Nature of Turbulence
UCSB
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Some "Non"s in Turbulence
-- Nonlinear, Nonlocal, Nonequilibrium,…

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Some Non's in the Nature of Turbulence

(I) Non-linearity

(II) Non-infinity
finiteness of Re & L/η

(III) Non-locality
in the energy transfer

(IV) Non-equilibrium

This talk may present another Non?

(I) Non-Linearity

K. Morishita &K.Moffatt

Reynolds number

Reynolds number $R_e \equiv \frac{u' L}{\nu}$,

Reynolds Number $= \frac{|convection\ term|}{|viscous\ term|} \equiv R_{loc}$,

$$R_{loc} = \frac{|(\mathbf{u} \cdot \nabla)\mathbf{u}|}{|\nu \nabla^2 \mathbf{u}|}$$

Question $R_{loc} \overset{?}{\sim} R_e$

DNS of Homogeneous Isotropic Turbulence

$R_e \approx 5300, \ R_\lambda \approx 429$

$\langle R_{loc} \rangle \approx 39.8$

The normalized average $\langle|(\mathbf{u}\cdot\nabla)\mathbf{u}|\rangle/(u'v_\eta/\eta)$ is 0.708 in the DNS and 0.847 in the random field. Thus the average is smaller in the DNS field. The smallness in the NS field is consistent with the phenomenon known as the depression of nonlinearity. The averages of $|\mathbf{u}\times\omega|^2$ and $|\mathbf{u}\times\omega-\nabla[p+(1/2u^2)]|$ are known to be smaller than their Gaussian counterparts

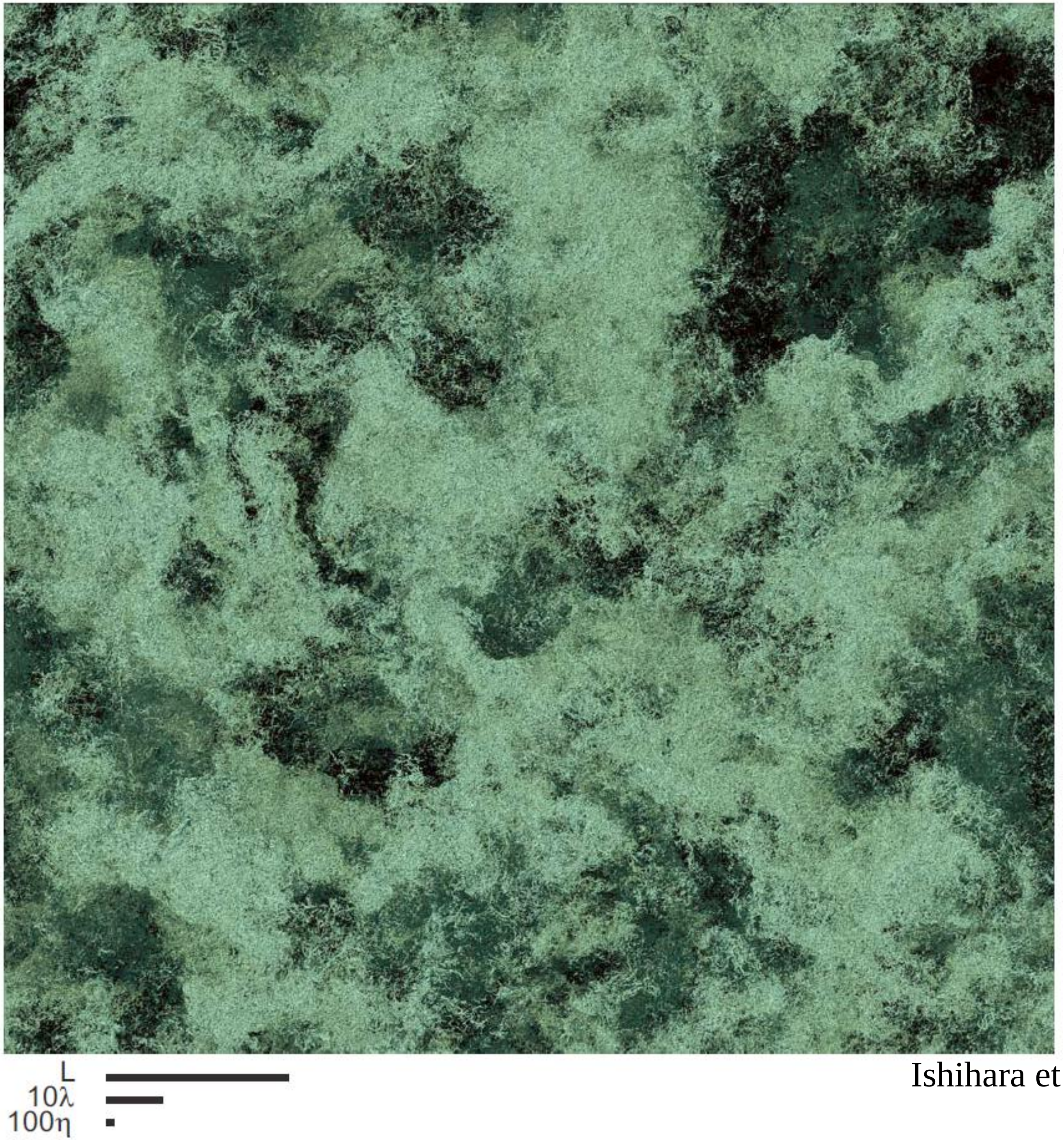
(Kraichnan & Panda P1988; Chen et al.1989; She et al.1991; Tsinober 2009)

$$|(\mathbf{u}\cdot\nabla)\mathbf{u}|\sim u'v_\eta/\eta \qquad |\nu\nabla^2\mathbf{u}|\sim \nu v_\eta/\eta^2$$

$$\langle R_{loc}\rangle \sim \frac{u'\eta}{\nu}\propto R_\lambda^{1/2}$$

→10.5

(Orszag,1977)



Ishihara et al.(Ann.Rev.F.M,2009)

Conclusion of (I)

- **a Hypothesis**

The nature of turbulence dislikes

too high local Reynolds number, i.e.,

too much unbalance between nonlinear vs. viscous terms.

and NS dynamics works against such a state, so that

strong-small-eddy structures are so organized

to suppress R_{loc}, by e.g., instability ?

(II) Non-Infinity

non-stationarity,
non-isotropy

Kraichnan (1991)

The Kolmogorov theories have profoundly shaped and Illuminated thinking about turbulence. But, in one respect, this influence has been unfortunate: relatively little attention has been developed to the prediction of turbulence statistics at finite Reynolds number.

…..

Moreover it is likely that the question of intermittency corrections to K41 can be resolved only when detailed understanding of the dynamics at finite Reynolds number has been achieved.

….

Batchelor(1961)

The theory is an asymptotic one, and its predictions hold with increasing accuracy (if the theory is correct) as $R_\lambda \to \infty$, but no theoretical estimate has been made of the actual value of R_λ needed for a given degree of accuracy.

Lecture by Antonia(2007)

Question:
Where is the inertial subrange ?

Where is the range, $L >> r >> \eta$?

How large $\frac{L}{r}, \frac{r}{\eta}, R_e$ need be ?

Kolmogorov's 4/5 law:

$$\frac{D_{LLL}(r)}{\varepsilon r} = -\frac{4}{5} \quad \text{for} \quad L >> r >> \eta$$

$$D_{LLL}(r) \equiv <(\delta u_L(r))^3>$$

Karman Howarth Eq.

$$D_{LLL}(r) = -\frac{4}{5}\bar{\epsilon}r + 6\nu\frac{\partial D_{LL}(r)}{\partial r} - \frac{3}{r^4}\int_0^r \frac{\partial D_{LL}(r')}{\partial t}r'^4 dr' + F(r)$$

Viscosity, Non-Stationarity, Forcing,

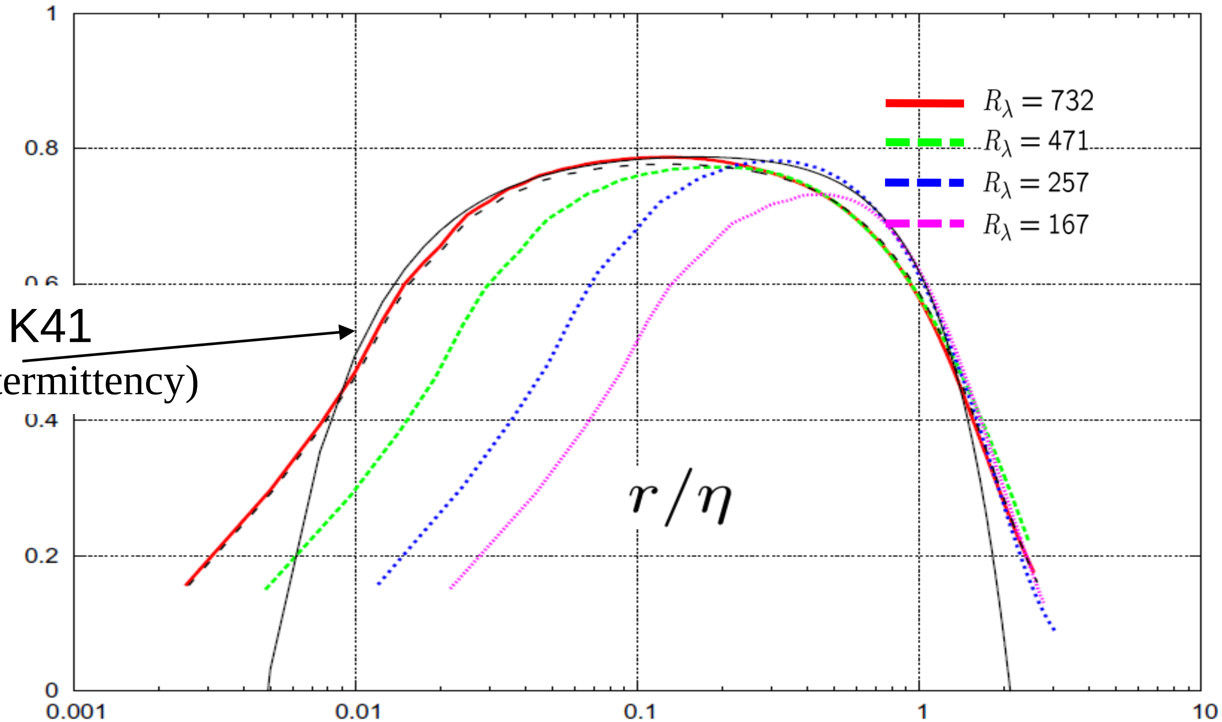
$$L \gg r \gg \eta$$

Anisotropy

Let's check by DNS data …

4/5 Law

$$\frac{-D_{LLL}(r)}{\bar{\varepsilon} r} \qquad D_{LLL}(r) \equiv \langle (\delta u_L(r))^3 \rangle$$



Theory a la K41

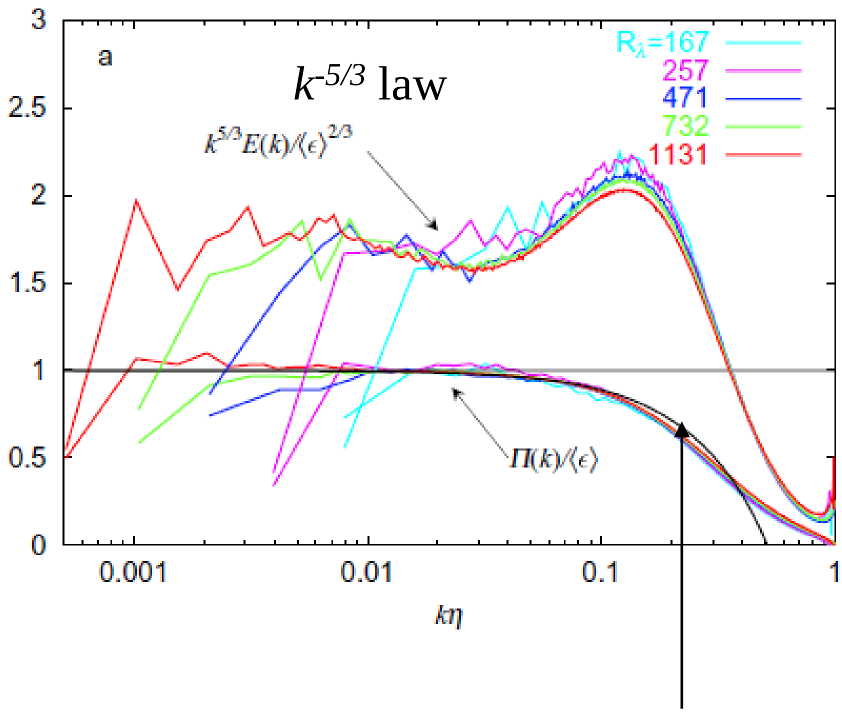
(neglecting intermittency)

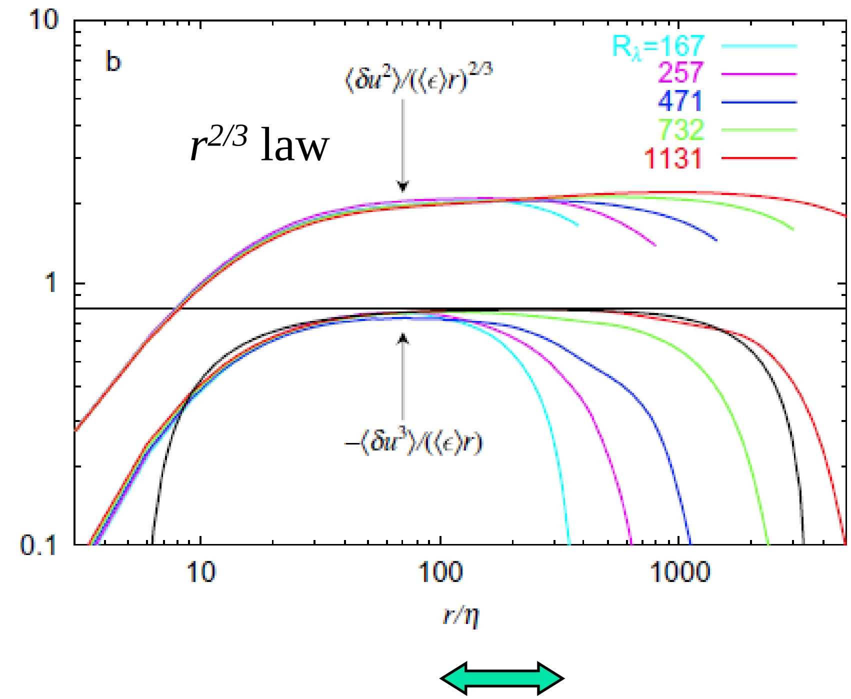
YK,Yoshino, Ishihara(JPSJ,2008)

Where is the IR ?

Wave number space
E(k) and energy flux $\Pi(k)$

Real space
2^{nd} and 3^{rd} order moments





Theory a la K41

Ishihara et al.(Ann.Rev.F.M,2009)

4/5 Law

$$\Delta(r) \equiv \frac{4}{5} + \frac{D_{LLL}(r)}{\bar{\varepsilon} r} = C_v \left(\frac{r}{\eta}\right)^{-4/3} + C_f \left(\frac{r}{L}\right)^2 ,$$

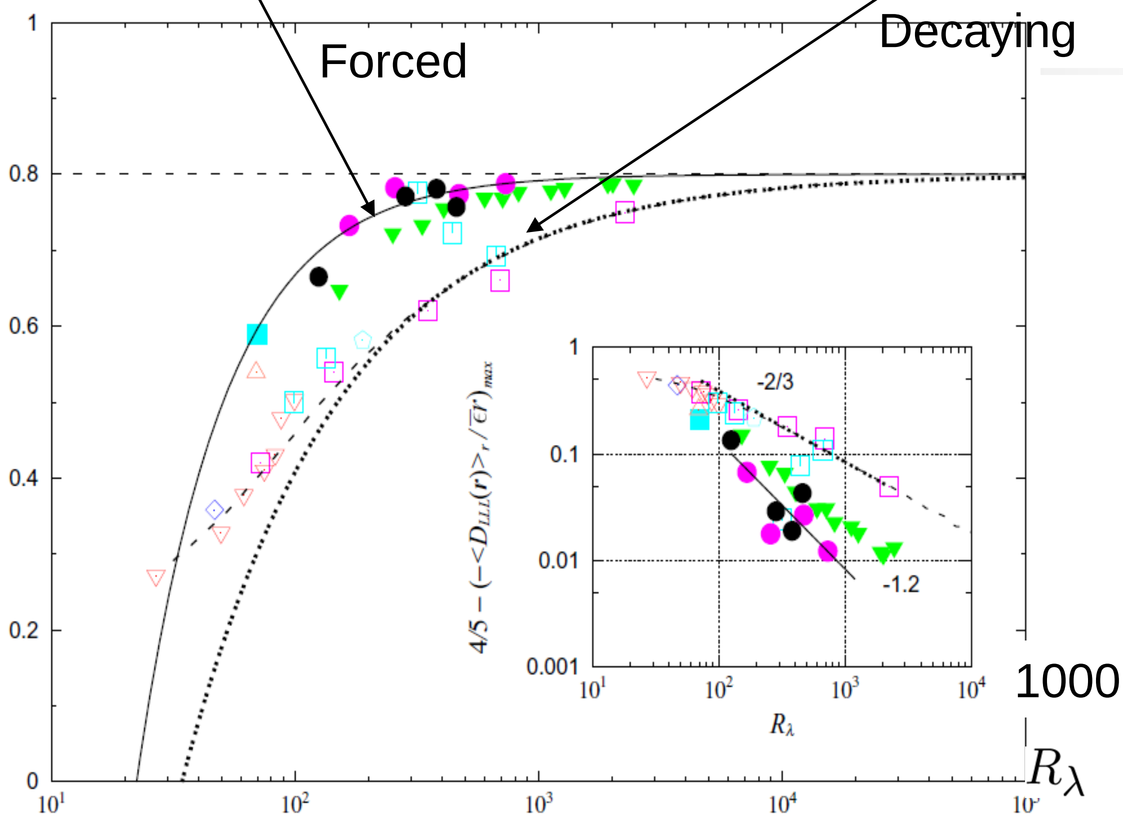
$$\Delta_{min} \propto R_\lambda^{-6/5}, \quad \frac{r_{min}}{\eta} \propto \left(\frac{L}{\eta}\right)^{3/5} \propto R_\lambda^{9/10}$$

Limdborg,1999;Qian,1999;Lundgren,2003;
Davidson 2004, Kaneda et al.,2008

theory

$$\frac{4}{5} + \left[\frac{1}{\bar{\epsilon} r}\langle D_{LLL}(\mathbf{r})\rangle_r\right]_{max} \propto R_\lambda^{-6/5}$$

$$\frac{4}{5} + \left[\frac{1}{\bar{\epsilon} r}\langle D_{LLL}(\mathbf{r})\rangle_r\right]_{max} \propto R_\lambda^{-2/3}$$



- present
- Fukayama *et.al 2000*
- Gotoh et. al 2002
- Orlandi & Antonia 2002
- Moisy *et. al* 1999
- Mydlarski & Warhaft 1996
- Zhou & Antonia 2000
- Gagne *et. al 2004*
- van de Water & Herweijer 1999
- Model by Antonia & Burattini 2005

R_λ = 1,000 (forced)
50,000(decaying)

YK,Yoshino, Ishihara, JPSJ(2008)

Where is IR ?→ Range satisfying

$$\text{Viscous \& Forcing effects on } \Delta < \delta$$

$$r_{min} < r < r_{max}$$

Forced turbulence $$r_{min} \equiv \eta C_v^{3/4} \delta^{-3/4}, \quad r_{max} \equiv L C_f^{-1/2} \delta^{1/2},$$

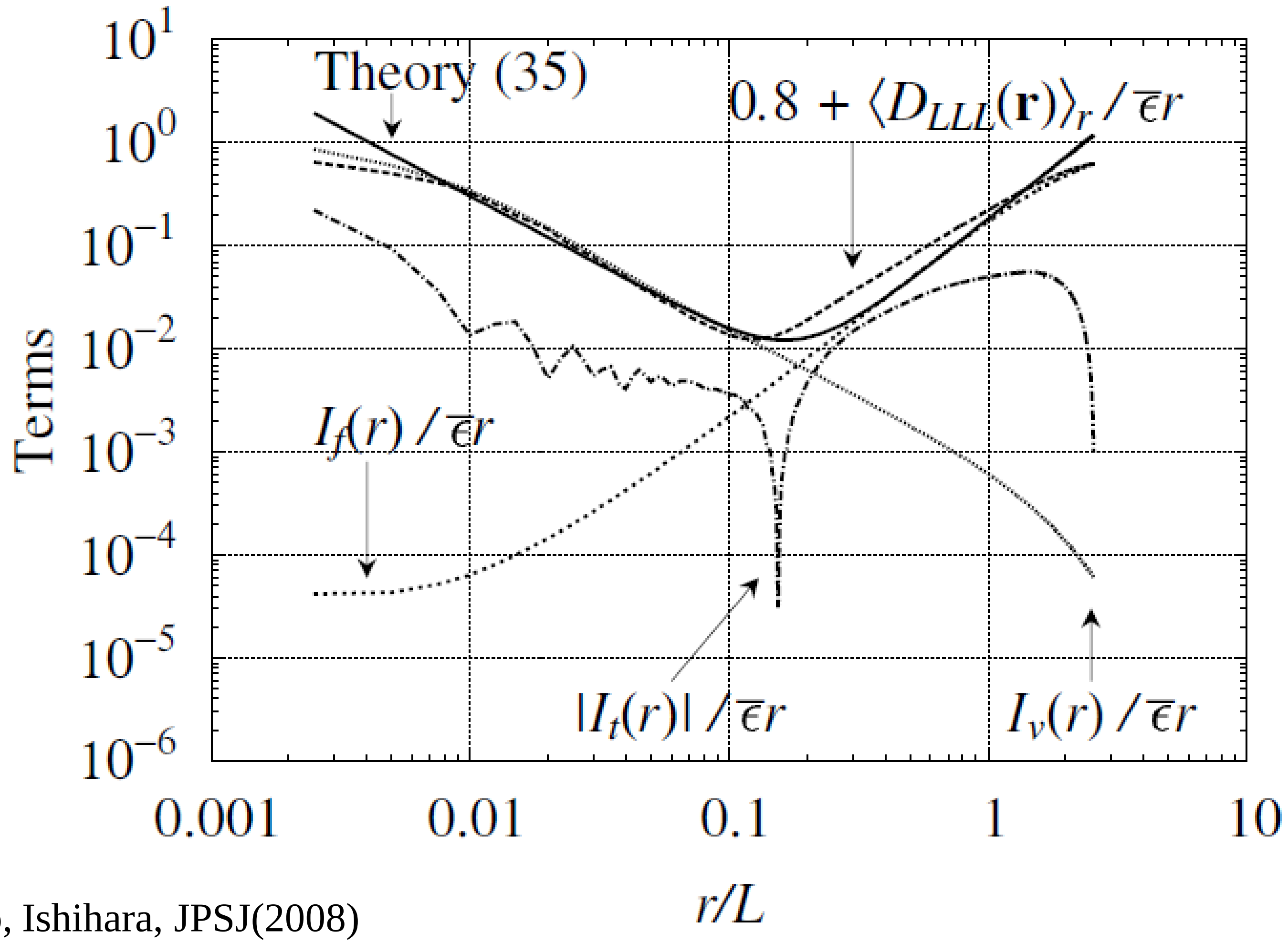
$$\frac{r_{max}}{r_{min}} = C \frac{L}{\eta} \delta^{5/4} \approx 0.12 \frac{L}{\eta} \delta^{5/4} \approx 5.0 \times 10^{-3} R_\lambda^{3/2} \delta^{5/4}$$

Example: $\delta = 0.1$ $\quad \frac{r_{max}}{r_{min}} > 10 \longrightarrow R_\lambda > 1100$

Decaying turbulence $$\frac{r_{max}}{r_{min}} = C' R_\lambda^{3/2} \delta^{9/4}$$

Wavenumber space $$\frac{k_{max}}{k_{min}} \propto R_\lambda^{3/2}$$ **for both decaying and forced**

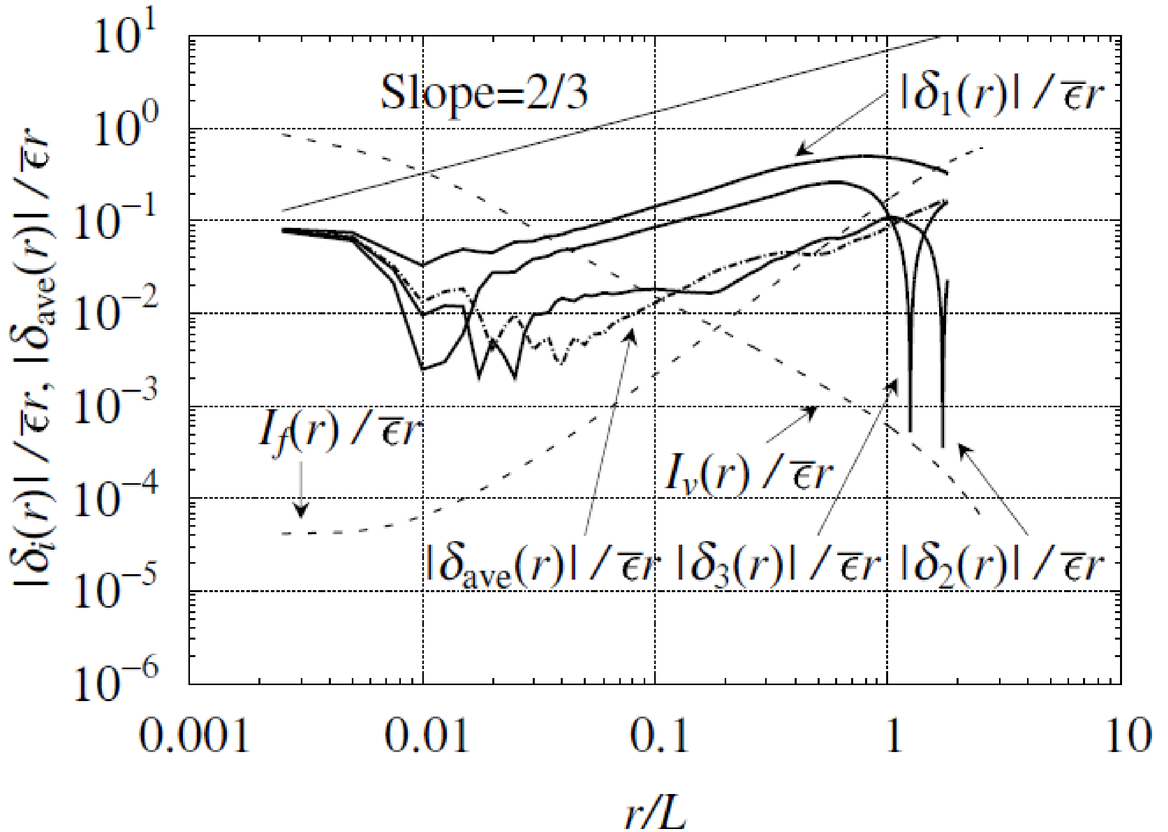
Non-Stationarity



YK,Yoshino, Ishihara, JPSJ(2008)

Non-isotropy

$$\delta_i(r) \equiv D_{LLL}(r\mathbf{e}_i) - \langle D_{LLL}(\mathbf{r}) \rangle_r \,, \quad \delta_{\rm ave}(r) = \frac{1}{3}\sum_{i=1}^{3}[D_{LLL}(r\mathbf{e}_i) - \langle D_{LLL}(\mathbf{r})\rangle_r]$$



YK,Yoshino, Ishihara, JPSJ(2008)

Fig. 7. Anisotropy measure $|\delta_i(r)|/(\bar{\epsilon}r)$ (solid lines), the average $|\delta_{\rm ave}(r)|/(\bar{\epsilon}r)$ (dashed-dotted line), $I_f(r)/(\bar{\epsilon}r)$ (dashed line) vs. r/L, in Run 2048.

Conclusion of II)

- **a Lesson**

There several “Non”’s which affect/contaminate/pollute “inertial range” statistics, such as scaling, intermittency corrections.

Their influence, which vanishes at infinite Re,
should not be confused with
genuine intermittency effects which remain finite at infinite Re

(III) Non-Locality
in Energy Transfer (in scale-space)

Statistics of Energy Transfer

Energy transfer from Grid to Sub-Grid scales:

$$T = -\tau_{ij}\overline{S}_{ij},$$

$$\overline{S}_{ij} = \frac{1}{2}\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right)$$

$$\tau_{ij} = (\overline{u_i u_j} - \overline{u_i}\ \overline{u_j}) - \frac{2}{3}\delta_{ij} q, \quad q = \frac{1}{2}(\overline{u_k u_k} - \overline{u_k}\ \overline{u_k}),$$

Pimeori, et al. PF(1990),
Pimeori, et al. PF(1991)
Domaradki, et al. PF(1993)
Cerutti & Meneeveau, PF(1998)
Chen et al.PRL(2003)

Spectral cut-off filter at k_c

Triad Interaction

NS equation →

second order nonlinearity

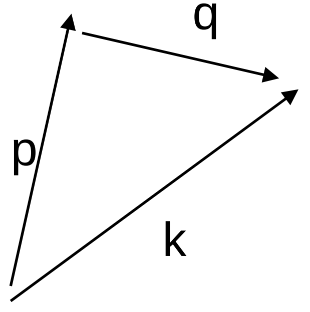
in the wave vector space

$$\frac{\partial}{\partial t} u \rightarrow uu \rightarrow \Sigma\, u(p)u(q) \qquad k=p+q$$

Triad Interaction

not

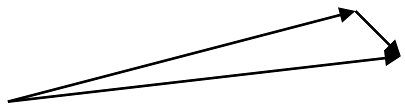
$$k \rightarrow k' \rightarrow k'' \rightarrow \ldots.$$



Local vs Non-Local Interactions

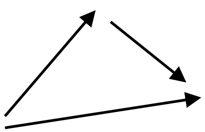
$$\alpha = \frac{\max(k,p,q)}{\min(k,p,q)}$$

$\alpha >> 1$



Distant/Non-Local Interaction

$\alpha \sim 1$



Local Interaction

$$\Pi(k) = <T(k,x)> = \int \frac{W(\alpha,k)}{\alpha} d\alpha$$

Dynamics:

Let u = U + u', and U ← Large scale (q) , u' ← small scale (k)

then → u u = U U + U u' + u' U + u'u'

If **distant interaction** is dominant Uu' + u'U >> u'u', (RDT like)

If **local interaction** is dominant u'u' >> Uu' + u'U

$$T = -\tau_{ij} \overline{S}_{ij},$$

function of k and x

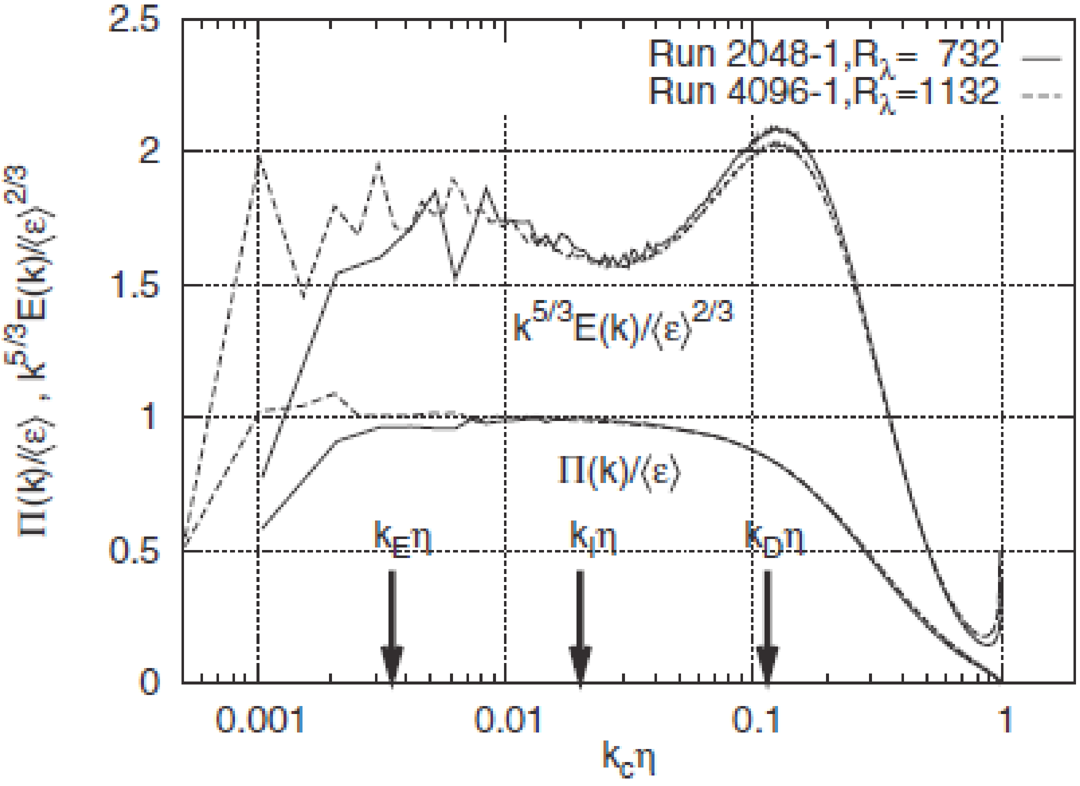


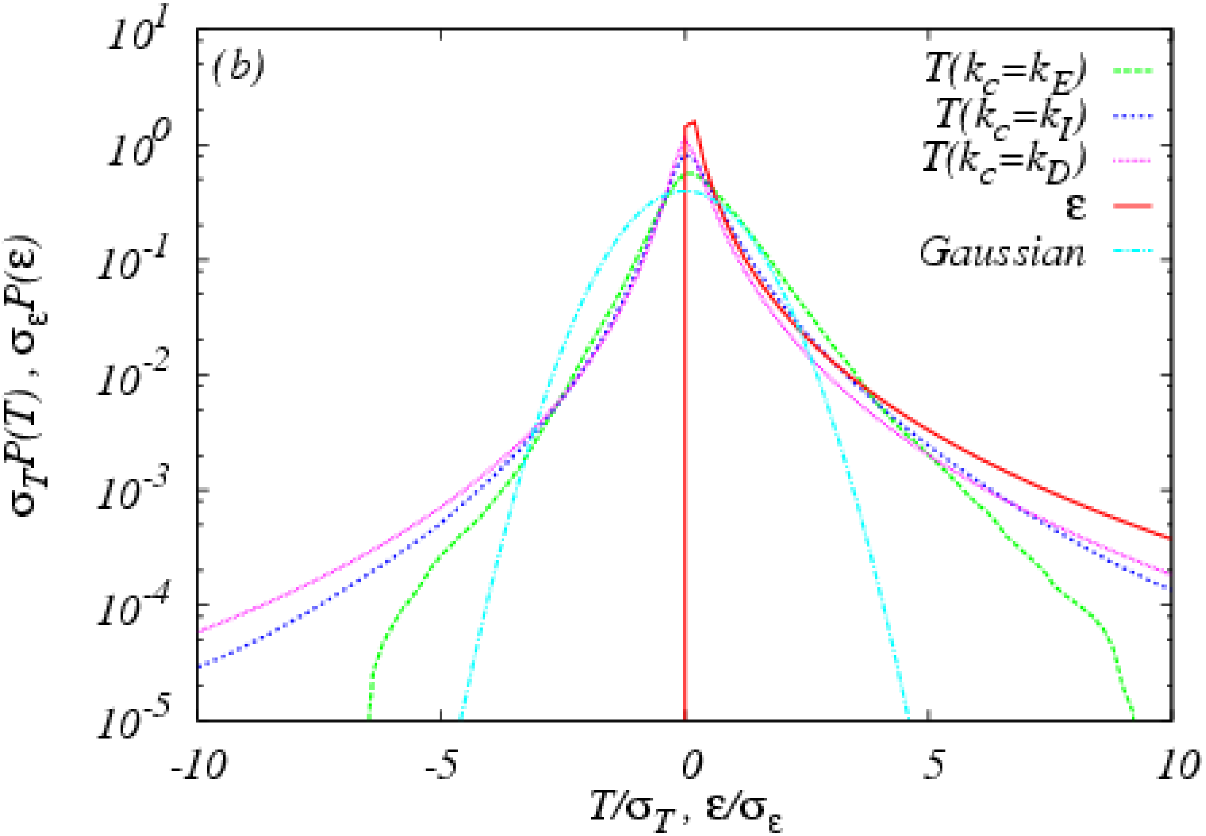
Fig. 4. $\Pi(k)$ and the positions of the cut-off wave numbers, in Run 2048-1 and Run 4096-1.

Aoyama et al. (2005)

PDF of T (& ε)

$$T = -\tau_{ij}\overline{S}_{ij},$$

$$\tau_{ij} = (\overline{u_i u_j} - \overline{u_i}\ \overline{u_j}) - \frac{2}{3}\delta_{ij} q, \quad q = \frac{1}{2}(\overline{u_k u_k} - \overline{u_k}\ \overline{u_k}),$$



Aoyama et al. (2005)

Interfaces -- Within turbulence DNS

energy dissipation rate enstrophy

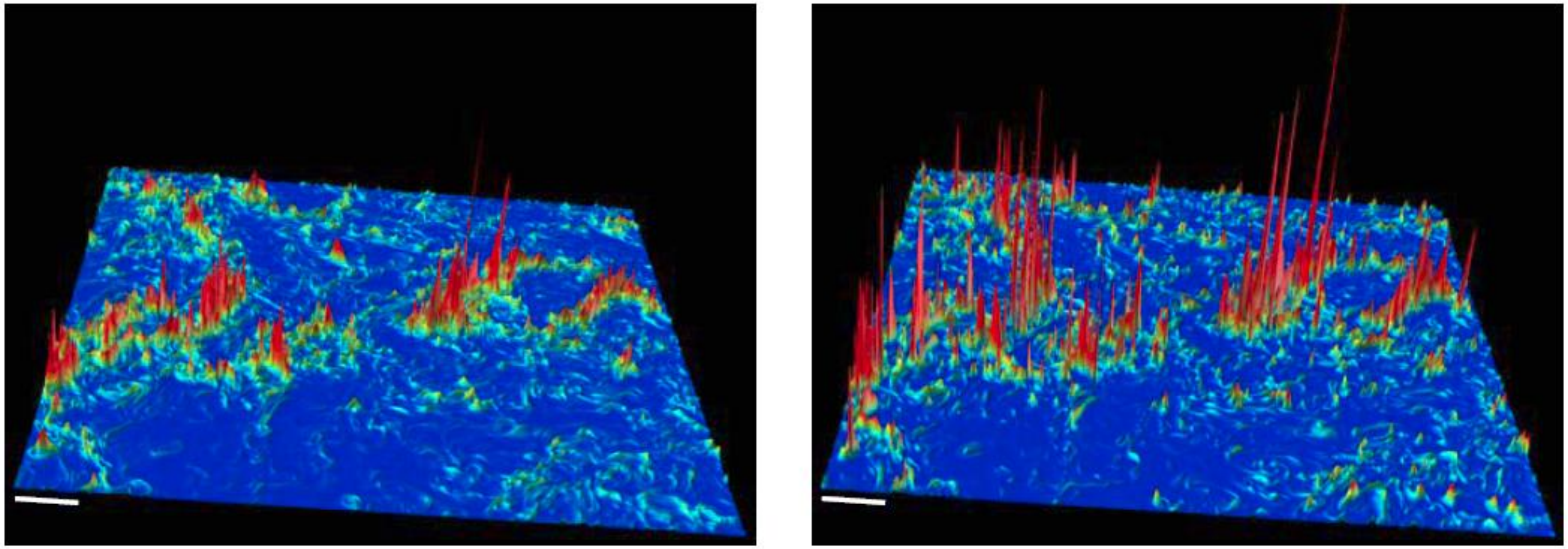


Figure 4: Snapshot of the intensity (in arbitrary unit) distributions of (a)the energy dissipation rate $\tilde{\epsilon} = \epsilon/(2\nu)$ and (b) the enstrophy $\Omega = \omega^2/2$ on a cross section in DNS-ES at $R_\lambda = 675$. The white bars at the bottom indicate 100η.

Ishihara etal. (Ann. Rev. Fluid Mech, 2009)

Conclusion of (III)

- **a Question**

The average of the energy transfer $\Pi(k)=\langle T(k,x)\rangle$
is dominated by local interaction.

But this does not imply that large deviation statistics (high order moments) of T(k,x) is also dominated local interaction.

?? Nonlocal interaction may be important??

(IV) Non-Equilibrium

Statistical Mechanics
for Systems at or near Thermal Equilibrium

Linear Response Theory (Nakano-Kubo)

Suppose that an external force X is applied to a thermal equilibrium system with the equilibrium density distribution ρ_e, then

$$\rho = \rho_e + \Delta\rho + \ldots,$$

$$\langle B \rangle = \langle B \rangle_e + \Delta \langle B \rangle + \ldots,$$

$$\Delta \langle B \rangle = cX,$$

$$J = CX,$$

$$J = CX$$

Generalized Flux vs. Generalized Force

e.g.

Density Flux vs. Density gradient ; J = C grad ρ

Heat Flux vs. Temperature gradient ; J = C grad T

Electric Current vs. External electric field ; J = σ E = σ grad ϕ , (I=E/R, Ohm's law)

Momentum Flux vs. Strain rate ; $\tau_{ij} = C_{ijmn} S_{mn},$

(Newton's law)

Universality in Response to disturbances, near equilibrium state

1905, Einstein, $D=\mu kT$,

the first example of FD-elation→ Perrin's experiment.

1928, Nyquist's theorem on thermal noise:

$$P(f)=4kT\ Re(Z(f))$$

1931, Onsager's reciprocal theorem:

$$J = C\,X, \qquad C={}^{T}C$$

generalized flux, generalized force

1950-60, Nakano, Kubo Linear Response Theory

Two kinds of universality in
not only a) Equilibrium state itself, like Boyle-Charles' law
but also b) Response to disturbance

Thermal Equilibrium system

Disturbance

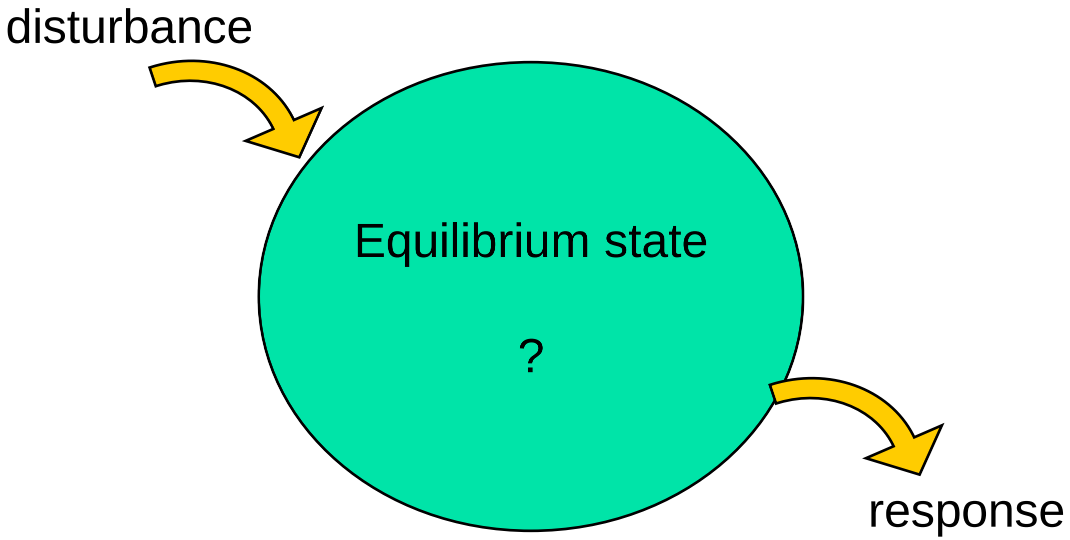
X
grad φ
grad T
grad c

Equilibrium state

Linear response

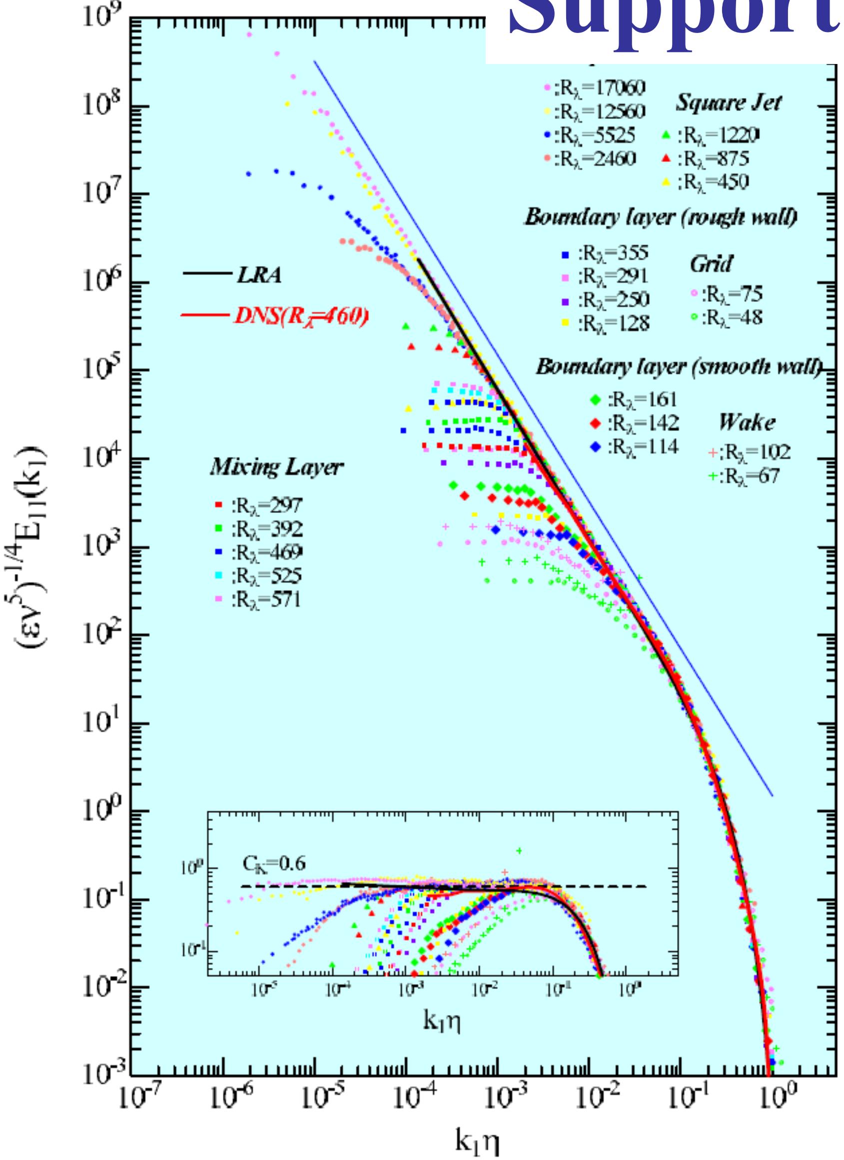
F=CX (Hooke's law)
J = C' grad φ= σE (Ohm's law)
J = C'' grad T (Fourier's law)
J = C''' grad c (Fick's law)

Turbulence ?



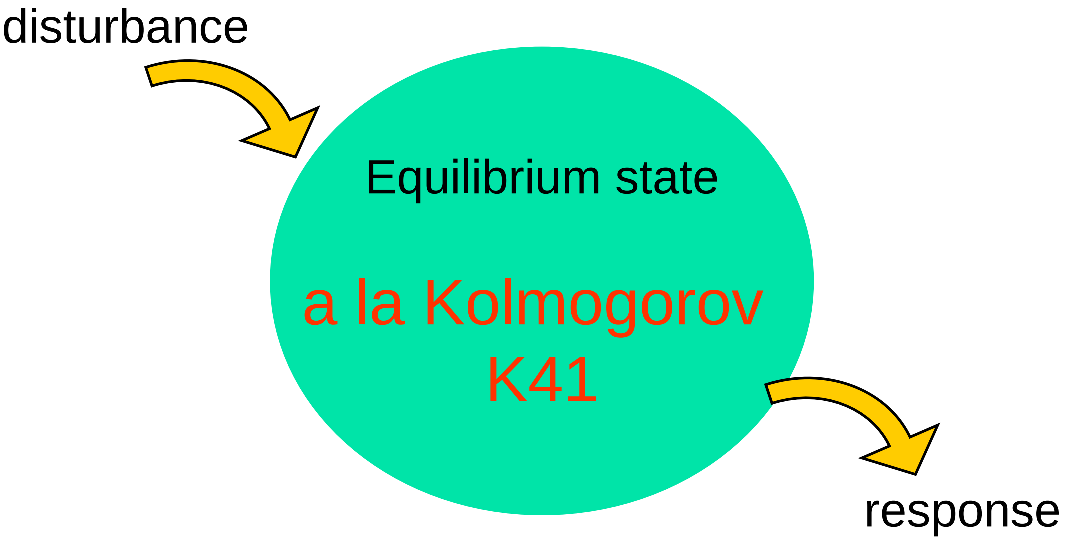
Supports for "Universality" -2

energy spectrum



Tsuji (FDR, 2009)

Figure 2. One-dimensional energy spectrum normalized by the Kolmogorov unit. The inset shows $\phi(k_1\eta)/(k_1\eta)^{-5/3}$. The flat region corresponds to Kolmogorov's constant C_K.


disturbance
Equilibrium state
a la Kolmogorov
K41
response

NS-equation

$$\frac{\partial}{\partial t}\boldsymbol{u}(\boldsymbol{x},t) = -(\boldsymbol{u}\cdot\nabla)\boldsymbol{u} - \nabla p + \nu\nabla^2\boldsymbol{u}$$

$$\nabla\cdot\boldsymbol{u} = 0.$$

Local co-ordinate

$$\boldsymbol{v} = <\boldsymbol{v}> + \tilde{\boldsymbol{v}}$$

Mean Shear

$$\frac{\partial}{\partial t}\tilde{\boldsymbol{v}}(\boldsymbol{r},t) = -(\tilde{\boldsymbol{v}}\cdot\nabla)\tilde{\boldsymbol{v}} - \nabla\boldsymbol{q} + \nu\nabla^2\tilde{\boldsymbol{v}} + \boldsymbol{M},$$

for $r \ll L$,

$$M_i = S_{mn} r_n \frac{\partial \tilde{v}_j}{\partial r_m} + S_{ij}\tilde{v}_j$$

$$\frac{\boldsymbol{M}}{(\tilde{\boldsymbol{v}}\cdot\nabla)\tilde{\boldsymbol{v}}} \sim \frac{S v_\ell}{v_\ell^2/\ell} = \frac{S\ell}{v_\ell} \propto S\ell^{2/3}/\epsilon^{1/3} \ll 1$$

cf. Kaneda & Yoshida (New J. Phys., 2004)

Anisotropic part

at small scale of turbulent shear flow

$$\langle B \rangle = \langle B \rangle_e + \Delta \langle B \rangle + \ldots,$$

$$\Delta \langle B \rangle = cX, \qquad J = CX,$$

$$\tau_{ij} = C_{ijmn} S_{mn},$$

(Stress-Strain rate relation for Newtonian Fluid)

$$<u_i(\mathbf{k})u_j(-\mathbf{k})> = Q_{ij}(\mathbf{k}) = Q_{ij}^{\ 0}(\mathbf{k}) + C_{ij\alpha\beta}(\mathbf{k}) S_{\alpha\beta}$$

equilibrium spectrum

Anisotropic part

at small scale of turbulent shear flow

isotropic quasi-equilibrium spectrum

response

$$\langle u_i(\mathbf{k})u_j(-\mathbf{k})\rangle = Q_{ij}(\mathbf{k}) = Q_{ij}{}^{0}(\mathbf{k}) + \underline{C_{ij\alpha\beta}(\mathbf{k})S_{\alpha\beta}}$$

$$C_{ij\alpha\beta}(\mathbf{k}) = \underline{a(k)}\left[P_{i\alpha}(\mathbf{k})P_{j\beta}(\mathbf{k}) + P_{i\beta}(\mathbf{k})P_{j\alpha}(\mathbf{k})\right] + \underline{b(k)}P_{ij}(\mathbf{k})\hat{k}_\alpha\hat{k}_\beta$$

$$\underline{a(k)} = A\varepsilon^{1/3}k^{-13/3},\ \underline{b(k)} = B\varepsilon^{1/3}k^{-13/3}$$

By K41

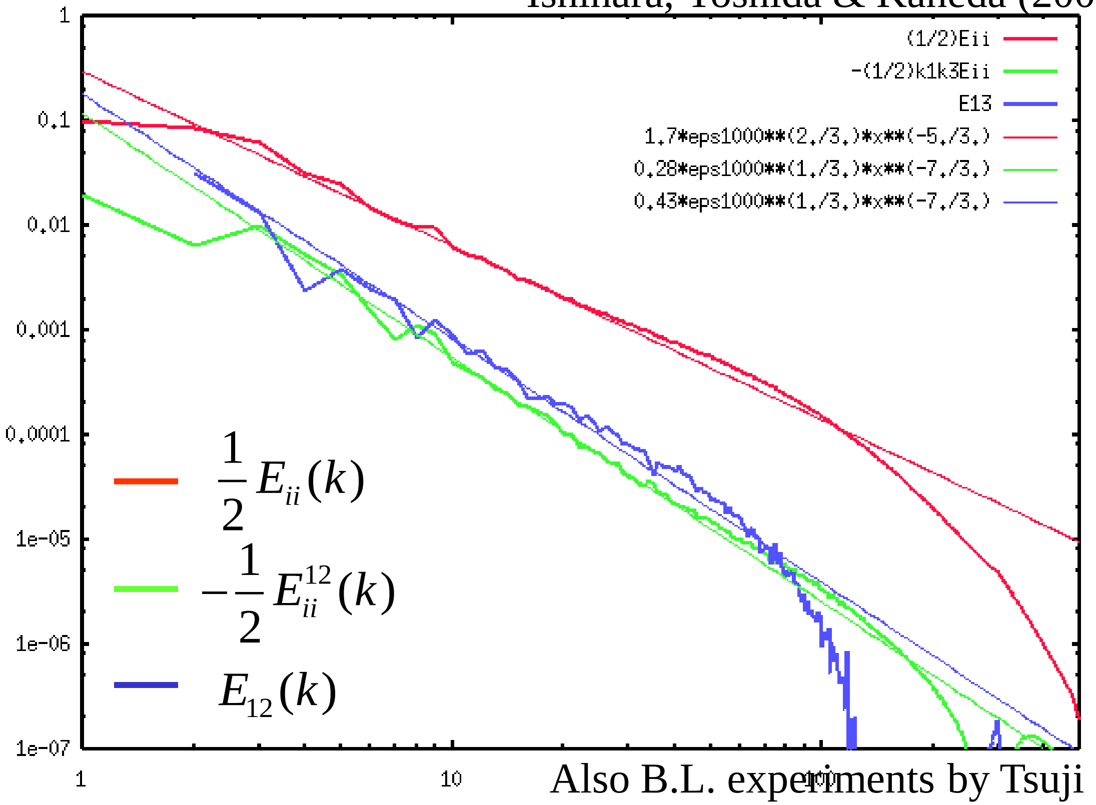
Only 2 (universal) parameters, A and B

Is this correct?
What are the values of A and B?

Anisotropic Energy spectrum in homogeneous turbulent shear flow -- Comparison with DNS

Mean Flow $\mathbf{U} = \begin{pmatrix} Sx_2 \\ 0 \\ 0 \end{pmatrix}$

Ishihara, Yoshida & Kaneda (2002)



Also B.L. experiments by Tsuji

$$E_{ij}(k) = \sum_{p=k} \left\langle u_i(\mathbf{p}) u_j(-\mathbf{p}) \right\rangle$$

$$E_{ij}^{ab}(k) = \sum_{p=k} \hat{p}_a \hat{p}_b \left\langle u_i(\mathbf{p}) u_j(-\mathbf{p}) \right\rangle$$

Theoretical predictions:

$$\frac{1}{2} E_{ii}^{12}(k) = \frac{4\pi}{15}(-A+B)\xi$$

$$E_{12}(k) = \frac{4\pi}{15}(7A-B)\xi$$

$$E_{12}^{11}(k) = E_{12}^{22}(k) = \frac{4\pi}{105}(13A-3B)\xi$$

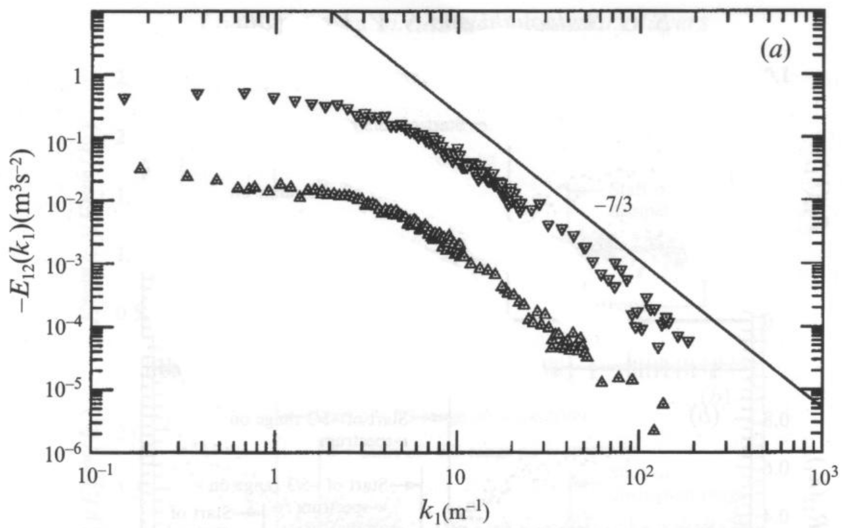
$$E_{12}^{33}(k) = \frac{4\pi}{105}(23A-B)\xi, \cdots$$

where $\xi = S\varepsilon^{1/3} k^{-7/3}$

Comparison with experiments

1D cross spectrum: $\bar{E}_{12}(k_1)$ $\left(\int_0^\infty \bar{E}_{12}(k_1)dk_1 = -\langle u_1 u_2 \rangle\right)$

$-\bar{E}_{12}(k_1)$ Saddoughi &Veeravalli(1994)



$$\bar{E}_{12}(k_1) = -C_0 \varepsilon^{1/3} k_1^{-7/3} S$$

$C_0 \approx 0.15$

Theory: $C_0 = \dfrac{36\pi}{1729}(-33A + 7B)$

$A \approx -0.16,\ B \approx -0.40$ from DNS

give $C_0 \approx 0.16.$

See Yoshida, Ishihara & Kaneda (PF,2003,14,pp.2385)

Stratified turbulence: Boussinesq approximation

$$\begin{aligned} \frac{\partial}{\partial t}\mathbf{u} &= -(\mathbf{u}\cdot\nabla)\mathbf{u} - \nabla p + \nu\nabla^2\mathbf{u} \boxed{-N\rho\mathbf{e}_3}, \\ \nabla\cdot\mathbf{u} &= 0, \\ \frac{\partial}{\partial t}\rho &= -(\mathbf{u}\cdot\nabla)\rho + \kappa\nabla^2\rho \boxed{+Nu_3}, \end{aligned}$$

Buoyancy by Stratification

$$P(\mathbf{k},t) = \frac{1}{(2\pi)^3}\int d^3\mathbf{r}\langle\rho(\mathbf{x}+\mathbf{r},t)\rho(\mathbf{x},t)\rangle e^{-i\mathbf{k}\cdot\mathbf{r}},$$

$$B_i(\mathbf{k},t) = -\frac{1}{(2\pi)^3}\int d^3\mathbf{r}\langle u_i(\mathbf{x}+\mathbf{r},t)\rho(\mathbf{x},t)\rangle e^{-i\mathbf{k}\cdot\mathbf{r}}.$$

(Yoshida & YK, New J. Phys.2004)

MHD turbulence;
Quasi-Static approximation

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \mathrm{grad})\, \boldsymbol{u} = -\frac{1}{\rho}\mathrm{grad} p + \nu \triangle \boldsymbol{u} + \frac{1}{\rho}\boldsymbol{F},$$

$$\mathrm{div} \boldsymbol{u} = 0,$$

Magnetic force

$$\frac{\partial \boldsymbol{B}}{\partial t} = (\boldsymbol{B} \cdot \mathrm{grad})\, \boldsymbol{u} - (\boldsymbol{u} \cdot \mathrm{grad})\, \boldsymbol{B} + \eta_{\mathrm{e}} \triangle \boldsymbol{B}$$

$$\mathrm{div} \boldsymbol{B} = 0$$

$$\boldsymbol{F} = \frac{1}{\mu_{\mathrm{e}}} (\boldsymbol{B} \cdot \mathrm{grad})\, \boldsymbol{B}$$

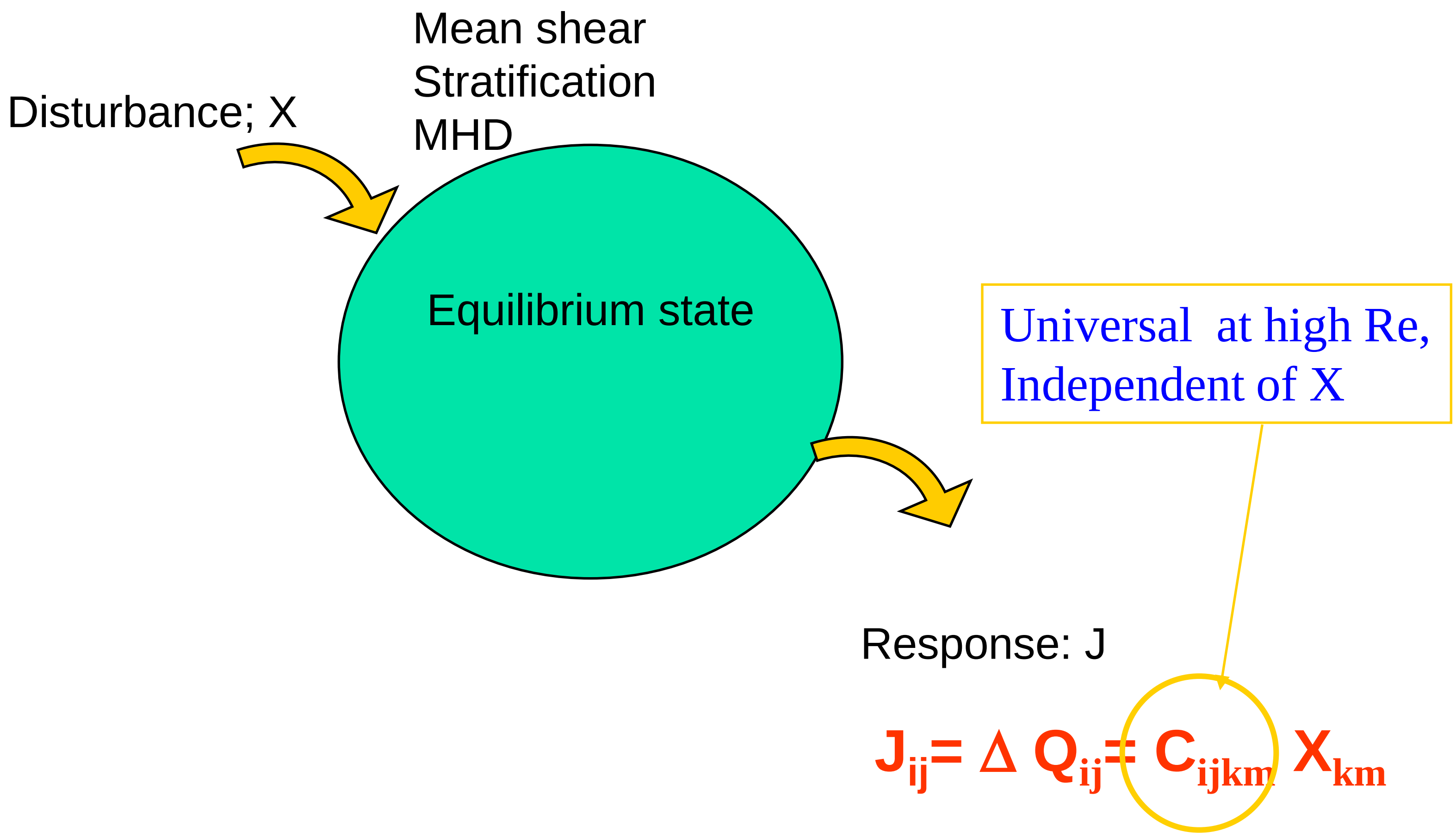
quasi-static approximation

$$\boldsymbol{F}' = \frac{1}{\mu_{\mathrm{e}}} (\boldsymbol{B} \cdot \mathrm{grad})\, \boldsymbol{B}$$

$$\simeq \frac{1}{\mu_{\mathrm{e}}} (\boldsymbol{B}_0 \cdot \mathrm{grad})\, \boldsymbol{b} \quad + O(b^2)$$

$$= -\frac{\sigma_{\mathrm{e}}}{\rho_{\mathrm{e}}} \triangle^{-1} (\boldsymbol{B}_0 \cdot \mathrm{grad})^2\, \boldsymbol{u}$$

(Ishida & YK, Phys. Fluids, 2007)



Mean shear
Stratification
MHD
Disturbance; X
Equilibrium state
Universal at high Re,
Independent of X
Response: J
$J_{ij} = \Delta Q_{ij} = C_{ijkm} X_{km}$

Conclusion of (IV)

- **a Conjecture**

Although we don’t know how to accurately specify the “equilibrium” state of turbulence, we may apply an idea similar to the linear response theory.

There may be a certain kind of universality
not only in the equilibrium state itself ,
but also in the response to disturbance added to the equilibrium state.

THE END

Thank you for your attention

Welcome your comments & questions