Matter-Wave Turbulence
Beyond kinetic scaling

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Center for Quantum Dynamics

EMMI

Thanks & credits to...


...my work group in Heidelberg:

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Jan Schole
Dénes Sexty

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Martin Gärttner
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Philipp Struck (→ Konstanz)
Kristan Temme (→ Vienna)

...my collaborators

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€€€...

Heisenberg-Programm
Deutsche Forschungsgemeinschaft
DFG

RUPRECHT-KARLS-UNIVERSITÄT HEIDELBERG

LGFG BaWue

DAAD Deutscher Akademischer Austausch Dienst
German Academic Exchange Service

Center for Quantum Dynamics

EMMI

HELMHOLTZ GEMEINSCHAFT

ULTRAcool...

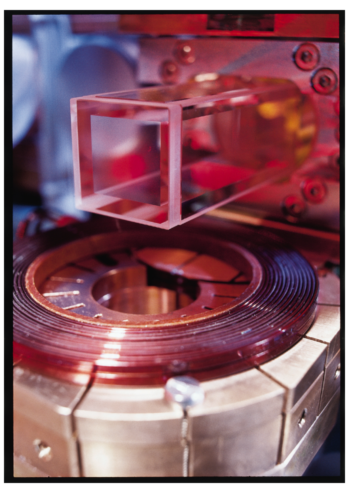
... atoms @ nanokelvins –

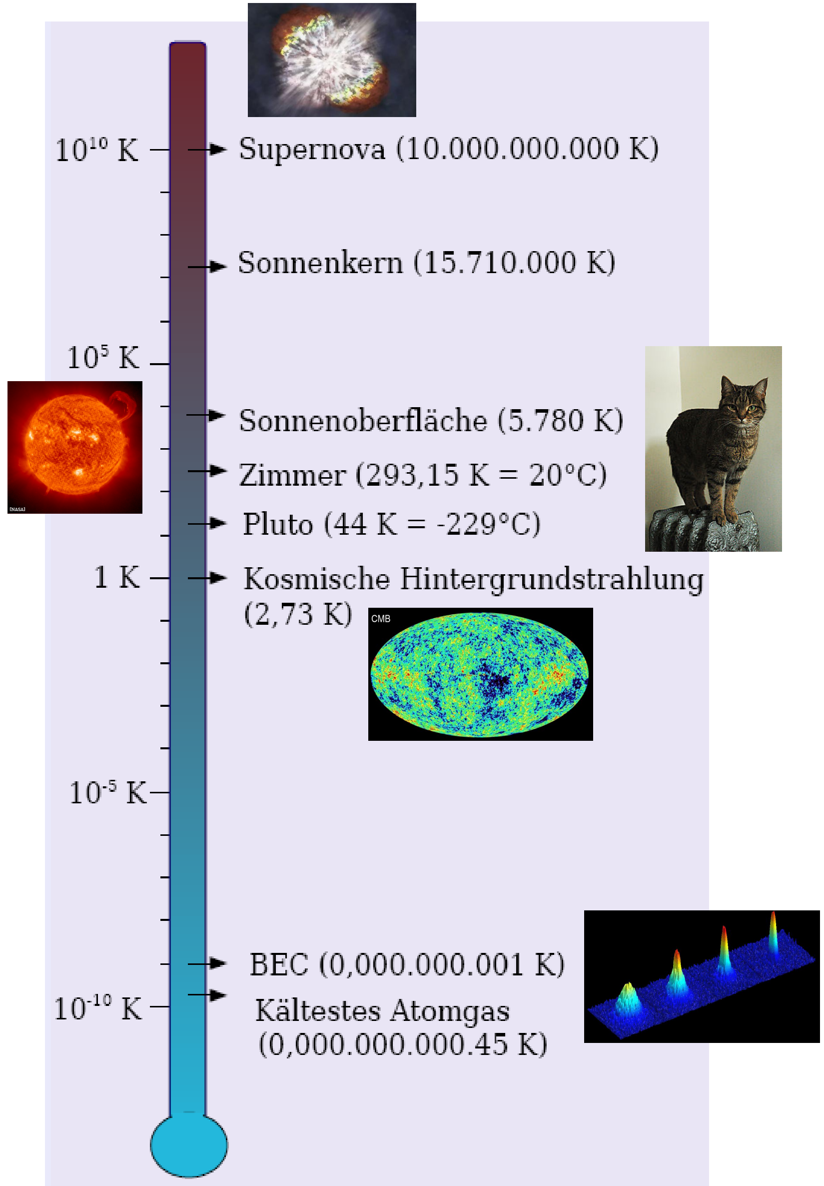
trapped only a few mm away from

glass cell @ room temperature

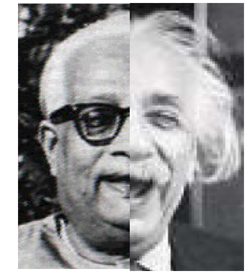
(vacuum of 10^{-12} Torr,
i.e. 10^{-15} bar,
or 10^{-10} Pa,

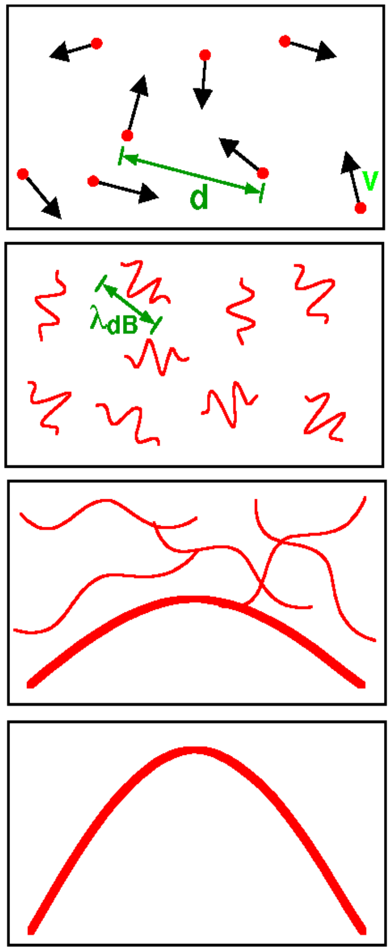
≈ atmospheric
pressure on the moon)





Bose-Einstein condensation





High Temperature T:
thermal velocity v
density d^{-3}
"Billiard balls"

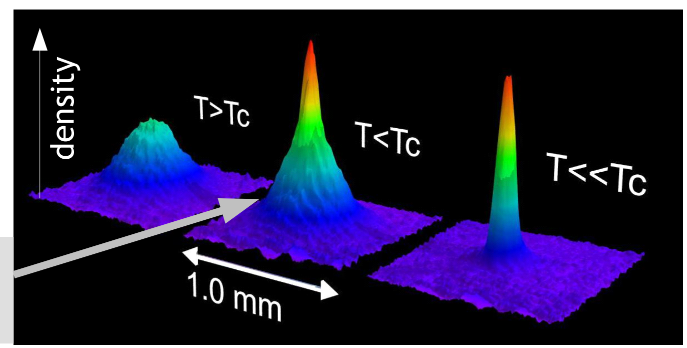
Low Temperature T:
De Broglie wavelength
$\lambda_{dB}=h/mv \propto T^{-1/2}$
"Wave packets"

$T=T_{crit}$: Bose-Einstein Condensation
$\lambda_{dB} \approx d$
"Matter wave overlap"

T=0: Pure Bose condensate
"Giant matter wave"

Experimental picture after free expansion of the trapped cloud:

Bose-Einstein condensation (BEC)



bimodal distribution

Vortices in a Na condensate

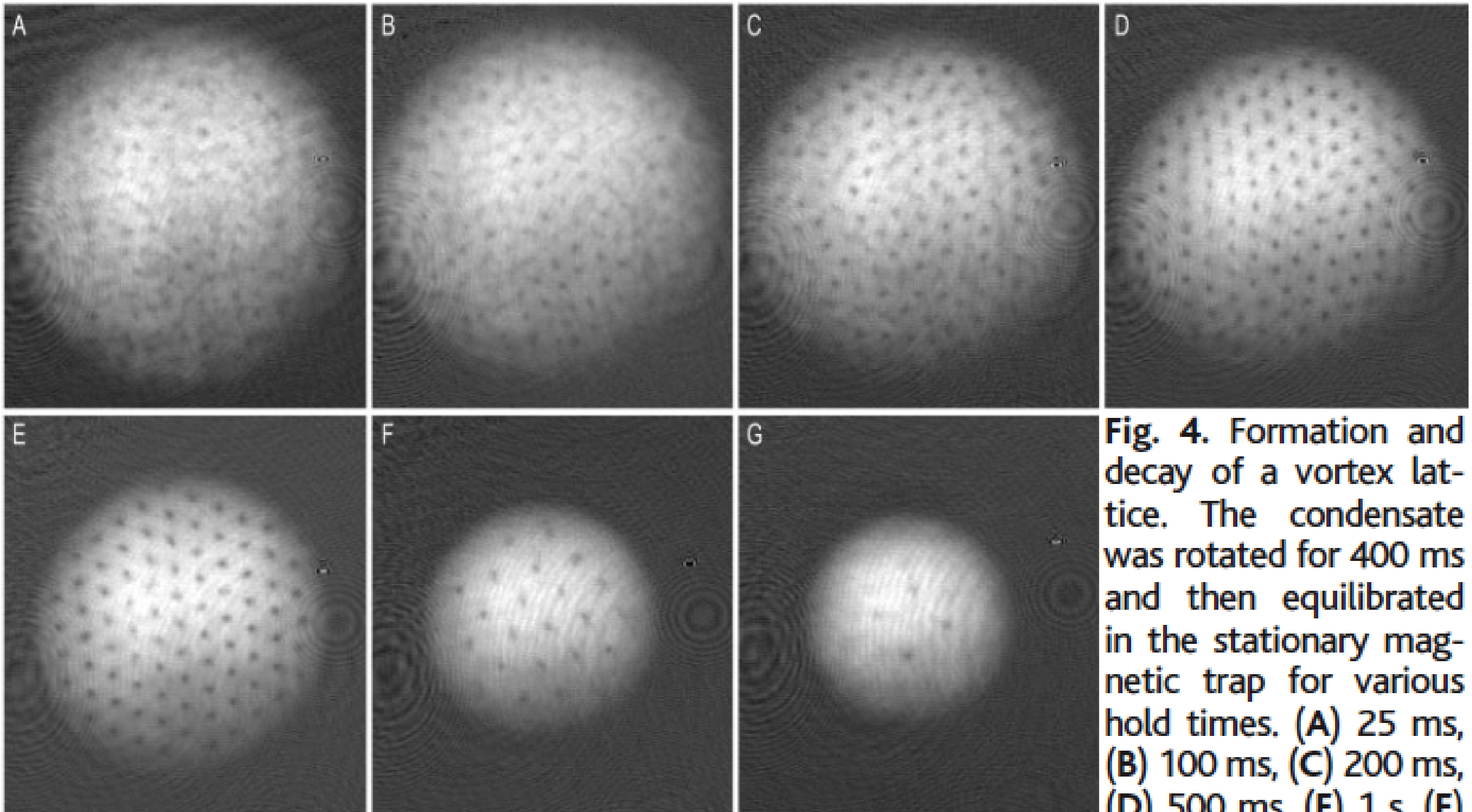
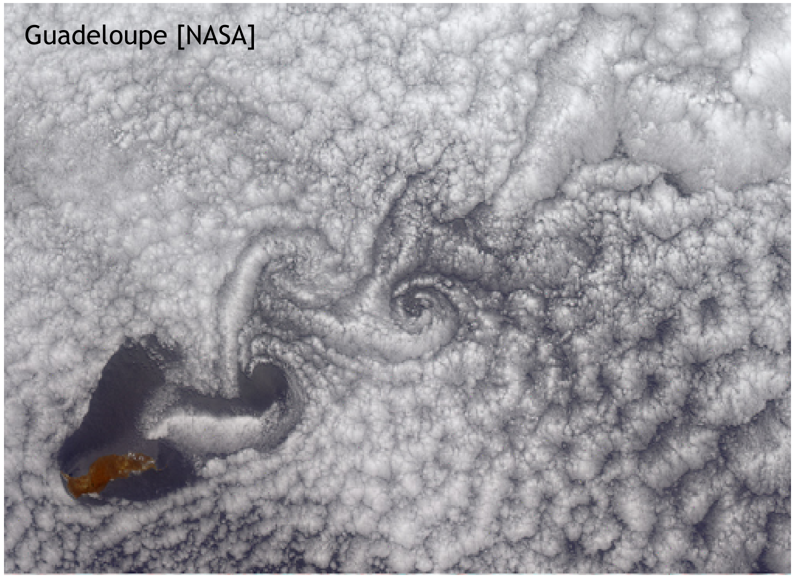


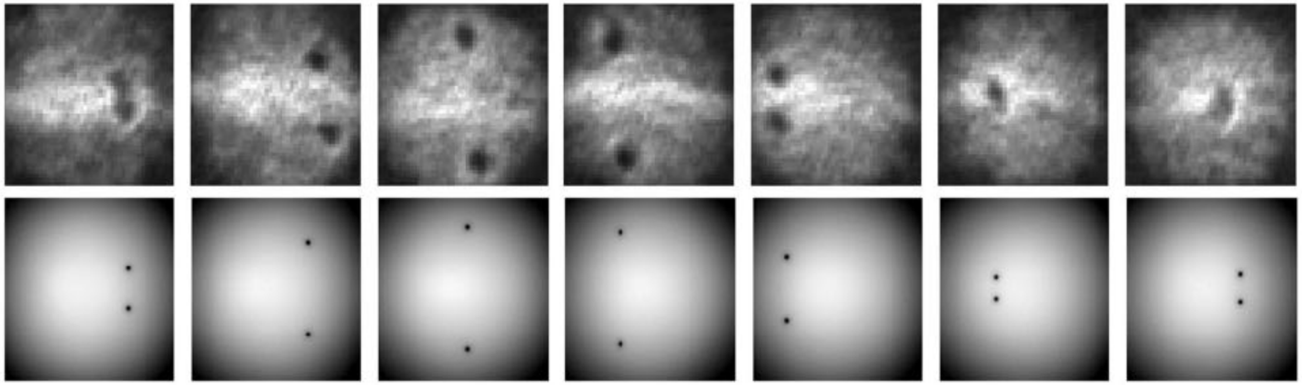
Fig. 4. Formation and decay of a vortex lattice. The condensate was rotated for 400 ms and then equilibrated in the stationary magnetic trap for various hold times. (**A**) 25 ms, (**B**) 100 ms, (**C**) 200 ms, (**D**) 500 ms, (**E**) 1 s, (**F**) 5 s, (**G**) 10 s,

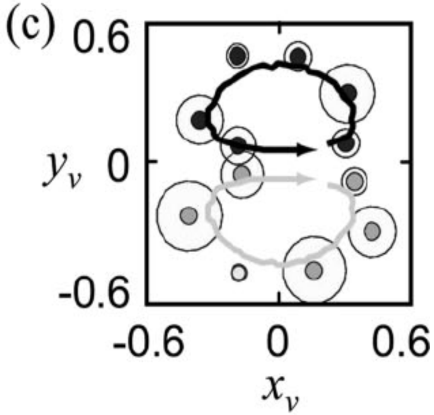
J. R. Abo-Shaeer, C. Raman, J. M. Vogels, W. Ketterle
20 APRIL 2001 VOL 292 SCIENCE

Vortex pairs



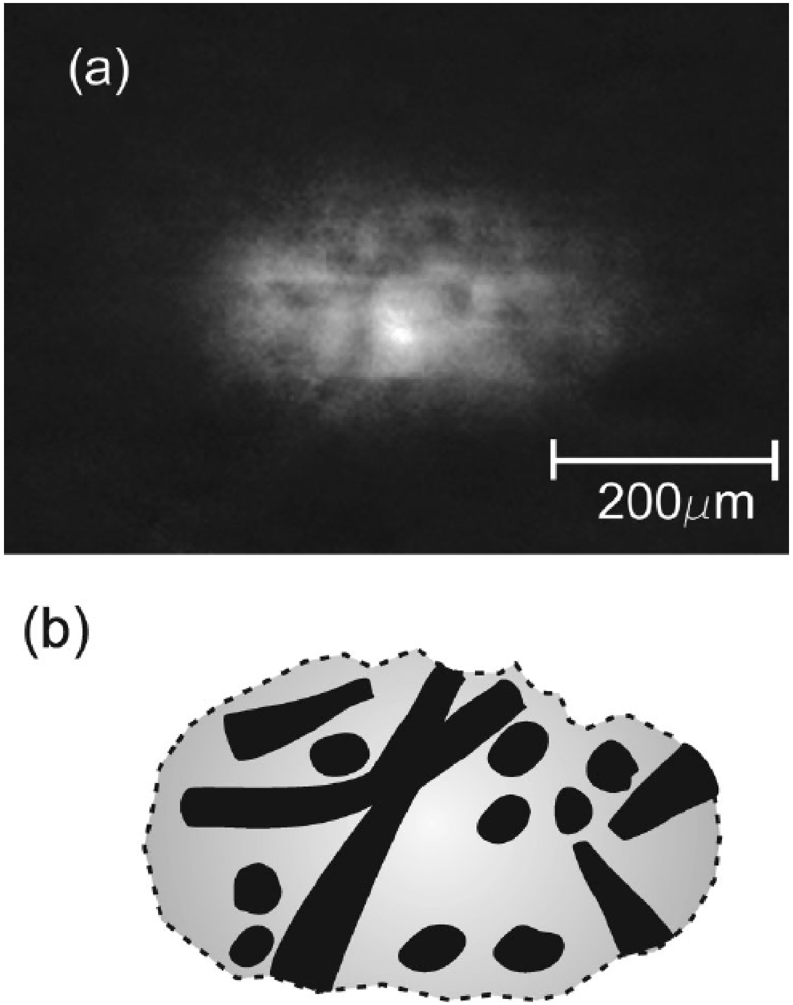
Tucson [AZ]



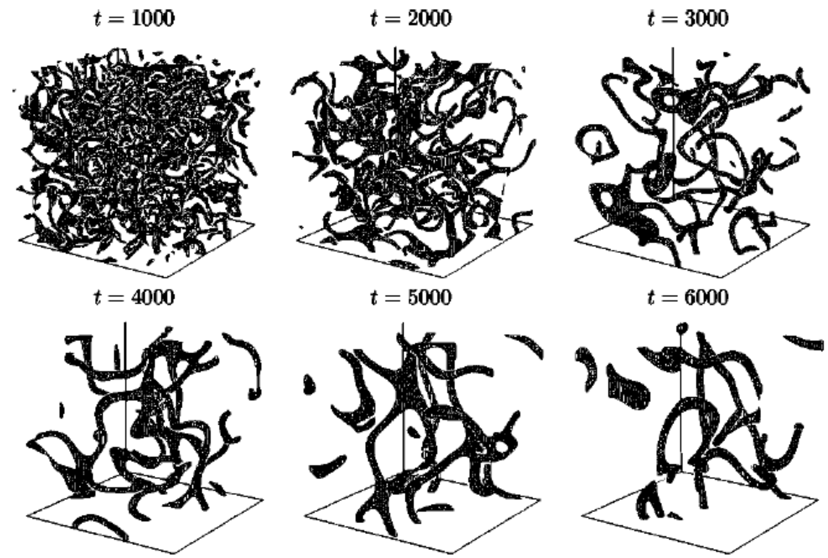


[T.W. Neely et al. PRL **104** (10)]

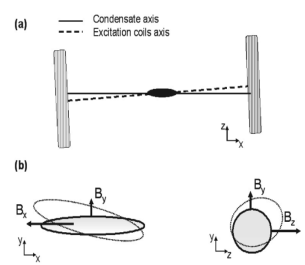
Vortex tangles in Bose Einstein Condensates



[E.A.L. Henn et al. PRL **103** (09)]

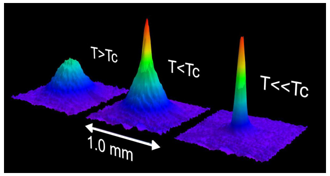


[N. Berloff & B. Svistunov, PRA (02)]



Quantum field theory of a Bose-Einstein gas

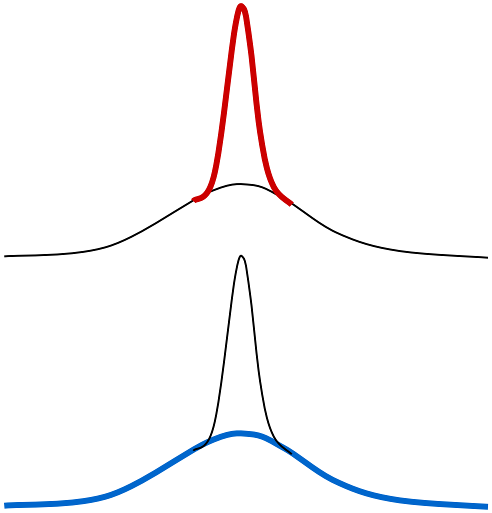
How to describe a condensate?



For **bosons**: $[\hat{\Phi}_{t,\mathrm{x}}, \hat{\Phi}^\dagger_{t,\mathrm{x}'}] = \delta(\mathrm{x}-\mathrm{x}')$

- Matter wave **mean field** $[x = (x_0, \mathrm{x}) = (t, \mathrm{x})]$

$$\phi_x = \langle \hat{\Phi}_x \rangle, \qquad |\phi_x|^2 = n_c(x) = \text{condensate density},$$



- Density of **non-condensed atoms** $(\hat{\Phi} = \phi + \widetilde{\Phi},\ \phi = \langle \hat{\Phi} \rangle)$

$$\langle \widetilde{\Phi}^\dagger_x \widetilde{\Phi}_x \rangle = n_{\mathrm{nc}}(x) \equiv n(x) - n_c(x),$$

How to describe a condensate?

For **bosons**: $[\hat{\Phi}_{t,\mathrm{x}}, \hat{\Phi}^\dagger_{t,\mathrm{x}'}] = \delta(\mathrm{x}-\mathrm{x}')$

- Matter wave **mean field** $[x = (x_0, \mathrm{x}) = (t, \mathrm{x})]$

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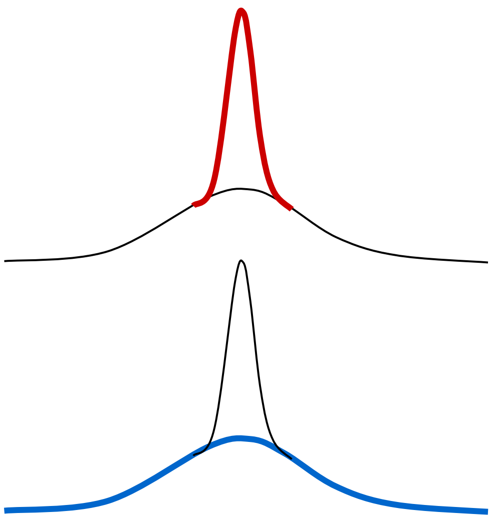

- Density of **non-condensed atoms** $(\hat{\Phi} = \phi + \widetilde{\Phi},\ \phi = \langle \hat{\Phi} \rangle)$

$$\langle \widetilde{\Phi}^\dagger_x \widetilde{\Phi}_x \rangle = n_{\mathrm{nc}}(x) \equiv n(x) - n_c(x),$$

- Total one-body **density matrix**

$$G_{11}(x,y) = \langle \widetilde{\Phi}^\dagger_x \widetilde{\Phi}_y \rangle \Rightarrow \text{spatial Fourier transform: momentum distribution } n(\mathrm{p}, t)$$

$\Rightarrow$ 1^{st}-order phase coherence $n^{(1)}(s) = n(\mathbf{R}+\mathbf{s}/2, \mathbf{R}-\mathbf{s}/2)$

($n(\mathbf{x}, \mathbf{y}; t) = \langle \Phi^\dagger(\mathbf{x}, t) \Phi(\mathbf{y}, t) \rangle$)

Gross-Pitaevskii dynamics

Many-body Hamiltonian:

$$H = \int d\mathrm{x}\, \hat{\Phi}^\dagger_\mathrm{x} \left[-\frac{\hbar^2}{2m}\Delta + V_\mathrm{ext}(\mathrm{x}) \right] \hat{\Phi}_\mathrm{x} + \frac{1}{2}\int d\mathrm{x}\, d\mathrm{y}\ \hat{\Phi}^\dagger_\mathrm{x} \hat{\Phi}^\dagger_\mathrm{y}\, V_{\mathrm{x}-\mathrm{y}}\, \hat{\Phi}_\mathrm{y} \hat{\Phi}_\mathrm{x}$$

Gross-Pitaevskii dynamics

Many-body Hamiltonian:

$$H = \int dx\, \hat{\Phi}^\dagger_x \left[-\frac{\hbar^2}{2m}\Delta + V_{\rm ext}(x) \right] \hat{\Phi}_x + \frac{1}{2}\int dx\, dy\, \hat{\Phi}^\dagger_x \hat{\Phi}^\dagger_y\, V_{x-y}\, \hat{\Phi}_y \hat{\Phi}_x$$

Gross-Pitaevskii, i.e. Classical Field Equation for “matter waves”, from vNE:

$$i\partial_t \langle \mathcal{O} \rangle_t = \langle [\mathcal{O}, H] \rangle_t$$

$$\Rightarrow \quad i\hbar\partial_t \phi_x = \left[-\frac{\hbar^2}{2m}\Delta + V_{\rm ext}(x) + \overbrace{g|\phi_x|^2}^{\text{from } \int V \langle \hat{\Phi}^\dagger \hat{\Phi} \hat{\Phi} \rangle} \right] \phi_x,$$

$$[\hat{\Phi}_{t,x}, \hat{\Phi}^\dagger_{t,x'}] = \delta(x - x')$$

Gross-Pitaevskii dynamics

Many-body Hamiltonian:

$$H = \int d\mathrm{x}\, \hat{\Phi}^\dagger_\mathrm{x} \left[-\frac{\hbar^2}{2m}\Delta + V_\mathrm{ext}(\mathrm{x}) \right] \hat{\Phi}_\mathrm{x} + \frac{1}{2} \int d\mathrm{x}\, d\mathrm{y}\, \hat{\Phi}^\dagger_\mathrm{x} \hat{\Phi}^\dagger_\mathrm{y}\, V_{\mathrm{x}-\mathrm{y}}\, \hat{\Phi}_\mathrm{y} \hat{\Phi}_\mathrm{x}$$

Gross-Pitaevskii, i.e. Classical Field Equation for “matter waves”, from vNE:

$$i\partial_t \langle \mathcal{O} \rangle_t = \langle [\mathcal{O}, H] \rangle_t$$

from $\int V \langle \hat{\Phi}^\dagger \hat{\Phi} \hat{\Phi} \rangle$

$$\Rightarrow \quad i\hbar\partial_t \phi_x = \left[-\frac{\hbar^2}{2m}\Delta + V_\mathrm{ext}(\mathrm{x}) + g|\phi_x|^2 \right] \phi_x ,$$

$$g = \frac{4\pi\hbar^2 a}{m}$$

typical scattering length: $a \simeq 5\ \mathrm{nm}$

typical bulk density: $n \simeq 10^{14}\ \mathrm{cm}^{-3}$

$\Rightarrow$ diluteness parameter: $na^3 \simeq 10^{-5}$

(GPE valid for $\sqrt{na^3} \ll 1$

$\Leftrightarrow$ small condensate depletion)

Beyond Gross-Pitaevskii

$$H = \int dx\, \hat{\Phi}^\dagger_x \left[-\frac{\hbar^2}{2m}\Delta + V_{ext}(x) \right] \hat{\Phi}_x + \frac{1}{2} \int dx\, dy\, \hat{\Phi}^\dagger_x \hat{\Phi}^\dagger_y V_{x-y} \hat{\Phi}_y \hat{\Phi}_x$$

Hartree-Fock-Bogoliubov (HFB) from $i\partial_t \langle \mathcal{O} \rangle_t = \langle [\mathcal{O}, H] \rangle_t$ and factorisation:

$$\langle \hat{\Phi}^\dagger_x \hat{\Phi}^\dagger_y \hat{\Phi}_z \hat{\Phi}_u \rangle_t \to \langle \hat{\Phi}^\dagger_x \hat{\Phi}^\dagger_y \rangle_t \langle \hat{\Phi}_z \hat{\Phi}_u \rangle_t + \langle \hat{\Phi}^\dagger_x \hat{\Phi}_z \rangle_t \langle \hat{\Phi}^\dagger_y \hat{\Phi}_u \rangle_t + \langle \hat{\Phi}^\dagger_x \hat{\Phi}_u \rangle_t \langle \hat{\Phi}^\dagger_y \hat{\Phi}_z \rangle_t$$

analogously for 3rd-order correlation function.

Beyond mean field

Dynamical Field Theory

$$\left[\Phi(t,\mathbf{x}),\Phi^{\dagger}(t,\mathbf{y})\right] = \delta(\mathbf{x}-\mathbf{y}) \qquad \text{(Bose)}$$

Our focus:

time dependence of the lowest-order “correlation” functions:

$$G_{ab}(x,y) = \langle \mathcal{T}\Phi_a(x)\Phi_b(y)\rangle \qquad x = (t,\mathbf{x})$$

Dynamical field theory

Alternative to BBGKY hierarchy:
Schwinger-Dyson...

$$G^{-1}_{ab}(x,y) \;=\; G^{-1}_{0,ab}(x,y;\phi) - \Sigma_{ab}(x,y;\phi,G)$$

or Kadanoff-Baym eqs.:

$$\int_z G^{-1}_{0,ac}(x,z)G_{cb}(z,y) = \delta_{ab}\delta_{\mathcal{C}}(x-y) + \int_z \Sigma_{ac}(x,z)\; G_{cb}(z,y)$$

Dynamical field theory

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with classical propagator defined by action

$$S[\varphi] = \frac{1}{2}\int_{xy} \varphi_a(x) iG^{-1}_{0,ab}(x,y)\varphi_b(y) \;-\frac{g}{8}\int_x \varphi_a(x)\varphi_a(x)\varphi_b(x)\varphi_b(x)$$

$$= \delta(x-y)[-i\sigma^2_{ab}\partial_{x_0} - H_{1\mathrm{B}}(x)\,\delta_{ab}]$$

Dynamical field theory

Alternative to BBGKY hierarchy:
Schwinger-Dyson...

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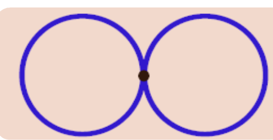
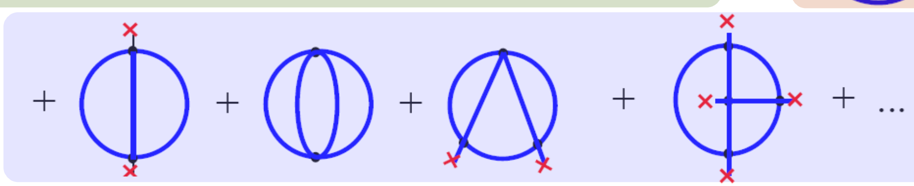
From two-particle irreducible effective action: [Luttinger, Ward (60); Baym (62); Cornwall, Jackiw, Tomboulis (74)]

$$\Gamma[\phi,G] = S[\phi] + \tfrac{i}{2}\mathrm{Tr}(\ln G^{-1} + G_0^{-1}(\phi)G) +$$





Dynamical field theory

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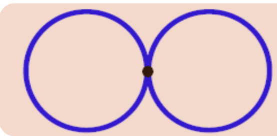
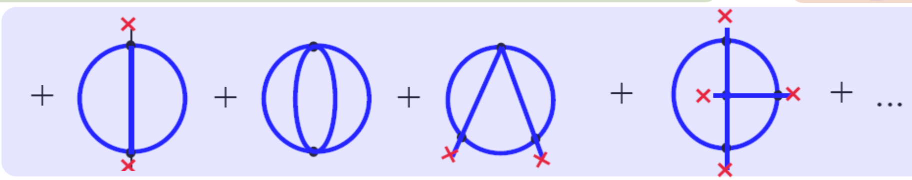




$$\frac{\delta\Gamma[\phi, G]}{\delta G(x,y)} = 0 \qquad\qquad \Sigma_{ab}(x,y;\phi,G) = 2i\,\frac{\delta\Gamma_2[\phi,G]}{\delta G_{ab}(x,y)}$$

Scaling from field dynamics

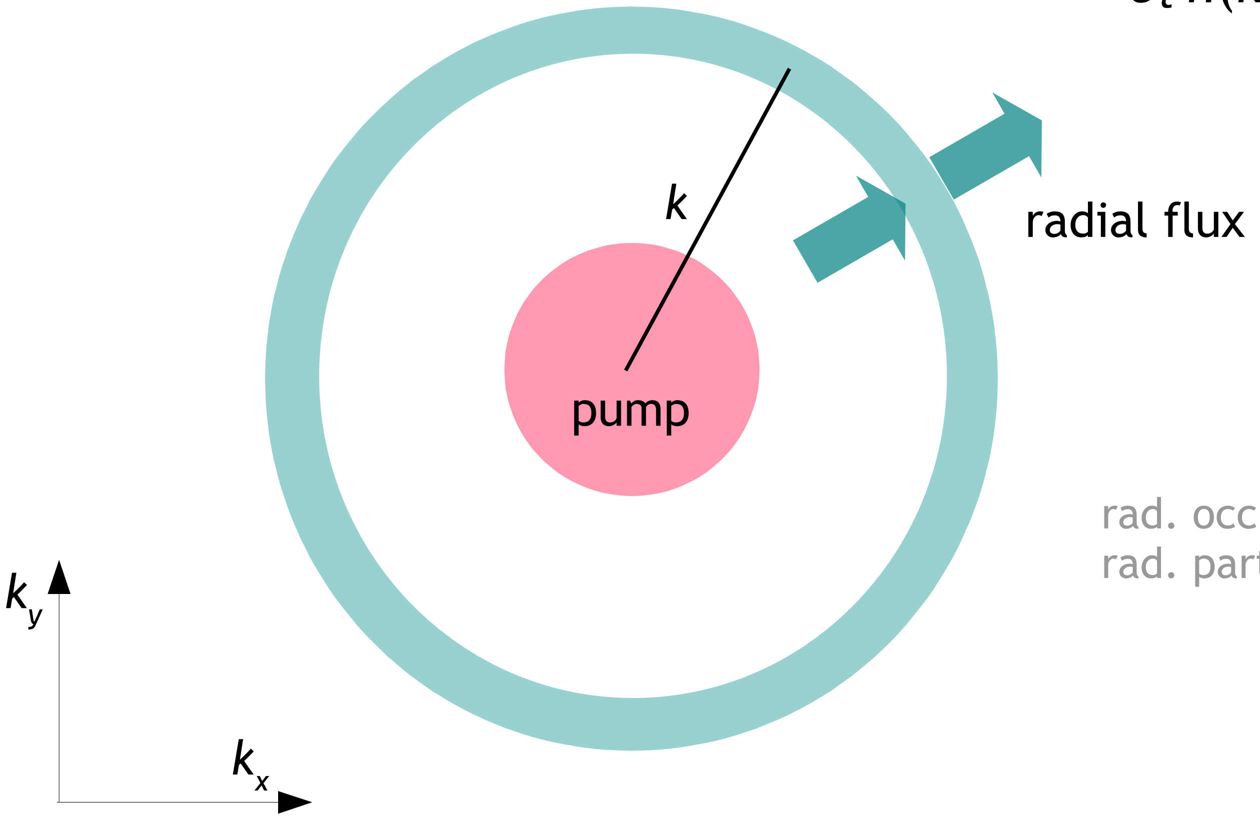
Many-body equilibration

Many-body equilibration

Local radial flux only

Balance equation for radial flux

$$\partial_t n(k) = -\partial_k Q(k)$$



rad. occupation no. n
rad. particle flux Q

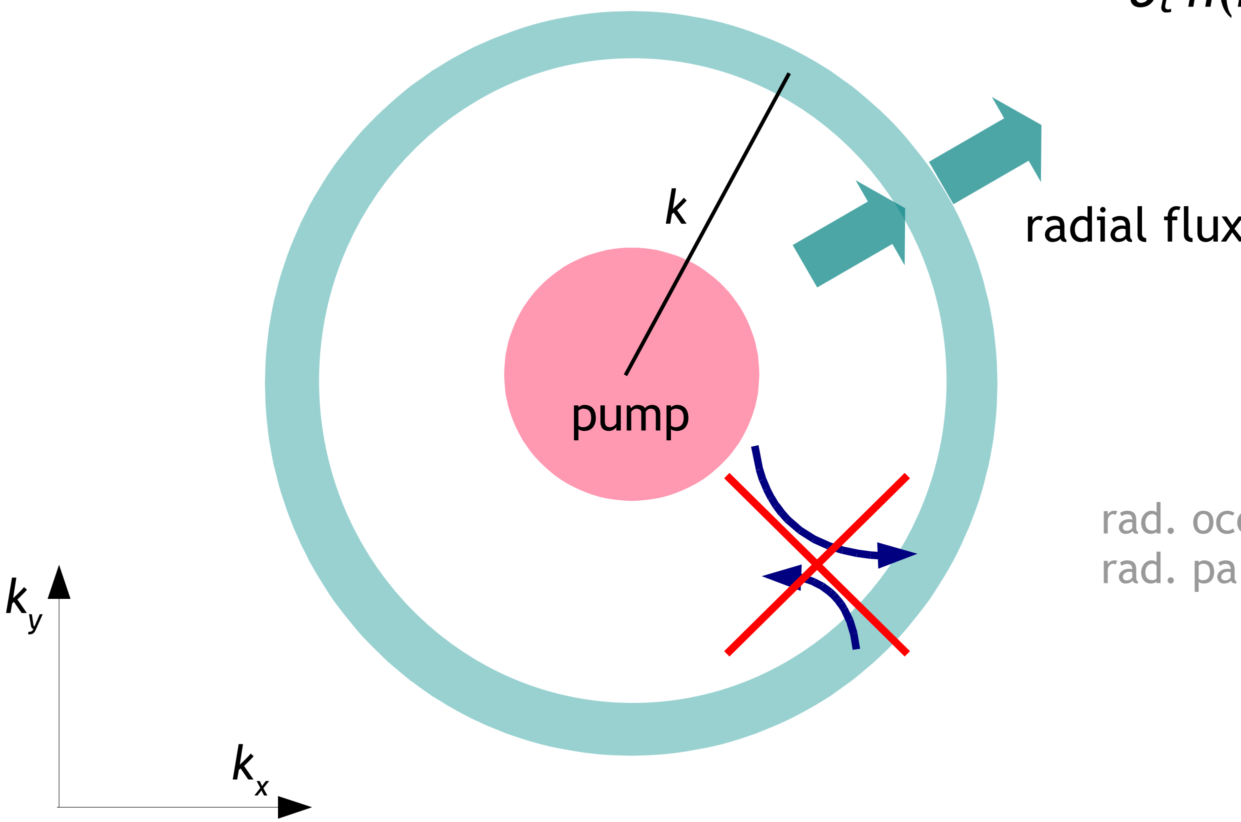
Local radial flux only

With kinetic (Boltzmann) eq.

Boltzmann scattering integral

$$\partial_t n(k) = -\partial_k Q(k)$$

$$\sim k^{d-1} J(k) \overset{!}{=} 0$$



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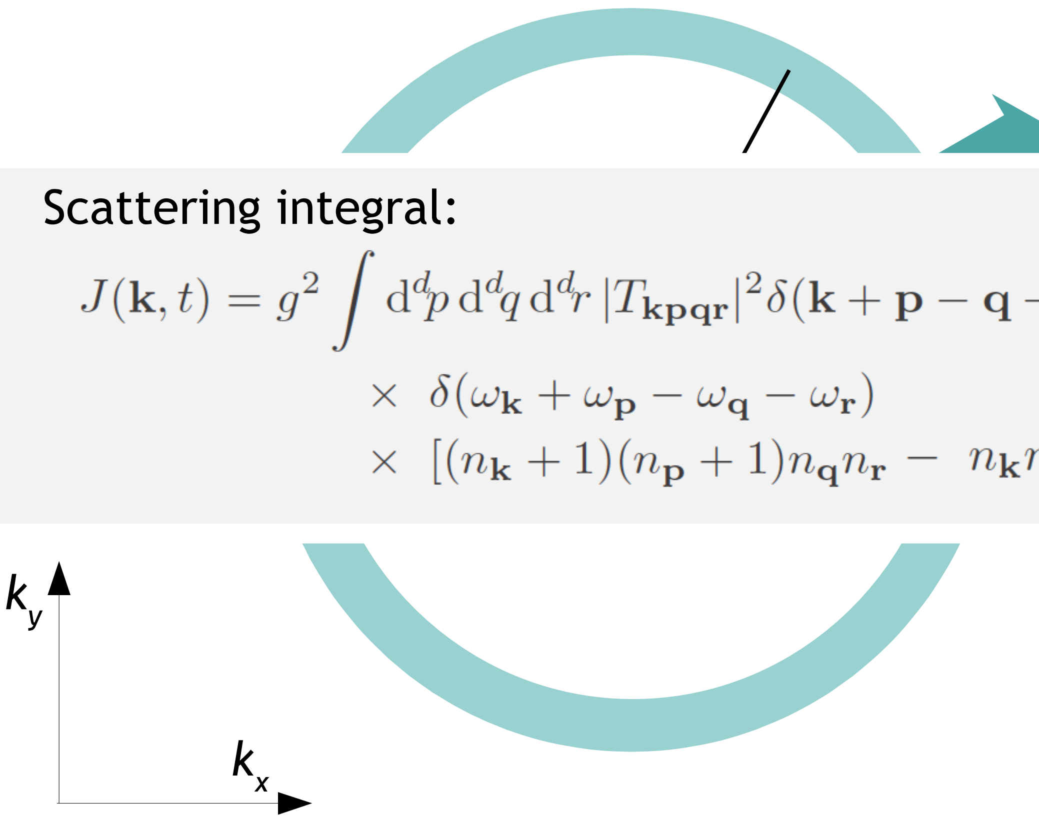


Scattering integral:

$$J(\mathbf{k},t) = g^2 \int \mathrm{d}^d p\, \mathrm{d}^d q\, \mathrm{d}^d r\, |T_{\mathbf{kpqr}}|^2 \delta(\mathbf{k}+\mathbf{p}-\mathbf{q}-\mathbf{r})$$
$$\times\ \delta(\omega_{\mathbf{k}} + \omega_{\mathbf{p}} - \omega_{\mathbf{q}} - \omega_{\mathbf{r}})$$
$$\times\ [(n_{\mathbf{k}}+1)(n_{\mathbf{p}}+1)n_{\mathbf{q}}n_{\mathbf{r}} - n_{\mathbf{k}}n_{\mathbf{p}}(n_{\mathbf{q}}+1)(n_{\mathbf{r}}+1)]$$

rad. particle flux Q

Local radial flux only

With kinetic (Boltzmann) eq.

$$\partial_t n(k) = -\partial_k Q(k) \sim k^{d-1} J(k)$$

Scattering integral:

$$J(\mathbf{k},t) = g^2 \int \mathrm{d}^d p\, \mathrm{d}^d q\, \mathrm{d}^d r\, |T_{\mathbf{kpqr}}|^2 \delta(\mathbf{k}+\mathbf{p}-\mathbf{q}-\mathbf{r}) \times \delta(\omega_\mathbf{k}+\omega_\mathbf{p}-\omega_\mathbf{q}-\omega_\mathbf{r}) \times [(n_\mathbf{k}+1)(n_\mathbf{p}+1)n_\mathbf{q}n_\mathbf{r} - n_\mathbf{k}n_\mathbf{p}(n_\mathbf{q}+1)(n_\mathbf{r}+1)]$$

$3d \;\; -d$

-2

-3ζ

Radial flux density is k-independent, $Q(k) \equiv Q$, if:

$$d - 1 + 1 + 3d - d - 2 - 3\zeta = 0$$

Local radial flux only

With kinetic (Boltzmann) eq.

$$\partial_t\, n(k) = -\,\partial_k\, Q(k) \quad \sim \quad k^{d-1}\, J(k)$$

Scattering integral:

$$J(\mathbf{k},t) = g^2 \int \mathrm{d}^d p\, \mathrm{d}^d q\, \mathrm{d}^d r\, |T_{\mathbf{kpqr}}|^2 \delta(\mathbf{k}+\mathbf{p}-\mathbf{q}-\mathbf{r}) \qquad 3d \;\; -d$$

$$\times\;\; \delta(\omega_\mathbf{k}+\omega_\mathbf{p}-\omega_\mathbf{q}-\omega_\mathbf{r}) \qquad -2$$

$$\times\;\; [(n_\mathbf{k}+1)(n_\mathbf{p}+1)n_\mathbf{q}n_\mathbf{r} - \; n_\mathbf{k}n_\mathbf{p}(n_\mathbf{q}+1)(n_\mathbf{r}+1)] \qquad -3\zeta$$

Radial flux density is k-independent, $Q(k)\equiv Q$, if:

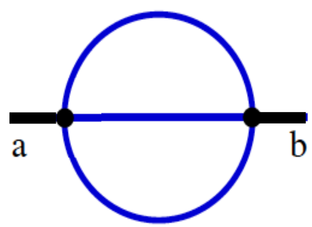
$$\cancel{d} - \cancel{1} + \cancel{1} + 3d - \cancel{d} - 2 - 3\zeta = 0 \;\Rightarrow\; \zeta = d - 2/3$$

Dynamical field theory

Kinetic (Quantum-Boltzmann) eq.: $\partial_t n_{\mathbf{k}} = J(\mathbf{k}, t)$

Dynamic (Schwinger-Dyson) eq.: (from 2PI effective action)

$$\partial_t n(p) \quad = \quad \Sigma^{\rho}_{ab}(p) F_{ba}(p) - \Sigma^{F}_{ab}(p) \rho_{ba}(p)$$

$\Sigma_{ab}(x,y) =$ 

$p = (p_0, \mathbf{p})$:

$F_{ab}(x, y) = \frac{1}{2}\langle\{\Phi_a(x), \Phi_b(y)\}\rangle_c$ Statistical function: Occupation

$\rho_{ab}(x, y) = i\langle[\Phi_a(x), \Phi_b(y)]\rangle.$ Spectral function: Available modes

$n_{\mathrm{BE}}(\omega) = 1/(e^{\beta(\omega-\mu)} - 1)$

$$F^{(\mathrm{th})}_{ab}(\omega, \mathbf{p}) = -i\left(n_{\mathrm{BE}}(\omega) + \frac{1}{2}\right)\rho^{(\mathrm{th})}_{ab}(\omega, \mathbf{p})$$

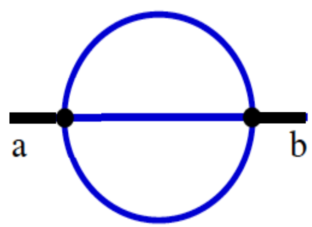
Dynamical field theory

Kinetic (Quantum-Boltzmann) eq.: $\partial_t n_{\mathbf{k}} = J(\mathbf{k},t)$

Dynamic (Schwinger-Dyson) eq.: (from 2PI effective action)

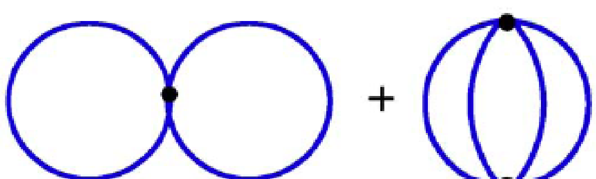
$$\partial_t n(p) \quad = \quad \Sigma^{\rho}_{ab}(p)F_{ba}(p) - \Sigma^{F}_{ab}(p)\rho_{ba}(p)$$

$\Sigma_{ab}(x,y) =$



$p = (p_0, \mathbf{p})$:

$\Gamma_2^{3\text{loop}}[\phi, G] =$



Scaling solutions

We look for scaling solutions fulfilling stationarity condition $J(p) = 0$

Scaling ansatz:

$$\rho_{ab}(s^z p_0, s\mathbf{p}) = s^{-2+\eta}\rho_{ab}(p_0, \mathbf{p})$$
$$F_{ab}(s^z p_0, s\mathbf{p}) = s^{-2-\kappa}F_{ab}(p_0, \mathbf{p})$$

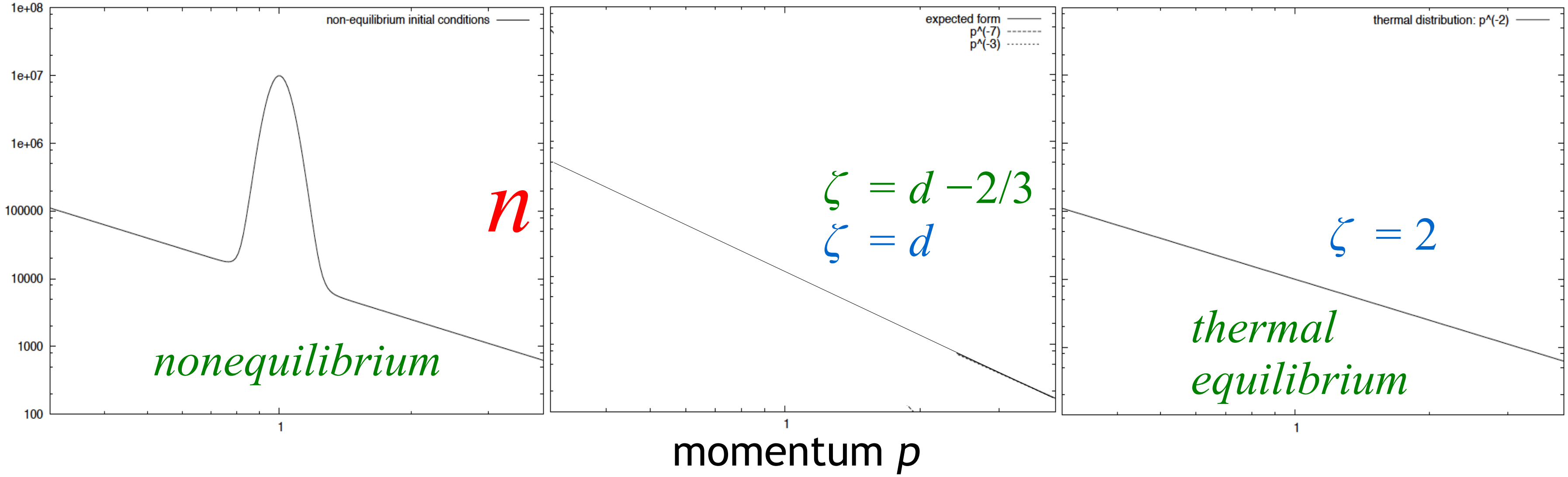
Implies scaling of the single-particle momentum distribution:

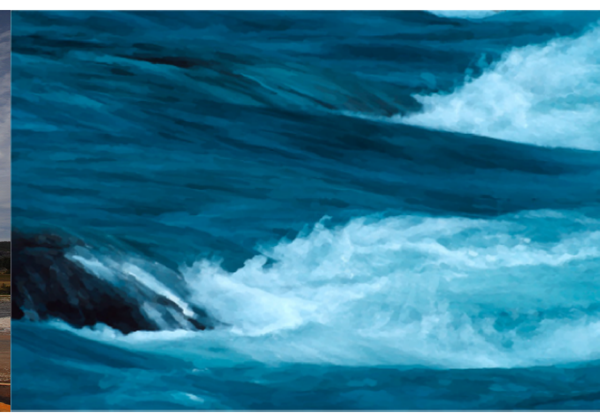
$$n(s\mathbf{p}) = s^{z-2-\kappa}n(\mathbf{p})$$

$= -\zeta$

Turbulent scaling in d+1 Dim.

$$n \sim k^{-\zeta}$$



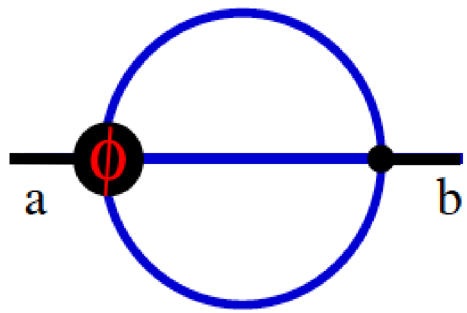


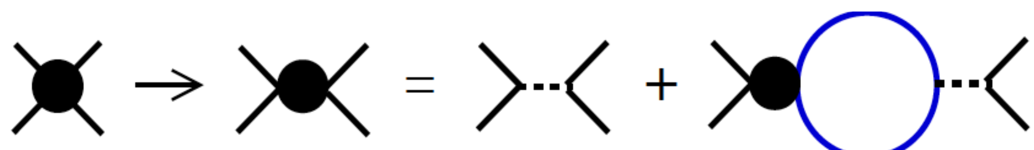
C. Scheppach, J. Berges, TG PRA **81** (10) 033611

Strong turbulence

$p = (p_0, \mathbf{p})$:

$$J(p) := \Sigma^{\rho}_{ab}(p) F_{ba}(p) - \Sigma^{F}_{ab}(p) \rho_{ba}(p) \stackrel{!}{=} 0$$

$\Sigma_{ab}(x,y) =$ 

2PI to NLO in $1/N$: Vertex: 

[Dynamics: J. Berges, (02); G. Aarts et al., (02); TG, Seco, Schmidt, Berges (05);
Kadan.Baym: “GW-Approximation”, Hedin (65)]

Local radial flux only

With kinetic (Boltzmann) eq.

$$T(k) \sim g \ / \ |1 + const. \times g\, k^{d-2} n(k)| \qquad 2 - d + \zeta$$

Scattering integral:

$$J(\mathbf{k},t) = g^2 \int \mathrm{d}^d p\, \mathrm{d}^d q\, \mathrm{d}^d r\, |T_{\mathbf{kpqr}}|^2 \delta(\mathbf{k}+\mathbf{p}-\mathbf{q}-\mathbf{r}) \qquad 3d \ -d$$

$$\times \ \delta(\omega_\mathbf{k} + \omega_\mathbf{p} - \omega_\mathbf{q} - \omega_\mathbf{r}) \qquad -2$$

$$\times \ [(n_\mathbf{k}+1)(n_\mathbf{p}+1) n_\mathbf{q} n_\mathbf{r} - n_\mathbf{k} n_\mathbf{p} (n_\mathbf{q}+1)(n_\mathbf{r}+1)] \qquad -3\zeta$$

Radial flux density is k-independent, $Q(k) \equiv Q$, if:

$$\cancel{d} - \cancel{1} + \cancel{1} + 3d - \cancel{d} - 2 - 3\zeta$$
$$+2(2 - d + \zeta) = 0$$

Local radial flux only

With kinetic (Boltzmann) eq.

$$T(k) \sim g \;/\; |1 + const. \times g\, k^{d-2} n(k)| \qquad 2 - d + \zeta$$

Scattering integral:

$$J(\mathbf{k},t) = g^2 \int \mathrm{d}^d p\, \mathrm{d}^d q\, \mathrm{d}^d r\, |T_{\mathbf{kpqr}}|^2 \delta(\mathbf{k}+\mathbf{p}-\mathbf{q}-\mathbf{r}) \qquad 3d \;\; -d$$

$$\times \;\; \delta(\omega_\mathbf{k} + \omega_\mathbf{p} - \omega_\mathbf{q} - \omega_\mathbf{r}) \qquad -2$$

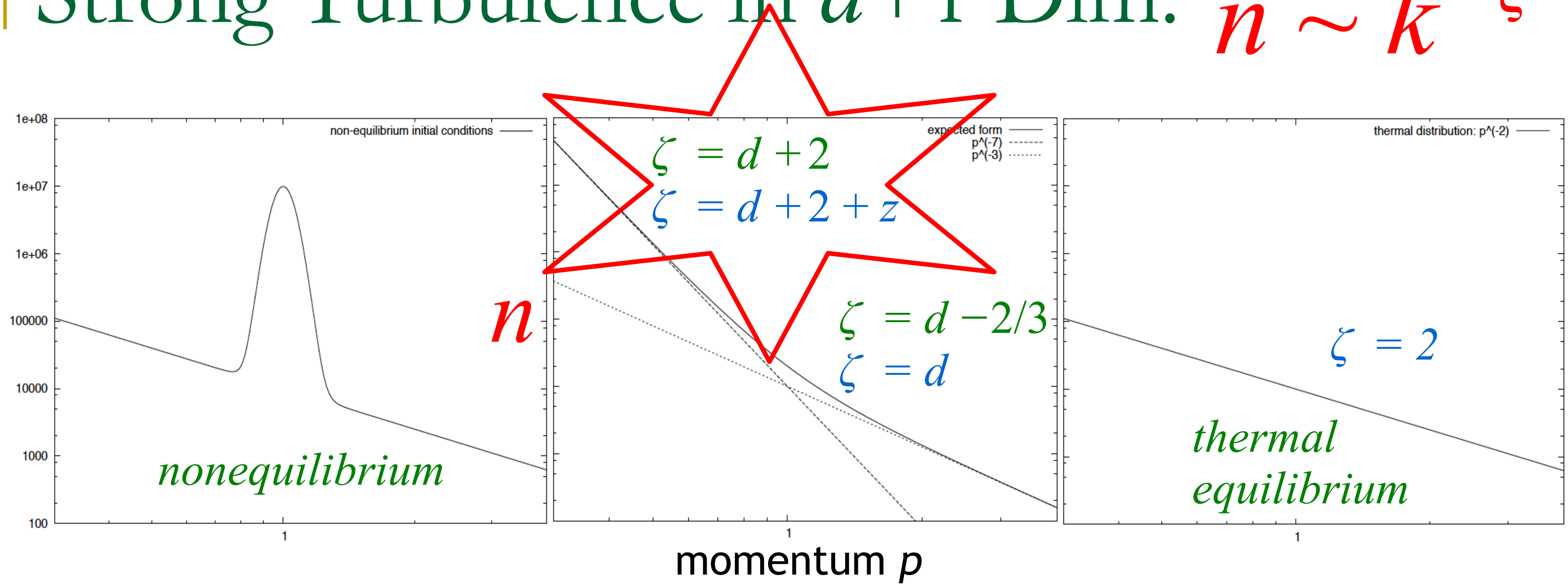
$$\times \;\; [(n_\mathbf{k}+1)(n_\mathbf{p}+1)n_\mathbf{q} n_\mathbf{r} - n_\mathbf{k} n_\mathbf{p}(n_\mathbf{q}+1)(n_\mathbf{r}+1)] \qquad -3\zeta$$

Radial flux density is k-independent, $Q(k) \equiv Q$, if:

$$d - 1 + 1 + 3d - d - 2 - 3\zeta + 2(2 - d + \zeta) = 0 \quad \Rightarrow \zeta = d + 2$$

Strong Turbulence in d+1 Dim.

$n \sim k^{-\zeta}$



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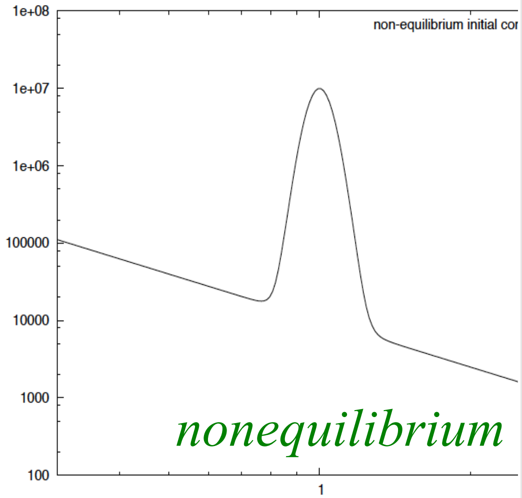
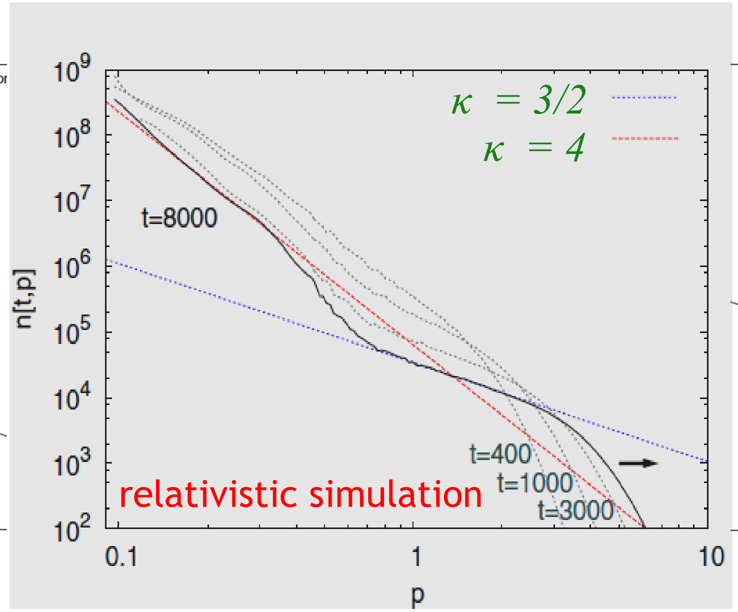
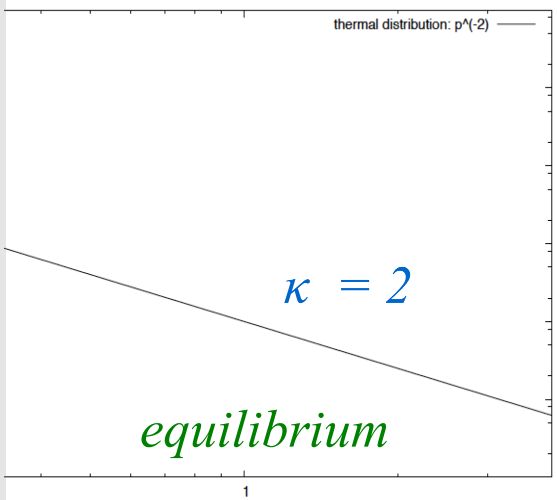
Summary

$$n(s\vec{p}) = s^{-\kappa} n(\vec{p})$$







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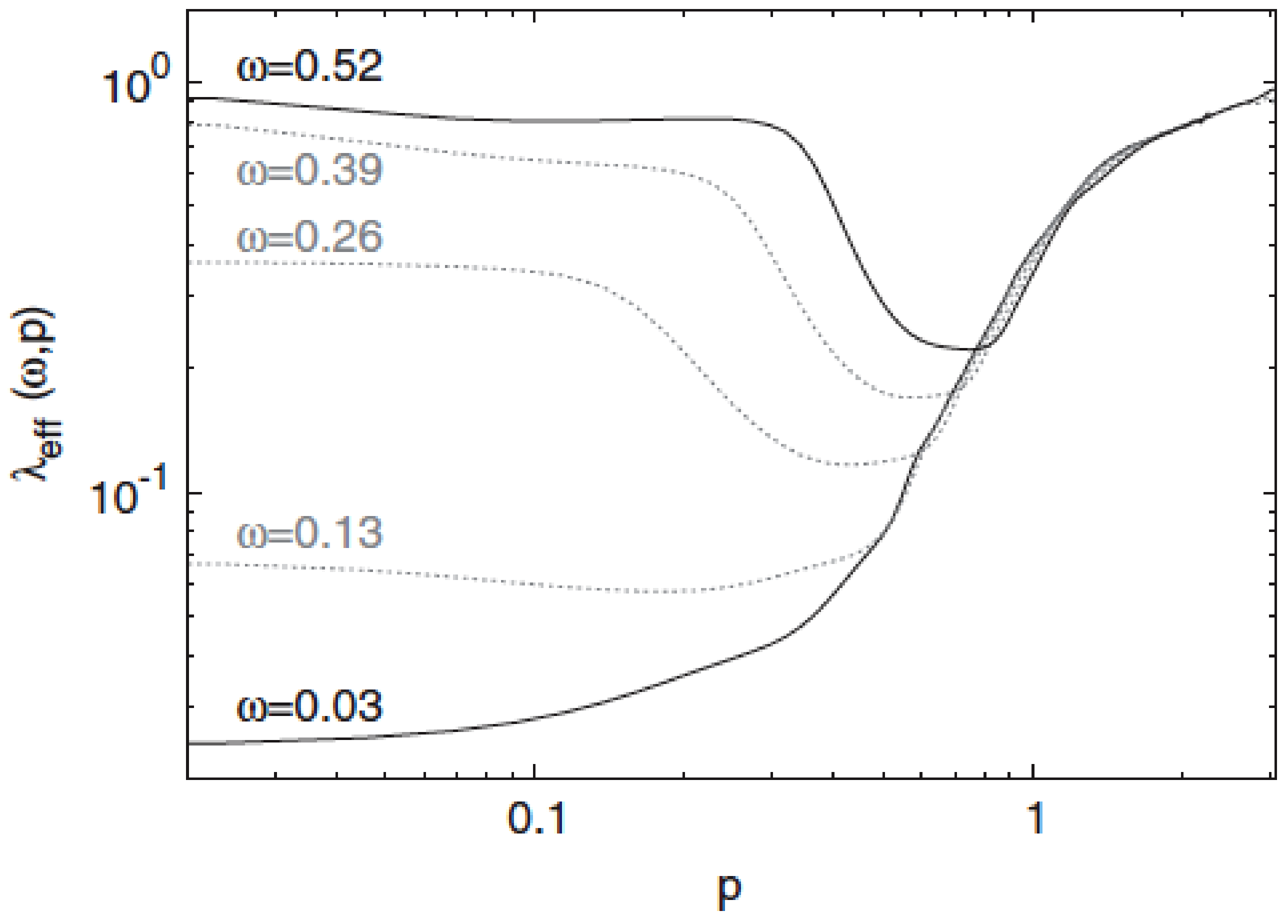
Effective coupling



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€€€...

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LGFG BaWue

DAAD Deutscher Akademischer Austausch Dienst
German Academic Exchange Service

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