Helicity and inertial waves in forced rotating turbulence

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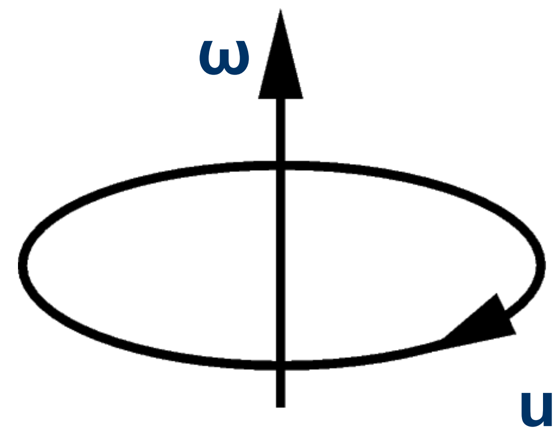
ROTATION

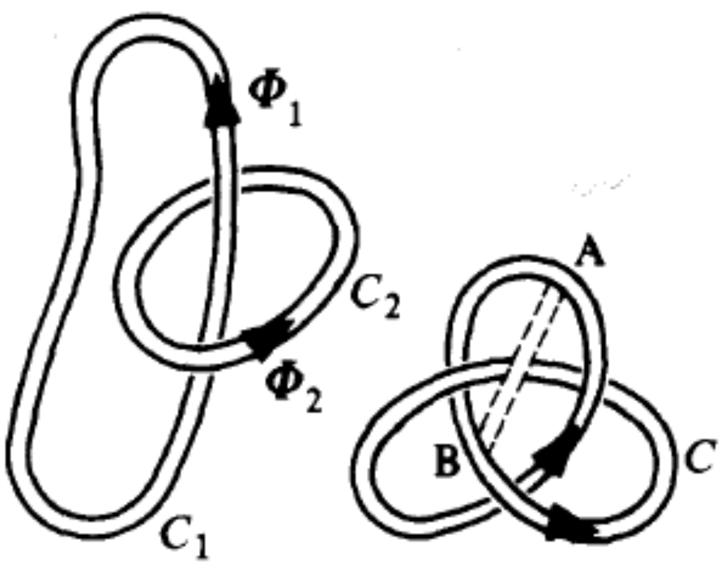
- Earth's atmosphere and oceans
- Frontogenesis, tornadoes, hurricanes
- *The planets, sun and stars, galaxies, …*
  and the origin of their magnetic fields

- Interplay between turbulent eddies & waves
- Role of symmetry breaking through helicity

Helicity
is a pseudo (axial) scalar

$$H = \int \boldsymbol{\omega} \cdot \mathbf{u} \, dV$$





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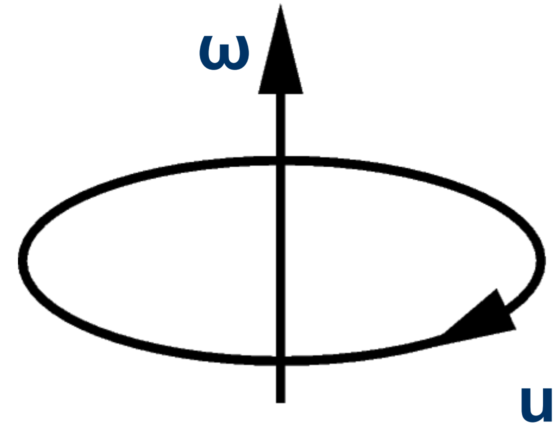
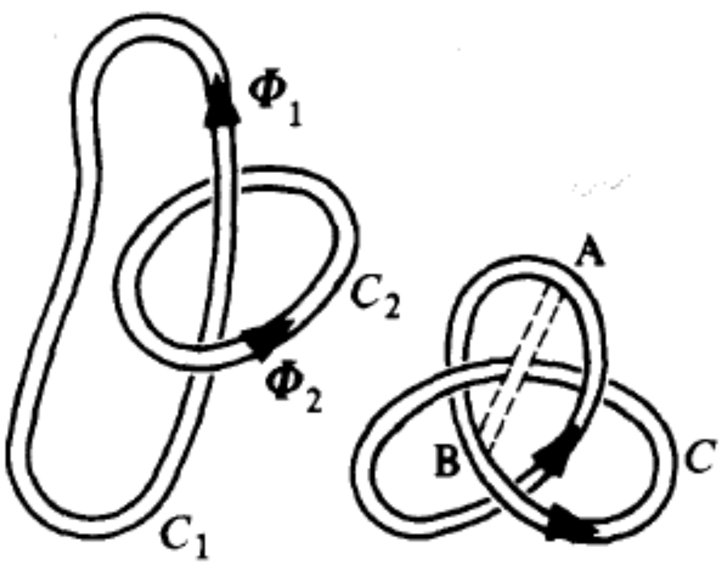


$$H = \int \boldsymbol{\omega} \cdot \mathbf{u} \, dV$$

$$<u_i(\mathbf{k})u_j^*(-\mathbf{k})>= U_E(|k|) \; P_{ij}(|k|)$$



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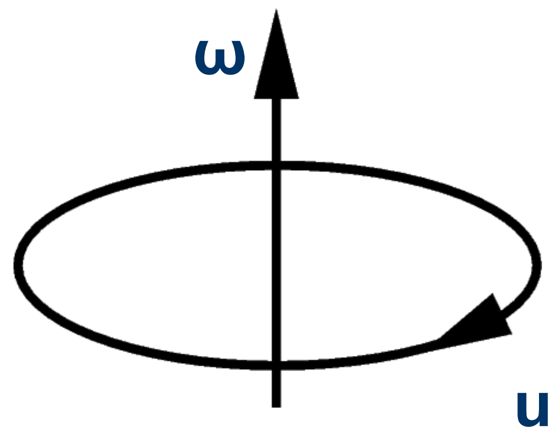
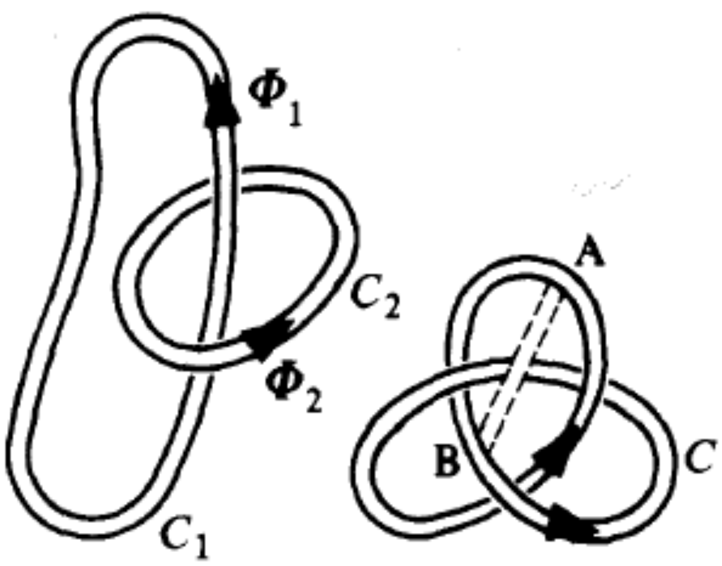


$$H = \int \boldsymbol{\omega} \cdot \mathbf{u} \, dV$$

$\langle u_i(\mathbf{k}) u_j^*(-\mathbf{k}) \rangle = U_E(|k|)\, P_{ij}(|k|) + \varepsilon_{ijl} k_l \, U_H(|k|)$



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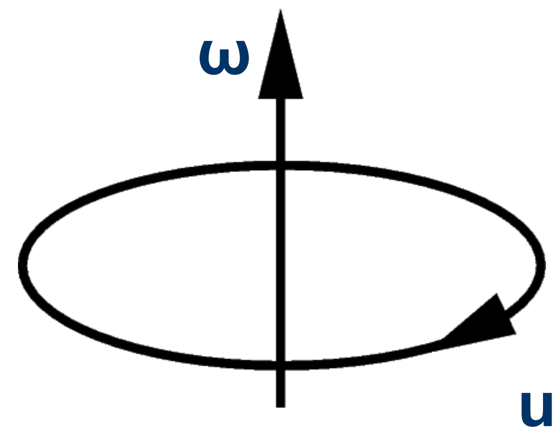

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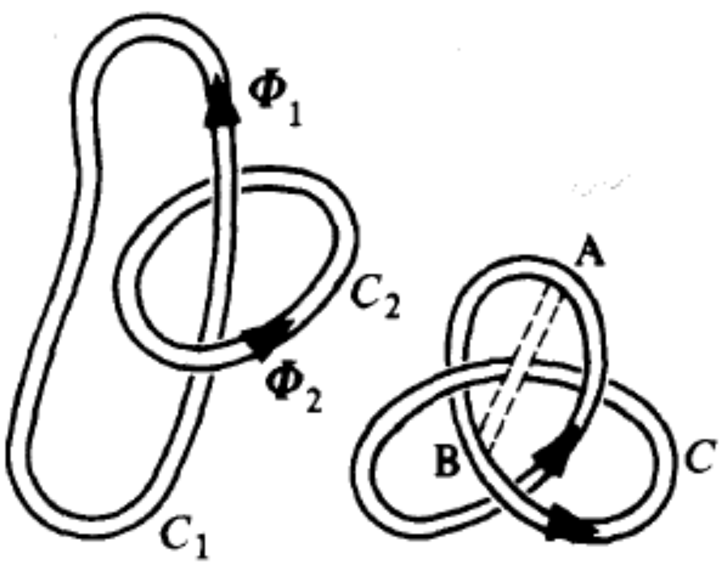
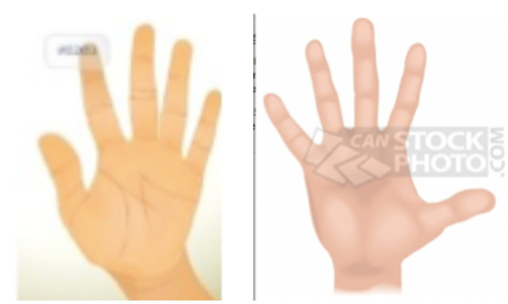


L



R

For particles, helicity is ***S.P***
where ***S*** *is spin vector and* ***P*** *is momentum,*
and helicity is chirality (Kelvin, 1873, 1904).
L,R: important differences: thalidomide, aspartame, …

Helicity
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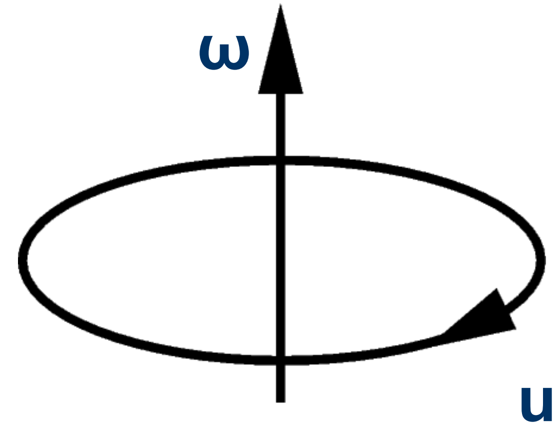
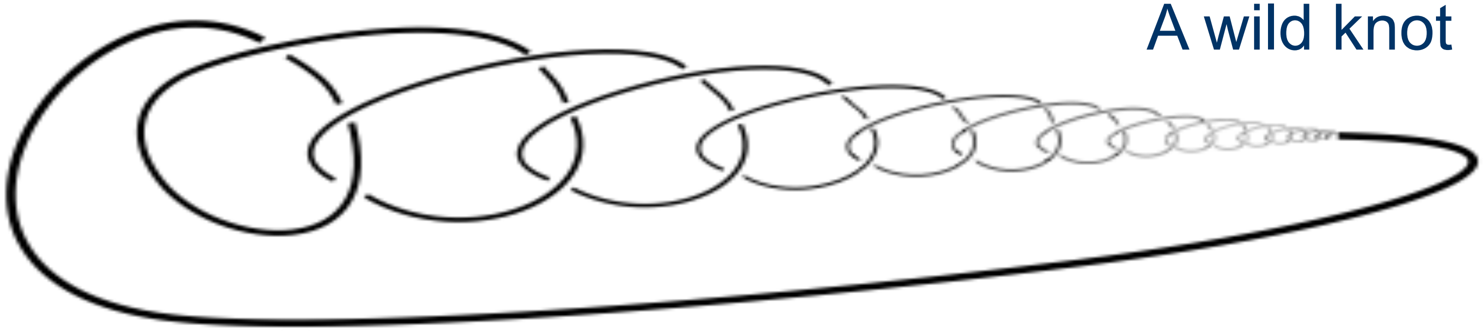
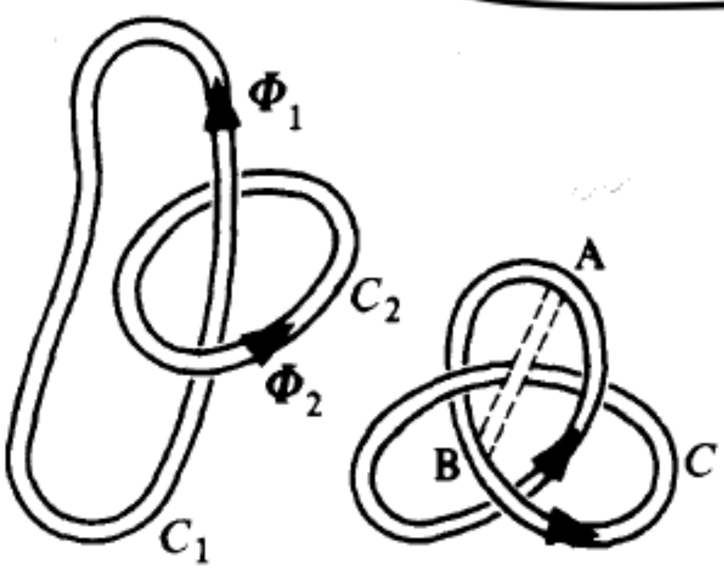


A wild knot





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Helicity in tropical cyclones

Molinari & Vollaro, 2010

Shear & helicity in the atmosphere

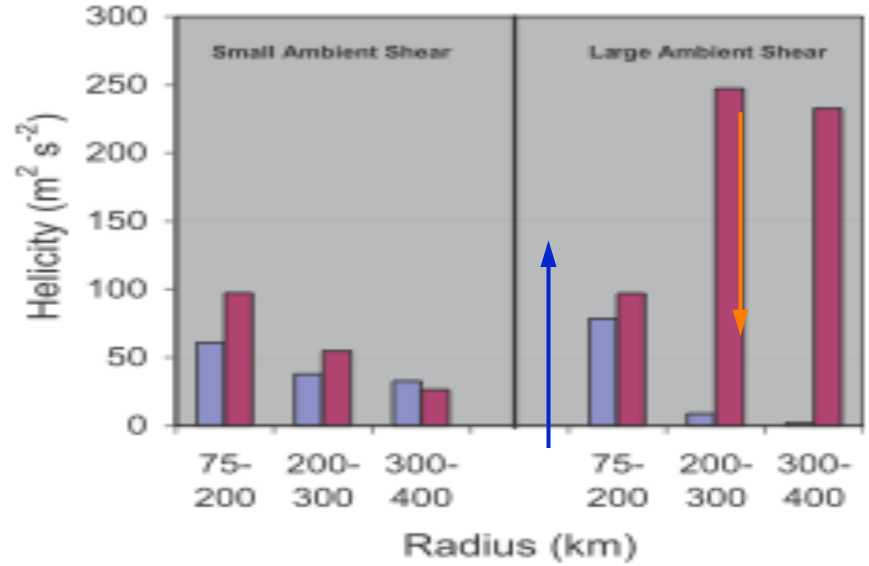
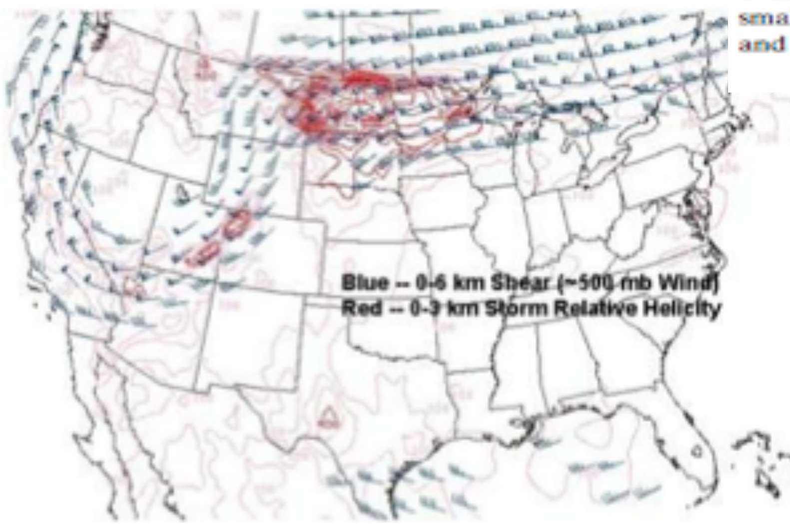


FIG. 4. Radial variation of 0–3-km helicity (m^2 s^{-2}) averaged over 75–200, 200–300, and 300–400 km for TCs experiencing (left) small and (right) large ambient shear. Upshear means are in blue and downshear in red.



Helicity spectrum in the Planetary Boundary Layer

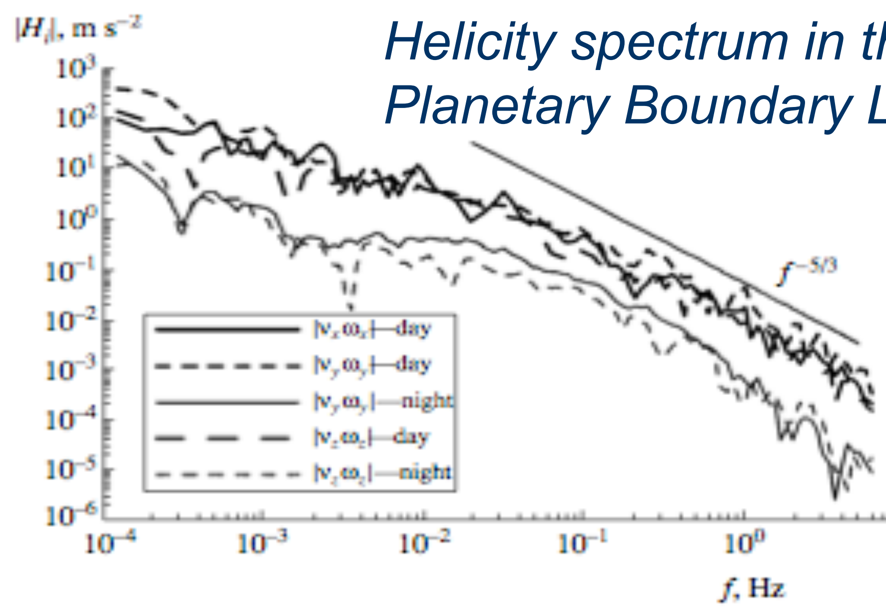
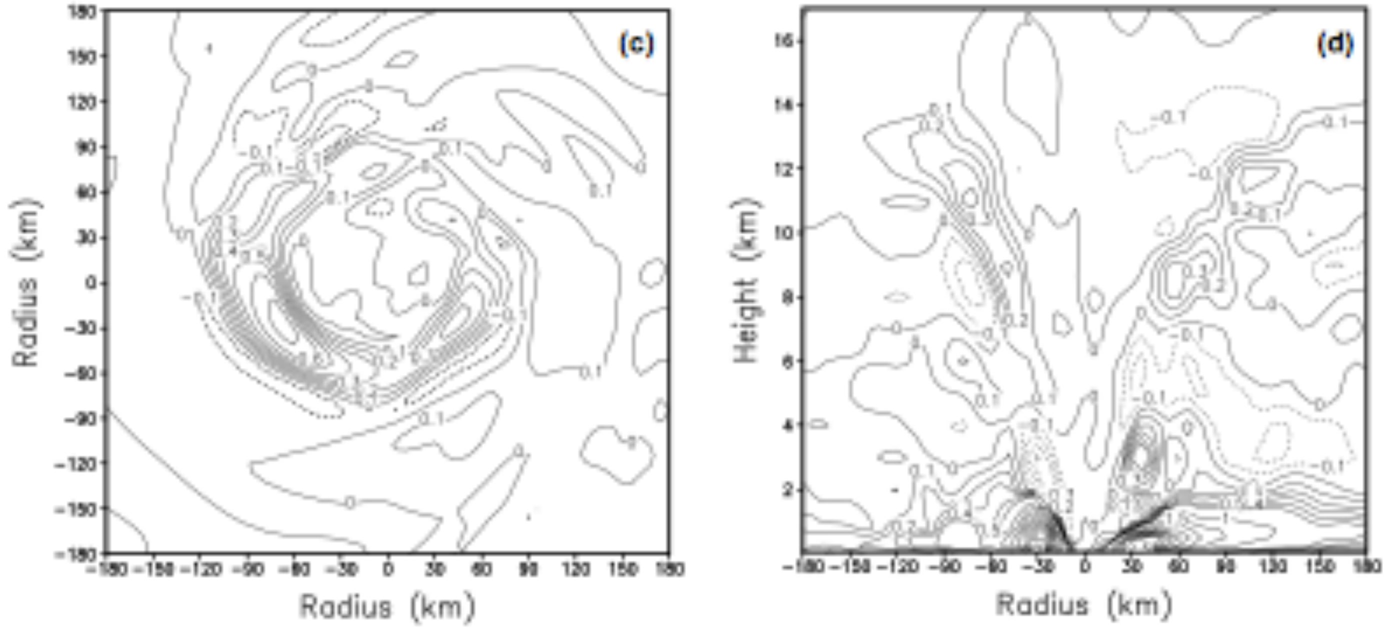


Fig. 4. Spectra of helicity components.

Koprov, 2005

Helicity in Hurricane Andrew, Xu & Wu, 2003



100 UTC 23 August 1992. Radius is
) at 5 km; (c) at 10 km; (d) vertical

Strong helicity where magnetic field is active, Komm et al., 2003

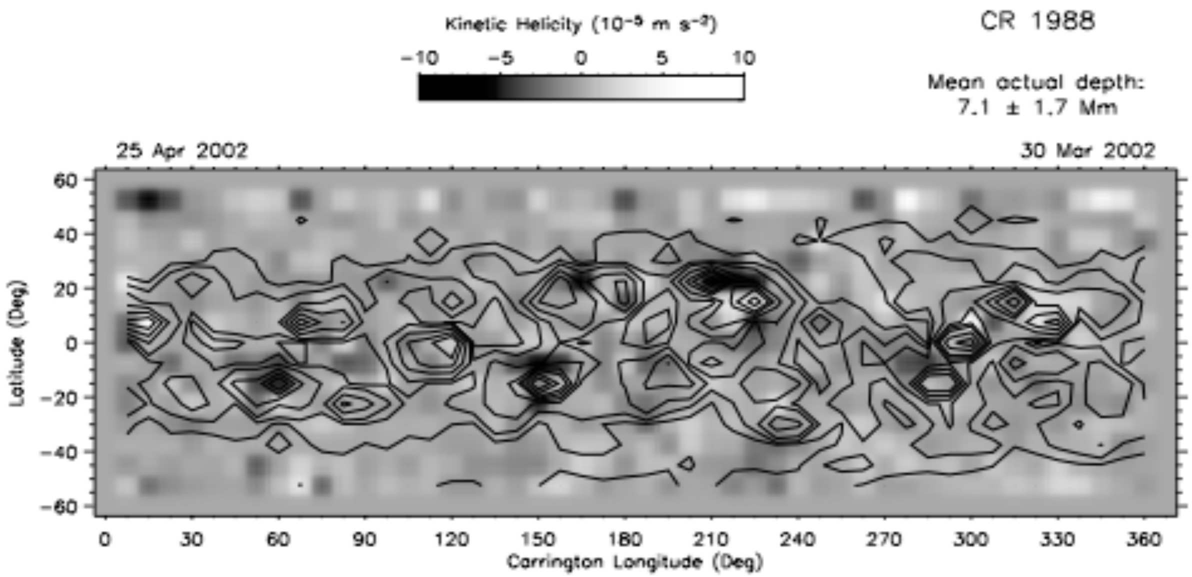


Fig. 12.— The kinetic helicity density at a depth of 7 Mm derived from MDI data. The contour lines indicate the magnetic flux (5, 10, 20, 40, 60, 80, 120, 160 G).

Helicity from dropsondes in Hurricane Bonnie-1998

Molinari and Vollaro, 2008

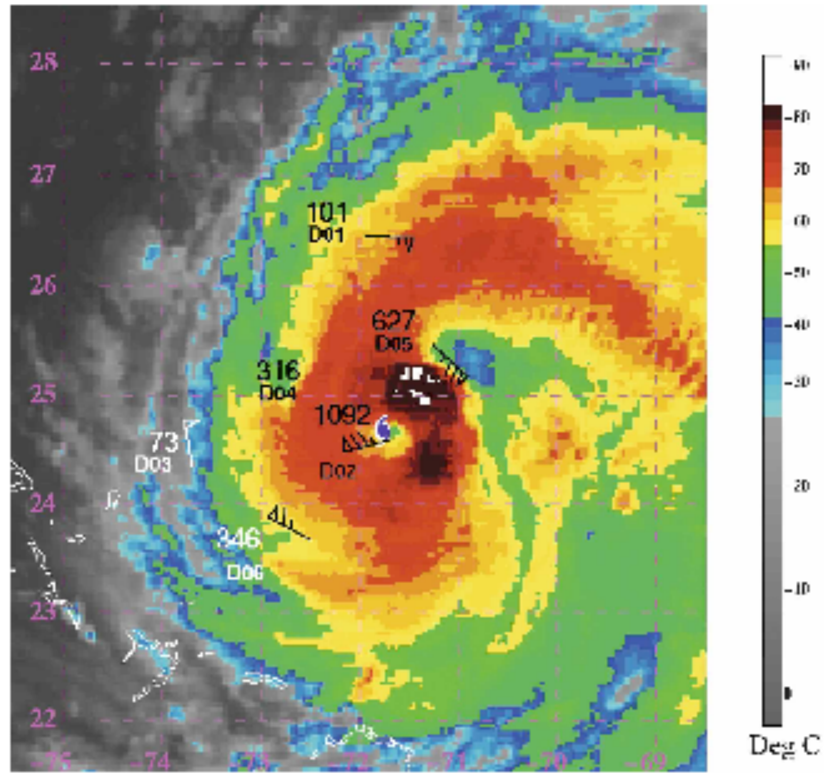


FIG. 4. Infrared satellite image at 2200 UTC 23 Aug, showing an intense local cell with a new cell forming upwind. The color bar represents brightness temperature (°C). Helicity values and mean winds over 0–6 km from sondes D1–D6 (released between 1845 and 2136 UTC 23 Aug) are also shown, plotted with respect to the moving center. These are shown in black or white depending on the background. Vertical wind shear was from the northwest at this time. The hurricane symbol represents the best-track center location.

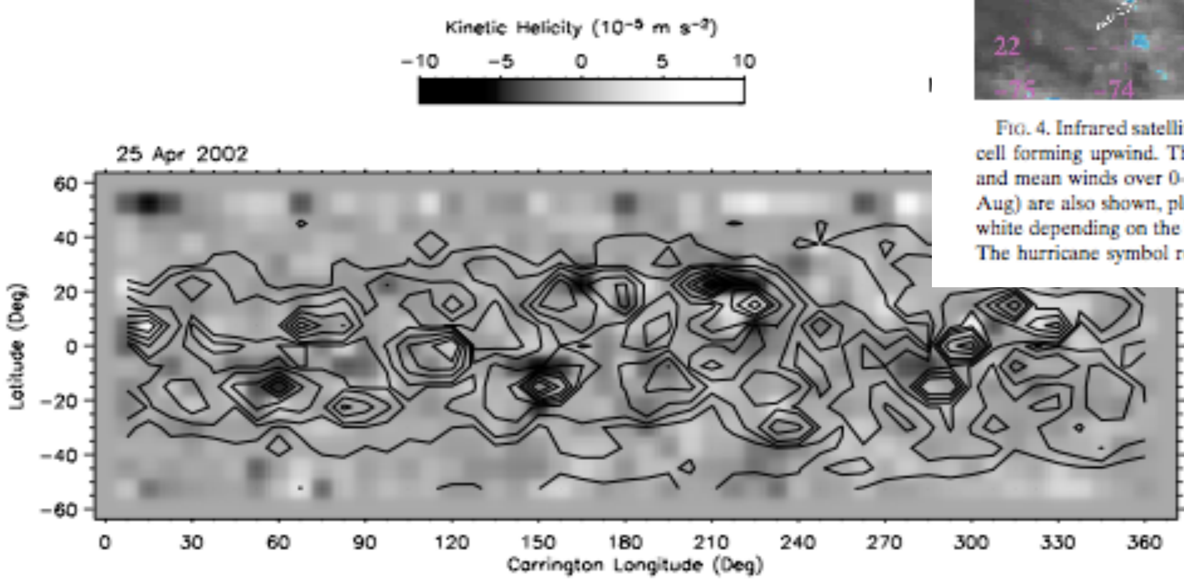


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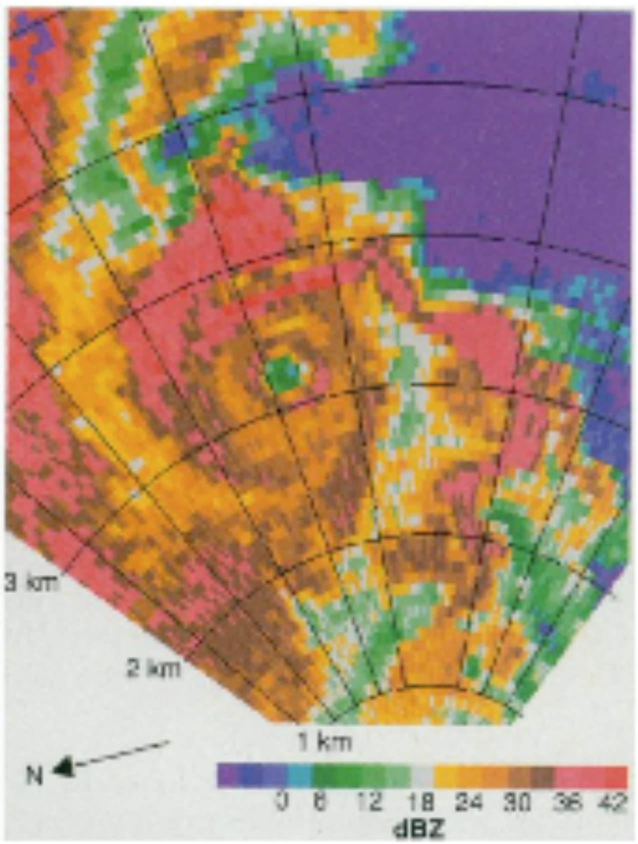
Fig. 12.— The kinetic helicity density at a depth of 7 Mm derived from MDI data. The contour lines indicate the magnetic flux (5, 10, 20, 40, 60, 80, 120, 160 G).

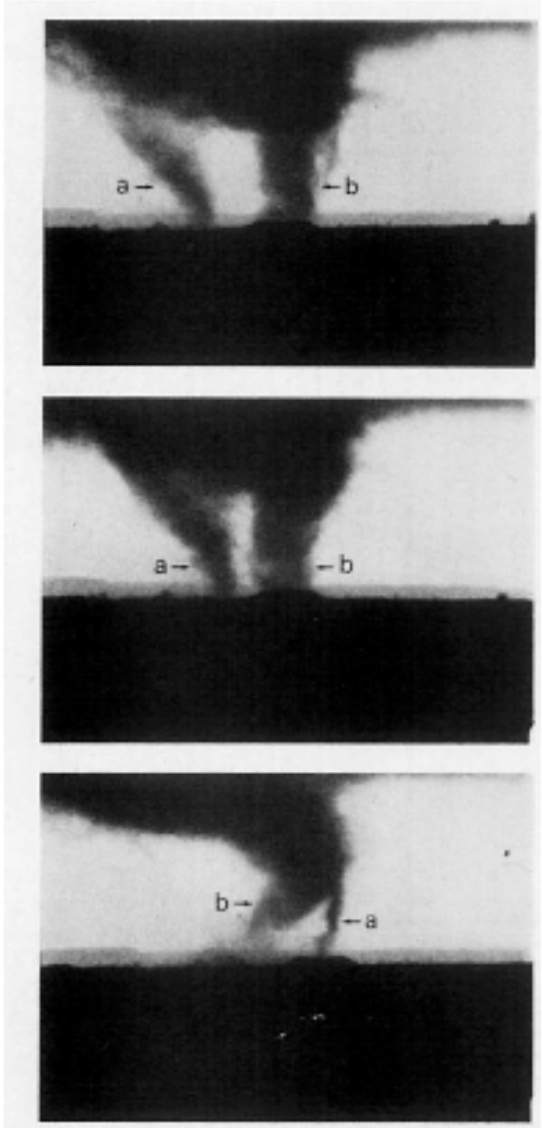
VORTEX I 1994-95

Verification of the Origin of Rotation in Tornadoes EXperiments



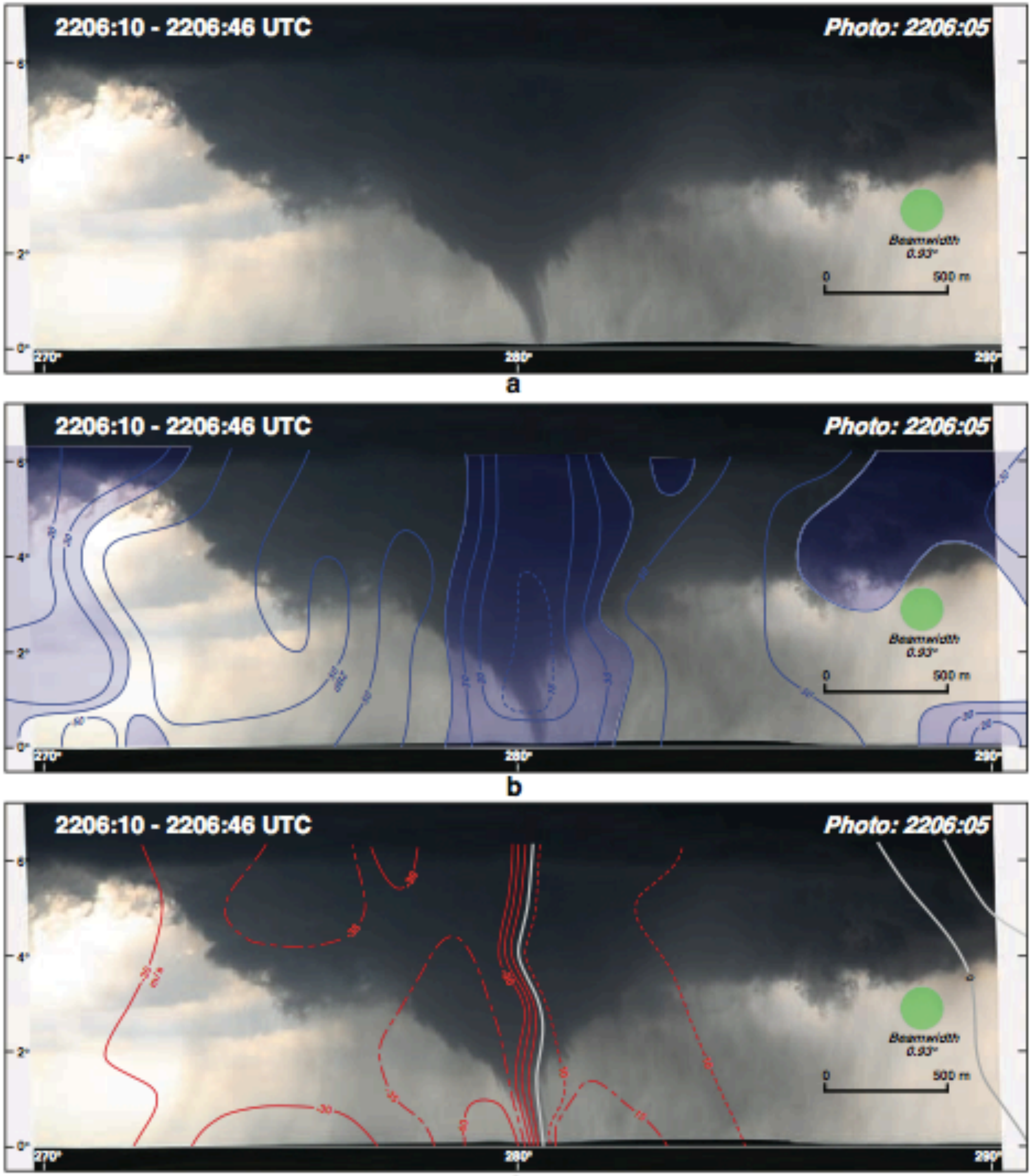
Radar reflectivity, range of 3 km

Wurman, Nature 1996



Multiple cores

VORTEX II 2009-10



Radar reflectivity

Velocity

Wakimoto et al. 2010

OUTLINE

- *The helical but non-rotating case (Beltrami ABC flow)*
- *The rotating but non-helical case (Taylor-Green flow)*

* Helicity and rotation (ABC forcing and Coriolis force)

  - Self-similarity of the direct energy cascade & conformal invariance
  - A new spectral law for fluid turbulence at small scale in the presence of both helicity and strong rotation
  - Strong helical well-ordered vortices amongst a tangle of vortex filaments
  - LES modeling to go beyond

- Discussion

- Magnetic helicity $H_m = \int \mathbf{B} \cdot \mathbf{A} dV$ & H_c = <u.B> are ideal invariants *(Woltjer, 1958)*
- Kinetic helicity $H = \int \boldsymbol{\omega} \cdot \mathbf{u} dV$ is an ideal (non-dissipative) invariant *(Moreau, 1961; Moffatt, 1969),* as well as energy E, with H(k) ≤ kE(k)

- Kraichnan, 1973: Absolute equilibria in the helical case

$$E(k) = \frac{k^2}{\alpha} \frac{4\pi}{1-\beta^2 k^2/\alpha^2}, \quad H(k) = \frac{k^4 \beta}{\alpha^2} \frac{8\pi}{1-\beta^2 k^2/\alpha^2},$$

→ No tendency for an inverse cascade of energy, unlike the two-dimensional case

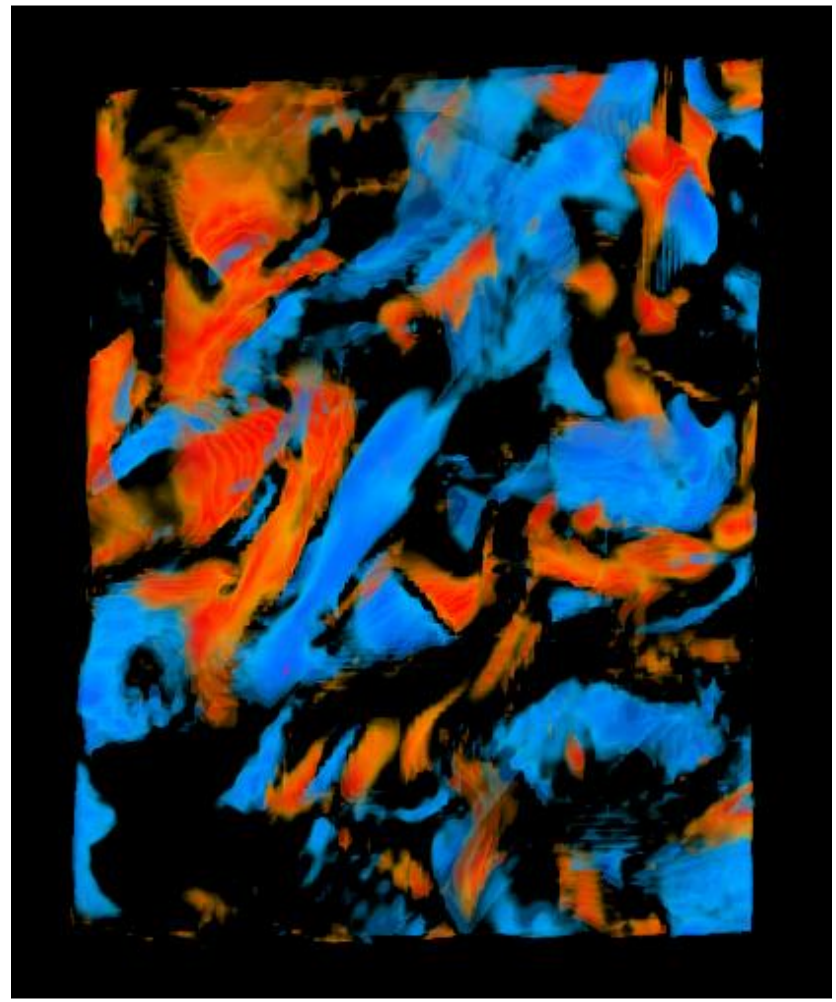
→ Also: H(k)/[kE(k)] ~ k grows with k … $E+=4\pi k^3/[\alpha -\beta k]$

Relevance of helicity to atmospheric flows: Lilly (1987), … , …

- Magnetic helicity & H_c=<u.B> are invariants *(Woltjer, 1958)*
- Kinetic helicity is an ideal (non-dissipative) invariant *(Moreau, 1961; Moffatt, 1969)*, as well as energy E, with $H(k) \leq kE(k)$

3D- Navier-Stokes, no rotation

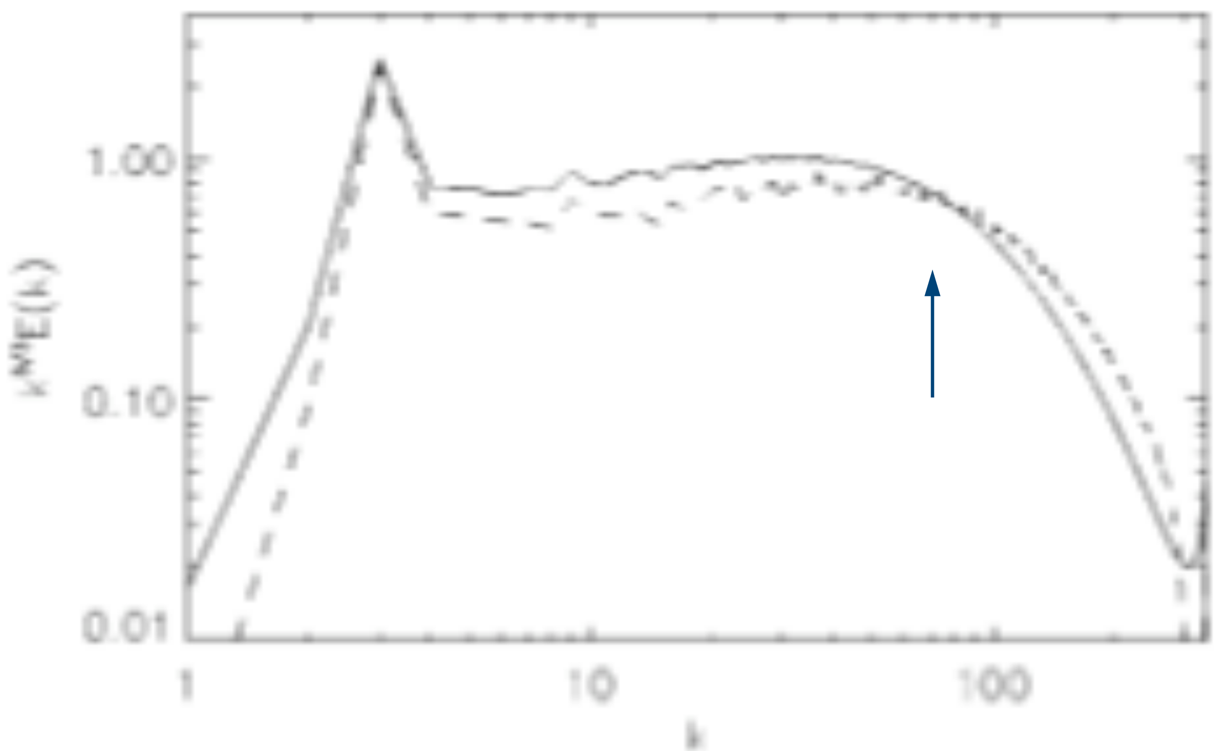
Zoom on relative helicity
$h_r = \cos(u, \omega)$



Blue, $h_r > 0.95$
Red, $h_r < -0.95$

No rotation, H≠0

Spectra of energy
-solid & helicity -dash,
both compensated by
a Kolmogorov 5/3 law



$E(k) \sim \varepsilon_E^{2/3}\, k^{-5/3}$
$H(k) \sim \varepsilon_E^{-1/3}\, \varepsilon_H\, k^{-5/3}$

Rates of transfer: $\varepsilon_E = dE/dt$, $\varepsilon_H = dH/dt$

1024^3 DNS

(two-point closures, André & Lesieur, 1977; and numerous direct numerical simulations, e.g. Chen et al. 2003)

Both E & H dissipate at the same wavenumber
(+ bottleneck)

Dynamical equations

$$\frac{\partial \mathbf{u}}{\partial t} + \omega \times \mathbf{u} + 2\Omega \times \mathbf{u} = -\nabla \mathcal{P} + \nu \nabla^2 \mathbf{u} + \mathbf{F}$$

$\nabla . \mathbf{u} = 0$

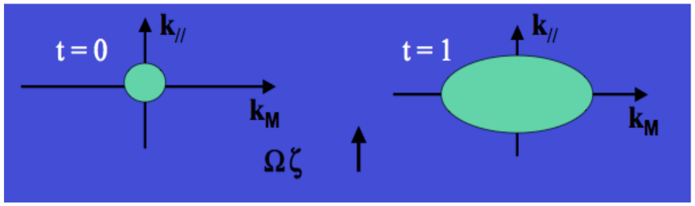
$Re = UL_0/v$; $Ro = U/2\Omega L_0$

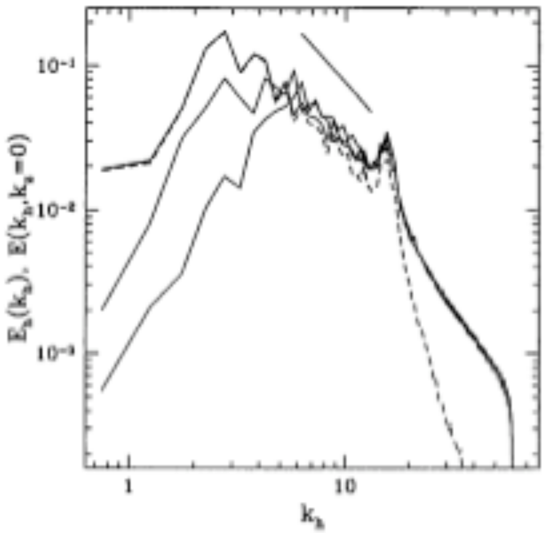
Reynolds nb. ; Rossby nb.

Frequency of inertial waves: $f_k = \pm k_{//} \Omega / k \sim 1/\Omega$

Numerous previous investigations

- Anisotropy of the resulting flow and tubular structures
- Weak turbulence of inertial waves *(Galtier, 2003)*
- Both a direct an inverse cascade of energy can coexist with three-dimensional forcing





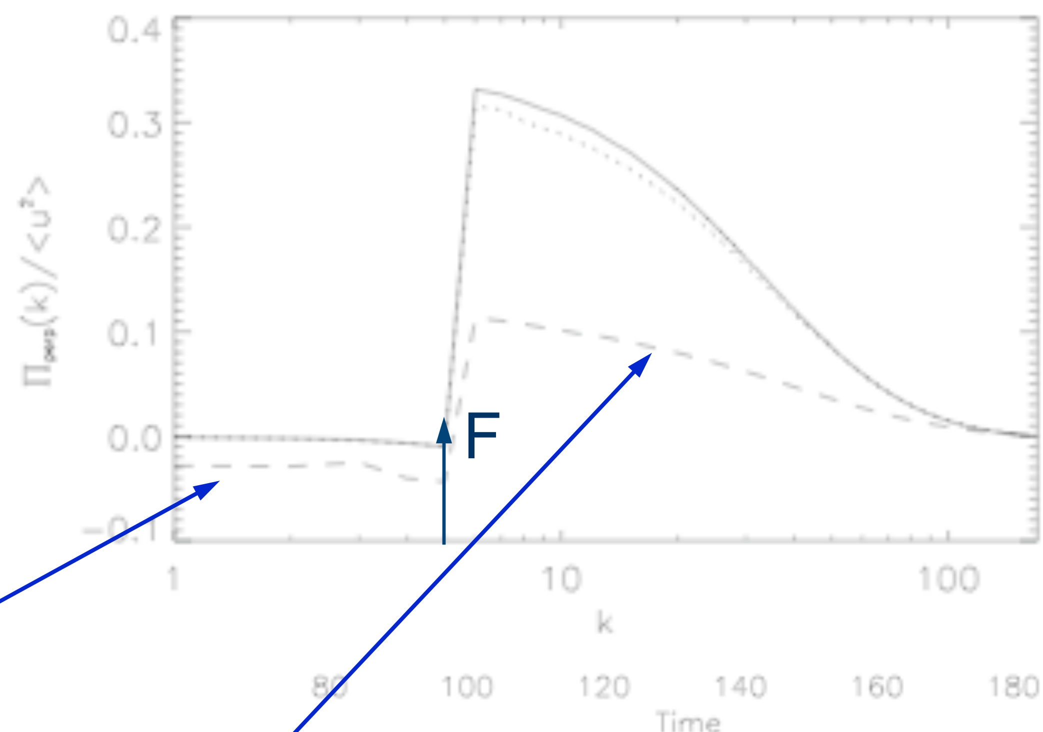
Smith et al., 1996

Energy **flux normalized** by U_{rms} for three Rossby numbers

512^3, k_F=4

Taylor-Green non-helical forcing

Solid: Ro= 1.4 , Dots: Ro= 0.35



Dash: Ro= 0.07

*Together with an inverse cascade (negative flux), there is a w**eaker direct cascade of energy at lower Rossby number** (positive flux)*

Mininni, Alexakis & AP 2009

Isotropic phenomenology of turbulence with waves

- Assumption: $\hat{\imath} = \tau_W / \tau_{NL} << 1$; transfer time T_{tr} evaluated as

$$T_{tr} = T_{NL} / \hat{\imath} = T_{NL} * (T_{NL}/T_W)$$ *with* $T_{NL} = l/u_l$ *and* $T_W = 1/\Omega$

- Constant energy flux: $\varepsilon = DE/Dt \sim k*E(k) / T_{tr}$

$$\longrightarrow \quad E(k) \sim [\hat{\imath}\Omega]^{1/2} k^{-2}$$

(Dubrulle & Valdetarro, 1992; Zhou, 1995)

Structure functions: $<\delta u(l)^p> \sim l^{\zeta p}$, $\zeta_p = p/2$

- At dissipation wavenumber k_d, $T_{diss} = [\nu k_d^2]^{-1} = T_{tr}$; this leads to

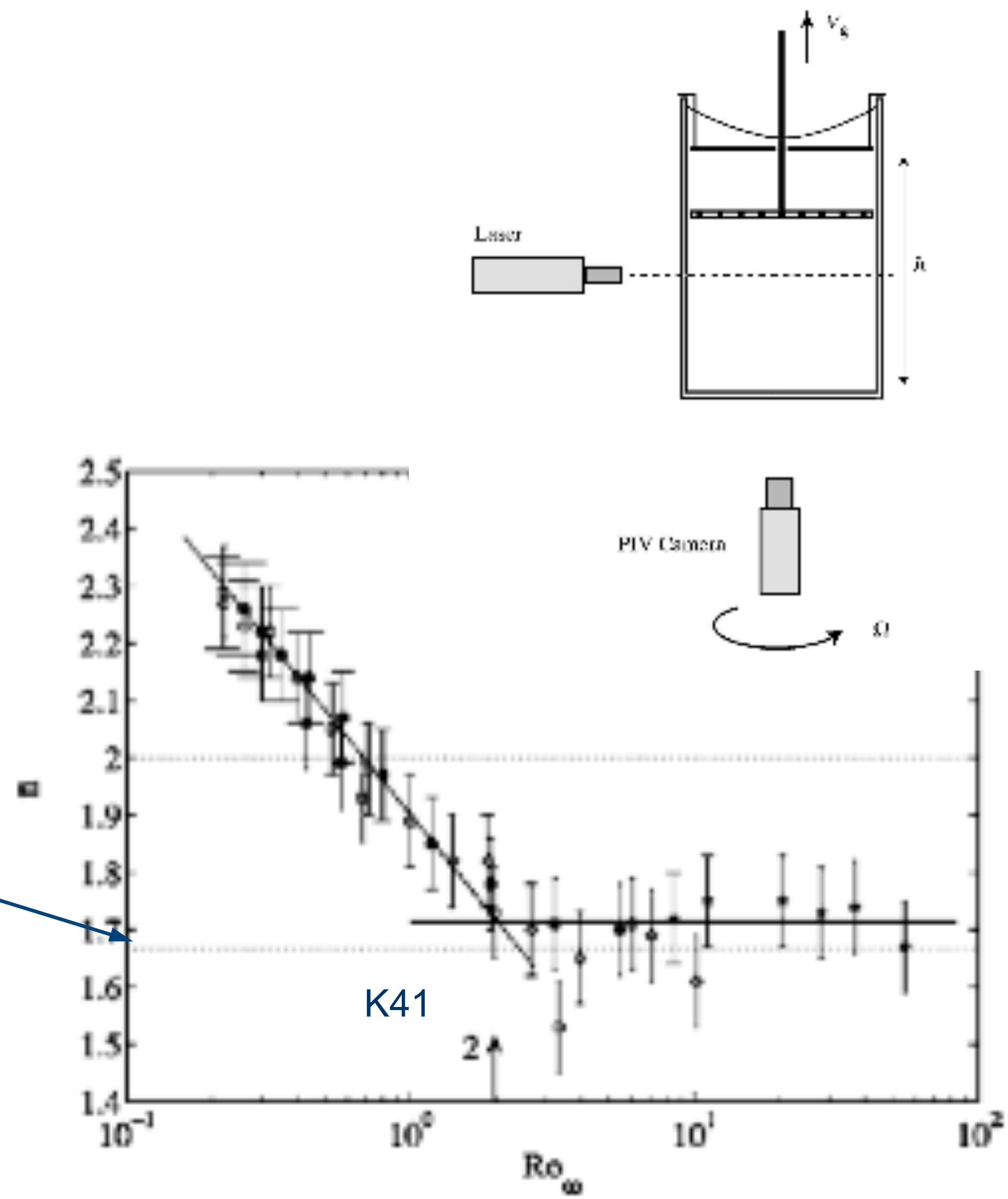
$$k_d / k_0 \sim [\varepsilon L_0^2 / \nu^2 \Omega]^{1/2} \sim Re * Ro$$

(Canuto & Dubovikov, 1997)

The scaling of the energy spectrum at high enough rotation rate

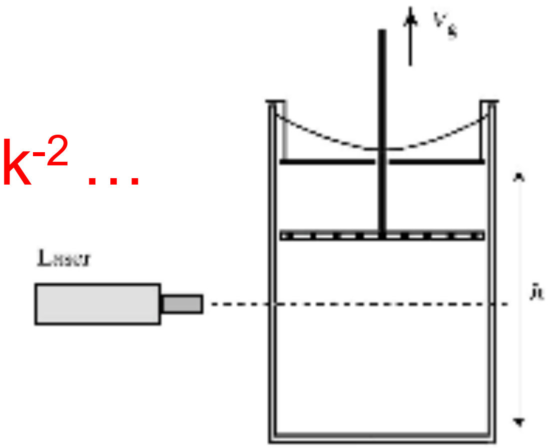
can differ from the classical Kolmogorov spectrum,

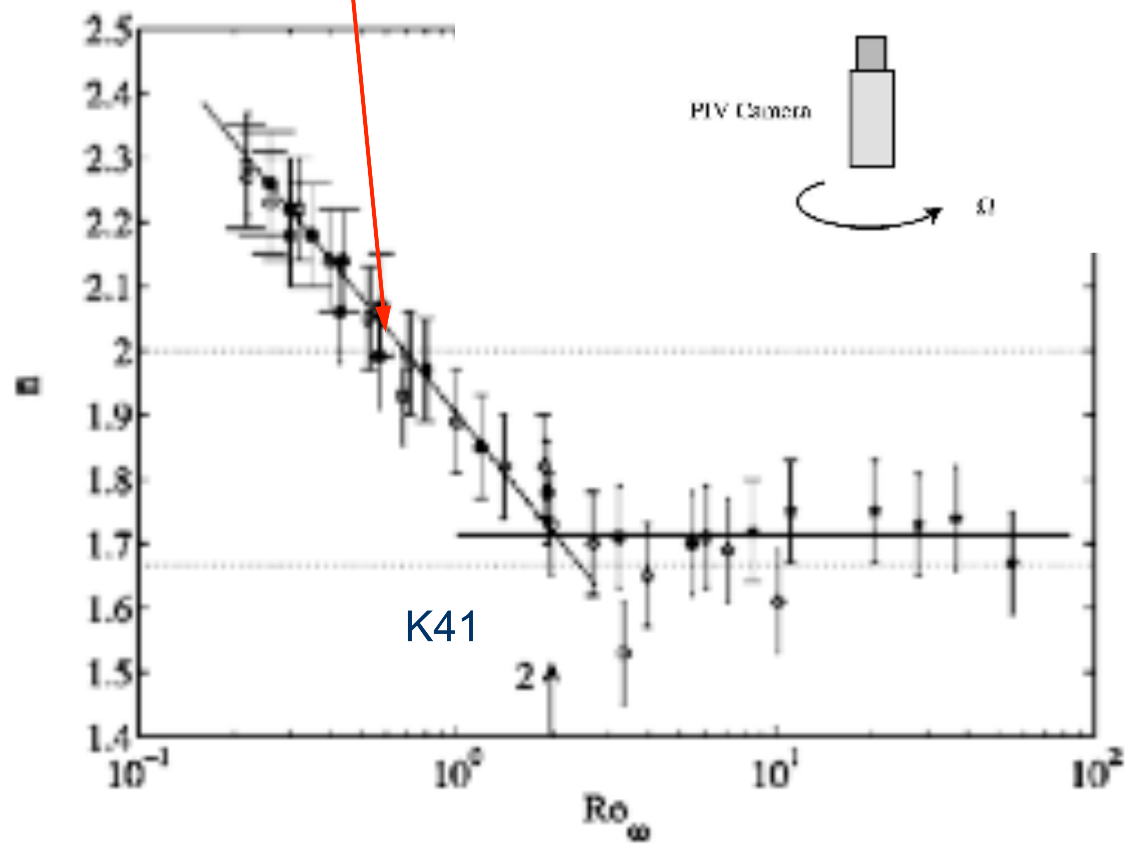
i.e. $E(k) \neq k^{-5/3}$



(Morize et al., 2005)

But it does not stop at k^{-2} …





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Is it stopping at ~ 2.3?

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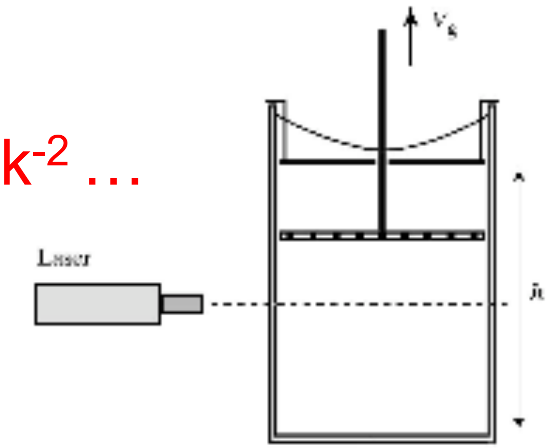

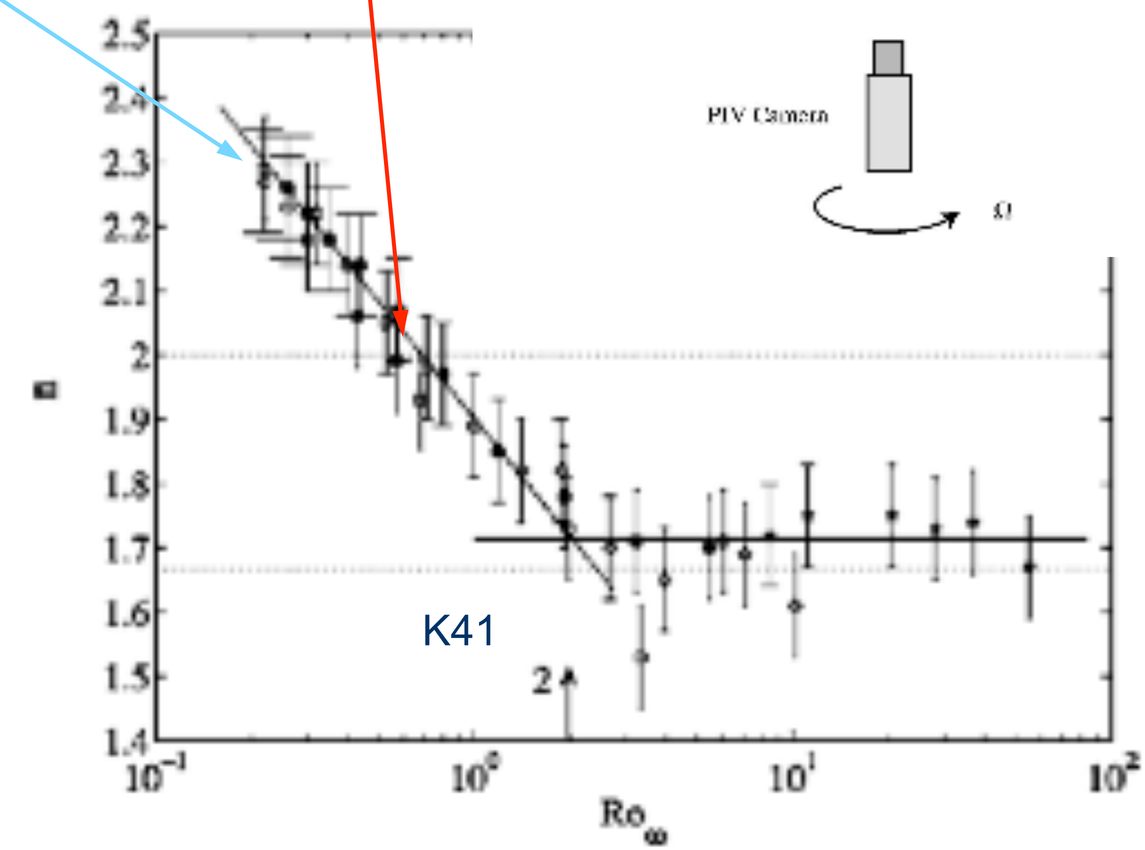

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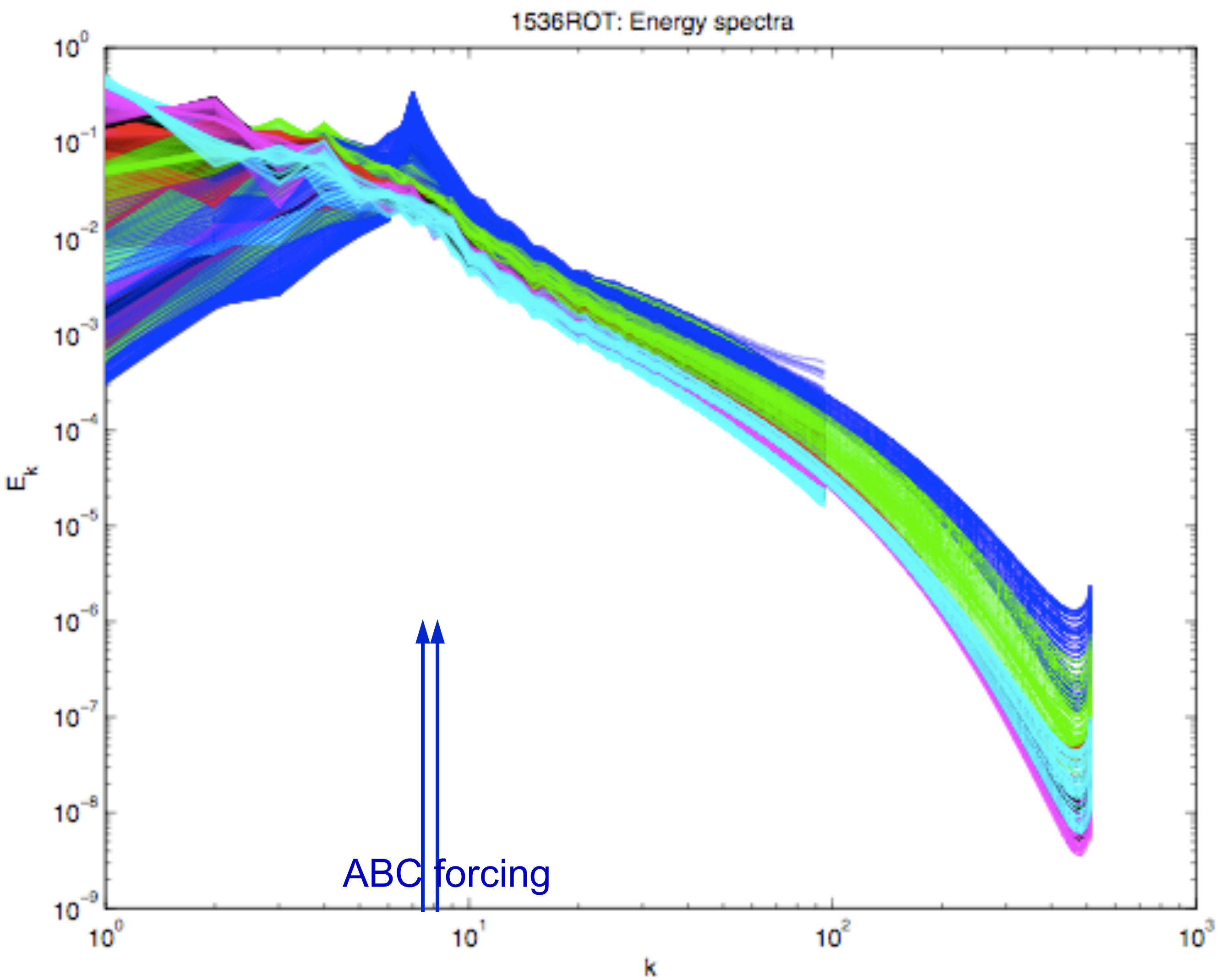


Initial conditions: fully developed non rotating Kolmogorov flow, 1536^3 grid

T=0 to T=30 *, going through dark blue, green, mauve, red, pink, pale blue* *+ LES*

Top view

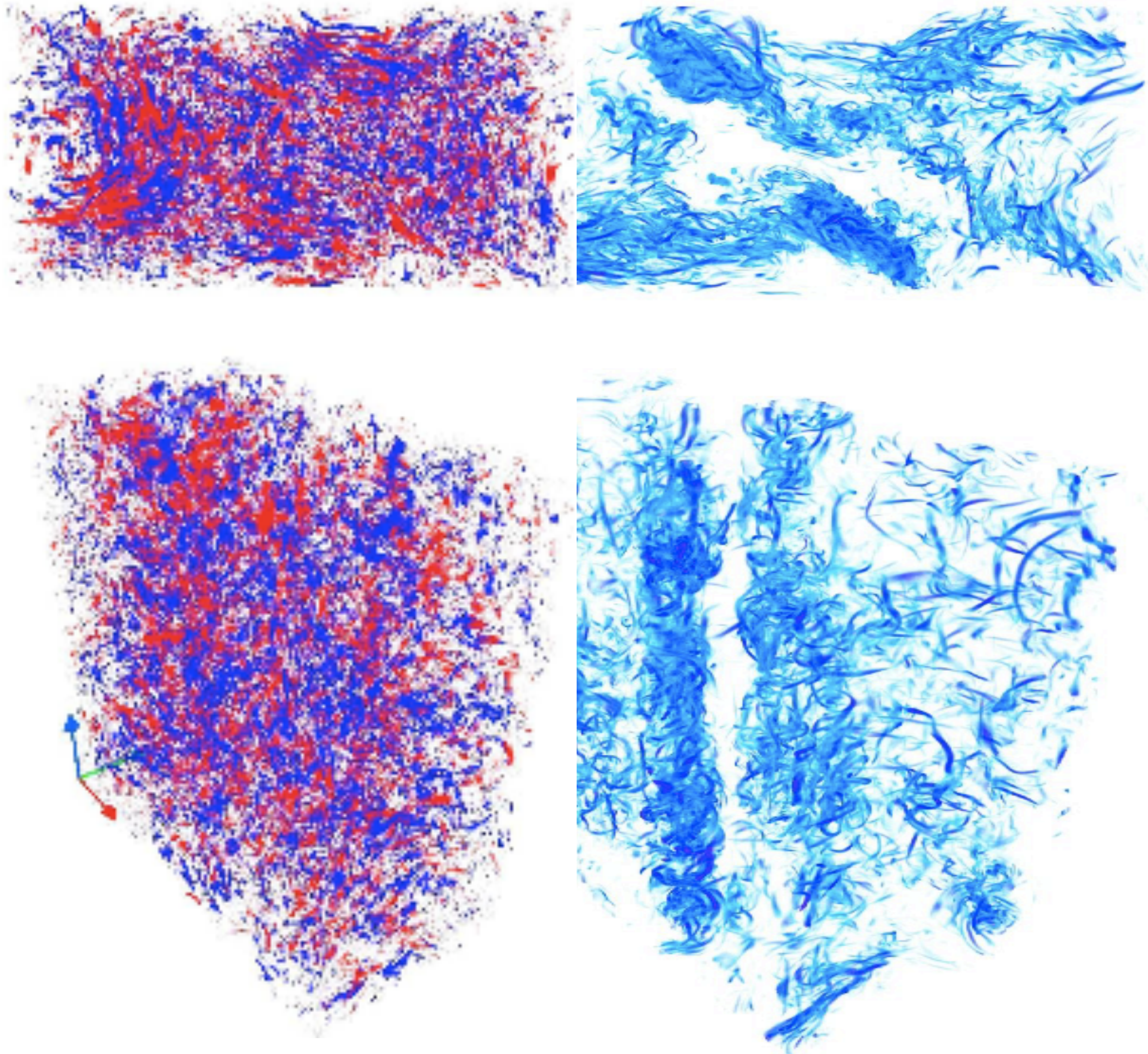
& side view of

(left) relative helicity

(*positive* or *negative*)

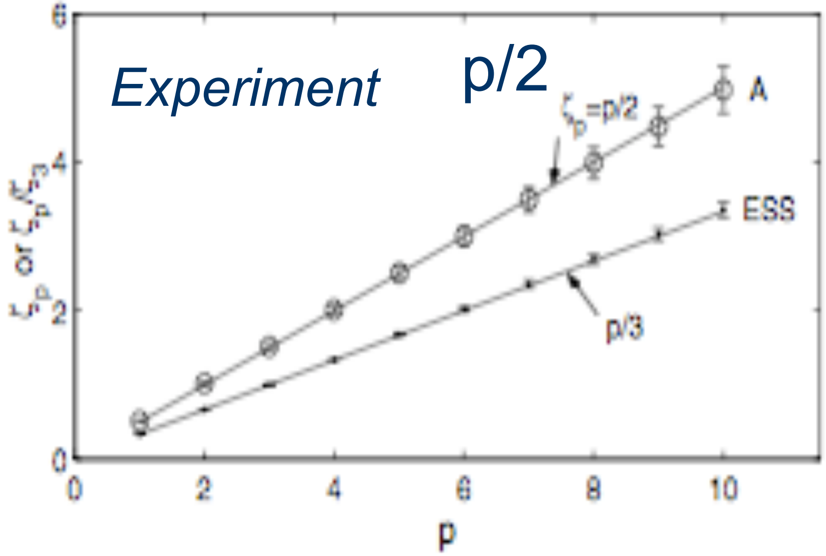
&

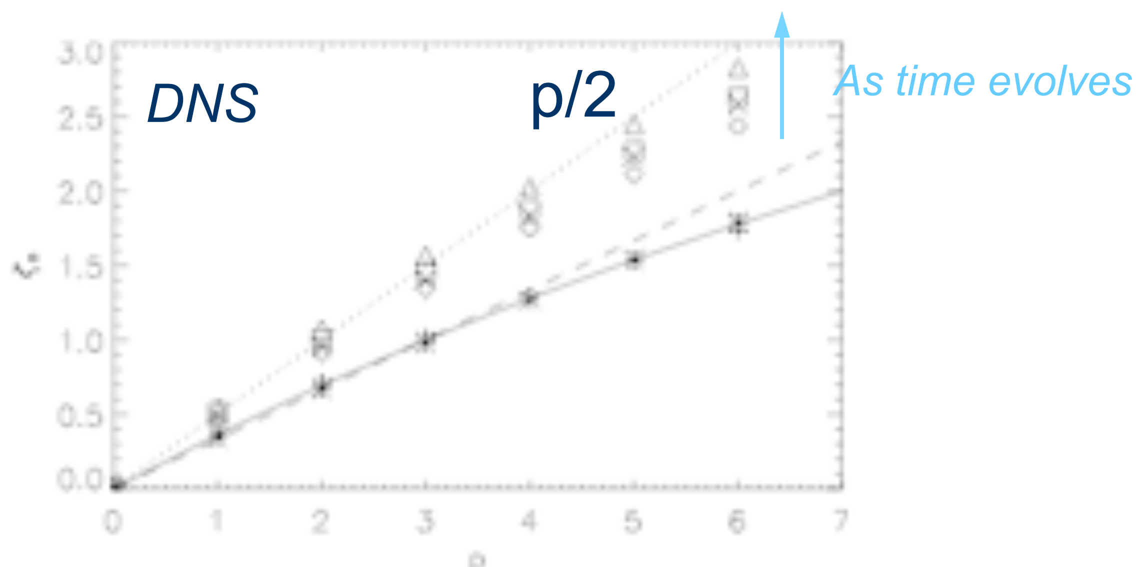
(right) vorticity



Taylor-Green non-helical forcing, k_0=4, 512^3, Ro=0.35

Scaling of structure functions in rotating turbulence

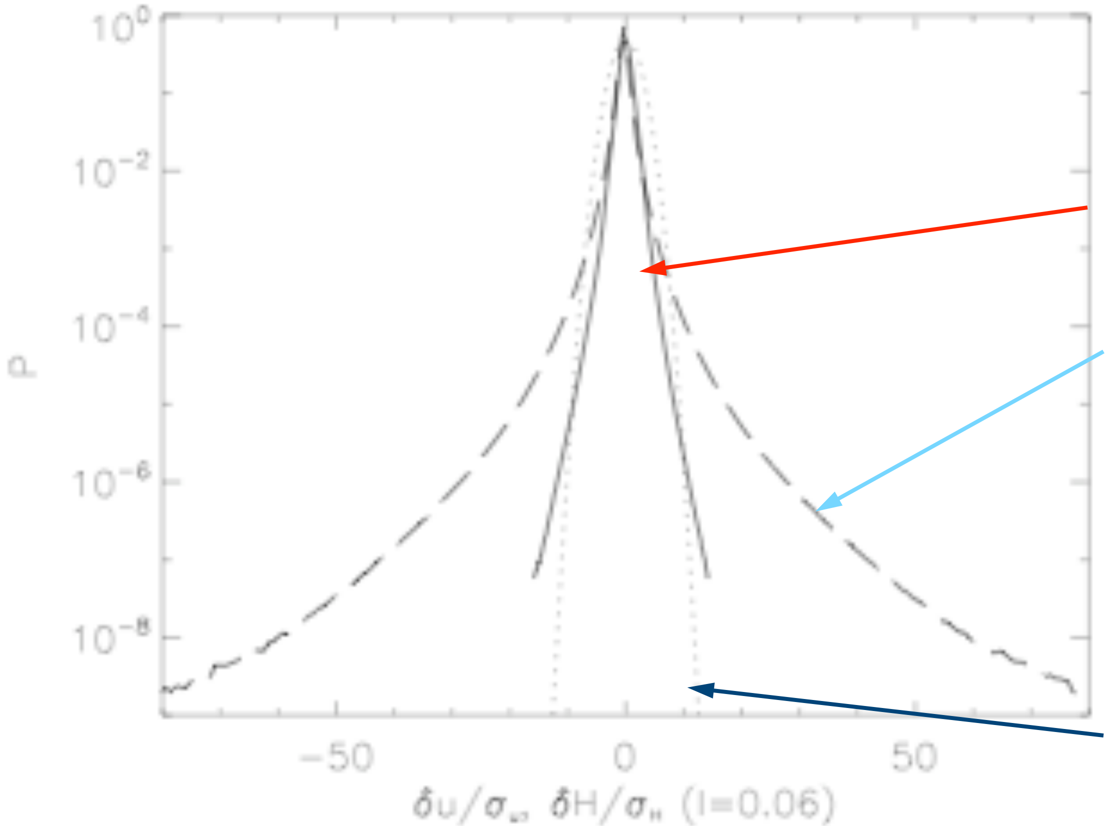




$\zeta_p = p/2$ *(Simand et al., '00; Baroud et al., '02; Mininni+AP, PRE **79** '09)*

From Taylor-Green forcing (globally non helical) to ABC forcing (Beltrami flow, fully helical) for rotating flows

Weakened intermittency in the **direct** energy cascade



PdF of normalized small scale

Solid: velocity gradients

Dash: helicity gradients

dots: Gaussian

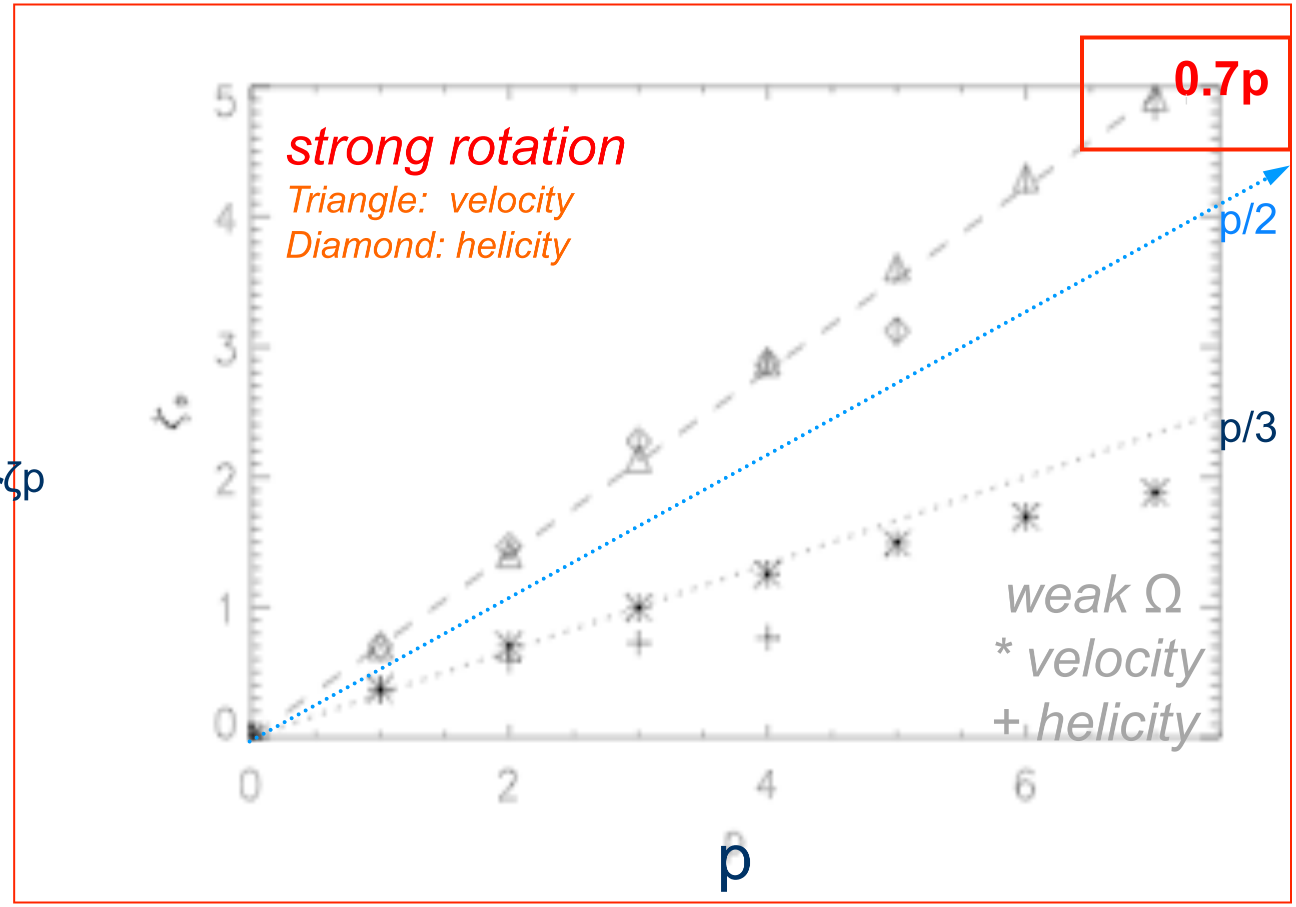
Scaling exponents of structure functions

$<\delta f=f(x+r)-f(x)]^p> \sim r^{\zeta p}$

of velocity

and helicity

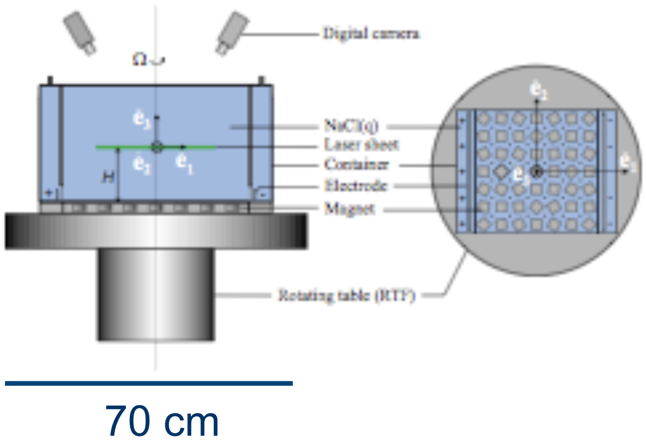
$E(k) \sim k^{-2.4}$

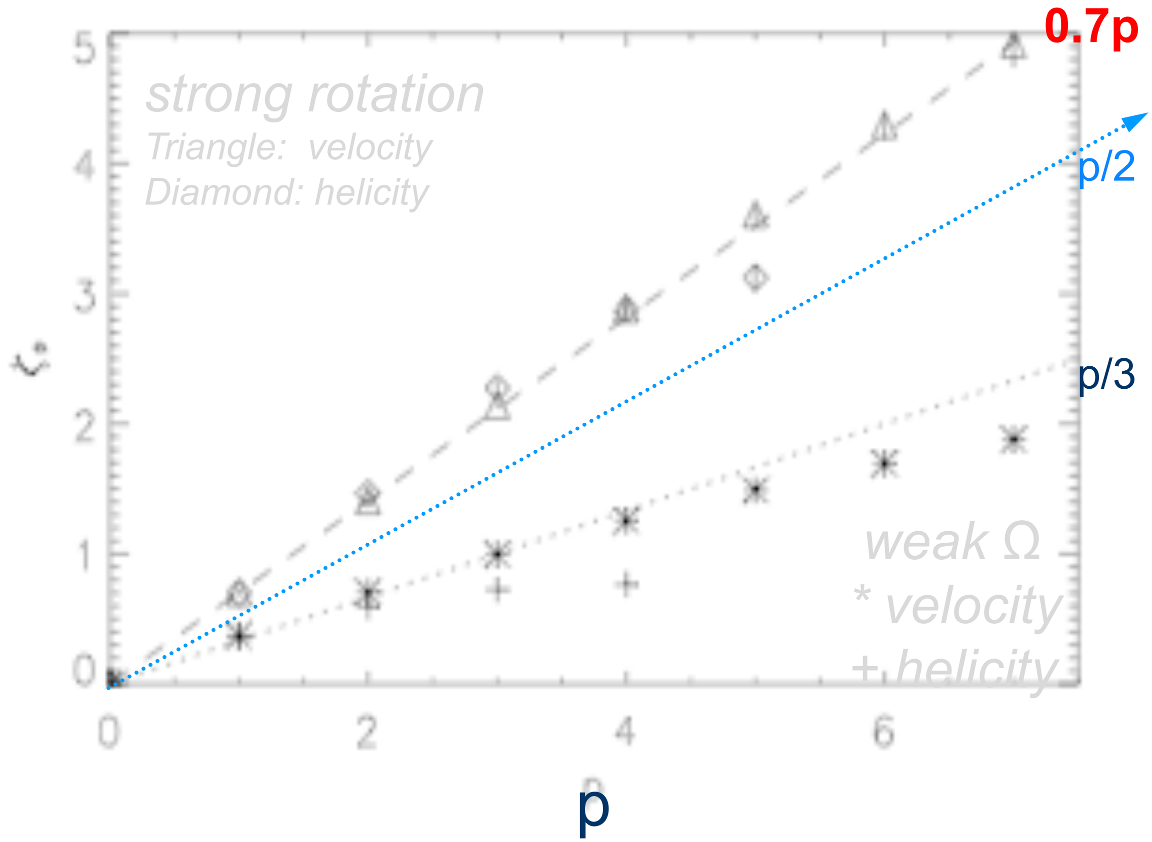


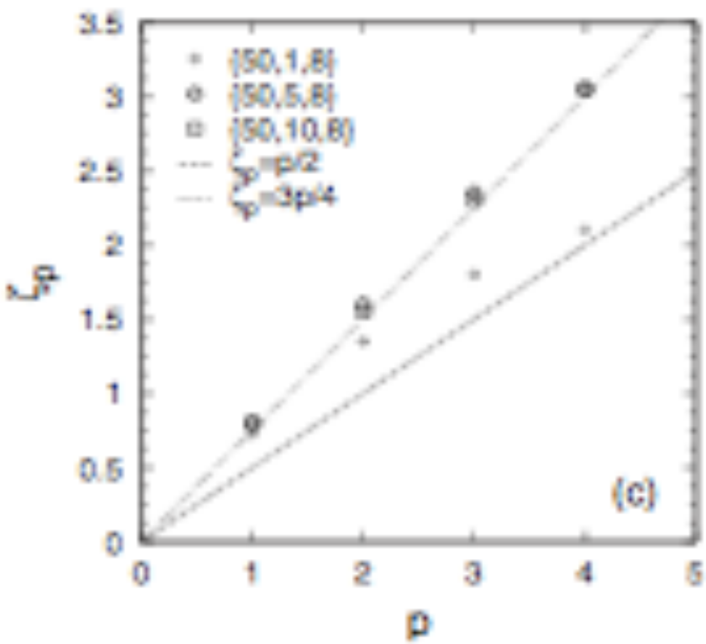
The energy in the <u>direct cascade is **self-similar**</u> for strong rotation,
whereas helicity displays some modicum of intermittency

ζ_p = p/2 for the non-helical case (Simand + '00; Baroud + '02; Mininni & AP '09) not observed here

Scaling exponents of structure functions







$\zeta_p \sim 3p/4$ at high Ω
experimentally
as well

van Bhokhoven et al. 2009

So, what's happening?

New spectral law for energy and helicity at high rotation

Isotropic phenomenology of turbulence with waves:

- Small parameter: î $= \tau_W / \tau_{NL}$; transfer time T_{tr} evaluated as:

  $T_{tr} = T_{NL}$ / î $= T_{NL}{}^{*} \, (T_{NL}/T_W)$ *with* $T_{NL} = l/u_l$ *and* $T_W = 1/\Omega$

- Constant helicity flux: $\eta = DH/Dt \sim k^{*}H(k) / T_{tr}$
- Assume $E(k) \sim k^{-\mathbf{e}}$, $H(k) \sim k^{-\mathbf{h}}$

→ **e + h = 4** in the helical case with rotation

Assuming maximal helicity [H(k)=kE(k)] leads to e=5/2

and structure functions: $\langle \delta u(l)^p \rangle \sim l^{\zeta_p}$, $\zeta_p = 3p/4$ *(Mininni & AP, 2009)*

But is maximal helicity a reachable solution?

$$E(k) = E_\Omega + E_K \sim \epsilon^a \, \tilde{\epsilon}^b \, \Omega^f \, k^{-e} + \epsilon^{2/3} \, k^{-5/3}$$

$$H(k) = H_\Omega + H_K \sim \epsilon^c \tilde{\epsilon}^d \Omega^g k^{-h} + \tilde{\epsilon} \epsilon^{-1/3} k^{-5/3}$$

$$\epsilon = dE/dt, \tilde{\epsilon} = dH/dt \quad , \; \mathcal{F}(a,b,c,d,e,f,g) = 0$$

Zeman wavenumber at which $\tau_W = \tau_{NL}$: $k_\Omega \sim \epsilon^\alpha \tilde{\epsilon}^\beta \Omega^\gamma$

Wavenumbers $k_{e,h}$ at which $E_\Omega, H_\Omega = E_K, H_K$

$$k_e \sim \epsilon^\delta \, \tilde{\epsilon}^{\,\phi} \, \Omega^{\frac{3(3a+3e-7)}{3e-5}} \; , \; k_h \sim \epsilon^\psi \, \tilde{\epsilon}^{\,\xi} \, \Omega^{\frac{-3(3a+3e-8)}{3e-7}}$$

$k_\Omega = k_E$ $\rightarrow$ $k_\Omega \sim \epsilon^{-1/2} \, \Omega^{3/2} \, \forall e$ and

$$E_\Omega \sim \epsilon^{\frac{3-e}{2}} \, \Omega^{\frac{3e-5}{2}} \, k^{-e} \quad (1)$$

$$H_\Omega \sim \epsilon^{\frac{-(3-e)}{2}} \, \tilde{\epsilon} \, \Omega^{\frac{7-3e}{2}} \, k^{-(4-e)} \quad (2)$$

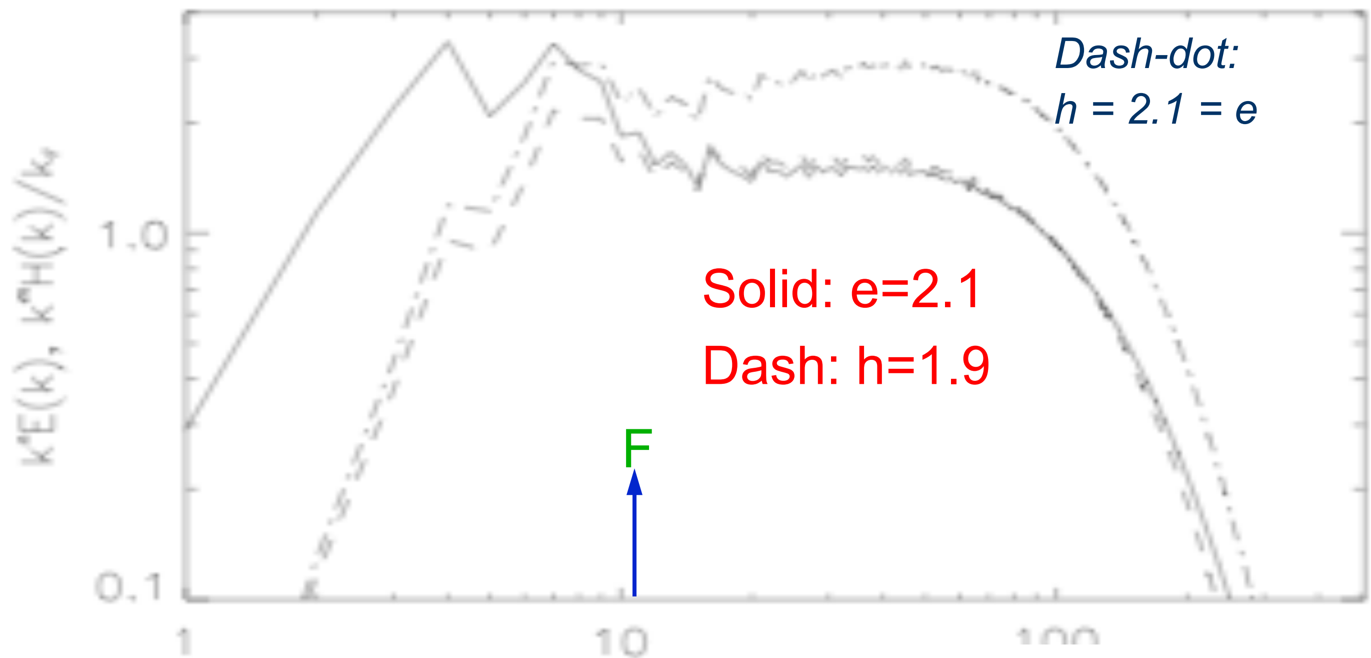
$\rightarrow e \leq 7/3$ if $k_{e,h,\Omega} \rightarrow \infty$ for $\Omega \rightarrow \infty$

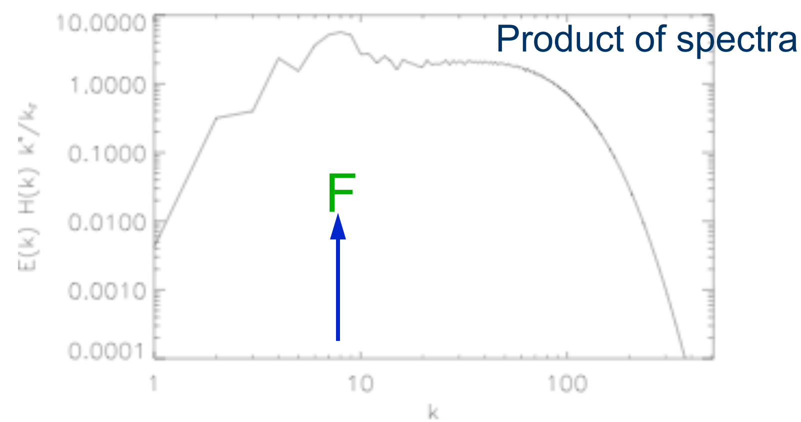
Chakraborty, 2007; Rosenberg et al. 2011

k^x - Compensated spectra for energy (x=e) & helicity (x=h)

1536^3 run

- $k_F=7$
- Re=5100
- Ro=0.06





Mininni & AP,
Phys. Fluids 2010

Compensated spectra for the new spectral law:
$k_{\perp}^{4}\, E(k)*H(k)/k_F$

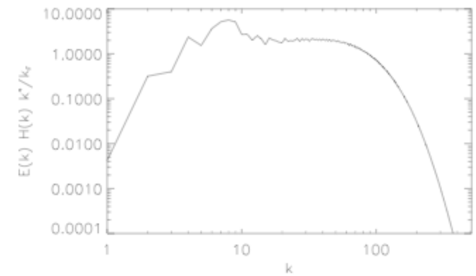
NORMALIZED RATIO OF HELICITY TO ENERGY TO SMALL SCALES

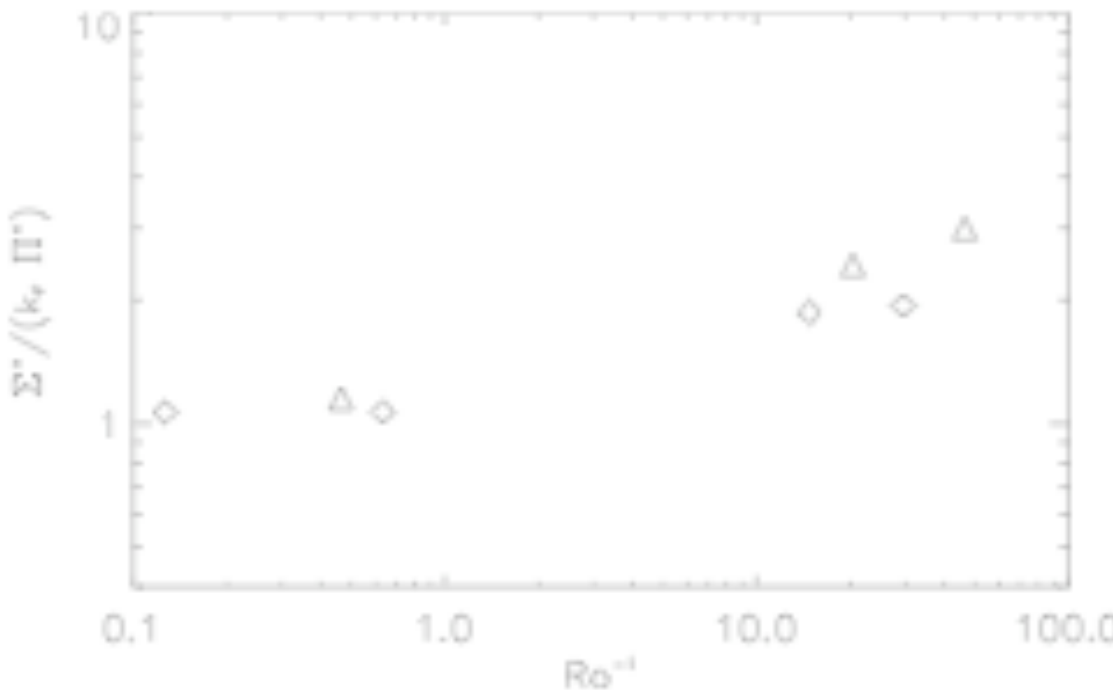
as a function of rotation

1536^3 run

k_F=7

Re=5100, Ro=0.06





Mininni & AP, Phys. Fluids 2010

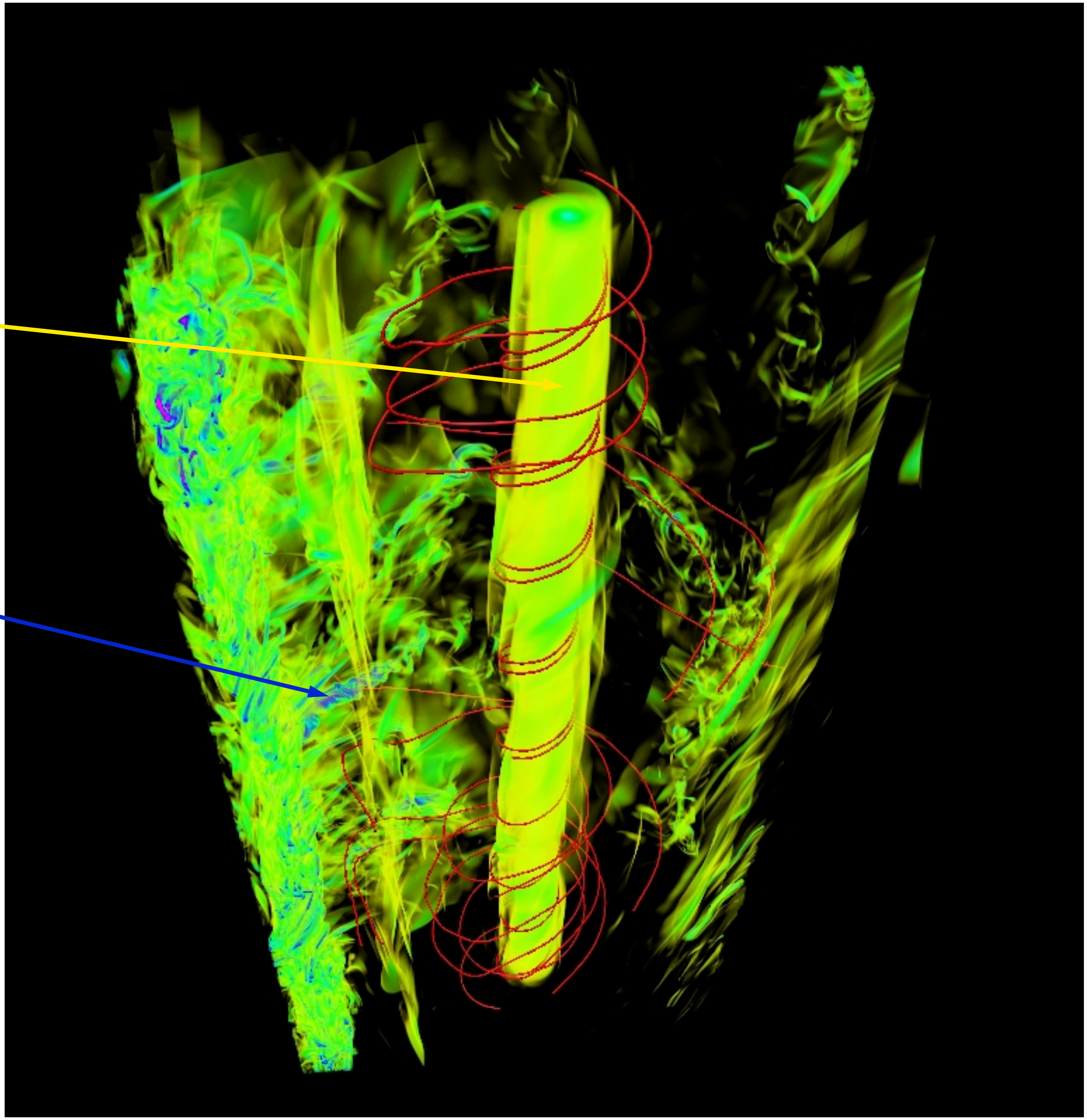
Mininni & AP,
Phys. Fluids 22 (2010)

Zoom on a
Beltrami core
vortex

amidst a tangle
of smaller-scale
vortex filaments

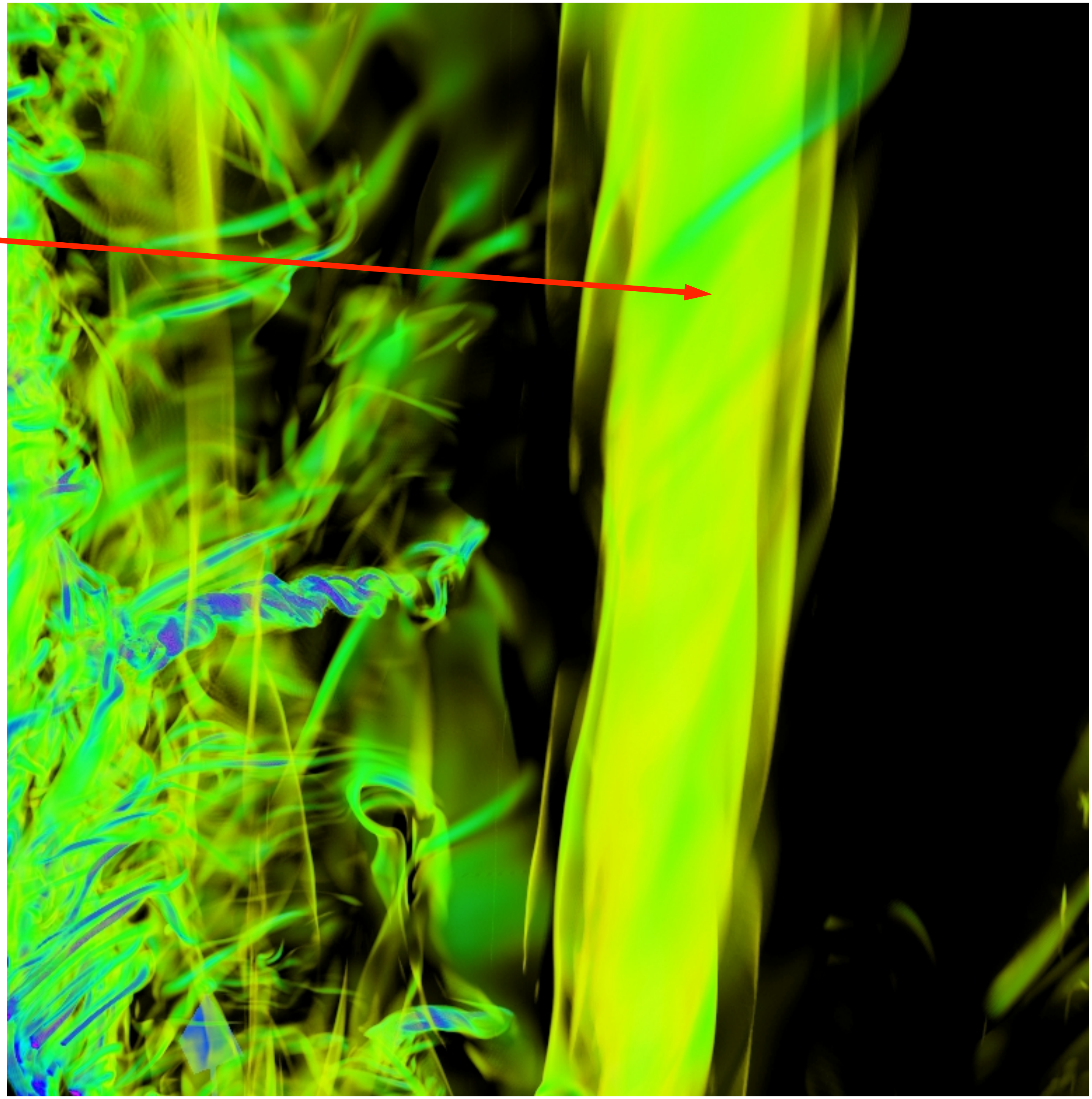
Together with
particle trajectories

1536^3 grid, $k_F=7$,
Re=5100,
Ro=0.06,



ZOOM on Vorticity:

Beltrami core vortices



Helical forcing at k_F=7

DNS on 1536^3 grid points

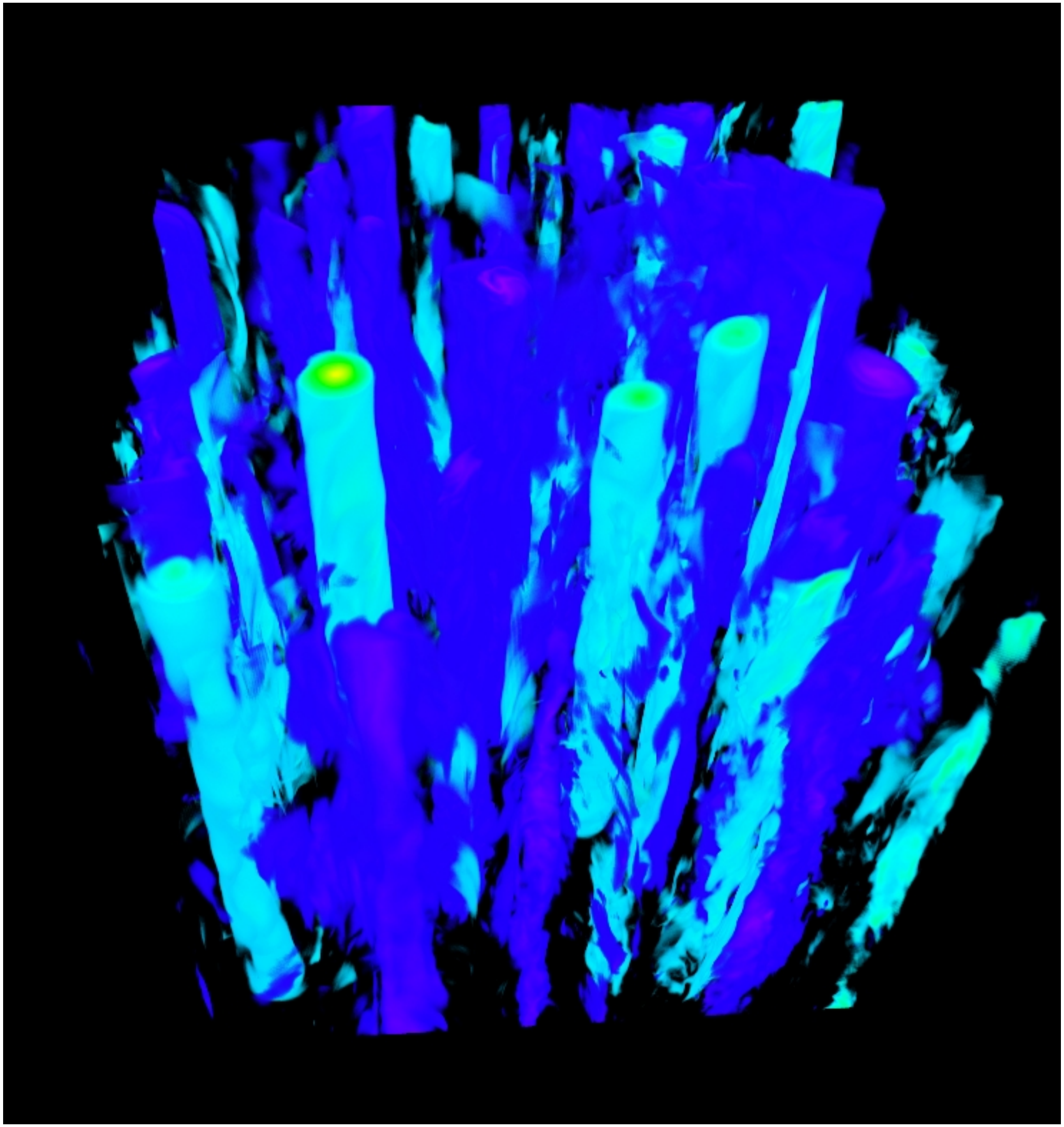
Re=5100, Ro=0.06

Updrafts, with H>0

- Mininni & AP, Phys. Fluids 22 (2010)

*Visualizations with VAPOR freeware (NCAR, Clyne et al., New J. Phys. **9**, 2008)*

Vertical velocity V_z, global view



1536^3 grid, k_F=7, Re=5100, Ro=0.06, helical forcing

With helicity, strong coherent structures form that are organized, fully helical and at relatively small scale: Beltrami core vortices

They are embedded in a complex tangle of vorticity, with also a large-scale component due to the inverse cascade

1536^3 grid, $k_F=7$,
Re=5100, Ro=0.06

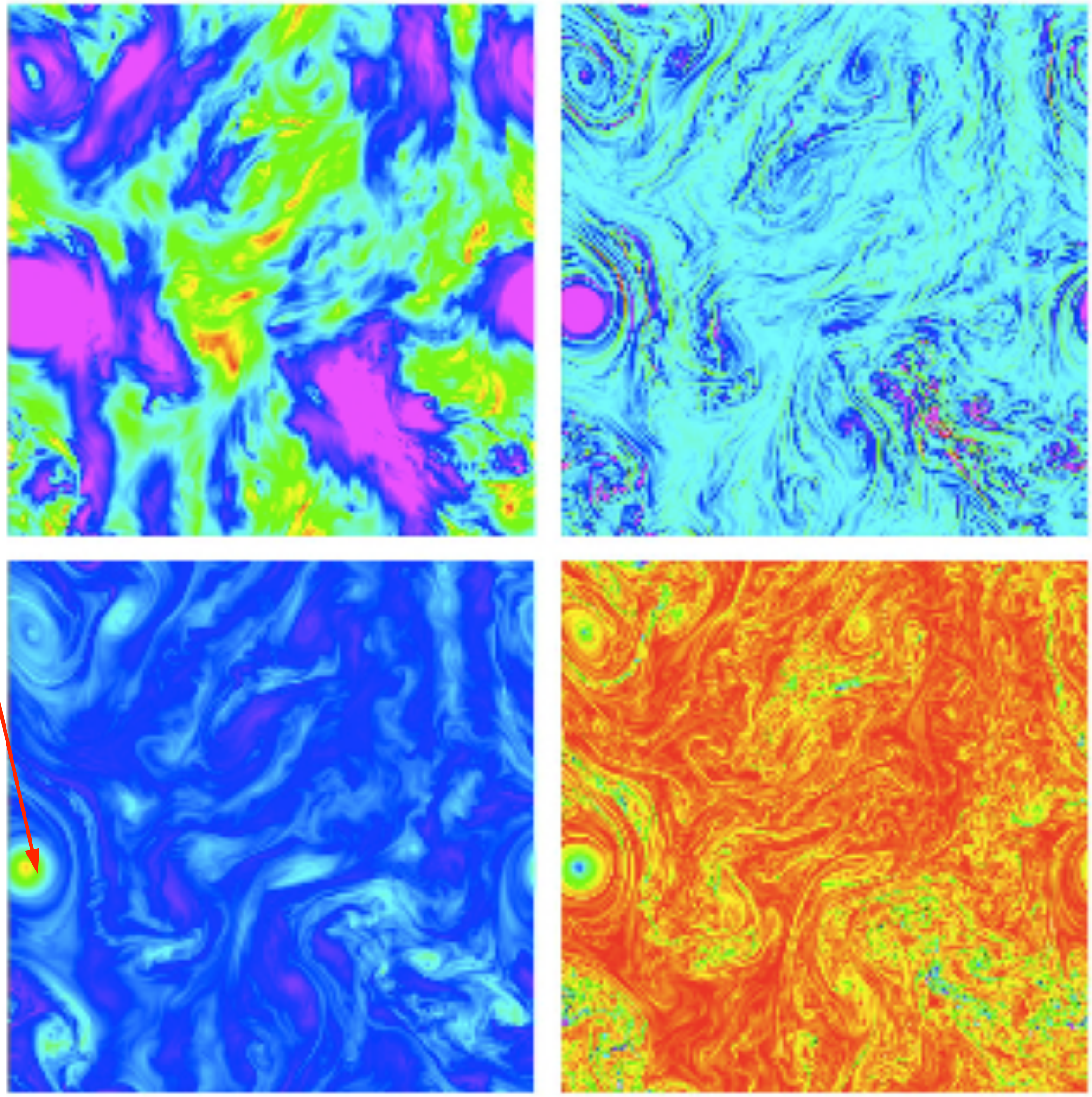
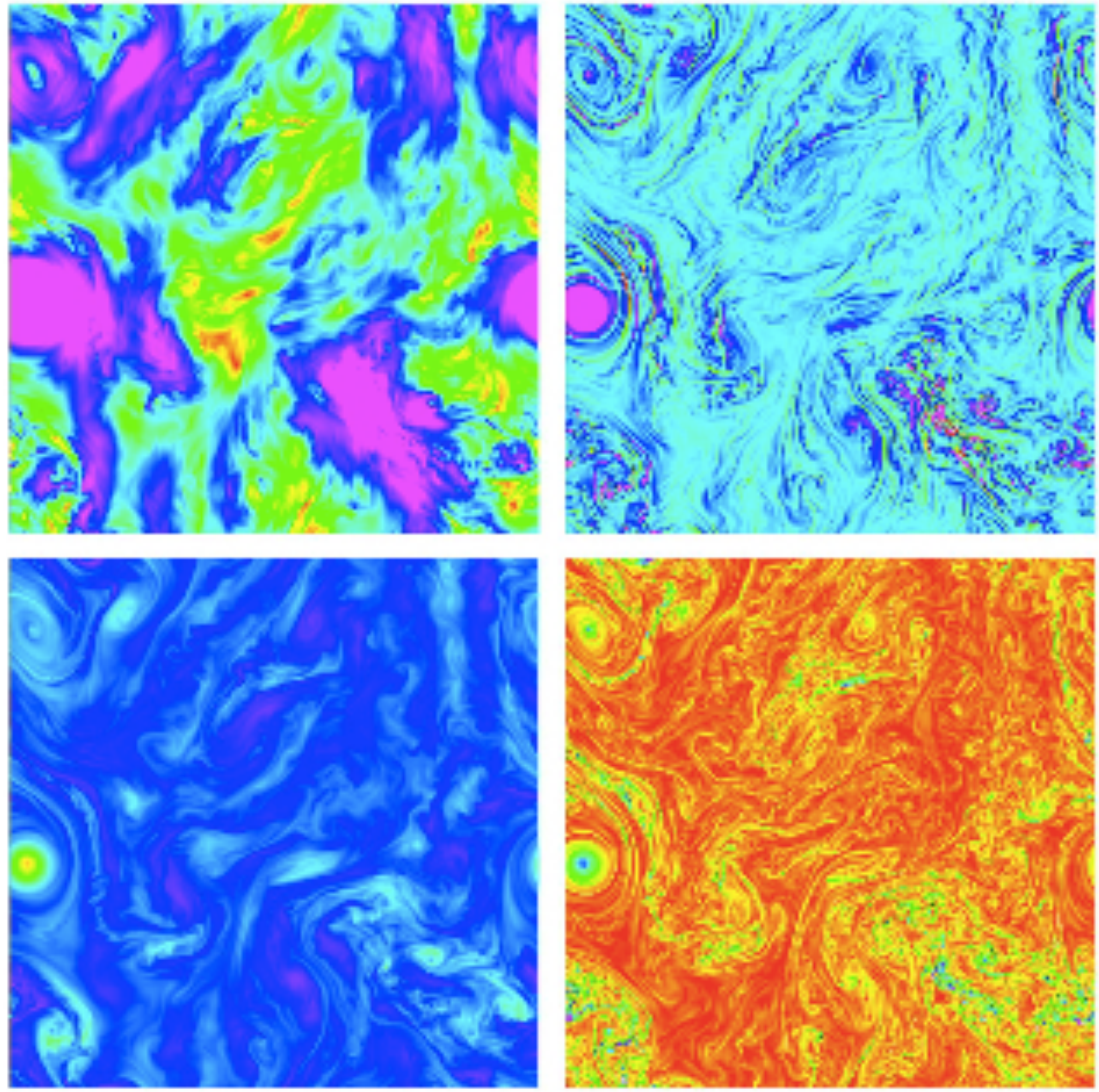


FIG. 9: From top to bottom and from left to right, slices of the energy density, vorticity intensity, z component of the velocity, and helicity density, in run B at $t \approx 30$.

With helicity, strong coherent structures form that are organized

Beltrami Core Vortices



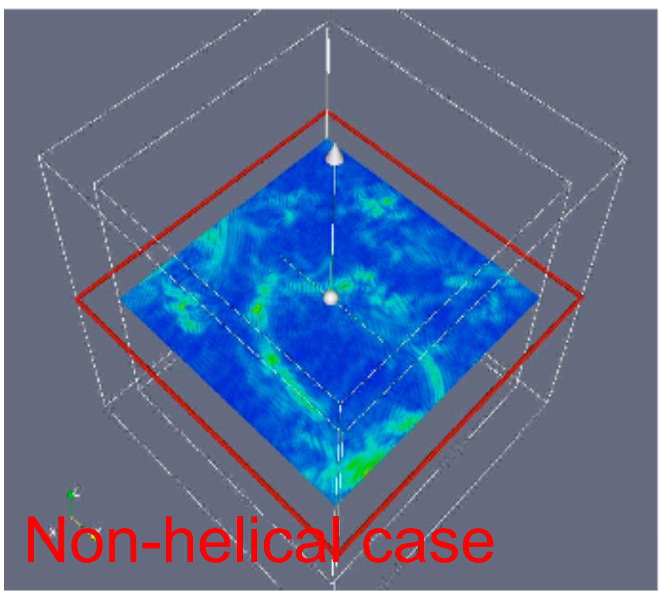
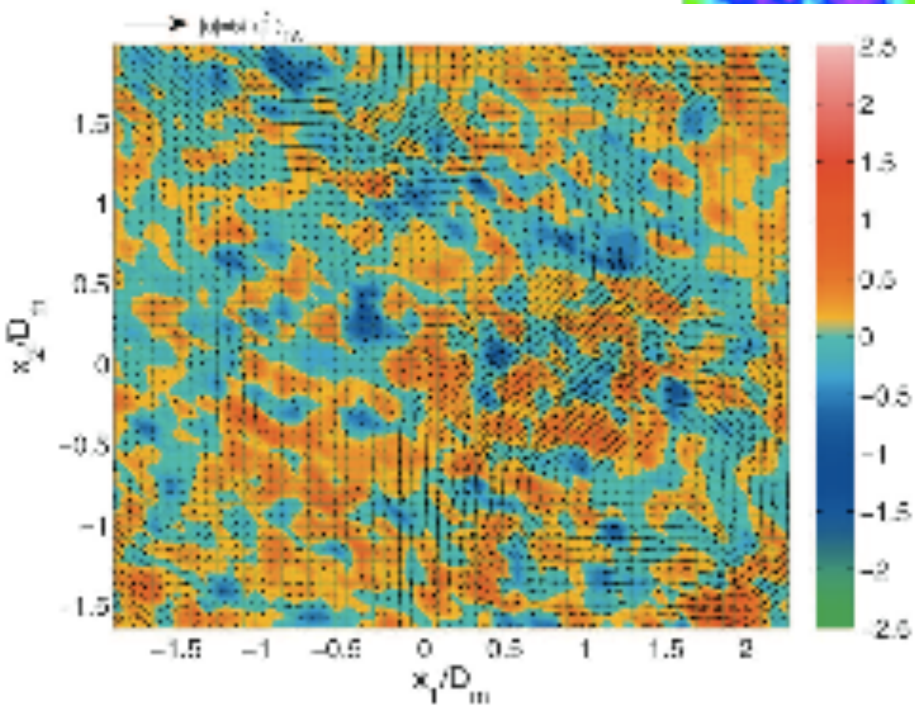


FIG. 9: From top to bottom and from left to right, slices of the energy density, vorticity intensity, z component of the velocity, and helicity density, in run B at $t \approx 30$.

With helicity, strong coherent structures



Experiment

(van Bokhoven et al. 2009)

color: v_z
arrows: v_H

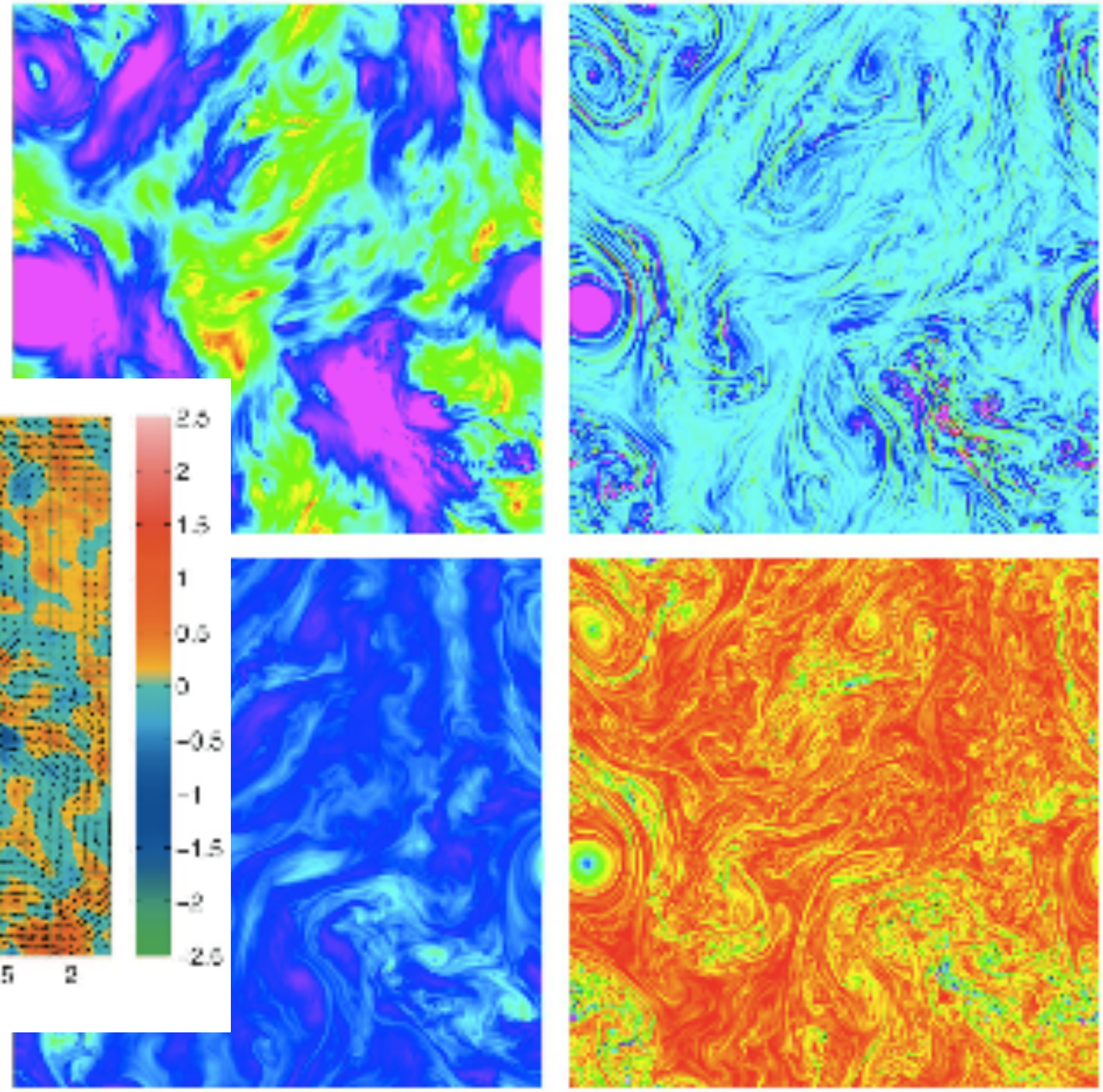
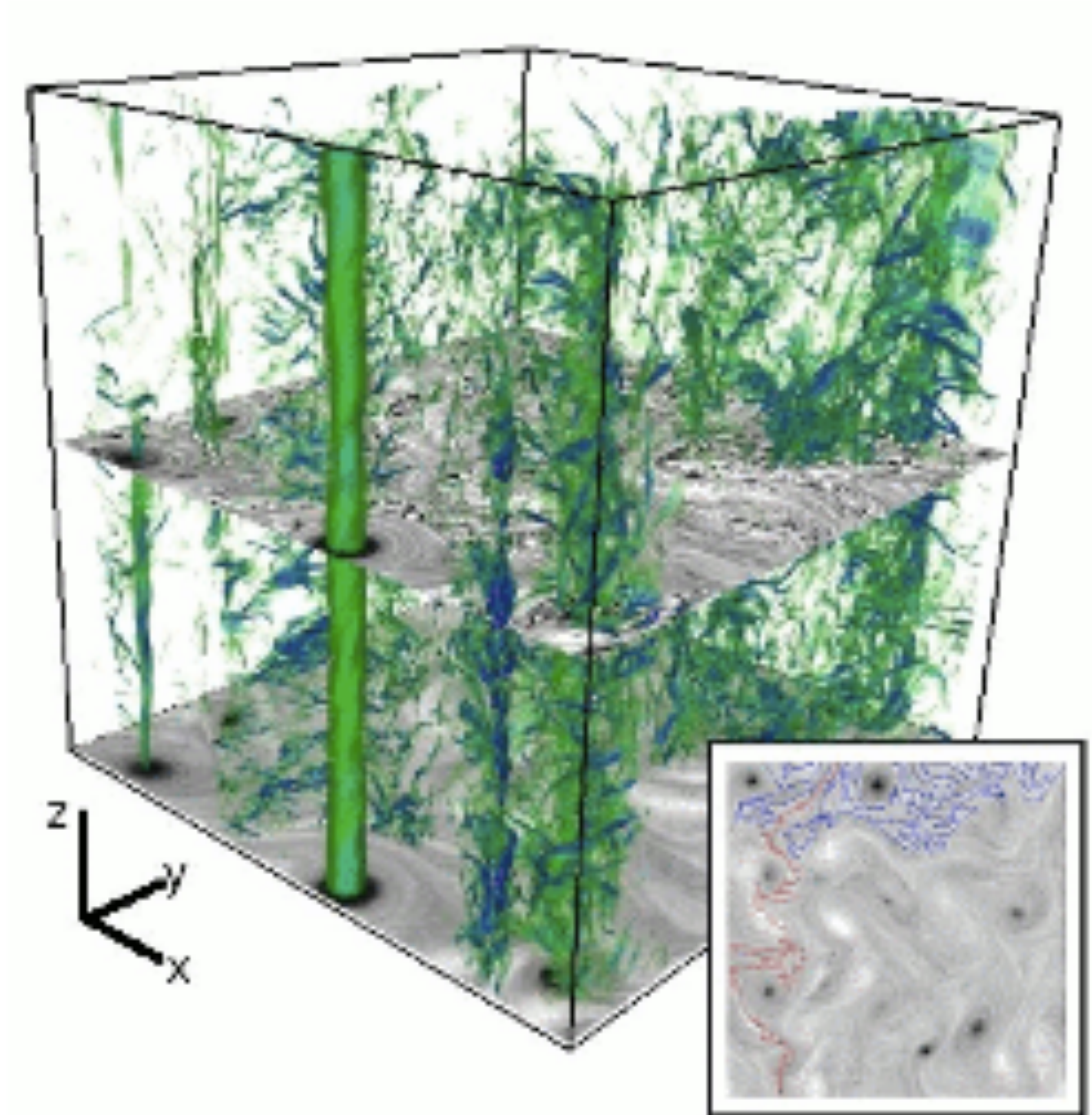


FIG. 9: From top to bottom and from left to right, slices of the energy density, vorticity intensity, z component of the velocity, and helicity density, in run B at $t \approx 30$.

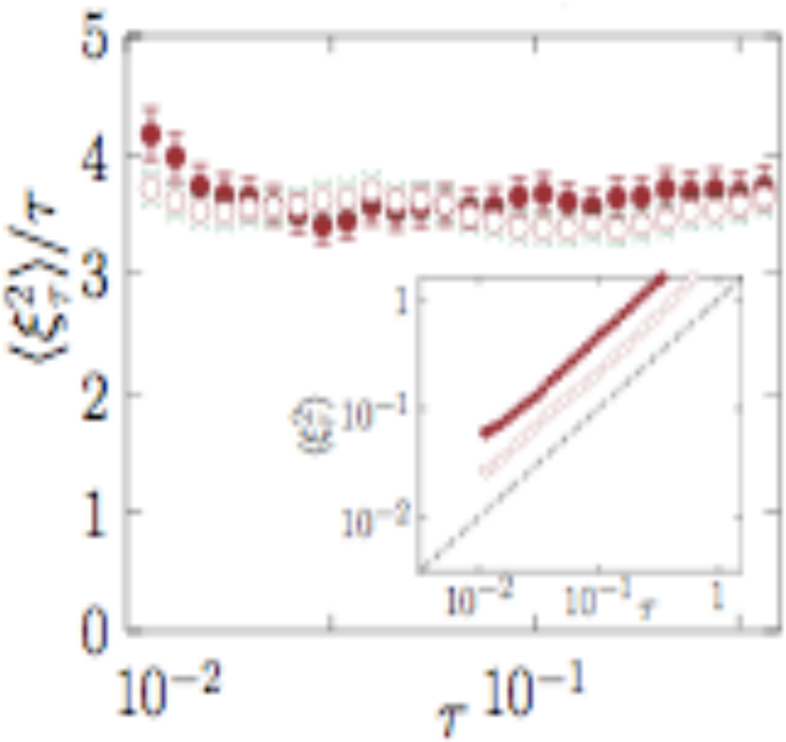
- Does the clear self-similarity of the direct cascade of energy in this quasi-2D flow imply conformal invariance, *à la Bernard et al. (2006)*, which these authors found in 2D NS in the inverse energy cascade?



Thalabard et al., PRL to appear, arXiv:1104.1658

Zero vorticity paths in z-averaged field

- Does the clear self-similarity of the direct cascade of energy in this quasi-2D flow imply conformal invariance, à la Bernard et al. (2006), which these authors found in 2D NS in the inverse energy cascade?



Yes, with κ=3.6 ± 0.4,

κ≠6 (2D NS inverse cascade)

Thalabard et al., PRL to appear, arXiv:1104.1658

Going beyond, at higher resolution

- What about recovery of isotropy at small scale beyond what we call the Zeman scale l_Ω at which $T_W = T_{NL} \rightarrow l_\Omega = [\varepsilon/\Omega^3]^{1/2}$
- Large run to resolve the inverse cascade, the wave-modulated anisotropic inertial range, the isotropic inertial range and the dissipation range
- 3072^3 grid points, Tera-grid allocation of 21 million hours on ~ 30,000 proc (i.e., 700 hours of clock time, or 6 weeks)

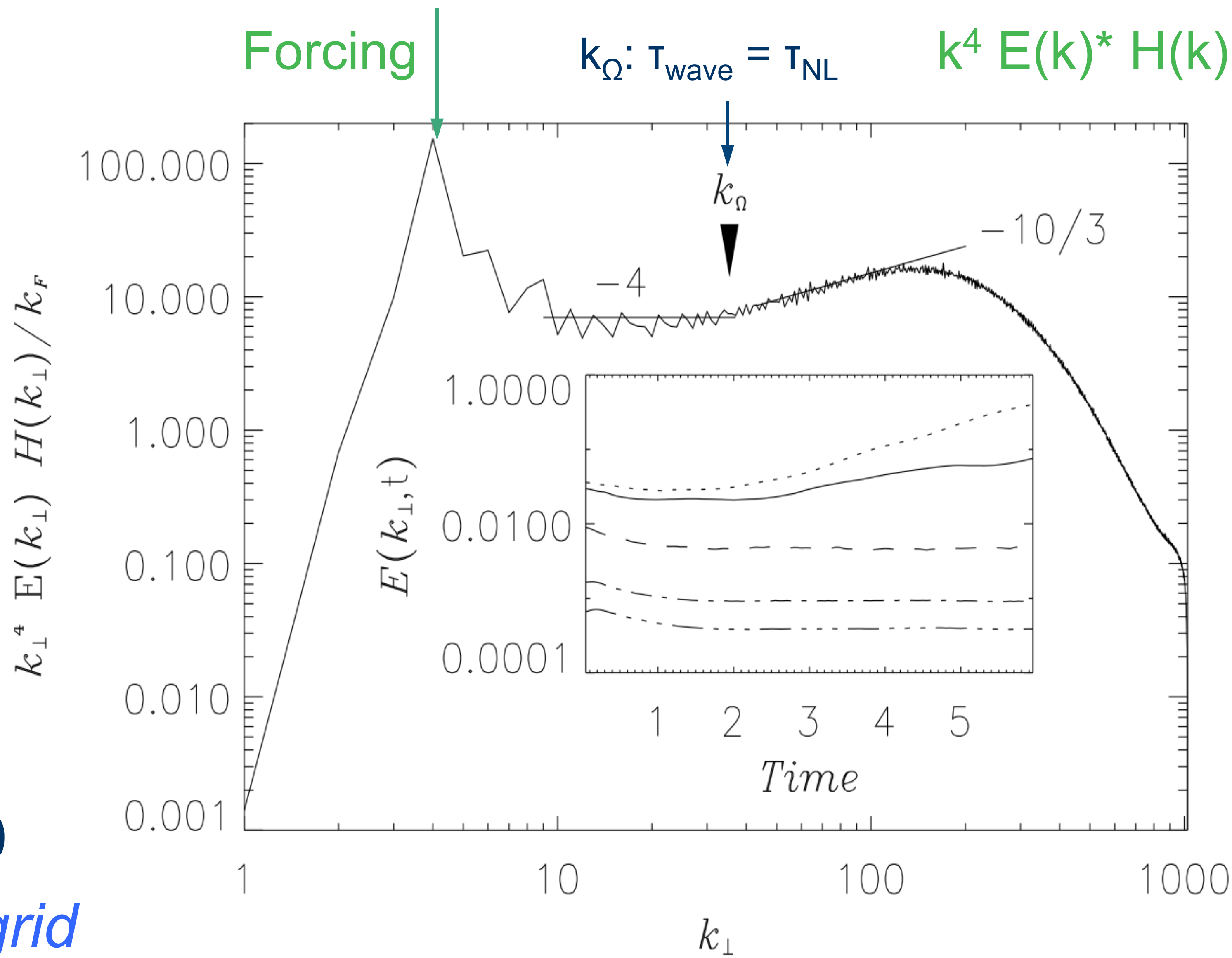
Return to isotropy in the small scales.

3072^3 grid

Ro ~ 0.07

Re ~ 24000

NSF Tera-grid



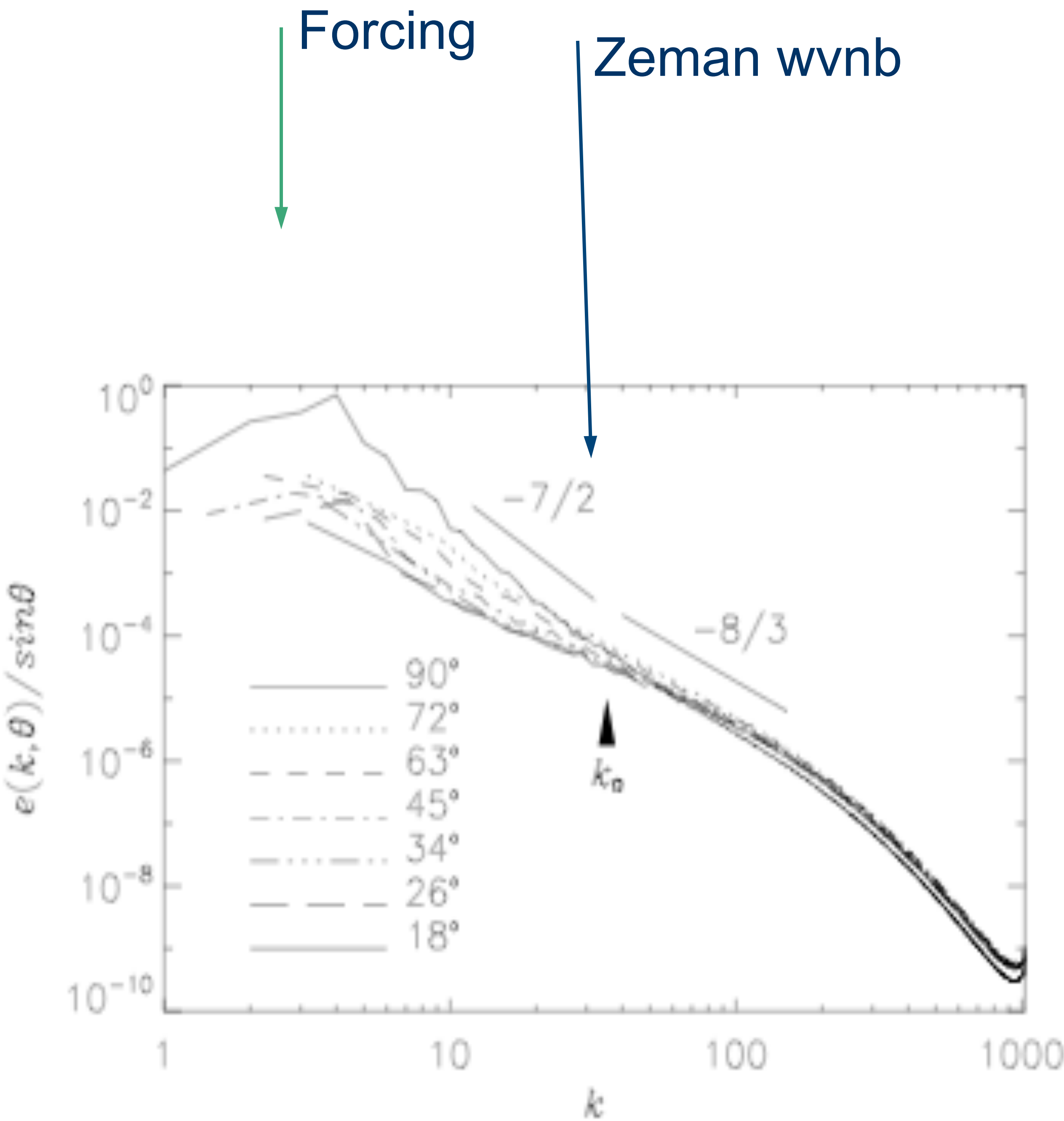
Return to isotropy in the small scales, angular dependence of spectra

3072^3 grid

Ro ~ 0.07

Re ~ 24000

NSF Tera-grid



Summary of results

- In the presence of helicity and rotation, the direct transfer to small scales is dominated by the helicity cascade and the energy cascade to small scales is quenched because of the inverse cascade

- This provides a *``small'' parameter* for the problem (the normalized ratio of energy to helicity fluxes), besides the small Rossby number
- The direct energy cascade is non-intermittent and conformal invariant (when properly averaged in the vertical direction). It is also different from (i) the non-helical case, and (ii) the (presumably) self-similar inverse cascade of energy to large scales.

- There is a change of inertial index in the small scales from a Kolmogorov law to a law steeper than what is predicted by a wave-induced non-helical model, with a possible breaking of universality and with a possible $e \leq 7/3$, $h \geq 5/3$ limit
- Isotropy recovers at small scale provided the Zeman scale is resolved
- The flow produces strong organized long-lived columnar helical structures, Beltrami Core Vortices, at scales slightly smaller than the injection scale, *with also a growth of structures at large scales*

Some questions

- Can helicity help in interpreting laboratory experiments or atmospheric data?
- Is there experimental evidence for this e+h=4 law?
- Is there experimental evidence for Beltrami Core Vortices?
- What about the large Reynolds number limit?
- How does the helicity cascade behave in non-helical rotating flows?

- How does the dynamics change in terms of the relative alignment between the velocity and the vorticity [relative helicity ρ (k)=*H(k)/kE(k)]?*

- Does the nature of the imposed forcing at large scale play a role?

  (helical or not: yes; random vs. deterministic? 2D vs 3D?)

- What happens locally in space? What structures transfer to small *vs.* large scales? What are the Beltrami Core Vortex structures made of? How do they evolve and interact to lead to both a direct and an inverse cascade? *(A. Fournier, local -wavelet- analysis; Amrik Sen, profile analysis; in progress)*

- Universality?
- Modeling: sotropic vs. anisotropic? Need/nature of helical contribution?

* Mininni *et al.,*``Scale interactions and scaling laws in rotating flows at moderate Rossby numbers and large Reynolds numbers," **Phys. Fluids, 21, 015108, 2009.**
• Mininni & Pouquet, ``Helicity cascades in rotating turbulence," **Phys. Rev. E 79, 026304, 2009.**
• Baerenzung *et al.,* ``Spectral Modeling of Rotating Turbulent Flows," **Phys. Fluids 22, 025104, 2010.**

• Mininni & Pouquet, Helical rotating turbulence. Part I. Global evolution and spectral behavior; and Part II. Intermittency, scale invariance and structures," **Phys. Fluids 22, 035105 & 035106, 2010.**
• Pouquet & Mininni, The interplay between helicity and rotation in turbulence: implications for scaling laws and small-scale dynamics. **Phil. Trans. Roy. Soc 368, 1635, 2010.**

• Pouquet *et al.,* ``Modeling of turbulent flows in the presence of magnetic fields or rotation," TI2009 Conference, to appear, Notes on Numerical Fluid Mechanics and Multidisciplinary Design, Springer Verlag, Michel Deville, Jean-Pierre Sagaut and Thien Hiep Eds. (2009). See also arXiv:**0904.4860.**

• Baerenzung *et al.,* ``Where we observe that helical turbulence prevails over inertial waves in forced rotating flows at high Reynolds and low Rossby numbers," *to appear,* ***J. Atmosph. Sci.,*** *s*ee also arXiv: **0912.3414.**

• Thalabard, Rosenberg, Pouquet & Mininni, Conformal invariance in three-dimensional rotating turbulence, to appear, Phys. Rev. Lett. (2011), see also arXiv:**1104.1658**, 2011.

• Thalabard, Auliel, Artana, Mininni & Pouquet, Experimental evidence of conformal invariance in soap film turbulent flows, submitted to Phys. Rev. Lett., see also arXiv:**1012.1725**, 2011.
• Mininni, Rosenberg & Pouquet, Two small-scale inertial ranges in rotating helical turbulence and the return to isotropy, submitted to J. Fluid Mech., see also arXiv:**1104.5519,** 2011.

• Mininni, Rosenberg, Reddy & Pouquet, An hybrid MPI-OpenMP scheme for scalable parallel pseudospectral computations for fluid turbulence, submitted to Parallel Computing, see also arxiv: **1003.4322** (2011).

Thank you for your attention

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Several sets of data & codes available, just contact us :)