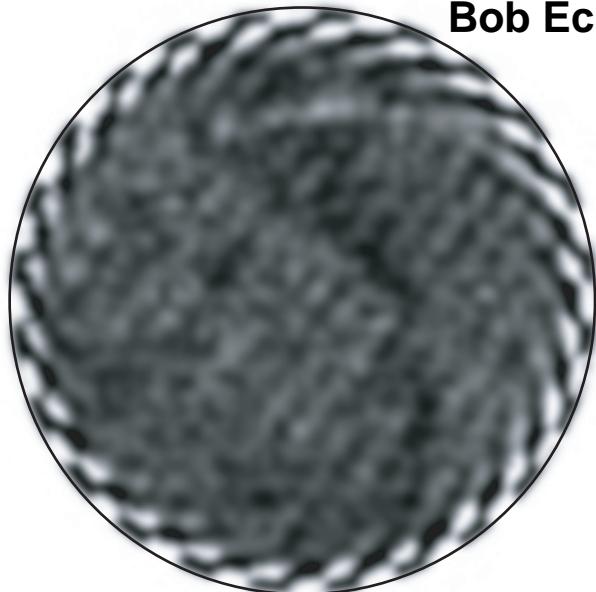
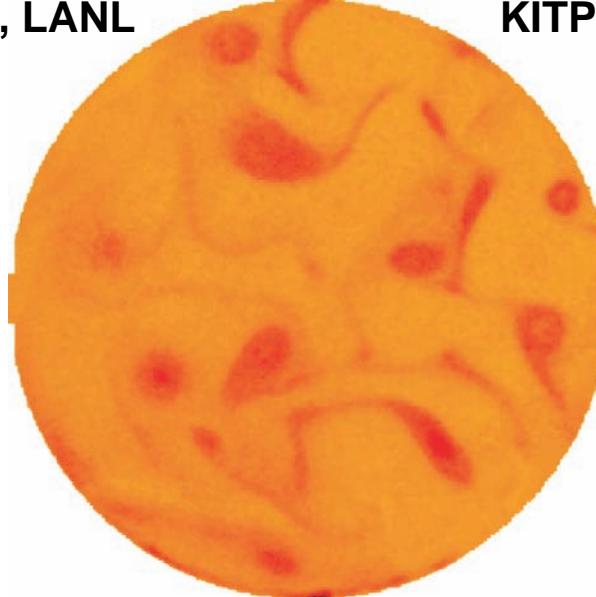


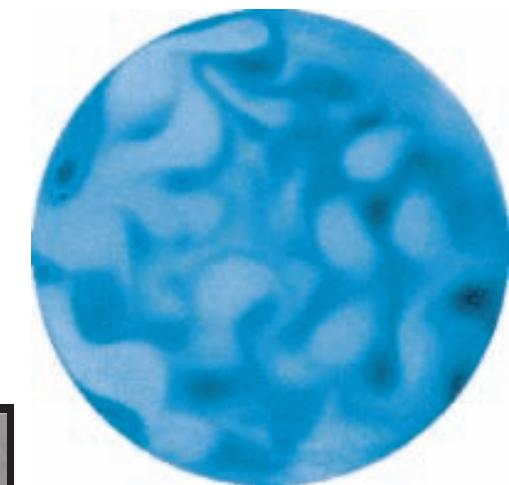
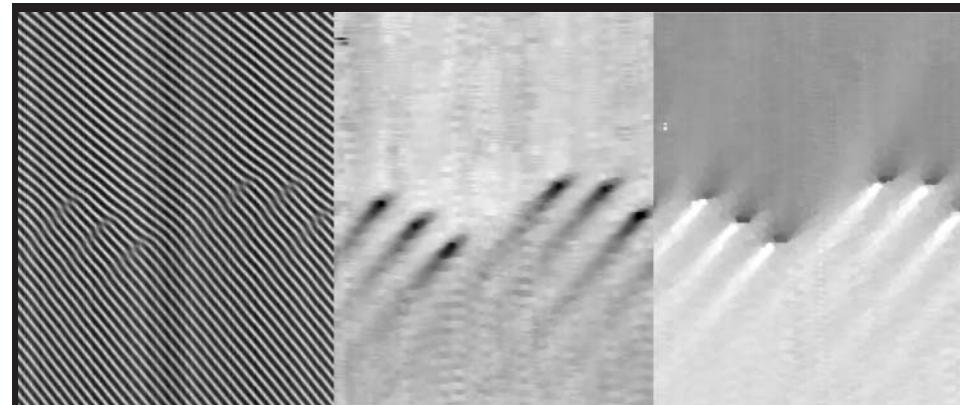
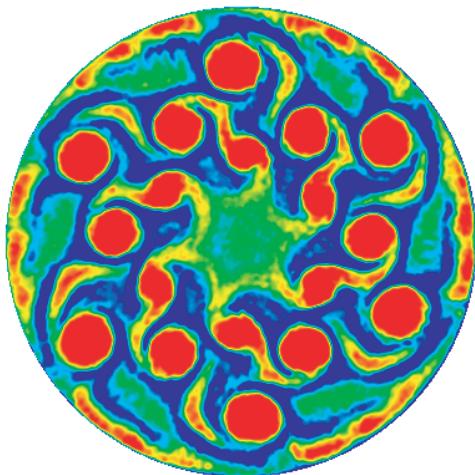
Bob Ecke, LANL



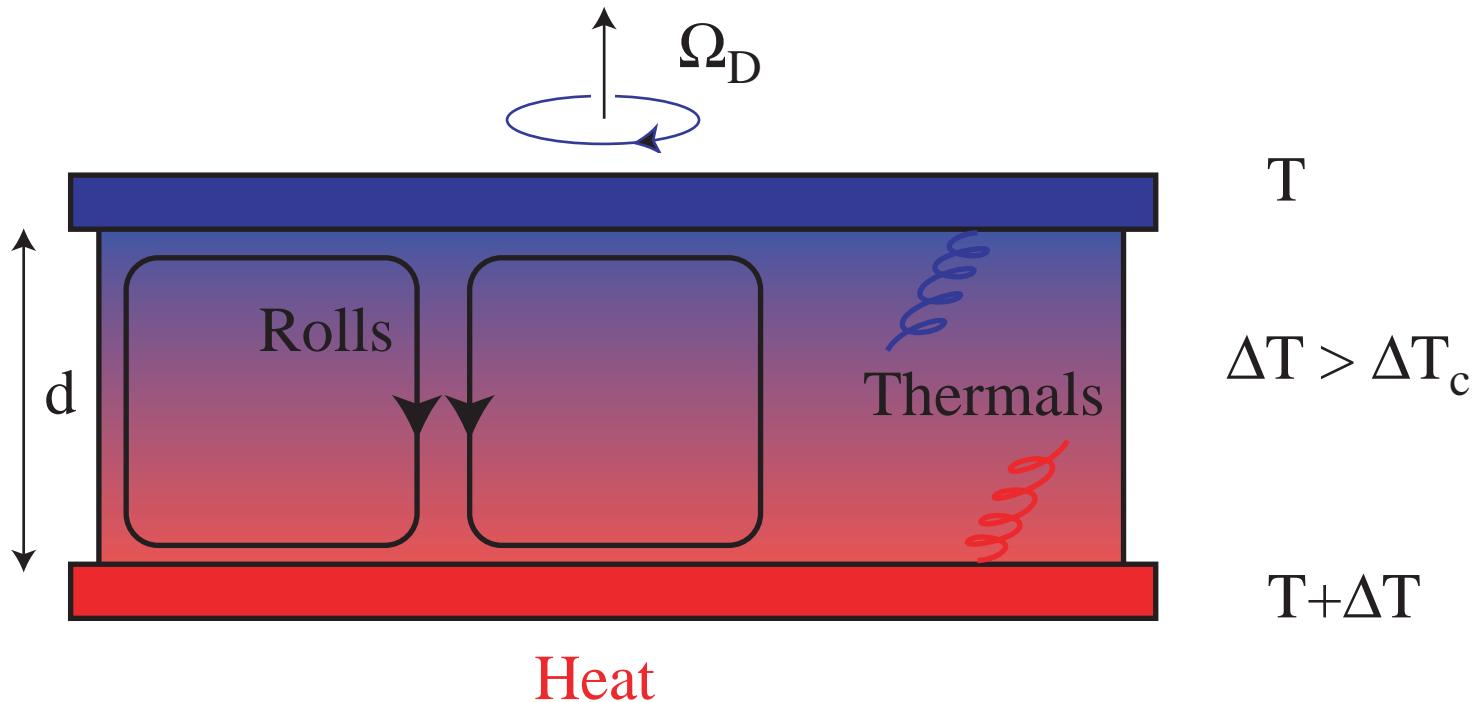
KITP, Apr 6, 2011



Rotating Convection Experiments: A Tour through Parameter Space



Rotating Rayleigh-Benard Convection



$$Ra = g\alpha d^3 \Delta T / \nu \kappa \sim \text{buoyancy/dissipation}$$

$$Pr = \nu / \kappa \sim \text{viscous to thermal dissipation}$$

$$Re = ud/n \sim \text{inertial/viscous dissipation}$$

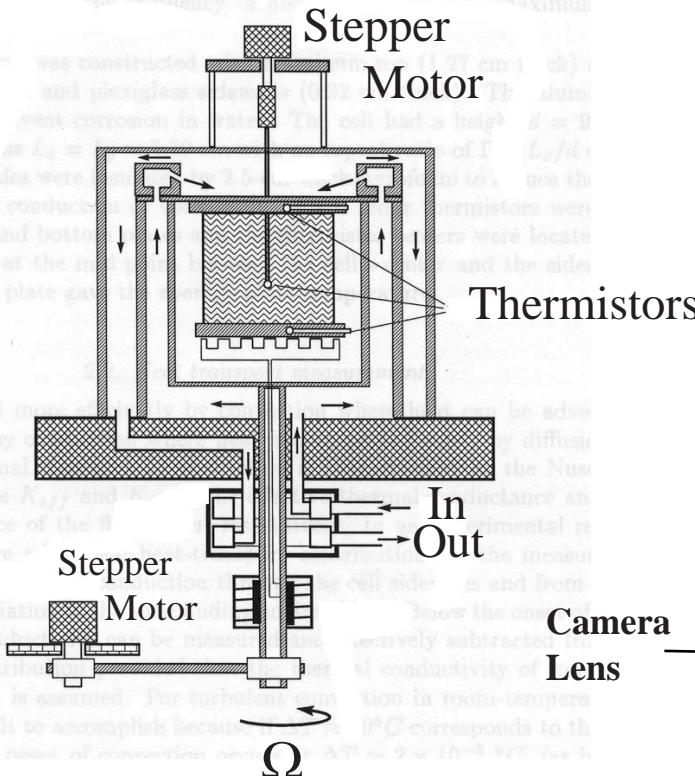
$$\Omega = \Omega_D d^2 / \nu \sim \text{rotation time/viscous diffusion time}$$

$$Ta = (2\Omega)^2$$

$$Ro = [Ra/(PrTa)]^{1/2} = (g\alpha \Delta T / d)^{1/2} / 2\Omega_D \sim \text{rotation time/buoyancy time}$$

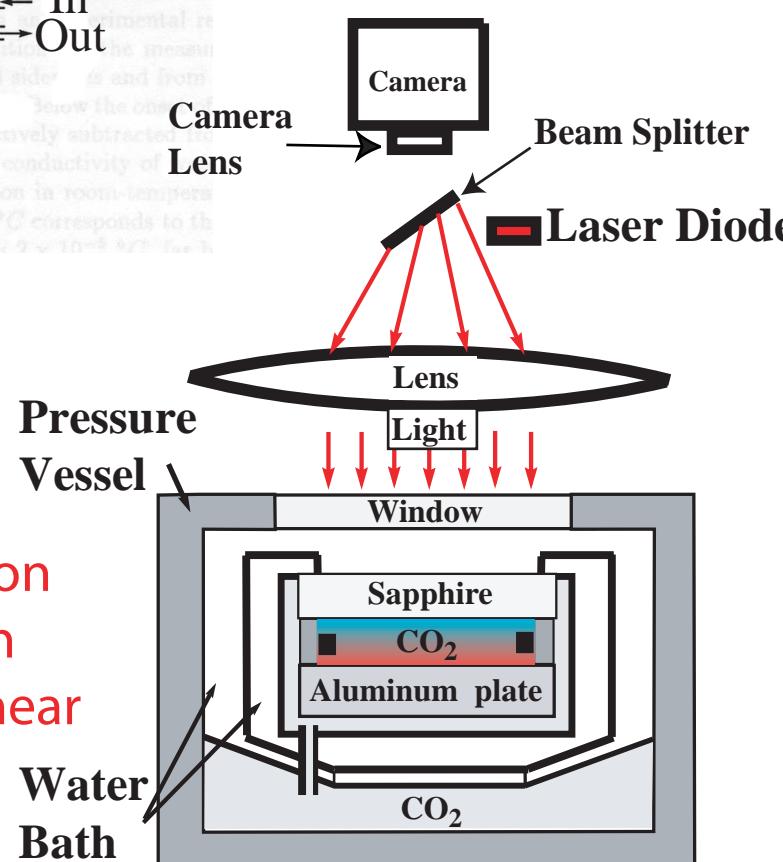
$$Fr = \Omega_D^2 R / g \sim \text{centrifugal acceleration/gravitational acceleration}$$

Heat Transport Temperature Fluctuations

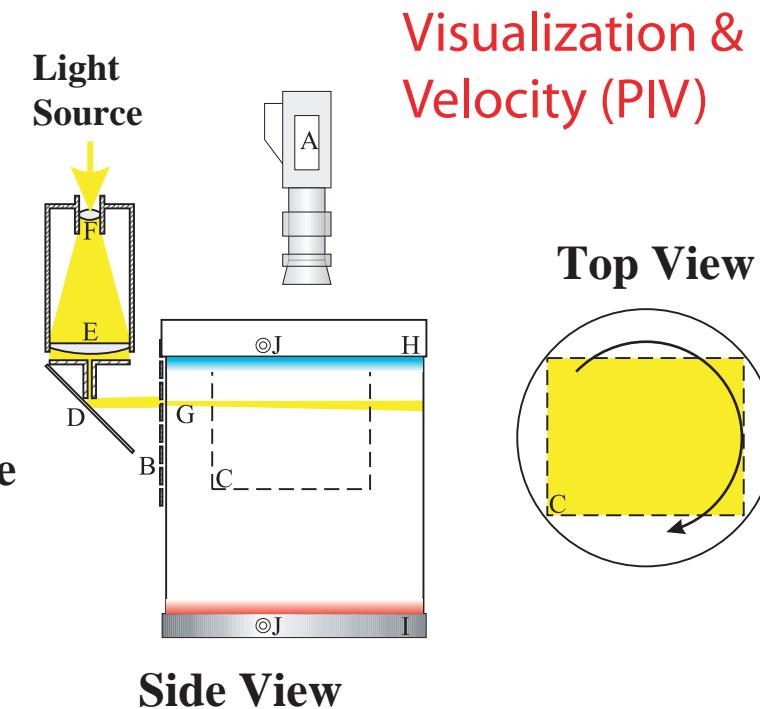


Gas Convection
Shadowgraph
Patterns & Linear
Stability

3

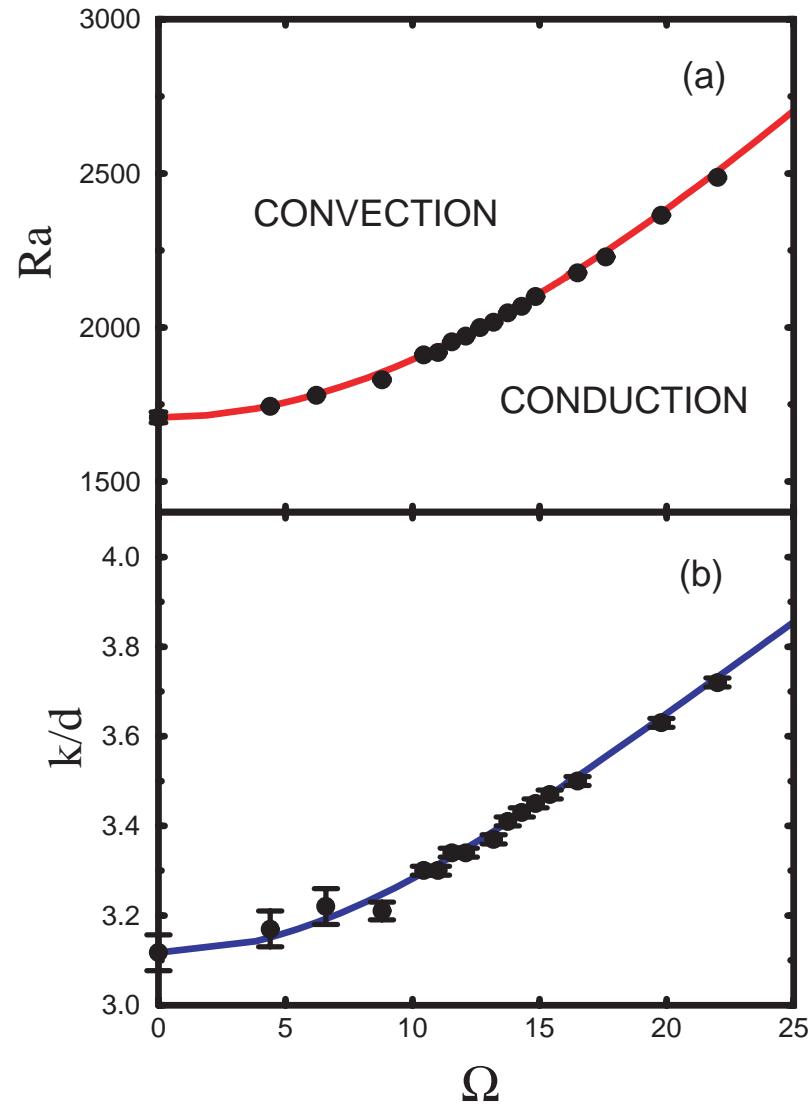
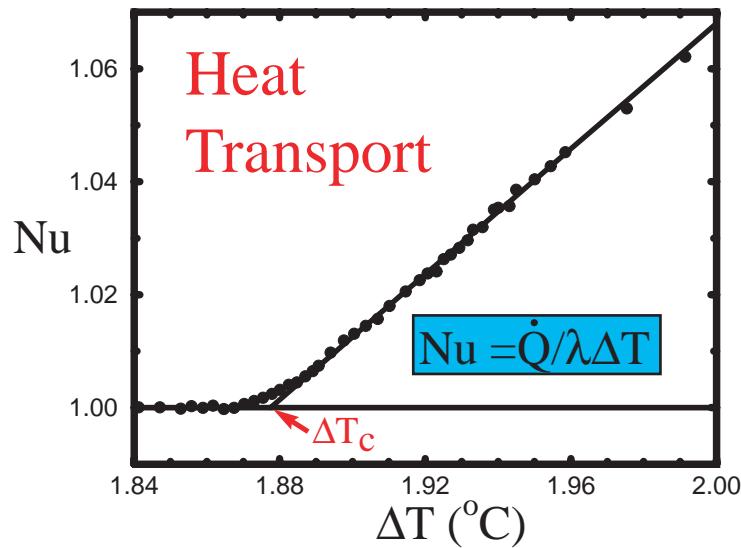


Apparatus: Heat Transport, Patterns, Particle Tracking



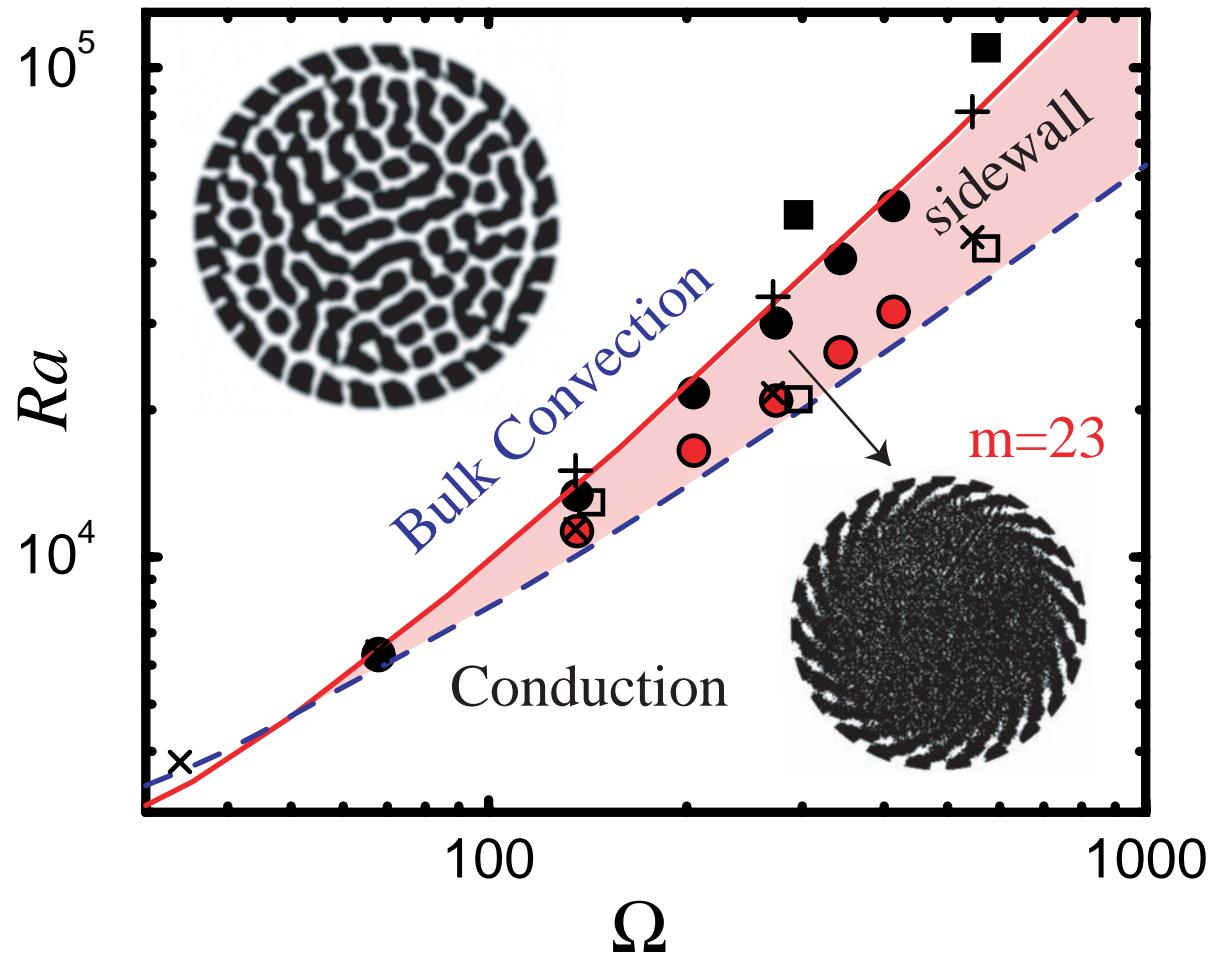
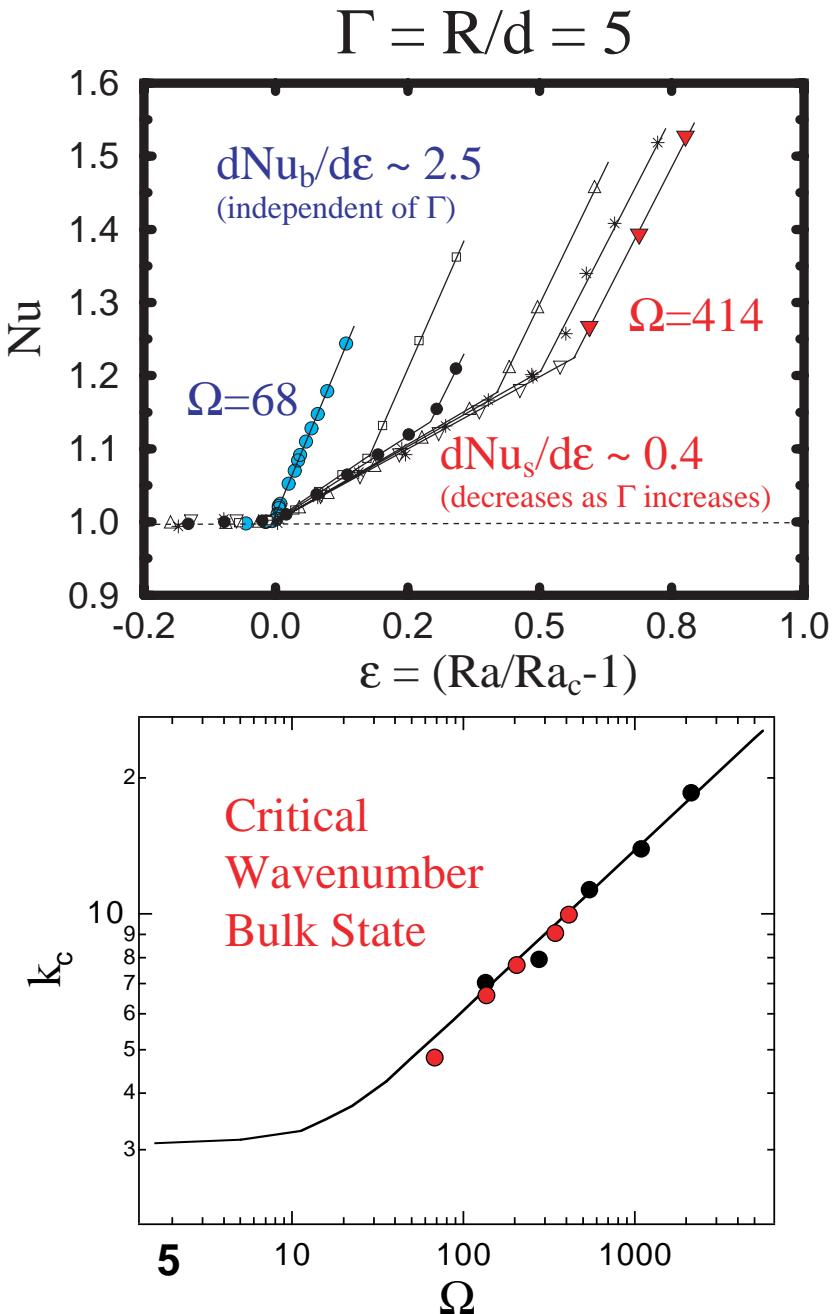
Rotating Rayleigh-Benard Convection

Linear Stability $\text{Pr} \sim 1, \Omega \leq 25$



Rotating Rayleigh-Benard Convection

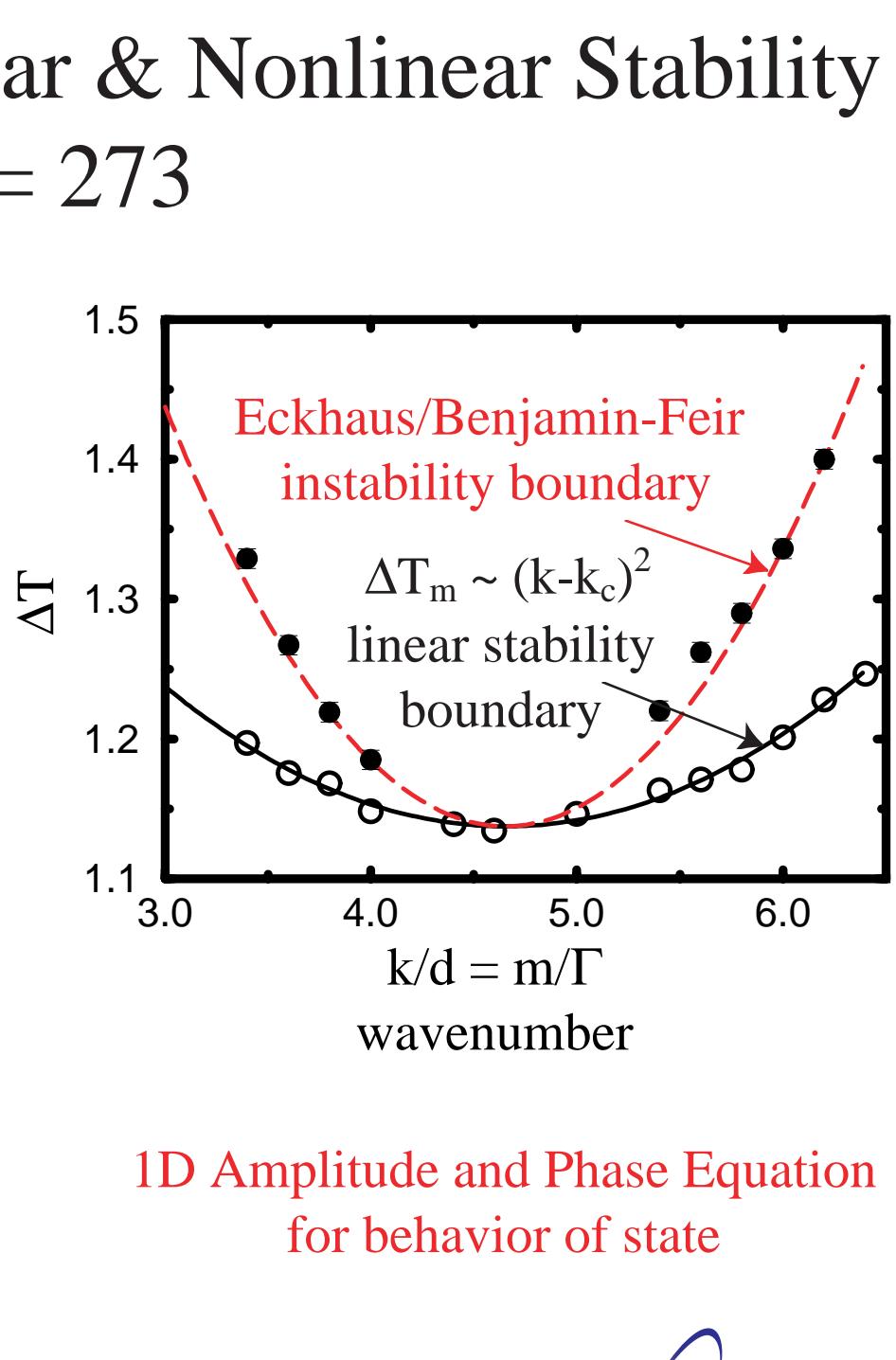
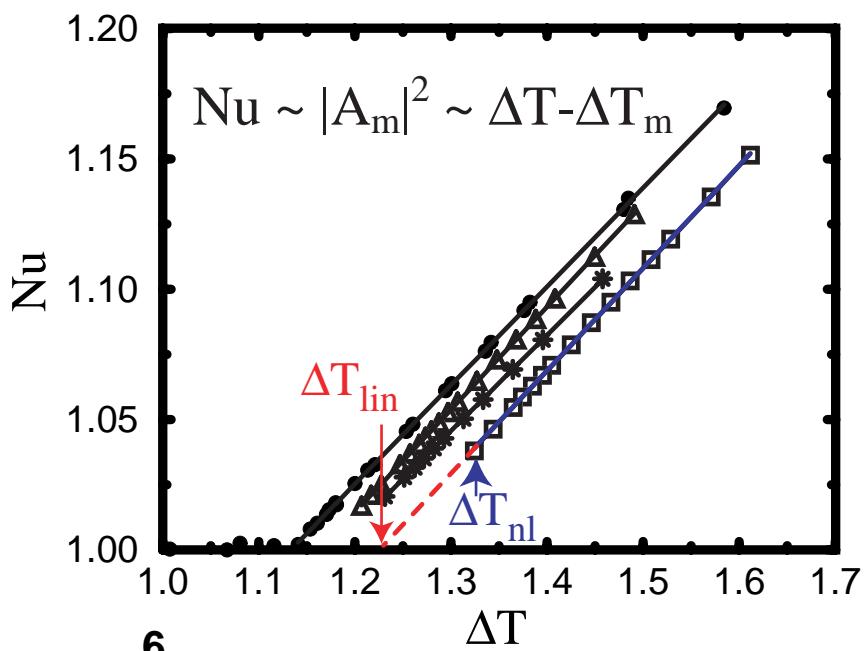
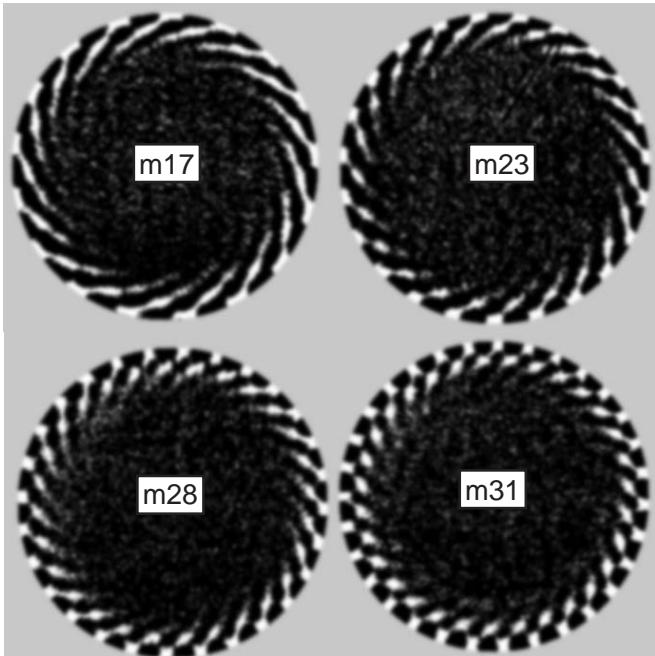
Linear Stability $\text{Pr} \sim 6, \Omega \geq 100$



Sidewall mode linearly unstable at smaller Ra than for the bulk state for $\Omega > 70$

Sidewall Mode - Linear & Nonlinear Stability

$$\Omega = 273$$

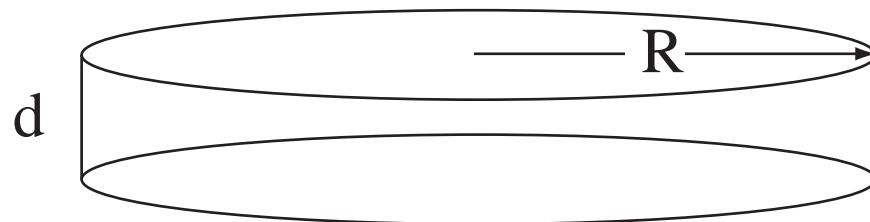


Liu & Ecke, *Phys. Rev. Lett.* **78**, 4391(1998); *Phys. Rev E* **50**, 4091 (1999).

$\text{Pr} \sim 6, \Omega \geq 100$

Dependence on Aspect Ratio

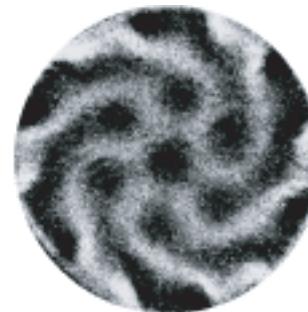
$$\Gamma = R/d$$



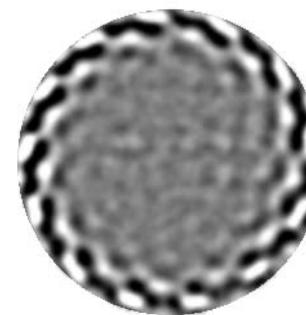
$$\lambda/d = 2\pi R/md = 2\pi\Gamma/m$$

$$\Rightarrow k = 2\pi/\lambda = \Gamma/m$$

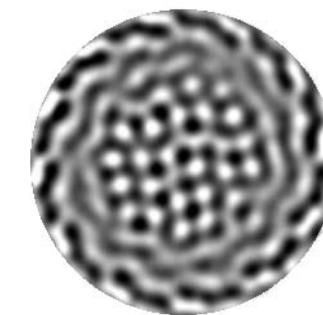
$$d\text{Nu}/d\Gamma \sim 2\pi R d / \pi R^2 \sim 1/\Gamma$$



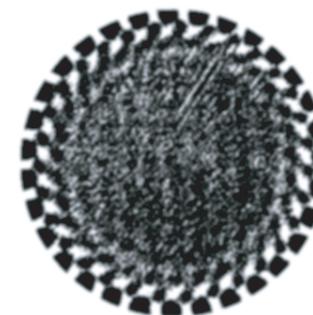
$$\Gamma = 1
m = 6$$



$$\Gamma = 2.5
m = 13$$



$$\Gamma = 5
m = 25$$



Sidewall Mode - Complex Ginzburg-Landau

$$U(x,t) = A(x,t)e^{i(k_c x + \Omega_c t)} + A^*(x,t)e^{-i(k_c x + \Omega_c t)}$$

where k_c is the critical wavenumber, ω_c is the Hopf frequency and $A(x,t)$ varies slowly in space and time. $\varepsilon = Ra/Ra_c - 1$.

$$\tau \partial_t A + s \partial_x A = (1+ic_0)\varepsilon A + (1+ic_2)\xi_0^2 \partial_{xx} A - g(1+ic_3)|A|^2 A + \text{HO Terms}$$

group velocity	linear growth	spatial diffusion frequency dispersion	nonlinear amplitude & frequency
----------------	---------------	--	---------------------------------

$$q = k - k_c \quad \omega = \Omega - \Omega_c \quad A(x,t) = A_0 e^{i(qx + \omega t)}$$

Steady State Solution: Perturbation expansion in small quantities - ε , q , ω

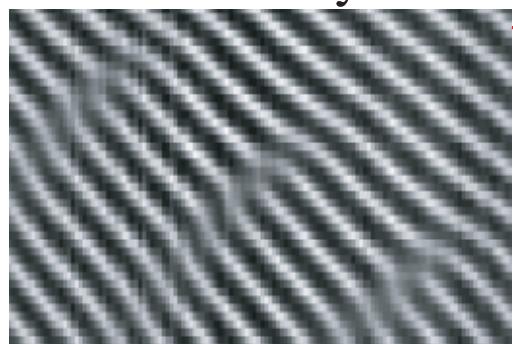
$$\text{Real Part: } \varepsilon - \xi_0^2 q^2 - g|A|^2 = 0 \Rightarrow |A|^2 = g^{-1}(\varepsilon - \xi_0^2 q^2)$$

$$\begin{aligned} \text{Imaginary Part: } \omega - sq &= c_0 \varepsilon - c_2 \xi_0^2 q^2 - g c_3 |A|^2 = (c_0 - c_3) \varepsilon - (c_2 - c_3) \xi_0^2 q^2 \\ \omega &= sq + (c_0 - c_3) \varepsilon - (c_2 - c_3) \xi_0^2 q^2 \end{aligned}$$

Sidewall Mode - Complex Ginzburg-Landau II

mode 31 \rightarrow 25 How?

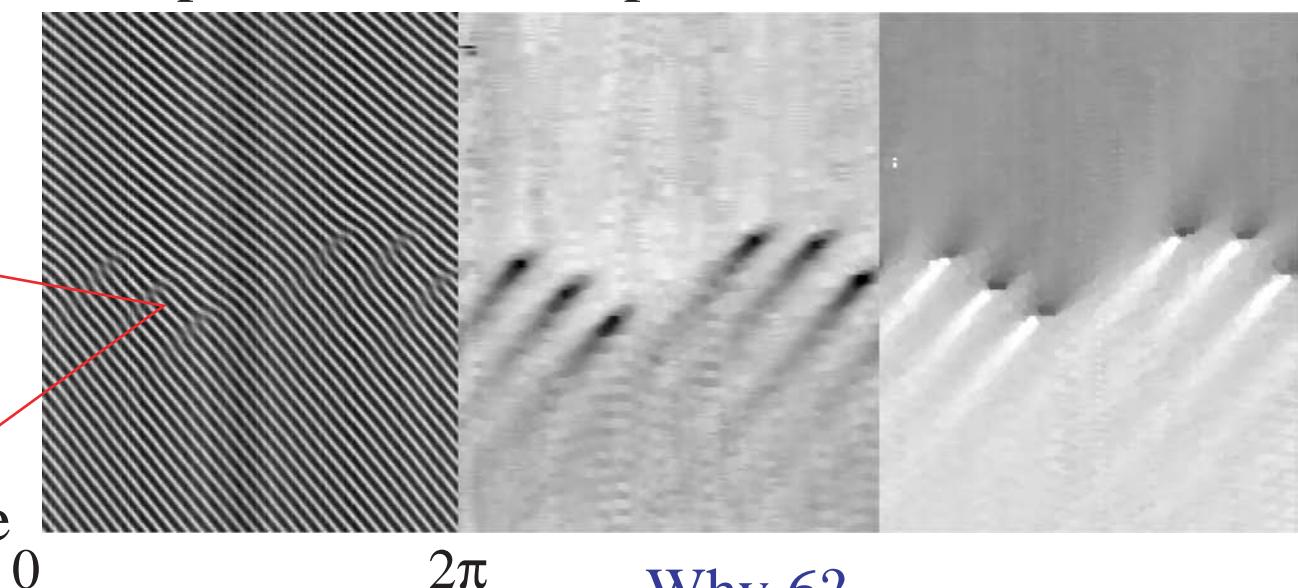
Space-Time Dislocation
Ekhaus/Benjamin-Feir
Instability



pattern

amplitude

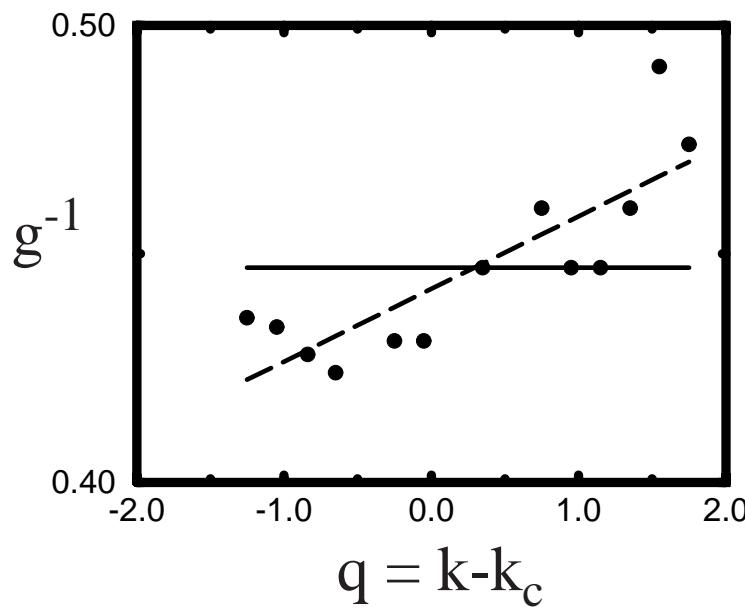
wavenumber



Why 6?

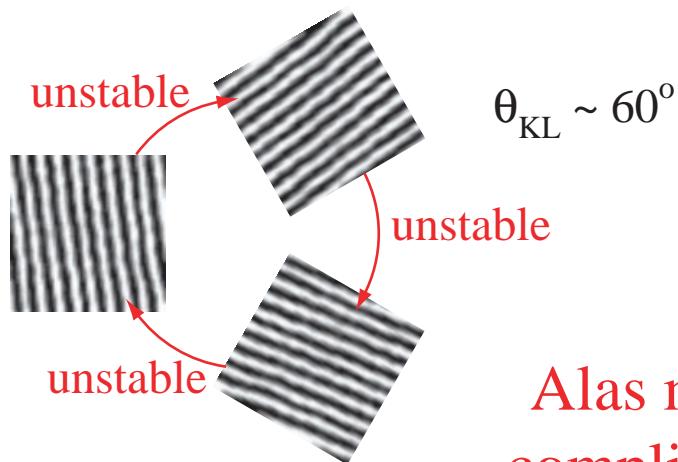
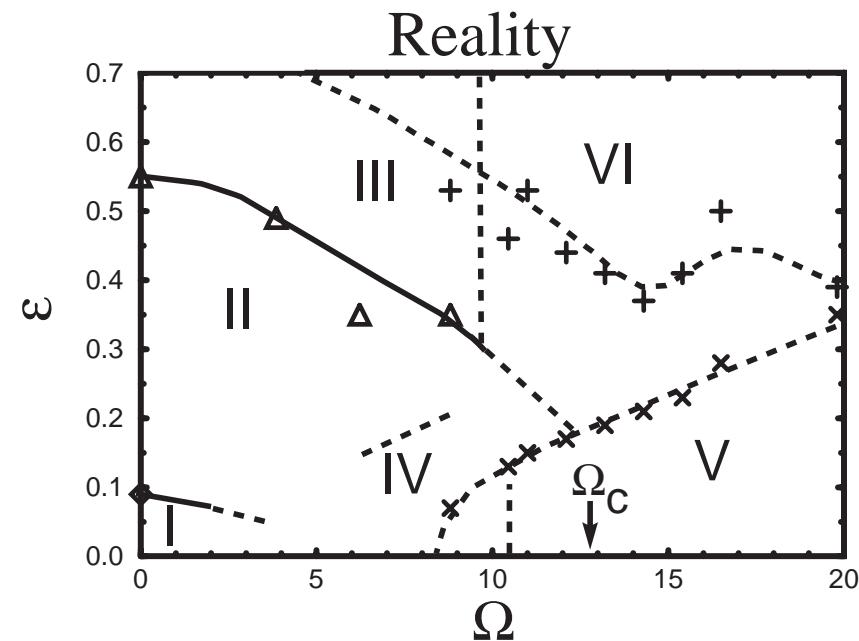
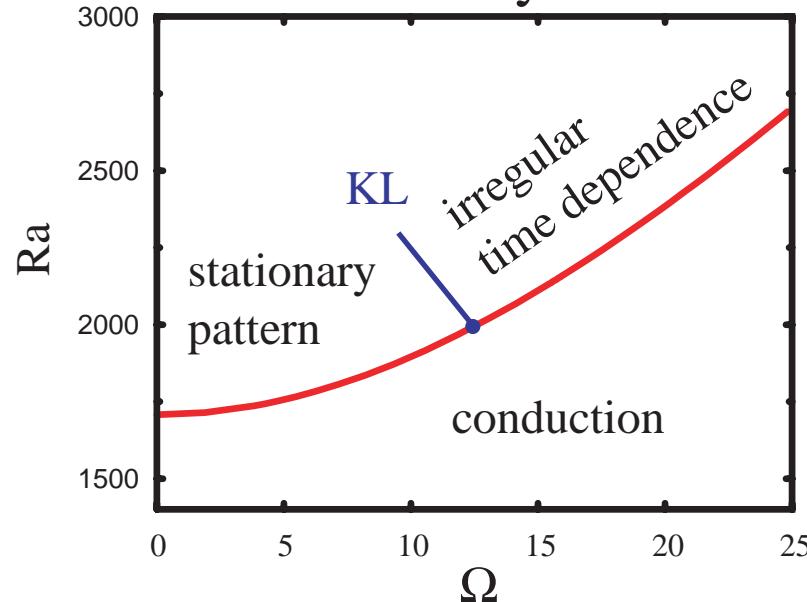
Example of Higher Order Term

CGL
 $|A|^2 \sim g^{-1}\epsilon$
should be
independent
of q



$$\begin{aligned}g^{-1} &= 0.44 + 0.05q \\ \Rightarrow \text{term} &\sim \partial_x A |A|^2 \\ &\sim q\epsilon\end{aligned}$$

Küppers-Lortz Instability Theory



Alas more
complicated
- spatial domains

Küppers-Lortz Instability

Large Ω , $Pr = 6$



Cellular Pattern

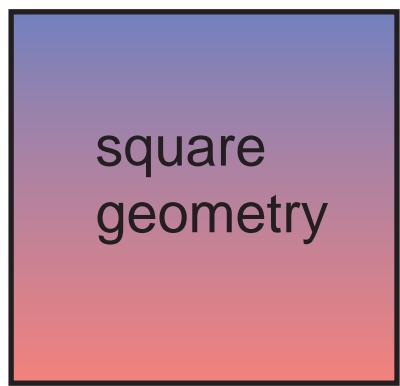
$$\theta_{KL} \sim 90^\circ$$

Square Pattern

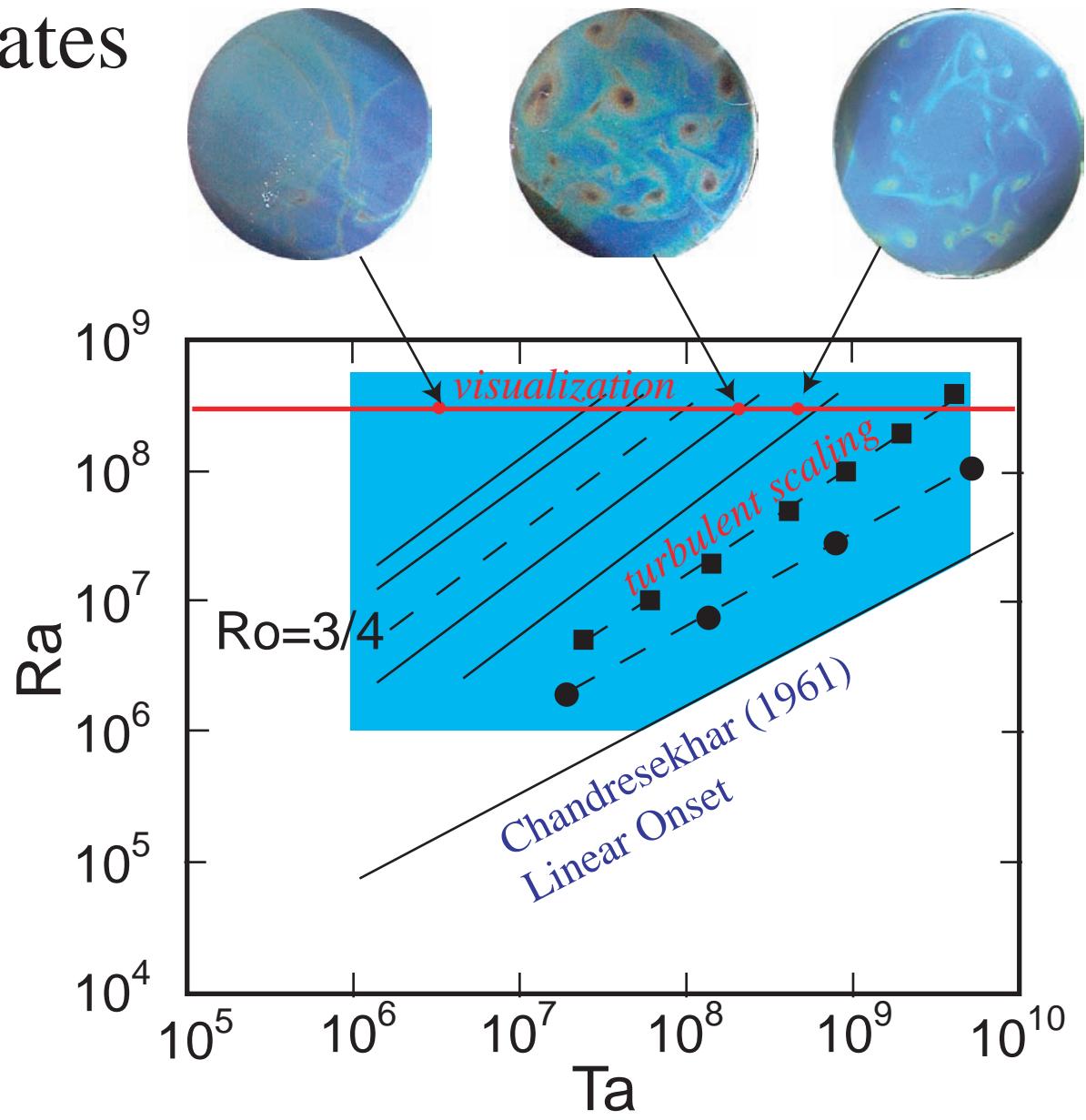
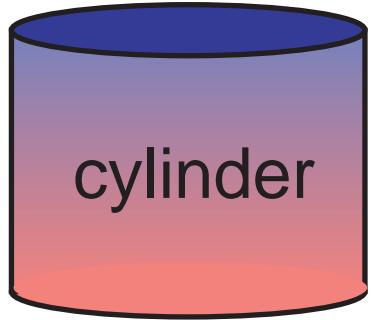
????

Rotating Rayleigh-Benard Convection High Rotation Rates

Heat Transport



Visualization

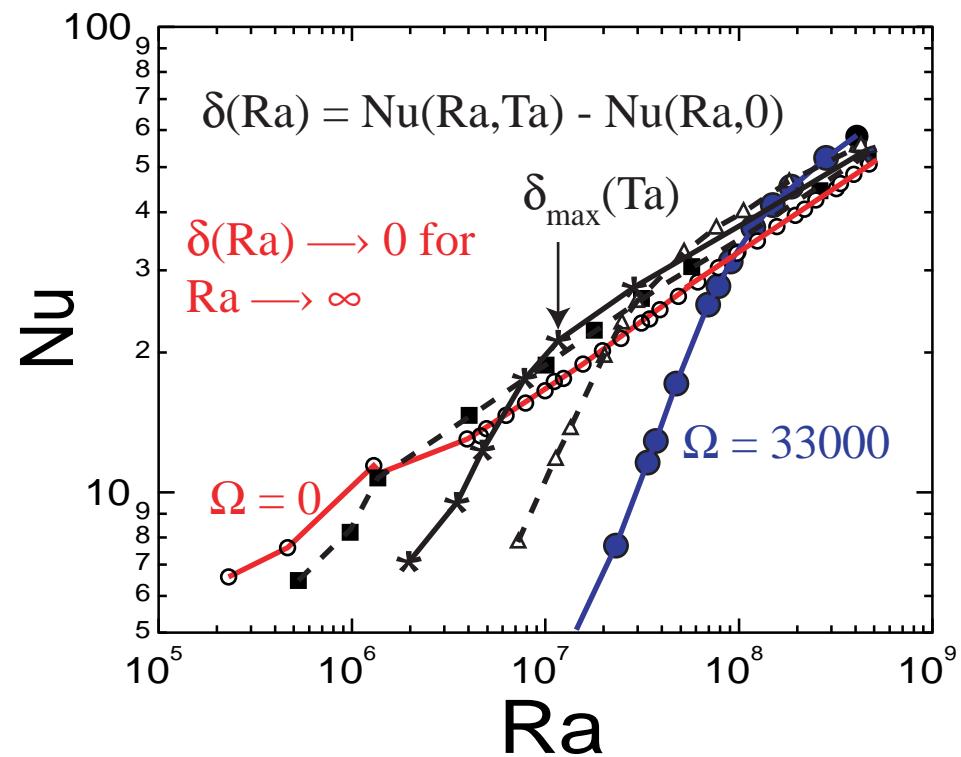
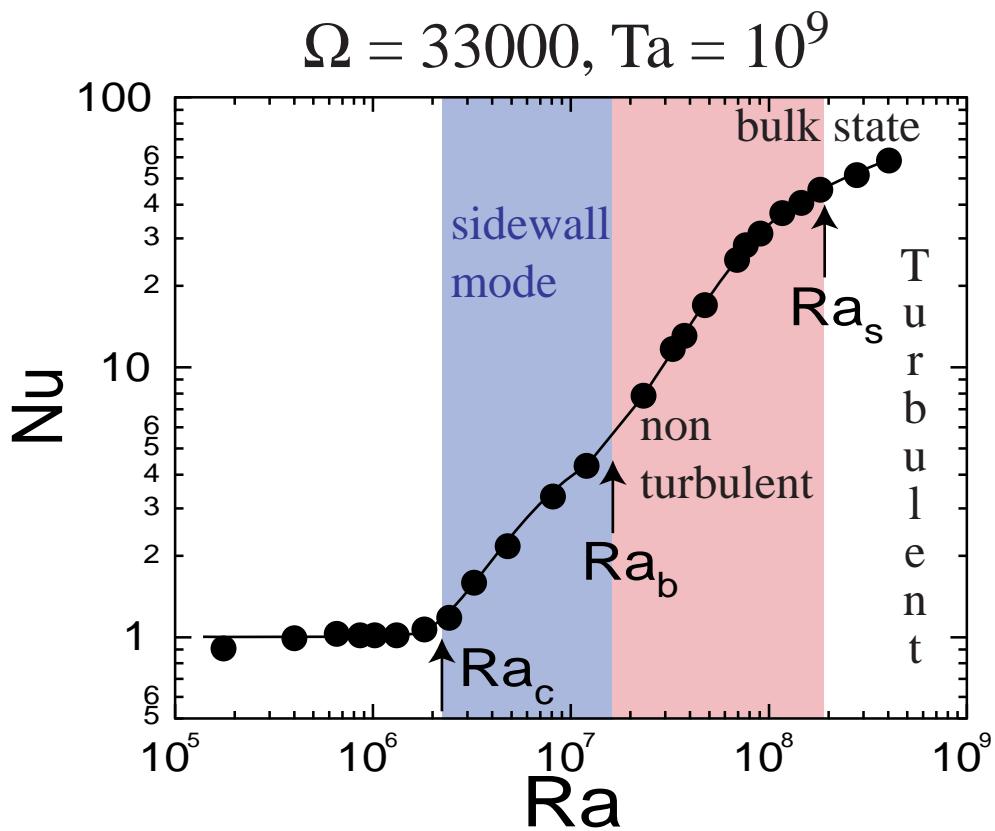


Lui & Ecke, Phys. Rev. Lett **79**, 2257(1997) and unpublished
Vorobieff & Ecke, J. Fluid Mech. **458**, 157 (2002).

Rotating Rayleigh-Benard Convection

Heat Transport at constant Ω

Wall mode → KL bulk state → Turbulent state



Turbulent scaling regime $\sim 10 Ra_{bulk}$

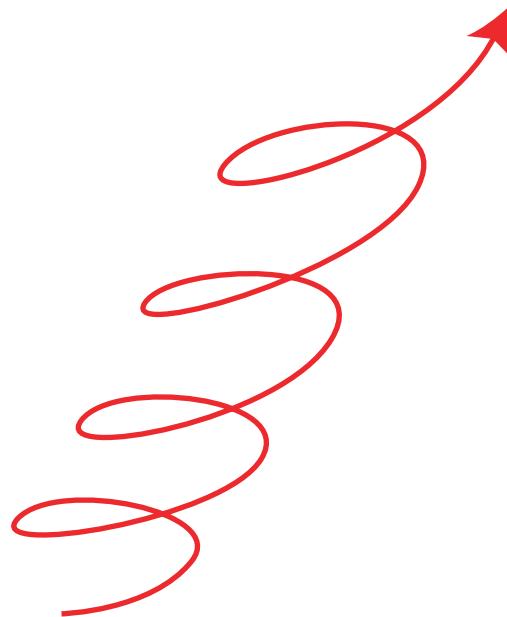
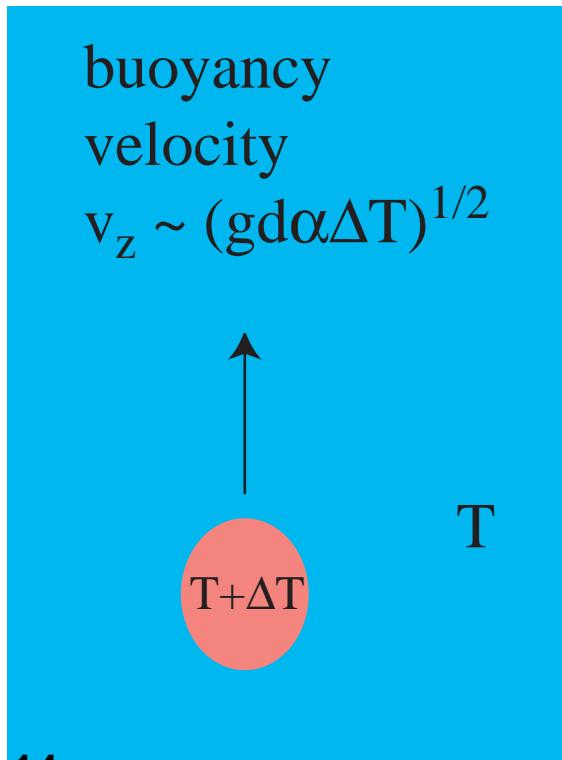
At constant Ω , buoyancy eventually wins out
over rotation so consider constant Ro

Turbulent Heat Transport Scaling Constant Rossby Number

Rossby Number Ro in convection measures
balance between buoyancy and rotation

$$\text{buoyancy } \tau_b \sim (g\alpha\Delta T/d)^{1/2}$$

$$\text{rotation } \tau_r \sim 1/\Omega_D$$

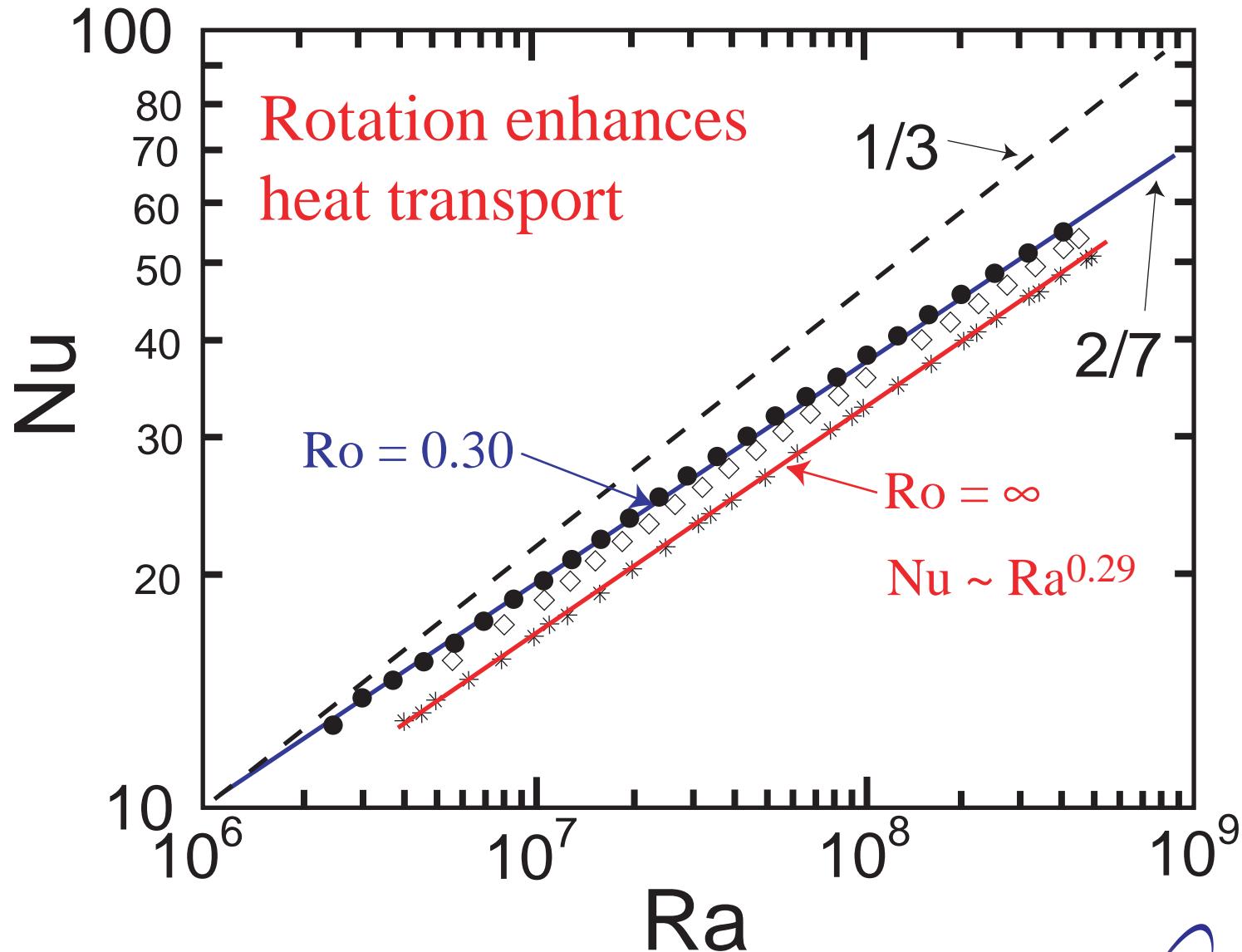


Note: Invisid Quantities

$$\text{Ro} = \tau_b / \tau_r = \sqrt{\frac{\text{Ra}}{\text{PrTa}}}$$

Turbulent Heat Transport Scaling

Constant Ro

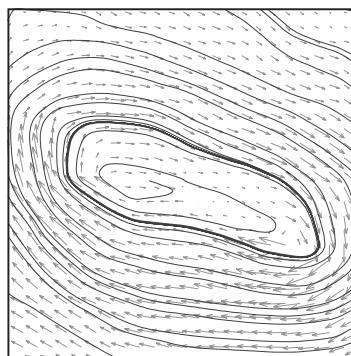


Why does rotation enhance heat transport?

Ekman pumping/suction in the boundary layer

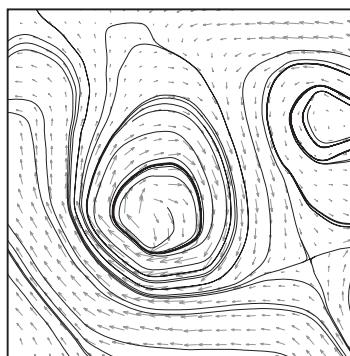
$$\Omega = 5000$$

$$Ro = 0.77$$



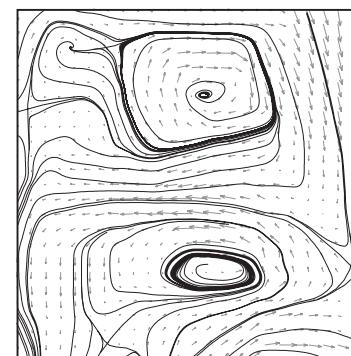
$$\Omega = 19000$$

$$Ro = 0.19$$



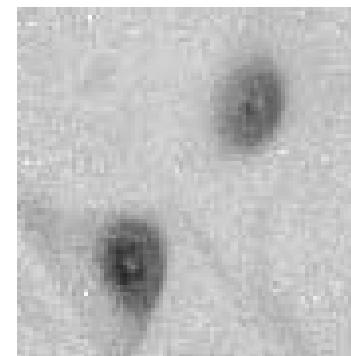
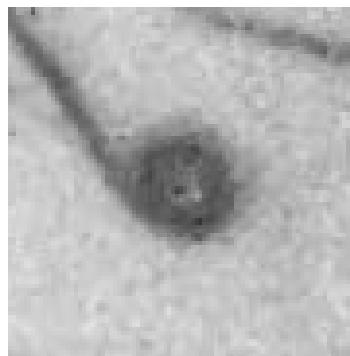
$$\Omega = 48000$$

$$Ro = 0.08$$

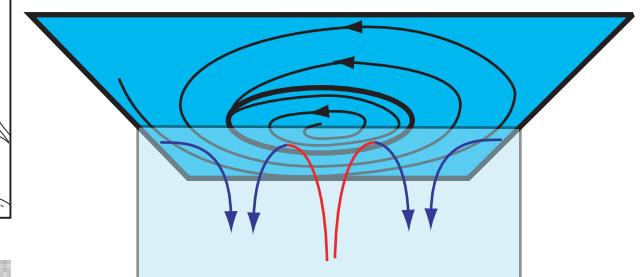


velocity
stream-
lines

temp



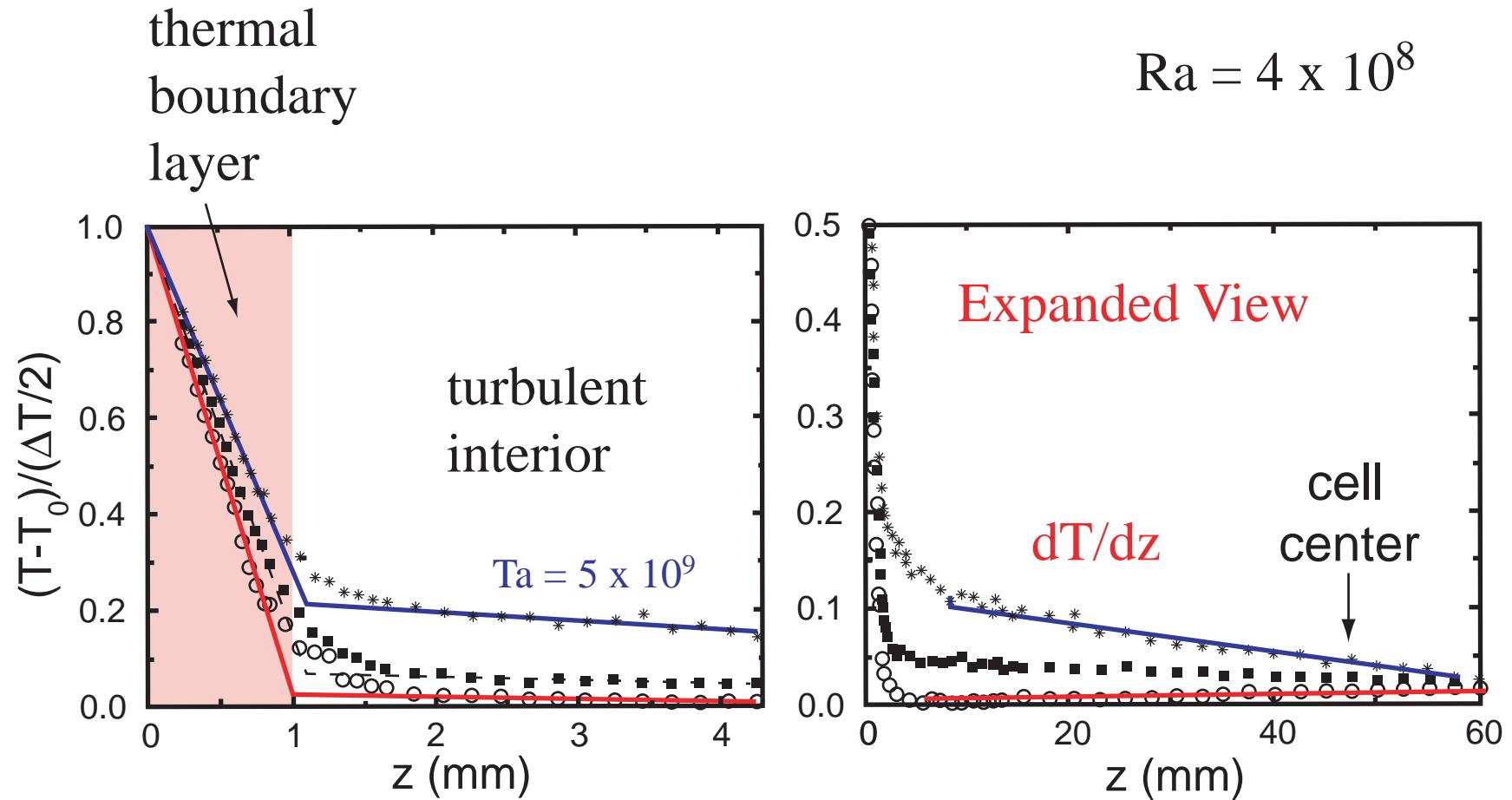
suction in core
pumping along ring



JLMW
Vortex structure

$$Ra = 3 \times 10^8$$

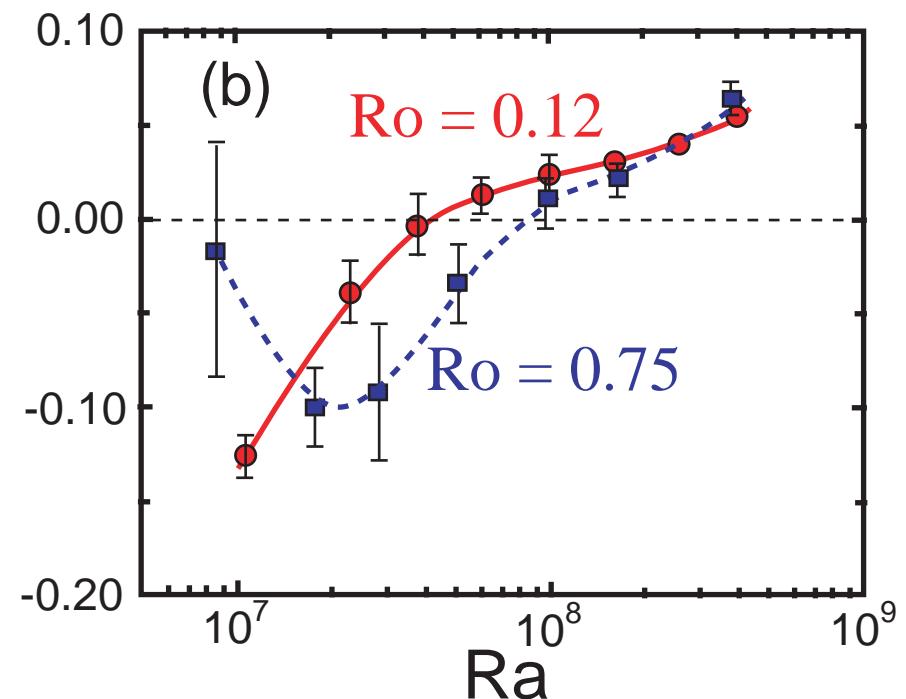
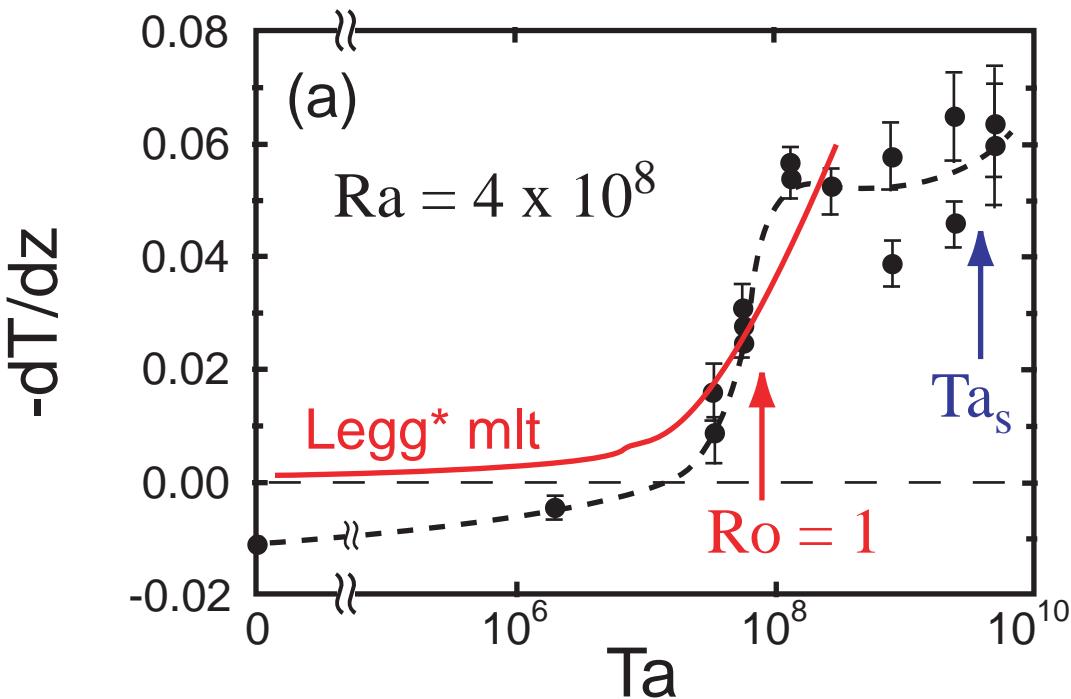
Vertical Temperature Profile



Interior vertically isothermal without rotation
but significant vertical temperature gradient with rotation

How does this depend on Ra and Ta?

Vertical Temperature Profile Dependence on Ra & Ta



As Ta increases at fixed Ra ,
 Ro decreases and $-dT/dz$
becomes positive.

$$\text{Legg* mlt: } -dT/dz \sim \Delta T/d \text{Ro}^{-1} \sim Ta^{1/2}$$

Temperature Statistics near Top

$\text{Ra} = 4 \times 10^8$

$\delta_T \sim 1\text{mm}$

$z \sim \delta_T/2$

Symmetric
hot & cold
fluctuations

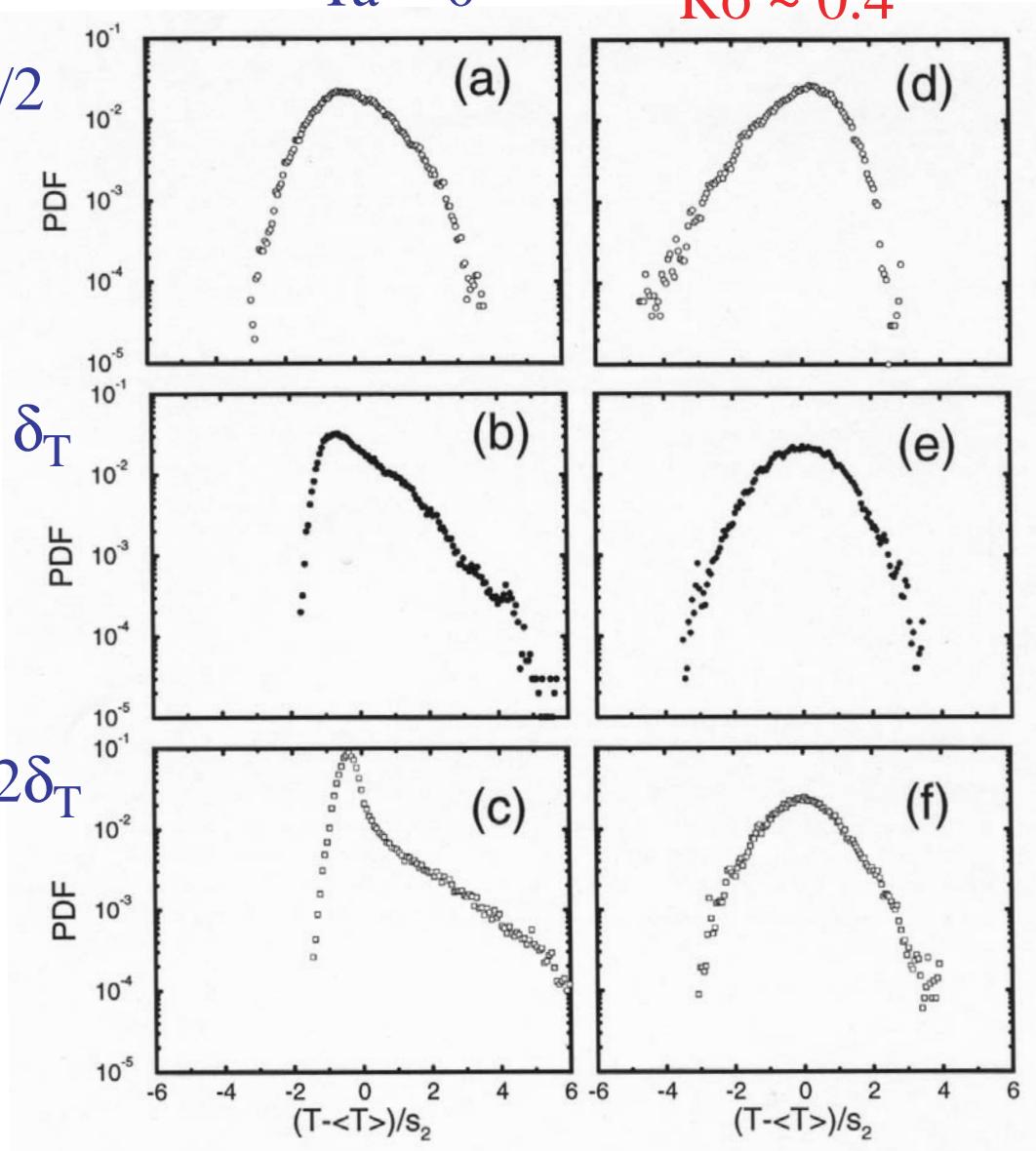
Lots of
hot plumes

$z \sim 2\delta_T$

Still many
hot plumes

$\text{Ta} = 0$

$\text{Ta} = 5 \times 10^8$
 $\text{Ro} \sim 0.4$

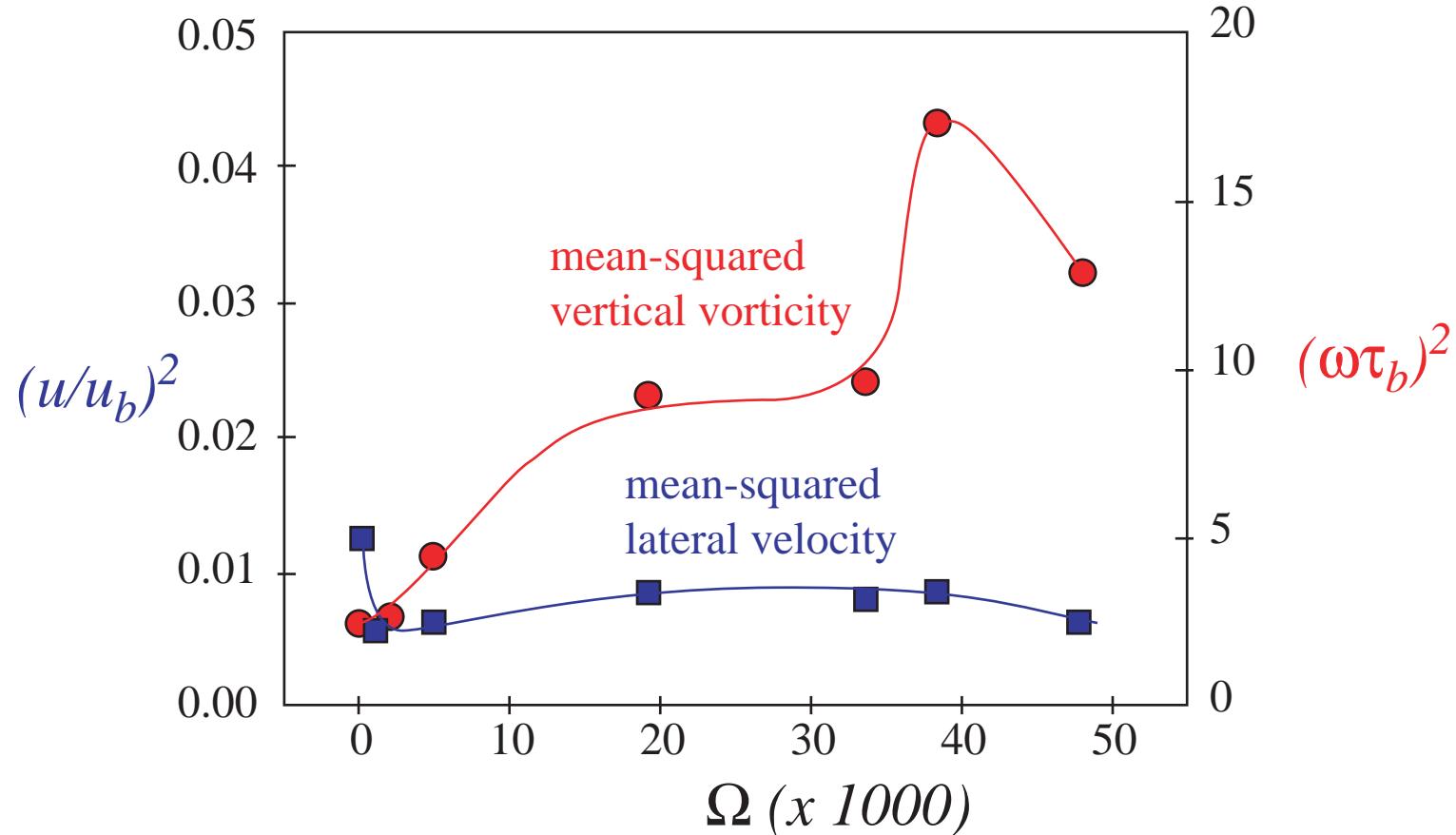


More cold
fluctuations
Ekman suction

Symmetric
rapid mixing

Very
symmetric

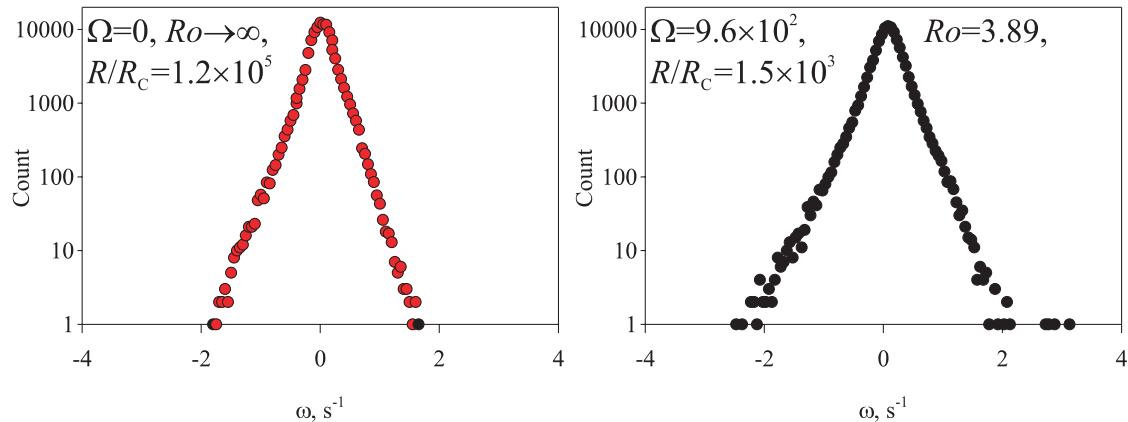
Velocity & Vertical Vorticity Near the Top Surface



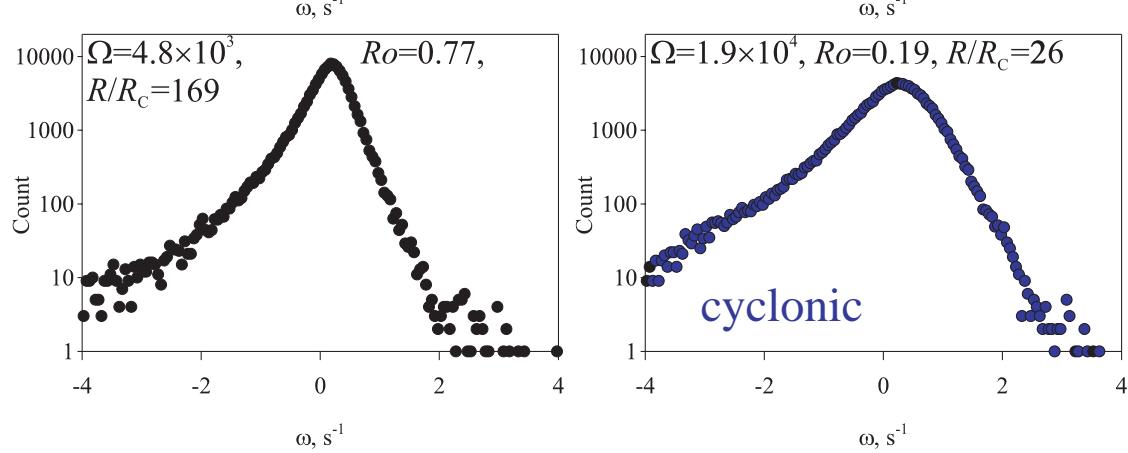
Injected kinetic energy pretty constant (neglect vert. velocity)

Injected vorticity rises rapidly with Ω

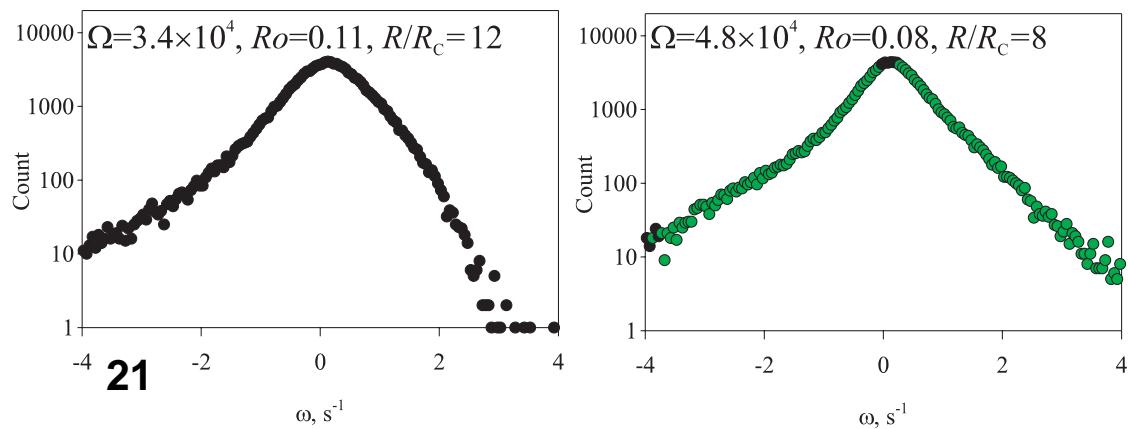
Vertical Vorticity Probability Distribution Functions (Top Surface)



Symmetric



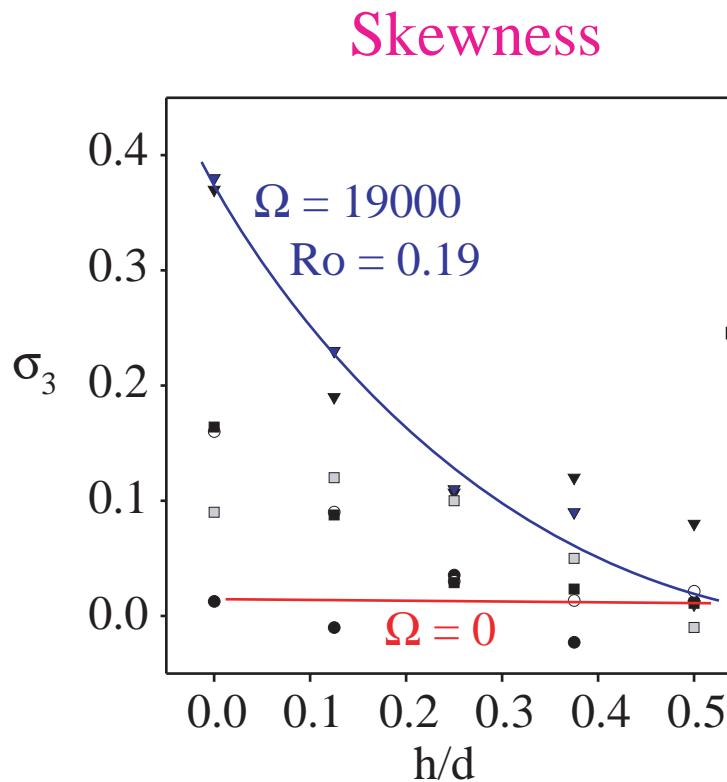
Strong Cyclonic
Tails



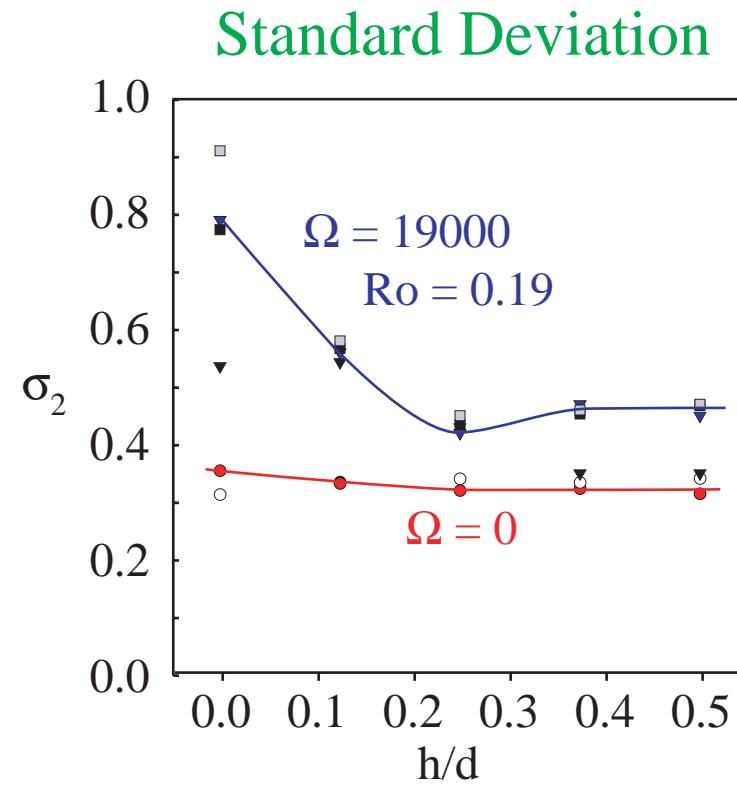
Symmetric
- near onset

Vertical Vorticity Statistics

Variation with Depth



Skewness shows strong cyclonic asymmetry that decays rapidly with depth



Vorticity fluctuations are larger with rotation even in the mid plane

Summary

Linear stability in rotating convection -
excellent agreement with experiment

1D sidewall traveling wave & complex
Ginzburg Landau amplitude equation

Complex pattern dynamics in rotating
convection with Kuppers-Lortz instability

Characterization of heat transport, temperature,
velocity and vorticity in rotating turbulent
convection.