Statistics of polymer extensions in turbulent channel flow

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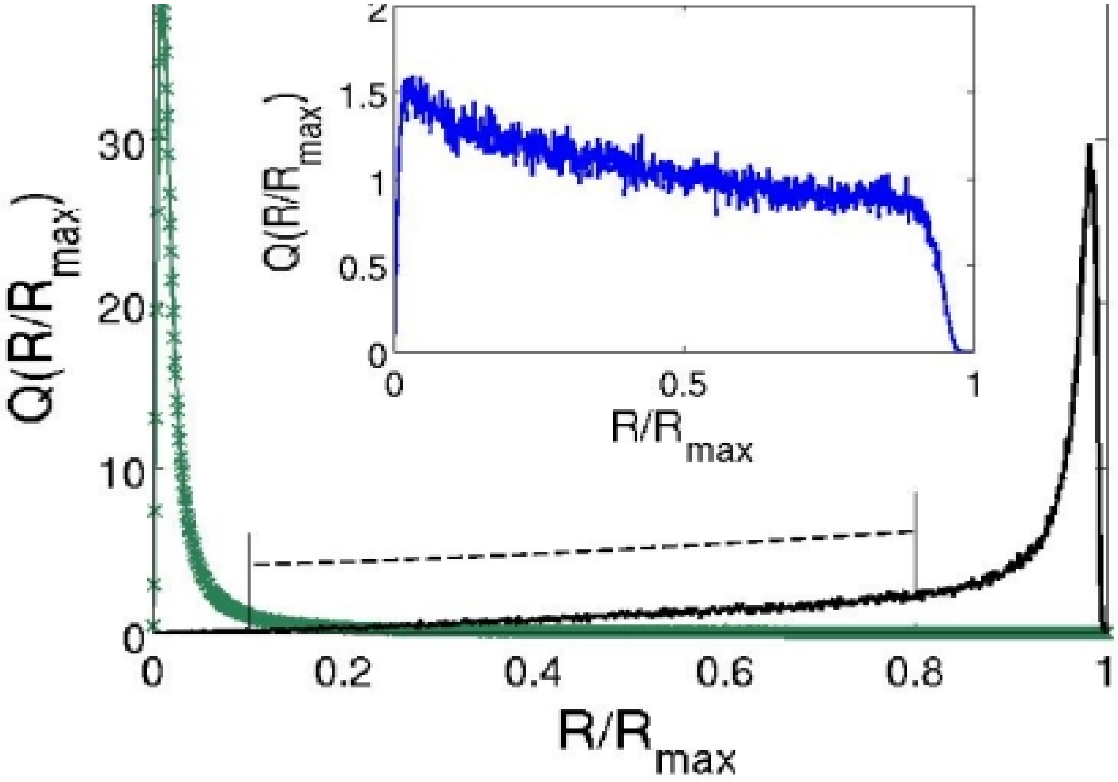
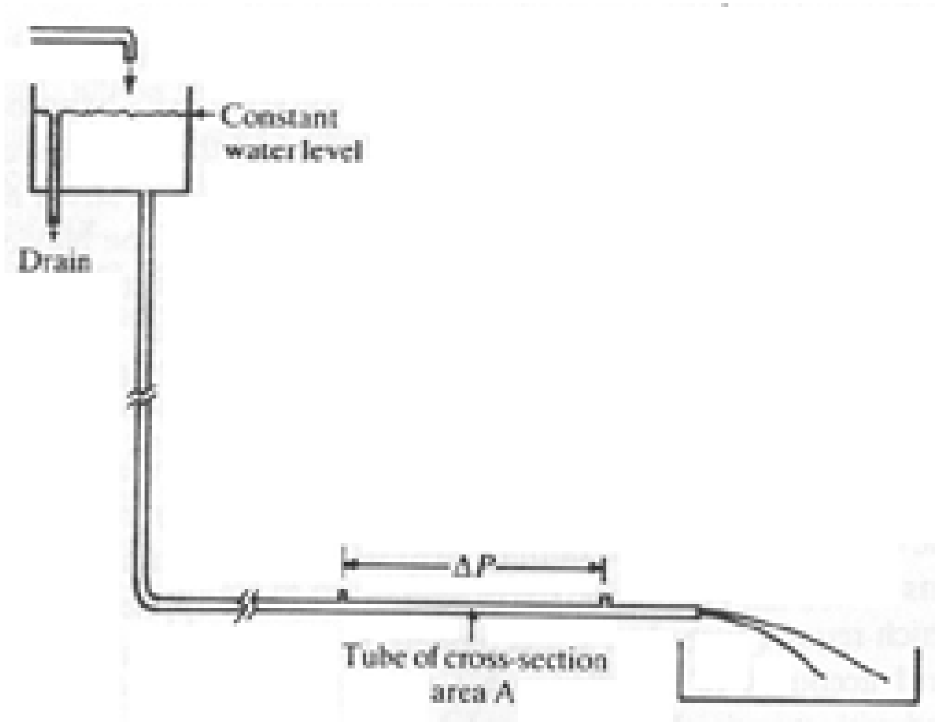


Turbulent Drag Reduction (Toms, 1946)

Addition of small amount of polymer gives rise to massive drag reduction.



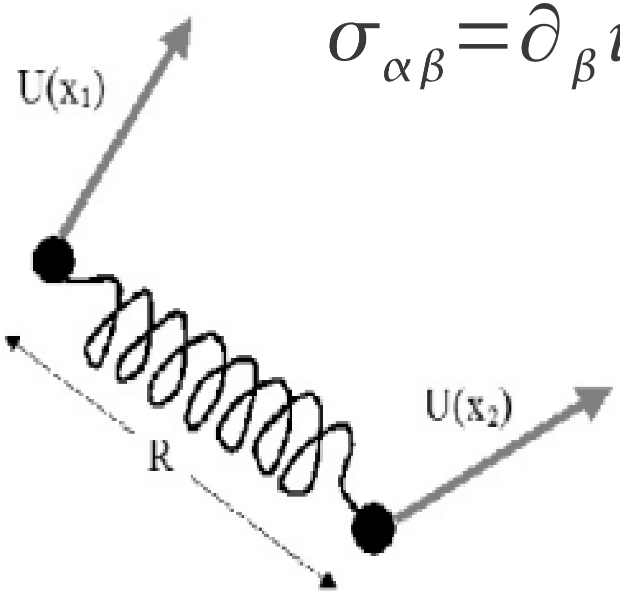
$$\% \, DR \equiv \frac{\Delta P^f - \Delta P^p}{\Delta P^f} \times 100$$

Simple model for single polymer

$$\partial_t R^{\mathrm{j}}_\alpha(t) = \sigma^{\mathrm{j}}_{\alpha\beta} R^{\mathrm{j}}_\beta + f(R^{\mathrm{j}}) + \sqrt{\frac{2R_0^2}{3\tau_{\mathrm{poly}}}}\, B^{\mathrm{j}}_\alpha.$$

$$\sigma_{\alpha\beta} = \partial_\beta u_\alpha$$

Gaussian, white Noise



Restoring force

$$f(R) = -R/\tau_{\mathrm{poly}}.$$

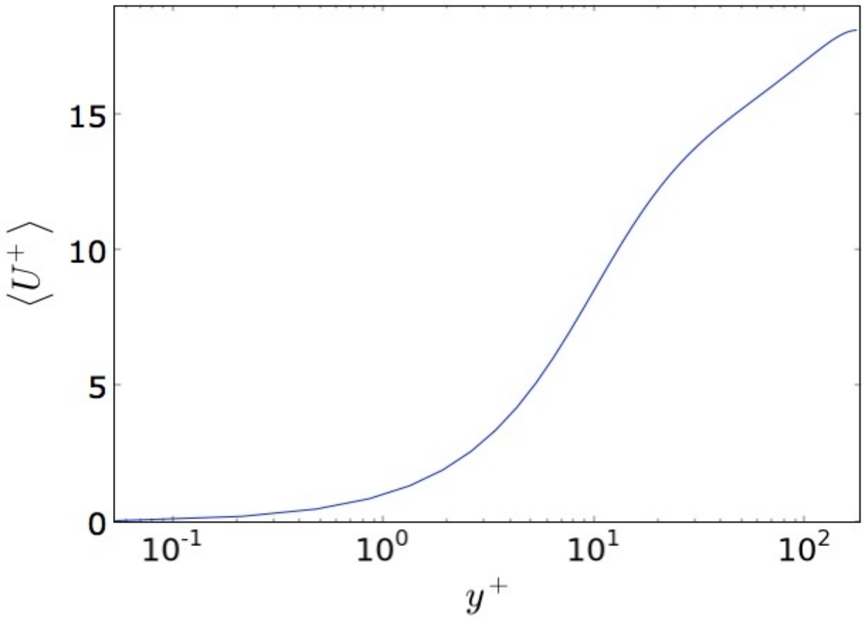
$$f(R) = -R/\tau_{\mathrm{poly}}\{1 - (R/R_{\max})^2\}.$$

Navier-Stokes equation

$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u} = \nu \nabla^2 \boldsymbol{u} + \boldsymbol{\nabla} p$$

with the incompressibility constraint,

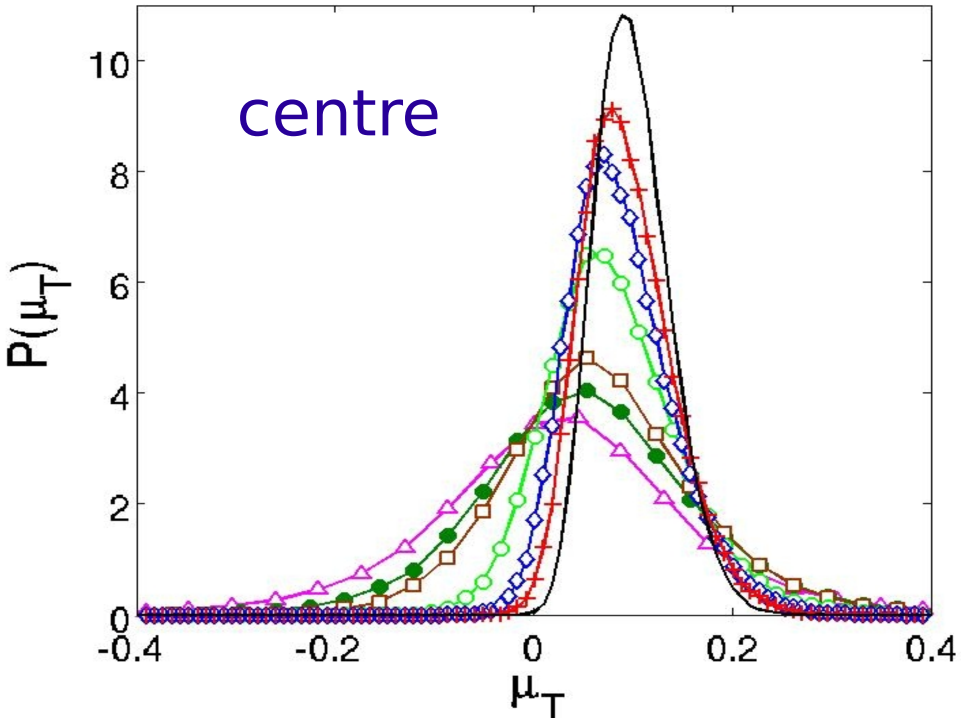
$$\boldsymbol{\nabla} \cdot \boldsymbol{u} = 0.$$

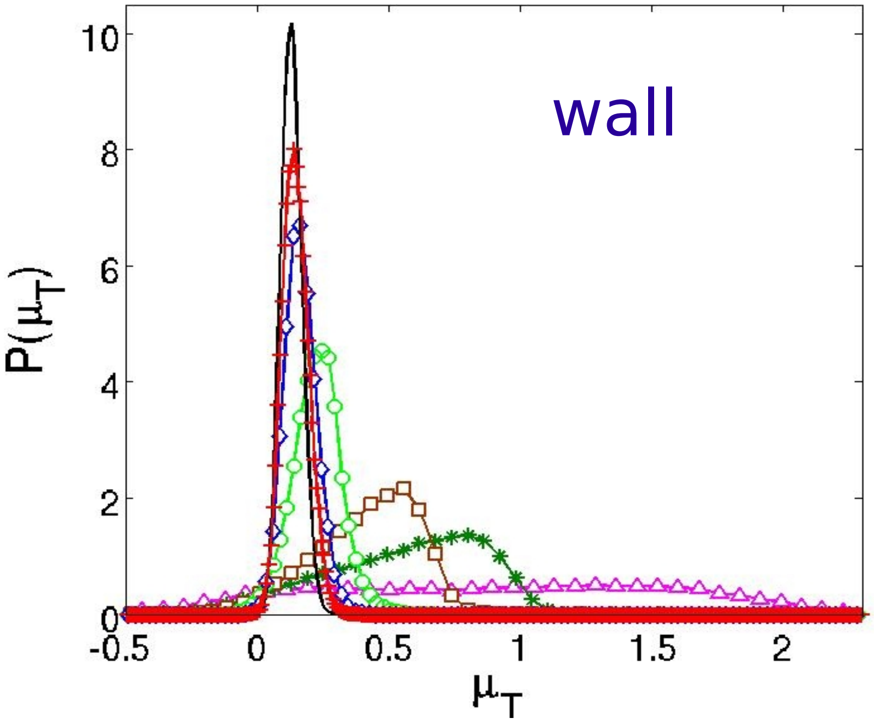


Lagrangian Particles

$$\partial_t \boldsymbol{r}^{\mathrm{j}}(t|t_0, \boldsymbol{r}_0^j) = \boldsymbol{v}^{\mathrm{j}}(t|t_0, \boldsymbol{r}_0^j).$$

Two particle separation in channel flow

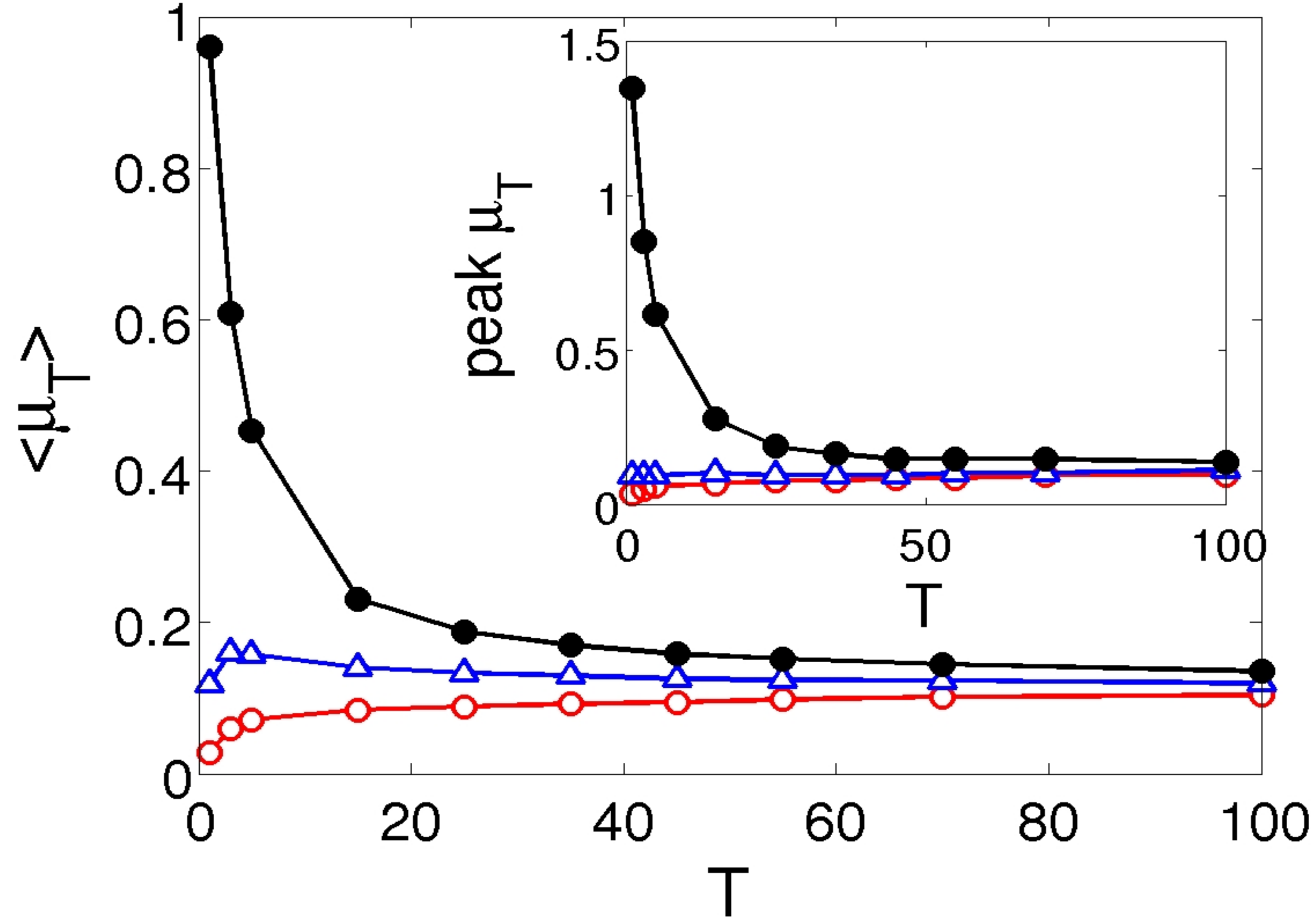




$$\partial_t \delta x^{\mathrm{j}}_{\alpha} = \sigma^{\mathrm{j}}_{\alpha,\beta} \delta x^{\mathrm{j}}_{\beta}.$$

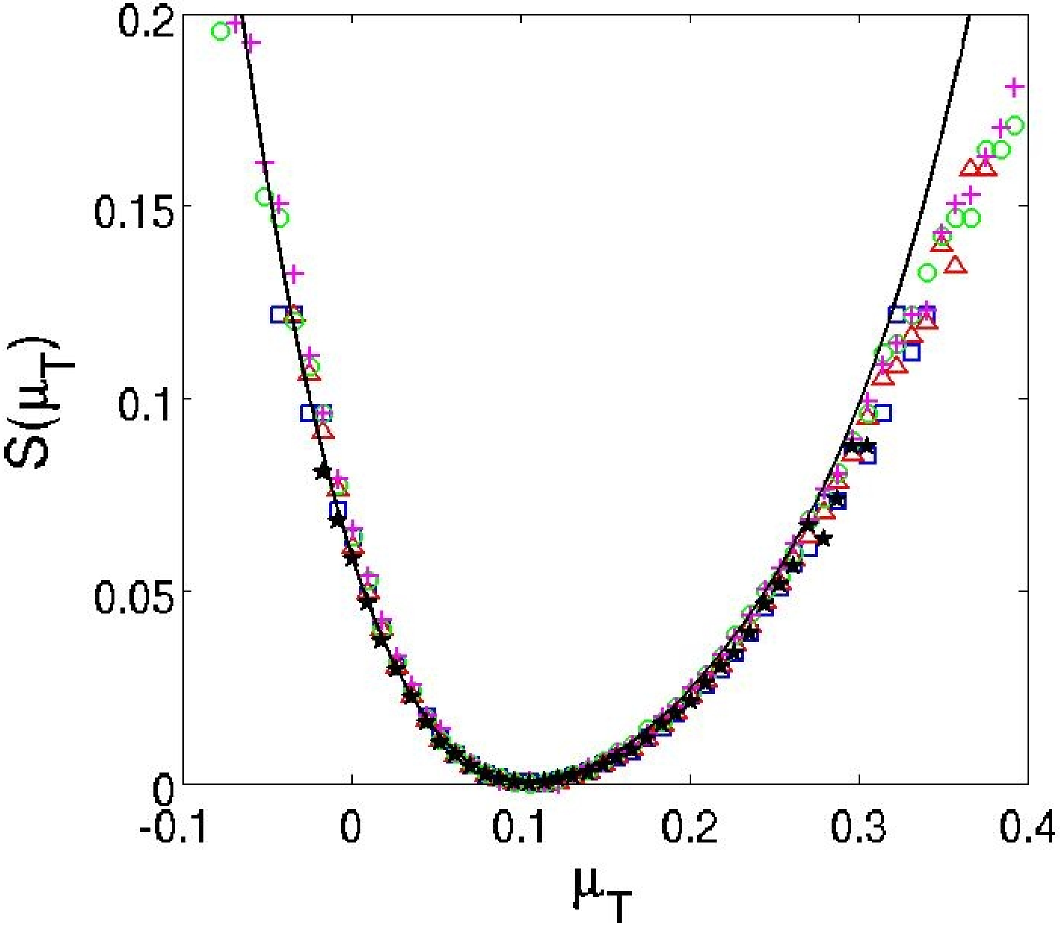
$$\mu^{\mathrm{j}}_{\mathrm{T}} = \frac{1}{T} \ln \left[\frac{|\delta x^{\mathrm{j}}(t)|}{|\delta x^{\mathrm{j}}(t-T)|} \right].$$

Change with time:



Cramer's function:

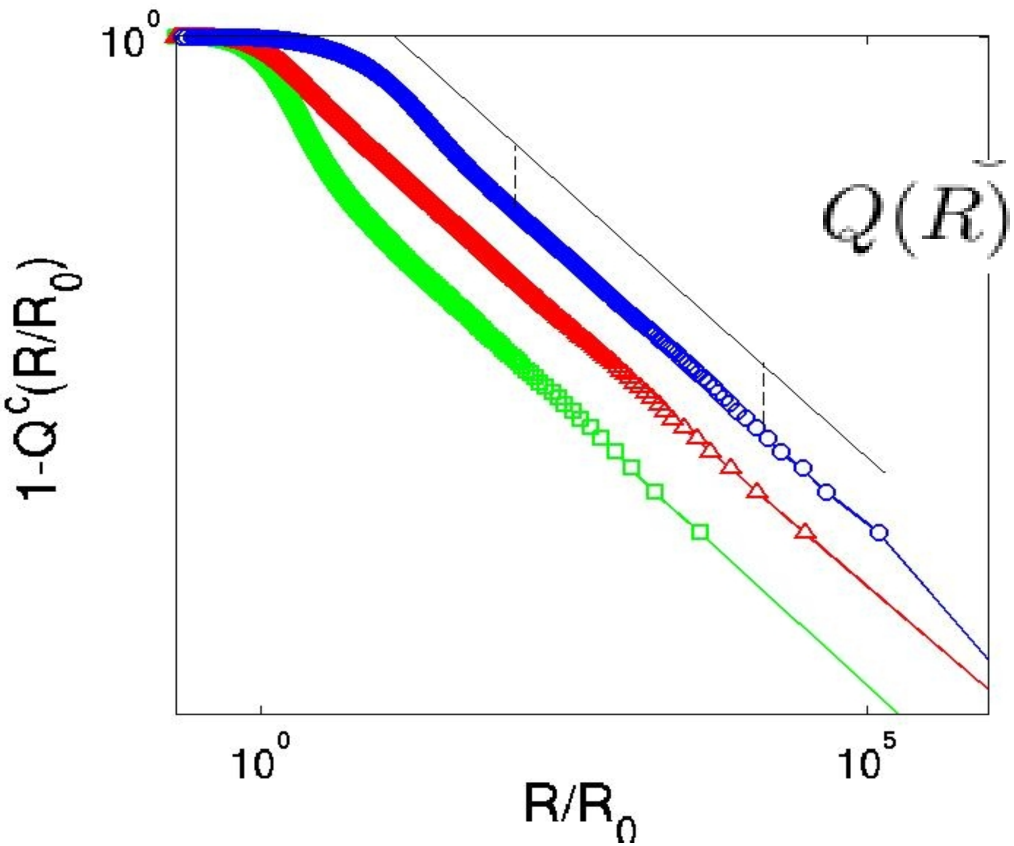
$$P(\mu_T) \sim \exp[-TS(\mu_T)]$$



$$\mathrm{Wi} \equiv \bar{\mu}\tau_{\mathrm{poly}}$$

$$S(\mu) = a_2(\mu - \bar{\mu})^2 + a_3(\mu - \bar{\mu})^3 + a_4(\mu - \bar{\mu})^4$$

Cramer's function and PDF of polymer extensions:



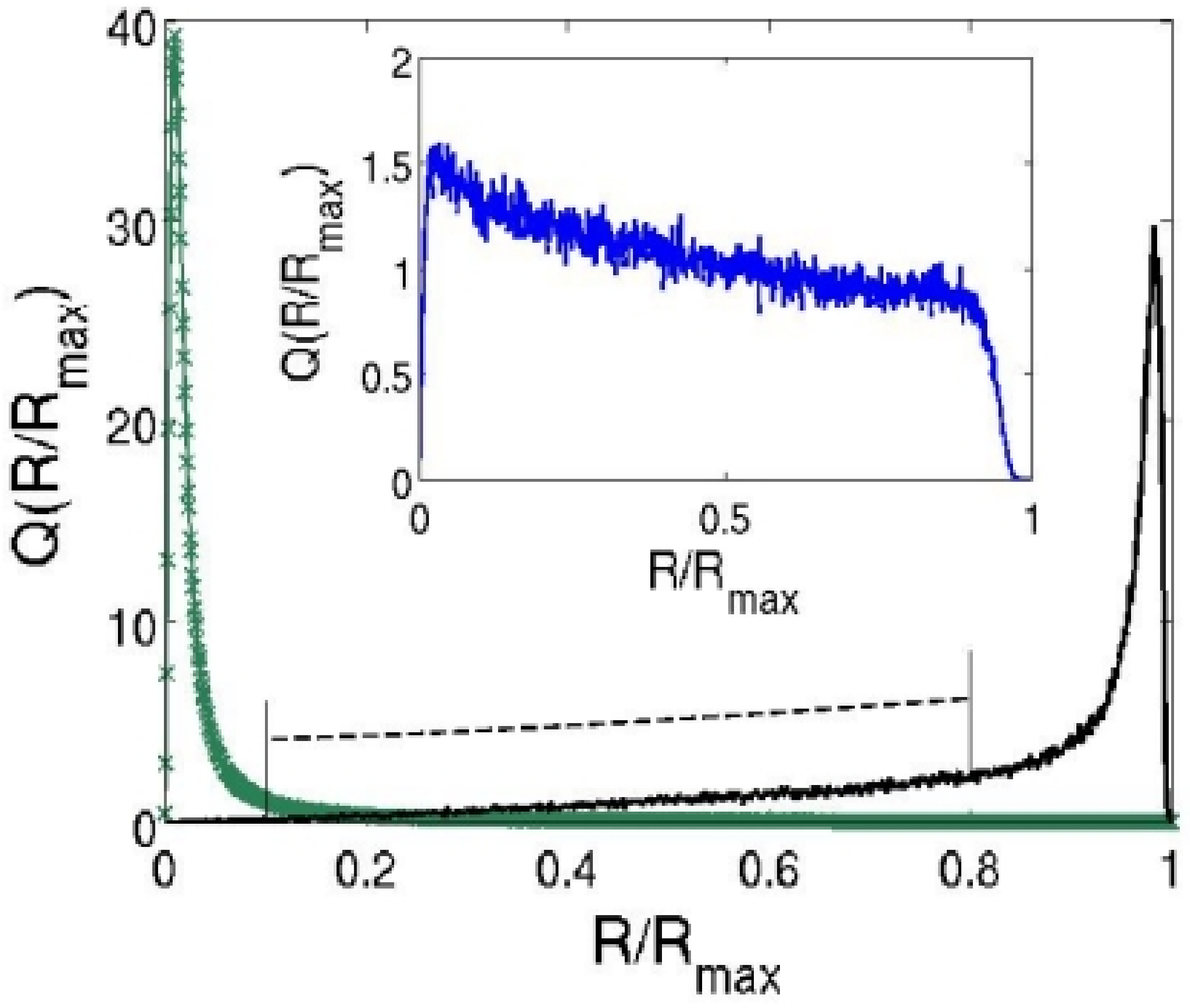
$$\mathcal{P}(R) \sim R^{-\alpha-1}$$

$$Q(\breve{R}) \sim \breve{R}^{-\alpha}$$

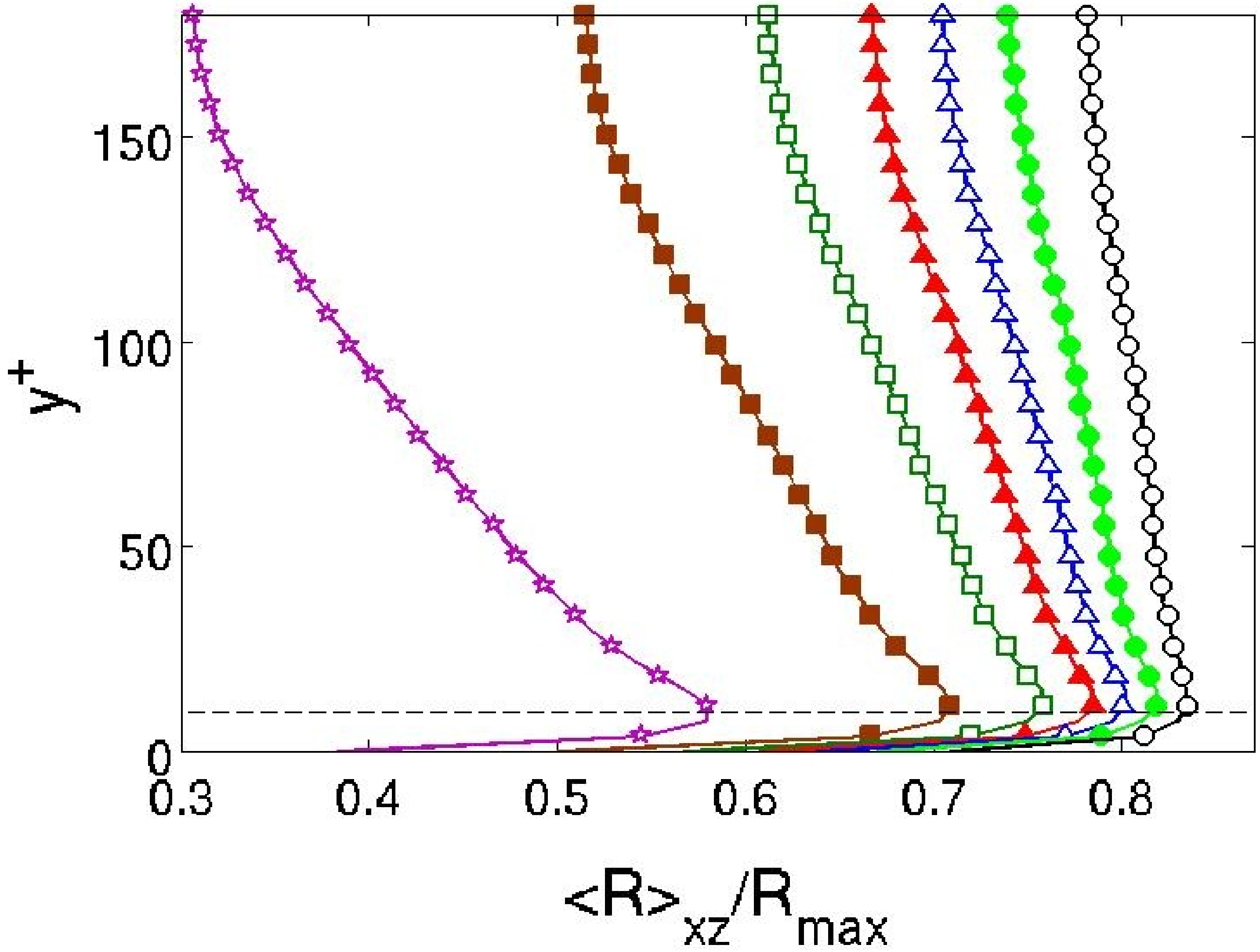
$$\alpha = S'(\beta + \frac{1}{\tau_{\rm poly}} - \bar{\mu})$$

$$S(\beta + \frac{1}{\tau_{\rm poly}} - \bar{\mu}) - \beta S'(\beta + \frac{1}{\tau_{\rm poly}} - \bar{\mu}) = 0$$

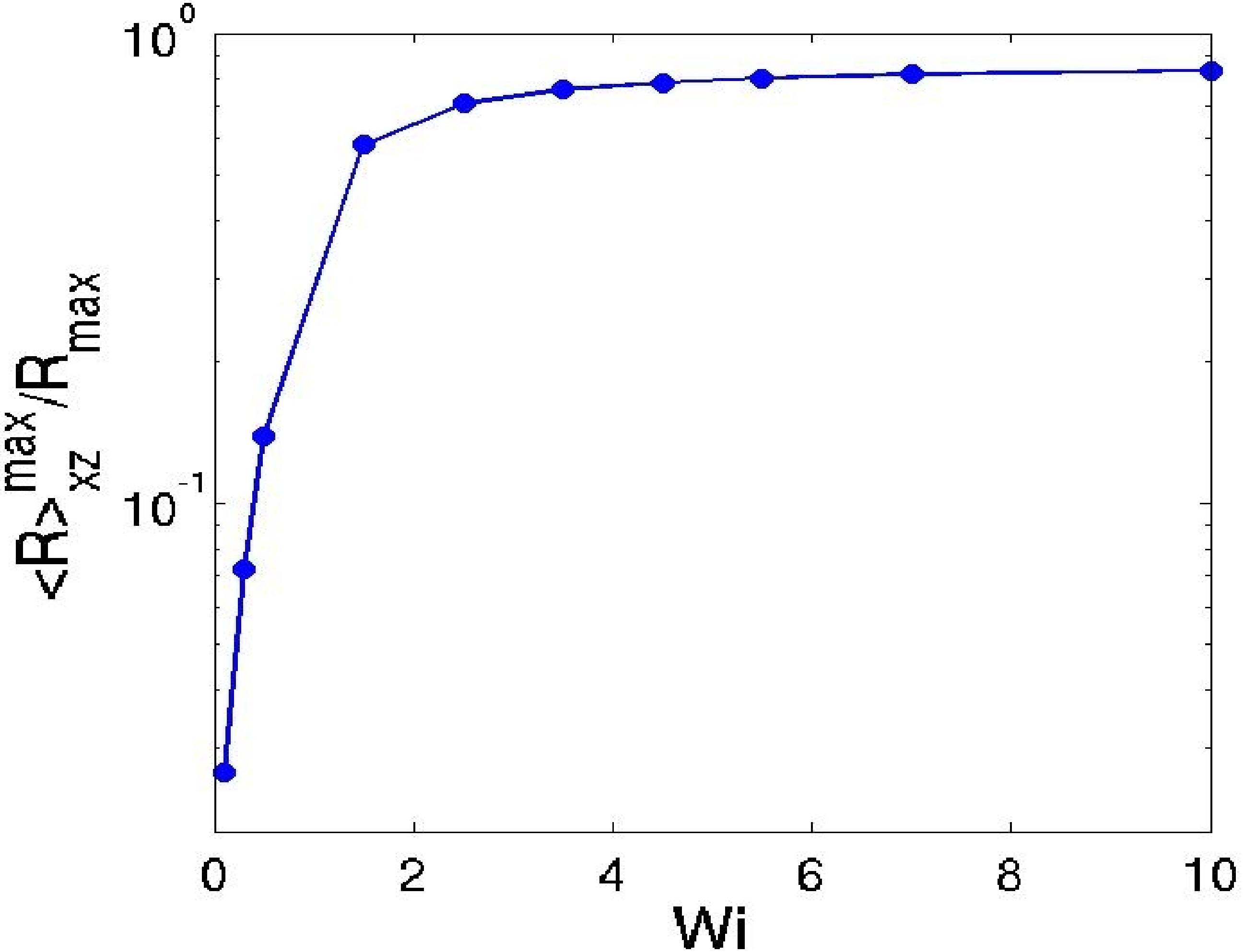
Coil-stretch Transition



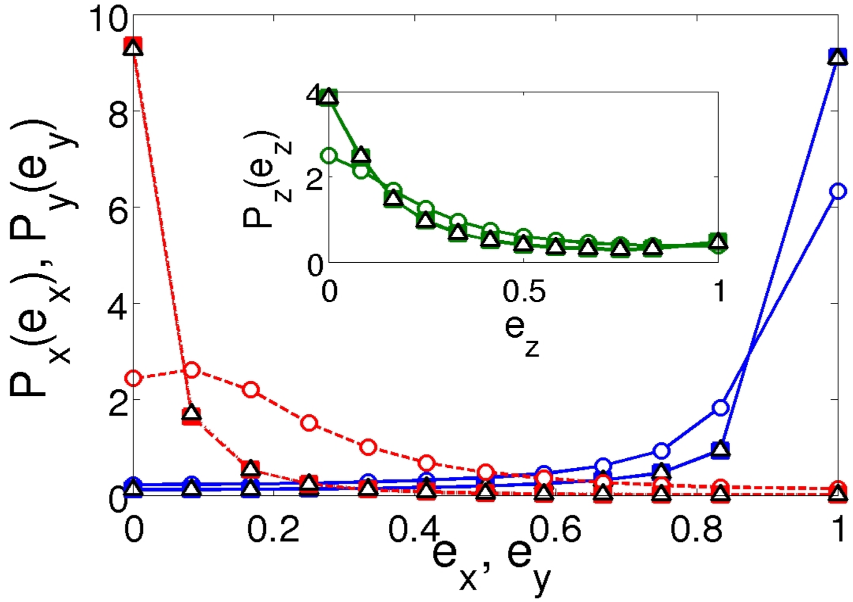
PDF of polymer extension



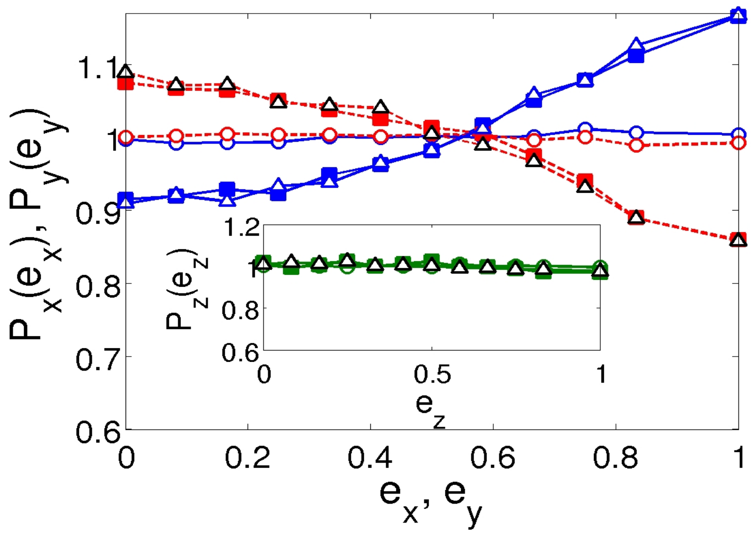
Maximum of polymer extensions



Orientation w.r.t. Channel geometry

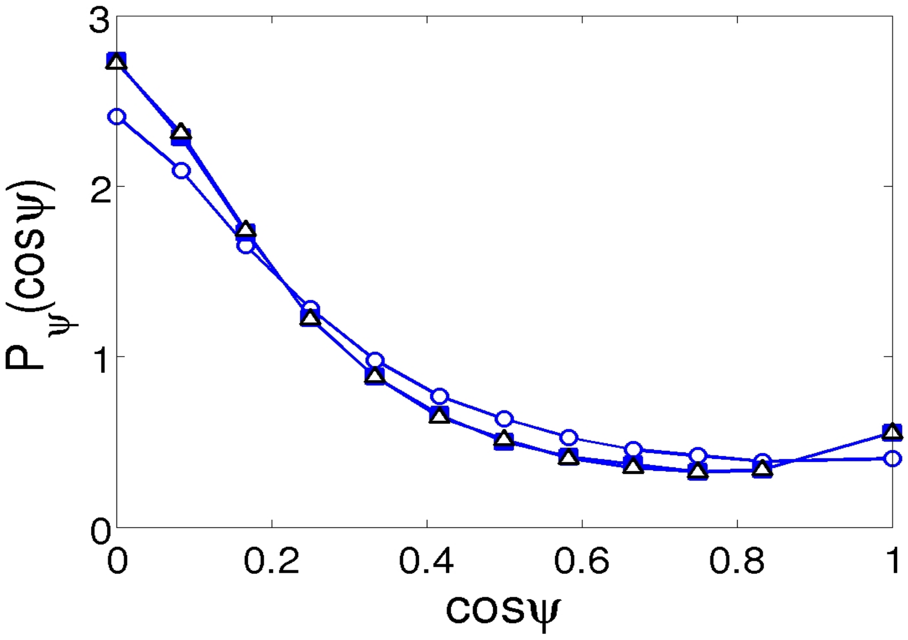


wall



centre

Orientation w.r.t. Local vorticity



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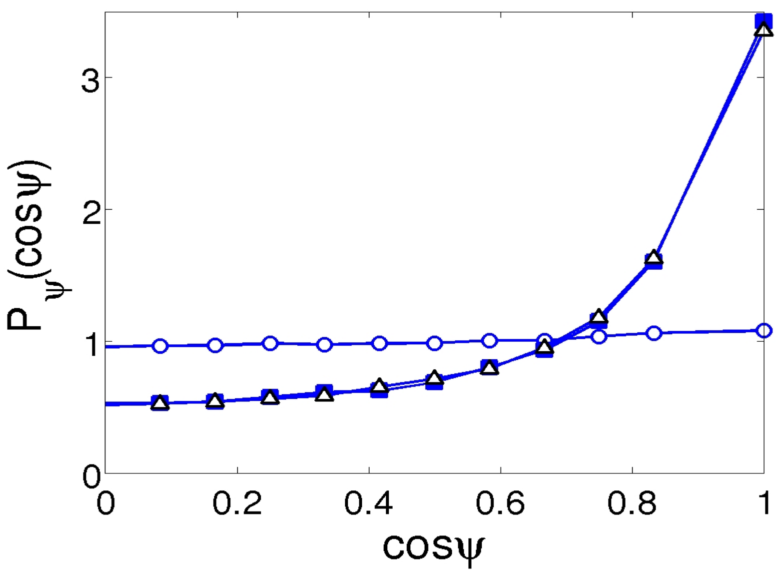

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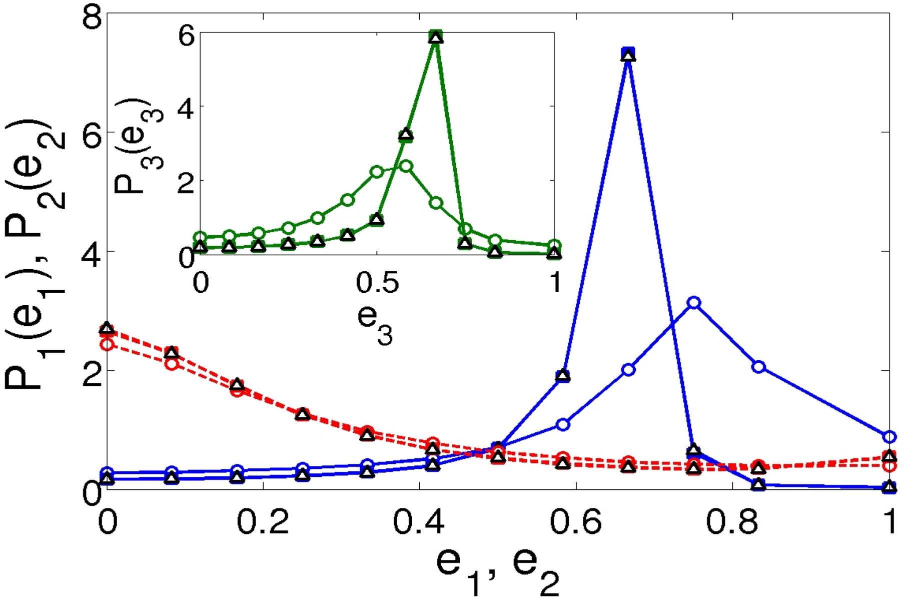
Orientation w.r.t. Local strain



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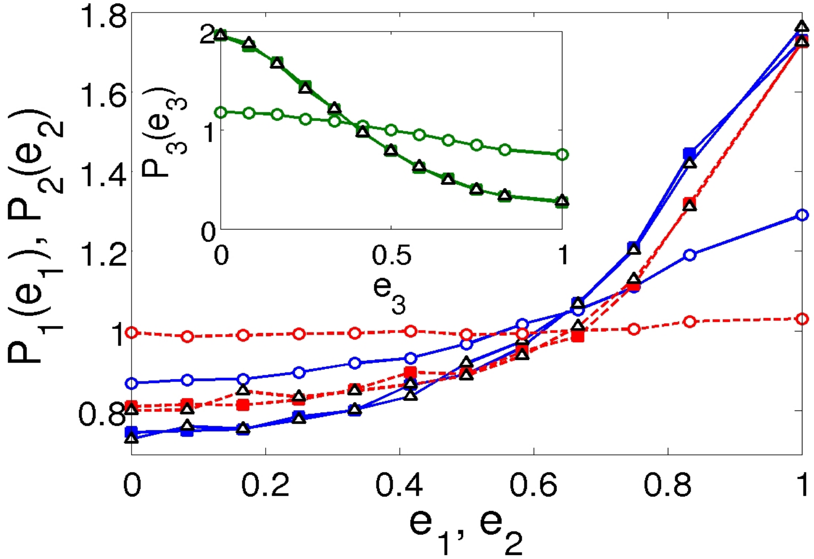

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Conclusion

First calculation of Cramer's function in channel flows

Verification of analytical theory of polymer stretching

Coil-stretch transition

Polymers are more stretched near the wall.

The orientation of the polymers is determined by the mean shear.