**CNR** National Research Council
**ISAC** Institute of Atmospheric Sciences and Climate

# Small scale properties of particles in turbulent flows

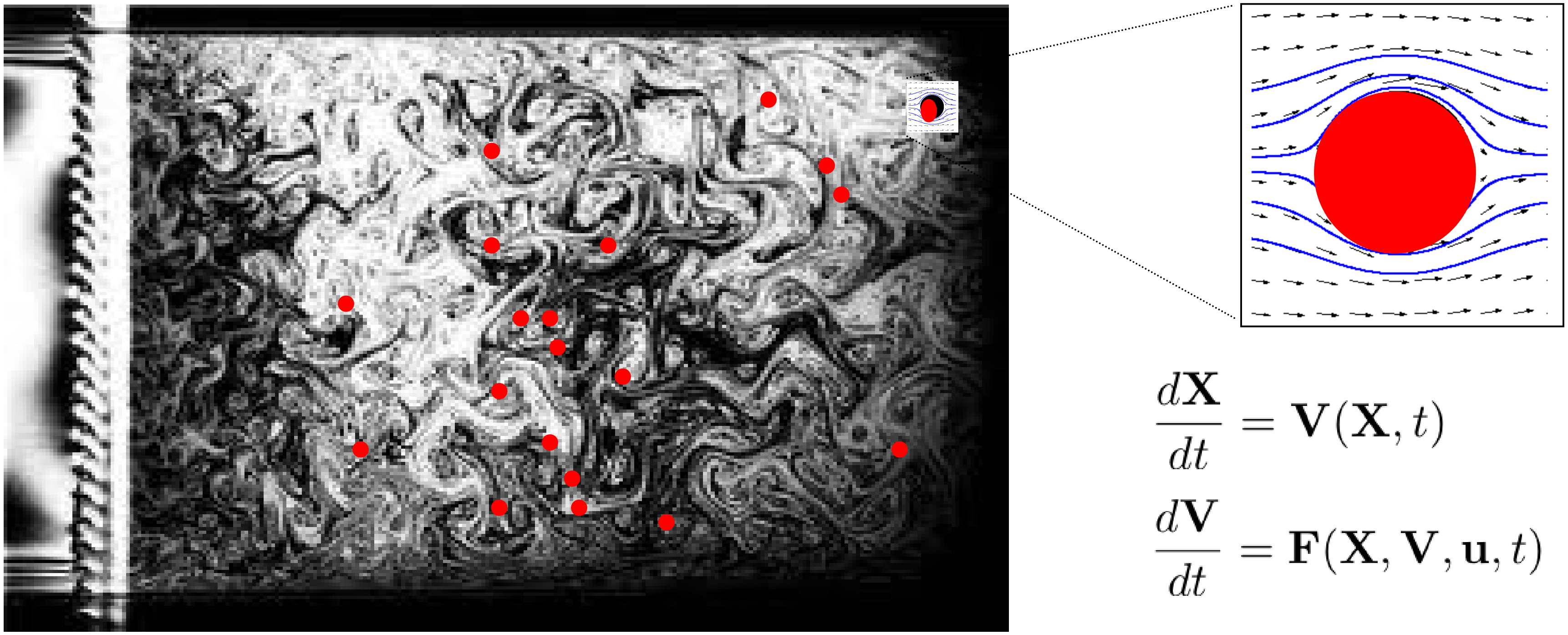
**Alessandra S. Lanotte**

**ISAC & KITP**

Jeremie Bec
Luca Biferale
Massimo Cencini
Stefano Musacchio
Andrea Scagliarini
Federico Toschi

*KITP The Nature of Turbulence* *May 17, 2011*

# General problem of two-phase dispersed flows



$$\frac{d\mathbf{X}}{dt} = \mathbf{V}(\mathbf{X}, t)$$

$$\frac{d\mathbf{V}}{dt} = \mathbf{F}(\mathbf{X}, \mathbf{V}, \mathbf{u}, t)$$

**Dispersed phase:**
very small spherical particles

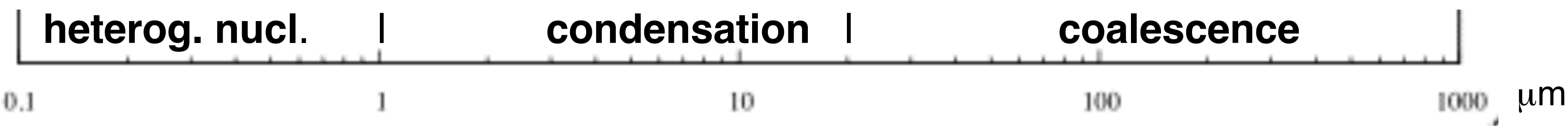
Particle/hydro-turbulence interactions

**Fluid phase:**
multi-scale flow

$$\mathbf{u} = \mathbf{u}(\mathbf{x}, t)$$

Ignore:
Particle/particle interactions
Turbulence modification by particles
Particle deformation, surface tension effects

# A way for liquid droplet size evolution




**Field description**:

Time-space evolution of local droplet distribution over size a: n(a,**X**,t)

**advection**

**condensation**

$$\partial_t n(a,\mathbf{X},t) + \nabla \cdot (\mathbf{v}\, n(a,\mathbf{X},t)) = -Q(s,\kappa,M)\,\frac{\partial}{\partial a}\,\frac{n(a,\mathbf{X},t)}{a}$$

$$+\int da' \left[\frac{K(a',a'')\,n(a')\,n(a'')}{2(a''/a)^2} - K(a',a)\,n(a')\,n(a)\right]$$

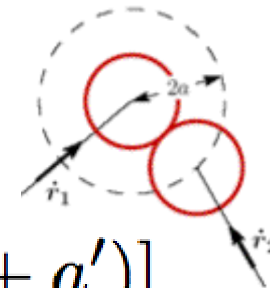
**coalescence**

- very complex due to interplay of inertia, gravity, microphysics, turbulence (Re, ε)
- need for better modeling/understanding of various regimes

# Collision rate: classical approaches

**~ Product of the local concentrations n(a) and n(a') of drops of size a and a', multiplied the target area, and the relative velocity.**

- Saffman & Turner (1956) estimate homogeneous distribution of droplets in a local mean shear $\gamma$:




$$K(a,a')n(a)\,n(a') \sim n(a)\,n(a')(a+a')^2\,[\gamma\,(a+a')]$$

! Valid for very small size: a ~ 5 $\mu$m

!! Relevant for energetic turbulence $\varepsilon > 0.1\ m^2/s^3$

- Eventually gravity dominates the dynamics

$$K_g(a,a')n(a)\,n(a') \sim E(a,a')\,n(a)\,n(a')(a+a')^2\,|V_g(a)-V_g(a')|$$

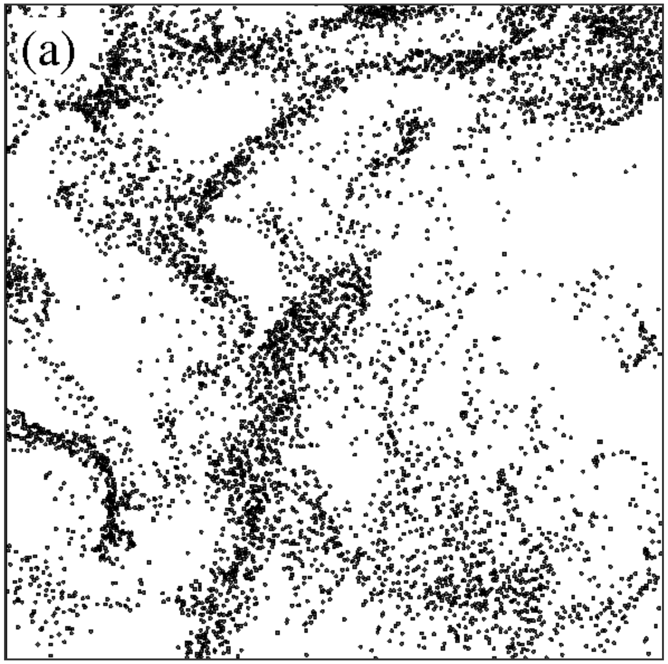
! Relevant for big particles and polidisperse

# Turbulence induced collisions for monodisperse suspension

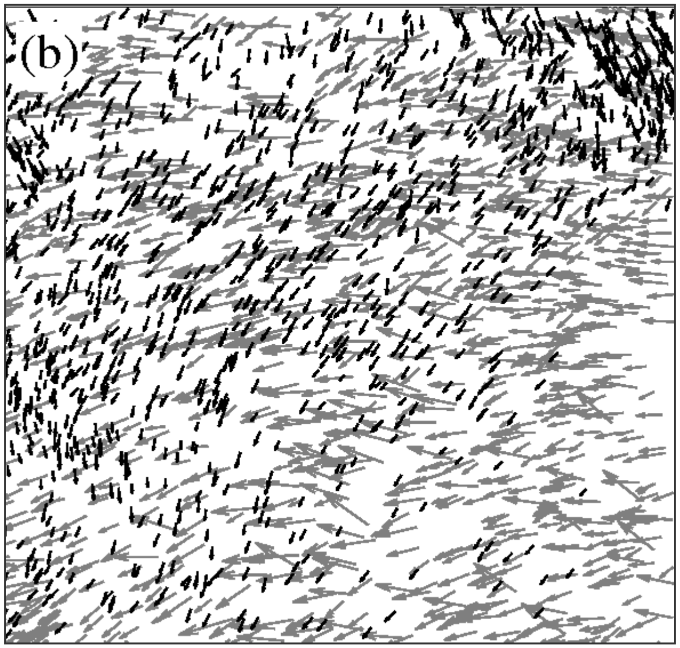
## Ghost collision approach

(Sundaram & Collins 1997)

$$K(a,a')n(a)\,n(a') \sim n^2(a)(2a)^2\,g(2a)\int \Delta V\,P(\Delta V|(a+a))\,d\Delta V$$




St=2




St=20

**PREFERENTIAL CONCENTRATION**

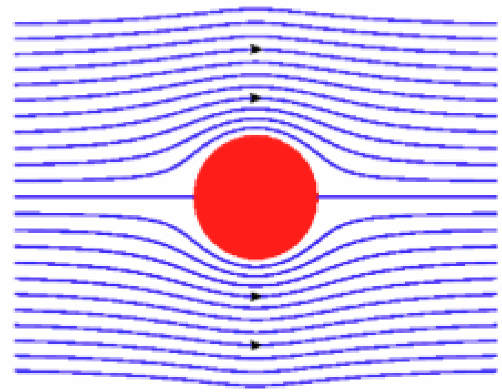
**VELOCITY DISTRIBUTION**

*Focus on : Reynolds and Stokes dependence*

# Point-droplet Model

**Spherical particles**:

Gatignol (1983), Maxey & Riley (1983)



- radius *a* much smaller than the Kolmogorov scale $\eta$
- dilute solution: no collisions
- they have a very small Reynolds number $Re_p = \frac{|V_a - u|a}{\nu} \ll 1$
- ***very heavy particles $\rho_a >> \rho_f$***

Spherical particles in an **unsteady Stokes flow**

$$m_a \frac{d\,\mathbf{V}_a}{dt} = 6\pi a(\rho_f\,\nu)[\mathbf{u} - \mathbf{V}_a] + (m_a - m_f)\mathbf{g}$$

*particle Stokes time* $\tau_a = \frac{2}{9}\frac{a^2\rho_a}{\rho_f\nu}$

**Advantages**

- Turbulent motion accurately solved
- Inertial small-scale effects are described
- Description is *valid* up to ~ big particles

**Shortcomings**

- Comput. demanding
- Moderate turbulence
- Finite-size effect missing

# 3D Direct Numerical Simulations

$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u} = -\boldsymbol{\nabla} P + \nu \nabla^2 \boldsymbol{u} + \boldsymbol{F}$$

$$\frac{d\mathbf{V}}{dt} = \frac{\mathbf{u} - \mathbf{V}}{\tau_p}$$

$$Re_\lambda \sim \frac{T_L}{\tau_\eta}$$

$$St = \frac{\tau_p}{\tau_\eta}$$

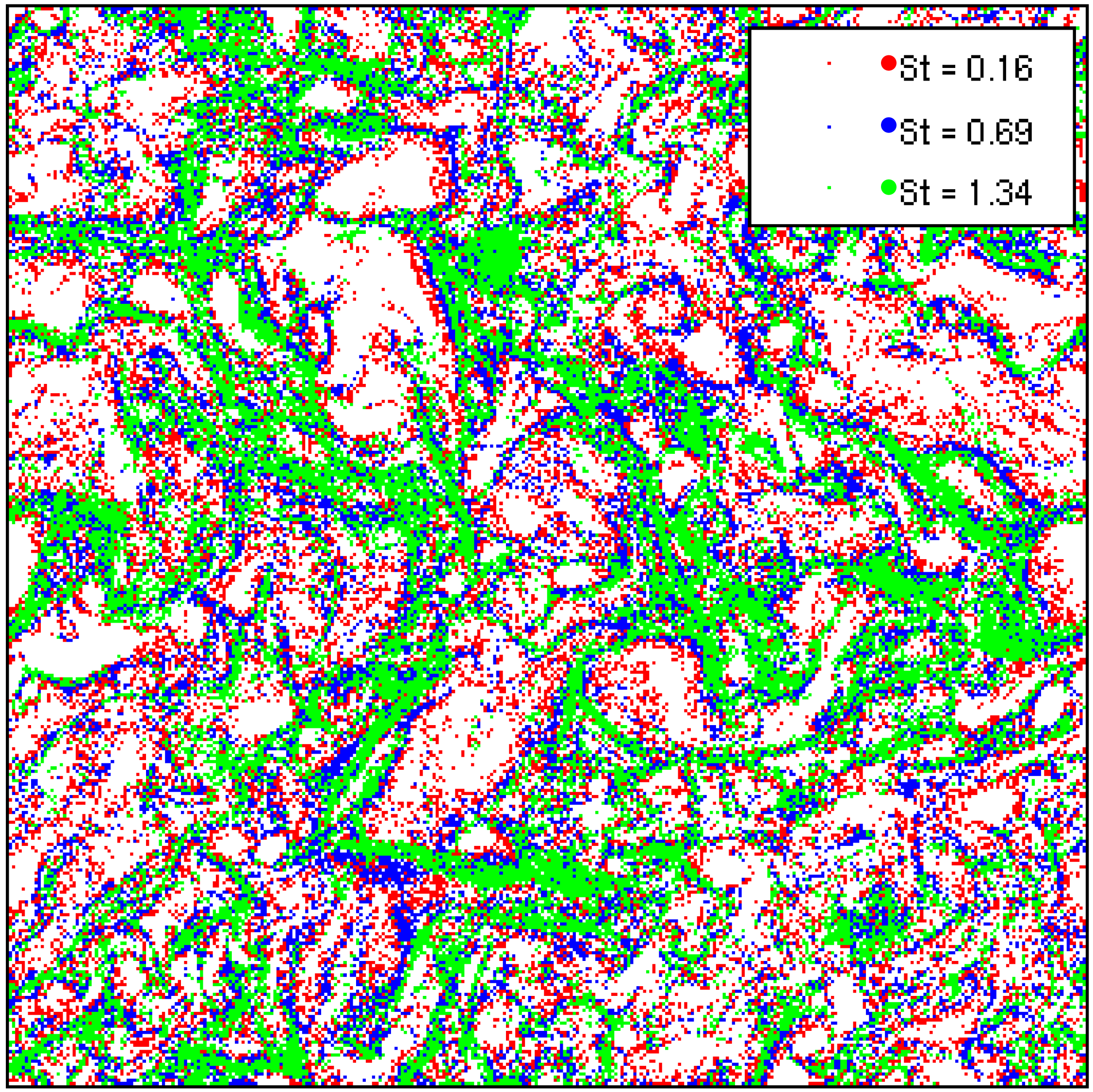
- Homogeneous & isotropic flow
- Passive heavy particles (1-way coupling)

| $N^3$ [ $Re_\lambda$ ] | $256^3$ [$Re_\lambda$ = 105] | $512^3$ [$Re_\lambda$ =185] | $2048^3$ [$Re_\lambda$ = 400] |
|---|---|---|---|
| St [range] | [0.16 - 3.3] | [0.16 - 3.3] | [0.16 - 70] |
| Slow dumps $10\tau_\eta$ | 2Million | 7.5Million | 100 Million |
| Fast dumps $0.1\tau_\eta$ | 250,000 | 500,000 | 200,000 |
| $L / \eta$ | 0.25m / 1mm | 0.5m / 1mm | 2m / 1mm |

# Spatial distribution of particles

Slice width ≈ 2.5η;

*L*




$$St = \frac{\tau_p}{\tau_\eta}$$

# Dissipative range clustering

## 3D Particle Radial distribution function

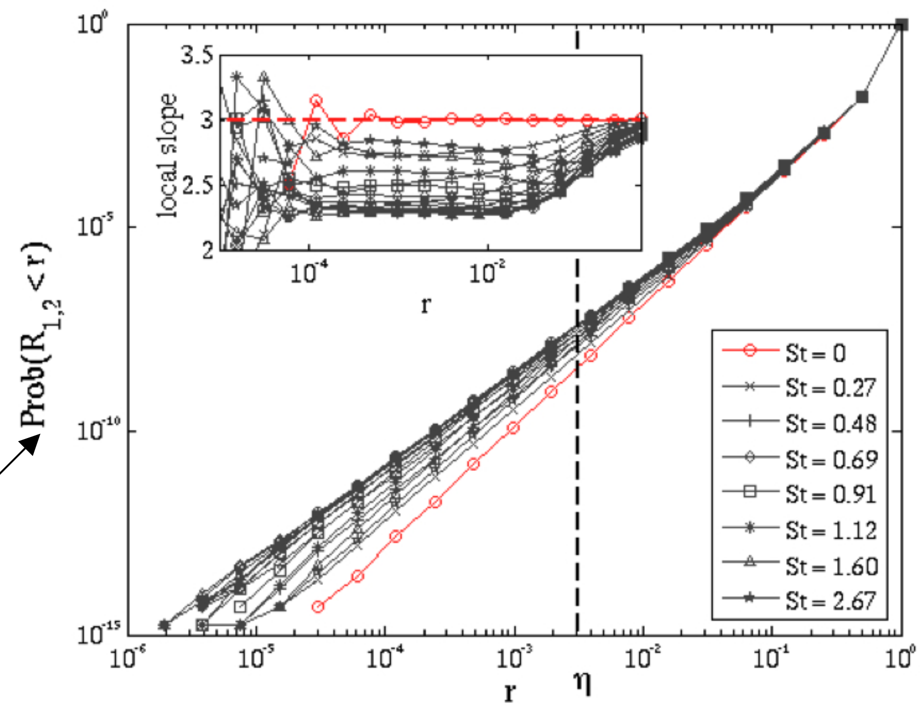
$$\begin{cases} g(r) \sim 1 & \text{Poissonian} \\ g(r) \sim \left(\frac{\eta}{r}\right)^{c_1} ; \quad c_1 > 0 & \text{Non homogeneous distribution} \end{cases}$$

Probability to find 2 particles at a distance smaller than r : $P(R_{12} < r) = P_2(r)$

$$g(r) = \frac{P_2(r)}{r^3} \sim \left(\frac{\eta}{r}\right)^{3-D_2}$$

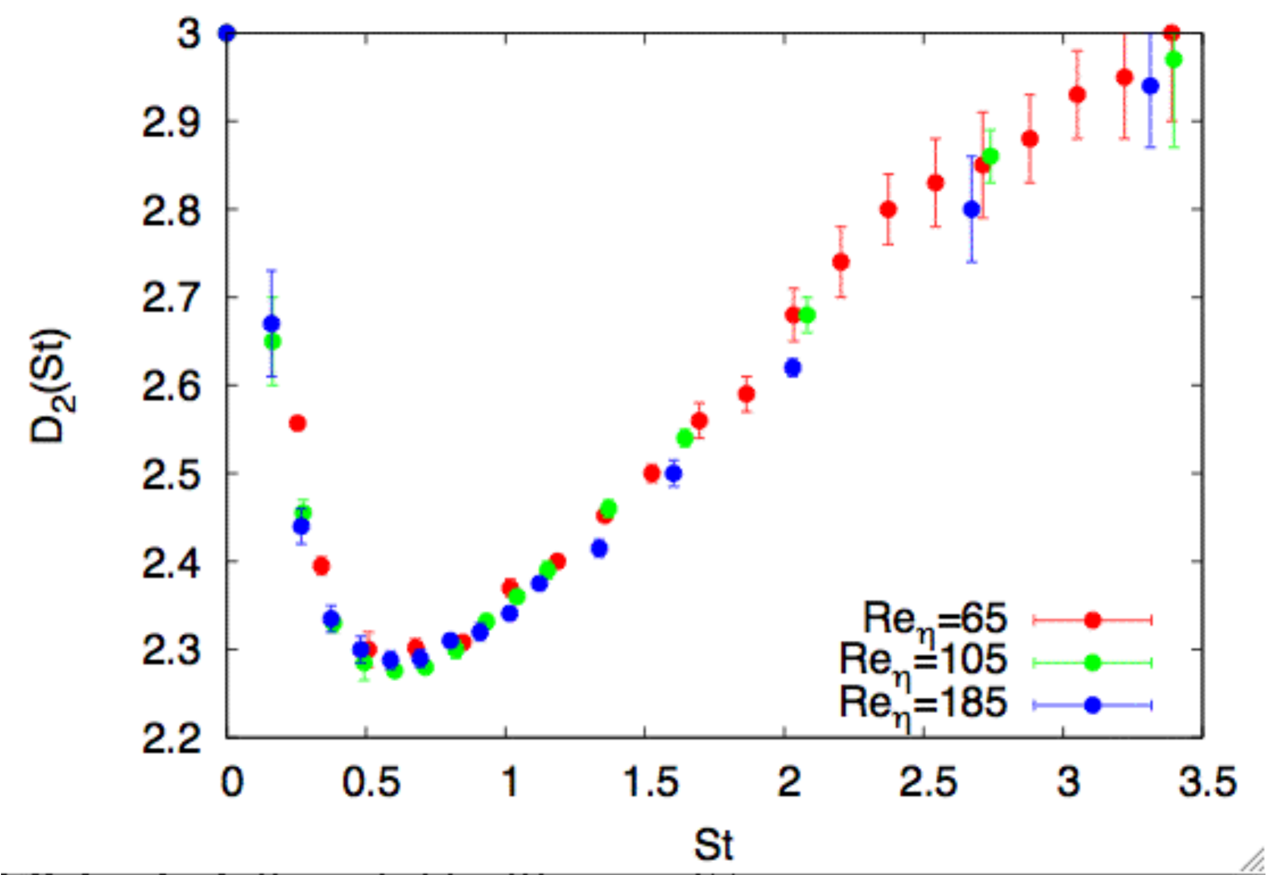
$D_2$ is the *correlation dimension*
(Grassberger 1983 ;
Hentschel & Procaccia,1983)

$$P_2(r) \sim r^{D_2}$$

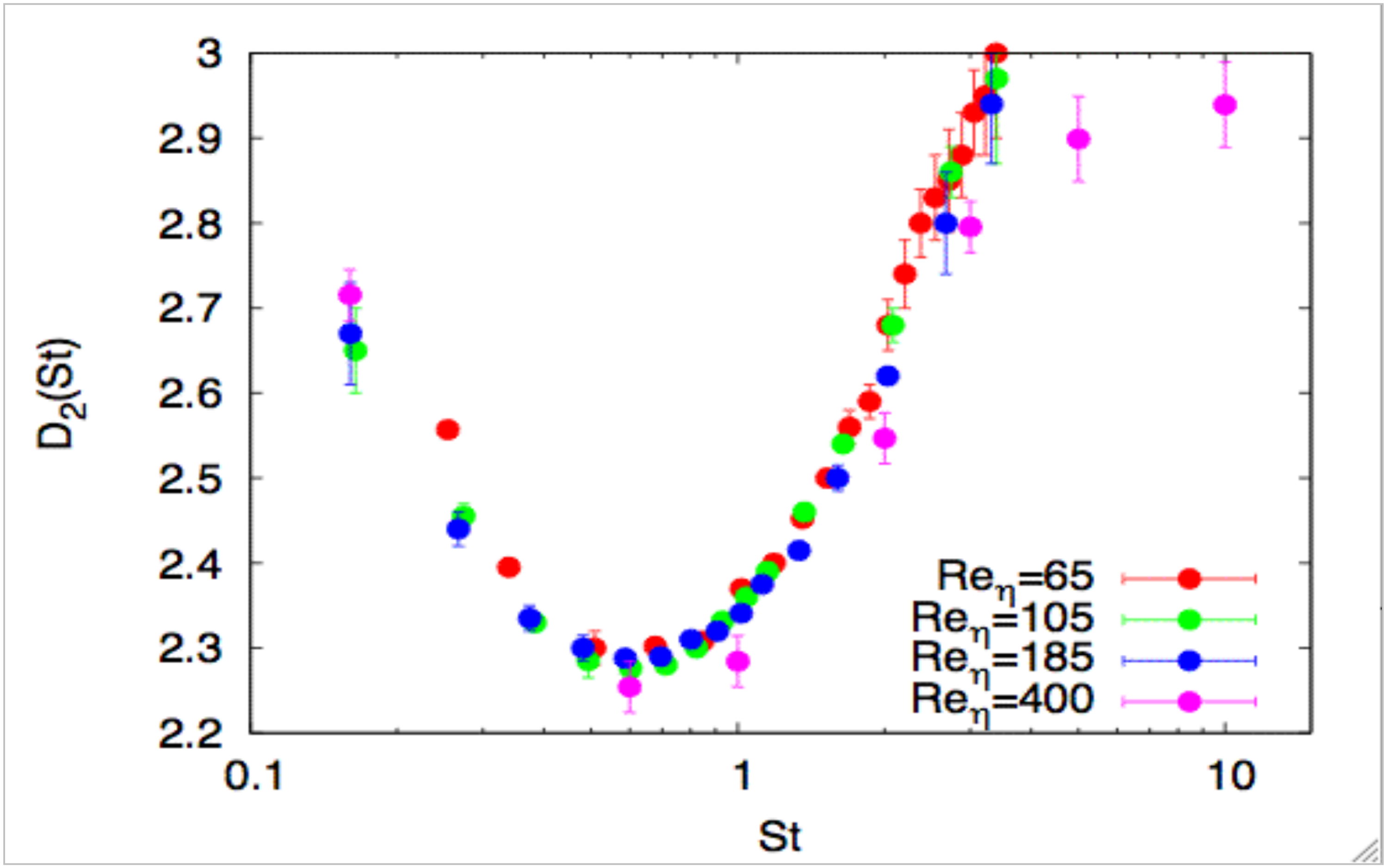


# Preferential concentration

| **Radius/ Kin energy dis** | a = 10 μm | a = 25μm | a = 50 μm | a =75 μm |
|---|---|---|---|---|
| $\varepsilon = 0.001 m^2/s^3$ | **St = 0.02** | **St = 0.14** | **St = 0.5** | **St = 1.2** |
| $\varepsilon = 0.01 m^2/s^3$ | **St = 0.06** | **St = 0.4** | **St = 1.7** | **St = 3.8** |



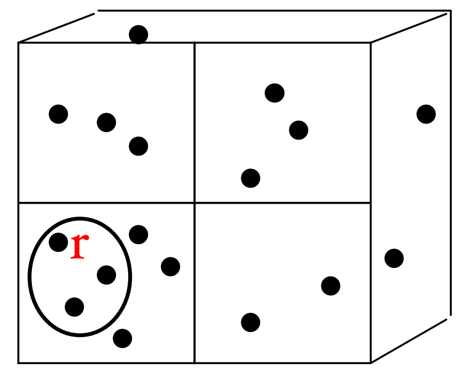
# Absent or weak Re effect




What about the fractal nature of the dynamic set where particles cluster?

# Dissipative range Mass distribution

**Generalise the idea:**
Estimate the probability to measure **q** particles randomly chosen, lying within a distance r.

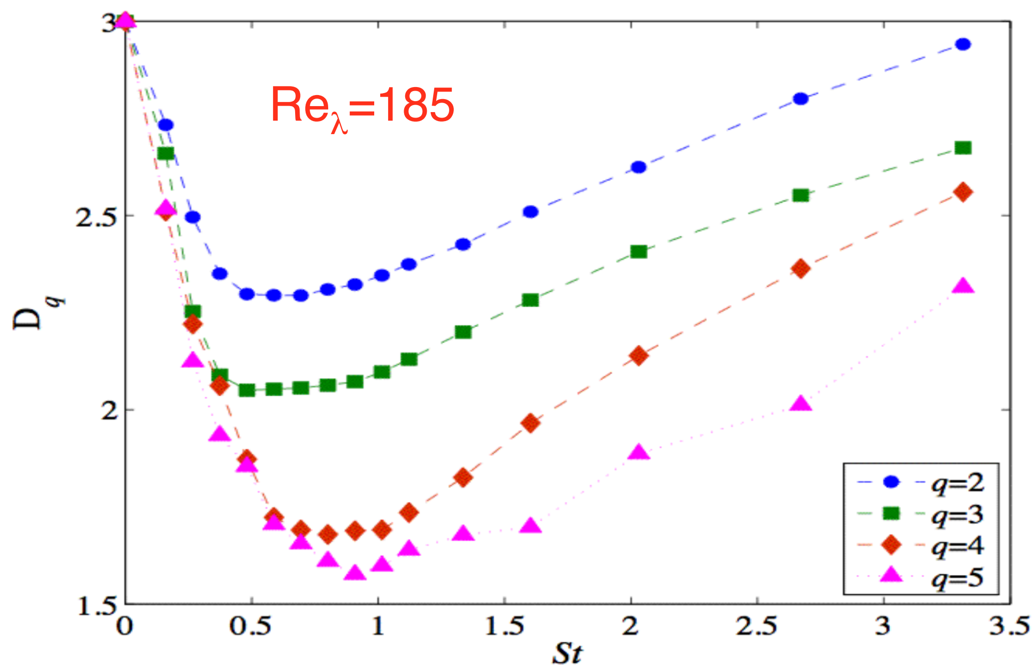



By Large deviation theory:

$$P_q(r;St) \sim r^{(q-1)D_q}$$

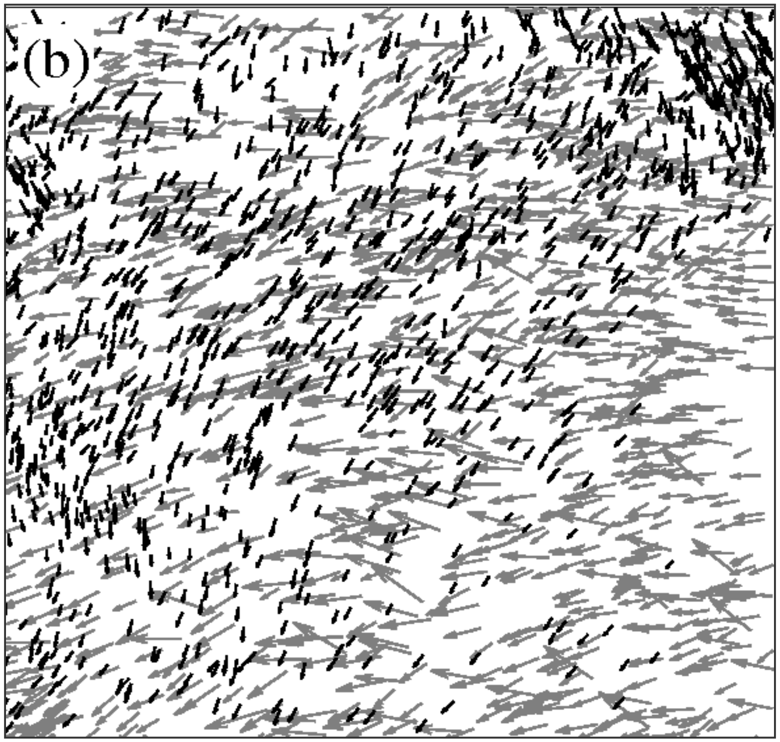
Multifractal particle mass distribution

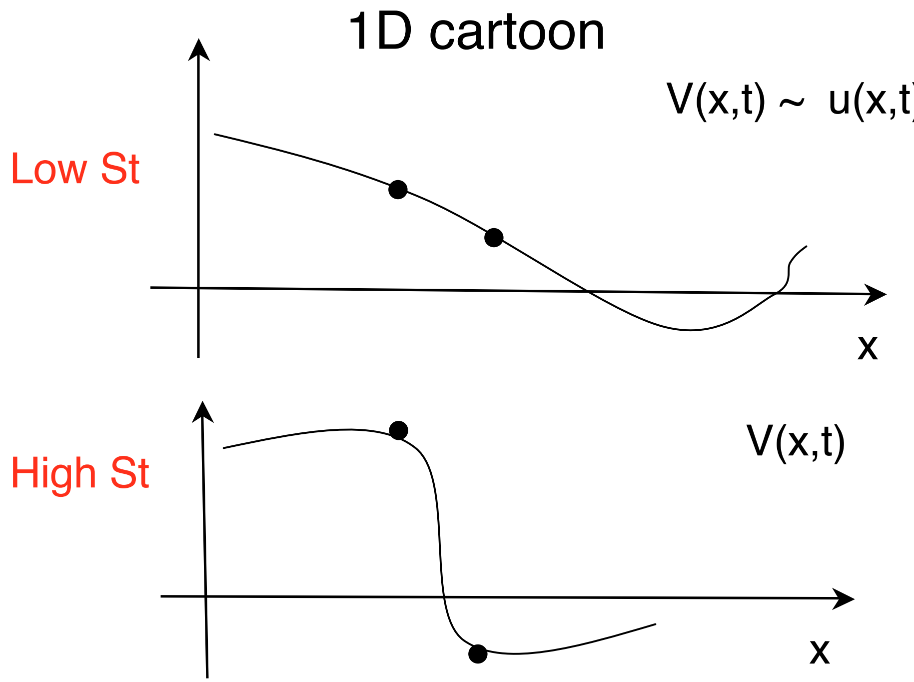
$$D_q \neq D_p$$




- At higher Reynolds, no differences are detectable
- Picture similar to random flows (Bec, Gawedzki, Horvai 2004)

# Heavy particles velocity behaviour




How are the velocity differences at changing Stokes and separation the scale r?

Are these singular events statistically relevant?
How do they appear & which is their statistical weight?
Do they exist at all Stokes numbers St>0 ?
Do they have a typical amplitude or not?

# Low & high inertia limits

limit St=0 **tracers limit**, particles follow fluid streamlines

limit St = ∞ **ballistic limit**, a gas of free particles unaffected by the flow

$$\frac{d\mathbf{V}}{dt} = \frac{\mathbf{u} - \mathbf{V}}{\tau_p}$$

$$\sigma_{ij} = \frac{\partial u_i}{\partial x_j}$$

Writing the evolution equation for particle velocity gradients

$$s_{ij} = \frac{\partial V_i}{\partial x_j}$$

$$\frac{ds_{ij}}{dt} + s_{ik}s_{kj} = \frac{\sigma_{ij} - s_{ij}}{\tau_p}$$

--> particle velocity gradients can blow up,
particularly if inertia is large $|s_{ij}| > \frac{1}{\tau_p}$

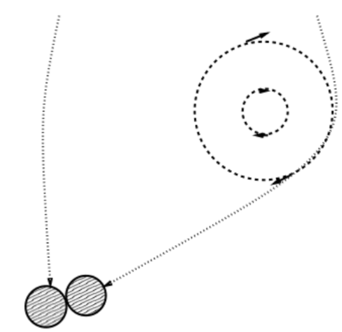
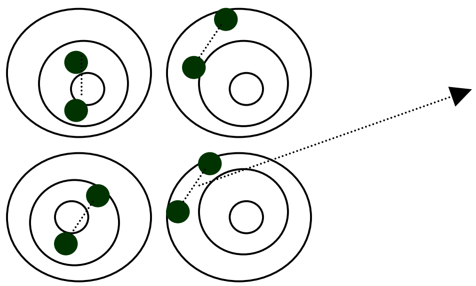


Figure 1: Sketch of the sling effect. Dotted lines show the trajectories of the droplets while broken lines show the streamlines of the vortex in the air flow.

**Particles might come close to each other with very different velocities,**
***sling effect*** ( Falkovich et al. Nature 2002
see also Wilkinson & Mehlig 2005)

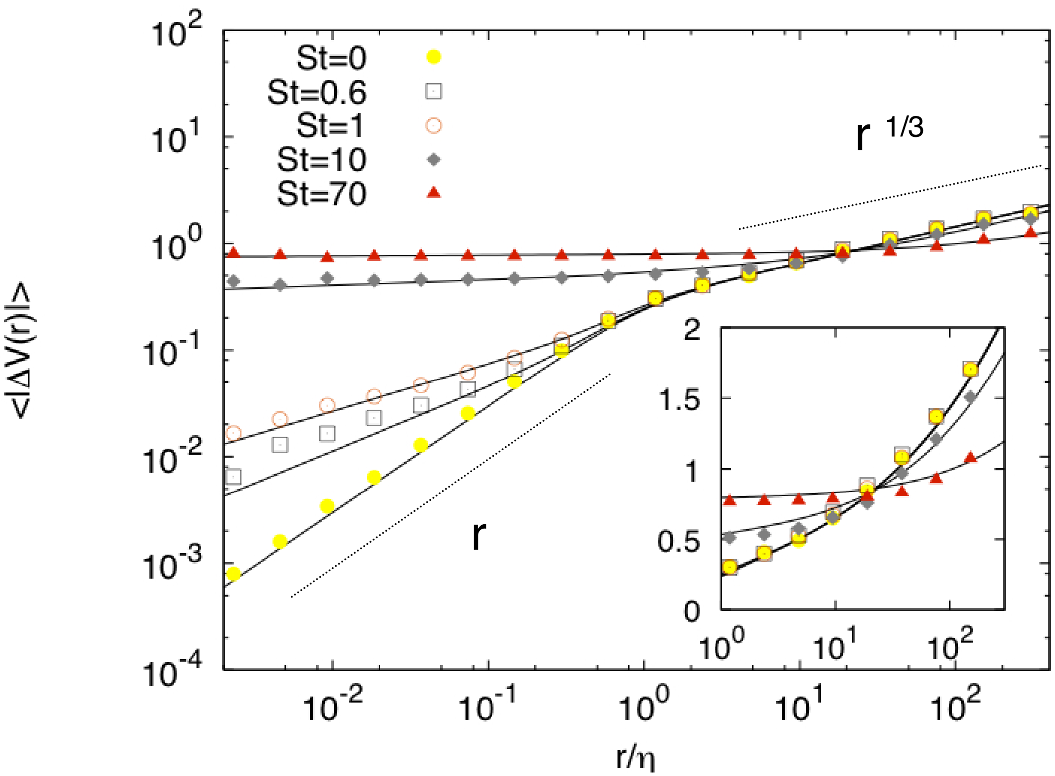
# Stationary distribution of heavy particles velocities



Take all pairs of particles at distance **r** and measure their average longitudinal velocity difference

$$S_p(r) = \langle |(\mathbf{V}(\mathbf{x}+\mathbf{r}) - \mathbf{V}(\mathbf{x})) \cdot \hat{\mathbf{r}}|^p \rangle$$

$Re_\lambda = 400$




$$\tau_p \simeq \tau_r$$

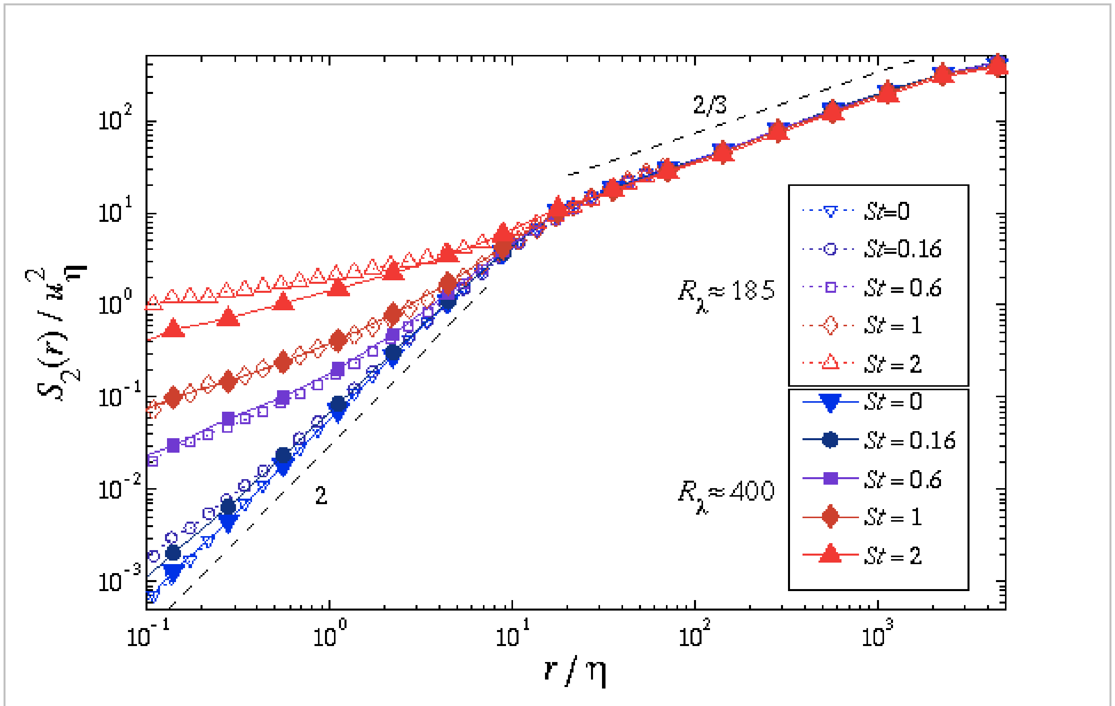
$$\downarrow$$

$$r^*(St) \sim \eta St^{3/2}$$

$r < r^*(St)$ Inertia dominates

$r > r^*(St)$ Inertia negligible

# Scale by scale behaviour




dissipative range

inertial range

First focus on small scale statistics r < h

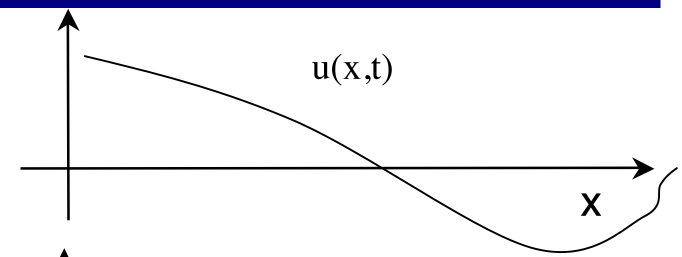
$$S_p(r, St) = \langle |\Delta_r V_{||}|^p \rangle \propto r^{\xi_p(St)}$$

# First moment of relative velocities

$$\langle|\Delta \mathbf{V}(r)_{\|}|\rangle \sim r^{\xi_1}$$

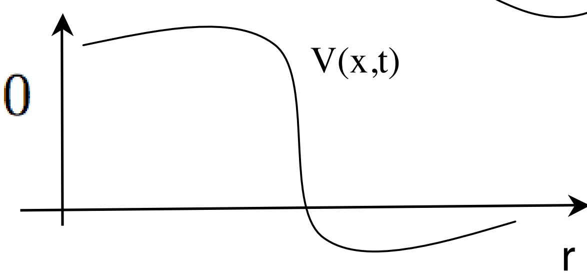
$< \Delta V(r) > \sim r; \quad \xi_1 = 1$

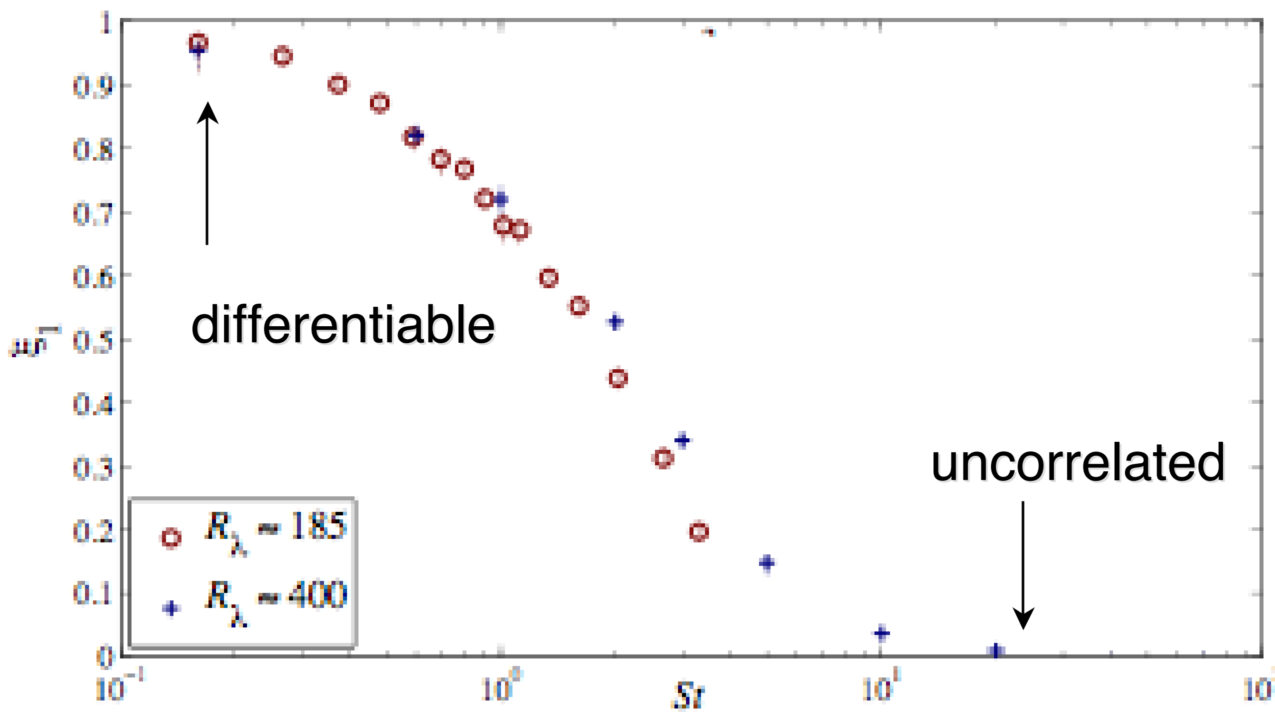
***Zero inertia limit***




$< \Delta V(r) > \sim cost; \quad \xi_1 = 0$

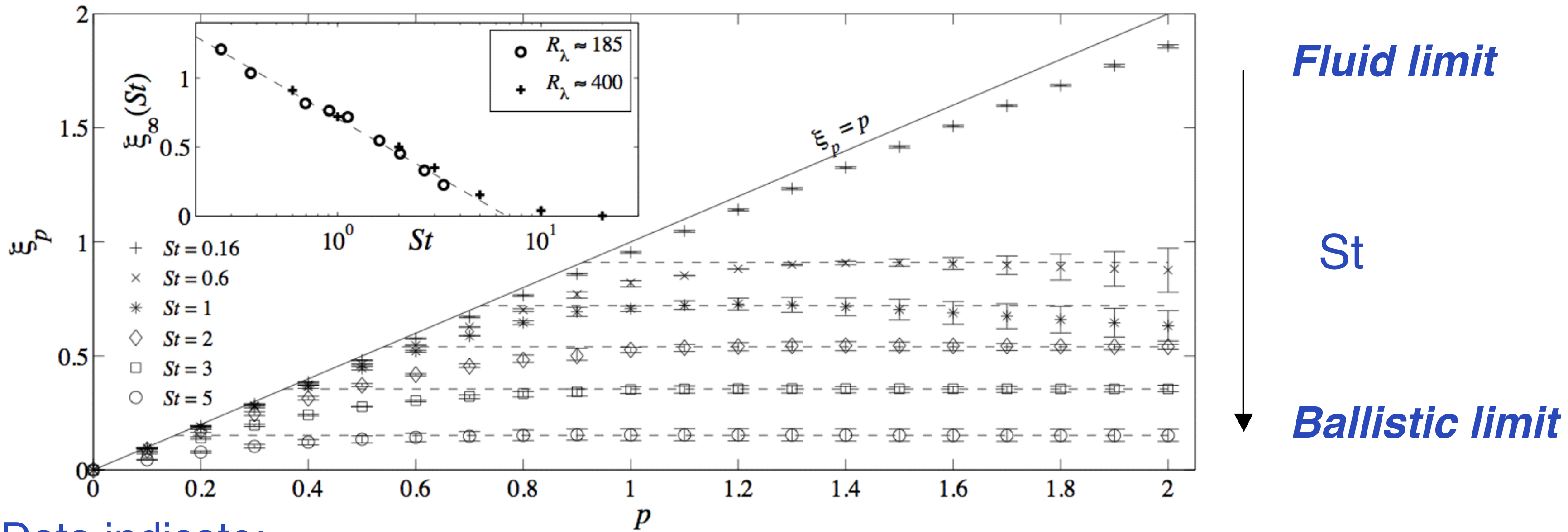
***Large inertia limit***



# Dissipative range statistics

$$S_p(r, St) = \langle |\Delta_r V_{\|}|^p \rangle \propto r^{\xi_p(St)}$$




Data indicate:

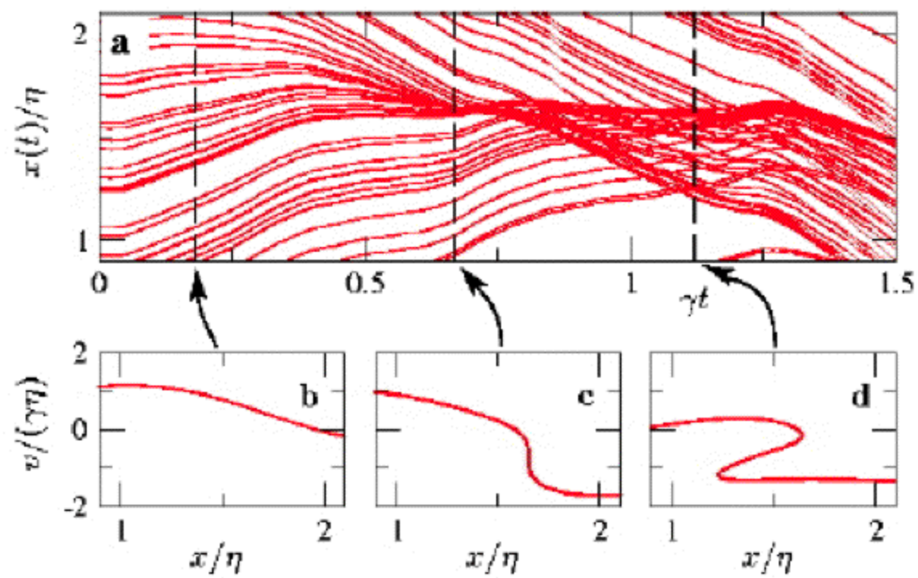
- weak Reynolds dependence;
- there is a saturation exponent, i.e caustics have a *typical* amplitude:

$$\begin{cases} \xi_\infty \propto \log(7/St), & St \leq 7 \\ \xi_\infty \simeq 0, & St \geq 7 \end{cases}$$

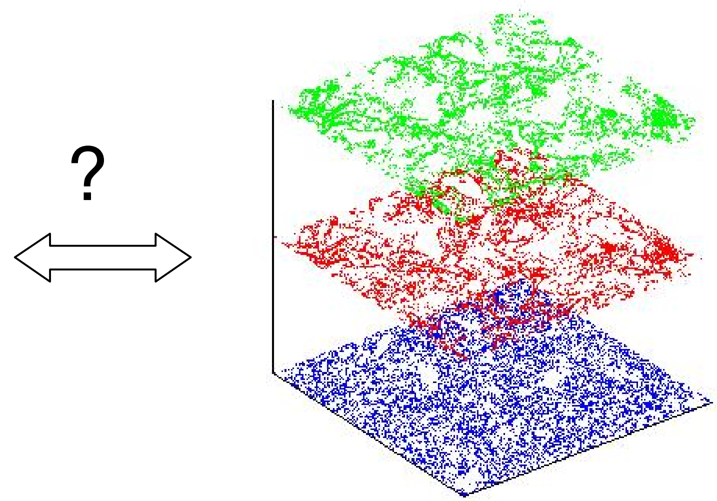
- Amplitude can be large for small Stokes, but caustics are rare
- The amplitude becomes small for large St (particle velocities decreasewith St), but caustics fill the space

$$\longrightarrow \quad P(r) \propto r^{3-\xi_\infty}$$

# How do *caustics* appear and do they relate to particle spatial properties?




?



We need to measure the steady-state joint probability of particles separation & particle relative velocity

$$\rho(\Delta r, \Delta_r V)$$

# Suggestion from 1D Model
## (Gustavsson & Mehlig 2010)

It has been suggested that

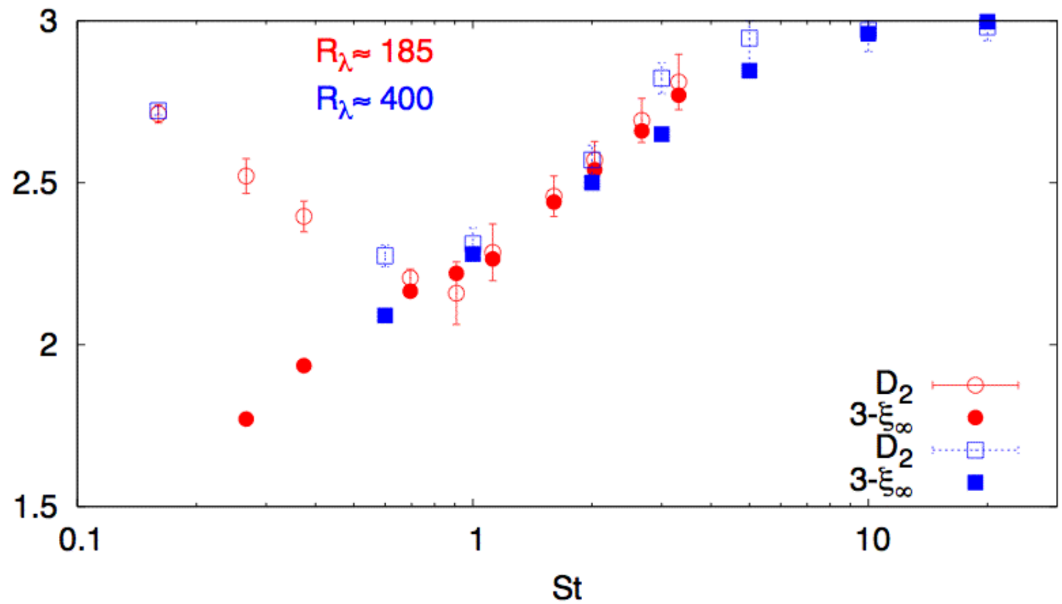
(see also Wilkinson & Mehlig 2005
Derevyanko et al. 2007)

$$S_p(r, St) = \langle (\Delta_r V_{\|})^p \rangle \sim B_p(St) \left(\frac{r}{\eta}\right)^p + e^{-A_p/St} \left(\frac{r}{\eta}\right)^{3-D_2(St)}$$

Smooth term,
with O(1) prefactor

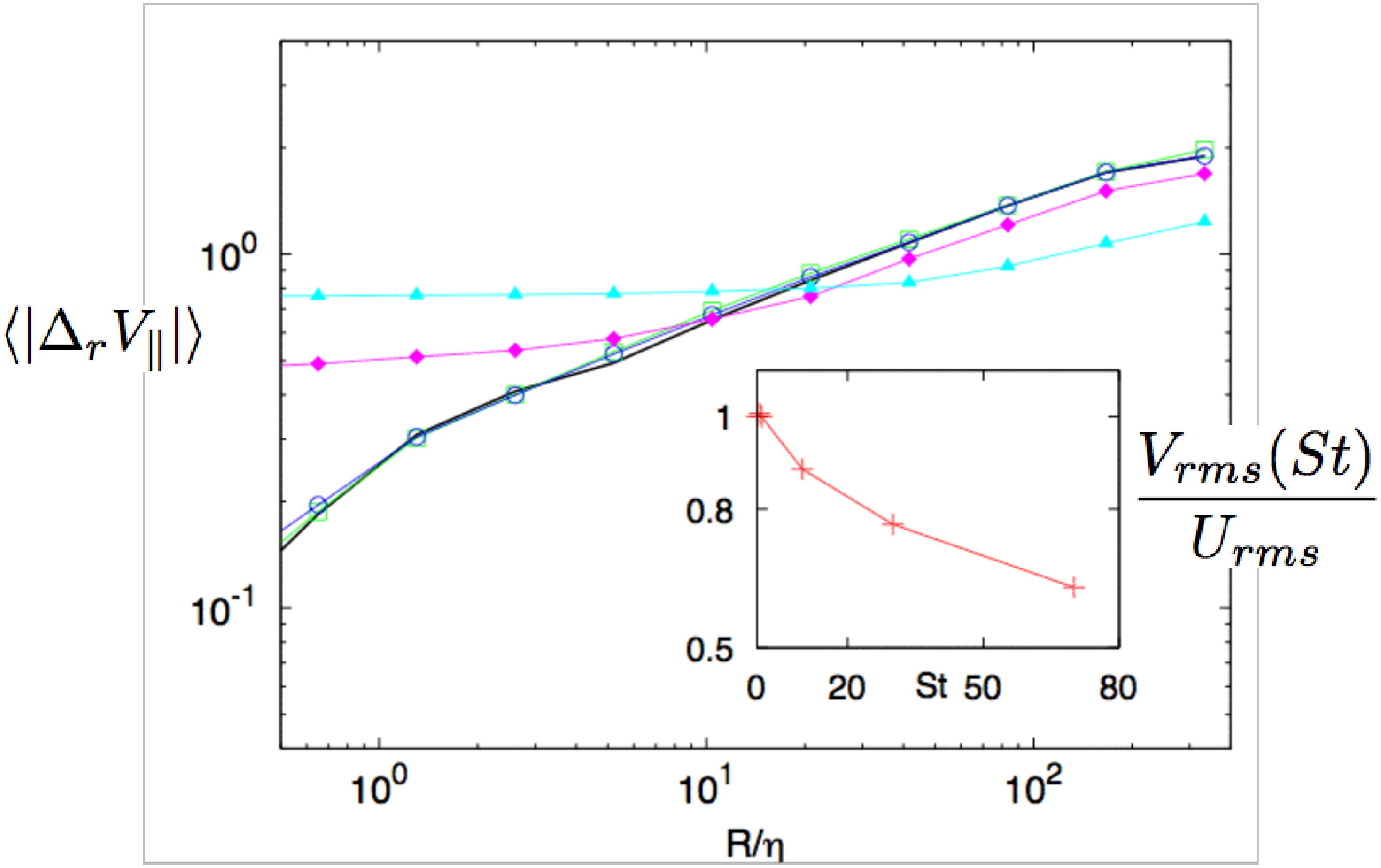
caustics are activated
as soon as St > 0.

$\xi_\infty$




Statistical relation seems to work for St>1, while it fails for small Stokes.
Reynolds effects are weak

# Rms particle velocity

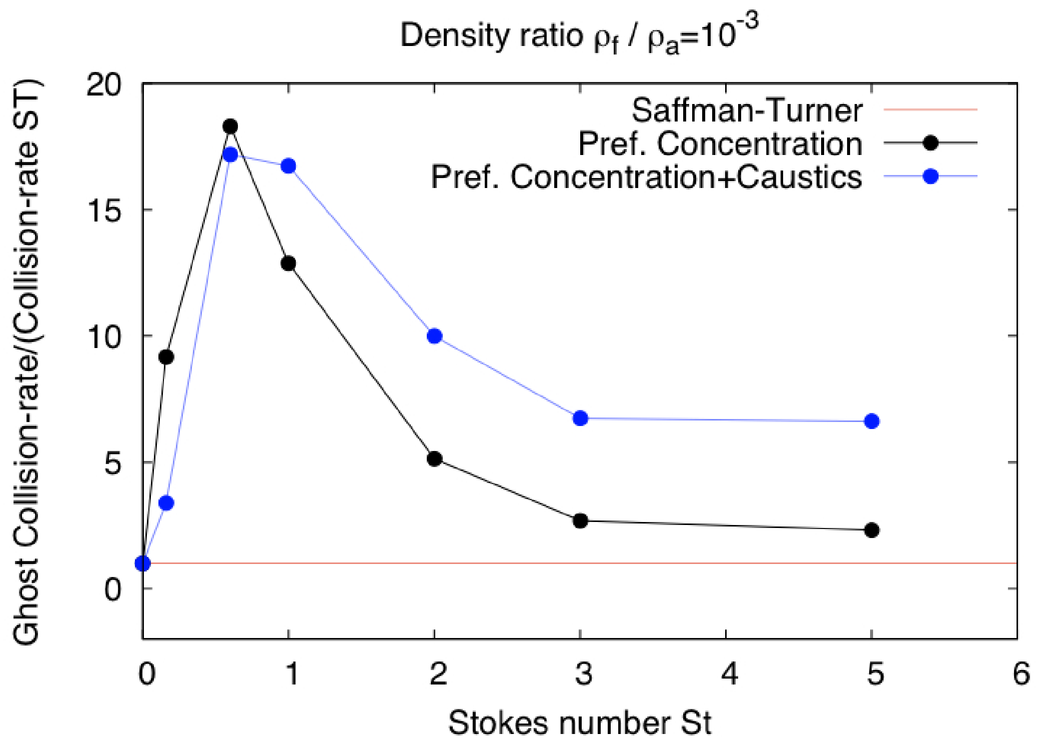


# A cartoon of ghost collisions in a monodisperse, *dilute* solution

Estimate the probability to find two **equal** droplets at distance 2a, with opposite velocities ( Stokes time $\tau_a \ll T_{coll}$)

$$N(a,a') = 2\pi(a+a')^2 n(a)n(a')\, g(a+a') \int \Delta V\, P(\Delta V|(a+a'))d\Delta V$$

$$N(a,a) \sim n^2(a)\,(2a)^2\, g(2a)\, \langle|\Delta V(2a)|\rangle$$



# Summarising

- **Preferential concentration**
  is relevant at small Stokes numbers St < 1, with a maximum around St=0.7 which seems independent of the Reynolds number.

- **Relative velocity statistics is very intermittent**
  "quasi-singularities" in the velocity stat. become more and more relevant as the Stokes number increases.
  For large St, relative velocity at very small scales is ~ O(1).
  Effective compressibiltity of particle velocity increases with inertia.
  "Caustics" can be understood as velocity jumps over nearby points of typical amplitude. In phase-space, they are associate with the folding of the manifold where particles velocities distribute.
  For St>1, velocity jumps and fractal preferential concentration of particle are related. Scaling exponents independent of Reynolds number.

- **Relevance for collisions & for particle separations**
  Pref. Concentration and caustics matter also for the way particles separate and explore different turbulent regions.

# Conclusions

We have discussed some properties of (a minimal model of)
inertial particles statistics in homogeneous and isotropic turbulence.
Even in this ***simple*** set-up, many questions are open:

- role of flow structures in ***small-scale clustering***
- nature of relative velocity singular behaviour:
  **caustics? complex folding in phase space?**
- how these effects combine into **real collisions**?
- is there any **Reynolds dependence** on caustics activation ?

**The present knowledge benefits from continuous exchange of experimental, theoretical and numerical results.**

*Applications to real phenomena is still a challenge.*

**Intermittency in the velocity distribution of heavy particles in turbulence**
*Journ. Fluid Mech*. **646**, 527 (2010).

**Turbulent pair dispersion of inertial particles** Journ. Fluid Mech. **645**, 497 (2010).