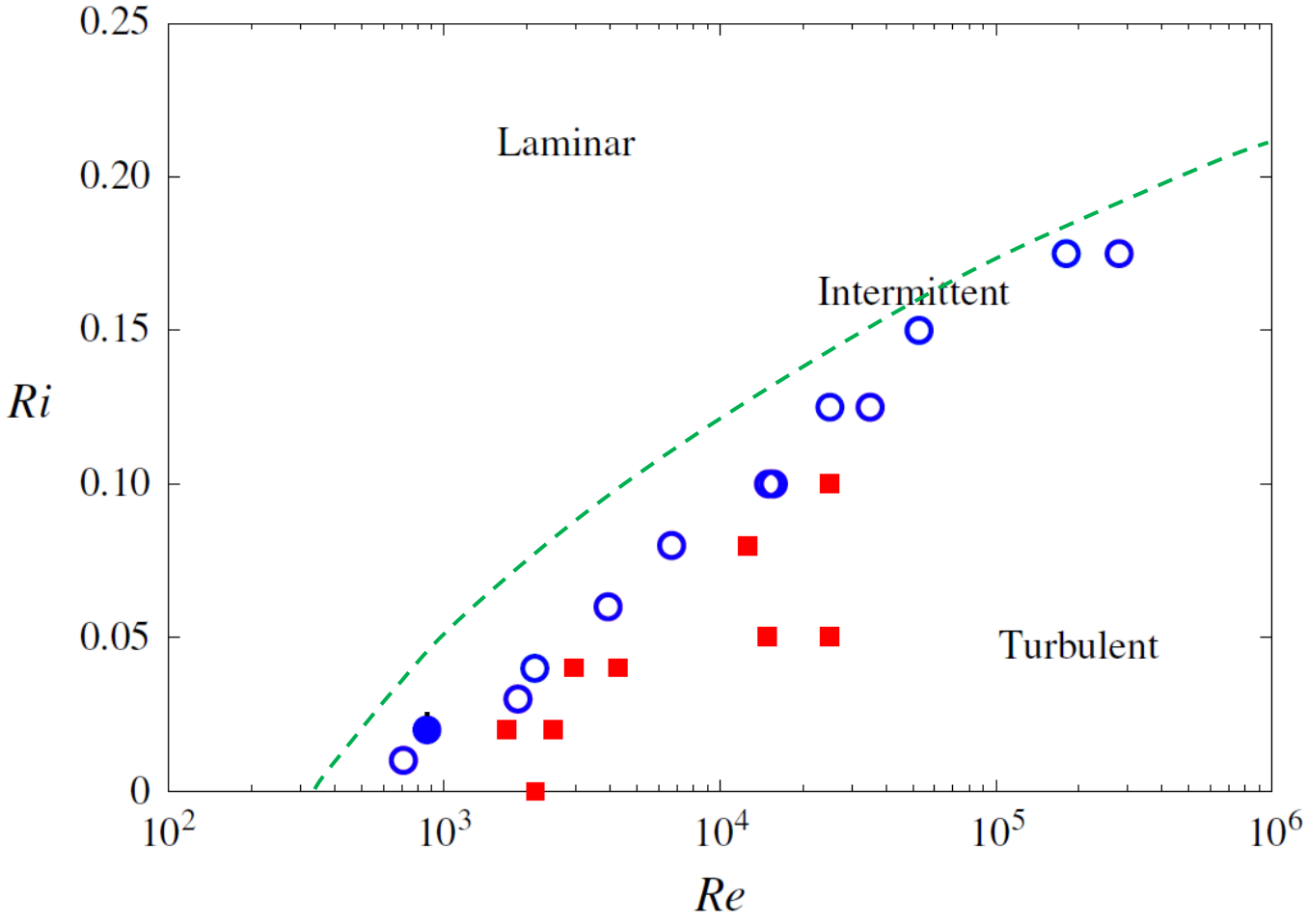
$$\nabla\cdot\mathbf{u} = 0,$$

$$\frac{\partial \rho}{\partial t} + \mathbf{u}\cdot\nabla\rho = \frac{1}{Re\;Pr}\nabla^2\rho$$

$$Ri_\mathrm{b} := \frac{\Delta\rho\, g\, h}{\rho_0\, U^2}, \qquad Re := \frac{U\,h}{\nu}, \qquad Pr := \frac{\nu}{\kappa}$$

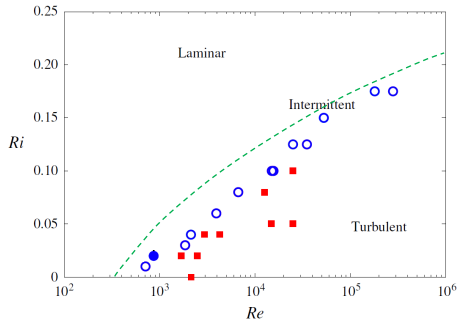
($Ri_b < 0$ --> Rayleigh-Benard convection + imposed shear $Ra = -Re^2\, Ri_b\, Pr$)

Pr = 0.7 (heated air) – 700 (salty water); Here Pr = 1 (parts I & II) or 0.7 (part III)



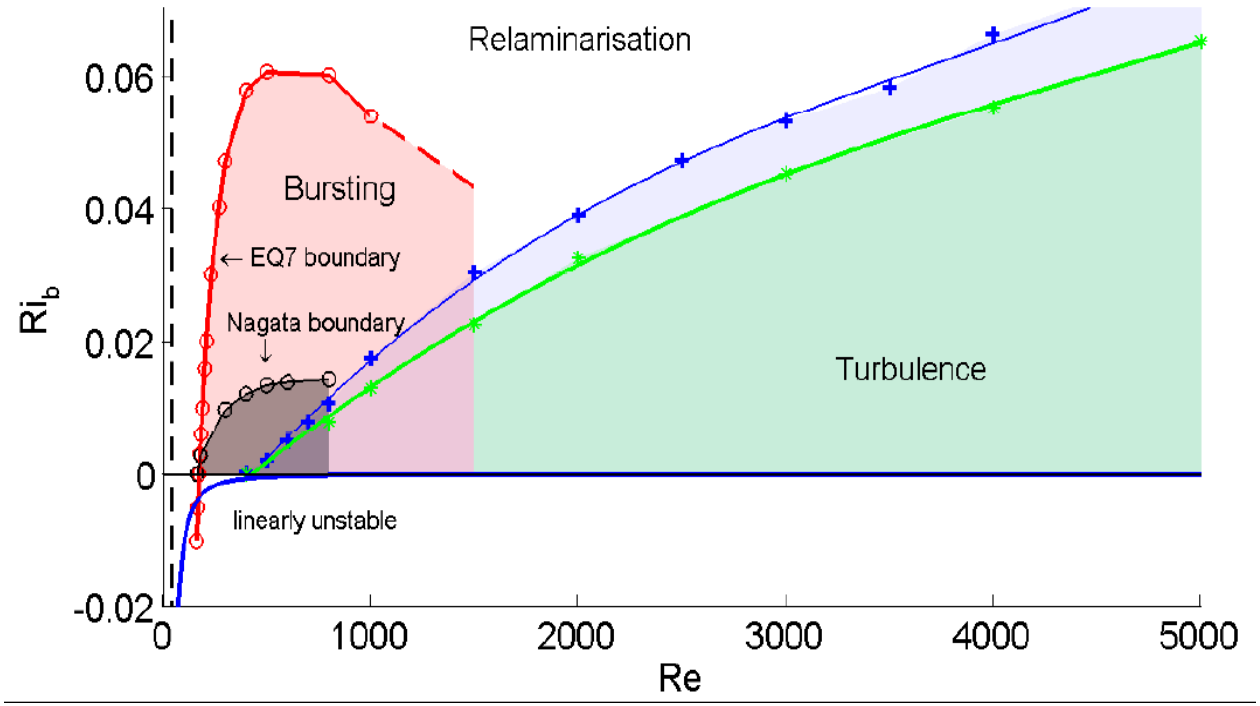
Brethouwer et al. (2012), Deusebio et al. (2015)

Three Questions



- Where do ECS exist? (with Daniel Olvera, Bristol)
- Have 'stratified' snakes got anything to teach us? (with Daniel Olvera, Bristol)
- Can turbulent spots be controlled by stratification? (with John Taylor, Enrico Deusebio & Colm Caulfield, all Cambridge)

Part I: ECS Existence



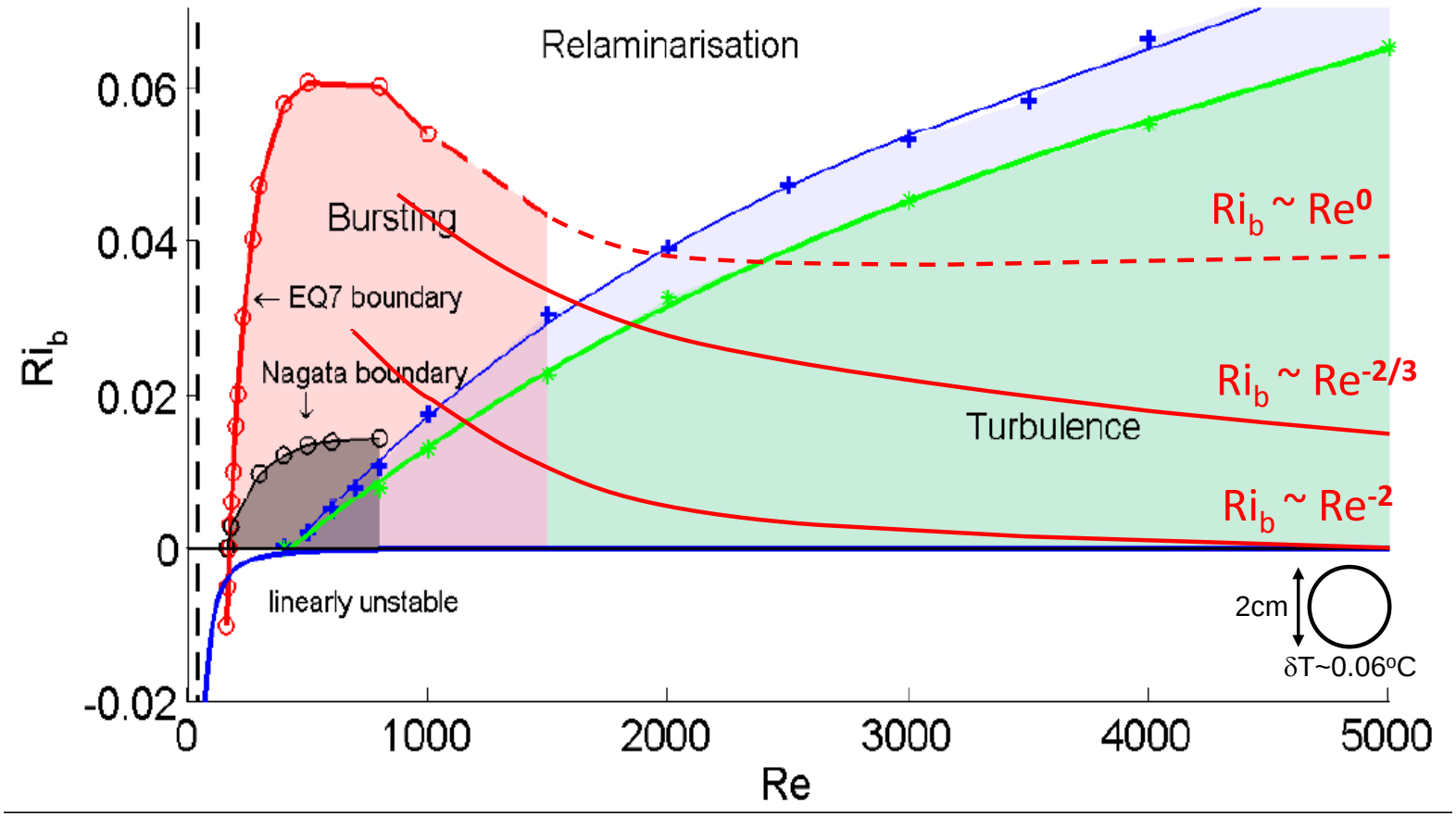
$2\pi \times 2 \times 2\pi$ & $2\pi \times 2 \times \pi$

'EQ7' Gibson et al. (2009)

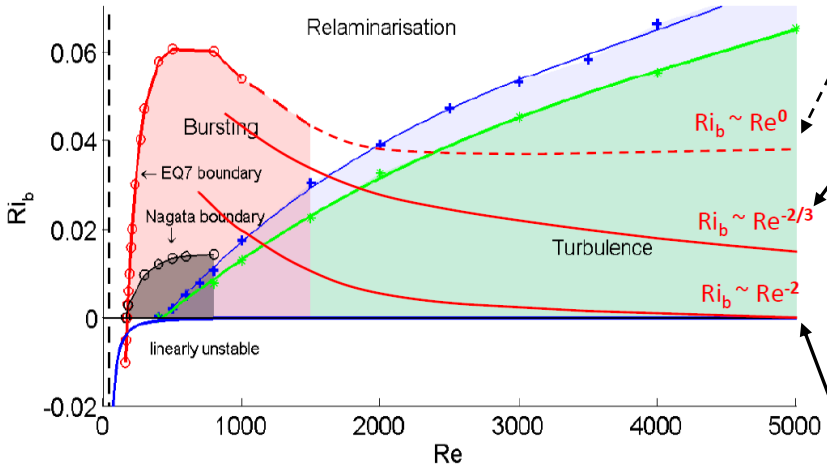
'HVS' Itano & Generalis (2009)

and 'mirror-symmetric mode' of Deguchi & Hall (2014,2015)

Part I: Scalings

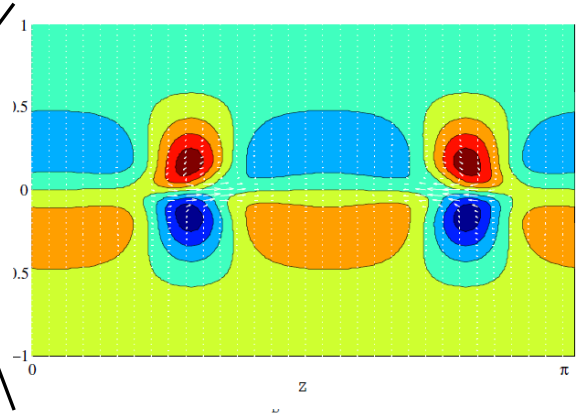


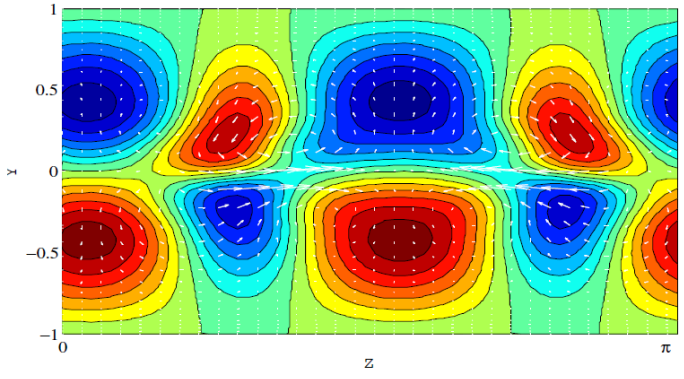
Part I: Regimes

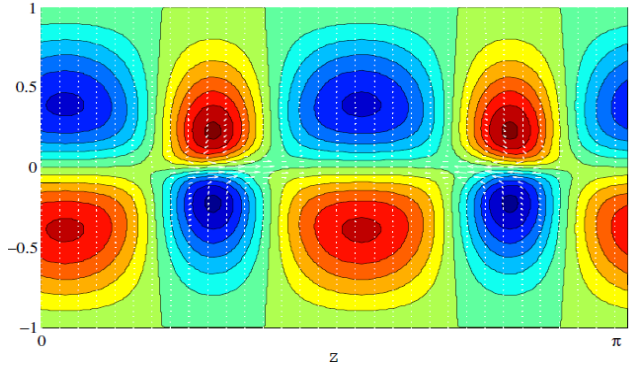


UNS (Deguchi & Hall 2014, Deguchi 2015)

BRE (Deguchi et al. 2013, Deguchi & Hall 2015)



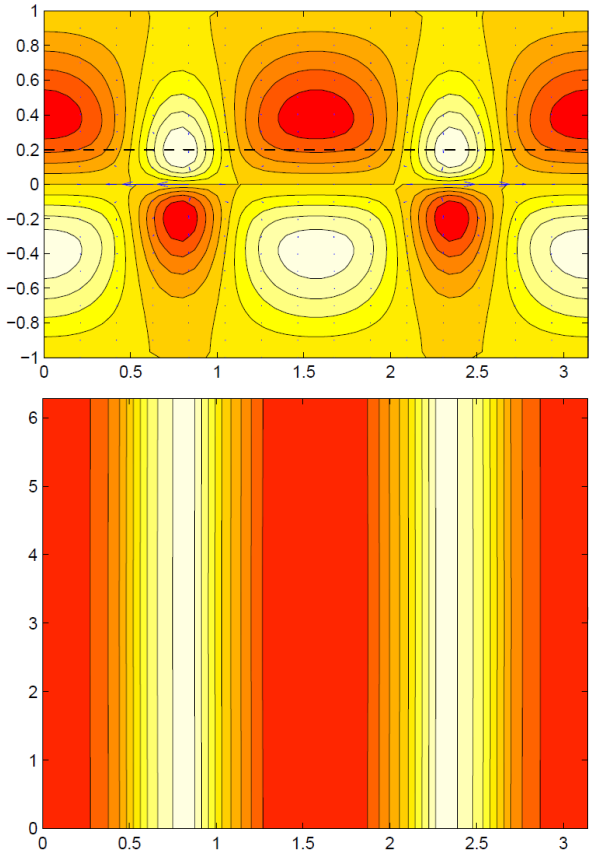




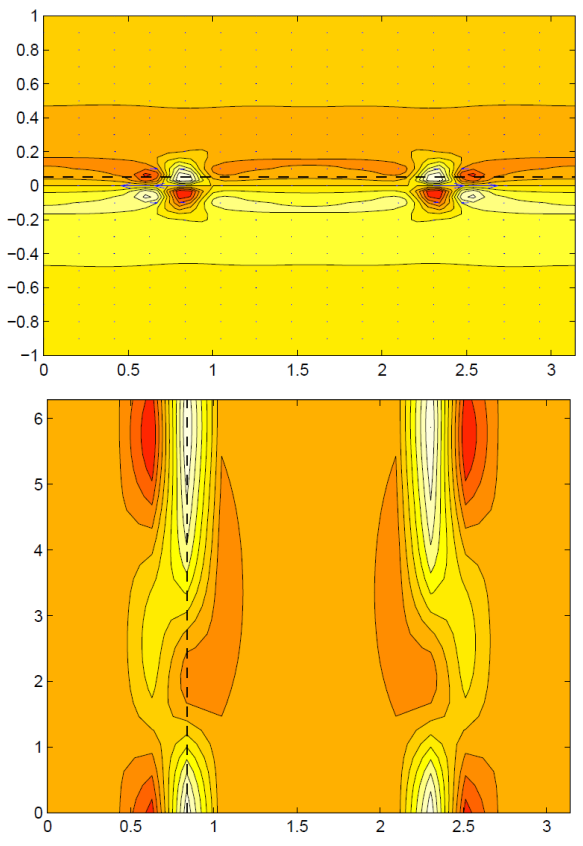
SSP/VWI (Waleffe 1997 / Hall & Smith 1991)

Part I: SSP/VWI vs BRE

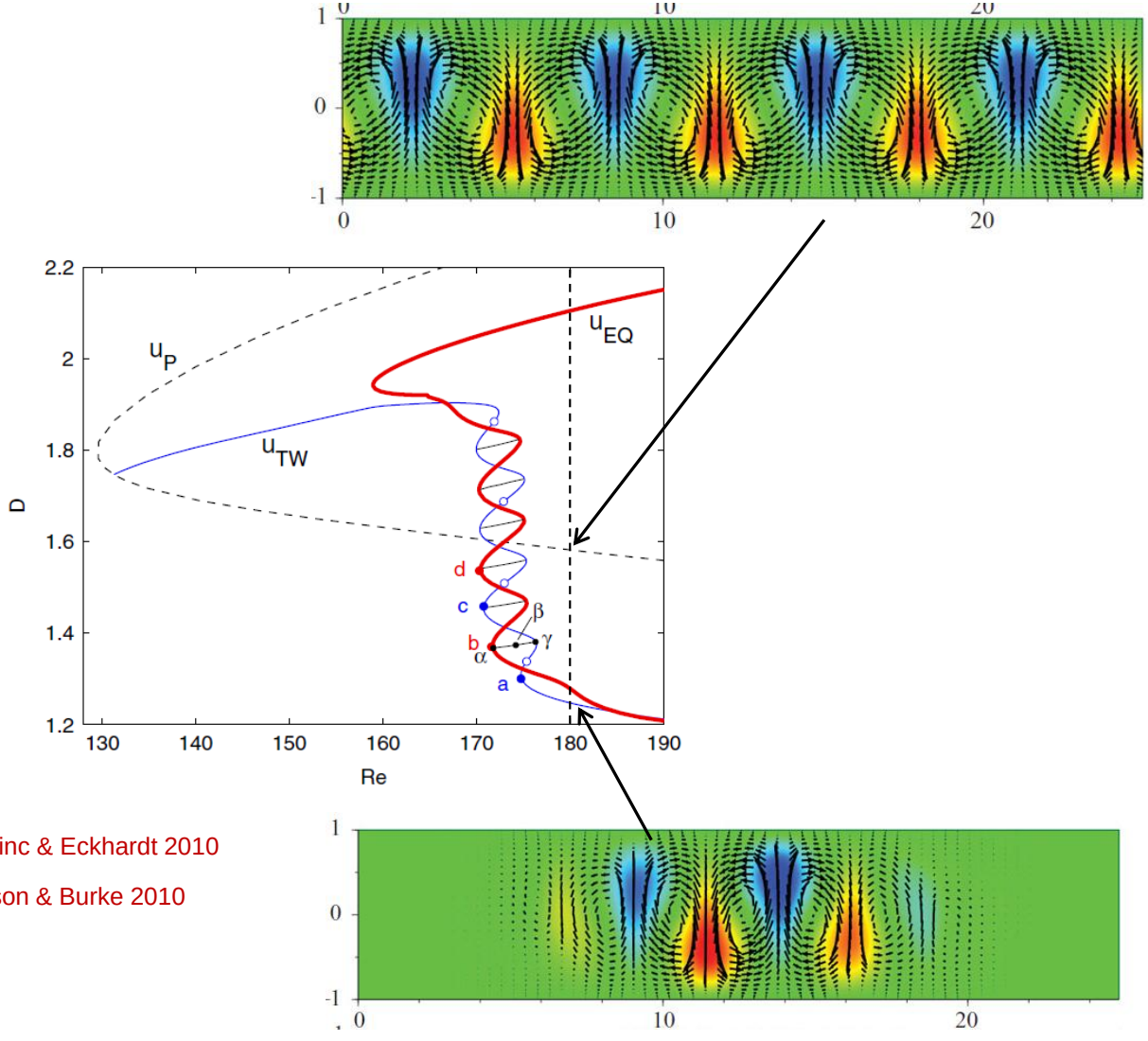
SSP/VWI



BRE



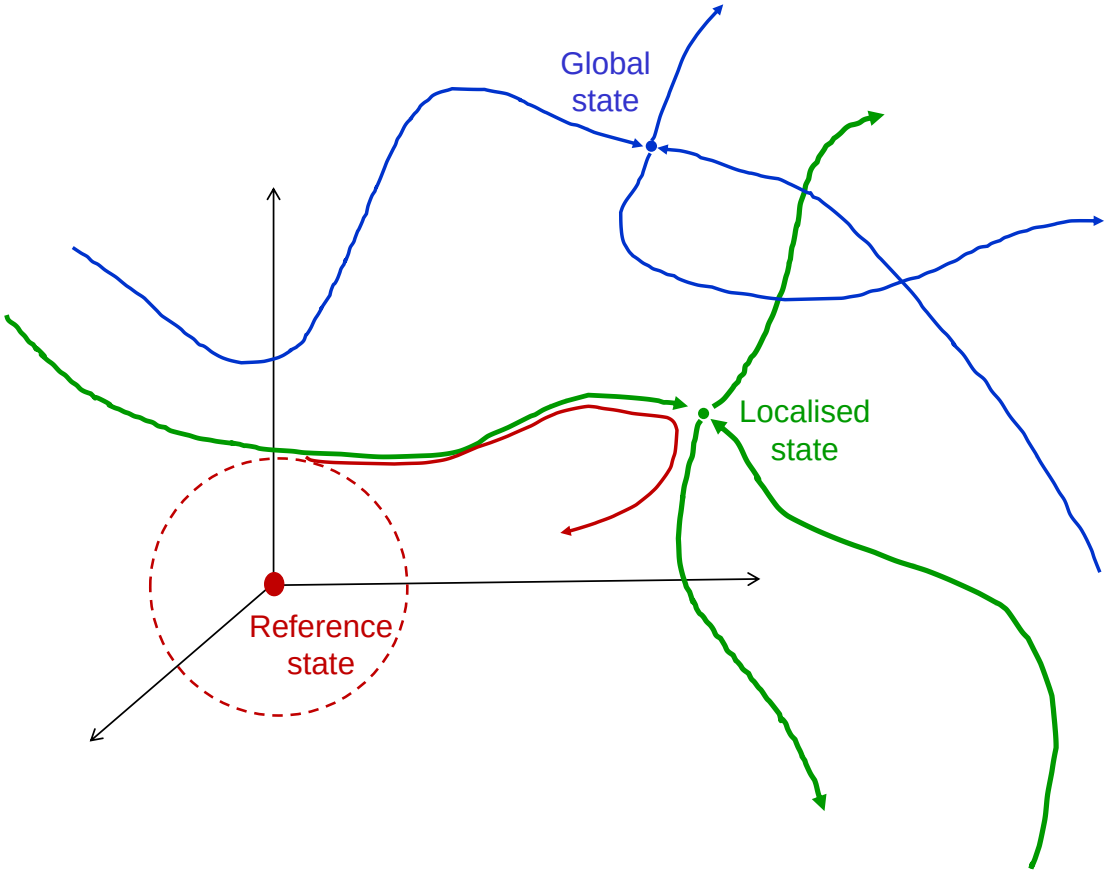
Part II: Unstratified Snakes



Schneider, Marinc & Eckhardt 2010

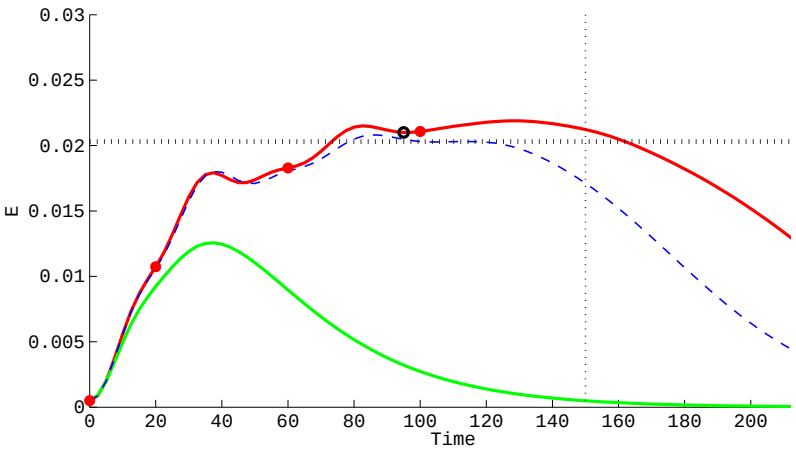
Schneider, Gibson & Burke 2010

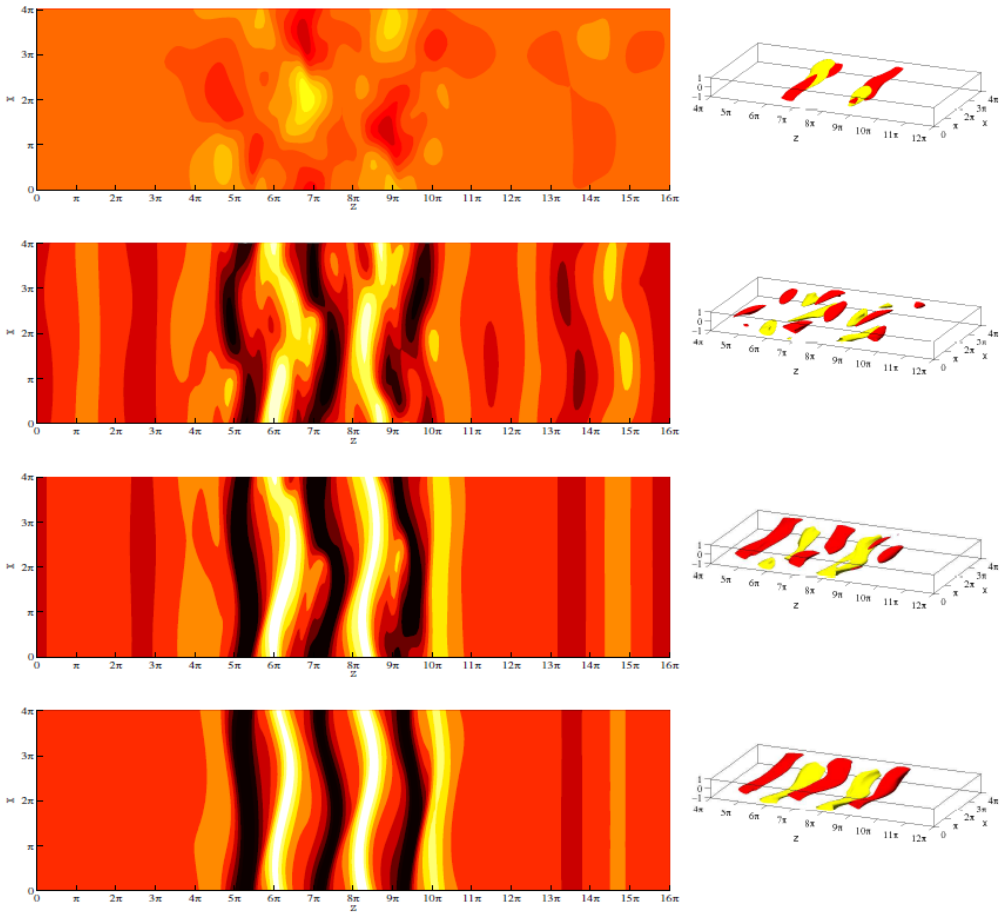
Part II: Energy Growth Optimisation



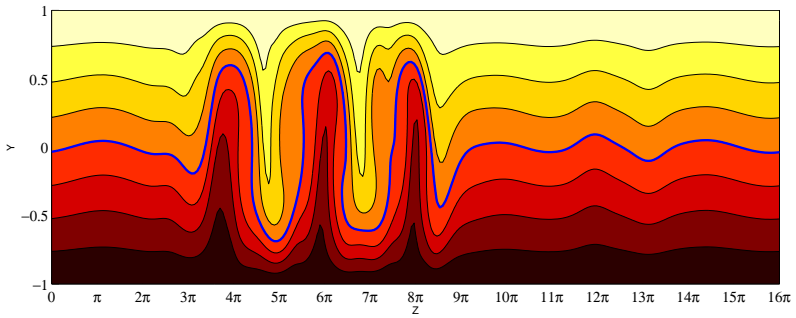
Pringle & K (2010), Cherubini et al (2010), Monokrousos et al. (2011), Rabin (2013)

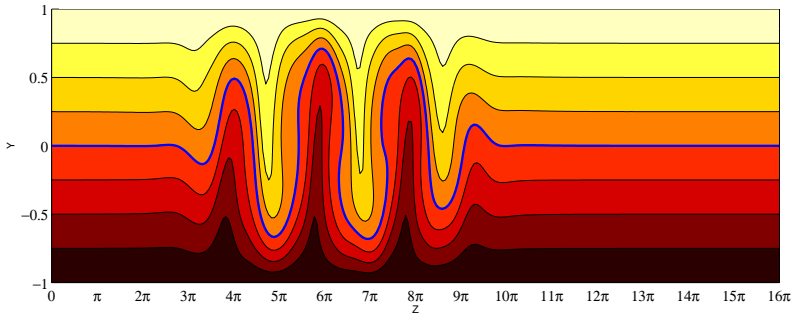
Part II: Evolution

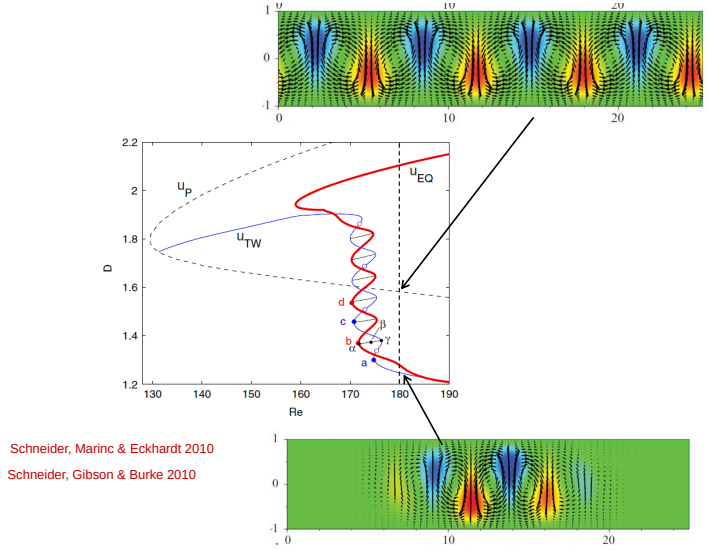




Part II: Convergence & Snaking in *Re*

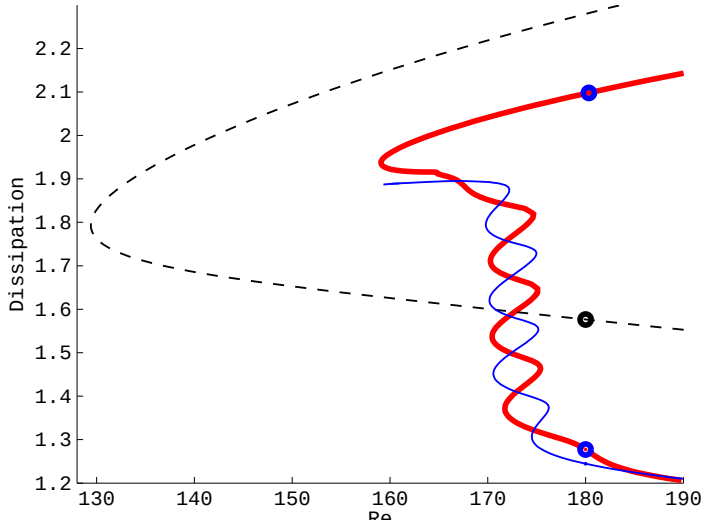




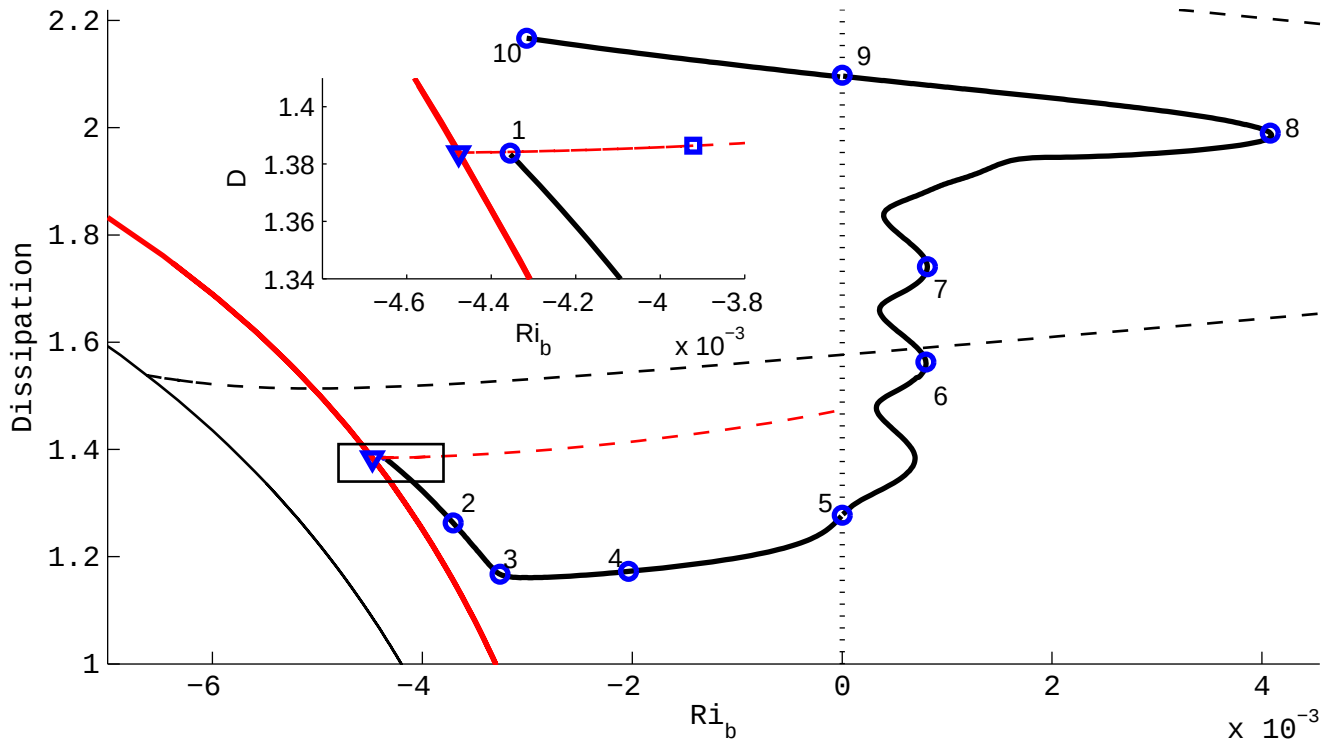


Schneider, Marinc & Eckhardt 2010

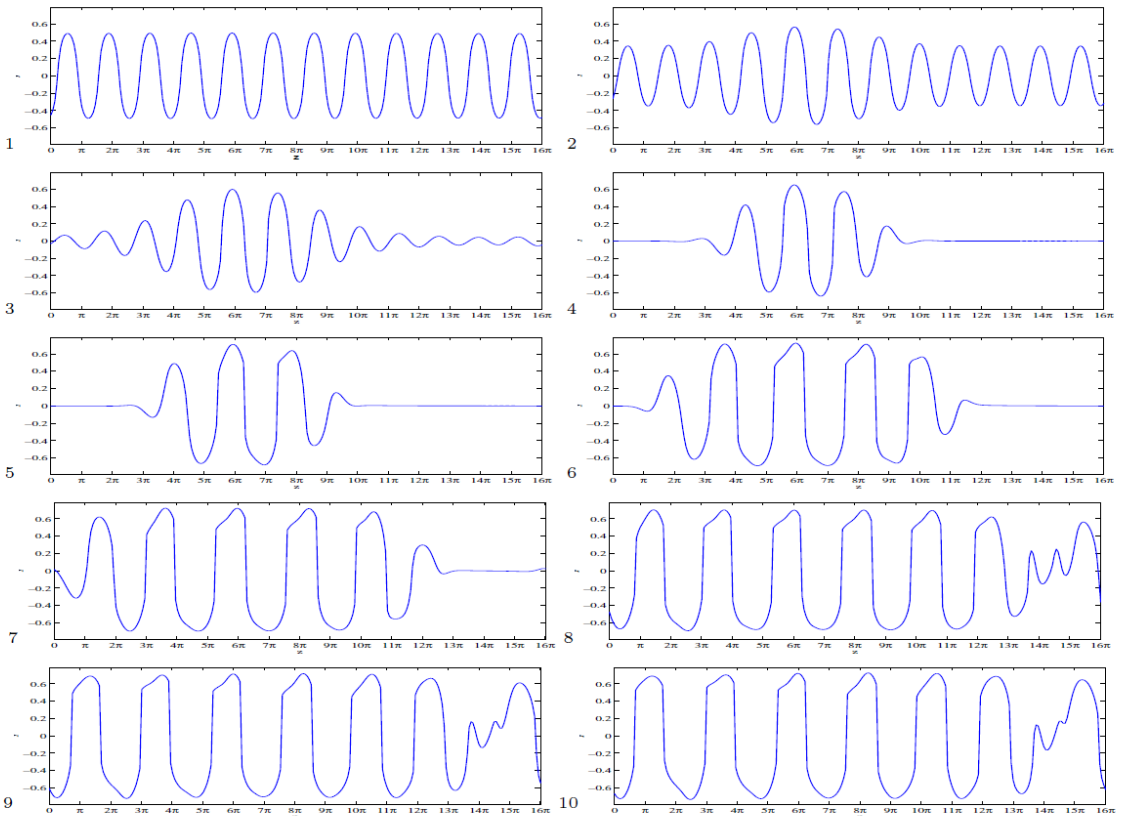
Schneider, Gibson & Burke 2010

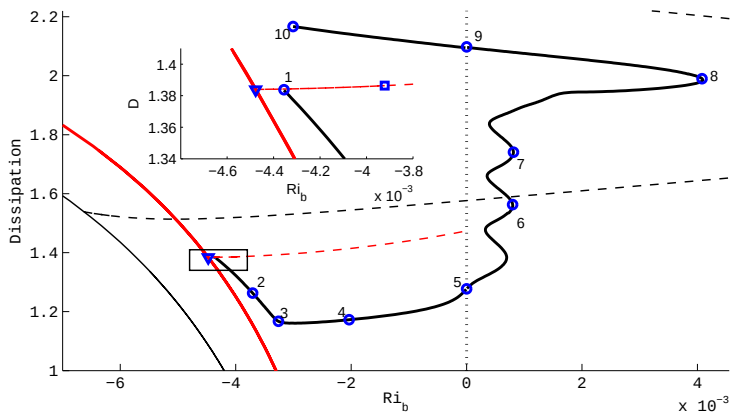


Part II: Snaking in Ri_b



Part II: Delocalisation

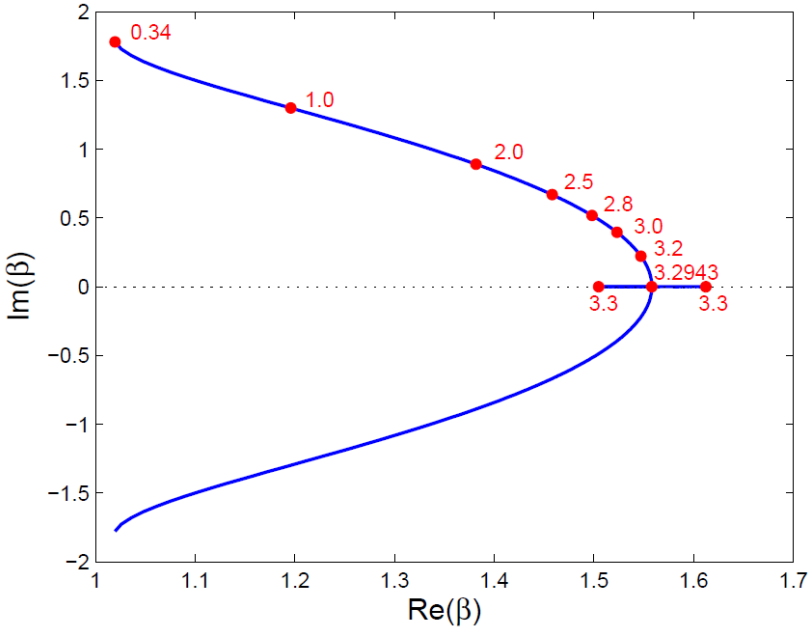


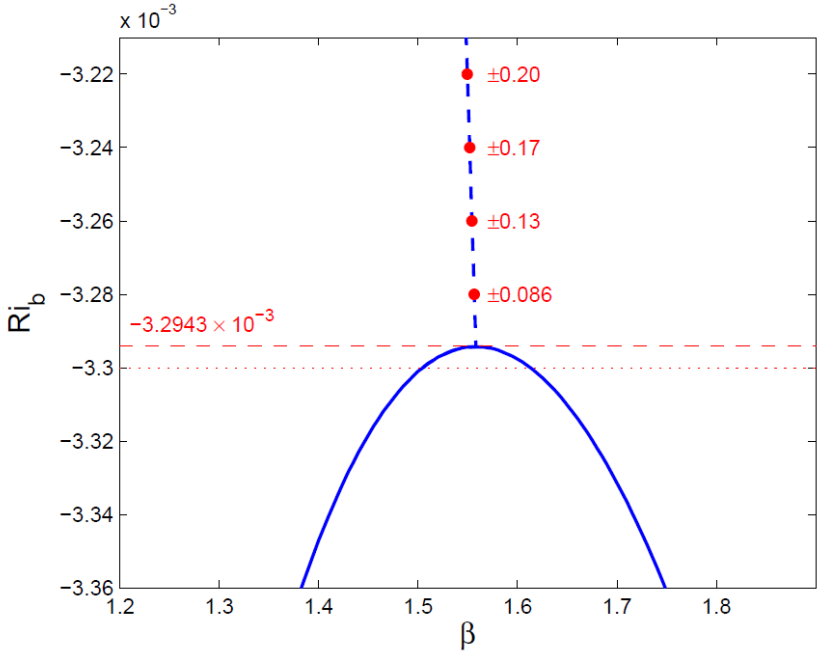


Part II: Linear Theory

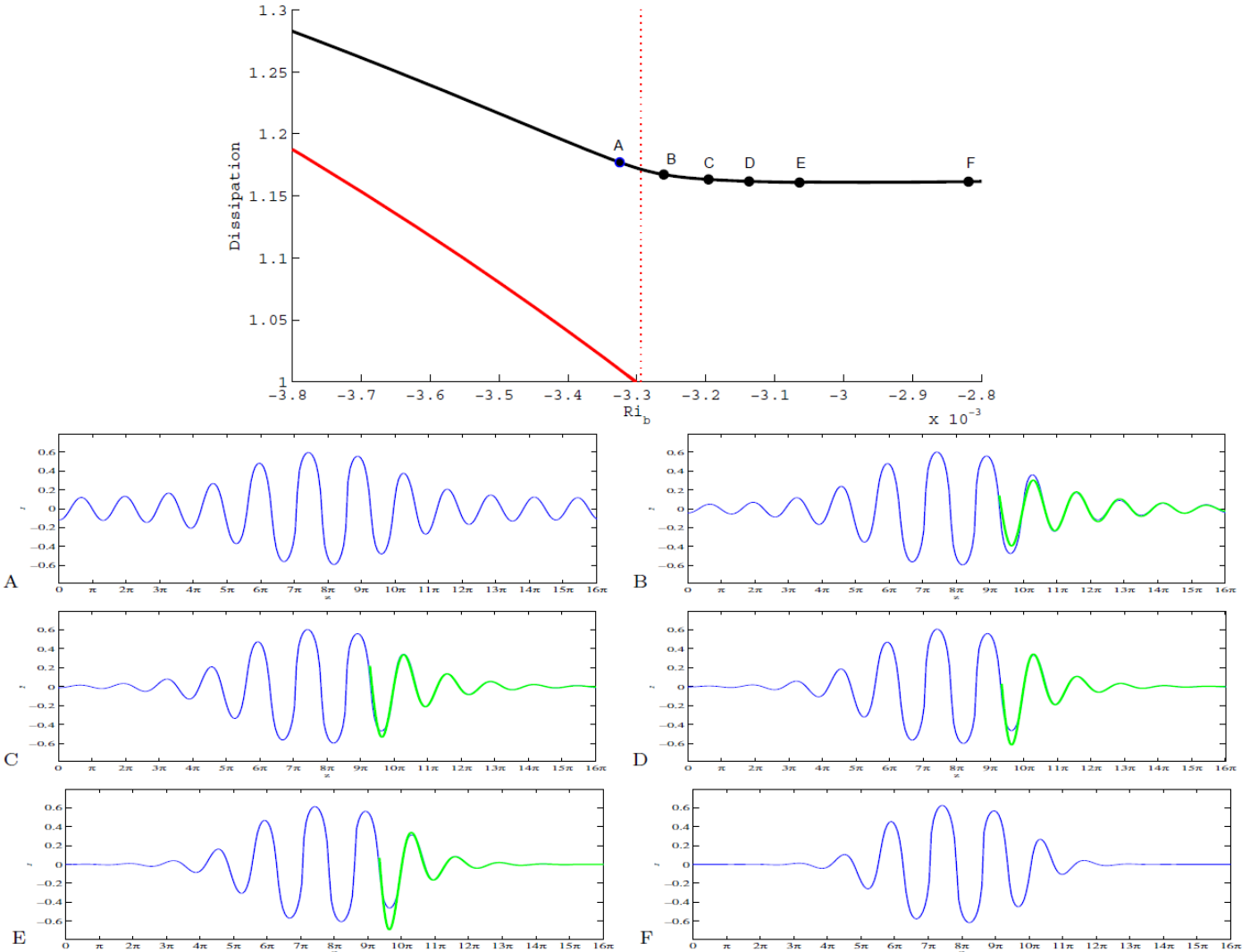
$$(\tilde{\mathbf{u}}, \tilde{\rho}, \tilde{p}) = (\,\tilde{\mathbf{u}}(y), \tilde{\rho}(y), \tilde{p}(y)\,)e^{i\beta z+\sigma t}$$

Gibson & Brand (2014)

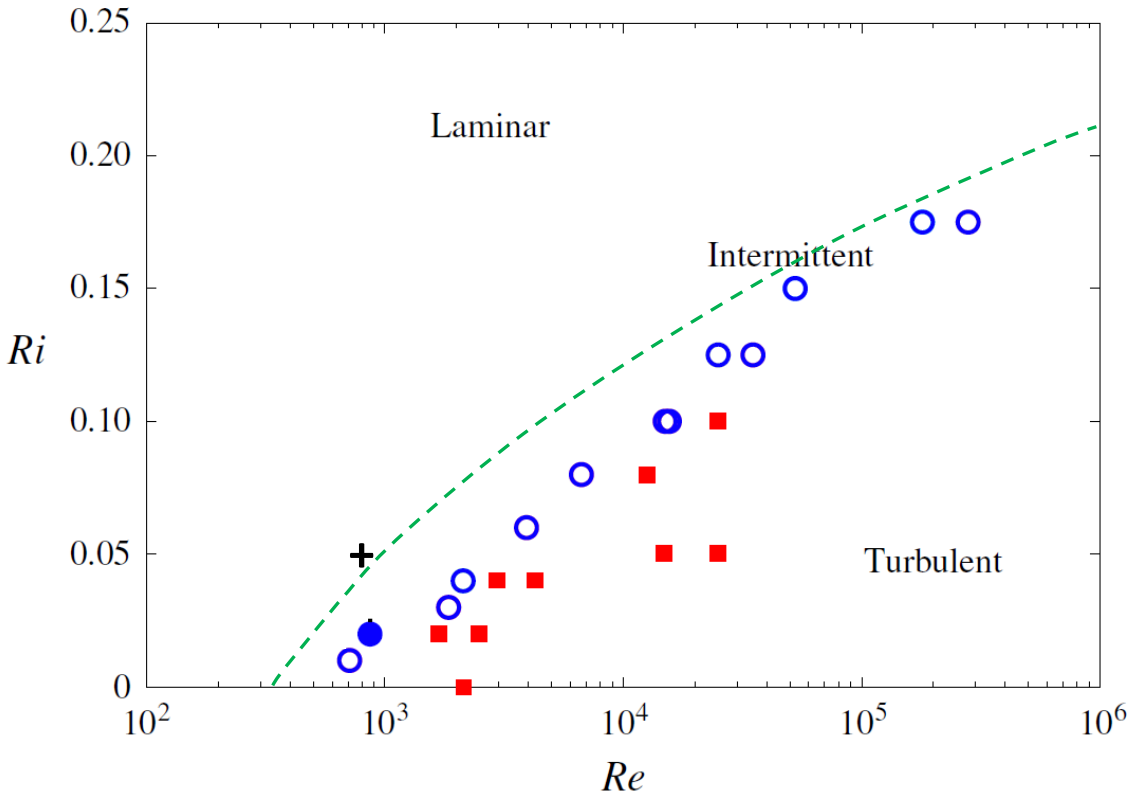




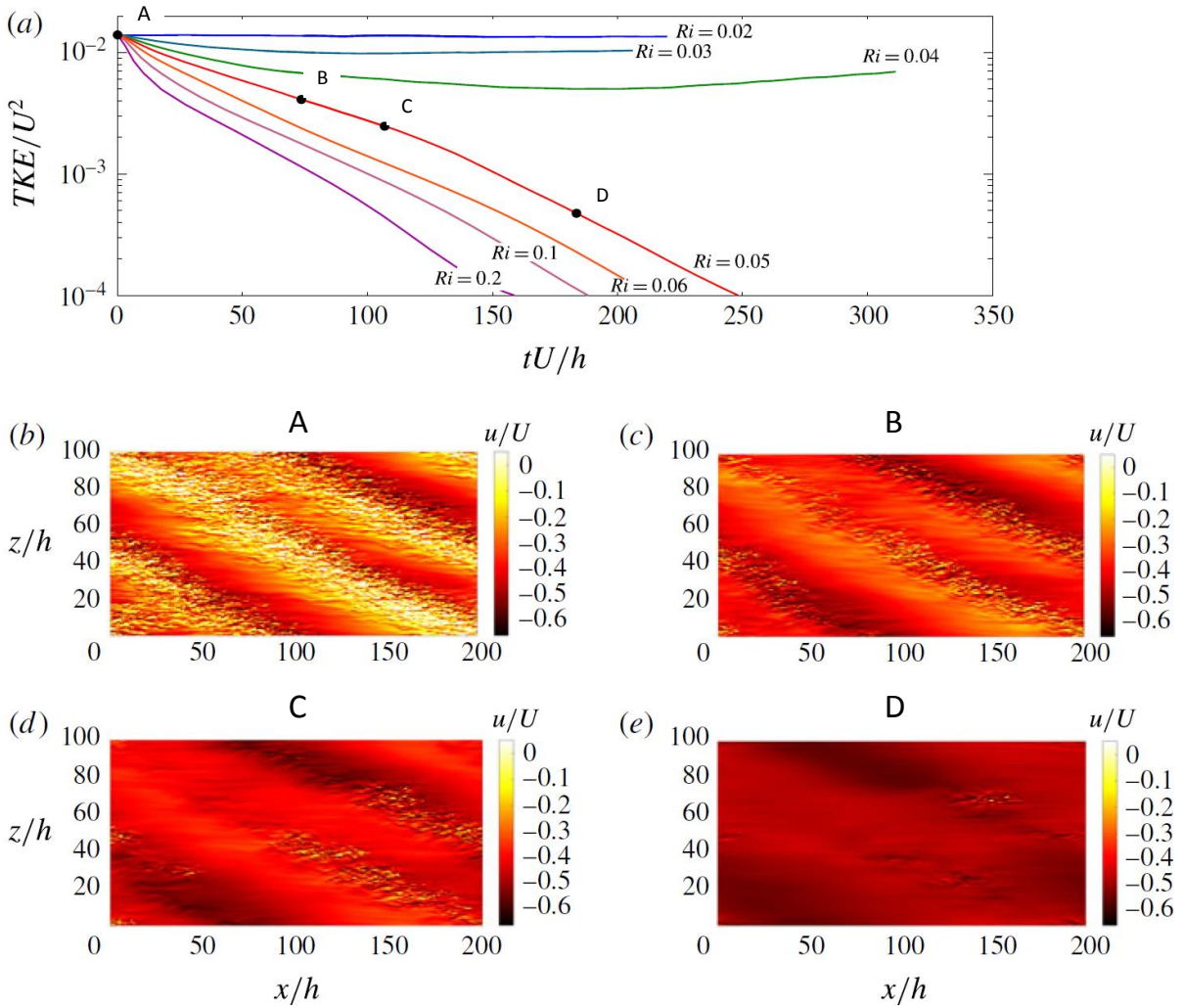
Part II: Comparison



Part III: Turbulent Spots



Part III: Run Down



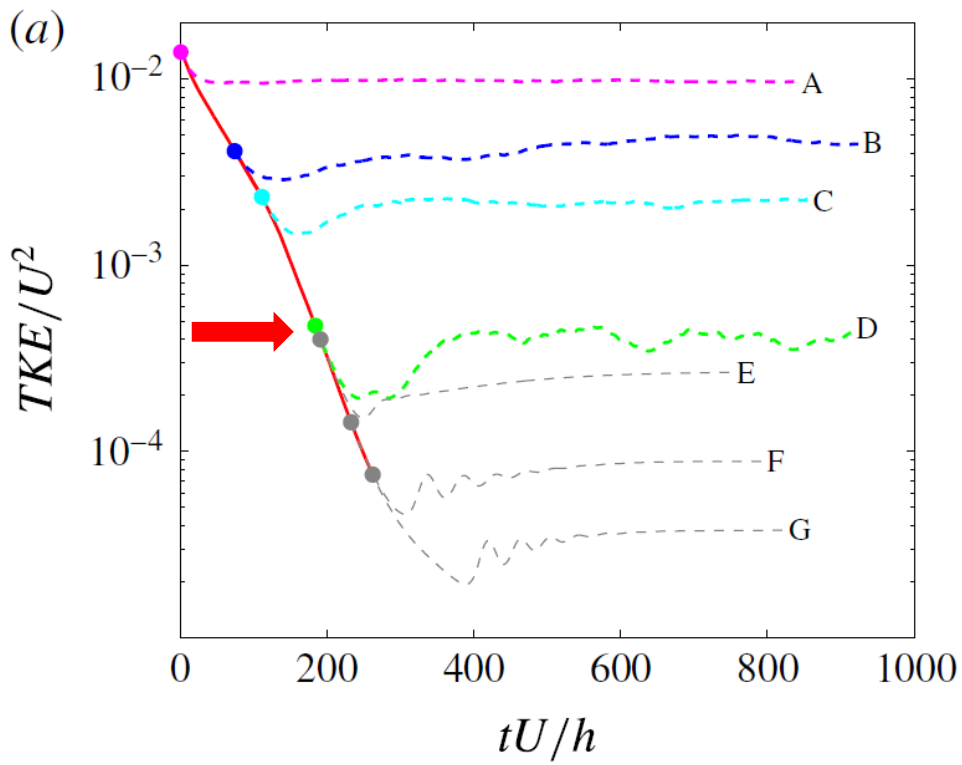
Geometry $64\pi \times 2 \times 32\pi$, Resolution $1024 \times 64 \times 1024$, $Pr = 0.7$ $Re = 865$.

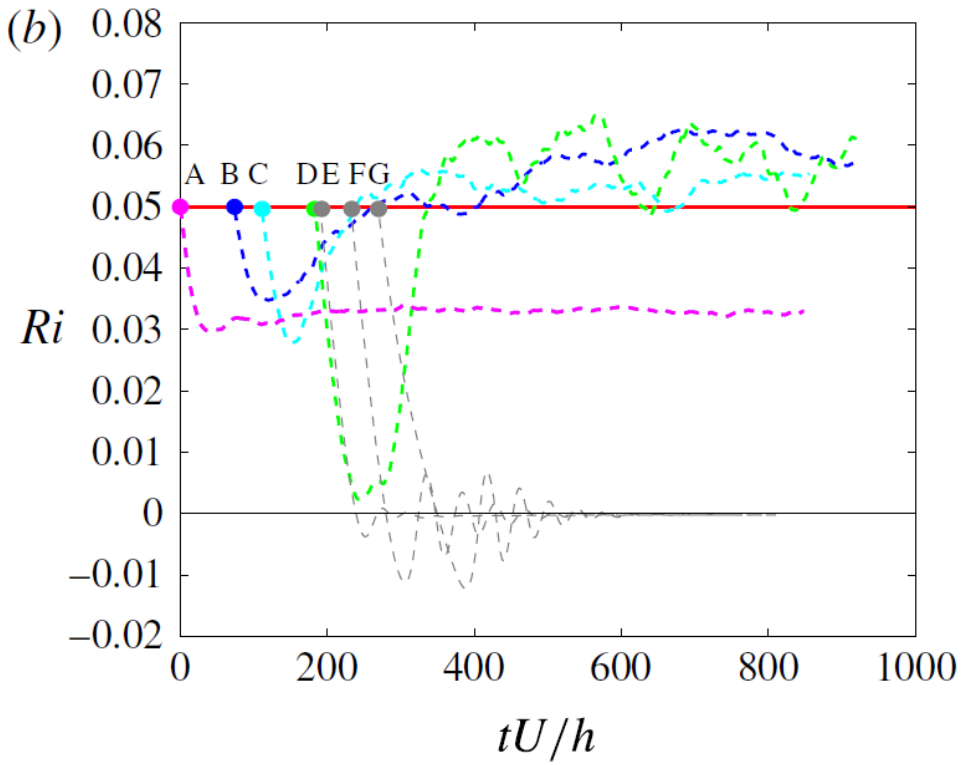
Part III: Control

$$\frac{d}{dt}\left[Ri_b(t) - c\,\log\left(\frac{TKE(t)}{TKE_0}\right)\right] = 0$$

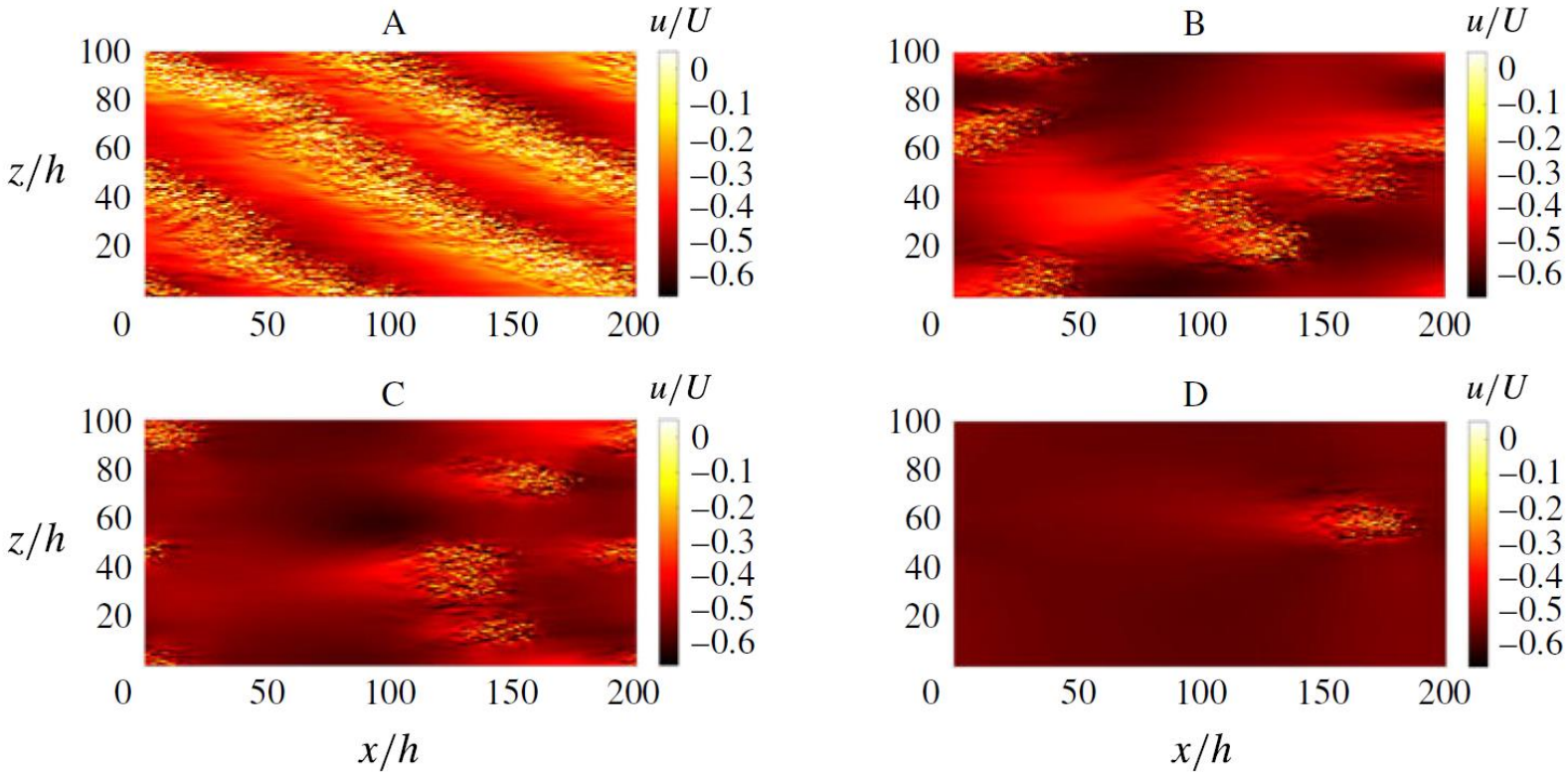
- $c = 0$ recovers Boussinesq equations
- $c > 0$ de/increasing TKE controlled by de/increasing Ri_b
- c small Ri_b 'slow' to react

Part III: Controlled Turbulence

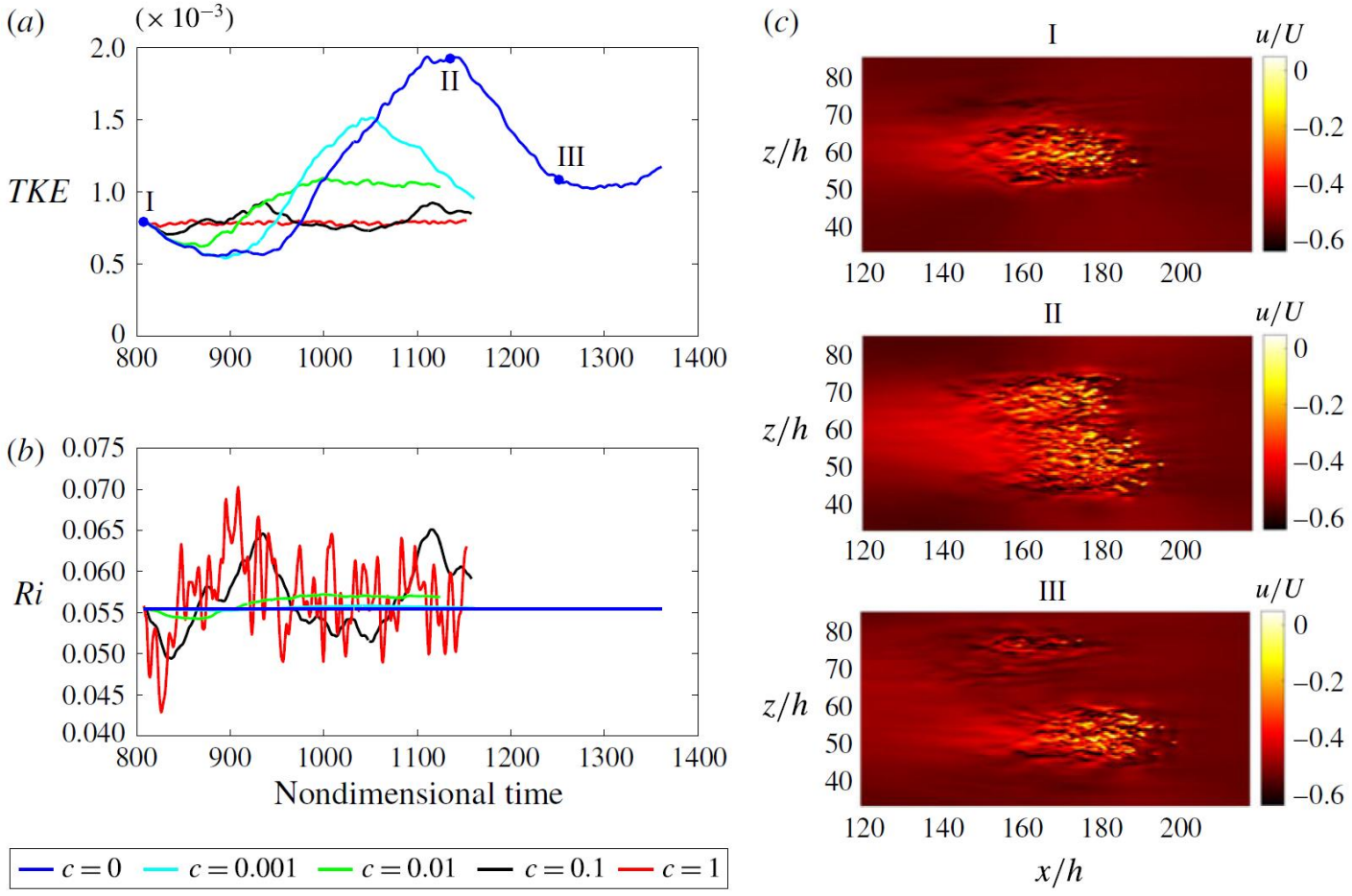




Part III: End Product



Part III: Varying C



Conclusions

- Stratification has a strong effect on ECS - 2 regimes captured, a third likely for $Ri_b = O(1)$ as $Re \to \infty$.
- Stratified snakes reveal a simple localisation mechanism.
- Turbulent spots can be controlled by Ri_b - opens up possibilities for study (e.g. UPOs).