Homogeneous isotropic turbulence with polymer additives

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Homogeneous isotropic turbulence
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References


- Numerical studies of three dimensional turbulence with polymer additives and two dimensional turbulence in thin films, Prasad Perlekar, Ph.D. Thesis, Indian Institute of Science, Bangalore,


Outline

- Overview of turbulence with polymers.
- Modelling polymer solutions.
- Direct numerical simulations (DNS): Decaying and Forced Turbulence.
- Conclusions.
Drag Reduction

- Toms (1946): Monochlorobenzene with 0.25% (by weight) of polymethylmethacrylate

- Reduction in the pressure gradient across the pipe, on the addition of polymers, for the same volumetric flow rate

- Drag Reduction (in percentage) \( DR \equiv \left( \frac{\Delta P_s - \Delta P_p}{\Delta P_s} \right) \times 100 \)
Reduction of small scale structures

- Turbulent jet of water with 50ppm polyethylene oxide at \( Re \sim 225 \)
Energy spectra


- Grid Reynolds number $Re_M = 7.6 \times 10^3$;
- For low polymer concentrations (50 and 100 ppm) there is no significant change in the energy spectrum; at somewhat higher concentrations (500 and 1000 ppm) the spectra fall more steeply.
Eigenvalues of the strain tensor


- Length: 140mm, Width: 120mm, Disk Dia.: 40mm, Observation volume: 10 x 10 x 10mm, $Re_{\lambda} = 38$.
- Regions of large strains reduced on the addition of polymers.
Structure function: $S_2(r)$


- $c = 5 \text{ppm}, Re_\lambda = 290, Wi = 3.5,$
- Small scale structures are modified on the addition of polymers.
Polymer Properties

Typical drag-reducing polymer: Polyethylene oxide $N \times [-\text{CH}_2\text{-CH}_2\text{-O-}]$

- Degree of polymerization ($N$) $\approx 10^4$
- Molecular weight $\approx 4 \times 10^6$ amu
- Zimm relaxation time $\approx 10^{-4}$s
- RMS end-to-end distance at maximal extension $\approx 34 \mu m$
Modelling polymer solutions

- Navier-Stokes (NS) with Polymer Additives:
  3D, unforced, incompressible, NS with additional stress because of polymers:

\[
\frac{\partial u}{\partial t} + (u \cdot \nabla)u = -\nabla p + \nu \nabla^2 u + \nabla \cdot T,
\]

where

- \( u(x, t) \): fluid velocity; point \( x \); time \( t \);
- \( \nu \): Kinematic viscosity of the fluid;
- \( T \): polymer contribution to the fluid stress;

\[ \nabla \cdot u = 0 \] enforces incompressibility.
Modelling polymer solutions

- Finitely Extensible Nonlinear Elastic-Peterlin (FENE-P) model

\[
\frac{\partial C_{\alpha\beta}}{\partial t} + (u_\gamma \partial_\gamma)C_{\alpha\beta} = (\partial_\gamma u_\alpha)C_{\gamma\beta} + C_{\alpha\gamma}(\partial_\gamma u_\beta) - \frac{1}{\mu} T_{\alpha\beta}.
\]

["Dynamics of polymeric liquids", Bird, et al.]

- \( c = \mu/(\nu + \mu) \); \( c = 0.1 \sim 100 \text{ppm} \) of PEO

- \( We = \tau_{\text{poly}} \sqrt{\epsilon(t_m)/\nu} \); \( t_m \) is the time corresponding to the peak in \( \epsilon \) for \( c = 0 \)

[Vaithianathan, et al., JCP, 187, 1 (2003).]
Direct Numerical Simulations
Solve NS and FENE-P numerically

\[
\frac{\partial u_\alpha}{\partial t} + (u_\gamma \partial_\gamma)u_\alpha = -\partial_\alpha p + \nu \partial_\gamma u_\alpha + \partial_\gamma T_{\alpha\gamma}, \\
\partial_\gamma u_\gamma = 0,
\]

\[
\frac{\partial C_{\alpha\beta}}{\partial t} + (u_\gamma \partial_\gamma)C_{\alpha\beta} = (\partial_\gamma u_\alpha)C_{\gamma\beta} + C_{\alpha\gamma}(\partial_\gamma u_\beta) - \frac{1}{\mu} T_{\alpha\beta}.
\]
Decaying Turbulence
Results: Initial Condition

- Start from an initial energy spectrum with energy concentrated in the first few Fourier modes and the polymers unstretched.
- Monitor the decay of the energy dissipation rate and the energy spectrum for the fluid with and without polymer additives.
Energy Dissipation Rate

\[ N = 256, \, \nu = 10^{-3}, \, \tau_{\text{poly}} = 1 \]

- The energy dissipation rate \( \epsilon(t) \) as a function of time \( t \) for different values of \( c \).
- The peak in \( \epsilon(t) \) decreases as \( c \) increases.
Dissipation Reduction (DR)

\[ N = 96, \ \nu = 10^{-2} \]

Natural definition of dissipation-reduction

\[ \%DR = \left( \frac{\epsilon^{f,m} - \epsilon^{p,m}}{\epsilon^{f,m}} \right) \times 100; \]

- \( f \) and \( p \) stand, respectively, for the fluid without and with polymers.
- An increase in \( c \) enhances the dissipation reduction DR (cf., earlier shell-model study).
- \( DR \) decreases marginally with an increase in \( We \).
Fluid energy spectrum

\[ N = 192, \ \nu = 10^{-2}, \ \tau_{poly} = 1 \]

\[ E_f(k) = \sum_{k-1/2 < k' < k+1/2} |u(k')|^2 \] at \( t_m \) for polymer concentrations \( c = 0.0, c = 0.1, c = 0.4 \).

Energy spectrum at cascade completion changes significantly for large Fourier modes.

This had not been resolved by earlier, high-\( Re \) simulations!
The change in the spectra and $\epsilon$ can be understood in terms of an additional, effective, scale-dependent viscosity $\Delta \nu(k) \equiv -\mu \sum_{k-1/2 < k' \leq k+1/2} u_{k'} \cdot (\nabla \cdot J)_{-k'} / [\tau_{\text{poly}} k'^2 E^{p,m}(k')]$.

Since $\Delta \nu$ becomes negative, polymers pump energy into the fluid around $k \approx 10$. 

Scale-dependent viscosity

$N = 192, \nu = 10^{-2}, \tau_{\text{poly}} = 1$
Structure Functions

Order-$p$ equal-time, longitudinal velocity structure function.

\[ S_p(r) \equiv \langle \delta u(r, t)^p \rangle, \]
\[ \delta u_{\parallel}(r, t) \equiv [\bar{u}(\vec{x} + \vec{r}, t) - \bar{u}(\vec{x}, t)] \cdot (\vec{r}/r). \]
Second order structure function $S_2(r)$

Experiments (Ouellette et al.)

- $c = 5\, \text{ppm}$, $Re_\lambda = 290$, and $We = 3.5$

Our DNS

- $N = 128$, $\nu = 0.01$, and $\tau_P = 1.5$

Figure: $c = 5\, \text{ppm}$, $Re_\lambda = 290$, and $We = 3.5$
PDF of $|\omega|$

- Probability distribution of the modulus of the vorticity ($P(|\omega|)$) at cascade completion ($c=0$, $c=0.4$).
- Addition of polymers leads to a decrease in the regions of large vorticity.
Isosurfaces of $|\omega|$ for $N = 256$, $\nu = 10^{-3}$, $\tau_{poly} = 1$.

- Iso-$|\omega|$ surfaces for $|\omega| = \langle |\omega| \rangle + 2\sigma$ for $c = 0$ (left) and $c = 0.4$ (right) at $t_m$.
- Small-scale structures are suppressed on the addition of polymers.
Stretching of Polymers: Cumulative distribution (CDF)

\[ N = 256, \quad \nu = 10^{-3}, \quad \tau_{\text{poly}} = 1 \]

- \( c = 0.1 \) (dashed line), \( c = 0.4 \) (line).
- An increase in \( c \) leads to a decrease in the polymer extension.
- A decrease in \( \nu \) leads to turbulent flows and large polymer extensions.
Summary of Results: Decaying turbulence

- Polymer additives lead to a decrease in small-scale structures.
- Polymers decrease the energy of the turbulent fluid at intermediate length scales and increase it at small scales.
- Dissipation reduction is the analogue in homogeneous, isotropic turbulence of drag-reduction in wall-bounded turbulence.
- An effective scale-dependent viscosity leads to a natural explanation of our results.
- This points toward an increase in the effective viscosity, but one that is scale-dependent.

Forced Turbulence

Time evolution of $E$ and $\epsilon$

$N = 256$, $Re_\lambda \approx 80$, $c = 0.1$

- Time averaged $E$ decreases with an increase in $We$
- Time averaged $\epsilon$ decreases with an increase in $We$
- $We = 3.5$ (blue circles), $We = 7.1$ (black dashed line), NS (red)
PDF of $|\omega|$ and $\epsilon_{\text{loc}}$

$N = 256$, $Re_\lambda \approx 80$, $c = 0.1$

$\omega \equiv \sqrt{\sum_{i,j} \omega_{ij} \omega_{ij}}$, $\epsilon_{\text{loc}} = \nu s^2 \equiv \sum_{i,j} S_{ij} S_{ij}$,

$s = (\nabla u + (\nabla u)^T)/2$, $\omega = \nabla \times u$

- Regions of large strain and vorticity decrease on the addition of polymers
PDF of $|\omega|$ and $\epsilon_{loc}$

$N = 256, \ Re_\lambda \approx 80, \ c = 0.1$

$\omega \equiv |\sqrt{\sum_{i,j} \omega_{ij}\omega_{ij}}|$, $\epsilon_{loc} = \nu s^2 \equiv \sum_{i,j} S_{ij}S_{ij}$,

$s = (\nabla u + (\nabla u)^T)/2$, $\omega = \nabla \times u$

- Regions of large strain and vorticity decrease on the addition of polymers
Isosurfaces of $|\omega|$

$N = 256, \ Re_\lambda \approx 80, \ c = 0.1$

- Iso-$|\omega|$ surfaces for $|\omega| = \langle |\omega| \rangle + 2\sigma$ for $c = 0$ (left) and $c = 0.1, \ We = 7.1$ (right).
- Small-scale structures are suppressed on the addition of polymers.
\( \mathbf{QR \ plots} \)

\( N = 256, \ Re_\lambda \simeq 23, \ We = 7.1, \ c = 0.1 \)

- Left: \( \text{NS} \); Right: \( \text{Polymer (} c = 0.1, \ We = 7.1 \) \)}
(Left) $N = 256, \ Re_\lambda = 80$
(Right) $N = 512, \ Re_\lambda = 20$
Polymer extensions

$N = 256, \ Re_\lambda \simeq 23, \ c = 0.1$

- $We = 7.1, \ c = 0.1$(line); $We = 3.5, \ c = 0.1$(dashed line);
- Polymer extensions larger in comparison to decaying turbulence
- At fixed $c$, polymer extension increases with an increase in $We$
Conclusions

- Our simulations show that the addition of polymers to flows that display homogeneous isotropic turbulence leads to dissipation reduction in both decaying and statistically steady turbulence; this dissipation reduction is the analogue of drag reduction in wall-bounded flows.

- Our numerical results agree with the experimental results of (a) Liberzon, et al., op. cit. and (b) Ouellette et al., op. cit.

- Polymers decrease the energy of the turbulent fluid at intermediate length scales and increase it at small scales; a scale-dependent viscosity provides a natural means of understanding our results.